



Abstract

It is well known in the industry that measuring the effectiveness of various marketing channels is difficult and that makes allocating the marketing budget optimally an arduous task. Nowadays, marketers have powerful statistical tools to help with analysis and decision making, one of which is Marketing Mix Modelling (MMM).

In our project we have conducted an MMM analysis on DT Mart's marketing data. Several regression-based models are built with R. Lag and adstock transformation are applied to the variables to reflect the lag and decay effect of advertising, respectively. As severe multicollinearity in the data is detected, two methods are exploited to tackle this challenge: variable selection and ridge regression. Subset regression models are built and analyzed after variable selection. The thirteen models built using training data are then validated using test data. Prediction accuracies are compared using Mean Squared Prediction Error (MSPE) and Precision Error (PM). The ridge regression model is selected as the best performing model and used to analyze the most and least effective media channels. A media investment optimization is proposed based on the ridge regression's suggestion and would lead to a 50% growth in revenue.

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1. Introduction

1.1 Background

The marketing mix is an essential business theory, which is the set of actions, or tactics, that a company uses to promote its brand or product in the market [1]. Price, Product, Promotion and Place are the 4Ps that make up a typical marketing mix. The 4Ps of the marketing mix influence each other. They set up the business plans for companies, to pursue their marketing objectives in the target market [2].

Marketing mix modeling (MMM) is a statistical modeling that uses historic data, to determine market attributions, evaluate the impact of each marketing channel, and then forecast the impact of future sets of tactics [3].

1.2 Literature Review

To build and apply MMM, Yuan J. (2020) introduced the guide with dataset using Python, R, and Excel. The article describes common data types for MMM, based variables, incremental variables, and others. Trent Y. (2022) built a simple MMM and make prediction using R. This paper introduced the difference between MMM and Multi Touch Attribution (MTA). For the MTA model, last click attribution and first click attribution and time decay were discussed. Then a MMM was built using R. Correlation between variables were checked using the PerformanceAnalytics package. Adstock was defined and applied. Tripathi H. (2021) referred to MMM as the butterfly effect, since a small change in the advertising strategy could make a high impact on the investments. This paper talked about the objective of MMM. Marketing mix elements were discussed. A blog introduced the definitions of marketing mix and marketing mix modeling. It compared the differences between marketing mix modeling and attribution modeling. Technologies such as linear regression model, multiplicative regression models were covered.

2. Project Overview

2.1 Problem Statement and Objective

In this project, DT Mart's marketing and sales data using marketing mix modeling will be studied. DT Mart is an Indian retailer for electronic products including cameras, gaming hardware, and entertainment products. We aim to find out the most and least effective marketing channels, try to optimize the marketing investment allocation, and see how much revenue growth can be achieved by doing so.



2.2 Approach

Our main approach is to combine the marketing and sales data first, split the data into training and test sets. Then we build and train models based on the training data, compare the models' performance using the test data. Finally, we'll optimize the marketing budget allocation as suggested by the best model and analyze the influence on revenue.

2.3 Hypothesis

The main hypothesis is that given the nature of DT Mart's business, and their target clientele, digital marketing channels such as online marketing, affiliates, and SEM should be more effective than traditional marketing channels such as TV and radio.

3. Data Transformation and Exploratory Data Analysis (EDA)

3.1 Data Cleaning and Transformation

We have two data sets, both from Kaggle. These are raw data, so preparing and cleaning data is an important job we should do before we start our model building and analyzing. In this process we use python to help us refine our job. After a preliminary inspection, we make the following assumptions about our data:

- Currency: we assume that the currency is use is Indian Rupee (INR) as the data suggests that the promotions occur on Indian public holidays
- Media investment and revenue amount: The values seen in the media investment data set are very small. A rule of thumb for marketing investment is that it should be 2% 5% of the revenue. Working backwards, we deduce that the media investment amount must be in the hundreds of thousands (00,000) so that the total media investment of DT Mart is 2.1% of the total revenue. As such, we will transform the "gmv_new" data into hundreds of thousands (gmv_new / 100000) to keep the units consistent

A quick overview of the original data can be seen in Table 3-1.

