UNIT 1 : RANDOM VARIABLE .

Total Card (58)

RED 36 — Blue (26)

Diamond (13) (♦) | Hearts (13) (♥) | clubs (13) (♣) | Spades (13) (♠)

K, Q, J, A 2, 3, A, 5, 6, 7, 8, 9, 10

n object : how many chase of choosing object

$$\boxed{n_r = \frac{n!}{(n-r)!\, r!}}$$

TRIAL :

Consider an experiment of throwing a coin ~~When tossing~~ a coin we may get a Head (H) or tail (T). Here tossing a coin is called a trial and getting head or tail is called Event.

EXHAUSTIVE EVENT :

Is the total no of possible outcomes in any trial ~~is~~ known ~~as~~. Example : In tossing a coin there is two possible (Head , tail).

In tossing dice there is possible 6 (1, 2, 3, 4, 5, 6)

MATHEMATICAL DIFINITION OF PROBABILITY :

(CLASSICAL OR PRIORITY)

Probability : (Is a happening of an events) =

$$P(E_n) = \frac{\text{no of favourable case}}{\text{Total no of exhaustive case}}$$

Example. In tossing a coin. What is the probability of getting head?

Answer

Coin = {H, T}

number of favourable case = 1

total no of possible outcome = 2

$$\text{Probability (of getting head (H))} = \frac{\text{no of favourable case}}{\text{total no of possible ou}}$$

$$= \frac{1}{2}$$

$$= 0.5 =$$

Eg: In throwing dice, what is the probability of getting 2?

Soln

no of favourable case = 1

total no of possible outcome = 6

$$P(E) = \frac{1}{6}$$

Eg 3: Find the probability of throwing

a) 4 b) an odd no c) even no, with ordinary die (

Soln

$$a)\ P(E_4) = \frac{1}{6}$$

Dice = 1, 2, 3, 4, 5, 6

a) Even no = {2, 4, 6} = 3
odd no = {1, 3, 5} = 3

a) P(Even no) = $\frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$

b) P(odd no) = $\frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$

Eg 4: Find the probability of throwing 7 with two dice.

Soln

	1	2	3	4	5	6	7
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
2			(3,4)	(2,5)			
3							
4	(4,3)						
5	5,2						
6	6,1						

1st dice = {1, 2, 3, 4, 5, 6}
2nd dic = {1, 2, 3, 4, 5, 6}

∴ (1,6), (2,5) {3,4}, (5,2), {6,1}

no of favourable case = 6
Total no of possible outcome = 36

P(no 7) = $\frac{6}{36} = \frac{1}{6}$

Find the probility of throwing 7 with three dice?

Ans:

total no of possible out come = 216.

no of favourable case =

Eg: A bag containing 6 red & 7 black balls. Find the probability of having red ball.

Soln

Probability (throwing red ball) = $\frac{\text{no of favourable case}}{\text{total no of possible outcom}}$

$$^{n}C_r = \frac{n!}{(n-r)!\ r!}$$

$$^{6}C_1 = \frac{6}{(6-1)! \times 1!} = \frac{6 \times 5!}{5! \times 1!} = 6$$

Probability (of throwing red ball) = $\frac{^6C_1}{^{(6+7)}C_1} \Rightarrow \frac{6}{13}$

NOTE

$$\boxed{^{n}C_r = n}$$

$$^{n}C_1 = n$$

$$^{n}C_n = 1$$

$$^{n}C_0 = 1$$

Eg: Find the probability that if a card drawn at random from ordinary pack its a diamond

Soln:

There are 13 diamond to get 1 diamond

1 diamond = $^{13}C_1$

Total card - 52 to get

1 diamond — $^{52}C_1$

Probability = $\frac{^{13}C_1}{^{52}C_1} = \frac{13}{52} = \frac{1}{4}$

chance = probability

ii) Find the probability of getting queen or king

$$\text{Probability (of getting king)} = \frac{4C_1}{52C_1} = \frac{4}{52} = \frac{1}{13}$$

Eg) Four persons are choosen at random from a group containing 3 men, 2 women and 4 children. Show that the ~~chance thate~~ chance that exactly two of them will be children is $10/21$?

