GTU
Electronics Engineering

ELEC 331
Electronic Circuits 2

Fall Semester

Instructor: Assist. Prof. Önder Şuvak

HW 4
Questions and Answers

Updated October 20, 2017 - 13:40

Assigned:

Due:

Answers Out:

Late Due:

Contents

Transfer Function – High Frequency Response **Sedra 6.40**

6.40 Consider an amplifier whose $F_H(s)$ is given by

$$F_H(s) = \frac{1}{\left(1 + \frac{s}{\omega_{P1}}\right)\left(1 + \frac{s}{\omega_{P2}}\right)}$$

with $\omega_{P1} < \omega_{P2}$. Find the ratio ω_{P2}/ω_{P1} for which the value of the 3-dB frequency ω_H calculated using the dominant-pole approximation differs from that calculated using the root-sum-of-squares formula (Eq. 6.36) by:

(a) 10%.
(b) 1%.

$$\omega_H \cong 1 \Bigg/ \sqrt{\left(\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} + \cdots\right) - 2\left(\frac{1}{\omega_{Z1}^2} + \frac{1}{\omega_{Z2}^2} + \cdots\right)} \qquad (6.36)$$

Necessary Knowledge and Skills: Bode plots, poles and zeros, dominant pole approximation, half-power or cut-off or corner or -3 dB frequency, root sum of squares formula and derivation

$$F_H(s) = \frac{1}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)}$$

Sedra 6.40

$\rightarrow$ if $w_{p1} < w_{p2} \Rightarrow$ Dominant pole approx says $w_H \approx w_{p1}$

$\rightarrow$ With (6.36) we have

$$w_H \cong \frac{1}{\sqrt{\left(\frac{1}{w_{p1}}\right)^2+\left(\frac{1}{w_{p2}}\right)^2}} = \frac{w_{p1}}{w_{p1}\sqrt{\cdots}}$$

$$= \frac{w_{p1}}{\sqrt{1+\left(\frac{w_{p1}}{w_{p2}}\right)^2}}$$

The difference between the two formulas

$$\Delta w_H = w_{p1} - \frac{w_{p1}}{\sqrt{1+\left(\frac{w_{p1}}{w_{p2}}\right)^2}}$$

$$\frac{\Delta w_H}{w_{p1}} = 1 - \frac{1}{\sqrt{1+\left(\frac{w_{p1}}{w_{p2}}\right)^2}}$$

$$= 1 - \frac{1}{\sqrt{1+\frac{1}{n^2}}} \quad \text{where } n = \frac{w_{p2}}{w_{p1}}$$

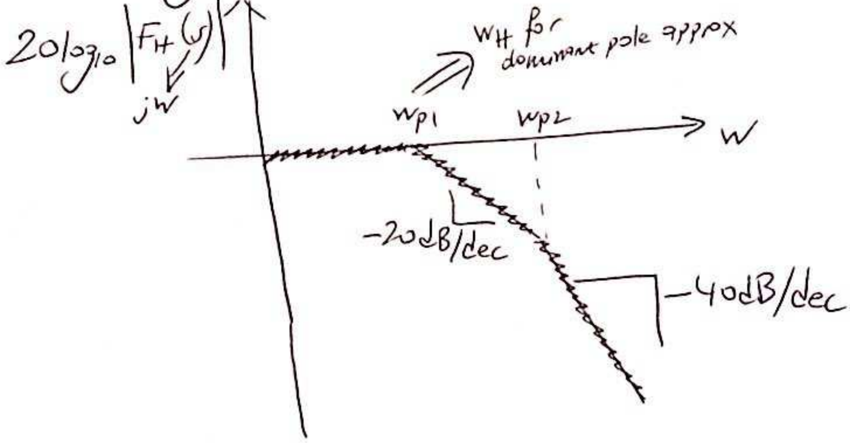
(1)

