Maschinelles Lernen 1 - Assignment 3

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1 Flipping the Coins

a)

$$Pr(D|p) = \prod_{i=1}^{7} Pr(x_i|p) = p^{\#head} * (1-p)^{\#tail} = p^5 * (1-p)^2 = p^5 - 2p^6 + p^7$$

b)

The maximum likelihood yields:

$$\nabla Pr(D|p) \overset{!}{=} 0 \Leftrightarrow 5p^4 - 12p^5 + 7p^6 = 0 \Leftrightarrow p^4(5 - 12p + 7p^2) = 0 \Leftrightarrow p^4 = 0 \vee 5 - 12p + 7p^2 = 0$$

From our given sequence of events we know that $p \in]0, 1[$

$$\Rightarrow 5 - 12p + 7p^2 = 0 \Leftrightarrow p^2 - \frac{12}{7}p + \frac{5}{7} = 0 \Leftrightarrow p_{1,2} = \frac{6}{7} \pm \sqrt{\frac{36}{49} - \frac{35}{49}} \Leftrightarrow p_{1,2} = \frac{6}{7} \pm \sqrt{\frac{1}{49}} \Leftrightarrow$$

$$p_{1,2} = \frac{6}{7} \pm \frac{1}{7} \Leftrightarrow p_1 = \frac{5}{7} \wedge p_2 = \frac{7}{7} = 1$$

Again we know that $p < 1$

$$\Rightarrow p = \frac{5}{7}$$

We are now looking for the probability of the given sequence $D_1 = (x_8, x_9) = (head, head)$:

$$Pr(D_1|\frac{5}{7}) = \prod_{i=8}^{9} Pr(x_i|\frac{5}{7}) = Pr(\{x_8 = head\}) * Pr(\{x_9 = head\}) = \frac{5}{7} * \frac{5}{7} = \frac{25}{49}$$

The probability that the next two tosses are "head" with the given unfair coin is $p = \frac{25}{49}$.

2 Biased Boundaries

b)

Our Maximum Likelihood function for $P(D_1|\mu_1)$:

$$l(\mu_1) = \ln P(D_1|\mu_1) = \sum_{i=1}^{n} \ln P(x_i|\mu_1)$$

Under the Gaussian generative assumtpion we get:

$$P(x_i|\mu_1) = \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{1}{2\sigma^2}(x_i-\mu_1)^2}$$

Applying the logarhitmus for convenience:

$$\ln P(x_i|\mu_1) = -\frac{1}{2}\ln 2\pi\sigma - \frac{1}{2\sigma^2}(x_i - \mu_1)^2$$

Computing the derivate:

$$\frac{d \ln P(x_i|\mu_1)}{d\mu_1} = \frac{1}{\sigma}(x_i - \mu_1)$$

For the dataset D we get:

$$\sum_{i=1}^{n} \frac{1}{\sigma}(x_i - \hat{\mu}_1) \overset{!}{=} 0$$

$$\rightarrow \hat{\mu}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \square$$