Disclaimer

This summary is part of the lecture “ETH Semiconductor Devices” by Prof. Dr. Colombo Bolognesi (FS19). It is based on the lecture.

Please report errors to huettern@student.ethz.ch such that others can benefit as well.

The upstream repository can be found at https://github.com/noah95/formulasheets

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ETH Semiconductor Devices 2019

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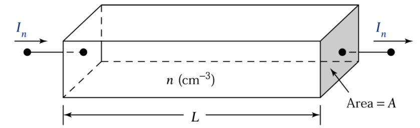
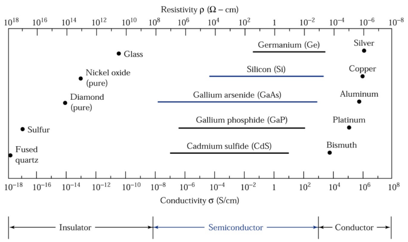
1 Introduction

1.1 Electric resistivity/conductivity

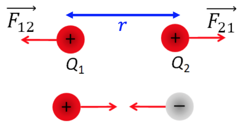


Conductivity σ is a material property describing how easily certain material can conduct electrical current. Resistivity $\rho = 1/\sigma$ describes how much a material opposes the current flow. The resistance of a square/round piece of metal is:



$$R = \rho \frac{l}{A} = \rho \frac{l}{r^2 \pi}$$

1.2 Electron motion



- Electric Force Definition

$$\vec{F}_e = Q\vec{E} \quad \vec{E} : \frac{\text{Newtons}}{C} = \frac{V}{m}$$

- Magneti Force Definition:

$$\vec{F}_m = Q\vec{v} \times \vec{B} \quad \text{not used in course}$$

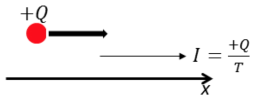
- Definition of electric field:

$$F_{12} = F_{21} = k\frac{Q_1 Q_2}{r^2} \quad E_{21} = \frac{F21}{Q_2} = k\frac{Q_1}{r^2}$$

- Definition: Current

$$I = \frac{\mathrm{d}}{\mathrm{d}t} Q$$

1.3 Current flow



1.4 Moore's Law

Gordon Moore predicted that the number of transistors on an integrated circuit doubles about every two years. This is described using exponential growth:

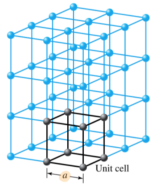
$$p(t) = p_0 \cdot b^{t/\tau}$$

Where

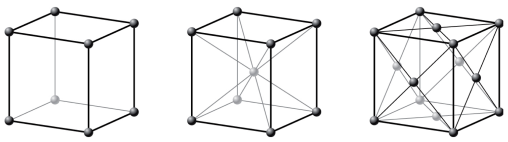
$$p(t) = \text{population at given time}$$
$$p_0 = \text{initial population}$$
$$b = \text{growth rate per time constant}$$
$$\tau = \text{time constant}$$

2 Solid state physics

2.1 Crystal structures



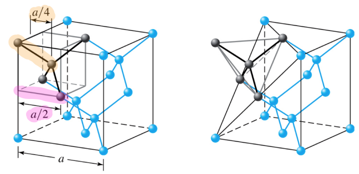
Coordination Number: Is the number of nearest neighbours any atom has in a given crystal lattice. By definition, a crystal lattice is periodic in 3D. a is the lattice constant.



(a) (b) (c)

(a)	SC: Simple cubic	Po
(b)	BCC: Body-centered cubic	Li, Na, K, Cr, Fe, NB
(c)	FCC: Facae-centered cubic	Al, Ar, Ni, Cu, Kr, Pd

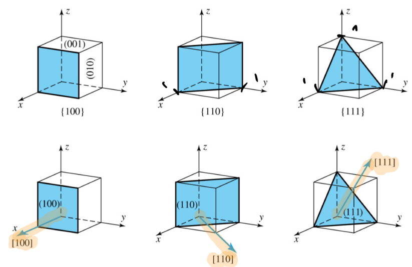
2.2 Silicon



Diamond unit cell: (a) the cubic unit cell, and (b) the inherent tetrahedral structure. The diamon crystal structure is epecially important in semiconductors. For Silicon, the nuclear diameter is 7.2fm $= 2.7 \cdot 10^{-6}$nm: Matter is impressively "empty".

$$a = \frac{4}{\sqrt{3}} \cdot \text{nearest neighbour distance}$$

2.3 Crystal Planes and Directions



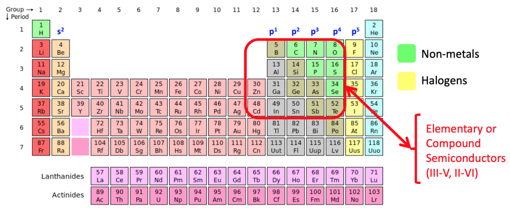
Things don't look the same in all directions. The crystal has different periodicities in different directions, *i.e.* **it does not look the same in all directions**. Different crystal planes and directions generally have different properties.

Miller indices

$$[abc] = \left[\frac{1}{p}, \frac{1}{q}, \frac{1}{r}\right]$$

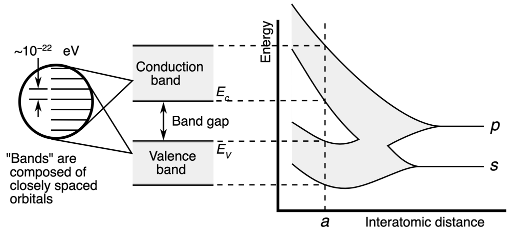
Where p, q, r are the intersections with the x, y, z axis. Miller indices describe the crystal plane.

2.4 Elements



Electronegativity: Tendency to attract electrons. Increases from Bottom to Top / Left to Right.

2.5 Band Gap



In solids, we are concerned about the atoms' outermost (*i.e.* valence) electrons because they determine the bonding and the electronic properties. On the far right in the figure, atoms are well seperated and non-interacting. Their energy states are sharp (atomic-like). As they get closer, the outermost (valence) electrons begin to interact and their energy levels start to shift with respect to the isolated value. **The number of states is conserved**. This is a consequence of the **Pauli Exclusion Principle**. Bands can be seperated by **energy gaps** where no electron is permitted to exist.

Partially Filled / Empty Band

Are associated with electrical **conduction**. As atoms approach each other to form solid, valence electron distributions overlap. Equilibrium distance maximum density of electrons for isolated atoms. Lowering of potential barriers between atoms allows electrons to move freely.

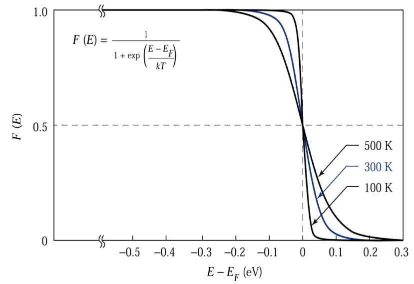
Full Band

Isolation, electrons are there but no net current. At equilibrium atomic separation, bands are seperated by a forbidden energy gap. At low T, valence band full, conduction band empty: *i.e.* no current.

