CAT 1

PART A:

The definition of probability

P(happening events) = $\frac{\text{no of favourable cases}}{\text{total no of exhaustive events}}$

P(throwing 10 with 2 dice) = $\frac{\text{no of f.c}}{\text{total no of e.v}}$

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

$= \frac{3}{36} = \frac{1}{12}$

P(getting queen) = $\frac{\text{no of f.c}}{\text{total no of e.e}}$

$= \frac{4C_1}{52C_1} = \frac{4}{52} = \frac{1}{13}$

8 redball
9 blackball
17 ball

P(drawing redball) = $\frac{8C_1}{17C_1} = \frac{8}{17}$

P(its diamond) = $\frac{^{13}C_1}{52C_1} = \frac{13}{52} = \frac{1}{4}$

6am The laws of additional probability:

8 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

9 Bayes theorem state that:
If $A_1, A_2, A_3 \dots A_n$ let n be mutually exclusive event
10 and exclusive events & B be independent event such that
$B \subset \bigcup_{i=1}^{n} A_i$ is the conditional probability of B given
that A has already occured.

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum P(A_i) \cdot P(B/A_i)}.$$

12 noon

1 pm PART B:

8a) 7 whiteball
2 6 red 2 ball drawn at random
5 Black
3 18 balls

$P(\text{both will be white}) = \frac{{}^{7}C_2}{{}^{18}C_2} = \frac{7}{153}$ = ~~0~~.0457

4

b) 3 men
2 women
6 4 children
We need to show the chance that exactly two of them will
7 be children 10/21

$P(\text{will be children}) = \frac{{}^{3}C_2 \times {}^{4}C_2}{{}^{9}C_4} = \frac{10}{21}$

9a) $P = \frac{{}^{26}C_3 \times {}^{26}C_3}{{}^{52}C_6} = \frac{130}{39151} = 3.32 \times 10^{-3}$

9.) 7 am 1 year = 365 days
1 Leap year = 366 days
8
1 leap year contain 52 sunday & two day extra
two day extra will be
Mon, Tue
10 Tue, Wed
Wed, thur
11 Thur, fri
Fri, sat
12 noon Sat, Sun }
Sun, Mon }
1 pm
$P(\text{selected contain 53 sunday}) = \frac{2}{7}$
2

10 a) $P(A) = 0.9$, $P(B/A) = 0.8$, $P(A \cap B) =$
from
4 $P(B/A) = \frac{P(A \cap B)}{P(A)}$
5 $P(A \cap B) = P(A) \cdot P(B/A) = 0.8 \times 0.9$
$= 0.72$

b) 6 $P(S \cup K) = ?$
~~$P(B/A) =$~~
7 $P(S) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52}$

$P(k) = \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{52}$

$P(S \cup K) = \frac{13}{52} + \frac{4}{52} - \left(\frac{13}{52} \times \frac{4}{52}\right)$

$= \frac{4}{13}$

15 Tuesday May 2018
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7 am PART C:

a) 4 Indian
8 5 Chinese 5, selected
6 Tanzanian
9 15

i) $P(3I \& 2\text{ chinese}) = \frac{{}^4C_3 \times {}^5C_2}{{}^{15}C_5} = \frac{40}{3003}$

10

ii) $P(2I, 1C, 2T) = \frac{{}^4C_2 \times {}^5C_1 \times {}^6C_2}{{}^{15}C_5} = \frac{450}{3003}$

11

iii) $P(5T) = \frac{{}^6C_5}{{}^{15}C_5} = 6/3003$

12 noon

11b) 1pm P(drawing Q & king)
P(B/A) =
2 $P(\text{Queen} \& \text{king}) = \frac{{}^4C_1}{{}^{52}C_1} \times \frac{{}^4C_1}{{}^{51}C_1} =$
3 $P(\text{king} \& \text{Queen}) = \frac{{}^4C_1}{{}^{52}C_1} \times \frac{{}^4C_1}{{}^{51}C_1} =$
4 $P(Q \& k \text{ in two draw}) = \frac{4}{52} \times \frac{4}{51} + \frac{4}{52} \times \frac{4}{51} = 8/663$

5

12 a) Beg 1 — Beg 2 — Beg 3
6 3 W — 2 W — 3 W
2 R — 3 R — 4 R
7 4 B — 5 B — 2 B

$P(A_1) = 1/3$ $P(A_2) = 1/3$ $P(A_3) = 1/3$

$P(B/A_1) = \frac{{}^3C_2 \times {}^2C_1}{{}^9C_3} = 1/14$

$P(B/A_2) = \frac{{}^2C_2 \times {}^3C_1}{{}^{10}C_3} = 1/40$

$P(B/A_3) = \frac{{}^3C_2 \times {}^4C_1}{{}^9C_3} = 1/7$

$P(A_1).P(B/A_1) = \frac{1}{3} \times \frac{1}{14} = 0.0238$

$P(A_2).P(B/A_2) = \frac{1}{3} \times \frac{1}{40} = 0.0083$

$P(A_3)P(B/A_3) = \frac{1}{3} \times \frac{1}{7} = 0.047$

$\sum P(A_i).P(B/A_i) = 0.0791$

$P(A_1/B) = \frac{0.0238}{0.0797} = 0.286$

$P(A_2/B) = \frac{0.0083}{0.0797} = 0.104$

$P(A_3/B) = \frac{0.047}{0.0797} = 0.597$

10

b)

Machine	$P(A_i)$	$P(B/A_i)$	$P(A_i) \cdot P(B/A_i)$
machine A	0.25	0.05	0.0125
Machine B	0.35	0.04	0.014
machine C	0.40	0.2	0.08

1 pm

2

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7 am CAT 2:

PART C:

8 1(a) $\mu = 12,$
$\delta^2 = 9 = 3$

9 $p(6 < x < 18)$ use chebychev inequality

from

$p\{-\delta k < |\mu - x| < k\delta\} < 1 - 1/k^2$

apply chebychev inequality.

11

$p\{\mu - k\delta < x < \mu + k\delta\} < 1 - 1/k^2$

12 noon by comparison

$p(6 < x < 18)$

1 pm $\mu - k\delta = 6$

$\mu + k\delta = 18$

2 $12 - k(3) = 6$

$3k = 12 - 6$

3 $3k = 6$

$\underline{k = 2}$

4

$p\{12 - 2(3) < x < 12 + 2(3)\} < 1 - 1/2^2$

5

$p(6 < x < 18) < 1 - \frac{1}{4}$

6

$p(6 < x < 18) < 3/4$.

7

11a) We need to derive mean & variance of Geometric distribution.

$P(x) = q^x p, \quad x = 0, 1, 2 \ldots (0 < p < 1)$

let $x = 0$

$p(x) = p + pq + pq^2 \ldots$

$E(x) = \sum^{n} x\, p(x)$

$E(x) = \sum_{x=0}^{n} x\, q^x p$

$= p \sum_{x=0}^{n} x q^x$

$= p(0 + q + q^2 + q^3 + \ldots q^n)$

$= pq(1 + q + q^2 \ldots q^n)$

$pq(1-q)^{-2}$

$pq/(1-q)^2$

$E(x)\ pq/p^2 = q/p$

$\left[\text{Mean} = E(x) = q/p.\right]$

Variance $\mu_2 = \sum_{x=1} x\, pq^x$

$= p \sum$

Q1a) Refer to UG
q_n - 18a)

12b) q_n - 19b).

PART B

8a) $P(X=x) = q^{x-1}p$, $x = 1,2,3$

We need to find the moment of generating function, mean and varience as

$$M_X(t) = \sum_{x=1}^{\infty} e^{tx} f(x)$$

$$= \sum_{x=1} e^{tx} \cdot q^{x-1} p$$

$$= p \sum_{x=1}^{\infty} e^{tx} q^{x} \cdot \frac{1}{q}$$

$$= \frac{p}{q} \sum_{1} e^{tx} q^{x}$$

$$= \frac{p}{q}\left[q + (e^tq)^2 + (e^tq)^3 + (e^tq)^3 + (e^tq)^4 \ldots\right]$$

$$= \frac{p}{q} \cdot e^tq\left[(1-e^tq)^{-1}\right]$$

$$M_X(t) = \frac{pe^t}{(1-e^tq)}$$

Mean $= M_X'(0)$

$$= \frac{d}{dt}\left(\frac{pe^t}{1-e^tq}\right)$$

$$= \frac{(1-e^tq)pe^t - pe^t(-e^tq)}{(1-e^tq)^2}$$

$$= \frac{pe^t - pqe^{2t} + pqe^{2t}}{(1-e^tq)^2}$$

$$\text{Mean} = \frac{pe^t}{(1-e^tq)^2}$$

but

Mean $= M_X'(0)$

$$= \frac{pe^0}{(1-e^0q)^2}$$

$$= \frac{p}{(1-q)^2}$$

Mean $= 1/p$

7 Variance $= Mx''(0) - (Mx'(0))^2$

$= \frac{p}{q^2}$

8

8b) $p = 0.1$ are defective

$n = 20$

10 $q = 1 - 0.1 = 0.9$

11) P(almost 3 are defective) $= P(X \leq 3)$

$P(X \leq 3) = {}^nC_x \, p^x q^{n-x}$

12 noon $= {}^{20}C_x (0.1)^x (0.9)^{20-x}$

for $X \leq 3$

1 pm $P(X=0) = {}^{20}C_0 (0.1)^0 (0.9)^{20-0}$

$=$

2 $P(X=1) = {}^{20}C_1 (0.1)^1 (0.9)^{20-1}$

$=$

3 $P(X=2) = {}^{20}C_2 (0.1)^2 (0.9)^{20-2}$

$=$

4 $P(X=3) = {}^{20}C_3 (0.1)^3 (0.9)^{20-3}$

$=$

5 $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$=$

6 $P(X \leq 3 =$

11) $P(X = 3) = {}^{20}C_3 (0.1)^3 (0.9)^{20-3}$

$=$

9b) $\hat{p} = \frac{325}{600} = 0.5417$, $n = 600$

$P(\text{smoker} + \text{non smoker}) = 1$

smoker $= \frac{1}{2}$

$Q = 1 - p$

$Q = 1 - 0.5 = 0.5$

H_0: Majority of men in city are smoker

H_1: majority of men are non smoker

from

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.5417 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{600}}}$$

$Z_{cal} = 2.04$

$Z_{cal} > Z_{tab}$ it reject H_0

∴ The no of smoker & non smoker are not equal in the city.

10a) ~~Refer~~ Refer to UE 15a.

b) Refer to UE 15b.

PART A:

1. Binomial distribution state

$$P(x) = {}^nC_x\, p^x q^{n-x}$$

2. $P(x=1) = \frac{3}{10}$ $\quad P(x=2) = \frac{1}{5}$

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\frac{1}{5} = \frac{e^{-\lambda}\lambda^2}{2!} \text{ -(ii)}$$

$$\frac{3}{10} = \frac{e^{-\lambda}\lambda^1}{1!} \text{ -(i)}$$

Divide eq (ii) by (i)

$$\frac{1}{5} \times \frac{10}{3} = \frac{e^{-\lambda}\lambda^2}{2!} \times \frac{1}{e^{-\lambda}\lambda}$$

$$\frac{2}{3} = \frac{\lambda}{2}$$

$$4 = 3\lambda$$

$$\lambda = \frac{4}{3} \ldots$$

i) $P(x=0) = \frac{e^{-\frac{4}{3}}\lambda^0}{0!} = e$

ii) $P(x=3) = \frac{e^{-\frac{4}{3}}\left(\frac{4}{3}\right)^3}{3!}$

$=$

~~3. Poisson~~

3. Exponential Distribution

$$p(x) = f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \\ 0, & \text{otherwise} \end{cases}$$

4) Population:

Refer to the group of individual under study

Sample:

Is a finite subset of statistical individual in population

5. F - test

$$F = \frac{S_1^2}{S_1^2}$$

S_1^2 = S.D greater

S_2^2 = S.D smaller

7 am. 6. t - test for difference of mean :

8 $t_{test} = \dfrac{\bar{x}_1 - \bar{x}_1}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$.

9

10

7. The formular for expected waiting time in the
11 queue for single server of infinite queue length
model

12 noon $W_q = \dfrac{L_q}{\lambda}$.

1 pm

2

3

4

5

6

7

7 am UE : 2024 :

PART A :

1. 8 Mathematical definition of probability

$$P(\text{happening events}) = \frac{\text{no of favourable cases}}{\text{total no of exhaustive event}}$$

9

2. 10 $P(\text{king}) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

11

3. Binomial Distribution:

12 noon $P(x) = {}^nC_x \, p^x q^{n-x}$.

1 pm

4 Exponential distribution state that

2

$$P(x) = f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

3

5) Population : Is a group of individual under study
Sample : Is a finite subset of statistical individual in population.

6 The t - test for difference of mean

7 $t_{test} = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

7. The formular for expected waiting time in the quee of a single server infinite queue length model

$$W_q = \frac{L_q}{\lambda}$$

8) The formula for expected waiting time in the system for single server finite queue length model is

8 $W_s = \frac{L_s}{\lambda_{eff}}$

9 $\lambda_{eff} = \lambda(1 - P_N)$

10) Markov Chain :

Let $X_0, X_1 \cdots X_n$ be square of discrete random variable then $\{X_n,$
11 $n \geq 0\}$ is a Markov chain if it satisfy markov property

$\therefore P(X_{t+1} = j \mid X_t = i, X_{t-1} = i_1, X_{t_0} = i_0)$

12 noon

10) Chapman Kolmogorov theorem :

1 pm Let $\{X_n, n \geq 0\}$ be homogeneous markov chain with TMP $(P) = P_{ij}$ and n-step TPM $(P^n) = P_{ij}^{(n)}$ then
2 n-step TPM $P^n = P_{ij}^{(n)}$ is equal to the n^{th} power of the one step TPM P

3 $\therefore P^n = (P_{ij}^{(1)})^n$

4 PART B

11a) 3 M

5 2 2 W

2 4 C

6 9 peoples

7 P(exactly 2 will be children) $= \frac{{}^5C_2 \times {}^4C_2}{{}^9C_4} = \frac{10}{21}$

b) $f(x) = \frac{3x^2}{10}, 0 \leq x \leq 1$, pdf

$\frac{3}{10}\int_0^1 x^2 dx = \frac{3}{10}[\frac{1}{3}x^3]_0^1$

$= \frac{1}{10}[1^3 - 0^3]$

$= 1/10$

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7 am $P(0.2 < x < 0.5) = \int_{0.2}^{0.5} \frac{3x^2}{10}\,dx$

8 $= \frac{3}{10}\int_{0.2}^{0.5} x^2 dx = \frac{3}{10}\frac{1}{3}x^3\Big/_{0.2}^{0.5}$

9 $= \frac{1}{10}\left[0.5^3 - 0.2^3\right]$

10 $= 0.0117.$

2a) $P(X = x) = q^{x-1}p \quad , X = 1, 2, 3 \ldots$

We need to find moment of generating function, mean & variance

This is discrete random variable.

1 pm $M_x(t) = \sum_{x=1}^{\infty} e^{tx} f(x)$

2 $= \sum_{x=1}^{\infty} e^{tx} \cdot q^{x-1} p$

3 $= \sum_{x=1} e^{tx} q^{x} \cdot q^{-1} p$

4 $= \sum_{x=1} e^{tx} q^{x} \cdot \frac{p}{q}$

5 $= \frac{p}{q} \sum_{x=1}^{\infty} e^{tx} q^{x}$

6 $= \frac{p}{q}\left[q + (e^t q)^2 + (e^t q)^3 + \cdots\right]$

7 $= e^t q \times \frac{p}{q}\left[(1-x)^{-1}\right]$

$= \frac{e^t}{q} p \cdot \frac{1}{(1-e^t q)^{1}}$

$= \frac{p e^t}{q(1-e^t q)^{-1}}$

then

Mean $= M_x'(0)$

Mean $= \frac{d}{dt}\left(\frac{p e^t}{\cancel{q}(1-e^t q)}\right) = \frac{vu' - uv'}{v^2}$

$= \frac{\cancel{t}pe^t(1-e^t q) + pe^t(t e^t q)}{(1-e^t q)^2}$

$= \frac{p\cancel{t}e^t - \cancel{t}pe^{2t}q - \cancel{t}pe^{2t}q}{(1-e^t q)^2}$

$= \frac{p\cancel{t}e^t}{(1-e^t q)^2}$

Mean $= M_x'(0) = \frac{p\cancel{t}e^0}{(1-e^0 q)^2} = \frac{p\cancel{t}}{(1-q)^2}$

Mean $= 1/p$

Variance $= q/p^2 = M_x''(0) - (M_x'(0))^2$

12b) By using binomial distribution

$n = 20$

$p = 10\% = 0.1$ (propability of defective)

$q = 1-p = 1-0.1 = 0.9$

1) P (at most 3 bulb are defective) $= P(X \leq 3)$

$P(X \leq 3) = \sum_{x=0}^{3} {}^{20}C_x (0.1)^x (0.9)^{20-x}$

$P(X=0) = {}^{20}C_0 (0.1)^0 (0.9)^{20-0} = 0.1216$

$P(X=1) = {}^{20}C_1 (0.1)^1 (0.9)^{20-1} = 0.27$

$P(X=2) = {}^{20}C_2 (0.1)^2 (0.9)^{20-2} = 0.285$

$P(X=3) = {}^{20}C_3 (0.1)^3 (0.9)^{20-3} = 0.190$

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$P(X \le 3) = 0.1216 + 0.27 + 0.285 + 0.196$

$P(X \le 3) = 0.8666$

8

ii) P(exactly 3 bulb are defective) $= P(X = 3)$

9 $P(X=3) = {}^{20}C_3 (0.1)^3 (0.9)^{20-3}$

$P(X=3) = 0.190$

10

13a) sample size $(n) = 26$

sample mean $(\bar{x}) = 990$

12 noon $S = 20$

population mean $\mu = 1000$

1 pm ~~t_{tab}~~ $= 1.708$

from

2 $H_0 : \mu = 1000$ (Sample is upto standard)

$H_1 : \mu < 1000$ (Sample is not upto standard)

3

$$t_{test} = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n-1}}} = \frac{990 - 100}{\frac{20}{\sqrt{26-1}}} = -2.!$$

4

5 $|t_{test}| = |-2.55| = 2.55$

6 $t_{cal} > t_{tab}$ it reject H_0

$\therefore$ The sample is not upto standard.

7

$H_0 : P = 0.5$, $H_1 : P > 0.5$

b) $p = 325/600 = 0.5417$

$Z_{tab} = 1.645$

$n = 600 > 30$ Large sample

P(smoker) $+ P$(non smoker) $= 1$

$\therefore P = 1/2$

CamScanner

7 am $Q = 1 - P = 1 - 0.5 = 0.5$

then

8 $Z_{test} = \frac{0.5417 - 0.5}{\sqrt{\frac{PQ}{n}}} = \frac{0.5417 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{600}}} = 2.04$

9

$Z_{cal} \geq Z_{tab}$ it reject H_0

10 ∴ The majority of men are smoker.

11 14b) $\lambda = 3/hr$

$\mu = 5/hr$

12 noon $\rho = \frac{\lambda}{\mu} = \frac{3}{5}$

1 pm P(there are 8 pple in system)

2 $P_n = P_0 \rho^n$ (finite queue)

$P_8 = P_0 \rho^8$

3 $P_0 = \frac{1 - \rho}{1 - \rho^{N+1}} = \frac{1 - \frac{3}{5}}{1 - \left(\frac{3}{5}\right)^{8+1}} = 0.404$

4

$P_8 = 0.404 \times \left(\frac{3}{5}\right)^8 = \underline{0.0068}$

5

14a) We need to derive P_0 value for single server

6 finite queue length N model.

from

7 $P_n = \rho^n P_0$

$P_0 + P_1 + P_2 \dots P_N = 1$

$P_0 + \rho P_0 + \rho^2 P_0 + \rho^3 P_0 + \dots \rho^N P_0 = 1$

$P_0 (1 + \rho + \rho^2 + \rho^3 + \dots \rho^N) = 1$

$P_0 \left(\frac{1 (1 - \rho^{N+1})}{1 - \rho} \right) = 1$

$P_0 = \frac{1 - \rho}{1 - \rho^{N+1}}$ proven

15a) $U = (\frac{1}{4}, 0, \frac{-3}{4}, \frac{1}{2})$

This is not probability vector because they are negat[ive]

8

ii) $V = (\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2})$

9 This is probability vector because

$= \frac{1}{4} + 0 + \frac{1}{4} + \frac{1}{2} = 1$

10

i) Sum of raw is 1

ii) They are non negative

15b) 12 noon $A = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

1 pm Its stochastic matic bcs sum of entities in raw is 1

They are non negative

2 We need to show its regular or not

3 $A = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

4 $A^2 = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

5 $= \begin{matrix} 0 + \frac{1}{2} & 0 + \frac{1}{2} \\ 0 + \frac{1}{4} & \frac{1}{2} + \frac{1}{4} \end{matrix}$

6

$A^2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$ Hence its regular

7

am PART C :

16a) First bag

3 W $P(A_1) = \frac{{}^3C_2 \times 2R_1}{{}^9C_3} =$

2 R

4 B $P(B/A) = \frac{1}{3}$

9 ball

Second bag :

2 W $P(A_2) = \frac{{}^2C_2 \times {}^3C_1}{{}^{10}C_3} =$

11 3 R

5 B $P(B/A_2) = \frac{1}{3}$

12 noon 10 ball

1 pm Third bag $P(A_3) = \frac{{}^3C_2 \times {}^4C_1}{{}^9C_3} =$

3 W

2 4 R

2 B $P(B/A_3) = \frac{1}{3}$

3 9 ball

Bag	$P(A_i)$	$P(B/A_i)$	$P(A_i) \cdot P(B/A_i)$
Beg 1	$\frac{1}{3}$		
Beg 2	$\frac{1}{3}$		
Beg 3	$\frac{1}{3}$		

b) $f(x) = 3x^2, \; 0 \le x \le 1$

i) $P(X \le a) = P(X > a)$

Soln

$P(X < a) = P(X > a)$

$P(X < a) + P(X > a) = 1$

$P(X \le a) = \frac{1}{2}, \; P(X > a) = \frac{1}{2}$

$P(X \le a) = \int_0^a f(x)\,dx = \frac{1}{2}$

$\frac{1}{2} = \int_0^a 3x^2\,dx$

$\frac{1}{2} = \frac{3}{3}x^3 \Big|_0^a$

$\frac{1}{2} = a^3 - 0^3$

$a = \left(\frac{1}{2}\right)^{\frac{1}{3}}$

ii) $P(X > b) = 0.05$

$0.05 = \int_b^1 3x^2\,dx$

$0.05 = x^3 \Big|_b^1$

$0.05 = 1^3 - b^3$

$b^3 = 1 - 0.05$

$b = (0.95)^{\frac{1}{3}}$

$1.00 - 0.05 = 0.95$

17a) $\mu = 12$

$\sqrt{\sigma^2} = \sqrt{9}, \; \sigma = 3$

$P(6 < X < 18)$ Use Chebyshev inequality

from $P\{|X-\mu| < k\sigma\} > 1 - \frac{1}{k^2}$

Apply Chebyshev inequality

$P(-k\sigma < X - \mu < k\sigma) > 1 - \frac{1}{k^2}$

$P(\mu - k\sigma < X < \mu + k\sigma) > 1 - \frac{1}{k^2}$

by comparison

$P(6 < X < 18)$

$\mu - k\sigma = 6$

$\mu + k\sigma = 18$

$12 - 3k = 6$

$12 - 6 = 3k$

$\frac{6}{3} = \frac{3k}{3}$

$k = 2$

then

$P(12 - 2(3) < X < 12 + 2(3)) > 1 - \frac{1}{2^2}$

$P(6 < X < 18) > 1 - \frac{1}{4} = \frac{3}{4}$

b)

Depth (x)	Frequency (f)	fx	Theoretical freq
0	122	0	121.306
1	60	60	60.65
2	15	30	15
3	2	6	3
4	1	4	0
		100	

$N = 200, \; \sum fx = 100$

i) Mean $= \frac{\sum fx}{N} = \frac{100}{200} = \frac{1}{2}$

Theoretical distribution $= N P(x)$

~~$f(x) = 200e^{-\lambda}$~~

$f(x) = N \frac{e^{-\lambda}\lambda^x}{x!}$

$f(x) = \frac{200e^{-0.5}(0.5)^x}{x!}$

Σfx

7 am Then :

$$f(0) = \frac{200 e^{-0.5} (0.5)^0}{0!} = 121$$

8

$$f(1) = \frac{200 e^{-0.5} (0.5)^1}{1!} = 61$$

9

$$f(2) = \frac{200 e^{-0.5} (0.5)^2}{2!} = 15$$

10

$$f(3) = \frac{200 e^{-0.5} (0.5)^3}{3!} = 3$$

11

$$f(4) = \frac{200 e^{-0.5} (0.5)^4}{4!} = 0$$

12 noon

1 pm) Sample size (n) = 10 $\quad S = \sqrt{7.122} = 2.67$

population mean (μ) = 50 $\quad$ then

2 t tab = 2.262

$H_0 : \mu = 50kg$ average packing is 50

$H_1 : \mu \neq 50$ average not equal

3 X	$\bar{X}$	$X - \bar{X}$	$(X - \bar{X})^2$
50	47.3	2.7	7.29
4 49	47.3	1.7	2.89
52	47.3	4.7	22.09
5 44	47.3	-3.3	10.89
45	47.3	-2.3	5.29
6 48	47.3	0.7	0.49
46	47.3	-1.3	1.69
7 45	47.3	-2.3	5.29
49	47.3	1.7	2.89
45	47.3	-2.3	5.29
473			64.1

then

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}}$$

$$t = \frac{47.3 - 50}{\frac{2.67}{\sqrt{10}}} = -3.198$$

$$|t_{cal}| = |-3.198|$$

$$t_{cal} = 3.198$$

$t_{cal} > t_{tab}$ it reject H_0

∴ The everage packing is not equal to 50 kg

$$\bar{X} = \frac{\Sigma x}{n} = \frac{473}{10} = 47.3$$

$$S^2 = \frac{\Sigma (x-\bar{x})^2}{n-1} = \frac{64.1}{9} = 7.122$$

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X_1	X_2	$\bar{X}_1$	$\bar{X}_2$	$x-\bar{X}_1$	$(\bar{x}-x_1)^2$	$x-\bar{X}_2$	$(x-\bar{X}_2)^2$
28	29	31.3	28.2	-3.3	10.89	0.8	0.64
30	30	31.3	28.2	-1.3	1.69	1.8	3.24
32	30	31.3	28.2	0.7	0.49	1.8	3.24
33	24	31.3	28.2	1.7	2.89	-4.2	17.64
33	27	31.3	28.2	1.7	2.89	-1.2	1.44
29	29	31.3	28.2	-2.3	5.29	0.8	0.64
34		31.3	28.2	2.7	7.29		
219	169				31.43		26.84

$$\bar{X}_1 = \frac{\Sigma x}{N} = \frac{219}{7} = 31.3$$

$$\bar{X}_2 = \frac{\Sigma x}{N} = \frac{169}{6} = 28.2$$

$$S_1^2 = \frac{\Sigma(x-x_1)^2}{n_1-1} = \frac{31.43}{7-1} = \sqrt{5.238} = 2.29$$

$$S_2^2 = \frac{\Sigma(x-x_2)^2}{n_2-1} = \frac{26.84}{6-1} = \sqrt{5.368} = 1.51$$

from

$$F\ test = \frac{S_1^2}{S_2^2} = \frac{5.368}{5.238}$$

$F_{cal} = 1.0248$

$F_{tab} = 4.39$

$\therefore$ $F_{cal} < F_{tab}$ its accept H_0

H_0: Horse the same running capacity

H_1: It does not have the sam

$\therefore$ Two horses have the same running capacity.

