GYTE
Electronics Engineering

ELEC 331
Electronic Circuits 2

Fall 2014

Instructor: Assist. Prof. Önder Şuvak

HW 14
Questions and Answers

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Assigned:	20141222
Due:	20141229
Answers Out:	20141230
Late Due:	20150105

Contents

BJT Current Source **Sedra 6.144**

D6.144 (a) For the circuit in Fig. P6.144, assume BJTs with high β and $v_{BE} = 0.7$ V at 1 mA. Find the value of R that will result in $I_O = 10\ \mu$A.

(b) For the design in (a), find R_o assuming $\beta = 100$ and $V_A = 100$ V.

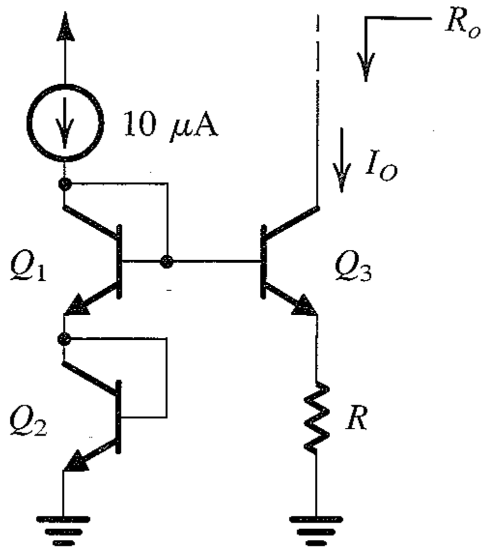

FIGURE P6.144

Notes: None.

Additional Tasks: None.

Necessary Knowledge and Skills: Current source, current reference, finite beta effect, output impedance, design question.

Sedra 6.144

$$I_{C_1} = I_S \exp\left(\frac{V_{BE1}}{V_T}\right)$$

$$I_{C2} = I_S \exp\left(\frac{V_{BE2}}{V_T}\right)$$

$$I_{C3} = I_S \exp\left(\frac{V_{BE3}}{V_T}\right)$$

a) $I_{C_1} = I_{C2} = 10\mu A$

$I_{C3} = 10\mu A$

$I_S \Rightarrow$ the same for Q_1 and Q_2 and Q_3

$V_T \Rightarrow$ the same for all

then

$V_{BE1} = V_{BE2} = V_{BE3} = V_{BE}$

$$V_{BE} = V_T \ln \frac{I_C}{I_S}$$

$$0.7V = 25mV \ln \frac{1mA}{I_S} \Rightarrow \text{compute } I_S$$

these calculation apply for $(V_{BE}, I_C) = (0.7V, 1mA)$

$$I_S = \frac{1mA}{\exp\left(\frac{0.7V}{25mV}\right)}$$

then

$$V_{BE} = 25mV \ln \frac{10\mu A}{I_S}$$

plug in this I_S

$\Downarrow$ for $I_C = 10\mu A$

compute!

Note that $RI_o = V_{BE2}$ since

$$V_{BE2} + \cancel{V_{BE1}} - \cancel{V_{BE3}} - RI_o = 0$$

these are the same, see above.

$$R = \frac{V_{BE2}}{I_o} = \frac{V_{BE2}}{10\mu A}$$

(1)

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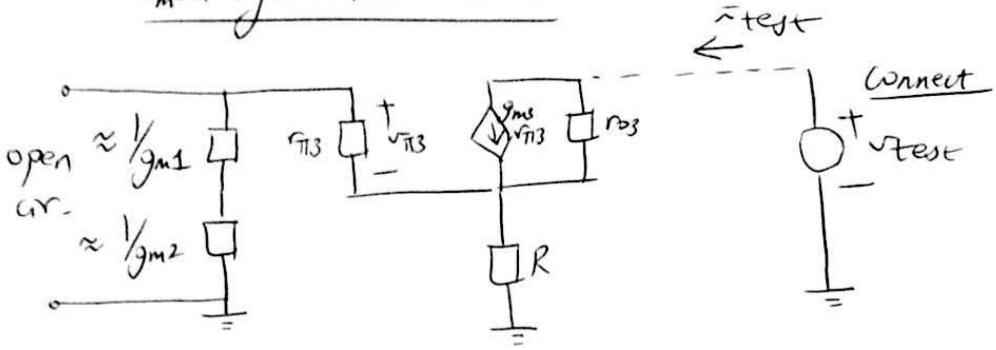
Sedra 6.144
cont.

