2 5 4 2

Random Variable :

$X \in (a,b) \Rightarrow a < x < b$

$X \in (a,b] \Rightarrow a < x \leq b$

$X \in [a,b) \Rightarrow a \leq x < b$

$X \in [a,b] \Rightarrow a \leq x \leq b$

FOR GAT 2

~~DISCRETE RANDOM VARIABLE :~~

~~If the Random variable takes on all value with acertain val, then the random variable is called "~~

DISCRETE RANDOM VARIABLE :

If the random variable takes the value only on the set {0, 1, 2, 3 .... n} is called "Discrete Random Variable"

CONTINUOUS RANDOM VARIABLE :

• If arandom variable takes all value with certain intervals, then the random variable is called Continuous Random Variable

NOTE $X = \{1, 2, 3, 4, \ldots, n\}$ x_i

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DISTRIBUTION FUNCTION OF RANDOM VARIABLE:

The distribution fxn of random variable X defined in $(-\infty, \infty)$ is given by,

$$\boxed{F(x) = P(X \leq x)}$$

PROBABILITY MASS FUNCTION:

The numbers $P(x_i)$, $i = 1, 2, 3 \ldots$ satisfy the following conditions

a) $P(x_i) \geq 0$

b) $\sum_{i=1}^{\infty} P(x_i) = 1$

This fxn "P" satisfying the above two conditions is called Probability Mass Function or Probability Function.

Eg 1.) A random variable X has the following probability fxn

Values of X, x	0	1	2	3	4	5	6	7	8
Probability $P(x)$	a	3a	5a	7a	9a	11a	13a	15a	17a

i) Determine the value of "a"

ii) Find $P(X < 3)$, $P(X \geq 3)$, $P(0 < X < 5)$

iii) Find the distribution fxn of X.

Solution

> Random variable 'X':

when

$x = 0$ $\quad$ $x = 1$ $\quad$ $x = 2$

$P(x=0) = a$ $\quad$ $P(X=1) = 3a$ $\quad$ $P(x=2) = 5a$

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15
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$x = 3$ $x = 4$ $x = 5$ $x = 6$

$x = 7$ $x = 8$

$P(x=7) =$

then

$$\sum_{x=0}^{\infty} P(x_i) = 1$$

$P(x_0) + P(x_1) + P(x_2) + P(x_3) + \cdots + P$
$= 1$

$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$81a = 1$

$a = 1/81$

ii) $P(x<3) = P(x=0) + P(x=1) + P(x=2)$
$= P(0) + P(1) + P(2)$
$= a + 3a + 5a$

$P(x<3) = 9a$
$= 9 \times \frac{1}{81}$

$P(x<3) = \frac{1}{9}$

$P(x \geq 3) = P(3) + P(4) + P(5) + P(6) + P(7) + P(8)$

$= 7a + 9a + 11a + 13a + 15a + 17a$

$= 72a$

$= 72 \times 1/81$

$P(x \geq 3) = 8/9$

OR

$P(0) + P(1) + P(2) + P(3) + \dots P(8) = 1$

$\frac{1}{8} + P(x \geq 3) = 1$

$P(x \geq 3) = 1 - \frac{1}{8}$

$P(0 < X < 5) = P(1) + P(2) + P(3) + P(4)$
$= 3a + 5a + 7a + 9a$
$= 24a$
$= 24 \times \frac{1}{81} = \frac{8}{27}$

$P(0 < X < 5) = \frac{8}{27}$

iv) The distribution fxn of X

x	$F(x) = P(X \leq x)$
0	$P(X \leq 0) = P(X = 0) = a = \frac{1}{81}$
1	$P(X \leq 1) = P(0) + P(1) = a + 3a = 4a = \frac{4}{81}$
2	$P(X \leq 2) = P(0) + P(1) + P(2) = 9a = \frac{9}{81} = \frac{1}{9}$
3	$P(X \leq 3) = P(0) + P(1) + P(2) + P(3) = 20a = \frac{20}{81}$
4	$P(X \leq 4) = P(0) + P(1) + P(2) + P(3) + P(4) = 33a$
5	$P(X \leq 5) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) =$
6	$P(X \leq 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$
7	$P(X \leq 7) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7)$
8	$P(X \leq 8) = P(0) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8)$

Eg 2) Suppose that the random variable 'X' assuming 3 value 0, 1, 2 with probabilities $\frac{1}{3}, \frac{1}{6}, \frac{1}{2}$ respectively. Find the distribution fxn of X.

Solution

x	0	1	2
$P(x)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$

33
15
4. 8
17
55

After

x	$F(x) = P(X \leq x)$
0	$P(X \leq 0) = P(0) = \frac{1}{3}$
1	$P(X \leq 1) = P(0) + P(1) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$
2	$P(X \leq 2) = P(0) + P(1) + P(2) = 1$

NOTE

RV

Discret RV	Continuos RV
- Probability mass fxn	- Probability density fxn
- distribution fxn	- Comulative distribution fxn
$P(x) \geq 0$, $\sum P(x) = 1$	$f(x) \geq 0$, $\int_{-\infty} f(x)\,dx = 1$

PROBABILITY DENSITY FXN: (P.d.f)

The probability density fxn for the continuos random variable "X" in the interval $[a, b]$ is given by

$$f(x) = \begin{cases} 0 & \text{for } x < a \\ \phi(x) & \text{for } a \leq x \leq b \\ 0 & \text{for } x > b \end{cases}$$

It satisfy two condition

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

7 am Commulative Distribution Fxn (Cdf).

8 If $\left[F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx \right]$

9

10 Eg 1) Is the fxn defined as the following fxn.

$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ $\Big|$ $[0, \infty)$ ~~0, x < 0~~ $(-\infty, 0)$

11 is p.df

12 noon

ii) $P(1 \leq x \leq 2)$

1 pm

iii) Find the cumulative distribution fxn (cdf) and

2 $F(2) = ?$

3 Solution:

1) In the interval $(1, 2)$

4 i) $f(x) \geq 0$

5 ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

6 then

$f(x) = e^{-x}$

7 $e^{-x} \geq 0$, $(1, 2)$

$\therefore$ Its p df its satisfy

ii)

NOTE $P(a \le x \le b) = \int_a^b f(x)\,dx$

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7 am

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x) + \int_{0}^{\infty} f(x)\,dx$$

8

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} e^{-x}\,dx$$

9

10

$$= \int_{0}^{\infty} e^{-x}\,dx$$

11

$$= -e^{-x}\Big|_0^{\infty}$$

12 noon

$$= -e^{0} - (-e^{\infty})$$

1 pm

$$= -(-1)$$

2

$$= 1$$

3

$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$ Hence its pdf

4

5

$P(1 \le x \le 2) = \int_1^2 f(x)\,dx$

6

$$= \int_1^2 e^{-x}\,dx$$

7

$$= -e^{-x}\Big|_1^2$$

$$= -(e^{-2} - e^{-1})$$

$$= \underline{e^{-1} - e^{-2}}$$

ii) Cdf

$$F(x) = P(x \le x) = \int_{-\infty}^{x} f(u)\,du$$

$$F(2) = P(x \le 2) = \int_{-\infty}^{2} f(x)\,dx$$

$x < 0$ $= 0$; $x \le 2$

$-\infty$ 0 1 2 $+\infty$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{2} e^{-x}\,dx$$

$$= 0 + (-e)^{-x}\Big/_{0}^{2}$$

$$= -(e^{-2} - e^{0})$$

$$= -(e^{-2} - 1$$

✓

Eg: A continuous random variable "x" has a pdf $f(x) = 3x^2$, $0 \le x \le 1$. Find 'a' and 'b' such that

i) $P(x \le a) = P(x > a)$ & ii) $P(x > b) = 0.05$

Solution

$P(x \le a) = P(x > a)$

$P(x \le a) + P(x > a) = 1$

$P(x \le a) + P(x > a) = 1$

$P(x < a) = 1/2 = P(x > a)$

~~$P(x \le a) = 1/2$~~

$\int_0^a f(x)\,dx = \frac{1}{2} \Rightarrow \int_0^a 3x^2\,dx = \frac{1}{2}$

8 $3\left(\frac{x^3}{3}\right)\Big/_0^a = \frac{1}{2}$

9 $a^3 - 0 = \frac{1}{2}$

$a^3 = \frac{1}{2}$

10 $a = \left(\frac{1}{2}\right)^{1/3}$

$P(x > b) = 0.05 \Rightarrow \int_b^1 f(x)\,dx = 0.05$

$0.05 = \int_b^1 3x^2\,dx$

12 noon

$0.05 = \frac{3}{3}x^3\Big/_b^1$

1 pm

$0.05 = 1^3 - b^3$

2 $b^3 = 1 - 0.05$

$b^3 = 0.95$

3 $b = (0.95)^{1/3}$

Eg: If $f(x)$ is pdf of random variable "x" is defined in the interval (a, b), then

i) Arithmetic mean $= \int_a^b x\,f(x)\,dx$

ii) Harmonic mean $= \int_a^b \frac{1}{x} f(x)\,dx$

iii) r^{th} - moment about origin $\mu_r' = \int_a^b x^r f(x)\,dx$

iv) r^{th} - moment about mean $\mu_r = \int_a^b (x - \text{mean})^r f(x)\,dx$

Varience, if $r = 2$;

$\mu_2 = \int_a^b (x - \text{mean})^2 f(x)\,dx$

$$= \int_a^b (x - \text{mean})^{(r)} dx$$

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$r = 2$

$$= \int_a^b$$

7 am

If $r = 3$

$$\mu_3 = \int_a^b (x - \text{mean})^3 f(x)\, dx$$

Eg A probability curve $y = f(x)$ has a range from 0 to ∞. If $f(x) = e^{-x}$, find the mean and varience and third moment about mean.

