11(a)

4 I
5 C
6 T
15 peoples

i) $P(3I \& 2C) = \frac{{}^4C_3 \times {}^5C_2}{{}^{15}C_5}$

$= 0.013$

ii) $P(2I \& 1C \& 2T) = \frac{{}^4C_2 \times {}^5C_1 \times {}^6C_2}{{}^{15}C_5}$

$= 0.149$

iii) $P(5T) = \frac{{}^6C_5}{{}^{15}C_5}$

$= 0.~~149~~0019$

12a) first bag: 3W, 2R, 4B — 9 ball

Second bag: 2W, 3R, 5B — 10 ball

Third bag: 3W, 4R, 2B — 9 ball

from bayes theorem

$P(B/A_i) = \frac{P(A_i) \times P(A_i/B)}{\sum P(A_i) \cdot P(A_i/B)}$

$P(A_1) = \frac{1}{3}$

$P(A_2) = \frac{1}{3}$ $\quad P(A_3) = \frac{1}{3}$

$P(A_1/B) = \dfrac{{}^3C_2 \times {}^2C_1}{{}^9C_3} = 0.071$

$P(A_2/B) = \dfrac{{}^2C_2 \times {}^3C_1}{{}^{10}C_3} = 0.025$

$P(A_3/B) = \dfrac{{}^3C_2 \times {}^4C_1}{{}^9C_3} = 0.143$

Bag	$P(A_i)$	$P(A/B)$	$P(A_i) \cdot P(A/B)$
1	$\frac{1}{3}$	0.071	0.0236
2	$\frac{1}{3}$	0.025	0.0083
3	$\frac{1}{3}$	0.143	0.0476
			0.0795

$P(B/A_1) = \dfrac{0.0236}{0.0795} = 0.296$

$P(B/A_2) = \dfrac{0.0083}{0.0795} = 0.104$

$P(B/A_3) = \dfrac{0.0476}{0.0795} = 0.599$

$P(B/A_1) = \dfrac{0.0125}{0.0345}$

$= 0.3623$

$P(B/A_2) = \dfrac{0.014}{0.0345}$

$= 0.405$

$P(B/A_3) = \dfrac{0.008}{0.0345}$

$= 0.232$

12b)

Machines	$P A_i$	$P(A/B)$	$P(A_i) \cdot P(A/B)$
A	0.125	0.05	0.0125
B	0.35	0.04	0.014
C	0.40	0.02	0.008
			0.0345

8a) PART B

7 W
6 R
5 B

18 ball

P(both whiteball) $= \frac{^7C_2}{^{18}C_2}$

$= \underline{0.137}$

b) 3 M
2 W
4 C

9 peop

P(children) $= \frac{4C_2 \times 5C_2}{9C_4} = \frac{60}{\cancel{60}126} = 0.471$

9a) P(6 pack card) $= \frac{^{26}C_3 \times 26C_3}{52C_6} = 0.332$

b) 1 Leap year = 366 days
52 sunday & 2 day extra
2 day extra calculated as

Mon, Tue
Tue, Wed
Wed, Thu
Thu, Fri
Fri, Sat
Sat, Sun
Sun, Mon

P(sunday) $= \frac{2}{7}$

10a) $P(A) = 0.9$

$P(B/A) = 0.8$

$P(A \cap B) = ?$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A) \times P(B/A)$$

$$= 0.9 \times 0.8$$

$$P(A \cap B) = \underline{0.72}$$

b) $P(S \cup K) = ?$

$$P(S \cup K) = P(S) + P(K) - P(S \cap K)$$

$$P(S) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52}$$

$$P(K) = \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{52}$$

$$P(S \cap K) = P(S) \times P(K)$$

$$= \frac{13}{52} \times \frac{4}{52}$$

$$= \underline{\frac{1}{52}}$$

$$P(S \cup K) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \underline{\frac{4}{13}}$$

PART A :

1.) P(happening event) = $\frac{\text{no of favourable case}}{\text{total no of exhustive events}}$

2.)

