

## Exercise Sheet 6

1.a)

$$\begin{aligned}
 f(\lambda, k) &:= -\ln \left( \prod_{i=1}^N p(v_i | \lambda, k) \right) \\
 &= -\sum_{i=1}^N \ln(p(v_i | \lambda, k)) \\
 &= -\sum_{i=1}^N \ln \left( \frac{k}{\lambda} \left( \frac{v_i}{\lambda} \right)^{k-1} \cdot \exp \left( -\left( \frac{v_i}{\lambda} \right)^k \right) \right) \\
 &= -\sum_{i=1}^N \ln \left( \frac{k}{\lambda} \right) + (k-1) \ln \left( \frac{v_i}{\lambda} \right) + \ln \left( \exp \left( -\frac{v_i}{\lambda} \right) \right) \\
 &= -N \cdot \ln \left( \frac{\lambda}{k} \right) - \sum_{i=1}^N (k-1) \ln \left( \frac{v_i}{\lambda} \right) - \left( \frac{v_i}{\lambda} \right)^k
 \end{aligned}$$

$$\Rightarrow f(\lambda, k) = N \cdot \ln \left( \frac{k}{\lambda} \right) + \sum_{i=1}^N (1-k) \ln \left( \frac{v_i}{\lambda} \right) + \left( \frac{v_i}{\lambda} \right)^k$$

1.b)

$$[\hat{\lambda}, \hat{k}] = \operatorname{argmin}_{\lambda, k} \sum_{i=1}^N (1-k) \ln \left( \frac{v_i}{\lambda} \right) + \left( \frac{v_i}{\lambda} \right)^k$$

1.e)

$$\mathbb{E}\{P\} = \int_0^{P(v_{\max})} P(v) \cdot p(P = P(v)) dv = \int_0^{v_{\max}} P(v) \cdot p(v) dv$$

$$\approx \sum_{i=1}^{N-1} \frac{P_{i+1} - P_i}{2} \cdot (p(v_{i+1}) - p(v_i))$$