Solution:

$$\text{Total} = 3C + 2C + 4C = 9$$

3	m
2	w
4	c
9	member

$9C_4$ = Total possible out come

number of favourable case $= 4C_2 \times 5C_2$ (total no of children; 5 choose total no of m + w)

$$\text{Probability} = \frac{\overset{6}{4C_2} \times \overset{10}{5C_2}}{9C_4\ (9\times7\times2)} = \frac{60}{126} = \frac{10}{21}$$

(12 - marks)

Eg) From a group of 3 mbezians, 4 Temekians and 5 kimarians. A sub committees of four people is selected by SJUIT. Find the probability that the subcommittee will consist of

i) 2 mbezians of 2 Temekian

ii) 1 mbezi an, 1 temekian & 2 kimarian

iii) 4 kimarian .

Solution :

3 mbezi

4 temeki

5 kimare

12 . people

Total possible outcome $= {}^{12}C_4$

no of favourable case $= {}^{3}C_2 \times {}^{4}C_2$

Probability (selecting 2 mbezi & 2 temekian) $= \frac{{}^{3}C_2 \times {}^{4}C_2}{{}^{12}C_4}$

$= \frac{2}{55}$

ii) Probability (selecting 1mbezian & 1temekian & 2kimarian) $= \frac{{}^{3}C_1 \times {}^{4}C_1 \times {}^{5}C_2}{{}^{12}C_4} = 8/33$

ii) Probability (selecting 4 kimarian) $= \frac{{}^{5}C_4}{{}^{12}C_4}$

Probability (selecting 4 kimarian) $= 1/99$

In card $\frac{26}{52}$

→ king $K = 4$

→ queen $Q = 4$

(Qn1) What is the probability that of 6 cards taken from a full pack, 3 will be black and 3 will be red?

Solution:

$$P(\text{happening events}) = \frac{{}^{26}C_3 \times {}^{26}C_3}{{}^{52}C_6} = \frac{\frac{26!}{(26-3)!\,3!} \times \frac{26}{(26-3)! \times 3!}}{\frac{52!}{(52-6)! \times 6!}}$$

$P(\text{happenning}) =$ ______

(Qn2) What is the probability of having ~~a queen~~ queen & king when two cards are drawn from a pack of 52 cards?

Solution

$$P(\text{king \& queen}) = \frac{{}^4C_1 \times {}^4C_1}{{}^{52}C_2} =$$

$P(\text{king \& queen}) =$ ______

Mutually Exclusive Events

Two events are said to be mutually exclusive when the occurrence of one doesn't influence in anyway the occurrence of the others.

Example: In tossing a coin the events header or tail are mutually exclusive, since both tail and head can not appear in the same time.

Equally Likely Events

Two events are said to be equally likely if one of them can not be expected in the preference to other.

Example: In tossing a coin, head or tail are equally likely events.

$P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$

7 am (Qn) What is the chance that a leap year selected at random will contain 53 sundays

8 Solution.

1 year = 365 days

9 1 leap year = 366 days

10 $(52 \times 7) + 2 = 366$ days
week, day

11

In 52 week there are 52 sunday for the 2 days

12 noon leave

Mon, tue thur, fri sun, mon

1 pm tue, wed frid, sat

wed, thur sat, sun

2

then

3 Prob $= \frac{2}{7}$ (sat, sun & sun mon)

4

(Qn) A bag contain 7 whiteball, 6 redball and 5 black ball. Two ball are drawn at random. Find the probability that they will both be white

6 Solution

$7W + 6R + 5B = 18$ balls

Prob (white ball) $= {}^{7}C_{2} / {}^{18}C_{2} = 21/153 = 7/52$

Correctly:

Total no of ball $= 7 + 6 + 5 = 18$

From 18 ball two ball can be drawn $= {}^{18}C_{2}$ ways

Total no of draw 2 ball $= \frac{18!}{(18-2)! \times 2!} = 153$

Then 2 white ball can be drawn from 7 whiteball in ${}^{7}C_{2}$ ways

$= \frac{7!}{(7-2)! \times 2!} = 21$ days

The number of favourable case $= 21$

P (drawn two white balls) $= \frac{\text{no of favourable case}}{\text{total no of possible outcome}}$

$= 21/153$

SET :

Universal set (S) $= \{1, 2, 3, 4, 5, 6, 7\}$

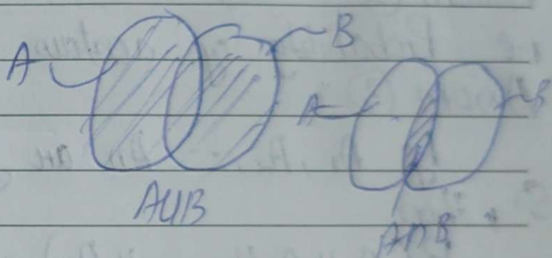
$A = \{1, 2, 3, 4, 5\}$

$B = \{3, 4, 5, 6, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cap B = \{3, 4, 5\}$

$A (A') = \{6, 7\}$



Commutative law :

$A \cup B = B \cup A$, $A \cap B = B \cap A$

Associative law,

$A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$

Distributive law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Complementary law

$A \cup A' = S$

$A \cap A' = \phi$

SAMPLE SPACE : (S)

i) Coin, $S = \{H, T\}$

ii) Dice, $S = \{1, 2, 3, 4, 5, 6\}$

iii) Two coin $S = \{HH, HT, TH, TT\}$

EVENTS :

Every non empty subset of sample space called EVENT

Subset $= \{\phi, \{H\}, \{T\}, \{H,T\}\}$

AXIOMATIC APPROACH TO PROBABILITY :

Axiom ① $P(A) \geq 0$ can never be negative.

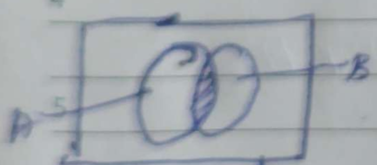
Axiom ② $P(S) = 1$

i.e Probability of a certain events is 1, $0 \leq P(A) \leq 1$.

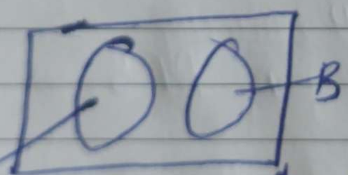
Axiom ③ :

If $A_1, A_2, \cdots A_n$ are finite number of disjoint events of S, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots P(A_n)$$



$A \cap B \neq \phi$



$A \cap B = \phi$ $P(A \cap B) = \phi = 0$

THEOREM 1 :

Probability of the impossible events is zero.

$$P(\phi) = 0$$

Proof :

$$S \cup \phi = S$$
$$P(S \cup \phi) = P(S)$$
$$P(S) + P(\phi) = P(S)$$
$$P(\phi) = 0$$

1) $P(\bar{A}) = 1 - P(A)$

Proof:

8 $A \cup A' = S$

$P(A \cup A') = P(S)$

9 $P(A) + P(A') = 1$

$[P(A') = 1 - P(A).]$

10

Law of Addition of probabilities:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Independent Events:

Two events A, B are independent if

1 pm $P(A \cap B) = P(A) \cdot P(B)$.

1) If from a pack of cards a single card is drawn, what is the probability that is it is either a spade or a king?

3 $P(A \cup B) = ?$

$P(\text{either } \underset{A}{\text{spade}} \text{ or } \underset{B}{\text{king}}) = ?$

4

$P(A) = P(\text{a spade card}) = {}^{13}C_{01} / {}^{52}C_{01} = 13/52$

5

$P(B) = P(\text{a king card}) = {}^{4}C_{01} / {}^{52}C_{01} = 4/52$

6

$= P(A) \cdot P(B)$

7 $P(A \cup B) = 13/52 + 4/52 - \left(13/52 \times 4/52\right) = 4/13$

$^3C_1 = 2$

$^7C_1 = 6$

CONDITIONAL PROBABILITY $P(A/B)$

The conditional probability of event A when event B has already happen is defined as

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

$$\boxed{P(B/A) = \frac{P(B \cap A)}{P(A)}}$$

NOTE:

$P(B) \neq 0$

$P(A) \neq 0$

Example:

A bag contains 3 red and 4 white ball. Two draws are made without replacement, whats the probability that both the balls are red.

