

Maschinelles Lernen 1 - Assignment 2

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1 Exercise

f = a house will be flooded	$\neg f$ = a house won't be flooded
x = house stands in a high risk area	$\neg x$ = house stands in a low risk area
$P(f) = 0.0005$	$P(\neg f) = 0.9995$
$P(x) = 0.04$	$P(\neg x) = 0.96$
$P(x f) = 0.8$	$P(\neg x f) = 0.2$
α_1 = buying insurance	α_2 = not buying insurance

We estimate the houses value is 210,000€, since the text states that 100,000€ is less than half the value.

$\lambda(\alpha_1 f) = 1100\text{€}$	$\lambda(\alpha_1 \neg f) = 1100\text{€}$
$\lambda(\alpha_2 f) = 110000\text{€}$	$\lambda(\alpha_2 \neg f) = 0\text{€}$

(a) Bayes Rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Compute $P(f|x)$:

$$P(f|x) = \frac{P(x|f)P(f)}{P(x)} = \frac{0.8 \cdot 0.0005}{0.04} = 0.01$$

(b) Compute $R(\alpha_1|x)$ and $R(\alpha_2|x)$:

$$P(\neg f|x) = 1 - P(f|x) = 0.99$$

$$R(\alpha_1|x) = \lambda(\alpha_1|f)P(f|x) + \lambda(\alpha_1|\neg f)P(\neg f|x) = 1100\text{€} \cdot 0.01 + 1100\text{€} \cdot 0.99 = 1100\text{€}$$

$$R(\alpha_2|x) = \lambda(\alpha_2|f)P(f|x) + \lambda(\alpha_2|\neg f)P(\neg f|x) = 110000\text{€} \cdot 0.01 + 0\text{€} \cdot 0.99 = 1100\text{€}$$

Since $R(\alpha_2|x) = R(\alpha_1|x)$ no option would be more viable than the other.

(c) insert text

2 Exercise

- (a) insert text
- (b) insert text

3 Exercise

- (a)

$$\begin{aligned}
& P(\omega_2|x) \leq P(\omega_1|x) \\
\Leftrightarrow & \frac{P(x|\omega_2)P(\omega_2)}{P(x)} \leq \frac{P(x|\omega_1)P(\omega_1)}{P(x)} \\
\Leftrightarrow & P(x|\omega_2)P(\omega_2) \leq P(x|\omega_1)P(\omega_1) \\
\Leftrightarrow & \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x+\mu)^2}{2\sigma^2}} P(\omega_2) \leq \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} P(\omega_1) \\
\Leftrightarrow & e^{-\frac{(x+\mu)^2}{2\sigma^2}} P(\omega_2) \leq e^{-\frac{(x-\mu)^2}{2\sigma^2}} P(\omega_1) \\
\Leftrightarrow & \frac{(x+\mu)^2}{2\sigma^2} + \ln(P(\omega_2)) \leq \frac{(x-\mu)^2}{2\sigma^2} + \ln(P(\omega_1)) \\
\Leftrightarrow & -(x+\mu)^2 + 2\sigma^2 \ln(P(\omega_2)) \leq -(x-\mu)^2 + 2\sigma^2 \ln(P(\omega_1)) \\
\Leftrightarrow & -x^2 - x\mu - \mu^2 + 2\sigma^2 \ln(P(\omega_2)) \leq -x^2 + x\mu - \mu^2 + 2\sigma^2 \ln(P(\omega_1)) \\
\Leftrightarrow & -x^2 - x\mu - \mu^2 + 2\sigma^2 \ln(P(\omega_2)) \leq -x^2 + x\mu - \mu^2 + 2\sigma^2 \ln(P(\omega_1)) \\
\Leftrightarrow & \ln(P(\omega_2)) \leq \frac{2x\mu}{2\sigma^2} + \ln(P(\omega_1)) \\
\Leftrightarrow & P(\omega_2) \leq e^{\frac{x\mu}{\sigma^2}} P(\omega_1)
\end{aligned}$$

Therefore:

$$\int_{x=-\infty}^{\infty} P(\text{error}|x)p(x)dx = \int_{x=-\infty}^{\infty} \min[P(\omega_1|x); P(\omega_2|x)] = \int_{x=-\infty}^{\infty} P(\omega_2|x)p(x)dx$$

if $P(\omega_2) \leq e^{\frac{x\mu}{\sigma^2}} P(\omega_1)$.

- (b) If we furthermore also assume $\mu_2 = -\mu_1$ for a Laplacian distribution, we need to make a differentiation.
 - for $x \geq \mu$, $P(\omega_2)$ does not depend on x any more.
 - for $x < \mu$, $P(\omega_2)$ does not depend on μ any more.

4 Exercise