Table 3-1 Overview of the Original Data

Dataset	Key Variables	Source	Cleaning Process
Media Investment	TV, Digital, Sponsorship, Content Marketing, Online Marketing, Affliates, SEM, Radio, Other	Kaggle - DT MART: Market Mix Modeling	Transform monthly data into weekly data, merge with Sales Revenue data
Sales Revenue (firstfile.csv)	gmv_new, special_sale	Kaggle - DT MART: Market Mix Modeling	Transform daily data into weekly data, merge with Media Investment data



The media investment dataset is relatively simple, displaying the monthly spending of each marketing channel, spanning July 2015 to June 2016. We can easily check that there are no null values, duplicates or obvious faults.

The values in this dataset need to be transformed from monthly to weekly, as our team will be using marketing mix modeling, where it is standard to use weekly data. Furthermore, the sales dataset has daily values, so it makes sense to unify them to a weekly time resolution. An excerpt of the transformed dataset can be seen in Table 3-2:

Table 3-2 An Excerpt of the Transformed Media Investment Data

+	+	+	⊦	+ -		+		+-	+	+
week_of_year	total_invest	TV	Digital	sponsorship	content_marketing	online_marketing	affiliates S	SEM	radio	other
1	3.42	0.04	0.5	1.48	0.0	0.26	0.1 1	.0	0.0	0.0
2	3.42	0.04	0.5	1.48	0.0	0.26	0.1 1	.0	0.0	0.0
3	3.42	0.04	0.5	1.48	0.0	0.26	0.1 1	.0	0.0	0.0
4	3.42	0.04	0.5	1.48	0.0	0.26	0.1 1	.0	0.0	0.0
5	3.42	0.04	0.5	1.48	0.0	0.26	0.1 1	.0	0.0	0.0
6	1.28	0.0	0.33	0.28	0.0	0.03	0.03 0.	63	0.0	0.0
7	1.28	0.0	0.33	0.28	0.0	0.03	0.03 0.	63	0.0	0.0
8	1.28	0.0	0.33	0.28	0.0	0.03	0.03 0.	63	0.0	0.0
9	1.28	0.0	0.33	0.28	0.0	0.03	0.03 0.	63	0.0	0.0
10	19.26	0.78	0.28	12.56	0.12	3.28	1.0 1.	24	0.0	0.0
11	19.26	0.78	0.28	12.56	0.12	3.28	1.0 1.	24	0.0	0.0
12	19.26	0.78	0.28	12.56	0.12	3.28	1.0 1.	24	0.0	0.0
13	19.26	0.78	0.28	12.56	0.12	3.28	1.0 1.	24	0.0	0.0
14	19.26	0.78	0.28	12.56	0.12	3.28	1.0 1.	24	0.0	0.0
15	42.55	11.53	3.15	1 21.18	0.85	6.1	1.75 7.	98	n . n i	0.01

The sales revenue dataset contains detailed information for each individual sale. We first check the dataset for null entries and duplications, then remove the columns that irrelevant to our analysis. After this, we proceed to accumulate the individual sales into weekly data, taking care that the column for gmv_new (revenue) is recognized as numeric.

After doing the above, the first few rows of the weekly sales data are displayed in Table 3-3:

Table 3-3 An Excerpt of the Transformed Sales Data

+	+ -	+
week_of_year	sales_name	gmv_new
1 2 3 3 4 4 5 5 6 6 7 7	No Promotion No Promotion Eid & Rathayatra No Promotion No Promotion No Promotion No Promotion No Promotion No Promotion	623671 41002829 12851746 34728672 45187859 31331539 20264 7434
7 8 8 9	Independence Sale No Promotion Independence Sale No Promotion Rakshabandhan Sale	4278 11440 689 18500 60269

A new boolean variable called special_sale will be used to indicate whether a special promotion has taken place in a given week.



Finally, after the two raw datasets have been cleaned and transformed, we use the week number as key to join the media investment data and sales data. The final data looks like so:

Digital sponsorship content_marketing online_marketing affiliates SEM radio other gmv_new week of year total invest TV special sale 3.42 0.04 0.5 1.48 0 0.1 0 6.23671 0 0 0 0 2 3.42 0.04 0.5 1.48 0.26 0.1 1 0 0 410.02829 3 3.42 0.04 0.5 1.48 0 0.26 0.1 1 0 475.80418 0.04 0.5 1.48 0 0.26 0.1 451.87859 0 5 0 1 0 313.31539 0 0.04 0.5 1.48 0.26 3.42 0.1 1.28 0 0.33 0.28 0 0.03 0.03 0.63 0 0.20264 0 1.28 0 0.33 0.28 0 0.03 0.03 0.63 0 0.11712 8 0 0 0.12129 0 0.33 0.28 0.03 0.03 0.63 0 1.28 1.28 0 0.33 0.28 0 0.03 0.63 0 0.78769 0.03 10 19.26 0.78 0.28 12.56 0.12 3.28 1 1.24 0 818.77355 0

Table 3-4 Cleaned Data

3.2 Exploratory Data Analysis

After data cleaning and preparation, exploratory data analysis was carried out using R. A timeseries chart of annual Gross Merchandise Value (GMV) over a year was created using ggplot. This is illustrated in Figure 3-1.

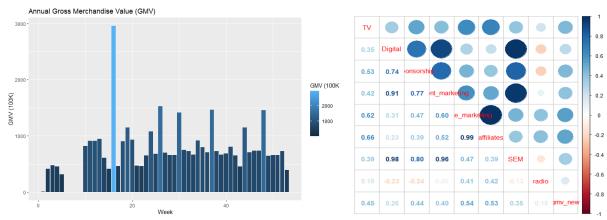


Figure 3-1 Annual Gross Merchandise Value

Figure 3-2 Correlation Between Variables

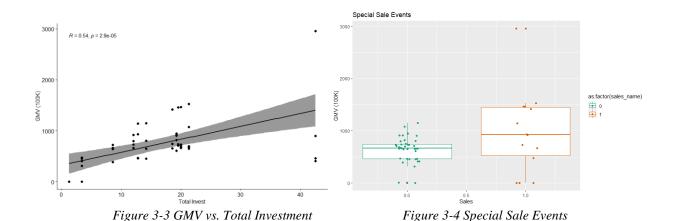
The GMV of 52 weeks in 100K are shown above. There are several values higher than the average that could be related to special holiday sales. The correlation between variables were checked using PerformanceAnalytics package. The results are shown in Figure 3-2.

All channels have a positive relationship with GMV. Significance ranges from 0.15 to 0.54. Online marketing and affiliates have a strong relationship with GMV. However, some collinearities between different channels are very high, since the factors affect each other. Multicollinearity can



make the model unstable because a lot of variances in the coefficient estimates were created. The relationship between the total investment and GMV were generated using ggplot, as illustrated in Figure 3-3.

With the total invest increases, the GMV also increases. The special sales were also analyzed. Sales without promotions were set as 0, and promotions were set as 1. The distributions were shown between. The chart in Figure 3-4. shows that the special sales have the effect on GMV.



4. Modeling

4.1 The Baseline Multiple Linear Regression Model

Multiple linear regression is the most common model used in MMM. As the correlation matrix suggests that most of the explanatory variables have linear relationships with the response variable, it is worth trying multiple linear regression (model_base) as the first step. The regression formula is as follows:

 $lm(formula = gmv_new \sim TV + Digital + sponsorship + content_marketing + online_marketing + affiliates + SEM + radio + other + special_sale, data = datatrain)$

According to the model summary, the overall regression is statistically significant. However, there are only three statistically significant variables, including sponsorship, content_marketing, and special_sale, at a 95% confidence interval. The adjusted R² value is 0.5484, which suggests that 54.84% of the variance in gmv_new could be explained by the media investments and special sales.



Four of the variable coefficients are negative, which means that increasing the investment in the relevant media channels, the revenue would decrease. In reality, having negative coefficients does not make sense as marketing spending should never cause the revenue to decrease. In our model, one explanation for the negative coefficients is that the corresponding investment is a very small percentage of the total investment. Another explanation would be the presence of multicollinearity in the data.

VIF value larger than 5 suggests multicollinearity. The VIF values of this model are mostly in hundreds or thousands, which suggests severe multicollinearity.

In marketing data, it is common to encounter multicollinearity because everything usually happens at the same time during marketing campaigns. For example, online marketing could be supplemented by SEM or affiliates. However, multicollinearity in our model generates high variance and weakens the statistical power of it, which is undesirable.

4.2 Lag and Adstock Transformed Models

Since advertising does not usually take effect immediately, we need to consider the "lag effect", illustrated in Appendix. It is also common that advertising or marketing events can have a lasting impact, also known as "decay", which we model by introducing the adstock transformation.

The formula of measuring adstock is as follows:

$$A_t = T_t + \lambda A_{t-1}$$
 $t = 1, 2, ..., n$ (Joy, 2006)

Where A_t is the Adstock at time t, T_t is the value of the advertising variable at time t, λ is the adstock rate, and n is the maximum number of weeks of decay.

To find the best value for these lag and adstock hyper-parameters, we iterated over a range of values and selected the ones which return the highest adjusted R² value for our model. The best combinations of the lag (in weeks) and adstock rate are displayed in Table 4-1. During research, we also found the "industry standard" lag and adstock rates for several marketing channels in this table, and our empirically determined values were very close to them.



Table 4-1 Lag and Adstock Hyper-parameters

Variables	Lag Weeks	Adstock Rate
TV	5	0.7
Digital	0	0.5
Sponsorship	7	0.25
Content_marketing	0	0.2
Online_marketing	6	0.2
Affiliates	0	0.5
SEM	2	0.25
Radio	0	0.2
Other	6	0.1

We used these lag and adstock hyper-parameters to transform the predictors and built two new multiple linear regression models, one with only lag transformed predictors and one with both lag and adstock transformed variables.

The overall regression is statistically significant for both models. The lag transformed model (model_lag) has improved in statistical significance as all the variables are significant and the adjusted R² increased to 0.6588, but multicollinearity still exists.

The lag and adstock transformed model (model_lag_ad) has two statistically insignificant variables and an adjusted R² value of 0.6197. It has done slightly worse in statistical significance, and multicollinearity is still a big problem, which leads us to 2 options: using variable selection to delete correlated variables and build subset models, or using ridge regression, a model tuning method to analyze data suffering from multicollinearity.

4.3 Variable Selection and Subset Models

Several methods have been used to select subsets of predictors, including best subset regression, stepwise, LASSO, and Elastic Net. The summary of the results is displayed in Table 4-2, including the best subset regression results based on Adjusted R² value, CP value and BIC value, and the results of stepwise, LASSO, and Elastic Net model selection procedures.