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

P(throwing 10 with 2 dice) = $\frac{3}{36} = \frac{1}{12}$

3.) P(getting queen) = $\frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

4.) 8 red
9 Black
17

P

P(redball) = $\frac{{}^8C_1}{{}^{17}C_1} = \frac{8}{17}$

5.) P(diamond) = $\frac{{}^{13}C_1}{{}^{52}C_1} = \frac{1}{4}$

6) Law of Additional probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

7) Bayes theorem

$$P(B/A_i) = \frac{P(A_i) \cdot P(A_i/B)}{\sum P(A_i) \cdot P(A_i/B)}$$

CAT II.

PART C:

11a) $\mu = 12$

$\sigma^2 = 9 = 3$

$P(6 < x < 18) = ?$

S.D

$P(-\sigma k < |x - \mu| < \sigma k) < 1 - \frac{1}{k^2}$

$P(\mu - \sigma k < x < \mu + \sigma k) < 1 - \frac{1}{k^2}$

by comparing

$\mu - \sigma k = 6$ --- (I)

$\mu + \sigma k = 18$ --- (II)

$12 - 3k = 6$

$12 - 6 = 3k$

$6 = 3k$

$\underline{k = 2}$

$P(12 - 3(2) < x < 12 + 3(2)) < 1 - \frac{1}{2^2}$

$P(6 < x < 18) < 1 - \frac{1}{4} = \underline{\frac{3}{4}}$

11b) from Geometric distribution

$P(x) = q^x p$

$P(x) = f(x) = \sum_{i=0}^{n} x\, p(x)$

$P(x) = \sum_{i=0}^{n} x\, q^{x} p$

$= p \sum_{i=0}^{n} x\, q^{x}$

$= p\,[0 + q + 2q^2 + \cdots$

$= pq\,[1 + 2q + 3q^2 \cdots$

$= pq\,(1-q)^{-2}$

$= \frac{pq}{(1-q)^2}$

but $p + q = 1$

$p = 1 - q$

$= \frac{\cancel{p}q}{p^{\cancel{2}}}$

Mean $= q/p$

Variance $= p/q^2$

1Ra) Sample size = 10
population mean μ = 50
t_{tab} = 2.262

$$t \text{ test} = \frac{\bar{X} - \mu}{\frac{S.D}{\sqrt{n}}}$$

$$S.D = \sum \frac{(x - \bar{x})^2}{n-1}$$

$\bar{X}$ - sample mean
μ - population mean
S.D - Standard deviation
n - sample size

$H_0 : \mu = 50$ (the average packing 80
$H_1 : \mu \neq 50$

X	$\bar{X}$	$X - \bar{X}$	$(X - \bar{X})^2$
50	47.3	2.7	7.29
49	47.3	1.7	2.89
52	47.3	4.7	22.09
44	47.3	-3.3	10.89
45	"	-2.3	5.29
48	"	0.7	0.49
46	"	-1.3	1.69
45	"	-2.3	5.29
49	"	1.7	2.89
45	47.3	-2.3	5.29
73			64.1

$$\bar{X} = \frac{\sum x}{n}$$

$$= \frac{473}{10} = 47.3$$

variance

$$S.D = \frac{\sum (x-\bar{x})^2}{n-1}$$

$$S.D^2 = \frac{64.1}{10-1} = 7.12$$

$$S.D = 2.67$$

$$t_{test} = \frac{\bar{x} - \mu}{\frac{S.D}{\sqrt{n}}}$$

$$= \frac{47.3 - 50}{\frac{2.67}{\sqrt{10}}}$$

$$|t_{test}| = |3.29|$$

$$t_{tab} = 3.19$$

$t_{cal} > t_{tab}$ it je reject H_0

Q2b) $N = 12$

$\lambda = 8/hr$

$\mu = 11/hr$

i) $P_0 = ?$

Finite queue

i) $P_0 = \frac{1-\rho}{1-\rho^{N+1}}$

$\rho = \lambda/\mu = 8/11 =$

$P_0 = \frac{1 - 8/11}{1 - (8/11)^{12+1}}$

$P_0 = \frac{3/11}{0.984}$

$P_0 = 0.2771$

$P_0 \approx 27.7\%$

ii) $P_0 + P_1 = P(\text{no queue})$

$P_1 = \rho P_0$

$P_1 = \frac{8}{11} \times 0.2771$

$P_1 = 0.2015 =$

$P(\text{no queue}) = 0.2015 + 0.2771$

no qu $= 0.4786 \approx 47.86\%$

$P_N = \rho^n P_0$

iii) $P(8 \text{ customer}) = \rho^8 P_0$

$= \left(\frac{8}{11}\right)^8 \times 0.2771$

$= 0.0217$

$P(8 \text{ custom} \approx 2.17\%$

PART B

$q^{x-1} \cdot p$

Sol $P(X=x) = q^{x-1}p, \; x = 1, \dots$

$M_x t = \sum e^{tx} P(x)$

$M_x(t) = \sum_{i=1}^{n} e^{tx} q^{x-1} p$

$= p \sum_{i=1}^{n} e^{tx} \frac{q^x}{q}$

$= \frac{p}{q} \sum_{i=1}^{n} e^{tx} q^x$

$= \frac{p}{q} [e^t q + (e^t q)^2 + \dots$

$= \frac{p}{q} \cdot e^t q [1 + (e^t q) + (e^t q)^2 \dots$

$= pe^t [(1 - e^t q)^{-1}]$

$\frac{u}{v} = \frac{u'v - v'u}{v^2}$

$M_x(t) = \frac{pe^t}{1 - e^t q}$

Mean $= M_x'(0)$

Mean $= \frac{d}{dt} \left(\frac{pe^t}{1 - e^t q} \right)$

$= \frac{\cancel{pe^t(-e^t q)} + (1 - e^t q) pe^t - pe^t(-e^t q)}{(1 - e^t q)^2}$

$= \frac{pe^t - \cancel{e^t q p} + \cancel{e^t} pq}{(1 - e^t q)^2} = \frac{pe^t}{(1 - e^t q)^2}$

Mean $= \dfrac{pe^0}{(1-e^0q)^2} = \dfrac{p}{(1-q)^2} = \dfrac{1}{p}$

Varience $= M_x''(0) - (M_x(0))^2$

$= \underline{q/p^2}$

8b) $P(x) = {}^nC_x\, p^x q^{n-x}$, $n = 20$

$p = 0.1$

$q = 1-p = 1-0.1 = 0.9$

$P(x) = {}^{20}C_x (0.1)^x (0.9)^{20-x}$

i) $P(x \le 3) = P(0) + P(1) + P(2) + P(3)$

$P(0) = {}^{20}C_0 (0.1)^0 (0.9)^{20-0} = \underline{0.1216}$

$P(1) = {}^{20}C_1 (0.1)^1 (0.9)^{20-1} = \underline{0.27017}$

$P(2) = {}^{20}C_2 (0.1)^2 (0.9)^{20-2} = \underline{0.2851}$

$P(3) = {}^{20}C_3 (0.1)^3 (0.9)^{20-3} = \underline{0.1901}$

$P(x \le 3) = 0.1216 + 0.27017 + 0.2851 + 0.1901$

$P(x \le 3) = 0.86697$

ii) $P(x = 3) = {}^{20}C_3 (0.1)^3 (0.9)^{20-3}$

$= \underline{0.1901}$

$n = 26$

9a) $\bar{x} = 990$

$SD = 20$

$\mu = 1000$

$H_0 : \mu = 1000$ (population is upto standard).

$H_1 : \mu < 1000$ population is not upto standard

$$t = \frac{\bar{x} - \mu}{\frac{S.D}{\sqrt{n}}} = \frac{990 - 1000}{\frac{20}{\sqrt{26}}}$$

$|t_{cal}| = |-2.5495|$

$t_{cal} = 2.5495$

$t_{tab} = 1.708$

$t_{cal} > t_{tab}$ it reject H_0.

$\therefore$ Sample is upto standard.

