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**Exercise 1**

■ Write the relation  $R$  as  $(x,y) \in R$

(a) The relation  $R$  on  $\{1, 2, 3, 4\}$  defined by  $(x,y) \in R$  if  $x^2 \geq y$ .

(b) The relation  $R$  on  $\{1, 2, 3, 4, 5\}$  defined by  $(x,y) \in R$  if 3 divides  $x-y$ .

INSPIRING CREATIVE AND INNOVATIVE MINDS

(a)  $R = \{(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

(b)  $R = \{(1,1), (1,4), (2,2), (2,5), (3,3), (4,1), (4,4), (5,2), (5,5)\}$

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**Exercise 12**

The relation  $R$  on the set  $\{1, 2, 3, 4, 5\}$  defined by the rule  $(x,y) \in R \text{ if } x \leq y$

(i) List the elements of  $R$   
(ii) Find the domain of  $R$   
(iii) Find the range of  $R$   
(iv) Is the relation  $R$  reflexive, symmetric, asymmetric, antisymmetric, transitive, and/or equivalence relation or partial order?

INSPIRING CREATIVE AND INNOVATIVE MINDS

(i)  $R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$

(ii) Domain =  $\{1, 2, 3, 4, 5\}$   
(iii) Range =  $\{1, 2, 3, 4, 5\}$   
(iv) The relation  $R$  is not reflexive because  $(5,5), (4,4) \notin R$

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_R^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R = M_R^T$$

∴ It is symmetric

$$(1,2) \in R, (2,1) \in R$$

∴ It is not antisymmetric

It is not irreflexive and antisymmetric

∴ It is asymmetric

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$(M_{25} = 0) \neq (n_{25} = 1)$$

$$(2,1), (1,5) \in R, \text{ but } (2,5) \notin R$$

∴ It is not transitive

For equivalence relation,

• It is symmetric but it's not transitive and reflexive.

∴ It's not equivalence relation

For partial order,

• It's not antisymmetric, transitive and reflexive.

∴ It's not partial order

The relation  $R$  on the set  $\{1, 2, 3, 4, 5\}$  defined by the rule  $(x, y) \in R$  if 3 divides  $x-y$

- List the elements of  $R$
- Find the domain of  $R$
- Find the range of  $R$
- Is the relation of  $R$  reflexive, symmetric, asymmetric, antisymmetric, transitive, and/or equivalence relation or partial order?

INSPIRING CREATIVE AND INNOVATIVE MINDS

$$(i) R = \{(1, 1), (1, 4), (2, 2), (2, 5), (3, 3), (4, 1), (4, 4), (5, 2), (5, 5)\}$$

$$(ii) \text{Domain} = \{1, 2, 3, 4, 5\}$$

$$(iii) \text{Range} = \{1, 2, 3, 4, 5\}$$

(iv) It is reflexive because  $(x, y) \in R$

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad M_R^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$M_R = M_R^T$$

$\therefore$  It is symmetric

$$(1, 4) \in R, (4, 1) \in R$$

$\therefore$  It is not antisymmetric  
It is not irreflexive and antisymmetric  
 $\therefore$  It is asymmetric

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$(M_{11} = 1), (A_{11} = 1)$$

$\therefore$  It is transitive because  $(1, 4), (4, 1) \in R, (1, 1) \in R$

For equivalence relation,

It is reflexive, symmetry and transitive.

$\therefore$  It is equivalence relation

For partial order,

It is reflexive, transitive but not antisymmetric.

$\therefore$  It is not partial order