# Maschinelles Lernen 1 - Assignment 2

Technische Universität Berlin WS 2013/2014

Christoph Conrads (315565) Antje Relitz (327289) Benjamin Pietrowicz (332542) Mitja Richter (324680)

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### 1 Exercise

f= a house will be flooded  $\neg f=$  a house won't be flooded x= house stands in a high risk area P(f)=0.0005  $P(\neg f)=0.9995$  P(x)=0.04  $P(\neg x)=0.96$  P(x|f)=0.8  $P(\neg x|f)=0.2$   $\alpha_1=$  buying insurance  $\alpha_2=$  not buying insurance

We estimate the houses value is  $210,000 \in$ , since the text states that  $100,000 \in$  is less than half the value.

$$\lambda(\alpha_1|f) = 1100 \in \lambda(\alpha_1|\neg f) = 1100 \in \lambda(\alpha_1|\neg f) = 1100 \in \lambda(\alpha_1|\neg f) = 0 \in \lambda(\alpha_1|\neg f) = 0$$

(a) Bayes Rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Compute P(f|x):

$$P(f|x) = \frac{P(x|f)P(f)}{P(x)} = \frac{0.8 \cdot 0.0005}{0.04} = 0.01$$

(b) Compute  $R(\alpha_1|x)$  and  $R(\alpha_2|x)$ :

$$P(\neg f|x) = 1 - P(f|x) = 0.99$$

$$R(\alpha_1|x) = \lambda(\alpha_1|f)P(f|x) + \lambda(\alpha_1|\neg f)P(\neg f|x) = 1100 \in *0.01 + 1100 \in *0.99 = 1100 \in$$

$$R(\alpha_2|x) = \lambda(\alpha_2|f)P(f|x) + \lambda(\alpha_2|\neg f)P(\neg f|x) = 110000 \in *0.01 + 0 \in *0.99 = 1100 \in$$

Since  $R(\alpha_2|x) = R(\alpha_1|x)$  no option would be more viable than the other.

(c) insert text

## 2 Exercise

- (a) insert text
- (b) insert text

## 3 Exercise

(a)

$$P(\omega_{2}|x) \leq P(\omega_{1}|x)$$

$$\Leftrightarrow \frac{P(x|\omega_{2})P(\omega_{2})}{P(x)} \leq \frac{P(x|\omega_{2})P(\omega_{2})}{P(x)}$$

$$\Leftrightarrow P(x|\omega_{2})P(\omega_{2}) \leq P(x|\omega_{2})P(\omega_{2})$$

$$\Leftrightarrow \frac{1}{\sqrt{2\pi}\sigma}e^{\frac{(x+\mu)^{2}}{2\sigma^{2}}}P(\omega_{2}) \leq \frac{1}{\sqrt{2\pi}\sigma}e^{\frac{(x-\mu)^{2}}{2\sigma^{2}}}P(\omega_{1})$$

$$\Leftrightarrow \frac{(x+\mu)^{2}}{2\sigma^{2}}P(\omega_{2}) \leq e^{\frac{(x-\mu)^{2}}{2\sigma^{2}}}P(\omega_{1})$$

$$\Leftrightarrow \frac{(x+\mu)^{2}}{2\sigma^{2}} + \ln(P(\omega_{2})) \leq \frac{(x-\mu)^{2}}{2\sigma^{2}} + \ln(P(\omega_{1}))$$

$$\Leftrightarrow -(x+\mu)^{2} + 2\sigma^{2}\ln(P(\omega_{2})) \leq -(x-\mu)^{2} + 2\sigma^{2}\ln(P(\omega_{1}))$$

$$\Leftrightarrow -x^{2} - x\mu - \mu^{2} + 2\sigma^{2}\ln(P(\omega_{2})) \leq -x^{2} + x\mu - \mu^{2} + 2\sigma^{2}\ln(P(\omega_{1}))$$

$$\Leftrightarrow -x^{2} - x\mu - \mu^{2} + 2\sigma^{2}\ln(P(\omega_{2})) \leq -x^{2} + x\mu - \mu^{2} + 2\sigma^{2}\ln(P(\omega_{1}))$$

$$\Leftrightarrow \ln(P(\omega_{2})) \leq \frac{2x\mu}{2\sigma^{2}} + \ln(P(\omega_{1}))$$

$$\Leftrightarrow P(\omega_{2}) \leq e^{\frac{x\mu}{\sigma^{2}}}P(\omega_{1})$$

Therefore:

$$\int_{x=-\infty}^{\infty} P(error|x)p(x)dx = \int_{x=-\infty}^{\infty} min[P(\omega_1|x); P(\omega_2|x)] = \int_{x=-\infty}^{\infty} P(\omega_2|x)p(x)dx$$
if  $P(\omega_2) \le e^{\frac{x\mu}{\sigma^2}}P(\omega_1)$ .

- (b) If we furthermore also assume  $\mu_2 = -\mu_1$  for a Laplacian distribution, we need to make a differentiation.
  - for  $x \ge \mu$ , condition for  $P(\omega_2)$  does not depend on x any more.
  - for  $x < \mu$ , condition for  $P(\omega_2)$  does not depend on  $\mu$  any more.

### 4 Exercise