Sedra 6.40 Continued

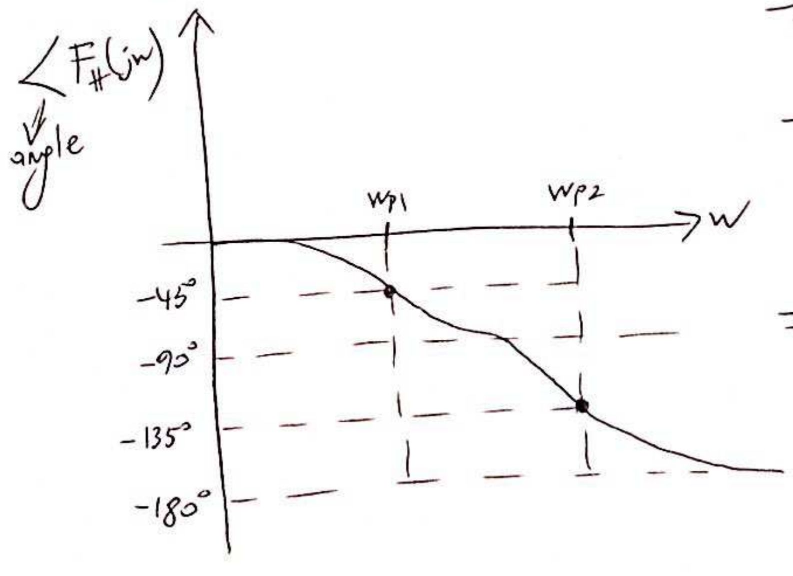
if $\frac{\Delta w_H}{w_{p1}} = 10\% = 0.1 \Rightarrow n = 2.07$

if $\frac{\Delta w_H}{w_{p1}} = 1\% = 0.01 \Rightarrow n = 7.02$

Asymptotic magnitude Bode plot of $F_H(s)$



Asymptotic phase Bodeplot for $F_H(s)$



→ each pole contributes −90° eventually

→ At the pole we have −45° phase shift in addition to the previous value of phase.

⇒ This plot is accurate if w_{p1} and w_{p2} are sufficiently wide apart.

2

Transfer Function – High frequency Response **Sedra 6.41**

6.41 The high-frequency response of a direct-coupled amplifier having a dc gain of −100 V/V incorporates zeros at ∞ and 10^6 rad/s (one at each frequency) and poles at 10^5 rad/s and 10^7 rad/s (one at each frequency). Write an expression for the amplifier transfer function. Find ω_H using:

(a) the dominant-pole approximation.

(b) the root-sum-of-squares approximation (Eq. 6.36).

If a way is found to lower the frequency of the finite zero to 10^5 rad/s, what does the transfer function become? What is the 3-dB frequency of the resulting amplifier?

$$\omega_H \cong 1 \Bigg/ \sqrt{\left(\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} + \cdots\right) - 2\left(\frac{1}{\omega_{Z1}^2} + \frac{1}{\omega_{Z2}^2} + \cdots\right)} \qquad (6.36)$$

Necessary Knowledge and Skills: Bode plots, transfer functions, poles and zeros, dominant pole approximation, half-power or cut-off or corner or -3 dB frequency, root sum of squares formula and derivation

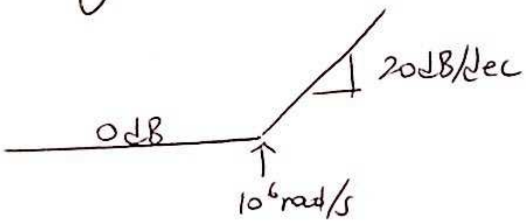
Sedra 6.41

$$H(j\omega) = -100 \, \frac{\overbrace{\cancel{\left(1 + \frac{j\omega}{\infty}\right)}}^{\text{This is } 1} \left(1 + \frac{j\omega}{10^6}\right)}{\left(1 + \frac{j\omega}{10^5}\right)\left(1 + \frac{j\omega}{10^7}\right)}$$

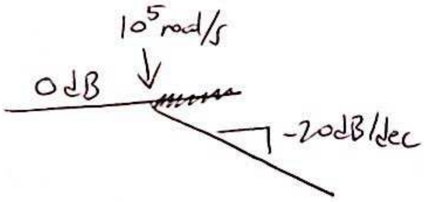
Components of the magnitude Bode plot (asymptotics)

$\Rightarrow$ $20\log_{10}|-100| = 20 \cdot 2 = 40\,\text{dB}$ (at each freq)