Solution

10

$$\text{mean} = \int_a^b x f(x)\, dx$$

11

$$= \int_0^\infty x e^{-x} dx$$

12 noon

from $\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3$

$$= -xe^{-x} - e^{-x} \Big|_0^\infty$$

1 pm

$$= -e^{-x}(x+1) \Big|_0^\infty$$

2

$$= -e^{-\infty}(\infty+1) - (-e^0(0+1))$$

$$= 0 - (-1(1))$$

3

$$= 1$$

$$\text{Varience } (\mu_2) = \int_a^b (x - \text{mean})^2 f(x)\, dx$$

4

$$= \int_0^\infty (x-1)^2 e^{-x} dx$$

5

from $\int u\,dv = uv - u'v_1 + u''v_2 + u'''v_3$

6

$$= (x-1)^2 e^{-x} - 2(x-1)e^{-x} + 2e^{-x} \Big|_0^\infty$$

7

$$= -(\infty-1)^2 e^{-\infty} - 2(\infty-1)e^{-\infty} - ((x-1)^2 e^{-0} - 2(0-1)$$

$$= 1 + 2 - 2$$

$$= 1$$

$\int uv\,dx = u\int v\,dx - \int(u'\int v\,dx)\,dx$

Variance $(\mu_2) = \int_a^b (x - mean)^2 f(x)\,dx$

8

$= \int_0^\infty (x-1)^2 e^{-x}\,dx$

9

then:

10 $u = (x-1)^2$, $dv = e^{-x}$

$u' = 2(x-1)$ $v = -e^{-x}$

11

$\int v\,du = uv - \int v\,du$

12 noon

$= -(x-1)^2 e^{-x} - 2\int (x-1)\,-e^{-x}\,dx$

1 pm

$= (x-1)^2 e^{-x} + 2\int (x-1)e^{-x}\,dx$

2

$u = x-1$, $dv = e^{-x}$

3 $u' = 1$ $v = -e^{-x}$

4 $= -(x-1)^2 e^{-x} + 2\left[\int (x-1)e^{-x} - \int -e^{x}\,dx\right]$

5 $= -(x-1)^2 e^{-x} + 2(x-1)e^{-x} + 2(e^{-x})$

6 $= -(x-1)^2 e^{-x} + 2(x-1)e^{-x} - 2e^{-x} \Big|_0^\infty$

7 $= 0 - ((-1)^2 + 2(-1)e^0 - 2e^0$

$= -(1 + 2 - 2)$

$= +1$

then:

3^{rd} moment about mean $(\mu_3) = \int_a^b (x - mean)^3 f(x)\,dx$

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$= \frac{2}{18}\left[\frac{x^2}{2}\right] + \frac{3x}{18}\Big/_2^4$

$\frac{x^2}{18} + \frac{3x}{18}$

$$I_3 = \int_0^{\infty} (x-1)^3 e^{-x}\,dx$$

$$= (x-1)^3(-e^{-x}) - 3\int (x-1)^2(-e^{-x})\,dx$$

$$= -(x-1)^3 e^{-x} + 3\int_0^{\infty} (x-1)^2 e^{-x}\,dx = 1$$

$$= -(x-1)^3 e^{-x}\Big/_0^{\infty} + 3(1)$$

$$= -(0 - ((-1)^3 e^0)) + 3$$

$$= -1 + 3$$

$$= \underline{\underline{2}}$$

Note :

$$\boxed{\int uv\,dx = u\int v\,dx - \int\left(u'\int v\,dx\right)}$$

Bernullies

$$\boxed{\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 + }$$

Eg: If the fxn defined as follow p.d.f ?

$$f(x) = \begin{cases} 0 & \text{if } x < 2 \\ \frac{3+2x}{18} & \text{if } 2 \leq x \leq 4 \\ 0 & \text{if } x > 4 \end{cases}$$

If so find the $P(2 \leq x \leq 3)$.

Solution

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

for $f(x) \geq 0$

$$f(x) = \frac{3+2x}{18}\Big/_2^4$$

$f(x) =$

7 am

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{2} f(x)\,dx + \int_{2}^{4} f(x) + \int_{4}^{\infty} f(x)dx$$

8

$$= 0 + \frac{3+2x}{18} + 0$$

9

$$= \int_2^4 \frac{3+2x}{18} = \frac{1}{8}\int_2^4 3+2x$$

10

11

$$= \frac{1}{18}\left[3x + x^2\right]\Big|_2^4$$

12 noon

$$= \frac{1}{18}\left[\left(3(4) + (4)^2\right) - \left(3(2) + (2)^2\right)\right]$$

1 pm

$$= \frac{1}{18}\left[28 - 10\right]$$

2

$$= \frac{1}{18} \times 18$$

3

$$= 1$$

$\therefore$ Its p.d.f bcs it satisfy $\int_{-\infty}^{\infty} f(x)\,dx = 1$

4

$$P(2 \le x \le 3) = \int_2^3 f(x)\,dx$$

5

$$= \int_2^3 \frac{3+2x}{18}\,dx$$

6

$$= \frac{1}{18}\left[3x + x^2\right]\Big|_2^3$$

7

$$= \frac{1}{18}\left[\left(3(3) + (3)^2 - \left(\left(3(2) + (2^2)\right)\right)\right)\right]$$

$$= \frac{1}{18}\left[18 - 10\right]$$

$$= \frac{8}{18} = \frac{4}{9}$$

Q) If a random variable "X" has the pdf

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & \text{if } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

find the mean and variance of X

Solution:

$$\text{Mean} = \int_a^b x f(x)\,dx = \int_{-1}^{1} x\left(\frac{x+1}{2}\right)dx$$

$$= \frac{1}{2}\int_{-1}^{1} x^2 + x\,dx$$

$$= \frac{1}{2}\left[\frac{1}{3}x^3 + \frac{1}{2}x^2\right]\Big|_{-1}^{1}$$

$$= \frac{1}{2}\left[\left(\left(\frac{1}{3}(1)^3 + \frac{1}{2}(1)^2\right) - \left(\frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2\right)\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{2} - \left(-\frac{1}{3}\right) + \left(\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{2+3}{6} + \frac{1}{3} - \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{5}{6} + \frac{2-3}{6}\right]$$

$$= \frac{1}{2}\left[\frac{5}{6} + -\frac{1}{6}\right]$$

$$= \frac{1}{2}\left[\frac{5-1}{6}\right]$$

$$= \frac{1}{2}\left[\frac{4}{6}\right]$$

Ans $= \frac{1}{3}$

7 Variance $(\mu_2) = \int_a^b (x - \text{mean})^2 f(x)\, dx$

8 $= \int_{-1}^{1} \left(x - \frac{1}{3}\right)^2 \left(\frac{x+1}{2}\right) dx$

9 $= \left(x-\frac{1}{2}\right)^2 \cdot \left(\frac{x+1}{2}\right) - 2\left(x - \frac{1}{3}\right)\left(\frac{x^2}{4} + \frac{1}{2}x\right)$

10 $UV = U'V_1 + U''V_2$

$U = \left(x - \frac{1}{3}\right)^2 \quad , \quad dv = \frac{x+1}{2}$

11

$U' = 2\left(x - \frac{1}{3}\right) \quad V = \frac{1}{2}\left(\frac{x^2}{2} + x\right) \quad , V_1 = \frac{1}{2}\left(\frac{1}{3}x^3 + \frac{x^2}{2}\right)$

12 noon

$= \frac{1}{2}\left(x - \frac{1}{3}\right)^2\left(\frac{x^2}{4} + \frac{x}{2}\right) - 2\left(x - \frac{1}{3}\right) \cdot \frac{1}{2}\left(\frac{1}{3}x^3 + \frac{x^2}{2}\right)\Bigg|_{-1}^{1}$

1 pm

$= \frac{1}{2}\int_{-1}^{1}\left(x^2 + \frac{1}{9} - \frac{2}{3}x\right)(x+1)\, dx$

2

$= \frac{1}{2}\int_{-1}^{1} x^3 + x^2 + \frac{x}{9} + \frac{1}{9} - \frac{2}{3}x^2 - \frac{2}{3}x\, dx$

3

$= \frac{1}{2}\left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2\times 9} + \frac{x}{9} - \frac{2}{3}\cdot\frac{x^3}{3} - \frac{2}{3}\cdot\frac{x^2}{2}\right]\Bigg|_{-1}^{1}$

4

5 $= \frac{1}{2}\left[\left(\frac{1}{4} + \frac{1}{3} + \frac{1}{18} + \frac{1}{9} - \frac{2}{9} - \frac{1}{3}\right) - \left(\frac{1}{4} - \frac{1}{3} + \frac{1}{18} - \frac{1}{9} + \frac{3}{9} - \frac{1}{3}\right)\right]$

6

$= \frac{1}{2}\left[\frac{2}{3} + \frac{2}{9} - \frac{4}{9}\right]$

7

$= \frac{1}{2}\left[\frac{2}{3} - \frac{2}{9}\right]$

$= \frac{2}{2}\left[\frac{3-1}{9}\right] = \frac{1}{2}\left[\frac{4}{9}\right] = \frac{2}{9}$.

Eg: If a continuous random variable 'X' is distributed over the interval [0,1] with pdf ax^2+bx, given whether a, b are constant. If the arithmetic mean of 'x' is 0.5, find the value of 'a' and 'b'

9 Solution:

$f(x) = ax^2+bx \quad [0,1]$

Mean = 0.5

then

$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_0^1 f(x)dx = 1$

12 noon $\int_0^1 ax^2+bx \, dx = 1$

1 pm $\left.\frac{ax^3}{3}+\frac{bx^2}{2}\right|_0^1 = 1$

2 $\frac{a(1)^3}{3}+\frac{b(1)^2}{2} - \left(a\left(\frac{0}{3}\right)^3 + b\left(\frac{0}{2}\right)^2\right) = 1$

3 $\frac{a}{3}+\frac{b}{2} - 0 = 1$

4 $\frac{a}{3}+\frac{b}{3} = 1$

5 $\frac{2a+3b}{6} = 1$

6 $2a+3b = 6 \quad \text{---} \ (1)$

Mean $= \int_0^1 x f(x)dx$

$0.5 = \int_0^1 x(ax^2+bx)dx = \cancel{0.5}$

$0.5 = \int_0^1 ax^3+bx^2 dx$

$0.5 = \left.\frac{ax^4}{4}+\frac{bx^3}{3}\right|_0^1$

12
5
6.0

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7 am $S = \left(\frac{a}{4} + \frac{b}{3}\right) - 0$

8 $S = \frac{a}{4} + \frac{b}{3}$

9 $S = \frac{3a + 4b}{12}$

10 $6 = 3a + 4b \quad ---(11)$

11 $3a + 4b = 6$
$2a + 8b = 6$ $\Big\} \times 2, \times 3$

12 noon

$6a + 8b = 12$

1 pm $6a + 9b = 18$

$-b = -6$

2 $\therefore b = 6$

then

3 $3a + 4(6) = 6$

$3a + 24 = 6$

4 $3a = 6 - 24$

$\frac{3a}{3} = \frac{-18}{3}$

5

$a = -6$

6

7 **MATHEMATICAL EXPECTATION (E(x))**

$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$ = Mean.

This is for continuous Random variable.

