



# Growth on Imperfect Crystal Faces

Monte-Carlo Method

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Computational Physics - AP3082

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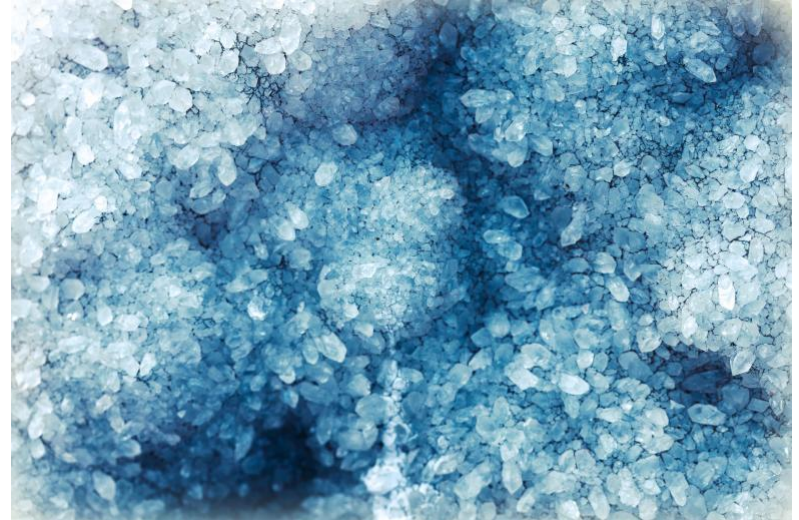
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# 1. Motivation

Crystal growth:

- Atomic bonding
- Atom mobility
- Surface impurities

Monte-Carlo computer simulations ideal



## 2. Theoretical Background

Interface interactions:

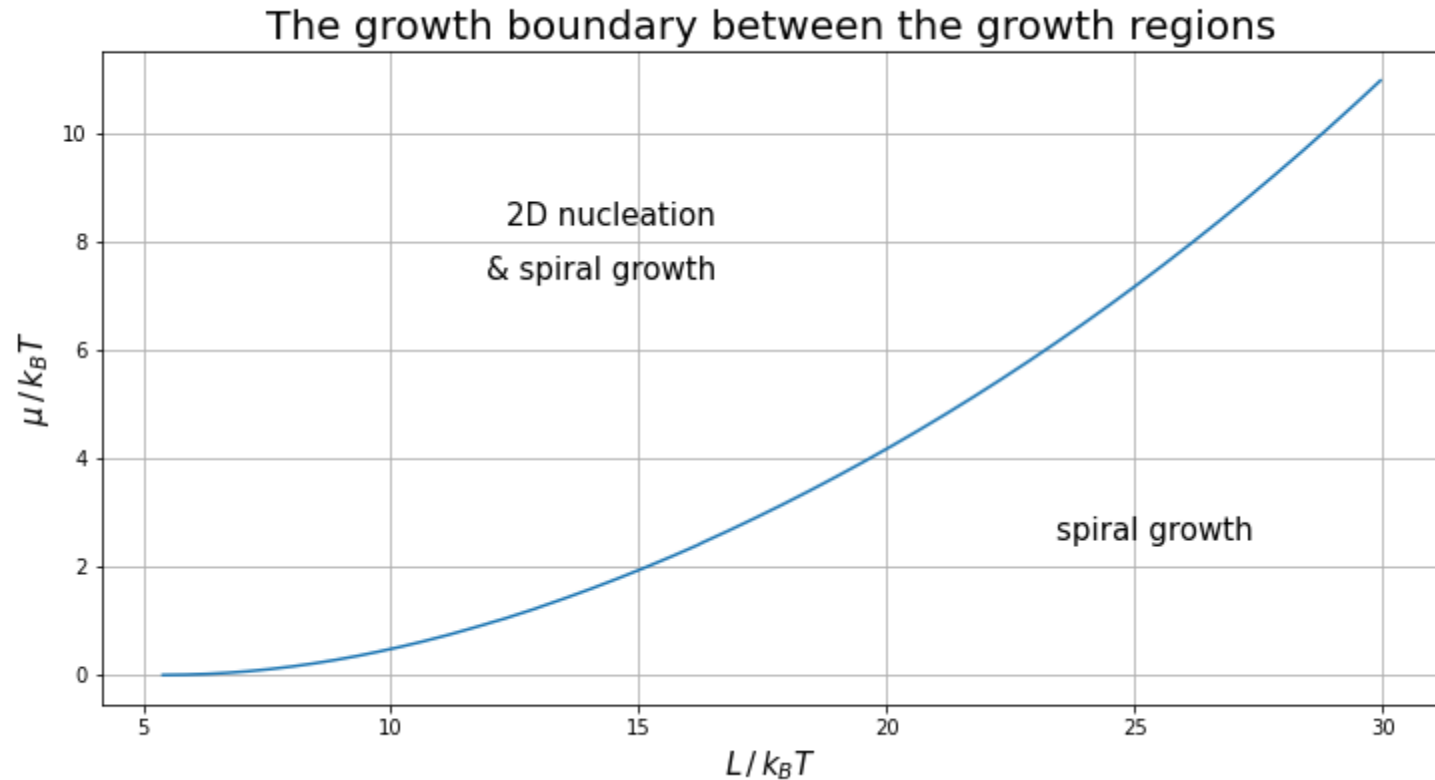
- Evaporation:  $k_n^- = v \exp(-n\phi/kT)$
- Impingement:  $k^+ = \exp(\Delta\mu/kT) k_3^-$
- Surface migration