b)

Small-signal equivalent cir.



$$r_{\pi 3} \gg \frac{1}{g_{m1}} + \frac{1}{g_{m2}}$$

$$R_o \cong \frac{v_{test}}{i_{test}} = \left(1 + g_{m3} r_{o3}\right)\left(R // r_{\pi 3}\right) + r_{o3}$$

Note that

$$r_{o3} = \frac{V_A}{I_{C3}} = \frac{100V}{10\mu A} = \frac{100}{10 \times 10^{-6}} = 10\,M\Omega$$

$$g_{m3} = \frac{I_{C3}}{V_T} = \frac{10\mu A}{25mV} = \frac{10 \times 10^{-6}}{25 \times 10^{-3}} = 0.4 \times 10^{-3}\,S$$
$$= 0.4\,mS$$

$$r_{\pi 3} = \frac{\beta}{g_{m3}} = \frac{\overset{250}{\cancel{100}}}{\cancel{0.4} \times 10^{-3}\,S} = 250\,K\Omega$$

$R \Rightarrow$ computed in part (a)

then compute R_o!

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BJT Differential Amplifier **Sedra 7.41**

7.41 For the differential amplifier shown in Fig. P7.41, identify and sketch the differential half-circuit and the common-mode half-circuit. Find the differential gain, the differential input resistance, the common-mode gain, and the common-mode input resistance. For these transistors, $\beta = 100$ and $V_A = 100$ V.

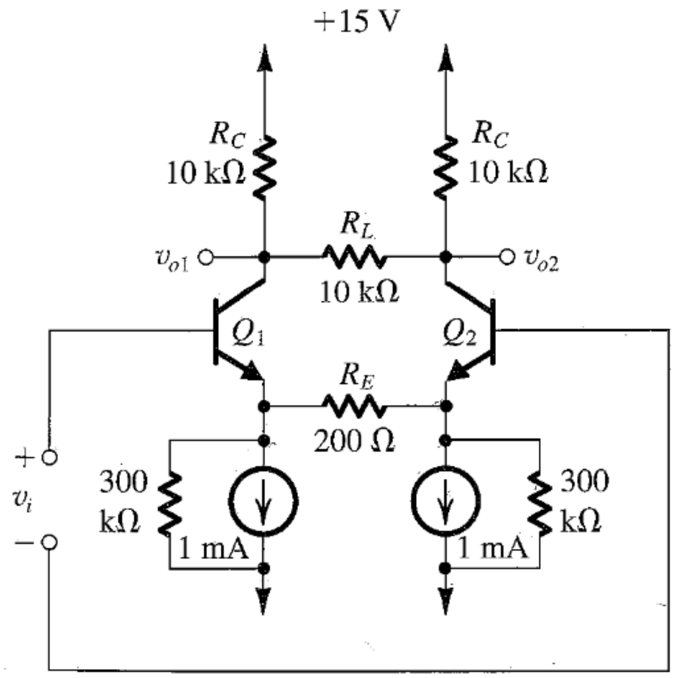


FIGURE P7.41

Notes: None.

Additional Tasks: Solve the following problem applied to the given circuit.

(d) If $v_{B1} = 0.1 \sin 2\pi \times 60t + 0.005 \sin 2\pi \times 1000t$ volts, $v_{B2} = 0.1 \sin 2\pi \times 60t - 0.005 \sin 2\pi \times 1000t$, volts, find v_o.

Necessary Knowledge and Skills: BJT symmetrical differential amplifier, common and differential mode gain, input impedances, CMRR.

Sedra 7.41

DC analysis

when $v_i = 0$ (no differential input)

$\Rightarrow$ No current over R_L and R_E

$\Rightarrow I_{C1} = I_{C2} \approx 1mA$

$\Rightarrow$ small signal parameter (applies to Q_1 and Q_2)

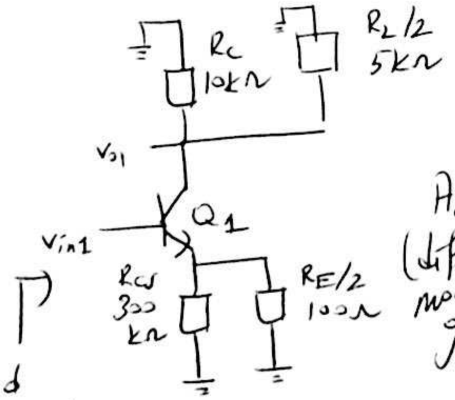
$$g_m = \frac{I_C}{V_T} = \frac{1mA}{25mV} = 40mS$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40mS} = 2.5K\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100V}{1mA} = 100K\Omega$$

Small signal analysis

Diff mode half cir $\Rightarrow$ In the middle of R_L and R_E we have virtual gnd.



$R_{i,d}$

(diff. mode input impedance)

Common emitter config

$$A_d = \frac{v_{o1}}{v_{in1}} \cong \left[- \frac{g_m}{1+g_m\left(R_{CS} /\!/ \frac{R_E}{2}\right)} \right] \left[R_C /\!/ \frac{R_L}{2} /\!/ \left[1+g_m\left(R_{CS} /\!/ \frac{R_E}{2}\right)\right] r_o \right]$$

(diff. mode gain)

$$R_{i,d} \cong r_{\pi 1}\left[1+g_m\left(R_{CS} /\!/ \frac{R_E}{2}\right)\right]$$

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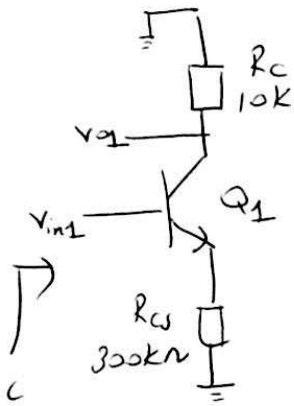
Common Mode Half Cir.

Sedra 7.41 contin.

With common mode input, there is no current over R_L or R_E.

common emitter config.



$$A_C = \frac{v_{o1}}{v_{in1}} \simeq \left[- \frac{g_m}{1 + g_m R_{CS}} \right] \left[R_C \,//\, \left[1 + g_m R_{CS} \right] r_o \right]$$

(common mode gain)

$R_{i,c}$

(common-mode input impedance)

$$R_{i,c} \simeq r_\pi \left[1 + g_m R_{CS} \right]$$

$$CMRR = \left| \frac{A_d}{A_c} \right|$$

If
$$v_{B1} = v_c + \frac{v_d}{2}, \quad v_{B2} = v_c - \frac{v_d}{2}$$
$$v_c = \frac{v_{B1} + v_{B2}}{2} = 0.1 \sin(2\pi \cdot 60t) \text{ V}$$
$$v_d = v_{B1} - v_{B2} = 0.01 \sin(2\pi \cdot 1000t) \text{ V}$$

Make use of superposition

$\Rightarrow$ Activate diff mode only

$$v_{o1} = A_d \frac{v_d}{2}$$
$$v_{o2} = -A_d \frac{v_d}{2}$$

$\Rightarrow$ Activate common mode only

$$v_{o1} = A_c v_c$$
$$v_{o2} = A_c v_c$$

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Sedra 7.41

contin-

By superposition

$$\Rightarrow v_{o1} = A_d \frac{v_d}{2} + A_c v_c$$

$$v_{o2} = -A_d \frac{v_d}{2} + A_c v_c$$

A_d, A_c computed above

$$\Rightarrow v_{o1} - v_{o2} = A_d v_d$$

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Practical Feedback Circuits **Malik 9.29**

9.29 For each circuit of Fig. P9.29, identify the type of feedback and find the numerical value of β using the appropriate two-port parameter of the feedback circuit. State whether the feedback is dc or ac. If both, find the two-port parameter values for both.

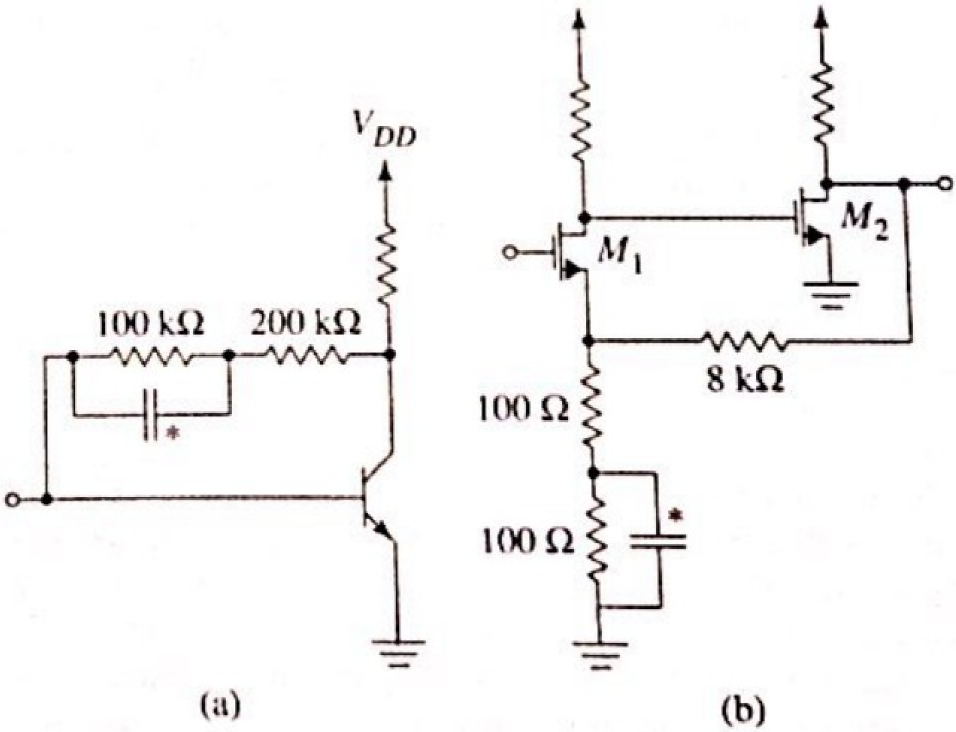


Figure P9.29

Notes: None.

Additional Tasks: Do full analyses of the feedback circuits in small signal.

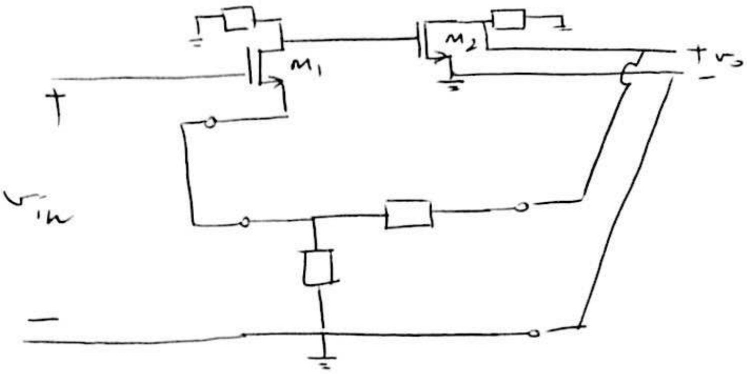
Necessary Knowledge and Skills: Type of feedback determination, distinguishing ac/dc feedback, two-port parameters of feedback networks.

Motik 9.29

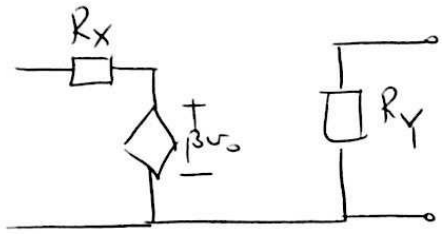
Feedback is both AC and DC in each of the circuits given.

Circuit b

Voltage-series feedback for a voltage amplifier



Two port parameters



in ac feedback

$$\beta = \frac{100}{8k} = \frac{1}{80}$$

$R_X = 100\Omega // 8k\Omega$

$R_Y = 8.1k\Omega$

in dc feedback

$$\beta = \frac{100+100}{8k} = \frac{1}{40}$$

$R_X = (100+100)\Omega // 8k\Omega$

$R_Y = 8.2k\Omega$

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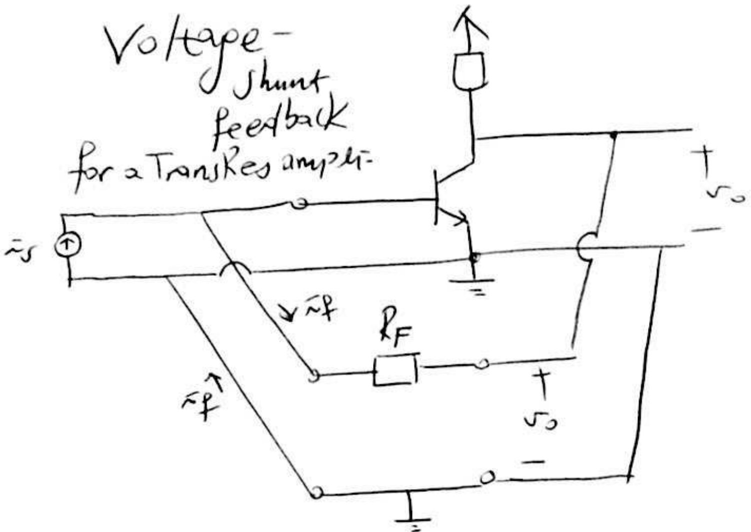
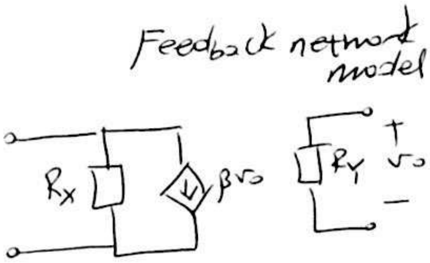
Malik 9.29

contin.

Circuit a





Two port parameters (see related	Formula	ac feedback	dc feedback
$\beta =$	$-\frac{1}{R_F}$	$-\frac{1}{200k\Omega}$	$-\frac{1}{300k\Omega}$
$R_X =$	R_F	$200k\Omega$	$300k\Omega$
$R_Y =$	R_F	$200k\Omega$	$300k\Omega$

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CMOS Basic Op-Amp **Rashid 14.19**

14.19 (D) The CMOS amplifier in Fig. 14.20 is operated at a biasing current of $I_Q = 50\ \mu A$. The parameters of the MOSFETs are $K_x = 10\ \mu A/V^2$, $|V_{M(NMOS)}| = V_{M(PMOS)} = 60$ V, $V_t = 1$ V, and $W/L = 80\ \mu m/10\ \mu m$, except for Q_7, for which $W/L = 160\ \mu m/10\ \mu m$. Assume $V_{DD} = V_{SS} = 5$ V.

a. Find V_{GS}, g_m, and r_o for all MOSFETs.

b. Find the low-frequency voltage gain of the amplifier A_{vo}.

c. Find the value of the external resistance R_{ref}.

d. Find the value of compensation capacitance C_x that gives a unity-gain bandwidth of 1 MHz and the corresponding slew rate.

e. Find the value of resistance R_x to be connected in series with C_x in order to move the zero frequency to infinity.

f. Find the common-mode input voltage range.

g. Find the output voltage range.

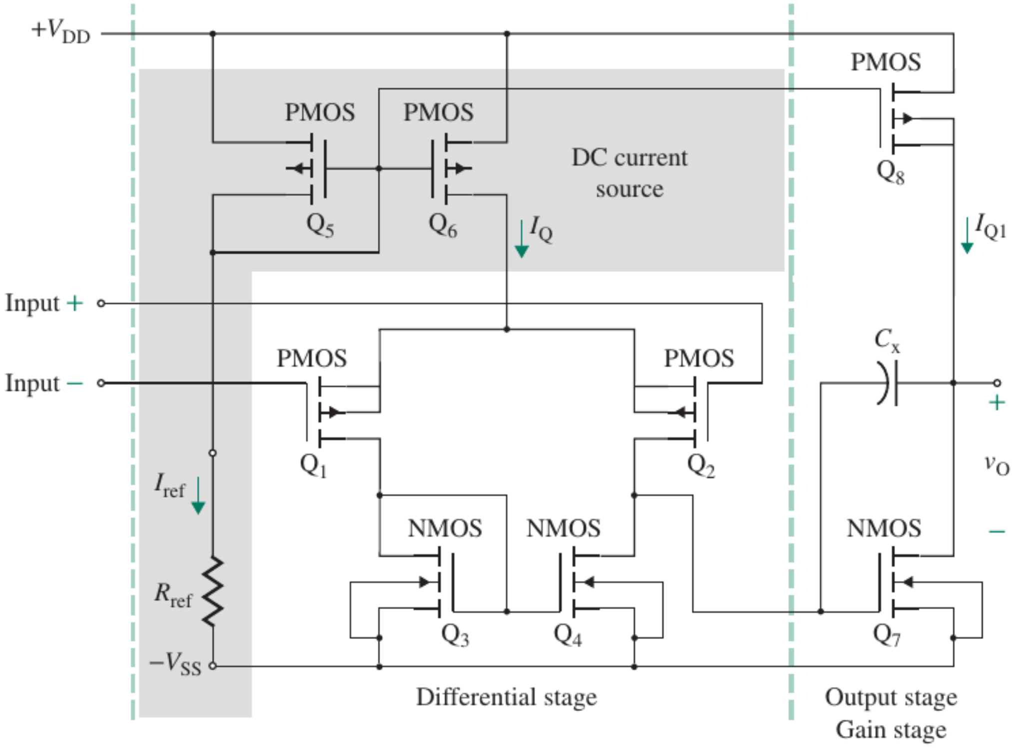

FIGURE 14.20 Schematic for op-amp MC14573 (Copyright of Motorola. Used by permission.)

CMOS Basic Op-Amp **Rashid 14.19**

Notes: See Rashid Section 14.4.2 for a similar example.

Additional Tasks: Point out the errors in the schematic.

Necessary Knowledge and Skills: CMOS basic OpAmp, voltage gain, slew rate, Miller effect, MOS current source, unity-gain bandwidth, gain-bandwidth product, voltage ranges to keep transistors in saturation.

Rashid
14-19

Errors in the schematic

→ Bulks of Q_5 and Q_6 should be connected to $+V_{DD}$
PMOS transistors

→ The Bulk of Q_8 should be connected to $+V_{DD}$
PMOS transistor

Systematic DC offset cancellation

See Sedra & Smith section 7.7.1 (or the answer to Sedra & Smith 9.2)

$$2\,\frac{\left(\frac{W}{L}\right)_8}{\left(\frac{W}{L}\right)_6} = \frac{\left(\frac{W}{L}\right)_7}{\left(\frac{W}{L}\right)_4}$$

which holds for the op-amp in this question.

part (a)

DC currents ⇒ $I_{D5} = I_{D6} = I_Q = I_{ref} = 50\mu A$
since $\left(\frac{W}{L}\right)$ ratios for Q_5 and Q_6 are the same.