Mean $E(x) = \sum_x x f(x)$ (for discrete random variable

Variance (μ_2) $= E(x^2) - [E(x)]^2$

8 $$\left[\mu_2 = E(x^2) - (E(x))^2\right]$$

Eg: If 'x' is a random variable and a is a constant then $E[ax+b] = aE(x) + b$: show

10 $E(ax+b) = \int_{-\alpha}^{\alpha} x f(x)\, dx$, but $x = ax+$

11 $= \int_{-\alpha}^{\alpha} (ax+b) f(x)\, dx$

12 noon $= \int_{-\alpha}^{\alpha} ax f(x)\, dx + b\int_{-\alpha}^{\alpha} f(x)\, dx$

1 pm $= a\int_{-\alpha}^{\alpha} x f(x)\, dx + b\int_{-\alpha}^{\alpha} f(x)\, dx = 1$

2 $E(ax+b) = a\,E(x) + b$ shown

3

4 # Moment Generating Fxn (mgf)

5

6 $$\boxed{M_x(t) = E(e^{tx})}$$

7 $M_x(t) = \int_{-\alpha}^{\alpha} e^{tx} f(x)\, dx$: for continuous random variable

$M_x(t) = \sum_{x=1}^{\infty} e^{tx} f(x)$ for descrete random variable

Eg: Find the mgf of the random variable with the probability Law $P(X=x) = q^{x-1} p$, $x = 1, 2, 3 \dots$

8

Solution

9 This is discrete random variable

10 $M_X(t) = \sum_{x=1}^{\infty} e^{t} f(x)$

11 $= \sum_{x=1}^{\infty} e^{tx} (q^{x-1} \cdot p)$

12 noon $= \sum_{x=1}^{\infty} e^{tx} \cdot q^{x} \cdot q^{-1} \cdot p$, $q^{-1} = \frac{1}{q}$

1 pm

$= \frac{p}{q} \sum_{x=1}^{\infty} e^{tx} \cdot q^{x}$

2

$= \frac{p}{q} \sum_{x=1}^{\infty} (e^{t} \cdot q)^{x}$ for $x = 1, \dots$

3

4

$= \frac{p}{q} \left(e^{t}q + (e^{t}q)^{2} + (e^{t}q)^{3} + (e^{t}q)^{4} \dots \right.$

5

$= \frac{p}{q} e^{t}q \left[1 + e^{t}q + (e^{t}q)^{2} + (e^{t}q)^{3} \dots \right]$ $(1-x)^{-1}$

6

$= \frac{p}{q} e^{t}q \left(1 - e^{t}q\right)$

7

$= pe^{t}(1 - e^{t}q)^{-1}$

$M_X(t) = \dfrac{pe^{t}}{1 - e^{t}q}$

17 Saturday February 201

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Formular of Mgf in mean & Variance

7 am

Mean = $M_x'(0)$

8

Variance = $M_x''(0) - (M_x'(0))^2$

9 $M_x''(0) - (\text{mean})^2$

from the qn

10

$M_x'(t) = \frac{d}{dt}(M_x(t))$

11

$= \frac{d}{dt}\left(\frac{pe^t}{1-e^tq}\right)$, $\frac{d}{du}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$

12 noon

1 pm $= \frac{(1-e^tq)\cdot pe^t - pe^t(-e^tq)}{(1-e^tq)^2}$

2

$= \frac{pe^t - \cancel{pqe^{2t}} + \cancel{pqe^{2t}}}{(1-e^tq)^2}$

3

4

$M_x'(t) = \frac{pe^t}{(1-e^tq)^2}$

5

then

6

Mean $= M_x'(0) = \frac{pe^0}{(1-e^0q)^2} = \frac{p}{(1-q)^2}$

7

$$\left\{\begin{aligned} M_x(t) &= \frac{pe^t}{1-e^tq} \\ M_x'(t) &= \frac{pe^t}{(1-e^tq)^2} \end{aligned}\right\}$$

7 am UNIT 2 : DISTRIBUTION THEORETICAL

* Discrete Random Variable
8 - Binomial distribution
- Poisson distribution
9 - Geometric distribution

* Continuous Random Variable
- Uniform distribution
11 - Normal distribution
- Exponential distribution

12 noon

I. Binomial distribution

1 pm

$$\boxed{p(x) = {}^nC_x \cdot p^x \cdot q^{n-x}}$$

2

not $p + q = 1$

3

mean $= E(x) = \sum_{x=0}^{n} x\, p(x)$

4

but

5 $p(x) = {}^nC_x\, p^x q^{n-x}$

6 $p(x) = \sum_{x=0}^{n} {}^nC_x\, p^x q^{n-x}$

7

mean $= np$

$\boxed{\text{Varience } (\mu_2) = npq}$

standard deviation $= \sqrt{\text{varience}}$

standard deviation $= \sqrt{npq}$

Example :
Find the binomial distribution for ~~is~~ the mean is 4 and varience is 3
Solution

mean $= np = 4$

Varience $= npq = 3$

$\frac{npq}{np} = \frac{3}{np}$

$q = \frac{3}{np}, \quad np = 4$

$q = \frac{3}{4}$

but

$p + q = 1$

$p = 1 - \frac{3}{4}$

$p = \frac{1}{4}$

and ,

$np = 4$

$n\left(\frac{1}{4}\right) \times 4 = 4 \times 4$

$n = 16$

Recall :

$p(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$p(x) = {}^{16}C_{x}\, p^{x} q^{16-x}$

2) With the usual notation find p for a binomial random variable 'x' if n = 6 and if

$9p(x=4) = p(x=2)$.

$p(x) = {}^nC_x\, p^x q^{n-x}$

condition :

$9p(X=4) = p(X=2)$

$9p(4) = p(2)$

$p(x) = {}^6C_x\, p^x q^{6-x}$

$9({}^6C_4\, p^4 q^{6-4}) = {}^6C_2\, p^2 q^4$

$9({}^6C_4\, p^2) = {}^6C_2\, q^2$

$9(15p^2) = 15q^2$, $p+q = 1$

$9p^2 = q^2$ $p = 1-q$

$q = 1-p$

$9p^2 = (1-p)^2$

$9p^2 = 1-2p+p^2$

$8p^2 + 2p - 1 = 0$

$p = -\frac{1}{2}$ or $p = \frac{1}{4}$, no -ve probabil

$\therefore$ The value of $p = \frac{1}{4}$

OR

$9p^2 = q^2$

$(3p)^2 = q^2$

$9p^2 = q^2$

$3p = 1-p$

$3p + p = 1$

$\therefore P = \frac{1}{4}$.

Qn) 10 coin are thrown simultaneously. Find the probability of getting atleast 7 head.

Solutions.

$n = 10, \quad p = \frac{1}{2}, \quad p+q = 1, \quad q = \frac{1}{2}$.

from

$$P(x) = {}^nC_x \, p^x \, q^{n-x}$$

$$P(x \geq 7) = ?$$

$$P(x \geq 7) = P(7) + P(8) + P(9) + P(10)$$

$$P(7) = {}^{10}C_7 \, p^7 \, q^{10-7}, \quad p = \frac{1}{2}, \quad q = \frac{1}{2}$$

$$P(7) = {}^{10}C_7 \, p^7 \, q^3$$

$$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3$$

$$= \frac{15}{128}.$$

$$P(8) = {}^{10}C_8 \left(\frac{1}{2}\right)^8 q^{10-8}$$

$$= {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2$$

$$P(8) = \frac{45}{1024}$$

$$P(9) = {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{10-9}$$

$$P(9) = \frac{5}{512}$$

$$P(10) = {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{10-10}$$

$$P(10) = \frac{1}{1024}$$

$P(x \geq 7) = \frac{15}{128} + \frac{45}{1024} + \frac{5}{512} + \frac{1}{1024}$

$= \frac{11}{64}$

$= 0.171875$

II : GEOMETRIC DISTRIBUTION :

$p(x) = q^x p, \quad x = 0, 1, 2, \quad (0 < p \leq 1)$

Let

$x = 0$

$p(x) = p, qp, q^2 p, q^3 p, \ldots$

Mean $E(x) = \sum_{x=0}^{n} x \, p(x)$

$= \sum_{x=0}^{n} x (q^x p)$

$= 0 + q^1 p + 2q^2 p + 3q^3 p + \ldots$

from $(1-x)^{-2} = 1 + 2x + 3x^3 + \ldots$

$= pq(1-q)^{-2}$

$= pqp^{-2}$

$= pq \cdot \frac{1}{p^2} = \frac{q}{p}$

$\left[\text{Mean} = q/p \right]$

Varience $M_x = p/q^2$

Standard distribution / deviation $= \sqrt{\text{varience}}$

$= \sqrt{q/p^2}$

$= \sqrt{q}/p$

MOMENT GENERATING FUNCTION (M.g.f):

$M_x(t) = E(e^{tx})$

$= \sum_{x=0}^{n} e^{tx} P(x)$

$= \sum_{x=0}^{n} e^{tx} q^x p$

$= \sum_{x=0}^{n} e^{tx} (q^x p)$

$= p + e^t qp + e^{2t}(q^2 p) + \cdots$

$= p[1 + (qe^t) + (qe^t)^2 + \cdots]$

from

$(1-x)^{-1} = 1 + x + x^2$

$= p(\cancel{1} - qe^t)^{-1}$

$M_x(t) = p/(1 - qe^t)$

7 am Mean $= M_x'(0)$

8 Varience $= M''(0) - (\text{mean})^2$

9 Problems:

A die is cast until 6 appear. What is the probability that it must be cast more than five time

Solution:

11 $p(x>5)$

$p(x) = q^x p$

12 noon

$p(x) = \left(\frac{5}{6}\right)^x \cdot \frac{1}{6}$; $p = \frac{1}{6}$. $q = 1 - \frac{1}{6} = \frac{5}{6}$.

1 pm

$p(x>5) = P(6) + P(7) + P(8) + \ldots$

2 or

$1 - P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

3

$P(0) = \frac{1}{6}$, $P(1) = \frac{5}{6} \cdot \frac{1}{6}$, $P(2) = \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$, P..

4

$P(3) = \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6}$, $P(4) = \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}$, $P(5) = \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6}$

5

$P(x>5) = 1 - \frac{1}{6}\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^4 + \left(\frac{5}{6}\right)^5\right)$

6

$P(x>5) = \frac{15625}{46656}$

7

$P(x>5) = 0.3349$

OR

$S = \frac{a(1-r^n)}{1-r}$

$S = \frac{1\left(1 - \left(\frac{5}{6}\right)^6\right)}{}$ $= 0.3349$

7 am III - 3 POISON DISTRIBUTION.

8 $P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$, $x = 0, 1, 2 \ldots$

9

Mean $= E(x) = \sum_{x=0}^{n} x\, p(x)$

10

11

12 noon

Eg: If X is a poisson variate $p(x=2) =$

2 $p(x=2) = 9p(x=4) + 90p(x=6)$

Find mean and varience of λ.

3 from:

$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$.

4

$P(X = x) = P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$.

5

$P(x = 2) = 9p(x = 4) + 90p(x = 6)$

6 $P(2) = 9p(4) + 90p(6)$

7 $\frac{e^{-\lambda}\lambda^{2}}{2!} = \frac{9e^{-\lambda}\lambda^{4}}{4!} + \frac{90e^{-\lambda}\lambda^{6}}{6!}$

$e^{-\lambda}\left(\frac{\lambda^{2}}{2!}\right) = e^{-\lambda}\left(\frac{9\lambda^{4}}{4!}\right) + e^{-\lambda}\left(\frac{90\lambda^{6}}{6!}\right)$.

$\frac{\lambda^{2}}{2!} = \frac{9\lambda^{4}}{4!} + \frac{90\lambda^{6}}{6!}$

7 am $\frac{1}{2}\cancel{\lambda^2} = \cancel{\lambda^2}\left(\frac{9\lambda^2}{4!} + \frac{90\lambda^4}{6!}\right)$.

8 $\frac{1}{2!} = \frac{3}{8}\lambda^2 + \frac{1}{8}\lambda^4$

9 $8 = 6\lambda^2 + 2\lambda^4$

Let $\lambda^2 = a$

10 $8 = 6a + 2a^2$

$2a^2 + 6a - 8 = 0$

11 $a^2 + 3a - 4 = 0$

$a = 1$, $\lambda^2 = a$

12 noon $\lambda^2 = 1$,

$\lambda = 1$

1 pm

Mean $= \lambda = 1$

2 Varience $= \lambda = 1$

3 S.D $= \sqrt{\text{varience}} = \sqrt{1} = 1$

4

Q2) If X is a poison varible such that $P(X=1) = 3/10$ and $P(X=2) = 1/5$, find $P(X=0)$ and $P(X=3)$

Solution.