## 2. Theoretical Background

Interface interactions:

- Evaporation:  $\tilde{k}_n^- = \exp(-n\tilde{T})$
- Impingement:  $\tilde{k}^+ = \exp(\Delta\tilde{\mu}) \tilde{k}_3^-$
- Surface migration

## 2. Theoretical Background



### 3. Implementation

Divide the surface atoms in subsets based on the number of neighbours of each atom

$$p_n = N_n(k_n^- + k^+) / \sum_{i=1}^5 N_i(k_i^- + k^+)$$

Select a subset with probability  $p_n$

Choose a lattice point from the subset

Choose from of interaction with a probability proportional to their interaction rate



### 3. Implementation

Simulate dislocation  
by introducing  
scanning matrices

At the boundaries  
periodic boundary  
conditions were  
applied

Dislocation  
↓

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Surface matrix

0	0	0	2	0
0	0	0	2	0
0	0	0	2	0
0	0	0	0	0
0	0	0	0	0

Forward scanning  
matrix  
→

0	0	-2	0	0
0	0	-2	0	0
0	0	-2	0	0
0	0	0	0	0
0	0	0	0	0

Backward  
scanning matrix  
←

### 3. Implementation

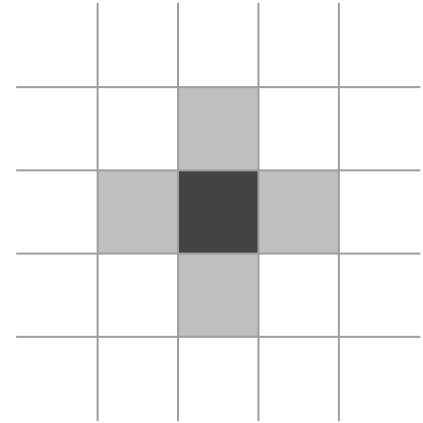
Optimize neighbour scanning by  
updating the number of neighbours of  
the lattice sites affected by evaporation,  
impingement or migration



Interaction site



Neighbour



### 3. Implementation

#### The growth rate:

Every  $N_{\text{cycles}}$  the crystal surface is stored

$$\Delta h_i = 1/N_{\text{cycles}} (\langle h \rangle_{i+1} - \langle h \rangle_i)$$

$$R/k^+d = 1/n \sum_{i=0}^n \Delta h_i$$

#### Growth rate error:

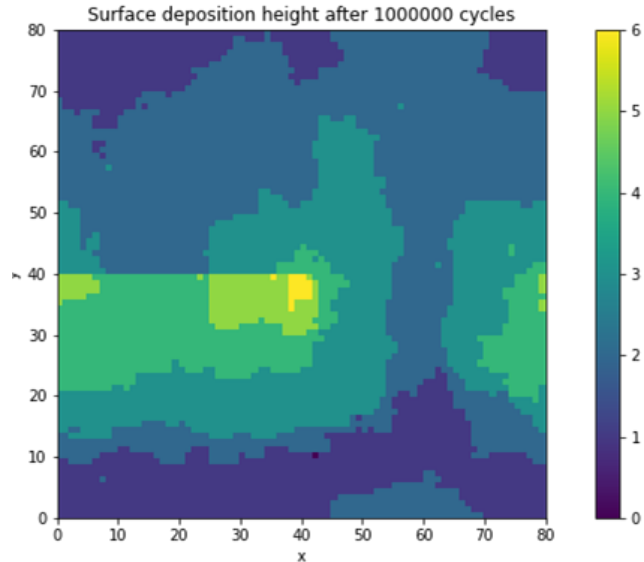
Growth rate is determined from one simulation - thus the data is correlated