Table 4-2 Subsets of Predictors Selected by Each Variable Selection Method

	_									
				,	Variables Selected					
Methods	TV	Digital	Sponsorships	Content Marketing	Online Marketing	Affiliates	SEM	Radio	Other	Special Sale
Best Subset Regression (Adjusted R-squared)		✓	✓	√	✓	√	√	✓	✓	✓
Best Subset Regression (CP value)			✓	✓	✓			✓		✓
Best Subset Regression (BIC value)			✓			√				✓
Stepwise Regression	Г	√	✓	✓	✓	√	√	√	✓	✓
LASSO			✓	✓	✓	√		√		✓
Elastic Net			✓	✓	✓	√		√		✓



Both best subset regression and stepwise regression selected the same nine variables: Digital, Sponsorships, content_marketing, online_marketing, Affiliates, SEM, Radio, Other, and special_sale. Both LASSO and Elastic Net selected the following six variables: Sponsorships, content_marketing, online_marketing, Affiliates, Radio, and special_sale. Best subset regression also selected five variables and three variables based on the CP value and BIC value respectively. Hence, we decided to build subset models with the nine, six, and five variables selected. We hoped to find the biggest subset model with multicollinearity dealt with so that we could analyze as many media channels as we could. We also added lag and adstock transformation to the subset models and generated six more subset models. The summary of the subset models is displayed in Table 4-3.

Table 4-3 Summary of Subset Models

Subset Models	Formula	Adjusted R-squared	Number of Insignificant Variables	Multicollinearity Dealt With
	Im(gmv_new ~ Digital + sponsorship + content_marketing +			
	online_marketing + affiliates + SEM + radio + other + special_sale,			
model_9	data = datatrain)	0.561	2	Х
	online_marketing + affiliates + radio + special_sale, data =			
model_6	datatrain)	0.519	4	Х
	Im(gmv_new ~ sponsorship + content_marketing +			
model_5	online_marketing + radio + special_sale, data = datatrain)	0.532	1	✓
	Im(gmv_new ~ Digital + sponsorship + content_marketing +			
	online_marketing + affiliates + SEM + radio + other + special_sale,			
model_9_lag	data = datatrain_lag)	0.551	6	Х
	online_marketing + affiliates + radio + special_sale, data =			
model_6_lag	datatrain_lag)	0.524	3	✓
	Im(gmv_new ~ sponsorship + content_marketing +			
model_5_lag	online_marketing + radio + special_sale, data = datatrain_lag)	0.524	1	✓
	Im(gmv_new ~ ads_digital + ads_sp + ads_cm + ads_om + ads_af +			
model_9_lag	ads_sem + ads_radio + ads_other + special_sale, data =			
_ad	datatrain_lag)	0.605	3	Х
model_6_lag	Im(gmv_new ~ ads_sp + ads_cm + ads_om + ads_af + ads_radio+	·		
_ad	special_sale, data = datatrain_lag)	0.515	3	✓
model_5_lag	Im(gmv_new ~ ads_sp + ads_cm + ads_om + ads_radio+	·		·
_ad	special_sale, data = datatrain_lag)	0.525	2	✓

We define "Multicollinearity Dealt With" as all the VIF values are smaller than or equal to five. "Number of Insignificant Variables" means the number of variables in the specific model which are not statistically significance at a 95% confidence interval. The overall regression in each model is statistically significant while some variables are not. The five-variable subset model shows good statistical significance and has multicollinearity dealt with. The six-variable subset model with lag transformation also has multicollinearity dealt with but has a little bit more insignificant variables according to the model coefficients.

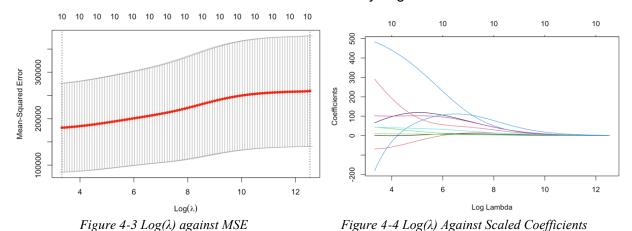
4.4 Ridge Regression Model

Ridge regression is a regularization method for analyzing data suffering from multicollinearity. When there is multicollinearity among the explanatory variables, the coefficient of one variable



depends on the others. By adding a penalty term λ , the coefficients of collinear variables would shrink, except for the significant variables.