6 $P(X=x) = P(X) = \frac{e^{-\lambda}\lambda^x}{x!}$

7 $P(1) = 3/10$

$P(1) = \frac{e^{-\lambda}\lambda^1}{1!} = \frac{3}{10}$ ... (1)

$P(2) = \frac{e^{-\lambda}\lambda^2}{2!} = \frac{1}{5}$ ... 2

Divide eqn 2 by eqn 1

$$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{1}{5}$$

$$\frac{e^{-\lambda}\lambda}{1!} = \frac{3}{10}$$

$$\frac{e^{-\lambda}\lambda^2}{2!} \times \frac{1!}{e^{-\lambda}\lambda} = \frac{1}{5} \times \frac{10}{3}$$

$$\frac{\lambda}{2} = \frac{2}{3}$$

$$3\lambda = 4$$

$$\lambda = \frac{4}{3}$$

$$P(x=0) = P0 = \frac{e^{-4/3}\lambda^0}{0!}$$

$$P(x=0) = e^{-4/3}$$

$$P(x=3) = P(3) = \frac{e^{-\lambda}\lambda^3}{3!}$$

$$P(x=3) = \frac{e^{-4/3}\left(\frac{4}{3}\right)^3}{3!}$$

$$P(x=3) = \frac{e^{-4/3}\left(\frac{4}{3}\right)^3}{6}$$

Eg: A car hire from has two cars w its hires out day by day the number of demands for a car on each day is distributed as poisson variable with mean is ~~0.5~~ given 1.5. Calculate the proportion of days on which

i) Neither car is used (both of them are not used $P(0)$).
ii) Some detemand is refused (same as $P>2$).

Solution.

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

where mean $(\lambda) = 1.5$

$$P(x) = \frac{e^{-1.5}(1.5)^{x}}{x!}$$

$$P(\text{of } x \text{ cars used}) = \frac{e^{-1.5}(1.5)^{x}}{x!}$$

$P(0) = ?$

$$P(0) = \frac{e^{-1.5}(1.5)^{0}}{0!}$$

$$= e^{-1.5}$$

$$P(0) = 0.2231$$

ii) Some detemand is refused, $P(x>2$ bcs we have two cars

$$P(x>2) = P(3) + P(4) + \cdots$$

$$= 1 - \left(P(0) + P(1) + P(2)\right)$$

where

$$P(0) = 0.2231$$

$$P(1) = \frac{e^{-1.5}(1.5)^{1}}{1!} = 0.3346$$

$\text{Mean} = \frac{\sum f(x)}{N}$

$\frac{185}{15}$
200

$P(2) = \frac{e^{-1.5}(1.5)^2}{2!}$

$P(2) = 0.2510$

then

$P(x>2) = 1 - (0.2231 +$
$= 0.191$

Note:

P(some demand is refused) = P(the no of demands to be more than 2)
$= P(x>2)$

Eg: Fit a poisson distribution to the following data and calculate the theoretical frequency

Death	0	1	2	3	4
frequency	122	60	15	2	1

Solution:

Death (x)	Frequency (f)	fx	theoretical frequency
0	122	0	f(0) = 121.306 ≈ 121
1	60	60	f(1) = 60.65 ≈ 61
2	15	30	f(2) = 15
3	2	6	f(3) = 3
4	1	4	f(4) = 0
	N = 200	$\sum fx = 100$	

$\text{Mean} = \frac{\sum fx}{N} = \frac{100}{200} = 0.5$

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formular

Theoretical distribution is $\left[N P(x) \right]$

$$f(x) = \frac{N e^{-\lambda} \lambda^{x}}{x!}$$

$$\left[f(x) = \frac{200\, e^{-0.5} (0.5)^{x}}{x!} \right]$$

then

$$f(0) = \frac{200\, e^{-0.5} (0.5)^{0}}{0!} = 121$$

$$f(1) = \frac{200\, e^{-0.5} (0.5)^{1}}{1!} = 61$$

$$f(2) = \frac{200\, e^{-0.5} (0.5)^{2}}{2!} = 15$$

$$f(3) = \frac{200\, e^{-0.5} (0.5)^{3}}{3!} = 3$$

$$f(4) = \frac{200\, e^{-0.5} (0.5)^{4}}{4!} = 0$$

CONTINUALS DISTRIBUTION.

i) Uniform distribution
ii) Normal distribution
iii) Exponential distribution. no change

1.) UNIFORM DISTRIBUTION:

A random variable 'X' is said to have a continuous uniform distribution if pdf is given by $p(x)$

$$p(x) = \begin{cases} K, & a < x < b \quad x \in (a,b) \\ 0, & \text{otherwise} \end{cases}$$

$$\text{pdf} = \int^{b} P(x)\,dx = 1 = \int_{a}^{b} K\,dx$$

7 am $= k\int_a^b dx$

8 $= k\, x\Big|_a^b$

9 $1 = k(b-a)$

10 $k = \frac{1}{b-a}$

$$P(x) = \begin{cases} b-a, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

12 noon

then:

1 pm

$$\text{Mean} = \int_a^b x\,P(x)\,dx = \int_a^b x\,\frac{1}{b-a}\,dx$$

2

3 $= \frac{1}{b-a}\int_a^b x\,dx$

4 $= \frac{1}{b-a}\,\frac{x^2}{2}\Big|_a^b$

5

6 $= \frac{1}{2(b-a)}(b^2-a^2)$

7 $= \frac{1}{2\cancel{(b-a)}}\cancel{(b-a)}(b+a)$

$$\text{Mean} = \frac{1}{2}(b+a)$$

$$\boxed{\text{Mean} = \frac{b+a}{2}}$$

$(b-a)(b+a)^2$

b: Expectation:

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$$Varience(U_2) = E(x^2) - (E(x))^2$$

$$Varience = E(x^2) - (Mean)^2$$

but

$$E(x^2) = \int_a^b x^2 p(x) dx$$

$$= \int_a^b x^2 \frac{1}{(b-a)} dx$$

$$= \frac{1}{b-a} \cdot \frac{1}{3} x^3 \Big/_a^b$$

$$= \frac{1}{3(b-a)} (b^3 - a^3)$$

$$= \frac{1}{\cancel{3(b-a)}} (\cancel{(b-a)}) (a^2 + ab + b^2)$$

$$E(x^2) = \frac{1}{3} (a^2 + ab + b^2)$$

$$Varience(U_2) = \frac{1}{3}(a^2+ab+b^2) - (Mean)^2$$

$$= \frac{1}{3}(a^2+ab+b^2) - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{1}{3}(a^2+ab+b^2) - \frac{1}{4}(a^2+ab+b^2)$$

$$= \frac{4(a^2+ab+b^2) - 3(a^2+ab+b^2)}{12}$$

$$= \frac{1}{12}(4a^2 + 4ab + 4b^2 - 3a^2 - 3ab + 3b^2$$

Varience $\mu_2 = \frac{1}{12}(a^2 + 2ab + b^2)$

$= \frac{1}{12}(a-b)^2$

$$\boxed{\text{Varience } (\mu_2) = \frac{1}{12}(b-a)^2}$$

Eg: If 'x' is uniformly distributed random variable with mean 1 and varience $4/3$ find $P(x<0)$

Solution

mean $= 1$

$\frac{a+b}{2} = 1$

$a + b = 2$ --- (i)

Varience $= 4/3$

$\frac{1}{12}(b-a)^2 = 4/3$

$(b-a)^2 = 16$

$a^2 + 2ab + b^2 = 16$ --- (ii)

$\sqrt{(b-a)^2} = \sqrt{16}^2$

$b - a = 4$ --- (iii)

$$\begin{cases} a+b = 2 \\ b-a = 4 \end{cases} \Rightarrow \begin{cases} a+b = 2 \\ -a+b = 4 \end{cases}$$

$2a = -2$

7 am $a = -1$

$a + b = 2$

8 $-1 + b = 2$

$b = 3$, $P(-1 < x < 3)$

9

then

10 $P(x < 0) = \int_{-1}^{0} P(x)\,dx$ same as

11 $P(x < a) = \int_{-\infty}^{a} P(x)\,dx$

12 noon $= \int_{-1}^{0} \frac{1}{b-a}\,dx$

1 pm $P(x < 0) = \frac{1}{b-a} \int_{-1}^{0} dx$

2

3 $= \frac{1}{b-a} \cancel{x^2} \Big/ \cancel{\delta}\,\Big|_{-1}^{0}$

4 $P(x < 0) = \frac{1}{3-(-1)} \times x\Big|_{-1}^{0}$

5 $= \frac{1}{4}(0 - -1)$

6 $P(x < 0) = \frac{1}{4}$

7

MOMENT GENERATING FXN.

$M_x(t) = \int_{a}^{b} e^{tx}\, p(x)\, dx$

$M_x(t) = E(e^{tx})$.

Eg: Show that for the uniform distribution $f(x) = \frac{1}{2a}$, $-a < x < a$, mgf is $\frac{\sinh at}{at}$.

Solution:

M.g.f $M_x(t) = E(e^{tx})$

$$= \int_{-a}^{a} e^{tx} f(x)\,dx$$

$$= \int_{-a}^{a} e^{tx} \cdot \frac{1}{2a}\,dx$$

$$= \frac{1}{2a}\int_{-a}^{a} e^{tx}\,dx$$

$$= \frac{1}{2a}\left[\frac{1}{t}e^{tx}\right]_{-a}^{a}$$

$$= \frac{1}{2at}\left[e^{at} - e^{-at}\right]$$

$$= \frac{1}{at}\left[\frac{e^{at} - e^{-at}}{2}\right] \quad \sinh at$$

$$= \frac{\sinh at}{at}$$

Mgf $M_x(t) = \frac{\sinh at}{at}$ shown.

