ST. JOSEPH COLLEGE OF ENGINEERING AND TECHNOLOGY
DEGREE CONTINUOUS ASSESSMENT TEST – I, MAY-2024
MA 2208 – PROBABILITY AND QUEUING THEORY

SEMESTER: IV **YEAR : II**
DURATION: 2 HOURS **MAX. MARKS: 50**

ANSWER ALL QUESTIONS

PART – A (7 x 2= 14)

1. Define mathematical definition of probability.
2. Find the probability of throwing 10 with two dice.
3. From a pack of 52 cards, one card is drawn at random. Find the probability of getting a queen.
4. A bag contains 8 red and 9 black balls. Find the probability of drawing a red ball.
5. Define continuous random variable.
6. Define mean and variance.
7. Define expectation of a random variable.

PART – B (3 x 4=12)

8.(a) A bag contains 7 white, 6 red and 5 black balls. Two balls are drawn at random. Find the probability that they will both be white.

(OR)

8.(b) Is the function defined as follows probability density function?

$$f(x) = \begin{cases} 0, & if\, x < 2 \\ \frac{3+2x}{18}, & if\, 2 \le x \le 4 \\ 0, & if\, x > 4 \end{cases}$$ If so find the $P(2 \le X \le 4)$.

9.(a) Find the binomial distribution for which the mean is 4 and variance is 3.

(OR)

9.(b) A machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws, what is the probability that there are (i) exactly three defectives (ii) not more than three defectives.

10.(a) A die is cast until 6 appears. What is the probability that it must be cast more than five times? (Apply Geometric Distribution)

(OR)

10.(b) *If X is a Poisson variate* $P(X = 2) = 9 P(X = 4) + 90\, P(X = 6)$, find (i) mean of X, (ii) variance of X.

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ANSWER ALL QUESTIONS

PART – A **(7 x 2= 14)**

1. State ψ^2 – test of goodness of fit.
2. Describe z - test for difference of means.
3. State the formula for expected number of people in the system for single server infinite queue length model.
4. State the formula for expected waiting time in the system for single server finite queue length model.
5. Define Markov chain.
6. Define transition probability matrix (TPM).
7. State Chapman-Kolmogorov theorem.

PART – B **(3 x 4=12)**

8.(a) A sample of 26 bulbs gives a mean life of 990 hours with a S.D. of 20 hours. The manufacturer claims that the mean life of bulbs is 1000 hours. Is the sample not up-to the standard? (Use tabulated value of t for 25 degrees of freedom at 5% level is 1.708).

(OR)

8.(b) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same, at 5% level. (Given tabulated z – value at 5% level is 1.96).

9.(a) For single server infinite queue length model, if arrival rate is 13 per hour and service rate is 15 per hour, then find the probability that there is no queue?

(OR)

9.(b) For single server finite queue length (N = 10) model, if arrival rate is 15 per hour and service rate is 18 per hour, then find the probability that the server is free?

10.(a) Check whether the following vectors are probability vectors. If not explain

(i) $$u = \left(\frac{1}{4},\ 0,\ -\frac{3}{4},\ \frac{1}{2}\right)$$

(ii) $$v = \left(\frac{1}{4},\ 0,\ \frac{1}{4},\ \frac{1}{2}\right)$$

(OR)

10.(b) Show that $A = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ is a stochastic matrix. Check whether it is regular?

PART – C

(2 x 12 = 24)

11.(a) Certain pesticide is packed into bags by a machine. A random sample of 10 bags is drawn and their contents are found to weigh (in kg.) as follows: 50, 49, 52, 44, 45, 48, 46, 45, 49, 45. Test if the average packing can be taken to be 50 kg. (Given tabulated t – value for 9 d.f. at 5% level is 2.262).

(OR)

11.(b) For single server finite queue length (N = 12) model), customers arrive at a barber shop at the rate of 8 per hour (poisson arrival) and the barber can serve at the rate of 11 per hour (exponential).

(i) What is the probability that a customer does not join the queue and walks into the barber's chair?

(ii) What is the probability that there is no queue?

(iii) What is the probability that there are 8 customers in the system?

12.(a) The transition probability matrix of a Markov chain $\{X_n\}$, three states 1, 2 and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $P^{(0)} = (0.7,\ 0.2,\ 0.1)$. Find (i) $P\{X_2 = 3\}$, (ii) $P\{X_1 = 3,\ X_0 = 2\}$, (iii) $P\{X_2 = 3,\ X_1 = 3,\ X_0 = 2\}$, and (iv) $P\{X_3 = 2,\ X_2 = 3,\ X_1 = 3,\ X_0 = 2\}$.

iv)

$2 \xrightarrow{(1)} 3 \xrightarrow{(1)} 3 \xrightarrow{(1)} 2$

$= 0.2 \times 0.2 \times 0.3 \times 0.4$

$= 0.0048$

$\frac{4\,73}{10}$

(OR)

12.(b) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B throws to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian and hence find the transition probability matrix P and also find P^2 and P^3.

iii) $P\{X_2 = 3, X_1 = 3, X_0 = 2\}$

$2 \xrightarrow{(1)} 3 \xrightarrow{(1)} 3$

$= P^{(0)}(2) \times P^1_{23} \times P^1_{33}$

$= 0.2 \times 0.2 \times 0.3 = 0.012$

i) $P(X_2 = 3)$

$P(X_n) = 3 = 0.1$

ii) $P(X_1 = 3, X_0 = 2)$

$2 \xrightarrow{(1)} 3$

$= P^0(2) \times P^1_{23}$

$= 0.2 \times 0.2$

ii) $= 0.4$