Solution:

$$P(A) = \frac{^3C_1}{^7C_1} = \frac{2}{6} = \frac{1}{3}$$

$$P(B) = \frac{^2C_1}{^7C_1} =$$

Qn) If $A \subseteq B$ then show that $P(A/B) \leq P(A)$.

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Sol $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\textcircled{A}) B \rightarrow A \subseteq B$

But $A \cap B = A$

$P(B/A) = \frac{P(B \cap A)}{P(A)}$

$\frac{2}{6} = \frac{P(B \cap A)}{3/7}$

$\frac{2}{6} \times \frac{7}{3} = P(B \cap A)$

$P(B \cap A) = \frac{1}{7}$

then

$P(A/B) = \frac{P(A)}{P(B)}$

$\boxed{P(A/B) \times P(B) \leq P(A)}$

Eg: A bag contain 5 white ball and 3 black balls, Two balls are drawn at random one after another without replacement. Find the probability that both balls drawn are blacks? $P(A \cap B)$?

Solution: Both means intersection

$5 + 3 = 8$

$P(A) = \frac{{}^3C_1}{{}^8C_1} = \frac{3}{8}$

then: during the 2^{nd} ball & the first ball drawn is black

$P(B/A) = \frac{{}^2C_1}{{}^7C_1} = \frac{2}{7}$, $P(B \cap A) = P(B/A) \times P(A)$

$P(B \cap A) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$

$P(B/A) = \frac{P(A \cap B)}{P(A)}$, $P(B/A) \times P(A) = P(A \cap B)$

Dependent

Independent

Qn) If $A \subseteq B$ then show that $P(A/B) \leq P(A)$

Solution

$P(A/B) = \frac{P(A \cap B)}{P(B)} =$

$P(A/B) = \frac{P(A)}{P(B)}$ $\therefore A \subseteq B$, $A \cap B$

$\boxed{P(A/B)\, P(B) \leq P(A)}$

$0 \leq P \leq 1$

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7 am

Q) If $P(A) = 0.9$, $P(B/A) = 0.8$. Find $P(A \cap B)$.

Solution.

8 $P(A) = 0.9$, $P(B/A) = 0.8$

from:

9 $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$.

10 $P(A \cap B) = P(A) \times P(B/A)$

$= 0.9 \times 0.8$

11 $P(A \cap B) = 0.72$

12 noon

Q) If A & B are independent events and $P(A) = 1/3$ and $P(B) = \frac{1}{4}$ find $P(A \cap B)$; $P(A/B)$.

1 pm

Solution:

2 If A & B are independent events

3 $P(A \cap B) = P(A) \cdot P(B)$.

$P(A \cap B) = \frac{1}{4} \times \frac{1}{3}$

4 $P(A \cap B) = \frac{1}{12}$

5

then

6 $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

7 $P(A/B) = \dfrac{1/12}{1/4} = \frac{1}{12} \times \frac{4}{\cancel{1}} = \frac{1}{3}$.

7 am BAYES THEOREM : (12 mark)

Let $A_1, A_2, A_3 \dots A_n$ be 'n' mutually exclusive events and exclusive events. Let B be independent events such that $B \subset \bigcup_{i=1}^{n} A_i$ is the conditional probability of B given that A has already occured. then

$$\left[P(A_i/B) = \frac{P(A_i)\ P(B/A_i)}{\sum_{i=1}^{n} (P(A_i) \cdot P(B/A_i))} \right]$$

If $A_1, A_2 \dots A_n$

$$\left[P(A_1/B) = \frac{P(A_1) \cdot P(B/A_1)}{\sum_{i=1}^{n} P(A_1) \cdot P(B/A_1)} \right]$$

Qn 1) The first bag contain 3 white ball, 2 red ball, and 4 black ball. Second beg contain 2 white, 3 red and 5 black ball. Third beg contain 3 white, 4 red and 2 black ball. One beg is choosen at random and from it 3 ball are drawn out of three balls, two balls ~~one~~ white and one red. What are the probabilities that they were taken from first beg, second bags, third beg.