Eg If "X" is uniformly distributed over $(-\alpha, \alpha)$, Find α so that

8 i) $P(x>1) = \frac{1}{3}$

9 ii) $P(|x|<1) \Rightarrow P(|x|>1)$

10 Solution

$P(x) = \frac{1}{b-a}$ if $a<x<b \Rightarrow -\alpha<x<b$

11

12 noon $P(x) = \frac{1}{\alpha-(-\alpha)}$

1 pm $P(x) = \frac{1}{2\alpha}$

2 then

i) $P(x>1) = \frac{1}{3}$

3 $\int_1^{\alpha} P(x)\,dx = \frac{1}{3}$

4

5 $\int_1^{\alpha} \frac{1}{2\alpha}\,dx = \frac{1}{3}$

6 $\frac{1}{2\alpha}\int_1^{\alpha} dx = \frac{1}{3}$

7 $\frac{1}{2\alpha}\, x\Big|_1^{\alpha} = \frac{1}{3}$

$\frac{1}{2\alpha}(\alpha-1) = \frac{1}{3}$

$3(\alpha-1) = 2\alpha$

$3\alpha - 3 = 2\alpha$

$3\alpha - 2\alpha = 3$

$\alpha = 3$

ii) $P(|x|<1) = P(|x|>1)$

But

$P(|x|<1) = -1<x<1$

and

$P(|x|>1) = 1 - P(x<1)$

then:

$P(-1<x<1) = 1 - P(-1<x<1)$

$2P(-1<x<1) = 1$

$2\int_{-1}^{1} P(x)\,dx = 1$

$2\int_{-1}^{1} \frac{1}{2\alpha}\,dx = 1$

$\frac{1}{\alpha}\int_{-1}^{1} dx = 1$

$$\frac{1}{\alpha}\, x \Big|_{-1}^{1} = 1$$

$$\frac{1}{\alpha}(1-(-1)) = 1$$

$$\frac{1}{\alpha}\, 2 = 1$$

$$\alpha = 2 .$$

NUMERICAL DISTRIBUTION:

A random variable "X" is said to follow numerical distribution with mean (μ) and varience δ^2.
⇒ Its density density function is given by the probability law:

$$P(x) = f(x) = \frac{1}{\delta\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\delta^2}}$$

where $-\infty < x < \infty$

for $\delta > 0$

$-\infty < \mu < \infty$

Recall:

$$\int_{-\alpha}^{\alpha} F(x)\, dx = 1$$

$$= \int_{-\alpha}^{\alpha} \frac{1}{\delta\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\delta^2}}\, dx$$

$$= \frac{1}{\delta\sqrt{2\pi}} \int_{-\alpha}^{\alpha} e^{-\frac{(x-\mu)^2}{2\delta}}\, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} dx$$

Assume

$$Z = \frac{x-\mu}{\sigma}$$

$$Z\sigma = x - \mu$$

$$\sigma dz = dx$$

Changing the limit:

$x = -\infty \Rightarrow Z = -\infty$

$x = \infty \Rightarrow Z = \infty$

then:

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}Z^2} \cancel{\sigma} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}Z^2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{Z^2}{2}} dz$$

NOTE:

1) $f(x)$ is an even function if $f(-x) = f(x)$

2) $f(x)$ is odd function if $f(-x) = -f(x)$

$$\boxed{\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{if } f(x) \text{ is even} \\ 0 & \text{if } f(x) \text{ is odd} \end{cases}}$$

7 am $x = -\infty \Rightarrow z = -\infty$

$x = \infty \Rightarrow z = \infty$

8 $= \frac{1}{8\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} (8\, dz)$

9 $= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz$

10

11 $= \frac{1}{\sqrt{2\pi}} \cdot 2 \int_{0}^{\infty} e^{-\frac{z^2}{2}} dz$

12 noon

1 pm $f(x)$ is an even fxn if $f(-x) = f(x)$

$f(x)$ is an odd fxn if $f(-x) = -f(x)$

2 $\therefore$

3 $$\int_{-a}^{a} f(x)\, dx = \begin{cases} 2\int_{0}^{a} f(x) & \text{if } f(x) \text{ is even} \\ 0 & \text{if } f(x) \text{ is odd.} \end{cases}$$

4

5

6 $= \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{z^2}{2}} dz$

7 $= \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\left(\frac{z}{\sqrt{2}}\right)^2} dz$

Assume $u = \frac{z}{\sqrt{2}}$ $\quad dz = \sqrt{2}\, du$

$du = \frac{1}{\sqrt{2}} dz$

when

$z = 0,\ u = 0$

$z = \infty,\ u = \infty$

$$= \frac{2}{\sqrt{2\pi}} \int_0^\infty e^{-u^2} \sqrt{2}\, du$$

$$= \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-u^2} du \quad \text{but } \int_0^\infty e^{-u^2} = \frac{\sqrt{\pi}}{2}$$

$$= \frac{2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2}$$

$$= 1 \quad \text{ID is probability density fxn}$$

For Funny

$$\int_0^\infty e^{-u^2} du = \frac{\sqrt{\pi}}{2}$$

Assume

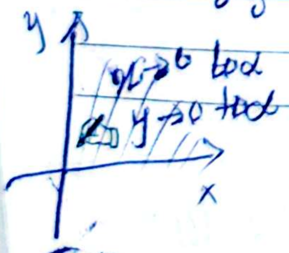
$$I = \int_0^\infty e^{-x^2} dx \qquad I = ?$$

$$I = \int_0^\infty e^{-y^2} dy$$

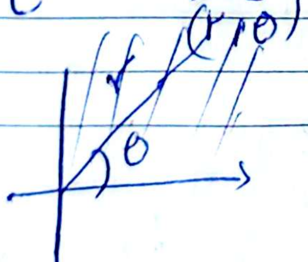
$$I^2 = \int_0^\infty \int_0^\infty e^{-x^2} \cdot e^{-y^2} dx dy$$

$$= \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$$

(r, θ)



Cartisian ordinate system



Polar coordinate system

$r = 0$ to ∞

$\theta = 0$ to $\pi/2$

cartisian to polar coordin

$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2\cos^2 + r^2\sin^2 = r$

$dxdy = \cancel{r} dr d\theta$

$dxdy = |J|\, dr d\theta$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial}{\partial r}(r\cos\theta) & \frac{\partial}{\partial \theta}(r\cos\theta \\ \frac{\partial}{\partial r}(r\sin\theta) & \frac{\partial}{\partial \theta} r\sin \end{vmatrix}$$

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7 am
$$= \begin{vmatrix} \cos\theta(1) & r(-\sin\theta) \\ \sin\theta(1) & r(\cos\theta) \end{vmatrix}$$

8
$$= r\cos^2\theta - (-r\sin^2\theta)$$

9
$$= r\cos^2\theta + r\sin^2\theta$$
$$= r(\cos^2\theta + \sin^2\theta)$$

10
$$= r(1) = r$$

11
$$= \int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty} e^{-r^2}\, r\,dr\,d\theta \quad ; \quad r^2 = t, \quad 2r\,dr = dt$$

12 noon

1 pm
$$= \int_0^{\pi/2}\int_0^{\infty} e^{-t}\,\frac{dt}{2}\,d\theta$$

2
$$= \int_0^{\pi/2}\left[-\frac{1}{2}e^{-t}\right]_0^{\infty}$$

3
$$= -\frac{1}{2}\int_0^{\pi/2}(\underbrace{e^{-\infty}}_{0} - \underbrace{e^{-0}}_{1=1})\,d\theta$$

4

5
$$= -\frac{1}{2}\int_0^{\pi/2}(-1)\,d\theta$$

6
$$= \frac{1}{2}\int_0^{\pi/2} d\theta$$

7
$$= \frac{1}{2}(\theta)\Big|_0^{\pi/2}$$

$$= \frac{1}{2}\left(\frac{\pi}{2} - 0\right)$$

$$= \frac{\pi}{4}$$

But
$$\sqrt{I^2} = \sqrt{\frac{\pi}{4}}$$
$$I = \sqrt{\frac{\pi}{2}} \qquad \text{Proven}$$

$$f(x) = \frac{1}{\delta\sqrt{2\pi}}\left(e^{-\frac{(x-\mu)^2}{2\delta^2}}\right)$$ - Normal distribution

$$\int_0^{\infty} f(x)\,dx = 1$$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} P(x)\,dx = \int_{x_1}^{x_2} \frac{1}{\delta\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\delta^2}}\,dx$$

$$= \frac{1}{\delta\sqrt{2\pi}}\int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\delta}\right)^2}\,dx$$

put $Z = \frac{x-\mu}{\delta} \Rightarrow dx = \delta\,dz$

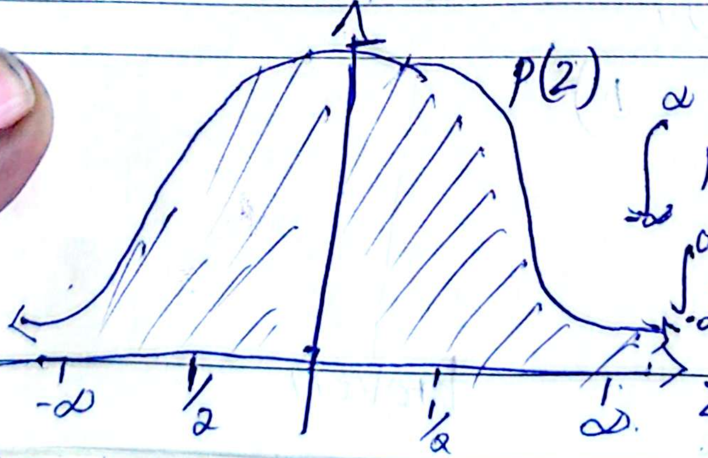
$\delta Z = x - \mu$

$$= \frac{1}{\delta\sqrt{2\pi}}\int_{x_1}^{x_2} e^{-\frac{z^2}{2}}\,\delta\,dz$$

$P(x_1 < x < x_2) =$ at $x = x_1 \Rightarrow Z = Z_1$

$x = x_2 \Rightarrow Z = Z_2$

$$\boxed{P(x_1 < x < x_2) = \frac{1}{\sqrt{2\pi}}\int_{Z_1}^{Z_2} e^{-\frac{z^2}{2}}\,dz}$$



$$\int_{-\infty}^{\infty} P(z)\,dz = 1$$

$$\int_{-\infty}^{0} P(z)\,dz + \int_0^{\infty} P(z)\,dz = 1$$

Varience (δ^2)
S.D (δ)

(Qn) If 'X' is normal distribution and mean of X is 12 and the S.D is 4 Find out the probability of the following
i) $x \geq 20$ $\quad P(x \geq 20)$
ii) $x \leq 20$ $\quad P(x \leq 20)$
iii) $0 \leq x \leq 12$ $\quad P(0 \leq x \leq 12)$

