## Problem Set 7

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1 Show that, under the assumption of homogeneous treatment effects, Wooldridge's Assumption  $FE_1$  implies the parallel trends assumption does not vary with g (for any  $t \ge 2$ ).

We compute the last expression and will make use of the different assumptions along the way to derive an expression which does not depend on g. The homogeneity of the treatment and the linear model yields

$$\mathbb{E}\left(Y_{g,t}(0) - Y_{g,t-1}(0)\right) = \mathbb{E}\left(\alpha_g + \lambda_t + \beta \cdot 0 + \varepsilon_{g,t} - \alpha_g - \lambda_{t-1} - \beta \cdot 0 - \varepsilon_{g,t-1}\right)$$
which simplifies to

$$\mathbb{E}\left(Y_{q,t}(0) - Y_{q,t-1}(0)\right) = \mathbb{E}\left(\lambda_t - \lambda_{t-1}\right) + \mathbb{E}\left(\varepsilon_{q,t} - \varepsilon_{q,t-1}\right)$$

The first term does not depend on g. The second is equal to zero, indeed, according to the law of iterated expectations,

$$\mathbb{E}\left(\varepsilon_{g,t}\right) = \mathbb{E}\left(\mathbb{E}(\varepsilon_{g,t}|\alpha_g, D_{g,t})\right) = 0$$

and similarly for t-1. We have proved the expected result.

- 3 Coding part
- a) Note that the treatment effect estimate for group 2007 and time 2004 is equal to the event study estimate for event time -3. Similarly, note that the treatment effect estimate for group 2004 and time 2007 is equal to the event study estimate for event time 3. Why is this so?

The only group for which we can study a treatment effect three years before treatment is the 2007 group. Therefore the -3 parameter in the event study is identical as the treatment effect for group 2007 in 2004. The same goes for group 2004 in 2007, it is the only group for which we can evaluate a treatment effect three years after treatment.

## b) How many never treated groups (or counties) are there?

Running a short piece of code and not forgetting to group by the *countyreal* variable we get 309 never treated counties.

c) Redo the analysis using the not yet treated group as control group. How do the results compare? Are you surprised (given your answer to the previous question)?

At first glance the results seem very similar (see Figure ??). The by default control group is the never treated groups. Thus, using the not yet treated group could increases the precision of results, as is it the case here (but not very much).

```
Group-Time Average Treatment Effects:
 Group Time ATT(g,t) Std. Error [95% Simult.
                                                Conf. Band]
             -0.0105
  2004 2004
                          0.0249
                                        -0.0773
                                                      0.0563
  2004 2005
                                                      0.0215
             -0.0704
                          0.0343
                                        -0.1624
                                                    -0.0293 *
  2004 2006
             -0.1373
                          0.0403
                                        -0.2452
  2004 2007
             -0.1008
                          0.0369
                                        -0.1996
                                                    -0.0020 *
  2006 2004
              0.0065
                          0.0240
                                        -0.0576
                                                      0.0707
  2006 2005
             -0.0028
                          0.0195
                                        -0.0550
                                                      0.0495
                                                      0.0430
  2006 2006
             -0.0046
                                        -0.0522
                          0.0178
                                        -0.0996
  2006 2007
             -0.0412
                          0.0218
                                                      0.0172
                          0.0146
  2007 2004
              0.0305
                                        -0.0085
                                                      0.0696
  2007 2005
             -0.0027
                                        -0.0485
                          0.0171
                                                      0.0430
  2007 2006
             -0.0311
                                        -0.0795
                          0.0181
                                                      0.0173
  2007 2007
             -0.0261
                          0.0166
                                        -0.0706
                                                      0.0185
Signif. codes: `*' confidence band does not cover 0
P-value for pre-test of parallel trends assumption:
Control Group: Never Treated, Anticipation Periods: 0
Estimation Method: Outcome Regression
```

```
Group-Time Average Treatment Effects:
                                                Conf. Band]
 Group Time ATT(g,t) Std. Error [95% Simult.
  2004 2004
             -0.0194
                          0.0248
                                       -0.0863
                                                     0.0475
  2004 2005
             -0.0783
                          0.0314
                                       -0.1630
                                                     0.0063
  2004 2006
             -0.1363
                          0.0391
                                       -0.2417
                                                    -0.0309 *
  2004 2007
             -0.1008
                          0.0354
                                       -0.1964
                                                    -0.0052 *
  2006 2004
                          0.0239
                                                     0.0618
             -0.0026
                                       -0.0669
  2006 2005
                                       -0.0544
                                                     0.0505
             -0.0019
                          0.0194
  2006 2006
              0.0047
                          0.0169
                                       -0.0410
                                                     0.0504
  2006 2007
             -0.0412
                          0.0203
                                       -0.0961
                                                     0.0137
  2007 2004
              0.0298
                          0.0144
                                       -0.0090
                                                     0.0685
  2007 2005
                                       -0.0435
             -0.0024
                          0.0152
                                                     0.0387
  2007 2006
             -0.0311
                          0.0180
                                       -0.0797
                                                     0.0175
  2007 2007
             -0.0261
                          0.0169
                                       -0.0715
                                                     0.0194
Signif. codes: `*' confidence band does not cover 0
P-value for pre-test of parallel trends assumption: 0.16814
Control Group: Not Yet Treated, Anticipation Periods:
Estimation Method: Outcome Regression
```

d) Define the treatment variable D equal to 1 if county g is treated in time t and 0 otherwise, i.e., replace the "..." with the appropriate code in the ExampleDID.R file. Then use the two-way fixed effects estimator to estimate the treatment effect. What do you find? How does your estimate compare to the corresponding estimate(s) obtained using the "did" package?

Table 1: Two-Way Fixed Effects Estimate

	Dependent variable:	
	lemp	
D	-0.296***	
	(0.072)	
Observations	2,500	
$R^2$	0.007	
Adjusted $R^2$	0.005	
F Statistic	$17.019^{***} (df = 1; 2494)$	
$\overline{Note}$ :	*p<0.1; **p<0.05; ***p<0.01	

By using the plm package we estimated the two-way fixed effects estimator (see Table ??).

- e) Implement the "classical" event study design using the two-way fixed effects estimator. What do you find? How do your estimates compare to the (sets of) estimates obtained using the "did" package?
- f) Run the twowayfeweights function. What do you find?

Treat. var: D	ATTs	Σ weights
Positive weights Negative weights	1198 638	1.6988 -0.6988
Total	1836	1

Figure 1: twowayfeweights

g) How do you interpret your findings in (d) and (f)?

The two way fixed effect model estimates a significant negative impact of the treatment on the dependent variable (lemp). However, the results from the two-way fixed effects weights method indicate that out of the 1836 estimated ATTs, 1198 receive a positive weight and 638 receive a negative weight. This suggests diversity in the average treatment effects observed in the sample.