Solution:

Beg 1 (A_1) $\begin{pmatrix} 3W \\ 2R \\ 4B \end{pmatrix} = 9$ ball $P(A_1)$

$P(A_1) = \frac{1}{3}$

$P(B/A_i)$

$\frac{{}^3C_2 \times {}^2C_1}{{}^9C_3}$ = ...

$P(B/A_1)$

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Beg 2 (A2) $\begin{pmatrix} 2W \\ 3R \\ 5B \end{pmatrix}$ = 10 ball

$P(A_2) = \frac{1}{3}$ $\left| \frac{{}^2C_2 \times {}^3C_1}{{}^{10}C_3} \right| = \frac{1}{40}$

P(B/A2)

beg 3 (A3) $\begin{pmatrix} 3W \\ 4R \\ 2B \end{pmatrix}$ 9 ball

$P(A_3) = \frac{1}{3}$

$\frac{{}^3C_2 \times {}^4C_1}{{}^9C_3} = P(B/A_3) = \frac{1}{7}$

then :

$P(A_i)$	$P\,B/A_1$	$P(A_i) \cdot P(B/A_1)$
Beg 1	1/14	$\frac{1}{3} \times \frac{1}{14} = \frac{1}{42} = 0.0238$
Beg 2	1/40	$\frac{1}{3} \times \frac{1}{40} = \frac{1}{120} = 0.0083$
Beg 3	1/7	$\frac{1}{3} \times \frac{1}{7} = \frac{1}{21} = 0.047$

$\sum P(A_1) \cdot P(B/A) = 0.0797$

from Bayes theorem

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum^3 P(A_i) \cdot P(B/A)}$$

then

1) First beg

$P(A_1/B) = ?$

Second beg

$P(A_2/B) = ?$

Third beg

$P(A_3/B) = ?$

then

$$P(A_1/B) = \frac{P(A_1)\cdot P(B/A_1)}{\sum P(A_1)\cdot P(B/A_1)} = \frac{0.0238}{0.0797}$$

$$= 0.286 = 28.6\%$$

$$P(A_2/B) = \frac{P(A_2)\cdot P(B/A_2)}{\sum (P(A_2)\cdot P(B/A_2)} = \frac{0.0083}{0.0797}$$

$$= 0.104 = 10.4\%$$

$$P(A_3/B) = \frac{P(A_3)\cdot P(B/A_3)}{\sum P(A_3)\cdot P(B/A_3)} = \frac{0.047}{0.0797} = 0.59$$

$$= 59.7\%$$

7 am ASSIGMENT : (due date 2nd May)

2 mark

1) Find the probability of drawing a queen and a king from a pack of cards in two consecutive draws the cards drawn not being replaced.

6 mark

2) 10 In a bolt factory machines A, B, C manufacture respectively 40%, 35% and 25% of the totals of their
11 output 2%, 4% and 5% are defective bolts.
A bolt is drawn at random from the posture and
12 noon found to be defective. What are probabilities that it was manufactured by making A, B and
1 pm (Bayes theorem).

2 Answer:

1) A standard deck of cards contain 52 cards in total with 4 queens and 4 king.
We are drawing two cards consequtively without replacement then the probability changes after the first draw

step 1: Probability of drawing a queen first:
There 4 queen in the deck of 52 cards so the probability is $P(\text{Queen first}) = \frac{4}{52} = \frac{1}{13}$

6

step: Probability of drawing a king second:
After drawing a queen, there are now 51 cards left with 4 king remain; so the probability is

$$P(\text{King second}/\text{queen first}) = \frac{4}{51}$$

step 3. Calculate the total probability:
Both events must occur consequtively, we multiply the probility: