GTU
Electronics Engineering

ELEC 331
Electronic Circuits 2

Fall Semester

Instructor: Assist. Prof. Önder Şuvak

HW 3
Questions and Answers

Updated October 20, 2017 - 13:39

Assigned:

Due:

Answers Out:

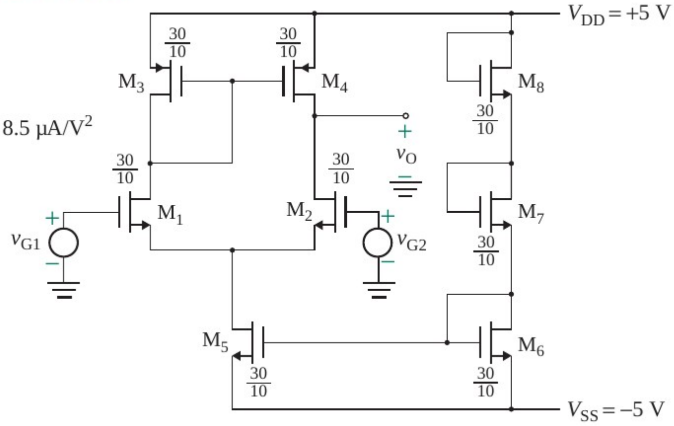
Late Due:

Contents

CMOS Differential Amplifier with Active Load **Rashid 9.14**

9.14 A CMOS amplifier is shown in Fig. P9.14. The parameters for the NMOS are $V_t = +2$ V, $V_M = -40$ V, and $V_{GS} = +4$ V at $I_D = 1$ mA; the parameters for the PMOS are $V_t = -3$ V, $V_M = 40$ V, and $V_{GS} = -6$ V at $I_D = 1$ mA. Calculate **(a)** A_d, A_c, and CMRR and **(b)** R_{id} and R_{ic}.

FIGURE P9.14



Necessary Knowledge and Skills: MOS differential amplifier, MOS active load, MOS current source, MOS nonlinear resistors (diode connected MOS), DC bias, voltage gain computation with active load, single ended amplifier with voltage gain the same as that of the fully differential amplifier, MOS small signal equivalent model, output impedance calculations.

Rashid
9.14

DC biasing of the current source

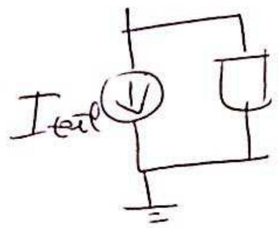
Since all of M_8, M_7 and M_6 are identical, we will have

$$V_{GS6} = V_{DS6}$$
$$= V_{GS7} = V_{DS7}$$
$$= V_{GS8} = V_{DS8} = \frac{V_{DD}-V_{SS}}{3} = \frac{10}{3} = 3.33 \text{ Volts}$$

All are in satur, because of the diode connection.

$$\frac{1}{2}\underbrace{\mu_n C_{ox}}_{8.5\,\mu A/V^2}\underbrace{\frac{W}{L}}_{\frac{30}{10}}(\underbrace{V_{GS}}_{V_{DS}=3.33V}-\underbrace{V_t}_{2V})^2 = I_D \Rightarrow \text{compute}$$

the tail current $I_{tail} = I_D$



$$R_{tail} = \frac{V_M}{I_D} = r_{o5}$$ (size is the same)

$V_M \to 40V$

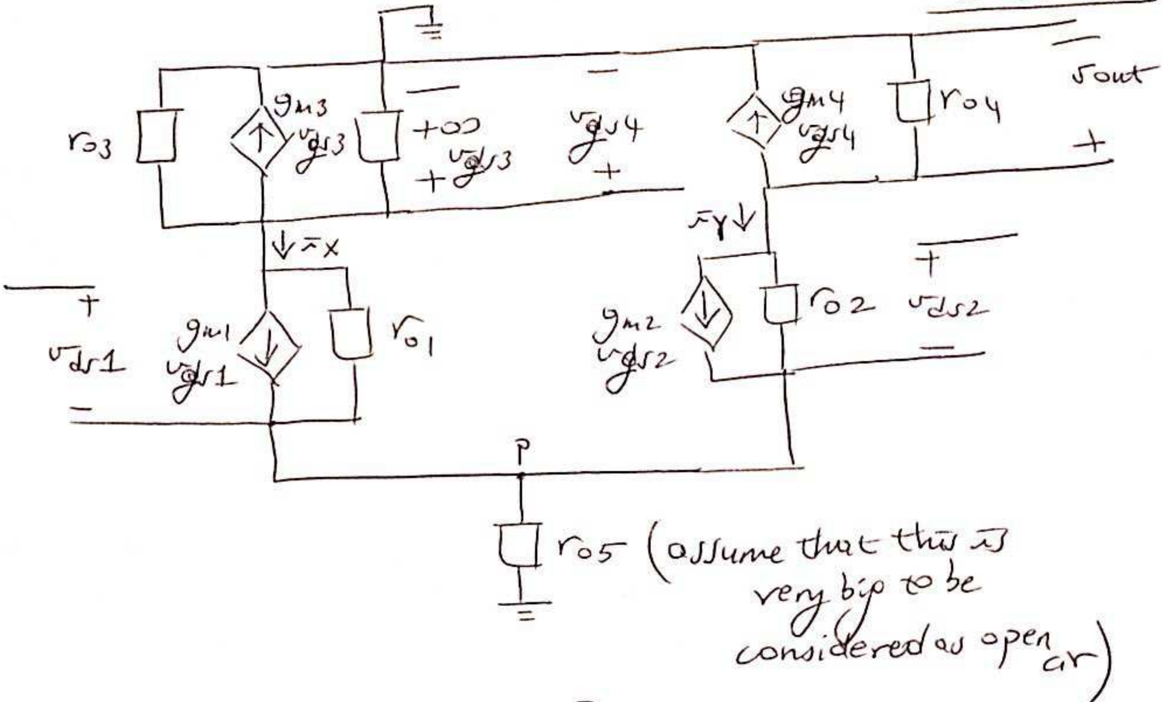
$R_{tail} \Downarrow$ compute

At the differential stage, we have an active loaded single ended diff amp.
analysis is different $\Rightarrow$ because it is asymmetrical.