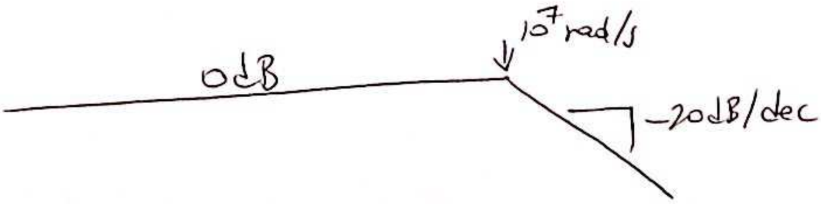
$\Rightarrow$



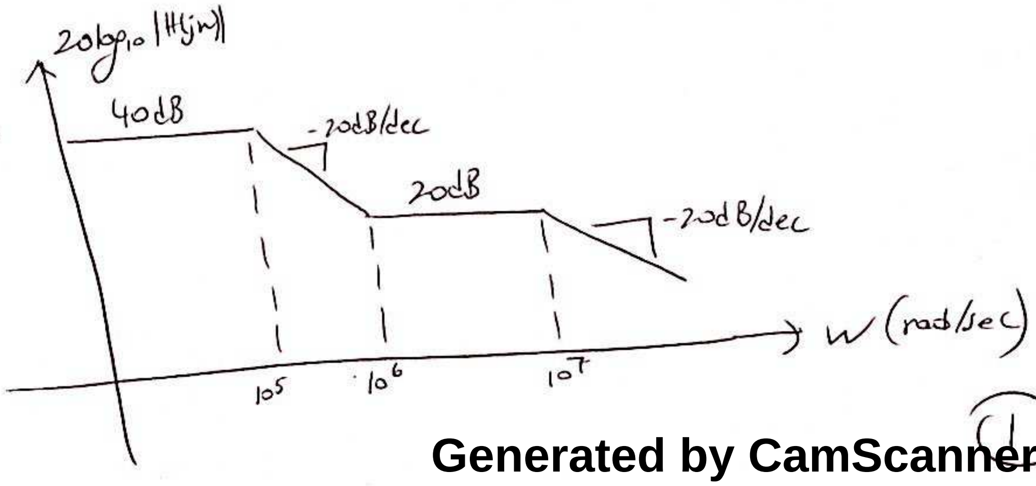
$\Rightarrow$



$\Rightarrow$



Add all of the above



1

Sedra 6.41 continued

Corner frequency calculation

$w_H \cong 10^5$ rad/s due to dominant pole approx

$$w_H \cong \frac{1}{\sqrt{\left(\frac{1}{10^5}\right)^2 + \left(\frac{1}{10^7}\right)^2 - 2\left(\frac{1}{10^6}\right)^2}}$$

$$= 1.01 \times 10^5 \text{ rad/s}$$

zero at 10^6 rad/s $\longrightarrow$ zero at 10^5 rad/s

$$H_{new}(jw) = -100 \frac{\cancel{\left(1+\frac{jw}{10^5}\right)}}{\cancel{\left(1+\frac{jw}{10^5}\right)}\left(1+\frac{jw}{10^7}\right)}$$

$$= \frac{-100}{1+\frac{jw}{10^7}}$$

then $w_{H,new} = 10^7$ rad/s

Sedra 6.41 continued

Derivation of (6.36), see pp 573–574 of Sedra.

$$F_H(s) = \frac{\left(1+\frac{s}{w_{z1}}\right)\left(1+\frac{s}{w_{z2}}\right)}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)}$$

~~Replace~~ substitute $s=jw$

$$\left|F_H(jw)\right|^2 = \frac{\left(1+\frac{w^2}{w_{z1}^2}\right)\left(1+\frac{w^2}{w_{z2}^2}\right)}{\left(1+\frac{w^2}{w_{p1}^2}\right)\left(1+\frac{w^2}{w_{p2}^2}\right)}$$

w_H is the 3 dB freq (where the power with respect to the DC power is halved)

DC power of $F_H(jw) = 1$

halved $\Rightarrow \frac{1}{2}$

Note that $10\log_{10}\left(\frac{1}{2}\right) \simeq -3\text{dB}$

Hence the name 3dB freq. for w_H (or corner freq)

 (2)

Sedra 6.41
Continued

put $w = w_H$

and $\left| F_H(jw_H) \right|^2 = \frac{1}{2}$ (power is halved at $w = w_H$)

$$\frac{1}{2} = \frac{\left(1+\left(\frac{w_H}{w_{z1}}\right)^2\right)\left(1+\left(\frac{w_H}{w_{z2}}\right)^2\right)}{\left(1+\left(\frac{w_H}{w_{p1}}\right)^2\right)\left(1+\left(\frac{w_H}{w_{p2}}\right)^2\right)}$$

Omit higher order terms.

$$w_H \simeq \frac{1}{\sqrt{\frac{1}{w_{p1}^2} + \frac{1}{w_{p2}^2} - \frac{2}{w_{z1}^2} - \frac{2}{w_{z2}^2}}}$$

(6.36) is the generalization of the last formula.

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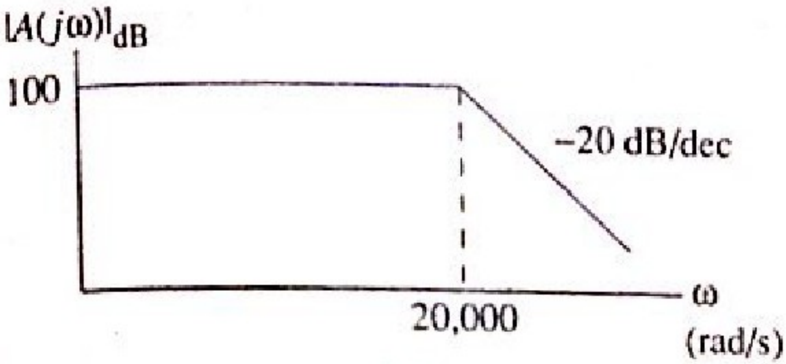
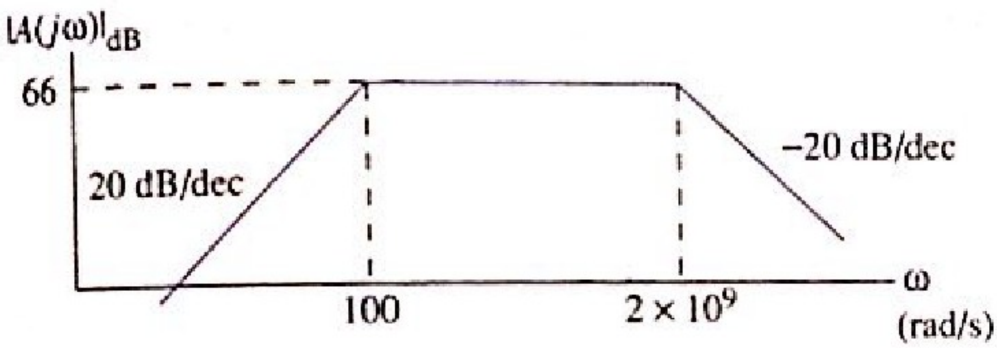
Magnitude Bode Plot **Malik 8.4**

8.4 Estimate the gain in dB at
(a) 10,000 rad/s for Fig. P8.3d.
(b) 4000 rad/s for Fig. P8.3d.
(c) 30,000 rad/s for Fig. P8.3a.
(d) 100,000 rad/s for Fig. P8.3d.

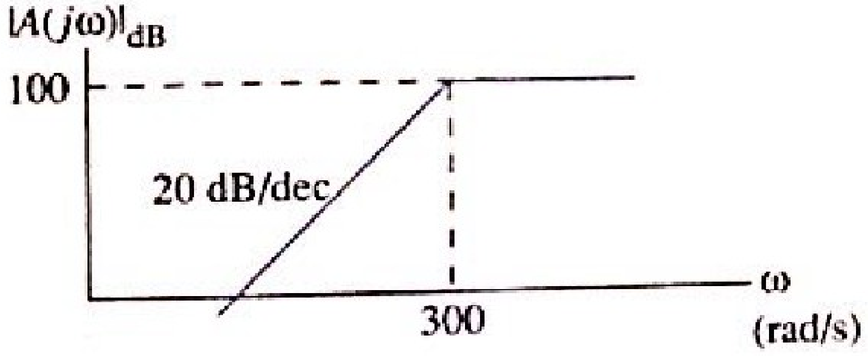


(a)

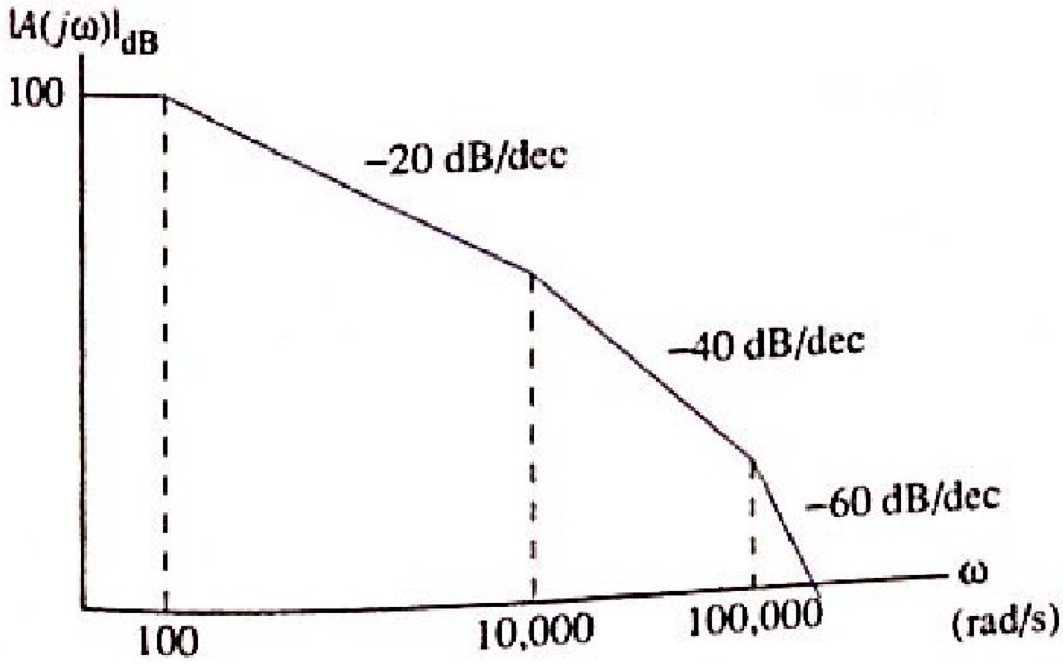


(b)

Magnitude Bode Plot **Malik 8.4**



(c)



(d)

Figure P8.3

Magnitude Bode Plot **Malik 8.4**

Necessary Knowledge and Skills: Bode plots, transfer functions, poles and zeros, half-power or cut-off or corner or -3 dB frequency, transfer function derivation from Bode plots, slope interpretations in Bode plots

Additional task: Derive the transfer function corresponding to each of the given magnitude Bode plots.

Molik 8.4

Find the transfer functions first.

8.3a $\Longrightarrow$ $A(j\omega) = 100 \dfrac{1}{1+\frac{j\omega}{20000}}$

8.3b $\Longrightarrow$ zero at 0 rad/s
pole at 100 rad/s
pole at 2×10^9 rad/s

$$A(j\omega) = \underbrace{B}_{\text{to be computed}} \frac{j\omega}{\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{2\times10^9}\right)}$$

Between $\omega = 100$ rad/s and $\omega = 2\times10^9$ rad/s

$$A(j\omega) \cong B \frac{j\omega}{\underbrace{\left(\frac{j\omega}{100}\right)}_{\left|\frac{j\omega}{100}\right| \gg 1} \underbrace{(1)}_{\left|\frac{j\omega}{2\times10^9}\right| \ll 1}} = 100B$$

$$20\log_{10} 100B = 66\text{dB}$$

$$\underbrace{20\log_{10} 100}_{40} + 20\log_{10} B = 66\text{dB}$$

then $B = 10 \wedge \left(\frac{26}{20}\right)$

power

Malik 8.4 Continued

8.3c zero at 0 rad/s
pole at 300 rad/s

$$A(j\omega) = B\frac{j\omega}{1+\frac{j\omega}{300}}$$

for $\omega \gg 300$ rad/s

$$A(j\omega) \simeq B\frac{j\omega}{\frac{j\omega}{300}}$$

$$= 300B$$

$$\underbrace{20\log_{10}300}_{40+20\log_{10}3} + 20\log_{10}B = 100$$

then $20\log_{10}B = 60 - 20\log_{10}3$

$$\log_{10}B = 3 - \log_{10}3$$

$$B = 10\wedge(3-\log_{10}3)$$

8.3d pole at 100 rad/s
〃 〃 10000 〃
〃 〃 100000 〃

$$A(j\omega) = B\frac{1}{\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{10000}\right)\left(1+\frac{j\omega}{100000}\right)}$$

$\omega \ll 100$ rad/s $\Rightarrow A(j\omega) \simeq B$

$$20\log_{10}B = 100$$

$$B = 10^5 = 10000$$

 (2)

Molik 8.4
continued

8.3d) gain at 10,000 rad/s

a)

count the decades

$$\underbrace{10^2 \quad 10^3}_{\text{one decade}}\underbrace{\quad 10^4}_{\text{another decade}} \Rightarrow -40\text{dB lost in total over 2 decades}$$

$100 - 40 = 60\text{dB}$ at

8.3d) gain at 4000 rad/s

b)

$$slope = -20\ \text{dB/dec} = \frac{A\left(\frac{j\omega}{4000}\right) - 100}{\log_{10}(4000) - \log_{10}(100)}$$

compute $A\left(\frac{j\omega}{4000}\right)$ using the eqn above

8.3d) gain at 30,000 rad/s

c)

$$slope = -20\text{dB/dec} = \frac{A\left(\frac{j\omega}{30000}\right) - 100}{\log_{10}(30000) - \log_{10}(20000)}$$

compute the numer. value.

8.3d) gain at 100,000 rad/s

d)

$A\left(\frac{j\omega}{10000}\right)$ computed in part (a)

count the decades

$\underbrace{10{,}000 \quad 100{,}000}_{\text{one decade}}$

40dB lost in a single decade

then $A\left(\frac{j\omega}{100000}\right) = A\left(\frac{j\omega}{10000}\right) - 40\ \text{dB} = 20\text{dB}$

(3)

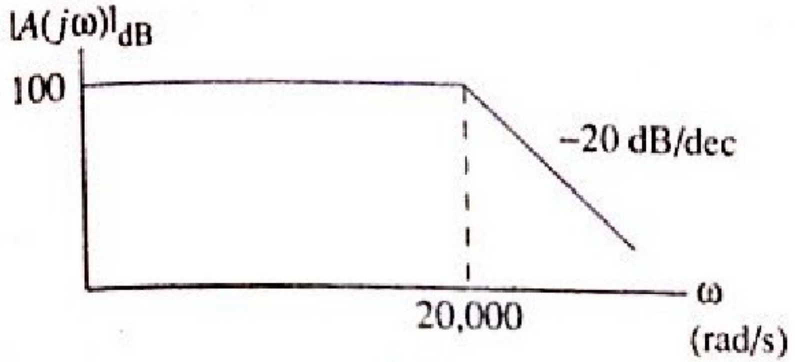
Magnitude Bode Plot **Malik 8.5**

8.5 Estimate the radian frequency where the gain is 0 dB for

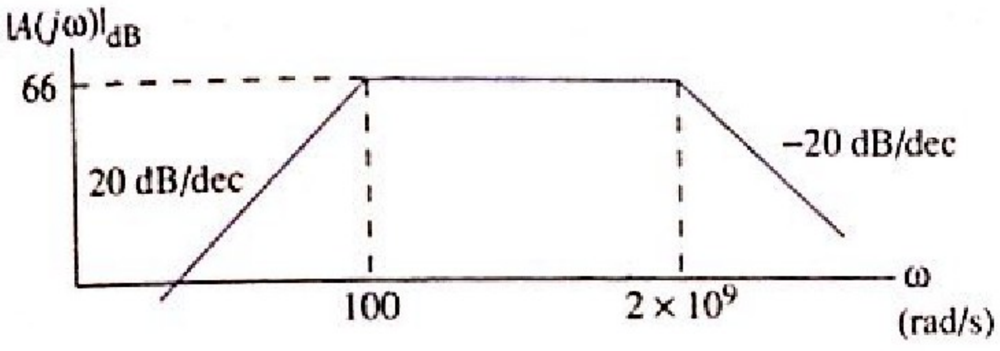
(a) Fig. P8.3a.

(b) Fig. P8.3c.

(c) Fig. P8.3d.



(a)



(b)

Magnitude Bode Plot **Malik 8.5**

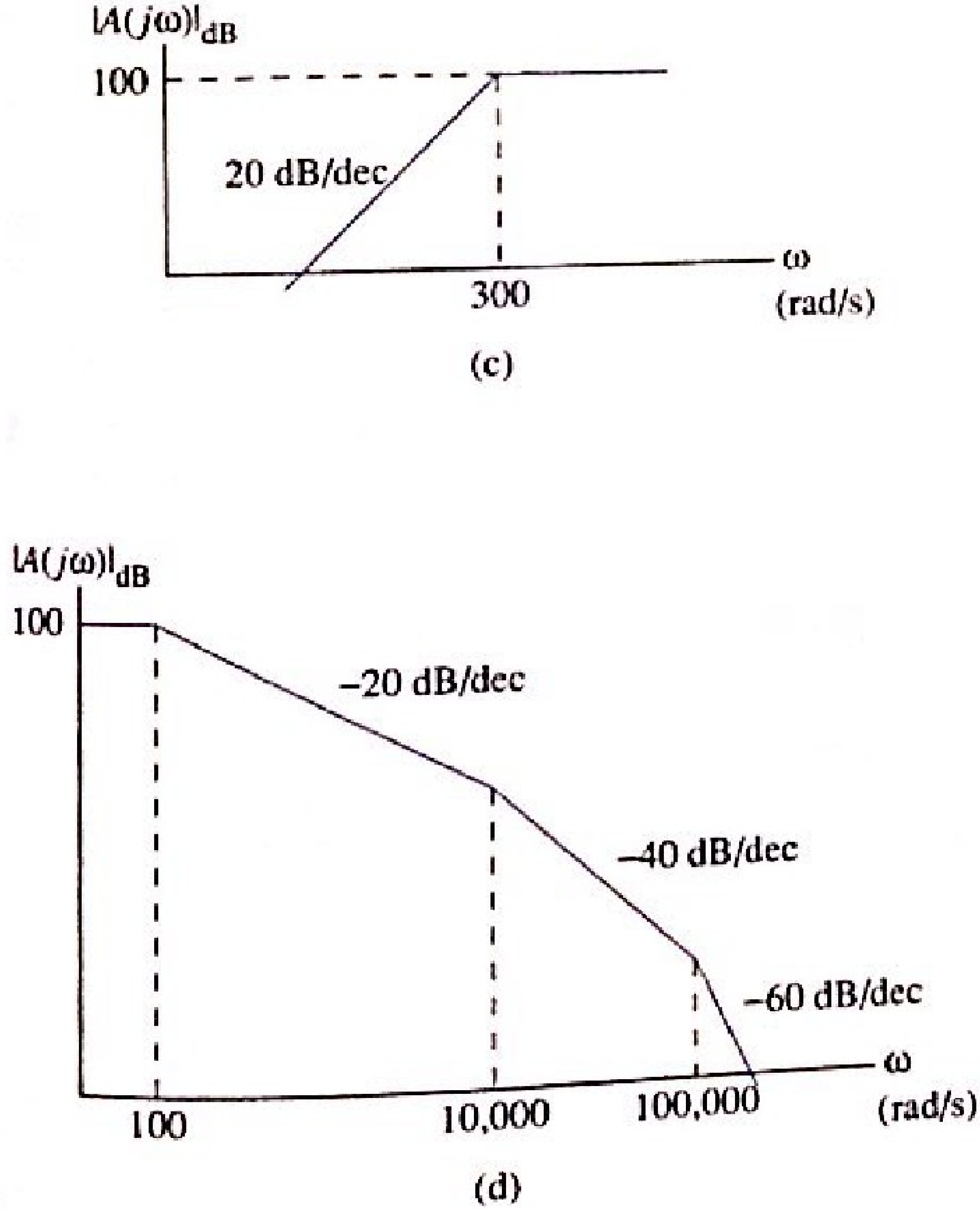


Figure P8.3

Magnitude Bode Plot **Malik 8.5**

Necessary Knowledge and Skills: Bode plots, transfer functions, poles and zeros, half-power or cut-off or corner or -3 dB frequency, slope interpretations in Bode plots

Moltk 8.5

8.3 a)

a)

$$slope = -20\,dB/dec \frac{0dB - 100dB}{\log_{10}(w_u) - \log_{10}(20{,}000)}$$

compute w_u

8.3 c)

b)

$$slope = +20\,dB/dec = \frac{100dB - 0dB}{\log_{10}(300) - \log_{10}(w_u)}$$

compute w_u

8.3d)
We know from Moltk 8.4d) that

c)

$$A\left(\frac{jw}{100{,}000}\right) \text{ in } dB = 20dB$$

then

$$slope = -60\,dB/dec = \frac{0 - 20dB}{\log_{10}(w_u) - \underbrace{\log_{10}(100{,}000)}_{5}}$$

$$-60\log_{10}(w_u) + 300 = -20$$

$$w_u = 10 \wedge \left(\frac{+320}{+60}\right) \text{ rad/sec}$$

Corner Frequency by OCTC **Razavi 11.13**

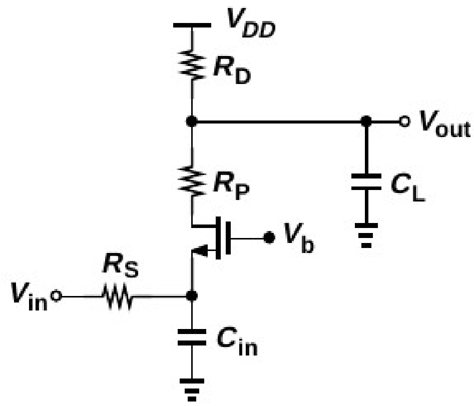


Figure 11.66

Assume the capacitors are small valued, comparable to the internal capacitances of the transistor. Compute the 3dB corner frequency by the OCTC method.

Necessary Knowledge and Skills: Open Circuit Time Constants (OCTC) method for approximating the high frequency cut-off point, small signal equivalent circuit for MOS, small signal impedance computations

Razavi
11.13

C_{in} sees R_{in} (C_L O.C. $v_{in} \to 0V$)

$$R_{in} = R_S \,//\, \frac{1}{g_m}$$

C_L sees R_L (C_{in} O.C. $v_{in} \to 0V$)

$$R_L = R_D \,//\, \left[R_P + r_o + (1+g_m r_o) R_S\right]$$

$$\omega_{3dB} = \frac{1}{C_{in} R_{in} + C_L R_L}$$

Corner Frequency by OCTC **Razavi 11.14**

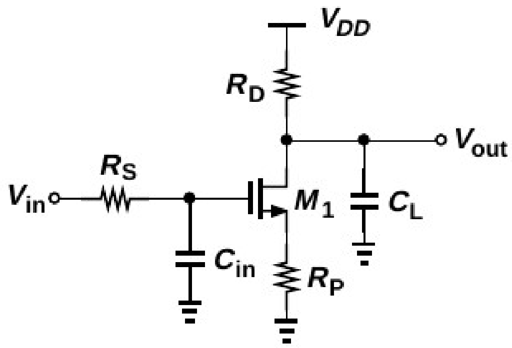


Figure 11.67

Assume the capacitors are small valued, comparable to the internal capacitances of the transistor. Compute the 3dB corner frequency by the OCTC method.

Necessary Knowledge and Skills: Open Circuit Time Constants (OCTC) method for approximating the high frequency cut-off point, small signal equivalent circuit for MOS, small signal impedance computations

Razavi
11.14

C_{in} sees R_{in} (C_L O.C. $V_{in} \to 0V$)

$R_{in} = R_S$

C_L sees R_L (C_{in} O.C. $V_{in} \to 0V$)

$R_L = R_D \,//\, [r_o + (1 + g_m r_o) R_P]$

$$W_{3dB} = \frac{1}{C_{in} R_{in} + C_L R_L}$$