To find the optimal amount of penalty, λ , we performed ten-fold cross validation using the cv.glmnet function in R to search for the λ that gives the minimum MSE (Mean-Squared Error). Figure 4-3 shows the MSE values for different λ values. The minimum MSE is achieved when λ = 27.89. The impact of different λ s in the estimated coefficients is displayed in Figure 4-4. The coefficients would be shrunk towards zero when λ is very large.



With λ = 27.89, ridge regression was performed using the function glmnet in R. The ridge regression (model_ridge) coefficient estimates are displayed in Figure 4-5, in comparison with the baseline multiple linear regression coefficients. The coefficients have shrunk a lot compared with the baseline model, which indicates that multicollinearity has been dealt with.

##		model_base	ridge
##	(Intercept)	-1193.8271	-43.4454
##	TV	401.5364	65.3841
##	Digital	5439.6274	-68.7930
##	sponsorship	115.0745	44.3046
##	content_marketing	-13698.5494	-180.4339
##	online_marketing	4037.1430	43.6812
##	affiliates	-11036.5046	103.1729
##	SEM	-1459.2110	1.8647
##	radio	13336.8821	288.6247
##	other	-954.5247	10.0537
##	special sale	661.4936	483.4719

Figure 4-5 Coefficients of the Ridge Regression Model and the Baseline Multiple Linear Regression Model



4.5 Model Comparison

Based on the training data, we have built 13 models. In order to compare these models' performance, we made predictions on the test data using these models and analyzed the prediction accuracy of each model. MSPE and PM were calculated and displayed together with adjusted R² in Table 4-4.

	Model	Adjusted R- Squared	MSPE	PM
	model_base	0.548	2932659.75	36.359
Full Models	model_lag	0.659	2258804.06	28.005
Full Widuels	model_lag_ad	0.620	232897.44	2.887
	model_ridge	0.439	86232.04	1.069
	model_9	0.561	2294132.98	28.443
	model_6	0.519	103598.40	1.284
	model_5	0.532	92979.20	1.153
	model_9_lag	0.551	270505.10	3.354
Subset Models	model_6_lag	0.524	87620.11	1.086
	model_5_lag	0.524	89556.69	1.110
	model_9_lag_ad	0.605	223024.83	2.765
	model_6_lag_ad	0.515	154677.59	1.918
	model_5_lag_ad	0.525	135058.48	1.674

Table 4-4 Adjusted R², MSPE, and PM Values of Each Model

This table shows that the baseline model has the highest prediction error, which affirmed our job in finding the best model and improving the model performance in the following steps. Among the models with all the variables, the ridge model performs the best while among the subset models, the model with 6 variables and with lag transformation performs the best.

Overall, model_ridge is the best performing model, although its adjusted R² is slightly lower. We would prefer a full model so that we could analyze all the media investment channels. Therefore, we would pick the ridge model for further analysis.

4.6 Ridge Regression Model Interpretation

With the coefficients estimated by the ridge regression, a formula of the response variable and predictors was generated:

Gmv_new = -43.445 +65.384*TV - 68.793*Digital + 44.305*Sponsorship - 180.434*Content_marketing + 43.681*Online_marketing + 103.173*Affiliates + 1.865*SEM + 288.625*Radio + 10.054*Other + 483.472*special_sale



The negative intercept value indicates that without media investment, DT Mart's revenue would decrease. The coefficients suggest that with 1INR increase in the specific media investment channel, the revenue will increase or decrease the amount of INR indicated by the relevant coefficients, holding all other variables constant. When there is a special sale, the revenue is predicted to increase 48.3M INR, holding all other variables constant. However, at this stage we focus our comparisons and analysis on the nine media investment channels.

According to the coefficient values, among the nine media investment channels, affiliates, TV, sponsorship and Online Marketing are the main drivers on revenue, while Digital and content marketing are the least effective as they have negative coefficients. Radio has a very high coefficient but as radio's spending is small and sporadic, it is unlikely to be a main driver of revenue.

