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# Exercise Sheet 6

## 1.a)

$$
\begin{aligned}
f(\lambda, k) :&= -\ln\left(\prod_{i=1}^{N} p(v_i|\lambda, k)\right) \\
&= -\sum_{i=i}^{N} \ln(p(v_i|\lambda, k)) \\
&= -\sum_{i=1}^{N} \ln\left(\frac{k}{\lambda}\left(\frac{v}{\lambda}\right)^{k-1} \cdot \exp\left(-\left(\frac{v_i}{\lambda}\right)^{k}\right)\right) \\
&= -\sum_{i=1}^{N} \ln\left(\frac{k}{\lambda}\right) + (k-1)\ln\left(\frac{v_i}{\lambda}\right) + \ln\left(\exp\left(-\frac{v_i}{\lambda}\right)^{k}\right) \\
&= -N \cdot \ln\left(\frac{\lambda}{k}\right) - \sum_{i=1}^{N}(k-1)\ln\left(\frac{v_i}{\lambda}\right) - \left(\frac{v_i}{\lambda}\right)^{k}
\end{aligned}
$$

$$
\boxed{\Rightarrow f(\lambda, k) = N \cdot \ln\left(\frac{k}{\lambda}\right) + \sum_{i=1}^{N}(1-k)\ln\left(\frac{v_i}{\lambda}\right) + \left(\frac{v_i}{\lambda}\right)^{k}}
$$

## 1.b)

$$
\boxed{[\hat{\lambda}, \hat{k}] = \underset{\lambda, k}{\operatorname{argmin}}\ \ln\left(\frac{k}{\lambda}\right) + \sum_{i=1}^{N}(1-k)\ln\left(\frac{v_i}{\lambda}\right) + \left(n\frac{v_i}{\lambda}\right)^{k}}
$$

with $\lambda > 0$, $v > 0$ and $k > 0$

## 1.e)

$$
\mathbb{E}\{P\} = \int_0^{P(v_{\max})} P(v) \cdot p(P = P(v))\mathrm{d}v = \int_0^{v_{\max}} P(v) \cdot p(v)\mathrm{d}v
$$

$$
\approx \sum_{i=1}^{N-1} \frac{P_{i+1} \cdot p(v_{i+1}) - P_i \cdot p(v_i)}{2} \cdot (v_{i+1} - v_i) \quad \text{(Trapezoidal rule)}
$$

with $v_N = v_{\max}$

Usually, the integral should go from $-\infty$ to $+\infty$, but since $P(v) = 0$ for $v \geq v_{\max}$ and $v > 0$, integrating from 0 to $v_{\max}$ is sufficient.