9b) $p = \frac{325}{600} = 0.542$

no of smoker + no of non smoker = 1

$P = 0.5$

$Q = 1 - 0.5 = 0.5$

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.542 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{600}}}$$

$Z = 2.058$

$Z_{cal} = 2.058$

$Z_{tab} = 1.645$

$Z_{cal} > Z_{tab}$ it reject H_0

10a) $U = \left(\frac{1}{4}, 0, -\frac{3}{4}, \frac{1}{2}\right)$ its not probability vector bcs its negative

ii) $V = \left(\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}\right)$ its probability, non negative vector = sum of row = 1

b) $A = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Its schoshatics matrix = $0 + 1 = 1$

$\frac{1}{2} + \frac{1}{2} = 1$

$$A^2 = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 + \frac{1}{2} & 0 + \frac{1}{2} \\ 0 + \frac{1}{4} & \cancel{0}\frac{1}{2} + \frac{1}{4} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

PART A :

1) Binomial Distribution

$P(x) = {}^nC_x \, p^x q^{n-x}$

2) $P(x=1) = \frac{3}{10}$

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(x=1) = \frac{e^{-\lambda}\lambda^1}{1!} = \frac{3}{10} \quad \text{--- (i)}$$

$$P(x=2) = \frac{1}{5}$$

$$\frac{1}{5} = \frac{e^{-\lambda}\lambda^2}{2!} \quad \text{--- (ii)}$$

ii ÷ i

$$\frac{1}{5} \times \frac{10}{3} = \frac{\cancel{e^{-\lambda}}\lambda^2}{2} \times \frac{1}{\cancel{e^{-\lambda}}\cancel{\lambda}}$$

$$\frac{2}{3} = \frac{\lambda}{2}$$

$$3\lambda = 4$$

$$\lambda = \frac{4}{3}$$

i) $P(x=0) = \dfrac{e^{-4/3} \cdot (4/3)^0}{0!} = e^{-4/3} = 0.126$

ii) $P(x=3) = \dfrac{e^{-4/3}(4/3)^3}{3!} = 0.104$

3. Exponential distribution

$$P(x) = f(x) = \begin{cases} \lambda e^{\lambda x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

5. $F_{test} = \dfrac{S_1^2}{S_1^2}$

S_1^2 - S.D greater

S_1^2 = smaller S.D

6. T test for difference mean

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

7. $W_q = \dfrac{L_q}{\lambda}$

$$t_{test} = \frac{\bar{X} - \mu}{\frac{S.D}{\sqrt{n}}}, \quad t_{te} = \frac{\bar{X}_1 - \bar{X}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$F\text{-test} = \frac{S_1^2}{S_2^2}$$

Z - test

$$Z = \frac{\bar{X} - \mu}{\frac{S.D}{\sqrt{n}}}, \quad \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \quad P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

CamScanner

UE

PART A

8.) $W_q = \frac{L_q}{\lambda_{eff}}$

$\lambda_{eff} = \lambda(1 - P_N)$.

9> Markov Chain, Let $\{X_0, X_1, X_2 \ldots\}$ be square of discrete random variables. Then $\{X_n, n \geq 0\}$ is a markov chain if it satisfy the markov Property.

$$P(X_{t+1} = J \mid X_t = i, X_{t-1} = i_1, X_t = i_2)$$

$$= P(X_{t+1} = J / X_t = i).$$

10> Chapman - kolmogorov theorem.

Let $\{X_n, n \geq 0\}$ be a homogeneous Markov chain with TPM $(P) = P_{ij}$ and n-step TPM.

$P^{(n)} = P_{ij}^{(n)}$ then n-step TPM $P^{(n)} = P_{ij}^{(n)}$ is equal to the nth power of the one-step TPM P.

$$P_{ij}^{(n)} = (P_{ij}^{(1)})^n.$$

PART B

11b) $P(x) = \int_{0.2}^{0.5} \frac{3x^2}{10} dx$

$= \frac{3}{10} \int_{0.2}^{0.5} x^2 dx$

$= \frac{3}{10} \left[\frac{1}{3} x^3 \right] \Big|_{0.2}^{0.5}$

$= \frac{1}{10} \left[0.5^3 - 0.2^3 \right]$

$P(x) = \frac{1}{10} \times 0.0117$

$P(x) = \int_0^1 \frac{3x^2}{10} dx$

$= \frac{3}{10} \left[\frac{1}{3} x^3 \right] \Big|_0^1$

$= \frac{1}{10}$

$S_n \cdot \frac{A_1(1-r^n)}{1-r}$

14a) P_0 single server for finite queue

$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$

$P_N = \rho^n P_0$

$P_0 = \frac{P_N}{\rho^N}$

$\Sigma P_N = \Sigma \rho P_0 + \rho^2 P_0 + \rho^3 P_0 \cdots \rho^N P_0$

$P_N = P_0 \rho (\cancel{1} + \rho + \rho^2 + \cdots \rho^N)$

$P_N = P_0 \rho \left(\frac{1(1-\rho^{N+1})}{1-\rho} \right)$

$P_N = \frac{P_0 \rho (1-\rho^{N+1})}{1-\rho}$

$P_0 = \frac{P_N (1-\rho)}{(1-\rho^{N+1})}$

$P_0 + P_1 + P_2 \dots = 1$

$\lim_{P_n} = \dfrac{1-\rho}{1-\rho^{N+1}}$

P_0

$\int_{-\infty}^{\infty} \int_0^1$

i) $\rho = \dfrac{\lambda}{\mu}$

$P_8 = \rho^8 P_0$

$P_0 = 1-\rho$

PART C :

16b) $f(x) = 3x^2$

i) $P(X \leq a) = P(X > a)$

$P(X \leq a) + P(X > a) = 1$

$P(X \leq a) = 0.5$

$\int_0^a 3x^2 dx = 0.5$

$x^3 \Big/_0^a = 0.5$

$a^3 - 0^3 = 0.5$

$a = (0.5)^{\frac{1}{3}} =$

ii) $P(x > b) = 0.05$

$\int_1^b 3x^2 dx = 0.05$

$x^3 \Big/_1^b = 0.05$

$b^3 - 1^3 = 0.05$

$b^3 = 0.05 + 1$

$b^3 = 1.05$

$b = (1.05)^{\frac{1}{3}}$

17b) $F(x) = N\,P(x)$

$P(x)$ - poisson

$P(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

x	f	f(x)	theoretical
0	122	0	~~0~~ 121
1	60	60	60.65
2	15	30	15.16
3	2	6	3 1.33
4	1	4	10 0.316
	200		

$f(x) = \dfrac{\sum fx}{N} =$ Mea

Mean $= \dfrac{100}{200} = 0.$

Mean $= \lambda = 0.$

$F(x) = N\,P(x)$

$$\lim_{P_n} = P_0 + P_1 + P_2 \cdots = 1$$

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} \qquad \int_{-\infty}^{\infty} \quad \int_0^1$$

b) $\rho = \frac{\lambda}{\mu}$

$P_8 = \rho^8 P_0$

$P_0 = 1 - \rho$

PART C :

16b) $f(x) = 3x^2$

i) $P(X \le a) = P(X > a)$

$P(X \le a) + P(X > a) = 1$

$P(X \le a) = 0.5$

$\int_0^a 3x^2 dx = 0.5$

$x^3 \Big/_0^a = 0.5$

$a^3 - 0^3 = 0.5$

$a = (0.5)^{1/3} =$ ______

ii) $P(x > b) = 0.05$

$\int_1^b 3x^2 dx = 0.05$

$x^3 \Big/_1^b = 0.05$

$b^3 - 1^3 = 0.05$

$b^3 = 0.05 + 1$

$b^3 = 1.05$

$b = (1.05)^{1/3}$

17b) $f(x) = N\,P(X)$

$P(x)$ – poisson

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

x	f	f(x)	theoretical
0	122	0	121
1	60	60	60.65
2	15	30	15.16
3	2	6	3.~~33~~
4	1	4	0.316
	200		

$$f(x) = \frac{\Sigma f x}{N} = \text{Mean}$$

$$\text{Mean} = \frac{100}{200} = 0.5$$

$\text{Mean} = \lambda = 0.5$

$f(x) = N\,P(x)$

18b)