4.7 Marketing Investment Optimization and Revenue Forecast

Based on the findings in 4.6, we adjusted the investment allocation of each marketing channel. The percentage of each media channel's investment is displayed in Figure 4-6.

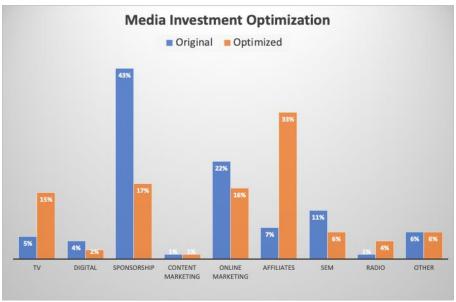


Figure 4-6 Media Investment Optimization Proposal

In this specific example, we increased the budget on TV and affiliates, and decreased the percentage of sponsorship, online marketing and SEM. Then we calculated the revenue using the formula generated by the ridge model in Section 4.6. Our model predicts that doing so the revenue would increase from 3884.73M INR to 5831.56M INR, a 50% growth.



5. Conclusion

5.1 Summary

With the datasets of DT Mart, we have conducted a thorough marketing mix modeling analysis. After training and validating the 13 models, we chose the ridge regression model as the best performing model for our data. Marketing budget allocation could be optimized and the influence on driving the revenue growth is significant.

There are a few benefits of the model. It could help marketers understand the most and least effective marketing channels and enable marketing budget allocation optimizing. It could also facilitate superior sales revenue forecasting.

There are also limitations. With only one year's data, after transformation into weekly data, we only have 53 observations and 10 variables. There is a risk of model overfitting. The data quality is not ideal. And there was severe multicollinearity issue. They all weaken the statistical power of the model. Besides, this model is more valuable in analyzing short term marketing effect on the revenue. Some variables may not be a strong driver of revenue in short term but could contribute significantly to the brand equity.

5.2 Future Work

Although the project is concluded here, there are several future improvements that can be made:

- The data quality could be improved. More data could be collected on a weekly basis at better quality. We could re-fit the model when more data becomes available.
- With at least 2 year's data, we would be able to include trend and seasonality in the model and would be able to predict revenue forecast for future.
- If attribution data could be provided, we could calculate the contribution of each media channel on revenue and perform an ROI analysis. Valuable insights could be gained for better decision making.

Reference

- [1] What is 'Marketing Mix', The Economic Times, Oct 06, 2022, https://economictimes.indiatimes.com/definition/marketing-mix;
- [2] Kotler, P., Marketing Management, (Millennium Edition), Custom Edition for University of Phoenix, Prentice Hall, 2001, p. 9;
- [3] A Complete Guide to Building a Marketing Mix Model in 2021, Terence Shin, Towards Data Science, Jul 24, 2021, https://towardsdatascience.com/an-complete-guide-to-building-a-marketing-mix-model-in-2021-fb53975be754
- [4] Marketing Mix Model Guide With Dataset Using Python, R, and Excel, Jacky Yuan, Analytics Vidhya, Dec 21, 2020, https://medium.com/analytics-vidhya/marketing-mix-model-guide-with-dataset-using-python-r-and-excel-4e319be47b4
- [5] *Advertising adstock theory*, Marko Mikes, Aug 27, 2016, Medium, https://medium.com/@Marko Mi/advertising-adstock-theory-85cc9e26ea0e
- [6] <u>Using R to Build a Simple Marketing Mix Model (MMM) and Make Predictions</u>, Trent Y., Towards Data Science, https://towardsdatascience.com/building-a-marketing-mix-model-in-r-3a7004d21239
- [7] Marketing Mix Modelling (MMM) a potential solution, Hitesh Tripathi, 2021, https://tripathitesh.medium.com/marketing-mix-modelling-mmm-a-potential-solution-48ba3a248de9
- [8]_A Deep Insight into Marketing Mix Modeling Definition, Methodology, Comparison, Techniques,
- https://blog.avada.io/resources/marketing-mix-modeling.html
- [9] The Science Behind the Art of Marketing-Mix Modeling, Derek L., Nakul P., and Sadashiv G., GAMMA—Part of BCG X, 2021, https://medium.com/bcggamma/the-science-behind-the-art-of-marketing-mix-modeling-ba721d6dab33
- [10] Advertising Adstock with Maximum Period Decay, Gabriel Mohanna, 2014 https://analyticsartist.wordpress.com/2014/02/26/advertising-adstock-with-maximum-period-decay/
- [11] *Understanding Advertising Adstock Transformations*, Joseph Joy, 2006, https://mpra.ub.uni-muenchen.de/7683/4/MPRA_paper_7683.pdf