2.6 Population of Electron States

Need to know the probability of finding electrons at a given energy to understand how they are distributed aming the various states (*i.e.* conduction and valence band). This is done by the **Fermi Dirac Statistics (FD)**. FD statistics enforce the Pauli Exclusion Principle and the monimaztion of energy.

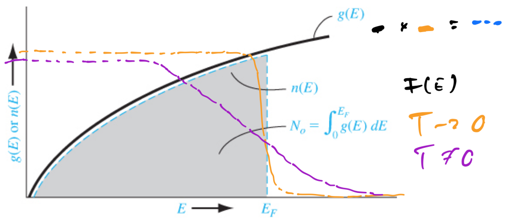
Probability of finding e^- at energy E is $F(E)$ Probability of finding a hole at energy E is $1 - F(E)$



$$F(E) = \frac{1}{1 + \exp\left(\frac{E-E_F}{kT}\right)}$$

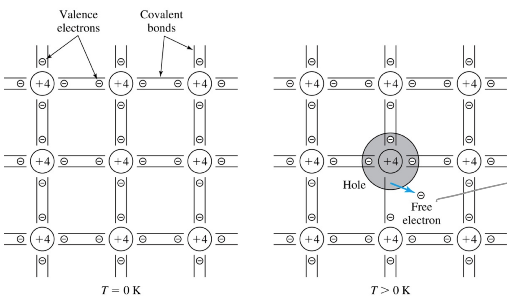
Fermi Level E_F: Energy at which the prob. of finding an e^-/hole is 50%

Density of states



Density of States $g(E)$: How many available states per volume at energy E.

2.7 Intrinsic carriers



Two types of carriers: electrons and holes.

Each Si atom is surrounded by 8 e^- at 0K, 4 come from the atom itself, 1 from each of its 4 nearest neighbours. As T increases, thermal energy eventually excites some e^- out of their bond. Equivalently, the FD distribution broadens around E_F, increasing the prob. that conduction band states become occupied. A empty state (a "hole") is left in the bonding (valence band) state. The valence band is then not full anymore, and it therefore becomes conductive too. **Consequence**: We now have two partially filled bands. Both the conduction and valence bands can now carry current because neither is completely full, not empty.

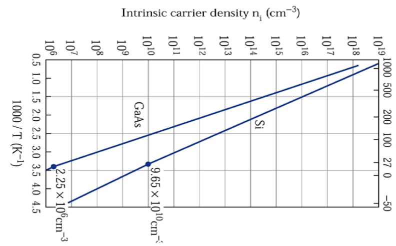
Detailed balance

In equilibrium, n_0 and p_0 are constant but the reaction still operates in both directions. This is called "detailed balance".

$$\boxed{n_0 \cdot p_0 = K = K_0 \exp\left(\frac{-E_a}{kT}\right) = n_i^2}$$

Free electron n_0 and hole p_0 density and activation energy E_a for bond breaking.

Intrinsic carrier density



The intrinsic carrier concentration depends exponentially on temperature T And on the energy gap of the material (Si: 1.12 eV, GaAs: 1.42 eV, @ 300K) Different energy gaps thus have a huge impact on n_i n_i becomes large at high T (limits high-T operation of devices)

Electric field

Electrons move opposite to electric field, holes move parallel to E-field.

3 Doping

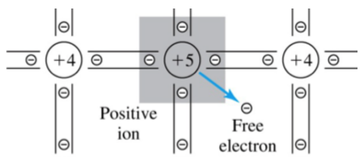
Doping is the introduction of impurities with different atomic valence in the pure crystal. These impurities are called *Extrinsic Carriers.*

E_D	Donor energy level/state
E_c	Conduction band edge
E_v	Valence band edge
E_F	Fermi energy
E_{Fi}	Intrinsic Fermi energy
E_g	Band gap width/energy
m^*	Effective mass of electron(n)/hole(p)
ϵ_0	Vacuum permittivity $8.854 \cdot 10^{-12}$F/m
n_i	Intrinsic electron concentration
p_i	Intrinsic hole concentration
	$n_i = p_i$
n_0	Thermal-equilibrium e^- concentration
p_0	Thermal-equilibrium hole concentration
n_d	Concenctraiton of e^- in donor state
p_a	Concenctraiton of holes in acceptor state
N_d	Concentration of donor atoms
N_a	Concentration of acceptor atoms
N_c	Effective density of states
N_v	Effective density of states
N_d^+	Conc. of pos. charged (ionized) donors
N_a^-	Conc. of neg. charged (ionized) acceptors

3.1 Impurity doping

It is distinguished between negative and positive doping.

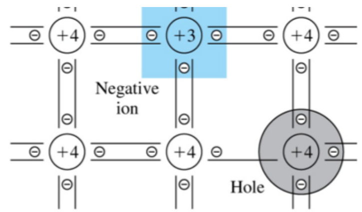
N-Doping



Taking donors from PT col V (P, As, Sb) that have 5 valence electrons. 4 make covalent bond, 5$^{\text{th}}$ is "not needed" and looks like a hydrogen atom. It is easily inozed/excited to conduction band. Its energy state is close to CB (E_c). ⚠ Overall, the solid is still charge neutral but with impurities ⚠

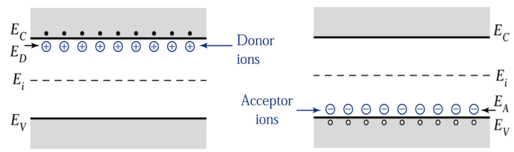
Donors introduce an energy state E_D near the conduction band edge E_c. e_- easily promoted to conduction band because E_c, E_D are close compared to E_c, E_v (E_g). Only $E_c - E_D$ is needed to free the electron. **Extra electrons are added without adding holes.**

P-Doping



Same as n-doping but by adding extra holes. Acceptors from PT col III (B, Al, Ga, In)

Energy Band



n-type doping | **p-type doping**

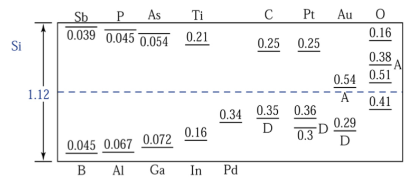


Image shows impurity levels in Silicon in eV. This amount of energy is required to move impurity to conductance/valence band.

Donors: treat as "Hydrogen-Like" atom inside solid. Its ionization energy E_D is modified by the dielectric constant and effective mass m^*.

Bohr radius

The most probable distance between nucleus and electron in a Hydrogen atom in ground state. For semiconductor we need to modify this value for ϵ_0 and m^*. This sphere corresponds to about 6735 Si atoms or 841 unit cells. Is thus very loosely bound to its "parent" impurity atom.

$$E_d \approx \frac{1}{\epsilon_r^2} \frac{m^*}{m_0} E_H$$

$E_{D,\text{Bohr}} = 27$meV a very good approximation.