Solution :
Mean $(\mu) = 12$
S.D $(\delta) = 4$

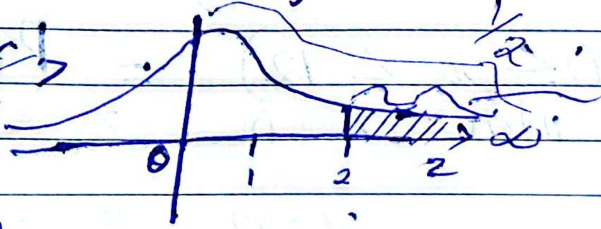
from

$$Z = \frac{x - \mu}{\delta}, \quad x = 20$$

$$Z = \frac{20 - 12}{4} = \underline{\underline{2}}$$

i) $P(x \geq 20) = P(Z \geq 2)$; when $x = 20$, $z = 2$

$P(Z \geq 2) =$ ~~P(>~~



$$= \int_{2}^{\infty} P(z)\,dz$$

$$= \frac{1}{2} - P(0 \leq Z \leq 2)$$

$$= 0.5 - 0.4772$$

$$P(Z \geq 2) = 0.0228$$

ii) 7 am $P(x \leq 20)$

$$Z = \frac{x - \mu}{8}$$

8

$$= \frac{20 - 12}{4} = 2$$

9

$P(x \leq 20) = P(Z \leq 2)$

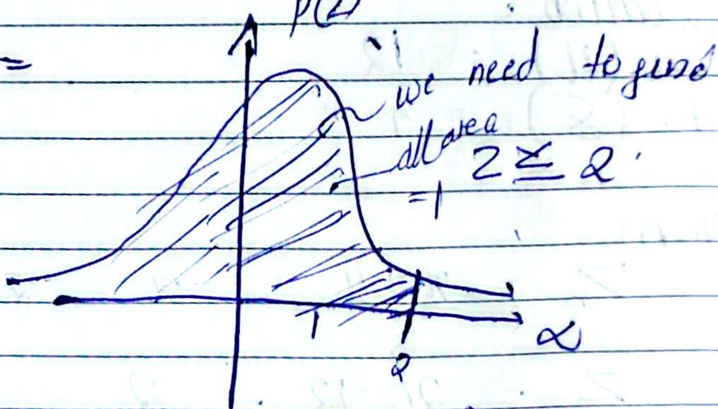
10

$P(Z \leq 2) =$

11

12 noon

1 pm



$P(Z \leq 2) = 1 - P(Z \geq 2)$

$= 1 - 0.0228$

3 $P(Z \leq 2) = 0.9772$

iii) 4 $P(0 \leq x \leq 12) = P(-3 \leq Z \leq 0)$

when $x = 0$, $Z = \frac{0 - 12}{4} = -3$

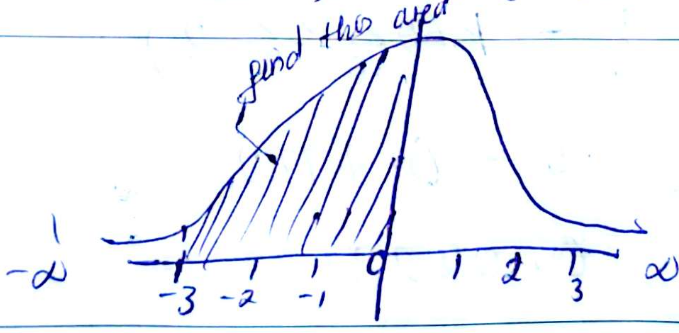
5

$x = 12$ $Z = \frac{12 - 12}{4} = 0$

6

then

7 $P(-3 \leq Z \leq 0) = P(0 \leq Z \leq 3)$ same



7am $P(-3 \le Z \le 0) = P(0 \le Z \le 3)$
$= 0.4987$

8

9 THE EXPONENTIAL DISTRIBUTION:

pdf

10 $p(x) = f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$

11

mgf

12 noon $M_x t = E(e^{tx})$

1 pm $= \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$

2 $= \int_{-\infty}^{\infty} e^{tx} (\lambda e^{-\lambda x})\, dx$

3 $= \lambda \int_{-\infty}^{\infty} e^{tx} e^{-\lambda x}\, dx$

4 $= \lambda \int_{0}^{\infty} e^{(t-\lambda)x}\, dx$

5 $= \lambda \dfrac{e^{(t-\lambda)}}{t-\lambda} \Big|_{0}^{\infty}$

6 $= \dfrac{\lambda}{t-\lambda} (e^{\infty} - e^{0})$

7 $= 0 - \dfrac{\lambda}{t-\lambda}$

$$\boxed{M_x(t) = \frac{\lambda}{t-\lambda}}$$

then

Mean $= E(x) = 1/\lambda$

Varience $(\delta^2) = E(x^2) - (E(x))^2$

$= 1/\lambda^2$

Eg: The mileage in car owners get with certain kind of radial tyre is RV having an exponential distribution with mean 4000 km. Find the propabilities that one of these tires will last

i) at least 2000 km $P(x > 2000)$

ii) at most 3000 km $P(x < 3000)$

Solution

$$\text{Mean} = 1/\lambda = 4000$$

$$\lambda = 1/4000 = 0.00025$$

$$\text{pdf } P(x) = f(x) = \frac{1}{4000} e^{-\frac{1}{4000}x}$$

i) $$P(x > 2000) = \int_{2000}^{\infty} \frac{1}{4000} e^{-\frac{1}{4000}x} dx$$

$$= \frac{1}{4000} \int_{2000}^{\infty} e^{-\frac{1}{4000}x} dx$$

$$= \frac{1}{4000} \cdot \frac{-4000}{1} e^{-\frac{1}{4000}x} \Big|_{2000}^{\infty}$$

$$= -e^{-\frac{1}{4000}x} \Big|_{2000}^{\infty}$$

7 am $= 0 - \left(e^{-\frac{1}{4000}\times 2000}\right)$

8 $= e^{-\frac{1}{2000}}$

$P(x>2000) = \dfrac{1}{\sqrt{e}}$

9

iii) Atmost 3000 km $P(x < 3000)$

$P(x<3000) = \dfrac{1}{4000}\int_0^{3000} e^{-\frac{1}{4000}x}\,dx$

12 noon

$= \dfrac{1}{4000} \cdot -4000\, e^{-\frac{1}{4000}x}\Big/_0^{3000}$

1 pm

$= -e^{-\frac{1}{4000}x}\Big/_0^{3000}$

2

$= -\left(e^{-\frac{1}{4000}\times 3000} - e^{-\frac{1}{4000}\times 0}\right)$

3

4 $= 1 - e^{-\frac{3}{4}}$

$P(x \le 3000) = 0.5276$

5

6 Eg: Amileage which car owners get with certain kind of radial tyre is RV having as

7 exponential distribution with mean

$\{|x-a| < b \Rightarrow -b < x-a < b$

7 am

CHEBYSHEVS INEQUALITY:

If X is a random variable with mean μ and varience δ^2, then for any positive number k, we have

9

$$P\{|X-\mu| \geq K\delta\} \leq 1/K^2$$

S.D

10

$$P\{|x-\mu| < K\delta\} > 1 - 1/K^2 \quad \text{or}$$

11

$$P\{-K\delta < X-\mu < K\delta\} > 1 - 1/K^2$$

Example:

A random variable X has mean $\mu = 12$ and varience $(\delta^2) = 9$ and unknown probability distribution

Find $P(6 < x < 18)$

Solution

2 Mean $(\mu) = 12$

$S.D(\delta) = \sqrt{\text{Varience}}$

3 $S.D(\delta) = \sqrt{9} = 3$

from

4

$$P\{|X-\mu| < K\delta\} > 1 - 1/K^2$$

5 By apply chebyshevs inequality

6 $P\{-K\delta < x-\mu < K\delta\} > 1 - 1/K^2$

7 $P\{\mu - K\delta < x < \mu + K\delta\} > 1 - 1/K^2$

by comparing

$P(6 < x < 18)$

$\mu - K\delta = 6$

$\mu + K\delta = 18$

A. SINGARAY V. SUNDARESAN
V. SIVA, SUBRAMANIAN

$12 - 3k = 6 \Rightarrow 6 = 3k \Rightarrow k = 2$

$12 + 3k = 18 \Rightarrow -6 = -3k \Rightarrow k = 2$

then

$P\{12 - 2(3) < X < 12 + 2(3)\} > 1 - \frac{1}{2^2}$

$P\{6 < X < 18\} > 1 - \frac{1}{4} = \frac{3}{4}$.

UNIT 3 : TESTING OF HYPOTHESIS :

POPULATION :

The group of individuals under study is called a population or universe.

SAMPLE :

A finite subset of statistical individuals in population

SAMPLE SIZE (n)

The number of individuals in a sample.

PARAMETERS :

i) Population mean (μ)

ii) Population variance (σ^2)

STATISTICS subset of population

i) Sample mean $(\bar{x})$

ii) Sample variance s^2

important

NULL HYPOTHESIS :

A definite statement about the population parameter. A hypothesis is usually ~~a hypothesis is usually~~ of such a hypothesis is usually of no difference and its denoted by H_0

ALTERNATIVE HYPOTHESIS: (H_1 or H_A)
Any hypothesis which is complementary to Null hypothesis is called Alternative Hypothesis.

Example:
1) The mean area of the several thousands apartments in a new development is advertised to be 1250 square feet. A tenant group think that the apartments are smaller than advertised. They hire an engineers to measure a sample of apartment to test their suspecion.

Solution
H_0: $\mu = 1250$ feet
H_1 or H_A = $\mu < 1250$ feet.

Then
1) $H_0 : \mu = \mu_0$
$H_1 : \mu < \mu_0$ or $\mu > \mu_0$ or $\mu \neq \mu_0$.

Testing of Hypothesis:
1/ Writing null hypothes and alternative hypothesis

Testing of Hypothesis

small sample
sample size ($n < 30$)
- Students t-tests
- F-test
- χ^2 test

Large sample
sample size ($n > 30$)
- Z-test
- small-test (SSSS)

7 pm Students t-test:
its formula $t = \dfrac{\bar{x} - \mu}{\frac{S.D}{\sqrt{x-1}}}$

8

9 statistic: $\left[t = \dfrac{\bar{x} - \mu}{S/\sqrt{x-1}} \right]$

10

11

Eg: The mean life time of a sample of 25 flourescent big
12 noon bulbs produced by a company is computed to be 1570 hours with S.D of 120 hours. The company
1 claims that the average life of the bulbs produced by the company is 1600 hours. Using the Level of the
2 significe of 0.05. is the claim claim acceptable?

Solution:

3 Sample size (n) = 25
sample mean ($\bar{x}$) = 1570
4 population mean (μ) = 1600
S.D (sample) = s = 120
5 Degree of freedom = n-1 = 25 - 1 = 24
Level of significance (α) = 0.05
6 $H_0 : \mu = 1600$
$H_1 : \mu \neq 1600$
7 then:

$$t_{cal} = \frac{\bar{x} - \mu}{\frac{S.D}{\sqrt{n-1}}} = \frac{1570 - 1600}{\frac{120}{\sqrt{25-1}}}$$

$$t_{cal} = \frac{-30}{\frac{120}{\sqrt{24}}} = -1.22$$

$|t_{cal}| = |-1.22| = 1.22$.

Calculated t value < tabulated t - value

Procedure

1) H_0 : δ μ

H_1 : $-\bar{x}$

S.D

statistic $t = \dfrac{\bar{x} - \mu}{S.D/\sqrt{n-1}}$

noon t calculated

If calculated t - value ≤ tabulated t value

if < accept H_0

if > ~~not~~ it reject H_0

then;

$|t_{cal}| = 1.22 <$ tabulated t value = 2.0

$1.22 < 2.06$ it accept H_0

Testing of Hypothesis procedure

1) Write the

$\mu =$

$\delta =$

$\bar{x} =$

$SD =$

$H_0 : \mu =$

$H_1 : \mu =$

statistic - $t = \dfrac{\bar{x} - \mu}{S.D/\sqrt{n-1}}$

Calculate t - value :
If calculate t - value < tabulated t value then accept H_0. If calculated t - value > tabulated t value, then reject H_0.

Eg 2 :
A sample of 26 bulbs give a mean life of 990 hour with a S.D of 20 hours. The manufacture claims that the mean life of bulbs is 1000 hours. Is the sample not upto the standard

Solution

Sample size (n) = 26 its small sample < 30
sample mean ($\bar{x}$) = 990
S.D (S) = 20
population mean (μ) = 1000

$H_0 : \mu = 1000$ (the sample is upto standard)
$H_1 : \mu \leq 1000$ or $\mu \neq 1000$ (the sample is not upto standard)

Then

Statistic $t_{cal} = \dfrac{\bar{x} - \mu}{S.D / \sqrt{n-1}} = \dfrac{990 - 1000}{20 / \sqrt{26-1}} = -10/4$

$t_{cal} = -2.5$

So

$|t_{cal}| = |-2.5| = 2.55$

t-tabulated = 1.708

Calculated t value $|t_{cal}|$ > tabulated

2.5 > 1.708

We reject the null hypothesis (H_0). Hence the sample is not upto standard.