Appendix

I. Lag and Adstock

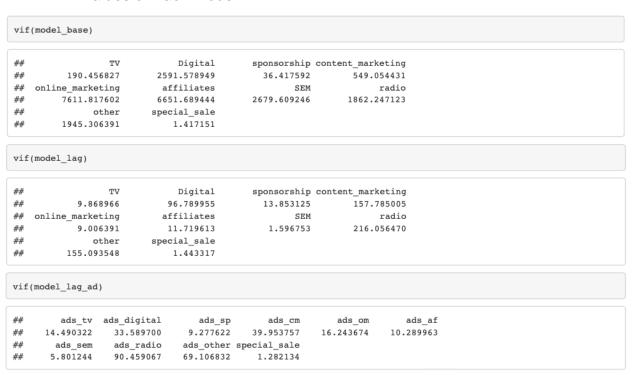




Figure I-1 Lag and Decay Effect (Yuan, 2020)

Figure I-2 Advertising Adstock (Mohanna, 2014)

II. VIF Values of Each Model





```
vif(model_9)
##
                        sponsorship content_marketing online_marketing
           Digital
      1066.472143
                        26.632958 536.857089 3783.281879
##
##
        affiliates
                             SEM
                                         radio
                                                            other
       2863.977311
                         620.982754 304.541898 196.468957
##
##
      special sale
##
          1.402502
vif(model 6)
                                                         affiliates
##
       sponsorship content_marketing online_marketing
##
         5.506919 7.727372
                                        450.976190
                                                         383.926866
##
            radio
                      special_sale
                   1.240833
##
         1.858690
vif(model_5)
        sponsorship content_marketing online_marketing
                                                             radio
                                                         1.835561
##
          4.380754
                         3.561388
                                         1.957405
##
       special sale
          1.239981
vif(model_9_lag)
           Digital
                        sponsorship content_marketing online_marketing
##
         51.868069
                         11.866599 85.581675 4.236638
                              SEM
##
        affiliates
                                          radio
                                                            other
##
          6.307113
                          1.519215 123.589568
                                                       83.648537
##
       special sale
         1.441451
vif(model_6_lag)
        sponsorship content_marketing online_marketing
                                                         affiliates
                         3.652673
                                                          4.087267
            radio
                      special_sale
          1.838191
##
                         1.239921
vif(model_5_lag)
       sponsorship content_marketing online_marketing
                                                             radio
                                                          1.602981
##
          4.094032
                          3.436531
                                          1.116647
##
       special_sale
          1.232694
##
vif(model_9_lag_ad)
## ads_digital
                                                      ads_af
                                                                ads_sem
                   ads_sp
                             ads_cm
                                          ads_om
##
    33.321697
                9.239091
                            38.702394
                                         6.098383
                                                    9.471711
                                                                5.787329
##
     ads radio
                ads other special sale
##
    77.791139
               55.389518
                            1.260230
```



##	ads_sp	ads_cm	ads_om	ads_af	ads_radio sp	ecial_sale
##	4.528237	3.790636	3.840905	5.431496	1.811970	1.233278
if(n	odel 5 lag a	d)				



III. Team Information



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