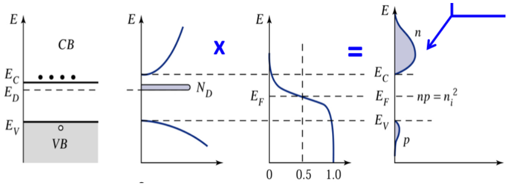
$$r'_{\text{Bohr}} = \frac{4\pi\epsilon_r\epsilon_0\hbar^2}{m^* m_0 q^2} = \frac{\epsilon_r}{m^*} 5.3 \cdot 10^{-9} = 31\text{Å}$$

3.2 Electro neutrality

Sum of atoms making up the semiconductor are elctrically neutral: The semiconductor thus has zero net charge. In general, both donor and acceptor impurities may be present.

$$n_0 + N_a^- = p_0 + N_d^+$$

3.3 Density of States (DOS)



Density of available quantum states times Fermi-Dirac distribution equals the density of electrons. $D(E_{\text{kin}})$ is the number of electronic states at energy E_{kin} in a range δe_{kin} per cm^3.

$$D(E_{\text{kin}}) = \frac{8}{h^3}\sqrt{2}\pi(m^*)^{3/2}(E_{\text{kin}})^{1/2}$$

3.4 Population of Electron States

Concentration of electrons and holes ($g(E)$ int Neamen p. 109) is sum of all density of states times Fermi-Dirac probability:

$$\int_{E=0}^{\infty} f(E)\cdot D(E_{\text{kin}})\,\mathrm{d}E_{\text{kin}}$$

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} \approx e^{\frac{E-E_F}{kT}} \quad \text{if} \quad E \gg E_F$$

$$n_0 = \frac{4\sqrt{2}(\pi m^* kT)^{2/3}}{h^3} e^{-(E_c-E_F)/(kT)}$$
$$= N_c \cdot e^{-(E_c-E_F)/(kT)}$$

For $E \gg E_F$ the Maxwell-Boltzmann approximation holds.

General case

$$n_0 = \int_{E=0}^{\infty} f(E)\times D(E_{kin})dE_{kin}$$

$$\boxed{\begin{aligned} x &= E_{kin}/kT \\ \eta &= (E_F - E_c)/kT \end{aligned}}$$

$$D(E_{kin}) = \frac{8\pi\sqrt{2}}{h^3}(m^*)^{3/2}(E_{kin})^{1/2} = g(E_{kin})$$

$$n_0 = \frac{8\sqrt{2}\pi}{h^3}(m^*)^{3/2}\int_{E_C=0}^{\infty}\frac{(E_{kin})^{1/2}}{1+exp[(E_{kin}-E_F)/kT]}dE_{kin}$$

$$n_0 = \frac{8\sqrt{2}\pi}{h^3}(m^*kT)^{3/2}\int_{x=0}^{\infty}\frac{(x)^{1/2}}{1+exp[x-\eta]}dx = \frac{2N_c\mathcal{F}_{1/2}(\eta)}{\sqrt{\pi}} = N_c\mathcal{F}_{1/2}(\eta)$$

When the Fermi level E_F is more than $3kT$ below E_c (or above E_v), the full Fermi-Dirac (FD) integral $F_{1/2}(\eta)$ is well approximated by the Maxwell-Boltzmann (MB) approximation e^{η}. When E_F is colser or even above E_c (below E_v), the material is said to be "degenerate".

Concentration of electrons and holes

Back to (MB).

$$n_0 = N_c \cdot e^{-\frac{E_c-E_F}{kT}} \qquad N_c = \frac{4\sqrt{2}(\pi m_n^* kT)^{3/2}}{h^3}$$

$$p_0 = N_v \cdot e^{-\frac{E_F-E_v}{kT}} \qquad N_v = \frac{4\sqrt{2}(\pi m_p^* kT)^{3/2}}{h^3}$$

We see that the intrinsic carrier concentration depends on the temperature and the energy gap of the semiconductor:

$$n_0 p_0 = N_v N_c \cdot e^{-\frac{E_c-E_v}{kT}} = n_i^2 = N_v N_c \cdot e^{-\frac{E_g}{kT}}$$

Remember:

$$\boxed{n_0 p_0 = n_i^2}$$

3.5 Book Chapter 4 Summary

Material	Si	Ge	GaAs
$n_i^2 (cm^{-6})$	$9.3\cdot 10^{19}$	$5.76\cdot 10^{26}$	$3.24\cdot 10^{12}$
$N_c (cm^{-3})$	$2.86\cdot 10^{19}$	$1.04\cdot 10^{19}$	$4.7\cdot 10^{17}$
$N_v (cm^{-3})$	$1.04\cdot 10^{19}$	$6.0\cdot 10^{18}$	$7.0\cdot 10^{18}$
$E_g (eV)$	1.12	0.66	1.42
m_n^*/m_0	1.08	0.067	0.55
m_p^*/m_0	0.56	0.48	0.37

Intrinsic Band Gap

$$E_g = -kT\ln\frac{n_i^2}{N_c N_v} \qquad n_i^2 = N_c N_v e^{-\frac{E_g}{kT}}$$

$$n_0 = N_c e^{\frac{-(E_c-E_F)}{kT}} \qquad p_0 = N_v e^{\frac{-(E_F-E_v)}{kT}}$$

Effective Mass

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \qquad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

Intrinsic Carrier Concentration

$$n_i^2 = N_c N_v e^{\frac{-(E_c-E_v)}{kT}} = N_c N_v e^{\frac{-E_g}{kT}}$$

Intrinsic Fermi Level

$$E_{Fi} - E_{\text{midgap}} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$E_{Fi} = \frac{E_v + E_c}{2} + \frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right)$$

Equilibrium distribution of el/holes

$$n_0 = n_i e^{\frac{E_F - E_{Fi}}{kT}} \qquad p_0 = n_i e^{\frac{-(E_F - E_{Fi})}{kT}}$$

Statistics of Donors and Acceptors

$$n_d = \frac{N_d}{1+\frac{1}{2}e^{\frac{E_d-E_F}{kT}}} \qquad p_d = \frac{N_a}{1+\frac{1}{2}e^{\frac{E_F-E_a}{kT}}}$$

Thermal-Equilibrium El. Concentration

n_0 for $N_d > N_a$ (n-type), p_0 for $N_a > N_d$ (p-type).

$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

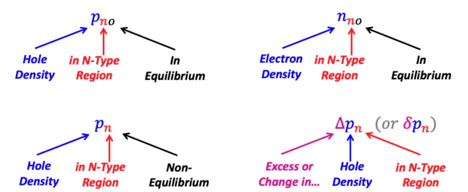
$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Position of Fermi Energy Level

Use the n_0 formula for n-type, the p_0 formula for p-type.