The error is determined using the autocorrelation function

$$\chi_A(t) = \frac{1}{\sigma_A^2} \sum_n (A_n - \langle A \rangle)(A_{n+t} - \langle A \rangle)$$

$$\sigma_A = \sqrt{\frac{2\tau}{N} (\langle A^2 \rangle - \langle A \rangle^2)}$$

## 4. Results

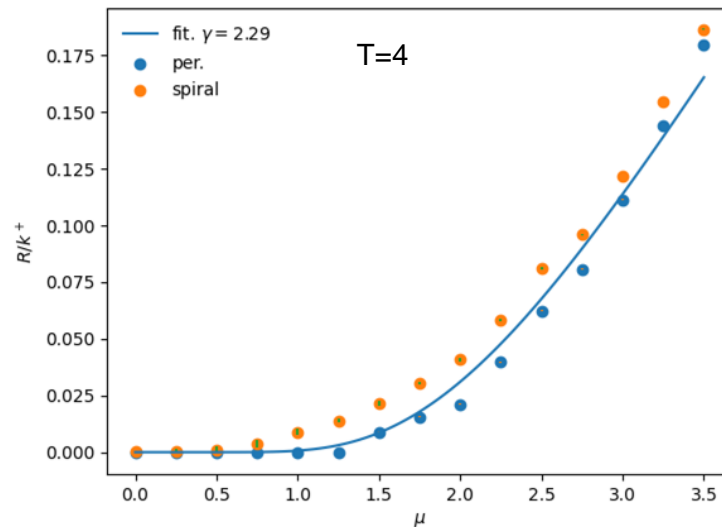
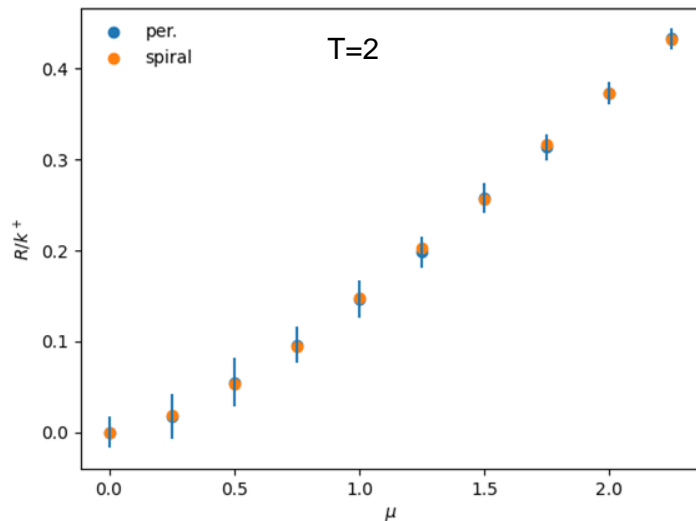


Spiral growth:

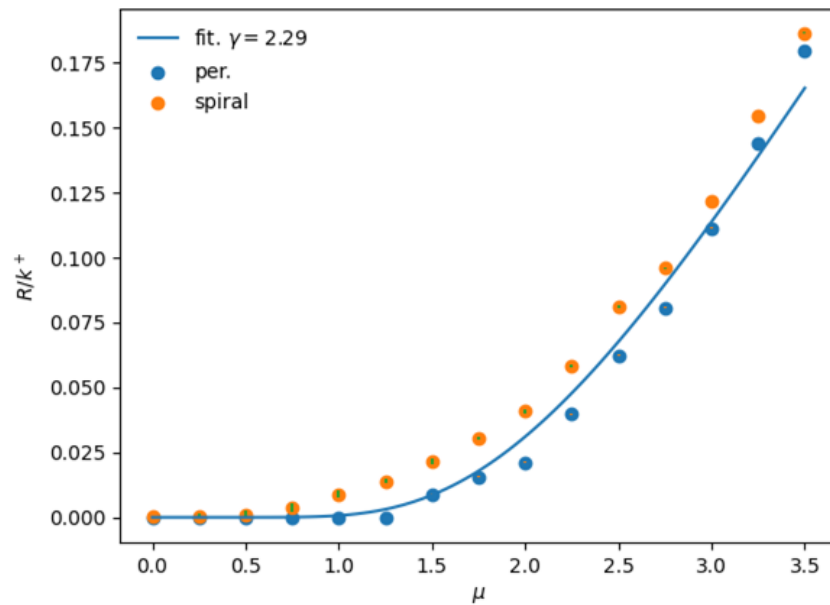
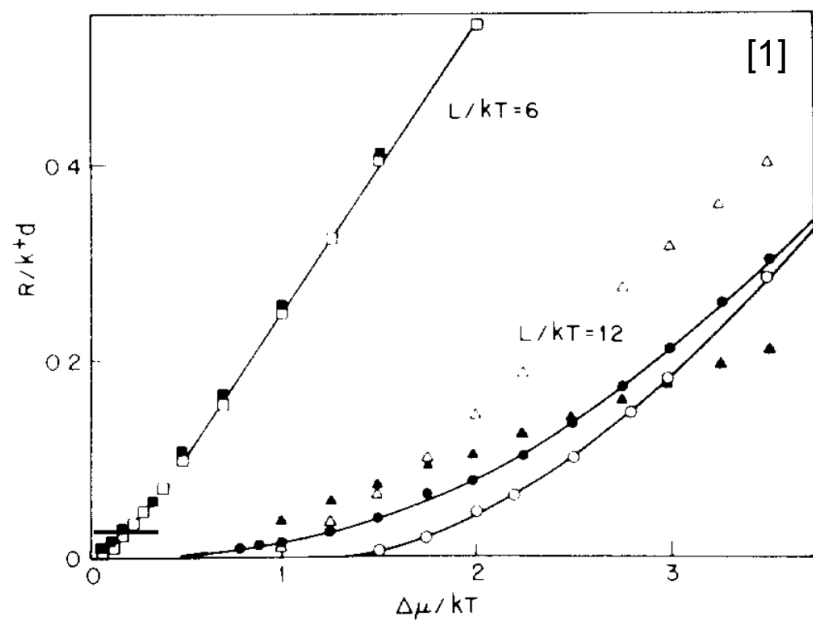
- when does a spiral grow
- what effect does it have

## 4. Results

Only in a small range of  $T$  do we see the contribution of spiral growth



## 5. Discussion



## 6. Conclusion & Outlook

- The results in the paper were replicated
- Spiral growth contributes to growth rate in a small window of temperatures.

With more time...

- ...we would like to have simulated with surface migrations and analysed these results

## 7. Short video

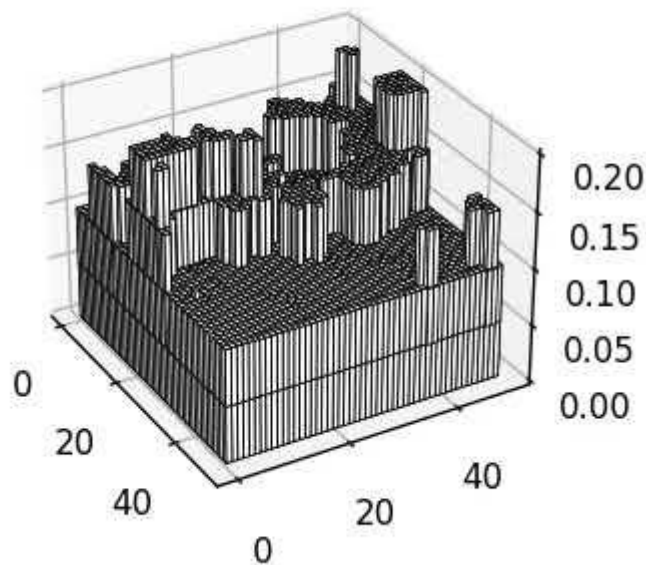
```
dims = [50,50]  
T = 6.5  
mu = 6  
N = 1e5 #attempts
```

defect introduced at  
[0,25]->[15,25]

Can see:

- Periodic BC
- Plateaus of growth
- Clumps of critical size stay

spirals can't be seen due to  
small simulation space





Source:

[1] G.H. Gilmer, *Growth on imperfect crystal faces: I. Monte-Carlo growth rates*,  
Journal of Crystal Growth, Volume 36, Issue 1 (1976), Pages 15-28,

Thank you for your attention

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