X_1	$\overline{X_1}$	$X-\overline{X_1}$	$(X-\overline{X_1})^2$	X_2	$\overline{X_2}$	$X-\overline{X_2}$	$(X-\overline{X_2})^2$
				29	28.1	0.9	0.81
28	31.29	-3.29	10.82	30	28.1	1.9	3.61
30	31.29	-1.29	1.66	30	28.1	1.9	3.61
32	31.29	0.71	0.50	24	28.1	-4.1	16.81
33	31.29	1.71	2.92	27	28.1	-1.1	1.21
33	31.29	1.71	2.92	29	28.1	0.9	0.81
29	31.29	-2.29	5.24	169			$\Sigma = 28.86$
34	31.29	2.71	7.34				
219			$\Sigma = 31.4$				

$$\overline{X_1} = \frac{\Sigma X}{n} = \frac{219}{7} = 31.29$$

$$\overline{X_2} = \frac{\Sigma X}{n} = \frac{169}{6} = 28.17$$

$$S.D = \frac{\Sigma(X-\overline{X_1})^2}{n_1 - 1} = \frac{31.4}{7-1} = 5.23$$

$$S.D = \frac{\Sigma(X-\overline{X_2})}{n-1} = \frac{28.86}{5} = 5.772$$

$$F = \frac{S_1^2}{S_2^2}$$

$$F = \frac{5.772}{5.23} = 1.104$$

1 Qa) P_0 = infinite

i) $P_0 = 1 - \rho$

$P_0 = 1 - \frac{\lambda}{\mu}$

arrival

$\lambda = 8/hr$

$\mu = 9/hr$

serve time

$P_0 = 1 - \frac{8}{9} = $ —

ii) P(no queue) $= P_0 + P_1$

$P_1 = \rho P_0 = \frac{\lambda}{\mu} P_0$

iii) $P_{10} = \rho^{10} P_0$

$= \left(\frac{\lambda}{\mu}\right)^{10} P_0$

2Qa) $P = \begin{pmatrix} \overset{11}{0.1} & \overset{12}{0.5} & \overset{13}{0.4} \\ \overset{21}{0.6} & \overset{22}{0.2} & \overset{23}{0.2} \\ \overset{31}{0.3} & \overset{32}{0.4} & \overset{31}{0.3} \end{pmatrix}$

$q_0 = (0.7, 0.2, 0.1)$

i) $P(X_2 = 3)$

$P(X_n = a) = q_n(a)$

$P(X_2 = 3) = q_2(3)$

$q_n = q_0 P^n$

$q_2 = q_0 P^2$

$P^2 = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.1 \end{pmatrix}^2$

$P^2 = \begin{pmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{pmatrix}$

$q_2 = (0.7, 0.2, 0.1) \times \begin{pmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{pmatrix}$

$q_2 = (\overset{11}{0.279}, \overset{12}{0.336}, \overset{13}{0.279})$

$P(X_2 = 3) = q_2(3)$

$= 0.279$

$\approx 27.9\%$

ii) $P(X_1 = 3, X_0 = 2) = \text{initial} \longrightarrow 2 \xrightarrow{1} 3$

$$= q_0(2) \times P_{23}^{(1)}$$

$$= 0.2 \times 0.2$$

$$= 0.04$$

$$\approx 4\%$$

iii) $P\{X_2 = 3, X_1 = 3, X_0 = 2\} = \text{initial} \longrightarrow 2 \xrightarrow{1} 3 \xrightarrow{1} 3$

$$= q_0(2) \times P_{23}^{(1)} \times P_{33}^{(1)}$$

$$= 0.2 \times 0.2 \times 0.3$$

$$= \underline{\hspace{3cm}}$$

iv) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

$$= \text{Initial} \longrightarrow 2 \longrightarrow 3 \longrightarrow 3 \longrightarrow 2$$

$$= q_0(2) \times P_{23}^{(1)} \times P_{33}^{(1)} \times P_{32}^{(1)}$$

$$= 0.2 \times 0.2 \times 0.3 \times 0.4$$

$$= \underline{\hspace{3cm}}$$