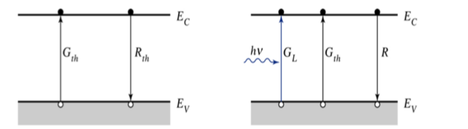
$$E_F - E_{Fi} = kT\ln\left(\frac{n_0}{n_i}\right) \qquad E_{Fi} - E_F = kT\ln\left(\frac{p_0}{n_i}\right)$$

4 Excess Carriers



G_{th}	Thermal generation rate
R_{th}	Thermal recombination rate
G_L	Photonic generation rate
τ_p, τ_n	Minority carrier lifetime
Δ_p, Δ_n	Excess carrier concentration to equi.
N_t	Density of recombination centers
σ	Recomb. center cross section **or** conductivity
v_{th}	Carrier mobility in thermal equilibrium
R_a, R_b	El. indirect capture and emission rate
R_c, R_d	Hole indirect capture and emission rate
e_n, e_p	El./Hole indirect emission probability
U	Net recombination rate
J_{diff}	Diffusion current dens.
D	Diffusion constant
q	electron charge
J_n	Electron diffusion current dens.
$v_{dr,n/p}$	Electron/hole drift velocity
$J_{dr,n/p}$	Electron/hole drift current dens.
$\mu_{n/p}$	Electron/hole mobility
$J_{n/p}$	Electron/hole total current dens.

4.1 Direct Generation/Recombination



Thermal (spontaneous) and external generation (e.g. light) accross the energy gap.

In steady state, at a given T: Electrons are continually generated due to thermal energy. Some electrons recombine with holes, so that on average n_0 and p_0 are constant. Rate is proportional to $n_0 \cdot p_0$.

$$G_{th} = R_{th} = \beta(n_0 \cdot p_0) = \beta n_i^2$$

This is because we always need 1 electron and 1 hole to "meet" for each recombination event.

$$R = \beta np \quad G = G_L + G_{th} \quad \Delta n = \Delta p$$
$$R = \beta \cdot n_n p_n = \beta(n_{n0} + \Delta n)(p_{n0} + \Delta p)$$
$$\frac{\mathrm{d}}{\mathrm{d}t}p_n = G - R$$

Mass action law ($n_0 p_0 = n_i^2$) states for n-type doping that:

$$p_{n0} \ll n_{n0}$$

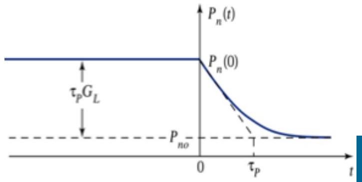
Low level injection is when

$$\Delta p \ll n_{n0}$$

Then the net recombination rate can be written as:

$$U = \beta(n_{n0} + p_{n0} + \Delta p)\Delta p \approx \beta n_{n0}\Delta p = \frac{\Delta p}{\tau_p}$$

The minority carrier lifetime describes how fast the excess carrier concentration decays towards equil. when excitation ends. Determined by majority carrier concentration.



$$\boxed{\tau_p = \frac{1}{\beta n_{n0}}}$$

Light on:

$$G_L = U = \frac{p_n - p_{n0}}{\tau_p} \qquad p_n(t \le 0) = p_{n0} + \tau_p G_L$$

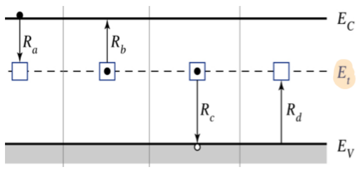
Light off:

$$\frac{\mathrm{d}p_n}{\mathrm{d}t} = G_{th} - R = -U = -\frac{p_n - p_{n0}}{\tau_p}$$

$$p_n(t) = p_{n0} + \tau_p G_L \exp\left(-\frac{t}{\tau_p}\right)$$

Generally: $\boxed{\Delta n = \tau_{n0} R_n}$

4.2 Indirect Generation/Recombination



Energy trap near midgap.

$$U \approx \frac{v_{th}\sigma_0 N_t \cdot \Delta p}{1 + \frac{2n_i}{n_{n0}} \cosh \frac{E_t - E_i}{kT}} \approx \frac{\Delta p}{\tau_p} \approx \frac{p_n - p_{n0}}{\tau_p}$$

Where $N_t v_{th} \sigma$ are the recombination events taking place per unit time, $v_{th}\sigma$ a cylindrical colume in material per unit time.

$$\frac{1}{2} m_n v_{th}^2 = \frac{3}{2} kT \qquad v_{th} \approx 10^7 cm \cdot s^{-1}$$

Electron capture (R_a) and emission (R_b) rate must be equal in therm. equi.

$$R_a = n N_t (1 - f) \cdot v_{th} \sigma_n \qquad R_b = e_n N_t f$$

The emission probability increases exponentially as E_t gets closer to conduction band edge:

$$e_n = \frac{v_{th}\sigma_n n(1-f)}{f} = v_{th}\sigma_n n_i e^{(E_t - E_i)/kT}$$

For holes:

$$R_c = p N_t f \cdot v_{th} \sigma_p \qquad R_d = e_p N_t (1 - f)$$

$$e_p = v_{th}\sigma_p n_i e^{(E_i - E_t)/kT}$$

These lead to the equation for $U = \dots$.

4.3 Diffusion

Result of concentration gradients. Equal probability of moving in any direction. Fick's first law of diffusion:

$$J_{\text{diff}} = -D\Delta N = -D\left(\frac{\partial N}{\partial x}\vec{x_u} + \frac{\partial N}{\partial y}\vec{y_u} \dots\right)$$

In thermal equi. and uniform distribution, free charge carriers are in constant motion. Net current is thus zero. Statistical mechanics show that particles at temp. T have avg. thermal energy of $3kT/2$ For a particle of mass m this corresponds to an avg. thermal velocity

$$\frac{1}{2} m v_{th}^2 = \frac{3}{2} kT \qquad J_{\text{diff},n} = -qF = qD_n \frac{\mathrm{d}n}{\mathrm{d}x}$$

In a crystal we must use the **effective mass** The electron diffusion current:

4.4 Drift

Result of an electric field as driving force. Zero field: Electrons move thermally (randomly) in all directions (no net flow) Non-zero field: There is a net drift of electrons, opposite to the E-field We can then define a drift velocity of electrons $v_{dr,n}$ Electrons do not accelerate indefinitely due to collisions

$$\begin{aligned} J_{dr,n} &= -qnv_{dr,n} & v_{dr,n} &= -\mu_n E \\ J_{dr,p} &= qpv_{dr,h} & v_{dr,h} &= \mu_p E \\ J_{dr,tot} &= \sigma E & \sigma &= q(n\mu_n + p\mu_p) \end{aligned}$$

4.5 Total current

Total current = drift + diffusion current = electron + hole current.

Electrons: $J_n = nq\mu\vec{E} + qD_n \frac{\mathrm{d}n}{\mathrm{d}x}$

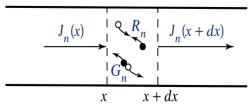
Holes: $J_p = \underbrace{pq\mu\vec{E}}_{\text{drift}} \underbrace{-qD_p \frac{\mathrm{d}p}{\mathrm{d}x}}_{\text{diffusion}}$

5 Excess Carriers

$G_{n/p}$	Generation rate of el/hole
$R_{n/p}$	Recombination rate of el/hole
$J_{n/p}$	Current density of el/hole
$L_{n/p}$	Minority carrier diffusion length
$D_{n/p}$	Diffusion constant
$\mu_{n/p}$	Carrier mobility

5.1 Continuity equation

Objective: Accounting for carrier densities when drift, diffusion and G/R take place.