Eg: 9 items of sample had the following value 45, 47, 52, 48, 49, 49, 53, 51. Does mean of nine item differ significantly from the assumed population mean 47.5

Solution

$n = 9$

$\overline{X} = \frac{\sum X}{n} = \frac{442}{9} = 49.11$

Varience $S^2 = \frac{1}{n-1} \sum (x-\overline{x})^2 = 6.859$

X	$\overline{X}$	$X - \overline{X}$	$(X - \overline{X})^2$
45	49.11	-4.11	16.89
47	49.11	-2.11	4.45
50	49.11	0.89	0.79
52	49.11	2.29	8.35
48	49.11	-1.11	1.23
47	49.11	-2.11	4.45
49	49.11	-0.11	0.012
53	49.11	3.87	15.13
51	49.11	0.89	3.57

$\Sigma x = 442$ $\Sigma(x-\overline{x})^2 = 54872$

$S = \frac{1}{9-1}(54872) = 6.859 = 2.62$

$H_0 : \mu = 47.5$

$H_1 : \mu \neq 47.5$

7 am Student test (t-test)

$$t = \frac{\bar{x} - \mu}{S.D/\sqrt{n}}$$

8

9

$$= \frac{49.11 - 47.5}{2.62/\sqrt{9}}$$

10

$$t = 1.84$$

11

Calculate t-value,

12 noon

$$t_{cal} = 1.84$$

$n = 9$

1 pm degree of freedom $= n-1 = 8$

2 8 degree of freedom at 5% level of significance

3 $t_{tab} = 2.31$

4 $t_{cal} < t_{tab}$ accept the mean of nine item do not differ significantly from population mean.

5

6

7

(Q) Students t - test for difference means :-
- To test the ~~difference~~ significant difference b/w two means $\bar{x}_1$ and $\bar{x}_2$ of samples size n_1 and n_2 use the statistics

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where

$$S^2 = \frac{\sum (x_1 - \bar{x}_1)^2 + (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

Or

$$S^2 = \frac{1}{n_1 + n_2 - 2}\left[n_1 S_1^2 + n_2 S_2^2\right]$$

S_1 and S_2 are sample standard deviation

$$\text{Degrees of freedom} = (n_1 - 1) + (n_2 - 1) = n_1 + n_2 - 2$$

(Q) Two horses A and B were test according to the time (in second) to run a particular tracks with the following results :-

x_1	Horses A	28	30	32	33	33	29	34
x_2	Horses B	29	30	30	24	27	29	

Test wheather you can discriminate between two horses [you can fact that 5% level significance value of t for 11 degrees of fraction is 2.2].

$\bar{x}_1 = \sum \frac{x_1}{n_1}$

Solution : Horse A :

x_1	$\bar{x}_1$	$(x_1 - \bar{x}_1)$	$(x_1 - \bar{x}_1)^2$
28	31.3	-3.3	10.89
30	31.3	-1.3	1.69
32	31.3	0.7	0.49
33	31.3	1.7	2.89
33	31.3	1.7	2.89
29	31.3	-2.3	5.29
34	31.3	2.7	7.21
219	~~31.3~~		31.43

~~$\sum \frac{219}{7} = 31.3$~~

$\sum \frac{x_1}{n_1} = \bar{x}_1$

~~$s^2 = \frac{\sum(x_1 - \bar{x}_1)^2 + \sum(x_2 - \bar{x}_2)}{n_1 + n_2 - 2}$~~

$\bar{x}_1 = \frac{219}{7} = 31.3 =$

Horse B :

x_2	$\bar{x}_2$	$x_2 - \bar{x}_2$	$(x_2 - \bar{x}_1)$
29	27.8	1.2	1.44
30	27.8	2.2	4.84
30	27.8	2.2	4.84
24	27.8	-3.8	14.44
27	27.8	-0.8	0.64
27	27.8	-0.8	0.64
$\sum$ 167			26.84

$\sum \frac{167}{6} = 27.8 = \bar{x}_2$

then $n_1 = 7, n_2 = 6$

Varience $(s^2) = \frac{\sum(x_1 - \bar{x}_1)^2 + \sum(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$

Varence = $\dfrac{31.43 + 26.84}{7+6-2}$

Varence = $\dfrac{58.27}{11}$

Varence = $5.29 = \sqrt{5.29} = 2.3$

10 then:

Ho : $\mu_1 = \mu_2$ [there is no (significance) discrimination difference between
11 two horses]

H_1 : $\mu_1 \neq \mu_2$

12 noon

Statistics

1 pm $t_{cal} = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{1/n_1 + 1/n_2}}$

2

$= \dfrac{31.3 + 27.8}{2.3\sqrt{1/7 + 1/6}} = \dfrac{3.5}{2.3\sqrt{0.3094}}$

3

4 Statistic t_{cal} = 2.73

then

Degree of freedom (df) = $n_1 + n_2 - 2$

= 7 + 6 - 2

6 = 11

t cal = 2.73

7 t tab = 2.2

then

t cal > t tab

2.73 > 2.2 it reject Ho

We reject the Null Hypothesis (Ho)

There is significant difference btwn two horses

7 am F - test :
To test whether is there is any significant difference b/w two estimates of population varience

9 $F = \frac{S_1^2}{S_2^2}$. $\frac{\text{greater varience}}{\text{smaller varience}}$ $[S_1^2 > S_2^2]$

10 where $S_1^2 = \frac{\sum (x_1 - \bar{x}_1)^2}{n_1 - 1}$, . n_1 first sample size

11 $S_2^2 = \frac{\sum (x_2 - \bar{x}_2)}{n_2 - 1}$ n_2 second sample size.

12 noon Fcal < Ftab it accept H_0
Fcal > Ftab it reject H_0.

1 pm

Eg: In one sample of 10 observation from normal population, the sum of the squares of the deviation of sample values from the sample mean is 102.1 and in another sample of 12 observation from another normal population, the sum of the square of the deviation of the sample value from the sample mean is is 120.5. Examine whether the two normal population have the same varience.

Solution

sample size $n_1 = 10$
$\sum (x_1 - \bar{x}_1)^2 = 102.4$
sample size $n_2 = 12$
$\sum (x_2 - \bar{x}_2)^2 = 120.5$.

then it is nothing but varience formular

$S_1^2 = \frac{\sum (x_1 - \bar{x}_1)^2}{n_1 - 1} =$

$S_2^2 = \frac{\sum (x_2 - x_2)^2}{n_2 - 1}$

7 am $H_0 : S_1^2 = S_2^2$ (there is no significant difference between two varience

8 $H_1 : S_1^2 \neq S_2^2$

then

9 $S_1^2 = \frac{102.84}{10-1} = \frac{102.4}{9} = 11.34$

10 $S_2^2 = \frac{120.5}{12-1} = \frac{120.5}{11} = 10.95$

11

$F_{cal} = \frac{11.34}{10.95}$

12 noon

$F_{cal} = 1.038$

1 pm

~~$F_{tab} =$~~

2 Calculated F-value $F_{cal} = 1.038$

[Tabulated F value at 5% level for (9, 11) degree

3 of fraction is 2.90]

$F_{tab} = 2.90$

4

$\therefore F_{cal} < F_{tab}$

5 $1.038 < 2.90$ it accept the H_0.

Hence there is no significance difference btwn two varience

7 [ASSIGMENT 11 :]

1) The weekly wages of 1000 workmean are normally distributed around a mean of 70 dollars with a S.D of 5 dollars. Estimate the number of workers ~~between~~ ~~whose~~ ~~69 dollars and 72 dollars~~

whose weekly wages will be
i) btw 69 dollars and 72 dollars
ii) less than 69 dollars
iii) more than 72 dollars

2) If x follows poisson distribution then prove that
$$P(x+1) = \frac{\lambda}{(x+1)} P(x)$$

3) Certain pesticide is packed into bag by machine. A random sample of 10 bags is drawn and their contents are found to weigh (in kg) as follow:
50, 49, 52, 44, 45, 48, 46, 45, 45, 49, 45.
Test if the average packing can be taken ~~into~~ to be 50 kg
t tab for dift = 2.262

4) ~~Two horses A and B~~

4) The random sample, were draw from two normal population and the following result were obtained

Sample 1	16	17	18	19	20	21	22	24	26	27	-	-
Sample 2	19	22	23	25	26	28	29	30	31	32	35	36

Obtain estimate of the variences of population and test whether the two population have the same varience

[F tab = 2.945] Due date June 6.

$$\psi^2 = \sum \frac{(O-E)^2}{E}$$

where O - observation frequency
E - expected frequency
$\psi^2_{cal} > \psi^2_{tab}$ - Accept H_0,
$\psi^2_{cal} < \psi^2_{tab}$ - reject H_0

Qn) The following fig show the distribution digit in nu chosen at random from telephone directory

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	1026	1107	997	966	107.5	933	1107	972	964	853

Test wheather the digit may be take to occur equally frequently in the directory.

$$\psi^2 = \sum \frac{(O-E)^2}{E}$$

Solution:

H_0: The digit occur equally in the directory

H_1:

Digit	(O) Frequency	Expected Mean frequency E	O - E	$(O-E)^2$	$(O-E)^2/E$
0	1026	1000	26	676	0.676
1	1107	1000	107	11449	11.449
2	997	1000	-3	9	0.007
3	966	1000	-34	1156	1.156
4	1075	1000	75	5625	5.625
5	933	1000	-67	4481	4.487
6	1107	1000	107	11449	11.449
7	972	1000	-28	784	0.784
8	964	1000	-36	1216	1.296
9	853	1000	-147	21609	31.609

58.542

7 $E = \frac{10000}{10} = \frac{F}{\text{no of frequency}}$ $\therefore E = 1000$.

8 $\psi^2 = \sum \frac{(O-E)^2}{E}$

9

10 $\psi^2_{cal} = 58.542$

11 $\psi^2_{tab} = 16.919$

12 noon $\therefore \psi^2_{cal} > \psi^2_{tab}$ reject H_0.

1 pm The digit occur equally frequently in the derectory.

2 The degree of freedom $df = n-1 = 10-1 = 9$

3 $\therefore$ 9 df of 5%.

4

5 LARGE SAMPLES

6 Sample Size $(n) > 30$

1) Z - sample test for single mean.

2) Z - test for difference of mean.

Z - test

$$Z = \frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}$$, δ - S.D of the population

Difference of Mean:

$$\boxed{Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}}$$

Eg: A sample of 100 members has a mean of 3.4 cms and SD 2.61 cm. Is the sample form a large population mean 3.25 cm and SD 2.61 cm. If the population is normal and the mean is uknown. Is the sample drawn from the population with mean 3.25? [Ztab = 1.96]

Solution

$n = 900 > 30$ (Large sample)

$\bar{x} = 3.4$
$S = 2.61$
$\mu = 3.25$
$SD(S) = 2.61$

Ho: the sample has been drawn from the population with mean $\mu = ?$

where
$\mu = 3.25$

from

$$Z = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} = \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}}$$

$Z_{cal} = 1.724$.

$Z_{cal} = 1.724 < Z_{tab} = 1.96$.

$\therefore$ It accept H_0.