1

Prashant 9.14 continued

Differential mode



PMOS devices are identical
NMOS 〃 〃 〃 } All carry the same DC current $= \frac{I_{tail}}{2}$

Therefore $g_{m3} = g_{m4} = g_{mP}$ $\quad$ $r_{o3} = r_{o4} = r_{oP}$

$g_{m1} = g_{m2} = g_{mN}$ $\quad$ $r_{o1} = r_{o2} = r_{oN}$

denote $V_A = v_{gs3} = v_{gs4}$

$v_{ds1} = v_1$

$v_{ds2} = v_2$

Note that $i_x + i_y = 0$

 (2)

Rashid
9.14
Continued

$$v_A = -i_X\left(\frac{1}{g_{mp}} \,//\, r_{op}\right)$$

$$i_Y + g_{mp}\underbrace{v_A}_{\Downarrow} + \frac{v_{out}}{r_{op}} = 0$$

substitute

$$-i_Y = \frac{v_{out}}{r_{op}} - g_{mp}\, i_X\left(\frac{1}{g_{mp}} \,//\, r_{op}\right)$$

$$= i_X$$

Then $$i_X = \frac{v_{out}}{r_{op}\left[1 + g_{mp}\left(\frac{1}{g_{mp}} \,//\, r_{op}\right)\right]}$$

(KVL) $$-v_A + (i_X - g_{mN}v_1)r_{oN} - (i_Y - g_{mN}v_2)r_{oN} + v_{out} = 0$$

Note that $v_{in,1} \neq v_1$

$v_{in,2} \neq v_2$

But $v_1 - v_2 = v_{in,1} - v_{in,2}$

and $i_X = -i_Y$

then $$-v_A + 2i_X r_{oN} - g_{mN}r_{oN}(v_{in1} - v_{in2}) + v_{out} = 0$$

③

Substitute v_A and i_X

Rashid 9.14 continued

$$\frac{v_{out}}{r_{op}\left[1+g_{mp}\left(\frac{1}{g_{mp}}//r_{op}\right)\right]}\left(\frac{1}{g_{mp}}//r_{op}\right)$$

$$+2r_{oN}\frac{v_{out}}{r_{op}\left[1+\underbrace{g_{mp}\left(\frac{1}{g_{mp}}//r_{op}\right)}_{\approx 1 \text{ since } 1/g_{mp} \ll r_{op}}\right]}+v_{out}=g_{mN}r_{oN}(v_{in1}-v_{in2})$$

$$v_{out}\left[\frac{\overbrace{\cancel{1/g_{mp}}}^{\text{neglected}}+2r_{oN}+2r_{op}}{r_{op}\left[1+g_{mp}\left(\frac{1}{g_{mp}}//r_{op}\right)\right]}\right]=g_{mN}r_{oN}(v_{in1}-v_{in2})$$

$$\frac{v_{out}}{v_{in1}-v_{in2}}=g_{mN}r_{oN}\frac{r_{op}\left[1+g_{mp}\left(\frac{1}{g_{mp}}//r_{op}\right)\right]}{2r_{oN}+2r_{op}}$$

$$\cong g_{mN}r_{oN}\frac{2r_{op}}{2r_{oN}+2r_{op}}$$

$$=g_{mN}\frac{r_{oN}r_{op}}{r_{oN}+r_{op}}=g_{mN}(r_{oN}//r_{op})$$

The circuit is single ended but the gain is approximately equal to that of the fully differential one.

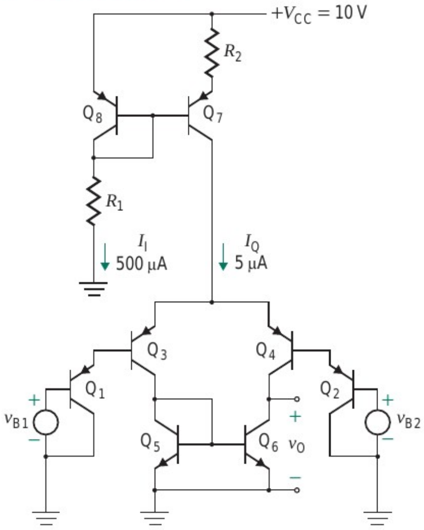
see pp 512-514 of Razavi

4

BJT Differential Amplifier with Active Load **Rashid 9.42**

9.42 A differential amplifier is shown in Fig. P9.42. The transistors are identical. Assume $V_{BE} = 0.7$ V, $V_T = 26$ mV, $\beta_{F(npn)} = 100$, $\beta_{F(pnp)} = 50$, $V_A = 40$ V, and $V_{CC} = 10$ V. Calculate the values of R_1, R_2, A_d, R_{id}, A_c, R_{ic}, and CMRR.

FIGURE P9.42



Necessary Knowledge and Skills: BJT input stages for BJT differential amplifier (for higher input impedance), BJT differential amplifier with current mirror active load, Widlar current source for tail current, single ended amplifier with the same gain as that of the fully differential amplifier, small signal equivalent model for BJT, voltage gain computations, input/output impedance computations.

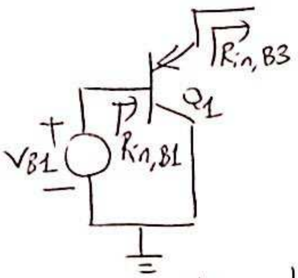
Rashid
9.42

Analysis and Design of the circuit

→ Design of the tail current source

→ Output impedance of the tail current source needs to be computed

→ Calculation of the impedances (small signal)



and similarly $R_{in,B4}$

$\Rightarrow$ Note $R_{in,B1} \approx (1+g_{m1} R_{in,B3}) \cdot r_{\pi 1}$

and $R_{in,B2} \approx (1+g_{m2} R_{in,B4}) r_{\pi 2}$

(the purpose here is to have larger input impedance)

$\Rightarrow$ Note here that

$$v_{B3} = v_{B1} \frac{R_{in,B3}}{R_{in,B3} + \frac{1}{g_{m1}}}$$

$$v_{B4} = v_{B2} \frac{R_{in,B4}}{R_{in,B4} + \frac{1}{g_{m2}}}$$

(Remember the gain of emitter follower stages)

$\Rightarrow$ $R_{in,B3}$ and $R_{in,B4}$ are going to be large, $\frac{1}{g_{m1}}$ and $\frac{1}{g_{m2}}$ are small

then $v_{B3} \approx v_{B1}$ and $v_{B4} \approx v_{B2}$

→ calculation of the differential mode gain (small signal)

→ " " " common " " (" ")

→ Compute CMRR

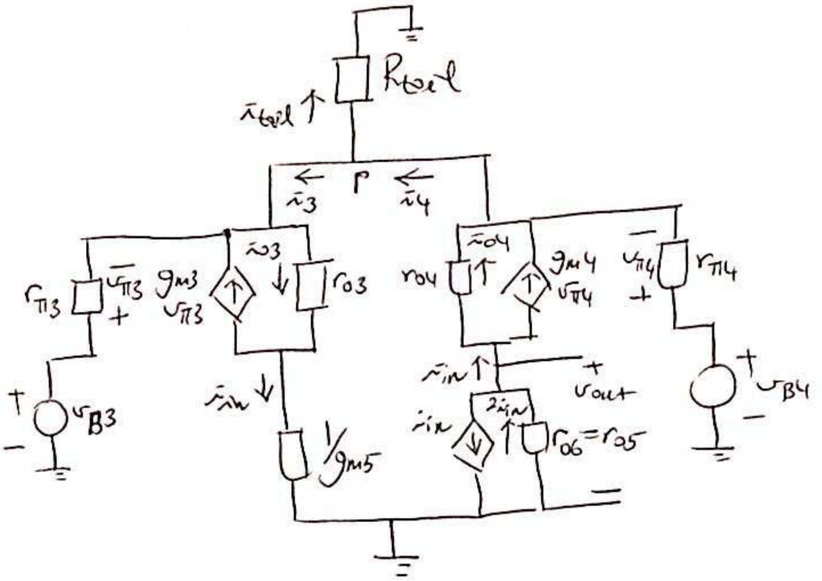
Above are the tasks that need to be completed in this question.

1

Simplified model of the differential stage in small signal

Rashid 9.42

cont-



$$v_{out} = -2 i_{in} r_{o5}$$

$$i_4 = i_3 + i_{tail}$$

$$v_p = i_{tail} \cdot R_{tail} = (i_4 - i_3) R_{tail}$$

$$-i_3 + i_{o3} - g_{m3} v_{\pi 3} + \frac{(i_4 - i_3) R_{tail} - v_{B3}}{r_{\pi 3}} = 0$$

$$v_{\pi 3} = v_{B3} - (i_4 - i_3) R_{tail}$$

$$i_{in} + g_{m3} v_{\pi 3} - i_{o3} = 0$$

$$-i_{in} + i_{o4} + g_{m4} v_{\pi 4} = 0$$

$$i_4 - i_{o4} - g_{m4} v_{\pi 4} + \frac{(i_4 - i_3) R_{tail} - v_{B4}}{r_{\pi 4}} = 0$$

$$v_{\pi 4} = v_{B4} - (i_4 - i_3) R_{tail}$$

$$v_{B3} - v_{\pi 3} = v_{B4} - v_{\pi 4} \Rightarrow v_{B3} - v_{B4} = v_{\pi 3} - v_{\pi 4}$$

(KVL) $$\underbrace{i_{o3}}_{\Downarrow\ (i_{in} + g_{m3} v_{\pi 3})} r_{o3} + i_{in} \frac{1}{g_{m5}} + 2 i_{in} r_{o5} + \underbrace{i_{o4}}_{\Downarrow\ (i_{in} - g_{m4} v_{\pi 4})} r_{o4} = 0$$

2

Rashid
9.42
cont-

Continue from the KVL

$r_{o3} = r_{o4}$

$g_{m3} = g_{m4}$

$$\left(i_{in} + g_{m3} v_{\pi 3}\right) r_{o3} + i_{in}\frac{1}{g_{m5}} + 2 i_{in} r_{o5} + \left(i_{in} - g_{m3} v_{\pi 4}\right) r_{o3} = 0$$

$$\underbrace{2 i_{in} r_{o5} + 2 i_{in} r_{o3} + i_{in}\frac{1}{g_{m5}}}_{\approx} + g_{m3} r_{o3}\underbrace{\left(v_{\pi 3} - v_{\pi 4}\right)}_{\parallel} = 0$$

$$2 i_{in}\left[\cancel{2} r_{o5} + \cancel{2} r_{o3}\right] + g_{m3} r_{o3}\left(v_{B3} - v_{B4}\right) = 0$$

$$\underbrace{\left[-2 i_{in} r_{o5}\right]}_{v_{out}}\left[\frac{-(r_{o5} + r_{o3})}{r_{o5}}\right] = -g_{m3} r_{o3}\left(v_{B3} - v_{B4}\right) = 0$$

$$\frac{v_{out}}{v_{B3} - v_{B4}} = g_{m3}\frac{r_{o3} r_{o5}}{r_{o5} + r_{o3}}$$

differential mode gain $= g_{m3}\left(r_{o3} \parallel r_{o5}\right)$

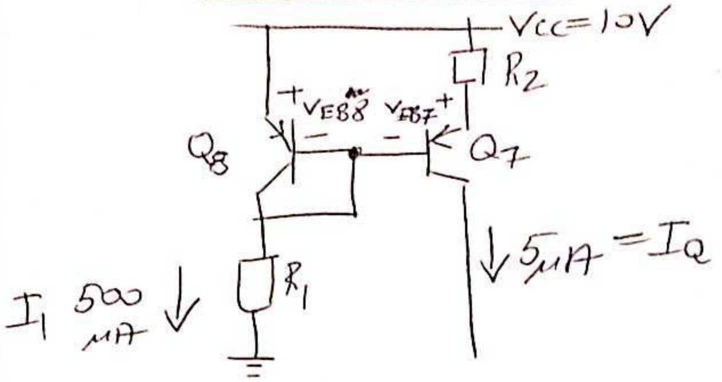
Common mode gain $\Rightarrow$ $\left[v_{B3} = v_{B4} \Rightarrow v_{out} = 0\right]$

CMRR in this approx is $+\infty$.

3

Rashid
9.42
cont-

Design of the tail current source



⇒ Neglect base currents

⇒ eqns

$$V_{EB8} + I_1 R_1 = V_{CC}$$
$$-V_{EB8} + V_{EB7} + I_Q R_2 = 0$$
$$I_1 = I_S \exp\left(\frac{V_{EB8}}{V_T}\right)$$
$$I_Q = I_S \exp\left(\frac{V_{EB7}}{V_T}\right)$$

4 eqns

4 unknowns: $\begin{bmatrix} V_{EB8} \\ V_{EB7} \\ R_1 \\ R_2 \end{bmatrix}$

R_{tail} = Output impedance of the tail current source

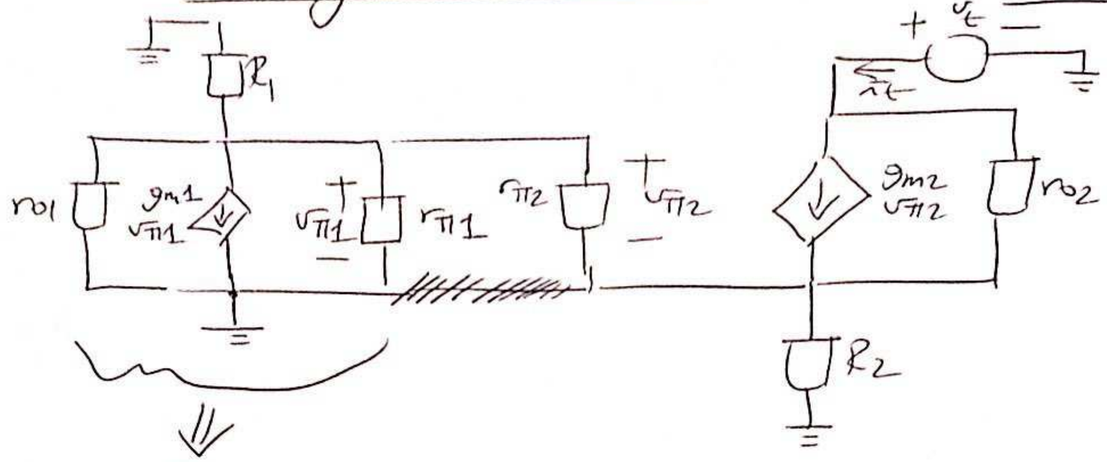
see the soln to problem 9.29 in Rashid.

(4)

Generated by CamScanner

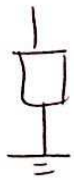
Rashid 9.29

Small-signal equivalent cir.



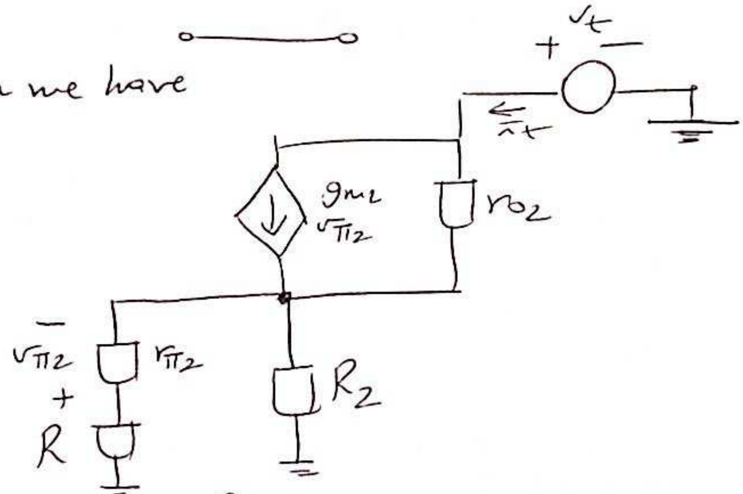
$\Downarrow$

$\equiv$



$R = R_1 // r_{o1} // \frac{1}{g_{m1}} // r_{\pi 1}$

then we have



$$-v_{\pi 2} = i_t \frac{R_2}{R_2 + r_{\pi 2} + R} \cdot r_{\pi 2}$$

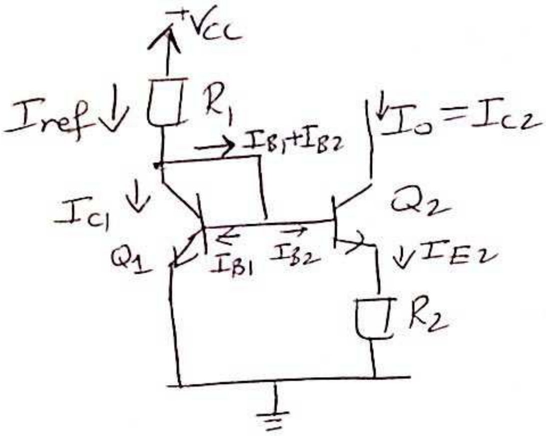
$$v_t = \left(i_t - g_{m2} v_{\pi 2}\right) r_{o2} + i_t \frac{R_2 \left(R + r_{\pi 2}\right)}{R_2 + r_{\pi 2} + R}$$

$$R_{out} = \frac{v_t}{i_t} = r_{o2} + g_{m2} r_{o2} \frac{R_2 r_{\pi 2}}{R_2 + r_{\pi 2} + R} + \frac{R_2 \left(R + r_{\pi 2}\right)}{R_2 + r_{\pi 2} + R}$$

Rashid
9.29
cont-

$$R_{out} = r_{o2} + (1 + g_{m2} r_{o2}) \frac{R_2 r_{\pi 2}}{R_2 + r_{\pi 2} + R} + \frac{R_2 R}{R_2 + r_{\pi 2} + R}$$

large signal analysis



(1)

$$I_{ref} = I_{C1} + I_{B1} + I_{B2}$$

$$= \boxed{(\beta+1) I_{B1} + I_{B2}} = \boxed{\frac{V_{CC} - V_{BE1}}{R_1} = \frac{V_{CC} - V_T \ln \frac{I_{C1}}{I_S}}{R_1}}$$

βI_{B1}

$$I_{C1} = I_S \exp\left(\frac{V_{BE1}}{V_T}\right)$$

$$I_{C2} = I_o = I_S \exp\left(\frac{V_{BE2}}{V_T}\right) = \beta I_{B2}$$

$$V_{BE1} = V_{BE2} + (\beta+1) I_{B2} R_2$$

$$V_T \ln \frac{I_{C1}}{I_S} = V_T \ln \frac{I_{C2}}{I_S} + (\beta+1) I_{B2} R_2$$

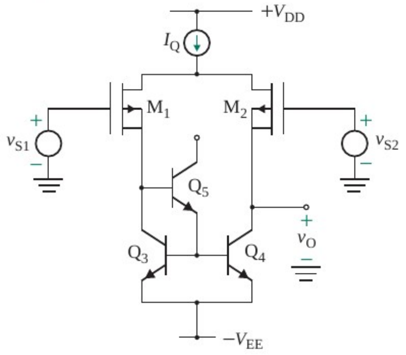
$$V_T \ln \frac{I_{C1}}{I_{C2}} = \boxed{(\beta+1) I_{B2} R_2 = V_T \ln \frac{I_{B1}}{I_{B2}}} \quad (2)$$

Make use of (1) and (2) and V_T, I_S and other parameters
$\Rightarrow$ solve for I_{B1} and I_{B2} (3)

PMOS Differential Amplifier with NPN BJT Active Load **Rashid 9.47**

9.47 A BiCMOS amplifier is shown in Fig. P9.47. The PMOS parameters are $V_t = -3$ V and $V_{GS} = -6$ V at $I_D = 1$ mA. The BJT parameters are $\beta_{F(npn)} = 100$, $\beta_{F(pnp)} = 50$, and $V_A = 40$ V. Assume $V_{DD} = -V_{EE} = 15$ V and $I_Q = 200$ μA. Calculate A_d, A_c, and CMRR.

FIGURE P9.47



Necessary Knowledge and Skills: PMOS differential amplifier, ideal current source for tail current, BJT current mirror for active load, smaller base current drawn by Q5 from the reference current at the expense of higher power and another VBE loss, single ended amplifier with a voltage gain the same as that of the fully differential amplifier, MOS and BJT small signal equivalent models, voltage gain and input/output impedance computations.

Extra Tasks: Point out the error in the schematic.

Rashid
9.47

Notes on the DC biasing of the differential stage

$$I_Q = 200\,\mu A$$

$$I_{D1} = I_{D2} = 100\,\mu A$$

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{SG} - |V_t|\right)^2 \quad \text{for PMOS transistors}$$

$$\underset{1mA}{\cancel{100\mu A}} = \frac{1}{2}\underbrace{\mu_p C_{ox}\frac{W}{L}}_{\text{compute}}\left(6V - |-3V|\right)^2$$

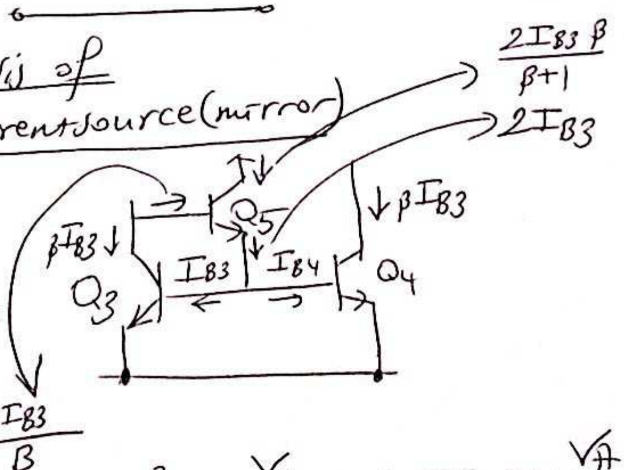
then

$$100\,\mu A = \frac{1}{2}\underbrace{\mu_p C_{ox}\frac{W}{L}}_{\text{computed above}}\left(V_{SG} - |-3V|\right)^2$$

V_{SG}: compute in this case (take the positive root that is larger than $|-3V|$)

Small signal analysis of the modified current source (mirror)

⇒ compute large signal currents and then small signal parameters

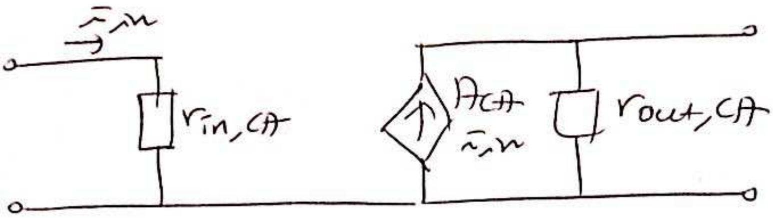


$$g_{m4} = g_{m3} = \frac{\beta I_{B3}}{V_T} \qquad r_{\pi 3} = r_{\pi 4} = \frac{\beta}{g_{m3}} = \frac{V_T}{I_{B3}} \qquad r_{o3} = r_{o4} = \frac{V_A}{\beta I_{B3}}$$

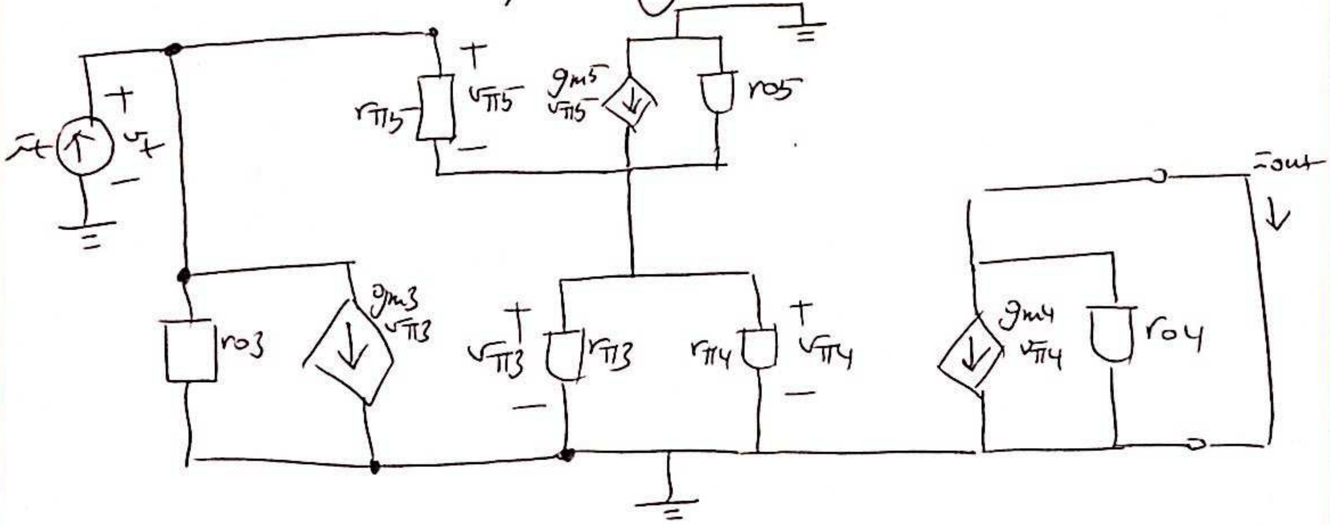
$$g_{m5} = \frac{2\beta I_{B3}}{(\beta+1)V_T} \qquad r_{\pi 5} = \frac{(\beta+1)V_T}{2I_{B3}} \qquad r_{o5} = \frac{(\beta+1)V_A}{2\beta I_{B3}}$$

Rashid 9.47 continued

Model the structure as a current ampli



$\Rightarrow$ compute $r_{in,CA}$ and A_{CA} with the following setup



$$v_t = v_{\pi3} + v_{\pi5}$$

$$-i_t + \frac{v_t}{r_{o3}} + g_{m3}v_{\pi3} + \frac{v_t - v_{\pi3}}{r_{\pi5}} = 0$$

$$-g_{m5}v_{\pi5} + \frac{v_{\pi3}}{r_{o5}} + \frac{v_{\pi3}}{r_{\pi3}} + \frac{v_{\pi3}}{r_{\pi4}} + \frac{v_{\pi3} - v_t}{r_{\pi5}} = 0$$

note that $g_{m3} \gg \frac{1}{r_{\pi5}}$

comparable cannot get rid of any

$$i_t = \frac{v_t}{r_{o3} \,//\, r_{\pi5}} + v_{\pi3}\left[g_{m3} - \cancel{\frac{1}{r_{\pi5}}}\right] = 0$$

$$r_{o3} = \frac{V_A}{\beta I_{B3}} \qquad r_{\pi5} = \frac{(\beta+1)V_T}{2I_{B3}}$$

typical values $V_A = 100V$, $\beta = 100$, $V_T = 25mV$

$$r_{o3} = \frac{100V}{100 I_{B3}} = \frac{1V}{I_{B3}}$$

$$r_{\pi5} = \frac{100(26mV)}{2I_{B3}} = \frac{1.3V}{I_{B3}}$$

2

Rashid 9.47

Cont.

$$\frac{v_{\pi 3}}{1/g_{m5} \,//\, \cancel{r_{o5}} \,//\, r_{\pi 3} \,//\, r_{\pi 4} \,//\, \cancel{r_{\pi 5}}} = \frac{v_t}{1/g_{m5} \,//\, r_{\pi 5}} = \frac{v_t}{r_{\pi 5}/(1+\beta)}$$

$1/g_{m5} << r_{o5}$ $\qquad$ $r_{\pi 3} = r_{\pi 4} << r_{\pi 5}$

LHS: we have $\dfrac{v_{\pi 3}}{1/g_{m5} \,//\, \frac{r_{\pi 3}}{2}}$

$$\frac{1}{g_{m5}} = \frac{(\beta+1) V_T}{2\beta I_{B3}} \simeq \frac{V_T}{2 I_{B3}}$$

(with $\beta >> 1$)

$$\frac{r_{\pi 3}}{2} = \frac{V_T}{2 I_{B3}}$$

LHS: we then have $\dfrac{v_{\pi 3}}{r_{\pi 3}/4}$

$$LHS = RHS \Rightarrow \frac{v_{\pi 3}}{r_{\pi 3}/4} \Rightarrow \frac{V_T}{4 I_{B3}} \simeq \frac{v_t}{r_{\pi 5}/(1+\beta)} \Rightarrow \frac{V_T}{2 I_{B3}}$$

$$\Rightarrow v_{\pi 3} \simeq \frac{v_t}{2}$$

3

Rashid 9.47 cont.

$$i_t = \frac{v_t}{r_{o3} // r_{\pi 5}} + \frac{v_t}{2/g_{m3}}$$

$$2/g_{m3} << r_{o3}$$

$$r_{\pi 5} = \frac{(\beta+1)V_T}{2I_{B3}} >> \frac{2}{g_{m3}} = \frac{2V_T}{\beta I_{B3}}$$

$$\frac{v_t}{i_t} \cong \frac{2}{g_{m3}} = r_{in,CA}$$

Compute now A_{CA}

$$v_{\pi 3} = v_{\pi 4} \cong \frac{v_t}{2}$$

$$g_{m4} v_{\pi 4} = -i_{out}$$

$$g_{m4} \frac{v_t}{2} = -i_{out}$$

$$\cancel{\frac{g_{m4}}{2}} \frac{\cancel{2} i_t}{\cancel{g_{m3}}} = -i_{out}$$

$$i_t = -i_{out} \Rightarrow A_{CA} = -1$$

Setup for computing $r_{out,CA}$

input is open cir.

$\Rightarrow$ connect this at the output

no independent source at the input stage $\Longrightarrow$ then $r_{out,CA} = r_{o4} = r_{o3}$

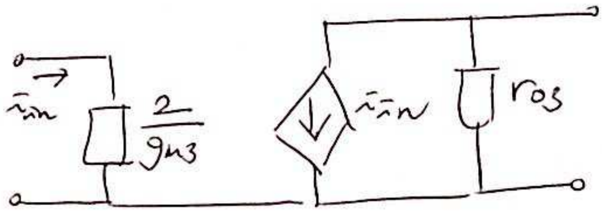
$v_t = v_{\pi 3} = 0$

4

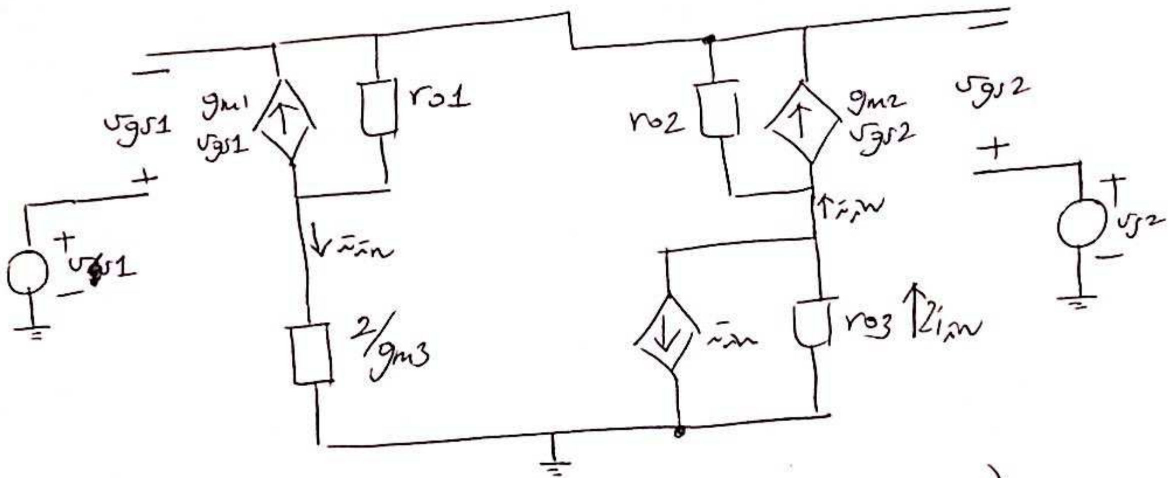
Rashid
9.47

we have for the small signal equivalent of the modified current mirror

$\Rightarrow$



$\Rightarrow$ small-signal equivalent of the differential amplifier with the active load



$\underline{\underline{KVL}}$: $\frac{2}{g_{m3}} i_{in} + 2 i_{in} r_{o3} + (i_{in} - g_{m2} v_{gs2}) r_{o2} + (i_{in} + g_{m1} v_{gs1}) r_{o1} = 0$

note $g_{m1} = g_{m2}$, $r_{o1} = r_{o2}$

also $v_{out} = -2 i_{in} r_{o3}$

neglect

$i_{in} \left[\cancel{\frac{2}{g_{m3}}} + 2r_{o3} + \underbrace{r_{o2} + r_{o1}}_{2r_{o1}} \right] + g_{m1} r_{o1} (v_{gs1} - v_{gs2}) = 0$

$2 i_{in} (r_{o3} + r_{o1}) = -g_{m1} r_{o1} (v_{gs1} - v_{gs2})$

3

Rashid
9.47
cont.

Note also that

$$v_{S1} - v_{gs1} = v_{S2} - v_{gs2}$$
$$v_{S1} - v_{S2} = v_{gs1} - v_{gs2}$$

$$\underbrace{\left[-2 i_{in} r_{o3}\right]}_{v_{out}} \left[-\frac{r_{o3}+r_{o1}}{r_{o3}}\right] = -g_{m1} r_{o1} (v_{S1} - v_{S2})$$

$$\frac{v_{out}}{v_{S1}-v_{S2}} = g_{m1}\left[\frac{r_{o1} r_{o3}}{r_{o1}+r_{o3}}\right]$$
$$= g_{m1}(r_{o1} // r_{o3})$$

In the case of common-mode excitation,

$v_{S1} = v_{S2} \Rightarrow i_{in} = 0 \Rightarrow v_{out} = 0$

Then approximately the common mode gain $\dfrac{v_{out}}{\frac{v_{S1}+v_{S2}}{2}} = 0 \Rightarrow CMRR = +\infty$

A more accurate analysis or simulation results are necessary to comment on the common-mode gain and CMRR.

(6)

CMOS Differential Amplifier **Razavi 10.68**

68. Calculate the common-mode gain of the circuit depicted in Fig. 10.87. Assume $\lambda > 0$, $g_m r_O \gg 1$, and use the relationship $A_v = -G_m R_{out}$.

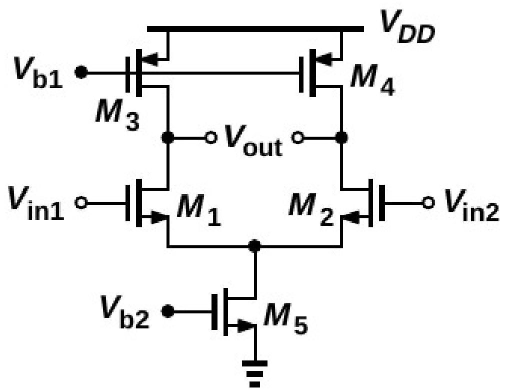


Figure 10.87

Necessary Knowledge and Skills: MOS differential amplifier, high impedance drain connections for load, single MOS transistor M5 for providing tail current, common mode gain computation, input/output impedance computation.

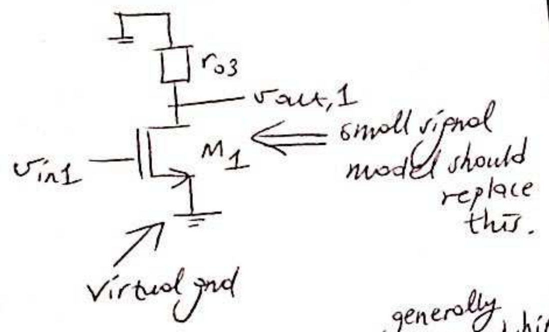
Razavi 10.68

M_5 is the current source.

$R_{out,CS} \Rightarrow R_{out}$ of the current source

$R_{out,CS} = r_{o5}$

Differential mode



generally neither is much higher than the other

$$\frac{v_{out,1}}{v_{in1}} = \underbrace{-g_{m1}(r_{o1} // r_{o3})}$$

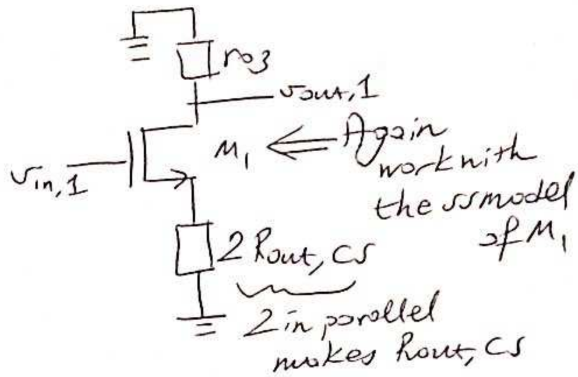
We can also prove that

$$A_d = \frac{v_{out,1} - v_{out,2}}{v_{in,1} - v_{in,2}} =$$

diff mode gain

since $g_{m1} = g_{m2}$
$r_{o3} = r_{o4}$
$r_{o1} = r_{o2}$

Common mode



TCA derivations:
(done in many other solutions)

$$A_c = \frac{v_{out1}}{v_{in,1}} = -\frac{g_{m1}}{1 + g_{m1} 2R_{out,CS}} \cdot \left[r_{o3} // r_{out,TCA} \right]$$

common mode gain

and

$$r_{out,TCA} = (1 + g_{m1} r_{o1}) 2R_{out,CS} + r_{o1}$$

(remember the cascode config)

$\Rightarrow$ Note here that $r_{o3} \ll r_{out,TCA}$

(1)

Razavi
10.68
continued

Common mode gain
A_c simplified (approx.)

$$A_c \approx -\frac{g_{m1} r_{o3}}{1+g_{m1} 2R_{out,CS}}$$

then
$$CMRR = \frac{A_d}{A_c} \cong \frac{-\cancel{g_{m1}} \frac{r_{o1} \cancel{r_{o3}}}{r_{o1}+r_{o3}}}{-\frac{\cancel{g_{m1}} \cancel{r_{o3}}}{1+g_{m1} 2R_{out,CS}}}$$

$$= \frac{r_{o1}(1+g_{m1} 2R_{out,CS})}{r_{o1}+r_{o3}}$$

2

BJT Differential Amplifier **Razavi 10.81**

81. The differential pair depicted in Fig. 10.95 must provide a gain of 5 and a power budget of 4

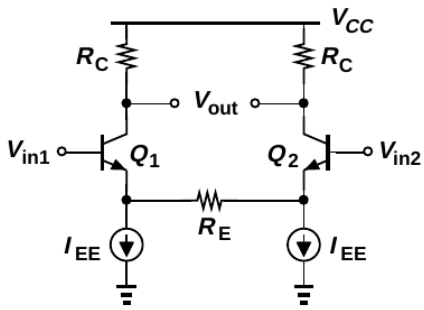


Figure 10.95

mW. Moreover, the gain of the circuit must change by less than 2% if the collector current of either transistor changes by 10%. Assuming $V_{CC} = 2.5$ V and $V_A = \infty$, design the circuit. (Hint: a 10% change in I_C leads to a 10% change in g_m.)

Necessary Knowledge and Skills: BJT differential amplifier with two tail currents, resistor loads, sensitivity computation, differential gain calculation, input/output impedance calculations, power budget in DC, design question.

Razavi 10.81

Power budget

in DC $\Longrightarrow$ $v_{in1} = v_{in2}$ (DC voltages)

No current thru R_E

then DC Power $= V_{CC} \cdot 2I_{EE}$

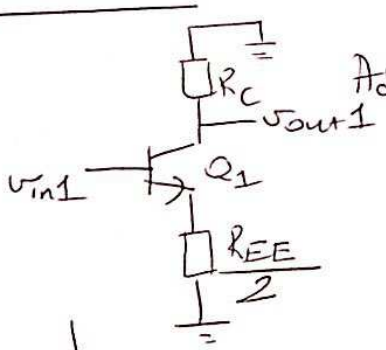
Power budget $= 4mW > V_{CC} \cdot 2I_{EE} = (2.5V)\, 2 \cdot I_{EE}$

Then $I_{EE} < \frac{4mW}{5V} = 0.8mA$

Differential Gain

(see TCA related derivations)

half cir



$$A_d = \frac{v_{out1}}{v_{in1}} \cong -\frac{g_{m1}}{1+g_{m1}\frac{R_{EE}}{2}} \cdot R_C$$

Note that $g_{m1} = \frac{I_{C1}}{V_T}$

$\cong \frac{I_{EE}}{25mV}$

$$\left|\frac{v_{out1}-v_{out2}}{v_{in1}-v_{in2}}\right| = A_d = 5 = \left| -\frac{\frac{I_{C1}}{V_T}}{1+\frac{I_{C1}}{V_T}\frac{R_{EE}}{2}} \cdot R_C \right|$$

specification

Also another requirement $\left|\frac{\Delta A_d}{A_d}\right| < 2\%$ when $\left|\frac{\Delta I_{C1}}{I_{C1}}\right| = 10\%$

this means $\left|\frac{\Delta A_d}{\Delta I_{C1}} \frac{I_{C1}}{A_d}\right| < \frac{1}{5} \Rightarrow \left|S^{A_d}_{I_{C1}}\right| < \frac{1}{5}$

sensitivity of A_d w.r.t. I_{C1}

Razavi
10.81
continued

$$S_{I_{C1}}^{A_d} = \frac{\partial A_d}{\partial I_{C1}} \frac{I_{C1}}{A_d}$$

where $A_d = -R_C \dfrac{2 I_{C1}}{2V_T + I_{C1} R_{EE}}$

$$\frac{\partial A_d}{\partial I_{C1}} = -R_C \left[\frac{2(2V_T + I_{C1} R_{EE}) - R_{EE}(2 I_{C1})}{(2V_T + I_{C1} R_{EE})^2} \right]$$

$$= -R_C \frac{4V_T}{(2V_T + I_{C1} R_{EE})^2}$$

$$\frac{\partial A_d}{\partial I_{C1}} \frac{I_{C1}}{A_d} = \left[\cancel{(-R_C)} \frac{\cancel{4}^{\,2} V_T}{(2V_T + I_{C1} R_{EE})^{\cancel{2}}} \right] \left[\cancel{\left(\frac{1}{-R_C}\right)} \frac{\cancel{(2V_T + I_{C1} R_{EE})}}{\cancel{2 I_{C1}}} \right] \cancel{I_{C1}}$$

$$= \frac{2V_T}{2V_T + I_{C1} R_{EE}}$$

(2)

Razavi
10.81
Continued

$$\left| S_{I_{C1}}^{A_d} \right| = \left| \frac{2V_T}{\underbrace{2V_T + I_{C1}R_{EE}}_{\text{already positive}}} \right| < \frac{1}{5}$$

$$\Longrightarrow \quad 10V_T < 2V_T + I_{C1}R_{EE}$$

$$8V_T < \underbrace{I_{C1}}_{\simeq I_{EE}} R_{EE}$$

$$8\underbrace{V_T}_{25mV} < I_{EE}R_{EE}$$

$$200mV < I_{EE}R_{EE}$$

Our constraints are

$$I_{EE} < 0.8mA$$

$$200mV < I_{EE}R_{EE}$$

$$|A_d| = R_C \frac{2I_{EE}}{2V_T + I_{EE}R_{EE}} = 5$$

choose $I_{EE} = 0.6mA$

$$R_{EE} > \frac{200mV}{0.6mA} = 333\Omega$$

choose $R_{EE} = 400\Omega$

3

Pnazani
10.8T
continued

then $$\frac{2\,(0.6\,mA)}{2(25\,mV)+(0.6\,mA)(400\,\Omega)}\,R_C = 5$$

$$\frac{1.2\,mA}{\underbrace{50\,mV}_{0.05\,V}+\underbrace{(0.6\,mA)(0.4k)}_{0.24\,V}}\,R_C = 5$$

$$R_C = \frac{5\,(0.29)\,V}{1.2\,mA}$$

$$= \frac{1.35}{1.2}\,k\Omega$$

(4)

BJT Differential Amplifier **Sedra 7.38**

7.38 Find the voltage gain and input resistance of the amplifier in Fig. P7.38 assuming that $\beta = 100$.

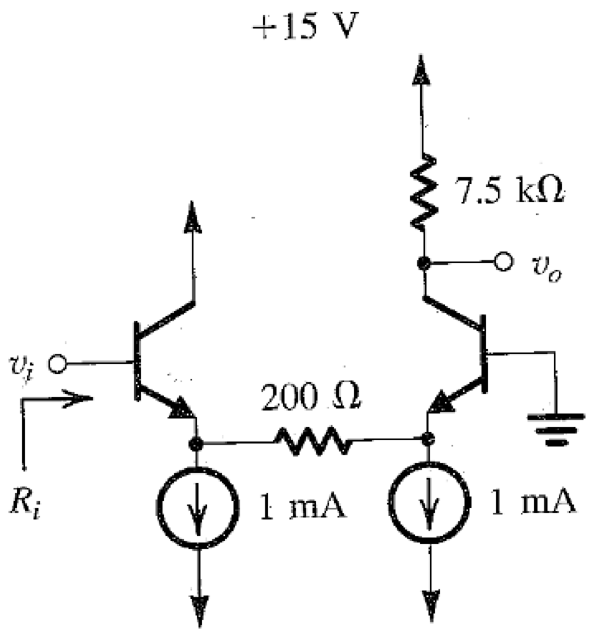


FIGURE P7.38

Necessary Knowledge and Skills: Cascaded amplifiers, common collector (emitter follower) followed by common base, input/output impedance computations, voltage gain calculations.

Sedra 7.38

DC biasing

With $v_i = 0\,V$ in DC

the two transistors Q_1 and Q_2 will almost be in balance (despite the 7.5kΩ resistance),

and $I_{C,Q1} \approx I_{C,Q2} \approx 1mA$ (in DC)

with the current over 200Ω ⇒ 0 V (in DC)

$$g_{m1} = \frac{I_{C,Q1}}{V_T} = \frac{1mA}{26mV}$$ (the same for g_{m2} (approx))

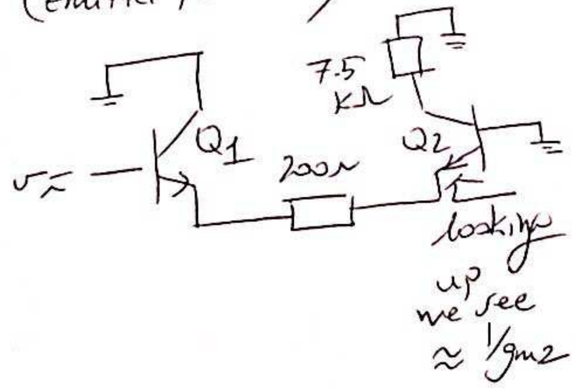
$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{g_{m1}}$$ ⇒ the same for $r_{\pi 2}$

$$r_{o1} \cong \frac{V_A}{I_{C,Q1}}$$ (can be computed if V_A is given) ⇒ the same for r_{o2}

Small signal analysis

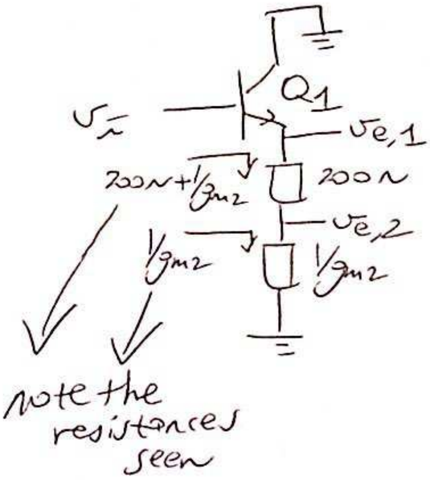
We have (in s.s.) a cascade of two amplifiers

common collector (emitter follower) + common base



(in this schematic, replace Q_1 and Q_2 by their s.s. equiv. cir) common ~~emitter~~ collector (emitter-follower) stage

Sedra 7.38 continued



$$\frac{v_{e,1}}{v_\pi} \simeq \frac{200\Omega + 1/g_{m2}}{200\Omega + 1/g_{m2} + 1/g_{m1}}$$

$$< 1$$

(see the derivations for common collector voltage gain)

$$\frac{v_{e,2}}{v_{e,1}} = \frac{1/g_{m2}}{200\Omega + 1/g_{m2}}$$ (voltage divider)

then
$$\frac{v_{e,2}}{v_\pi} = \frac{v_{e,2}}{v_{e,1}} \frac{v_{e,1}}{v_\pi}$$

$$= \frac{1/g_{m2}}{\cancel{200\Omega + 1/g_{m2}}} \frac{\cancel{200\Omega + 1/g_{m2}}}{200\Omega + 1/g_{m2} + 1/g_{m1}}$$

$$= \frac{1/g_{m2}}{200\Omega + 1/g_{m2} + 1/g_{m1}}$$

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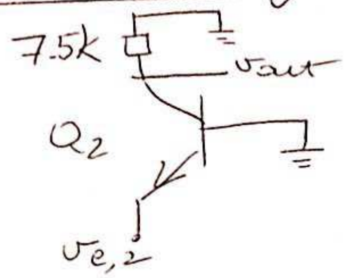
2

Common base stage

Sedra 7.38 continued



$$\frac{v_{out}}{v_{e,2}} \cong g_{m2}\,(7.5k\Omega)$$

(see the derivations for common base amplifiers)

then $\dfrac{v_{out}}{v_i} = \dfrac{v_{e,2}}{v_i}\,\dfrac{v_{out}}{v_{e,2}}$

$$\cong \frac{\cancel{1/g_{m2}}}{200\Omega + 1/g_{m2} + 1/g_{m1}} \cdot \cancel{g_{m2}}\,(7.5k\Omega)$$

$$= \frac{7.5k\Omega}{200\Omega + \underbrace{1/g_{m2}}_{} + \underbrace{1/g_{m1}}_{}} \approx \frac{7.5k\Omega}{250\Omega} = 30$$

$$\approx \left(\frac{1mA}{25mV}\right)^{-1} = 25\Omega$$

see TCA derivations for R_i (they apply even if we do not analyze a TCA here)

$$R_i \approx \left[1 + g_{m1}\left[200\Omega + 1/g_{m2}\right]\right] r_{\pi 1}$$

$$\approx \left[1 + g_{m1}200 + 1\right] r_{\pi 1} = 2r_{\pi 1} + 200 \cdot \underbrace{100}_{} \qquad 2r_{\pi1} \Rightarrow \beta(25\Omega)$$

$$= 5k + 20k \qquad g_m r_\pi = \beta$$

$$= 25k$$

3