Change in e $\quad \frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G_m - R_n)$

Change in hole $\quad \frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + (G_p - R_p)$

Written for **electrons** in p-type material

$$\frac{\partial n_p}{\partial t} = n_p \mu_n \frac{\partial \vec{E}}{\partial x} + \mu_n \vec{E} \frac{\partial n_p}{\partial x} + D_n \frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{p0}}{\tau_n}$$

Written for **holes** in n-type material

$$\frac{\partial p_n}{\partial t} = p_n \mu_p \frac{\partial \vec{E}}{\partial x} + \mu_p \vec{E} \frac{\partial p_n}{\partial x} + D_p \frac{\partial^2 p_n}{\partial x^2} + G_p - \frac{p_n - p_{n0}}{\tau_p}$$

Steady state

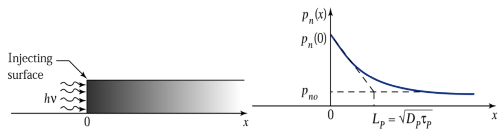
For steady state considerations:

$$\boxed{0 = D_n \frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{p0}}{\tau_n}}$$

Einstein relation

$$\boxed{D_p = \frac{kT}{q}\mu_p} \qquad \boxed{D_n = \frac{kT}{q}\mu_n}$$

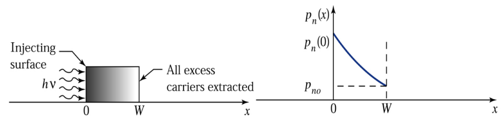
Semi-infinite sample



$$p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \exp\left(-\frac{x}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

Finite sample



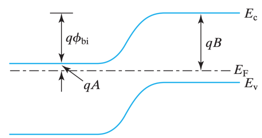
$$p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \frac{\sinh\left(\frac{W-x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}$$

Short diode

If $W \ll L_p$ then recombination is weak, few holes recombine in the time required for them to cross the region W. The excess carrier profile bebomes linear:

$$p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \frac{W - x}{W}$$

5.2 Equilibrium: constant fermi level



In equilibrium, the Fermi level mus be constant to balance transfer rates so that no net current flows.

$$\boxed{E_{F1} = E_{F2}}$$

5.3 Doping: Band bending

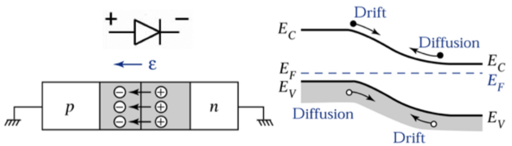
Near the transition region, the distance between the Fermi level and conduction/valence band edge changes. Far away from the junction, the material does not "know" there is a junction.

6 PN Junction

N_a	Acceptor concentration in p-region
N_d	Donor concentration in n-region
$n_{n0} = N_d$	Equi. maj. carrier e in n reg.
$p_{p0} = N_a$	Equi. maj. carrier h in p reg.
$n_{p0} = n_i^2/N_a$	Equi. min. carrier e in p reg.
$p_{n0} = n_i^2/N_d$	Equi. min. carrier h in n reg.
n_p	Total min. carrier e in p reg.
p_n	Total min. carrier h in n reg.
$n_p(-x_p)$	Min. carr. e in p-reg at depl-edge
$p_n(x_n)$	Min. carr. h in n-reg at depl-edge
Δn_p	Excess min. car. e in p-reg
Δn_n	Excess min. car. h in n-reg
V_{bi}	Builtin voltage at equilibrium
x_n	End of depletion region on n-side
x_p	Start of depletion region
W	Depletion region width
V_a	Forward applied voltage
J_s	Reverse saturation cur. dens.

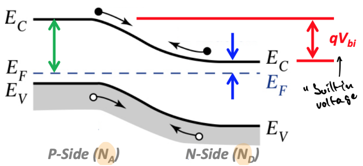
$$\Delta n_p = n_p - n_{p0} \qquad \Delta n_n = p_n - p_{n0}$$

6.1 Diode



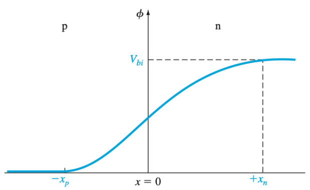
The amount of band bending balances the drift/diffusion currents.

6.2 Equilibrium



$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_a N_d}{n_i^2}\right)$$

6.3 Space Charge Width



$$x_n = \left\{\frac{2\epsilon_s V_{bi}}{q}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a+N_d}\right]\right\}^{1/2}$$

$$x_p = \left\{\frac{2\epsilon_s V_{bi}}{q}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a+N_d}\right]\right\}^{1/2}$$

$$W = \left\{\frac{2\epsilon_s V_{bi}}{q}\left[\frac{N_a+N_d}{N_a N_d}\right]\right\}^{1/2}$$

6.4 Forward Bias

Condition:

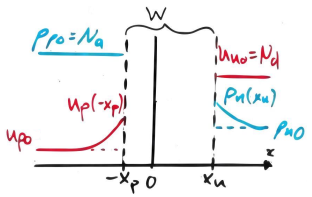
$$\left.\begin{matrix} n_p(-x_p) & > n_{p0} \\ p_n(x_n) & > p_{n0}\end{matrix}\right\}\text{forward biased}$$

Injection strength:

$$\left.\begin{matrix} n_p & < N_a \\ p_n & < N_d\end{matrix}\right\}\text{low level injection}$$

$$\left.\begin{matrix} n_p & > N_a \\ p_n & > N_d\end{matrix}\right\}\text{high level injection}$$

Applying an external forward voltage is called forward bias or minority carrier injection. This voltage subtracts from V_{bi}. Minority carriers at depletion edges increase by factor $\exp(qV_F/kT)$. Known as **Shockley Boundary Conditions**.



$$n_p(-x_p) = n_{p0}e^{\left(\frac{qV_a}{kT}\right)} \qquad p_n(x_n) = p_{n0}e^{\left(\frac{qV_a}{kT}\right)}$$

$$\boxed{n_{p0} = \frac{n_i^2}{N_a}} \qquad \boxed{p_{n0} = \frac{n_i^2}{N_d}}$$

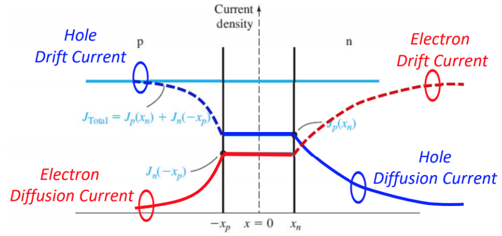
6.5 Ideal Junction Current

The ideal current J is calculated by:

$$J_s = \left[\frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n}\right] \qquad J = J_s\left[e^{\frac{qV_a}{kT}} - 1\right]$$

Recalling

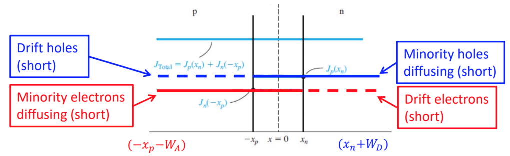
$$D_i = \frac{kT}{q}\mu_i = \frac{L_i^2}{\tau_i} \qquad L_i = \sqrt{D_i \tau_i} \qquad i \in \{n, p\}$$