19a) $\lambda = 8/hr$
$\mu = 9/hr$

$\rho = \frac{\lambda}{\mu} = \frac{8}{9}$

i) P(Customer don't join the queue) = P_0

$P_0 = 1 - \rho$

$= 1 - \frac{\lambda}{\mu}$

$= 1 - \frac{8}{9} = \frac{1}{9}$

$P_0 = 0.111$

ii) P(there is no queue) = $P_0 + P_1$

$P_1 = \rho P_0$

$P_1 = \frac{8}{9} \times \frac{1}{9} = \frac{8}{81}$

$P_1 = 0.0987$

P(there is no queue) = $\frac{1}{9} + \frac{8}{81}$

$= 0.2098$

iii) P(there are 10 custom in system) = P_{10}

$P_{10} = \rho^{10} P_0$

$= \left(\frac{8}{9}\right)^{10} \cdot \frac{1}{9}$

$P_{10} = 0.634$

19b) $\lambda = 8/hr$
$\mu = 11/hr$
$N = 12$

i) P(not join que) = P_0

$P_0 = \frac{1-\rho}{\ldots}$

$P_0 = \frac{1 - \frac{8}{11}}{1 - \left(\frac{8}{11}\right)^{12+1}} = 0.277$

i) P_0 = 27.70 service always free

ii) P(no que) = $P_0 + P_1$

$P_0 = 0.1529$

$P_1 = \rho P_0$

$= \frac{8}{11} \times 0.277$

$P_1 = 0.2015$

then:

P(no queue) = $0.277 + 0.2015$

$= \underline{0.4785}$.

iii) P(8 custom in system) = P_8

$P_8 = \rho^8 P_0$

$= \left(\frac{8}{11}\right)^8 \times 0.277$

$P_8 = 0.0217$

a)

$$P = \begin{array}{cc} & \text{Future} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{Present}\begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix} \end{array}$$

$$p(0) = (0.7, 0.2, 0.1)$$

i) $P(X_2 = 3)$

from

$$P(X_n = a) = q_n(a) \qquad q_n = q_0 P^n$$

$$P(X_2 = 3) = q_2(3)$$

$$q_2 = q_0 P^2$$

$$q_2 = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}^2 (0.7, 0.2, 0.1)$$

$$q_2 = (0.7, 0.2, 0.1) \begin{pmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{pmatrix}$$

$$q_2 = (0.279, 0.336, 0.279)$$

$$q_2(3) = 0.279 = 27.9\%$$

ii) $P\{X_1 = 3, X_0 = 2\} = \text{initial} \rightarrow 2 \xrightarrow{1} 3$

$$= q_0(2) \cdot P_{23}^{(1)}$$

$$= 0.2 \times 0.2$$

$$= 0.04$$

iii) $P\{X_2 = 3, X_1 = 3, X_0 = 2\}$

$$\text{initial} \rightarrow 2 \xrightarrow{1} 3 \xrightarrow{1} 3$$

$$q_0(2) \times P_{23}^{(1)} \times P_{33}^{(1)}$$

$$= 0.2 \times 0.2 \times 0.3$$

$$= \underline{0.012}$$

iv) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$

$$\text{initial} \rightarrow 2 \xrightarrow{1} 3 \xrightarrow{1} 3 \xrightarrow{1} 2$$

$$q_0(2) \times P_{23}^{(1)} \times P_{33}^{(1)} \times P_{32}$$

$$= 0.2 \times 0.2 \times 0.3 \times 0.4$$

$$= 0.0048$$