The sample has been drawn from the population with mean 3.25

~~Eg 2: The mean of 2 large samples 1000 of 2000 nos over~~

Eg 2: The mean yield of wheat from a district A has 210 pound with S.D 10 pound per acre form a sample of 100 plots. In another district, the mean yield was 220 pound with S.D 12 pounds from a sample of 150 plots. Assuming that the S.D of yield in the entire state was 11 pound, test whether there is any significant difference between mean yield of crops in the two districts).

Solution:

sample size

$n_1 = 100$

$\bar{x}_1 = 210$

$s_1 = 10$

$\bar{x}_2 = 220$, $n_2 = 150$, $s_2 = 12$.

H_0: There is no significant difference b/w the mean yield of crops in the two district $(\bar{x}_1 = \bar{x}_2)$

then H_1 : $\bar{x}_1 \neq \bar{x}_2$. , $s_1 = 11$.

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{210 - 220}{\sqrt{\frac{10^2}{100} - \frac{12^2}{150}}}$$

7 Note
If you don't know population S.D use the above formula. (unknown)

9 $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$ sample standard deviation

10

If you know the population S.D use the above formula

12 noon

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$ standard deviation

1 pm

2

then

3

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

4

5 $$Z = \frac{210 - 200}{11\sqrt{\frac{1}{100} - \frac{1}{150}}}$$

6

7 $Z_{cal} = 7.041$

$Z_{tab} = 1.96$

$Z_{cal} > Z_{tab}$ it reject H_0.

∴ There is significance difference btwn the mean $\bar{x}_1$ & $\bar{x}_2$

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

(ii) Test of significance for Single population:

Suppose a large sample of size n taken from a normal population. To test the significant difference between the sample proportion p and the population proportion P.

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}, \quad Q = 1 - P, \quad Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where

p - sample proportion

P - population proportional.

(Qn) In a big city men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers? ($Z_{tab} = 1.96$

Solution

$n = 600 > 30$ this is Large sample.

$$p = \frac{325}{600} = 0.5417$$

$\frac{1}{2}$ Smoker's + $\frac{1}{2}$ non smokers = 1

$\therefore P = \frac{1}{2}$ (population proportion of smokers in the city)

then: $Q = 1 - \frac{1}{2} = \frac{1}{2}$.

H_0: The no of smokers + non smokers are equal on the city

H_1: The no of smoker + non smoker are not equal on city

$$Z = \frac{0.5417 - 0.5}{\sqrt{\frac{\frac{1}{2} \times \frac{1}{2}}{600}}}$$

$Z_{cal} = 2.04$

$Z_{tab} = 1.96$

$Z_{cal} > Z_{tab}$ it reject H_0

∴ The no of smoker + non smoker are not equal in the city.

When you know the sample proportio & population proportion use the Z test formular

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}, \quad p + q = 1$$
$$q = 1 - p$$

Eg: Random samples of 400 men and 600 women were asked whether would like to have a fla[t?] near their residence 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions men and women in flavour of the proposal are same ($Z_{tab} = 1.96$ at 5% Level

Solution

$n_1 = 400$, $n_2 = 600$ (sample size > 30, Large sample)

$p_1 = \frac{200}{400} = \frac{1}{2}$, $p_2 = \frac{325}{600} = 0.541$

$p_1 = 0.5$

then

H_0 :- Assume there is no significant differen b/w option men & women as far as proporsal of favour is concerned.

then

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{400 \times 0.5 + 600 \times 0.541}{400 + 600}$$

$P = 0.5246$

$q = 1 - p = 1 - 0.5246 = 0.4754$

$$Z = \frac{0.5 - 0.541}{\sqrt{0.5246 \times 0.4754 \left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$|Z| = |-1.24|$

$|Z| = 1.24$

$Z_{cal} = 1.24$, $Z_{tab} = 1.96$

$Z_{cal} < Z_{tab}$ it accept H_0.

7 am

Test Hypothes

8

Small sample (n < 30	Large sample (n > 30
i) t - test for single mean (mean)	i) Z - test for single mean
ii) t - test for difference mean	ii) Z - test for difference mean
iii) F - test (varience)	iii) Z - test for single proportion
iv) ψ^2 = test	iv) Z - test for difference of propo

11

12 noon

Unit 4 : Queuing Theory :

1 pm

i) Expected number of people in the system (L_s)
ii) Expected number of people in the queue (L_q)
iii) Waiting time in the system (W_s).
iv) Waiting time in the queuing (W_q)

4

Queuing Models :

5

6

7 Single server Q.M   Multiple Server Q.M.

Server - Barber
Arriving.

Service rate = μ / hr
Arrival rate = λ / hour

System = queue + service.

Single Server Q.M:

Infinite queue length — Finite queue length

- Ls : expected no of pple in system — Ls
- Lq : expected no of pple in queue — Lq
- Ws : waiting time in system — Ws
- Wq : waiting time in queue — Wq.

that there are.

P_0 - probability zero people in the system.
P_1 - probability of one people in the system.
P_2 - probability that there are two people in the system.

P_n : Probability that there are n people in the system

7 am
1) INFINITE QUEUE LENGTH.
8
Consider
9
$$\lambda = \rho$$
10
11
$$P_n = \rho^n P_0 \; ; \; n = 1, 2, 3 \; --- \; n.$$

12 noon
$$P_1 = \rho^1 P_0$$
1 pm
$$P_2 = \rho^2 P_0$$
2
$$\vdots \qquad \vdots$$
3
4
5
$$\boxed{P_n = \rho^n P_0.}$$
6
7

geometric series

7am Infinite queue Length:

$P_0 + P_1 + P_2 + \cdots = 1$

8 $P_0 + \rho P_0 + \rho P_0^2 + \cdots = 1$

$P_0 (1 + \rho + \rho^2 \cdots) = 1$

9 $P_0 \left(\frac{1}{1-\rho}\right) = 1$

10 $\therefore a + ar + ar^2 + \cdots = \frac{a}{1-r}$

11 $\left[P_0 = 1 - \rho\right]$

12 noon but

$\rho = \lambda / \mu$

1 pm

but $P_n = \rho^n P_0$

2 $n=1$: $P_1 = \rho P_0 = \rho(1-\rho)$

$n=2$: $P_2 = \rho^2 P_0 = \rho^2(1-\rho)$

3 $n=3$: $P_3 = \rho^3 P_0 = \rho^3(1-\rho)$

4 $\therefore \left[P_n = \rho^n (1-\rho)\right]$

5 where

L_s = expected no of people in the system or Length of the system

6

7 $= \sum_{n=0}^{\infty} n P_n = 0 + 1P_1 + 2P_2 + 3P_3 + \cdots n P_n$

$L_s = P_1 + 2P_2 + \cdots n P_n$

$= \rho P_0 + 2\rho^2 P_0 + \cdots n(\rho^n P_0) + \cdots$

$= \rho P_0 (1 + 2\rho + \cdots + n\rho^{n-1} + \cdots)$

$\therefore = \rho P_0 \sum_{k=1}^{\infty} k \rho^{k-1}$

Sum of G.S $= \frac{\rho}{1-\rho}$

7 am $L_s = \rho P_0 \sum_{k=0}^{\infty} \frac{d}{d\rho} (\rho^k)$

8 $L_s = \rho P_0 \frac{d}{d\rho} \sum_{k=0}^{\infty} \rho^k$

9 $L_s = \rho P_0 \frac{d}{d\rho} (1+\rho + \rho^2 + \rho^3 + \ldots)$

10 $= \rho P_0 \frac{d}{d\rho} \left(\frac{1}{1-\rho} \right)$

11 $= \rho P_0 \left(\frac{\cancel{1-\rho} \cancel{/} - (\rho(-1))}{(1-\rho)^2} \right)$

12 noon $= \rho P_0 \left(\frac{-1}{(1-\rho)^2} \right) (-1)$

1 pm $= \frac{\rho P_0}{(1-\rho)^2}$ but $P_0 = 1-\rho$

2 $= \frac{\rho(1-\rho)}{(1-\rho)^2}$

3 $\boxed{L_s = \frac{\rho}{1-\rho}}$

4 $\boxed{P_n = \rho^n P_0}$ — L_s

5 $\boxed{P_0 = 1-\rho}$ — L_q

6 $\boxed{\frac{\lambda}{\mu} = \rho}$

7 $\boxed{L_s = \frac{\rho}{1-\rho}}$ $\boxed{L_q = L_s - \rho}$

L_s = expected no of people in the system.

L_s = expected no of people in the queue + expected no of people who are being served

$L_s = L_q + \lambda/\mu$

If no que no probability ppl get service directly.

7 am $[L_s = L_q + \rho]$ Littles equations
8 $[L_q = \lambda W_q]$

9 where
W_s = Is waiting time in the system
10 W_q - Is waiting time in the queue.

11 Eg: λ = 8/hour - arrival rate
μ = 9/hour - service rate.
12 noon i) what is the probability that there is no queues?
$(P_0 + P_1)$.

1 pm Solution:
$\rho = \lambda/\mu = \frac{8}{9} =$
2
$P_0 = 1 - \rho$
3 $= 1 - 8/9 = \frac{9-8}{9} = \frac{1}{9} = 0.111$

4 $P_1 = \rho P_0$
$= 8/9 \times 1/9$
5
$P_1 = 8/81$
6
$P(\text{there is no queue}) = P_0 + P_1$
7 $= 1/9 + 8/81$
$= \frac{9+8}{81} = 17/81 = 0.2098$
$= 20.98\%$.

Qn) Customers arrive at a clinic at the rate of 8/hour and doctor can serve at the rate of 9/hour.

i) What is the probability that a customer does not join the queue and walk into the doctor's room? (P_0 - no queueline)

ii) What is the probability that there 10 customer on the system? (P_{10})

iii) What is the expected no. of people in the system (L_s)

iv) What is the expected waiting time in the queue?

v) What is the expected waiting time in the system.

vi) What is the expected waiting no of people in the que.

vii) What is the probability that there is no queue?

Solution

ii) P_{10} = , i) P_0 , iii) L_s , iv) W_q , v) W_s , vi) L_q

vii) $P_0 + P_1$

$\rho = \lambda/\mu = 8/9 =$

i) $P_0 = 1 - \rho$

$P_0 = 1 - 8/9 = 1/9 = 0.111 = 11.1\%$

ii) $P_{10} = \rho^{10} P_0$

$P_{10} = \left(\frac{8}{9}\right)^{10} \times 0.111$

$P_{10} = 0.0341$

$P_{10} = 3.4\%$

$1h = 60min$
$0.888 = ?$

23 Monday April

2018
113 - 365 Week 17

7 am iv) $L_s = \frac{\rho}{1-\rho} = \frac{8/9}{1-8/9} = 8$

8

v) $W_q = ?$

9 $L_q = \lambda W_q$, $L_q = ?$

10 $L_q = L_s - \rho$

$= 8 - \frac{8}{9} = \frac{64-\cancel{8}}{9} = 7.11$

11

$W_q = \frac{L_q}{\lambda} = \frac{7.11}{8} = 0.888$ hour $\approx 53min$

12 noon

v) $W_s = \frac{L_s}{\lambda}$

1 pm

$= \frac{8}{8} = 1$ hour $= 60min$

2

vi) $L_q = ?$

3 $L_q = L_s - \rho$

4 viii) $P_0 + P_1 = \frac{1}{9} + \frac{8}{81} = 0.2098 = 20.98\%$

5

6

7