Short diode

For a short diode L_p, L_n become W_d, W_a respecively if $W_{a,d} \ll L_{n,p}$.

$$J_s = \left[\frac{qD_p p_{n0}}{W_d} + \frac{qD_n n_{p0}}{W_a}\right]$$



6.6 One sided junction

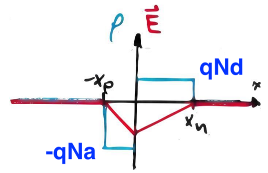
If one side of a junction is much more doped than the other:

$$N_a \gg N_d \Longrightarrow p^+n \qquad N_d \gg N_a \Longrightarrow n^+p$$

6.7 Energy, field, potential, etc.

$$\frac{d\vec{E}}{dx} = \frac{\rho}{\epsilon_s} \qquad \vec{E}(x) = \frac{1}{q}\frac{dE_V}{dx}$$

$$\left|\vec{E}_{max}\right| = \frac{qN_a x_p}{\epsilon_s} = \frac{qN_d x_n}{\epsilon_s} = \frac{2(V_{bi}+V_R)}{W} = \left[\frac{V}{m}\right]$$

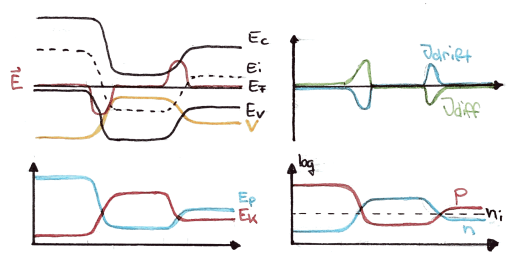


Draw the curves using following qualitative eqn. $E_P = E_C$ for e^- and $E_P = -E_C$ for hole.

$$V \propto E_F - E_C \qquad \vec{E} \propto -\frac{dV}{dx} \qquad E_K + E_P = \text{const.}$$

$$\ln(p) \propto E_i - E_F \qquad \ln(n) \propto E_F - E_i$$

$$J_{\text{drift}} \propto n\vec{E} \qquad J_{\text{diff}} \propto \frac{dn}{dx}$$

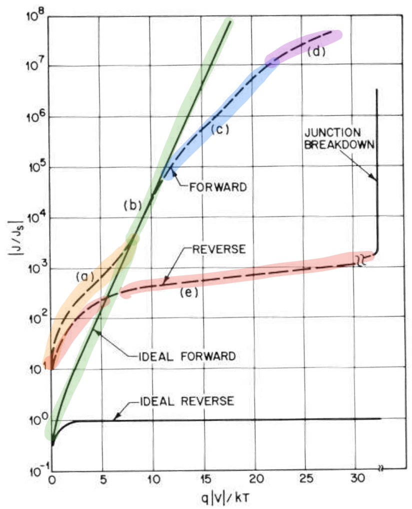


7 Complementary effects

C_j	Depletion capacitance
C_d	Diffusion capacitance
J_g	Current due to generation
J_r	Current due to recombination
$J_{\text{rev/fwd}}$	Non-ideal current densities
η	Ideality factor

7.1 Summary

a) Recombination in depletion region
b) Ideal injection
c) High-level injection
d) Series resistance
e) Generation in depletion region



7.2 Depletion Capacitance

For Silicon $\epsilon_r = 11.9$.

$$C_j = \frac{\epsilon_0 \epsilon_r}{W}$$

$$\frac{1}{C_j^2} = \frac{1}{\epsilon_0\epsilon_r}\frac{2(V_{bi}-V_f)}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)$$

7.3 Diffusion Capacitance

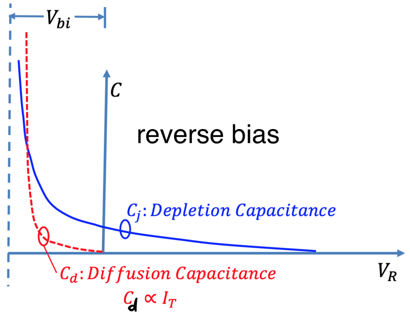
The stored charged Q_p due to the excess hole density contributes a capacitance C_d.

$$J = qD_p \frac{\mathrm{d}p_n}{\mathrm{d}x} = qD_p \frac{p_{n0}\left(e^{qV_f/kT}-1\right)}{W_n}$$

$$Q_p = qW_n \frac{p_{n0}\left(e^{qV_f/kT}-1\right)}{2}$$

$$C_d = \frac{\mathrm{d}Q_p}{\mathrm{d}V_f} = \frac{\tau}{r_d} = \frac{W_n^2}{2D_p}\frac{J}{kT/q}$$

C_j dominates in reverse bias, C_d becomes dominant in forward bias due to minority carrier charge storage.



7.4 Generation Current in reverse bias

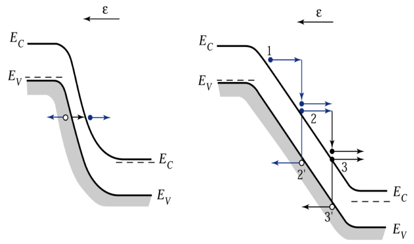
J_g adds to the ideal reverse saturation current density J_s due to electron hole pair generation.

$$J_g = \frac{qWn_i}{\tau_g} \qquad J_{\text{rev}} = J_s + J_g$$

7.5 Reverse Breakdown

Tunneling occurs in thin, highly doped junctions while avalanche multiplication arises in thick, more lightly doped junctions. Tunneling results in a sharp breakdown characteristic (Zener diode) and avalanch mult. in a more soft slope. Higher doping results in lower breakdown voltage and sharper characteristics.

$$N_a \approx N_d \rightarrow \text{Tunneling} \qquad N_a \neq N_d \rightarrow \text{Avalanche}$$



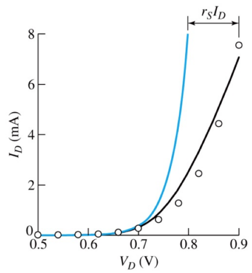
7.6 Recombination in forward bias

In forward bias the pn is greater than n_i^2: recombination takes place.

$$J_r = \frac{qWn_i}{2\tau_r}e^{qV_f/2kT} \qquad J_{\text{fwd}} = J_s + J_r$$

7.7 Series resistance

Ohmic losses occur in the undoped regions at high current which leads to voltage drop.



7.8 Ideality factor

The factor η characterizes the diode forward current ideality and is often called the *ideality factor*. From graph with two points $I_{f1,2}, V_{f1,2}$ η can be calculated.

$$J_{FT} \propto \exp\left(\frac{qV_F}{\eta \cdot kT}\right) \qquad \frac{I_{f1}}{I_{f2}} = \frac{e^{qV_{f1}/\eta kT}}{e^{qV_{f2}/\eta kT}} \rightarrow \eta = \dots$$

8 Depletion approximation

8.1 Depletion width correction

Using the Gummel correction. $2V_t$ for a 2-sided junction, V_t for single sided junction.

$$V_t = \frac{kT}{q} \quad \text{Thermal voltage}$$

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)(V_{bi}-2V_t)}$$

8.2 Debye Length

The length of the region where the doping concentration changes.

$$L_D = \sqrt{\frac{\epsilon_s V_t}{qN_d}}$$

8.3 Junction Capacitance

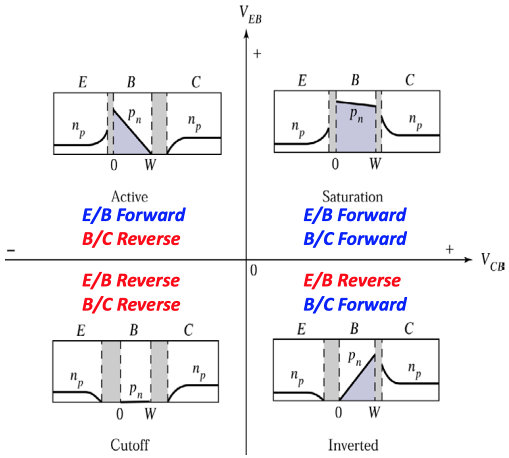
$V_t = 0$ for depletion approximation.

$$\Psi = V_{bi} + V_R \qquad C_j \approx \sqrt{\frac{\epsilon_s q N_d}{2(\Psi - V_t)}} \qquad [C_j] = \frac{F}{m^2}$$

9 Bipolar transistor

9.1 Modes of operation

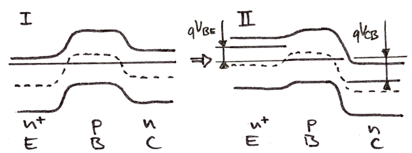
E/B junction injects minority carriers into base. B/C junction extracts minority carriers from base.



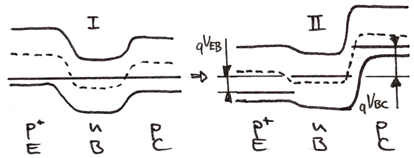
Normal "Active" Mode

E/B junction fowrard-biased: Minority carrier injection. B/C reverse-biased: Minority carrier extraction. Recombination in base consumes e/h pairs.

NPN transistor in I equilibrium and II forward active mode.

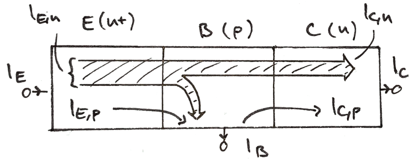


PNP transistor in I equilibrium and II forward active mode.



9.2 Current flow and gain

For pnp flip signs and change $n \leftrightarrow p$.



$$I_B = I_E - I_C = I_{E,p} + (I_{E,n} - I_{C,n}) - I_{C,p}$$

Emitter efficiency γ and base transport factor α_T:

$$\gamma = \frac{I_{E,p}}{I_{E,p}+I_{E,n}} \qquad \alpha_T = \frac{I_{C,p}}{I_{E,p}}$$

Common emitter current gain in forward- and reverse-active mode:

$$\beta_F = \frac{I_C}{I_B} = \frac{D_B N_E W_E}{D_E N_B W_B}\frac{A_C}{A_E}$$

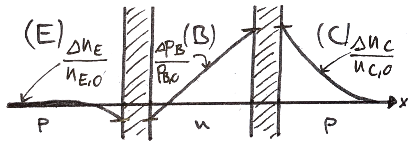
$$\beta_R = \frac{I_E}{I_B} = \frac{D_B N_C W_C}{D_C N_B W_B}\frac{A_E}{A_C}$$

Common base current gain:

$$\alpha = \frac{\beta}{\beta+1}$$

9.3 Doping

Focus is on minority carriers. NPN has pnp minority carriers and PNP npn structure. In this image the BJT is in reverse active mode.



$$\frac{\Delta n_E}{n_{E,0}} = \frac{n_E - n_{E,0}}{n_{E,0}} = e^{\frac{qV_{BE}}{kT}} - 1$$

$$\frac{\Delta p_B}{p_{B,0}} = \frac{p_B - p_{B,0}}{p_{B,0}} = e^{\frac{qV_{CB}}{kT}} - 1$$

In order to be useful as a transistor we want

$$I_{pC} = I_{pE} \gg I_{nE} = I_B$$

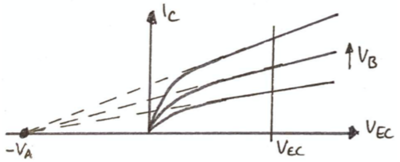
$$\frac{n_{iB}^2}{N_{DB}} \gg \frac{n_{iE}^2}{N_{AE}} \to N_{AE} \gg N_{DB}$$

$$J_C = \frac{qD_B}{W_B}\frac{n_i^2}{N_B}(e^{qV_{EB}/kT}-1) = I_S(e^{qV_{EB}/kT}-1)$$

$$g_M = \frac{\mathrm{d}I_C}{\mathrm{d}V_{EB}} = I_S e^{qV_{EB}/kT}\frac{q}{kT} \approx \frac{I_C}{kT/q}$$

9.4 Early Effect

Collector-emitter voltage decreases base width which increases collector current.



Calculate base width x_B at zero voltage and at two operating points with depletion correction.

$$W_{BE} = \sqrt{\frac{2\epsilon_i}{q}\frac{N_B+N_E}{N_BN_E}V_x} \quad V_x = V_{bi} - V_{BE}$$

$$W_{BC} = \sqrt{\frac{2\epsilon_i}{q}\frac{N_B+N_C}{N_BN_C}V_x} \quad V_x = V_{bi} + V_{BC}$$

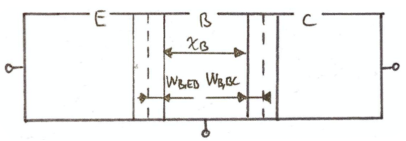
$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_BN_i}{n_i^2}\right) \quad i = E, C$$

$$x_B = x_{B0} + W_{B,BC,0} + W_{B,BE,0} - W_{B,BC,1} - W_{B,BE,1}$$

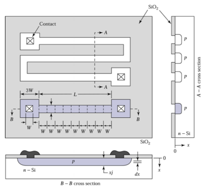
Then calculate collector currents at two operating points.

$$J_C = \frac{qn_i^2D_B}{N_Bx_B}\left(e^{qV_{EB}/kT}-1\right) \to V_{CE} = V_{BE} + V_{CE}$$

$$V_A = J_{C,1}\frac{V_{CE,2}-V_{CE,1}}{J_{C,2}-J_{C,1}} - V_{CE,1} \quad \frac{\mathrm{d}I_C}{\mathrm{d}V_{EC}} = \frac{I_C}{V_A+V_{EC}}$$



9.5 Diffused Resistor

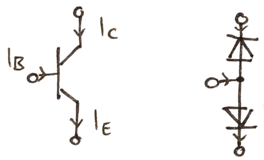


Resistance is per unit square.

$$R_{tot} = R_\square\frac{L}{W} + 2R_cR_\square \qquad R_\square = \frac{\rho}{x}$$

$$\rho = \frac{1}{qp\mu_p}, \quad p = N_a$$

9.6 BJT vs. Back to back diode



BJT:

$$I_C = \beta I_B = \frac{\beta}{1+\beta}I_E \quad I_E = I_S\left(e^{qV_{BE}/kT}-1\right)$$

Diodes:

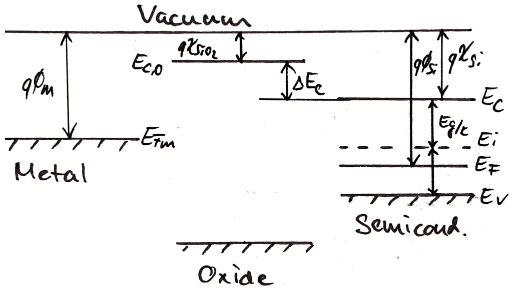
$$I_C = I_E - I_B, I_C = I_S \quad I_B = I_S\left(e^{qV_{BE}/kT}-1\right)$$

10 MOS Transistor

ϕ	Work function
χ	Electron affinity
ϵ_{Ox}	$= \epsilon_0\epsilon_{r,Ox}$
ϵ_s	$= \epsilon_0\epsilon_{r,s}$
Ψ_B	Bulk potential
Ψ_S	Si surface potential
ϕ_{ms}	Work function difference $\phi_m - \phi_s$
C_0	Oxide capacitance

10.1 Band structure

Workfunction ϕ is energy required to extract electron. In semiconductor materials the electrons get extracted from conduction band, so we use the electron affinity χ parameter.

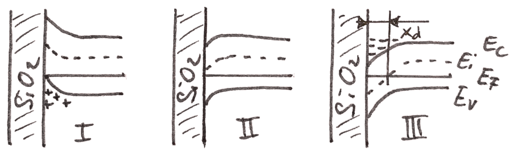


$$q\phi_s = q\chi_s + E_g/2 + \phi_{fn} \qquad q\chi_{SiO2} = q\chi_s - \Delta E_c$$

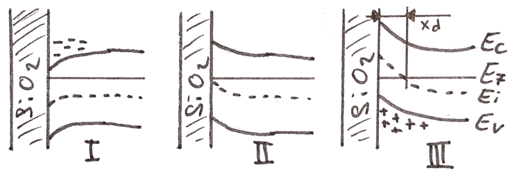
$$\phi_{fn} = (E_i - E_F) = kT\ln\frac{N_a}{n_i}$$

10.2 Gate votlage dependance

The gate voltage creates a depletion region or channel at the interface between Si and oxide depending on the regime. The regimes are called I accumulation, II depletion and III inversion mode. The **NMOS** forms a n-channel and therefore uses p-type substrate.



The **PMOS** forms a p-channel and therefore uses n-type substrate.



Inversion Surface potential is double the bulk potential.

$$\Psi_S = 2\cdot\Psi_B \quad \Psi_B = \frac{kT}{q}\ln\frac{N_a}{n_i}$$

Gate capacitance in accumulation:

$$C = C_{Ox} = \frac{\epsilon_{Ox}}{d}$$

in depletion:

$$C = (C_{Ox}^{-1} + C_j^{-1})^{-1} = \frac{\epsilon_{Ox}}{d + \frac{\epsilon_{Ox}}{\epsilon_S}}W$$

Depletion width in general x_d and at inversion transision x_{dt}:

$$\phi_{fp} = V_t\ln\frac{N_a}{n_i} \quad x_d = \sqrt{\frac{2\epsilon_s\phi_s}{qN_d}} \quad x_{dt} = \sqrt{\frac{4\epsilon_s\phi_{fp}}{qN_d}}$$

10.3 Operating mode

$$\begin{cases} V_{DS} \geq V_{GS} - V_t & \text{saturation} \\ V_{DS} < V_{GS} - V_t & \text{linear} \end{cases}$$

$$I_D = \frac{\mu C_{OX}}{2}\frac{W}{L}(V_{GS}-V_t)^2 \qquad \text{sat}$$

$$I_D = \frac{\mu C_{OX}}{2}\frac{W}{L}\left(2(V_{GS}-V_t)V_{DS} - V_{DS}^2\right) \qquad \text{lin}$$

10.4 E-Field

Applied voltage V_a. If intrinsic, then $\Psi_S = \Psi_B$.

$$V_a = \Psi_S + V_{Ox} \quad \Psi_B = \frac{kT}{q}\ln\frac{N_a}{n_i} \quad W = \sqrt{\frac{2\epsilon_s\Psi_S}{qN_a}}$$

$$E_S = \frac{qN_a}{\epsilon_s}W \quad E_{Ox} = \frac{E_S\epsilon_s}{\epsilon_{Ox}} \quad V_{Ox} = E_{Ox}d$$

10.5 Threshold Voltage

Work function difference ϕ_{ms} from table.

$$\phi_{ms} = \phi_m - \left(\chi_S + \frac{E_g}{2q} + \frac{E_i - E_F}{q}\right) \quad C_{Ox} = \frac{\epsilon_{Ox}}{d}$$

$$V_t = V_{Ox} + \Psi_S + \phi_{ms} \quad V_{Ox} = \frac{\sqrt{2q\epsilon_sN_a\Psi_S}}{C_{Ox}}$$

$\Psi_S = 2\Psi_B$ if inversion mode.

10.6 Frequency dependance

Cutoff frequency f_T

$$f_T = \frac{g_M}{2\pi(C_{gs}+C_{gd})} = \frac{\mu_nC_{OX}Z(V_{gs}-V_T)/L}{2\pi\cdot 2C_{OX}ZL/3} = \frac{3}{4}\frac{\mu_n}{\pi}\frac{(V_{gs}-V_T)}{L^2}$$

10.7 C-V Characteristics

Low gate voltage

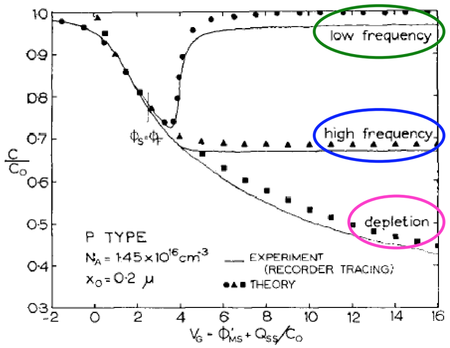
Accumulation: Only oxide thickness contributes to gate capacitance.

Medium gate voltage

Depletion: Attracted minority carriers widen the capacitor plates in a sense that the conducting bulk is further away - capacitance decreases.

High gate voltage

Inversion: At low f a conducting channel is formed and the capacitor plates are closer together meaning higher capacitance. At high f the charges in bulk have no time to recombine and the capacitor plates do not move closer resulting in lower capacitance.



$$\frac{C}{C_0} = \frac{1}{\sqrt{1 + \frac{2\epsilon_{Ox}^2 V}{q N_a \epsilon_s d^2}}} \qquad C_0 = C_{Ox}$$

10.8 Flatband

Flatband condition is met if fermi level through device is constant, this is the case at equilibrium. The applied voltage is composed of the oxide portential V_{Ox}, the silicon surface potential Ψ_s and the metal-silicon work function difference ϕ_{ms}.