⇒ $I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I_Q}{2} = 25\mu A$

⇒ $I_{D8} = I_{D7} = I_Q = 50\mu A$
since Q_6 and Q_8 have the same $\left(\frac{W}{L}\right)$ ratio.

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(1)

Rashid
14.19
contin-

The formula for the Drain current

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_t\right)^2 \quad \text{for NMOS}$$

part (a) continued

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{SG} - |V_t|\right)^2 \quad \text{for PMOS}$$

It is given in the question that

$$K_x = \frac{1}{2}\mu_n C_{ox} = \frac{1}{2}\mu_p C_{ox} = 10\mu A/V^2$$

We have the I_D and $\frac{W}{L}$ for each transistor

$\Rightarrow$ Calculate numerically $\left(V_{GS} - V_t\right)$ or $V_{SG} - |V_t|$ for each transistor

$\Rightarrow$ Calculate V_{GS} or V_{SG} for each transistor

$\Rightarrow$ $r_o = \dfrac{|V_A|}{I_D}$ $\quad |V_A| = 60V$ compute r_o for each transistor

$$\Rightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = 2K_x\frac{W}{L}\left(V_{GS} - V_t\right)$$

$$= \frac{2I_D}{V_{GS} - V_t} \quad \left(\text{or } \frac{2I_D}{V_{SG} - |V_t|} \text{ for PMOS}\right)$$

Calculate g_m numerically for each transistor

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2

Rashid
14.19
Contin-

part (b)

Since M_1 and M_2 are identical
(Q_1) (Q_2)
and biased with the same current:

$$g_{m1} = g_{m2}$$

$\Rightarrow$ gain of the differential stage

$$\frac{v_{g7}}{v_{in,+} - v_{in,-}} \cong -g_{m1}(r_{o2} // r_{o4})$$

(see the derivations for a diff amp with current mirror active load)

$\Rightarrow$ gain of the common source stage

$$\frac{v_{d7}}{v_{g7}} \cong -g_{m7}(r_{o8} // r_{o7})$$

$$\Rightarrow A_{vo} = \frac{\cancel{v_{g7}}}{v_{in,+} - v_{in,-}} \cdot \frac{v_{d7}}{\cancel{v_{g7}}}$$

calculate numerically

---

part (c)

(KVL)

$$V_{DD} - V_{SG,5} - I_{ref} R_{ref} = -V_{SS}$$

$V_{DD} = V_{SS} = 5V$

$V_{SG,5}$ computed in part (a)

$I_{ref} = I_Q = 50\mu A$ ($\left(\frac{W}{L}\right)$ ratios of Q_5 and Q_6 are the same)

$\Rightarrow$ Compute R_{ref}.

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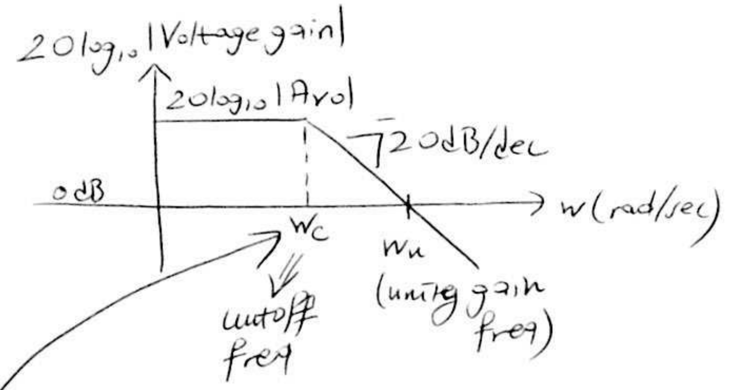
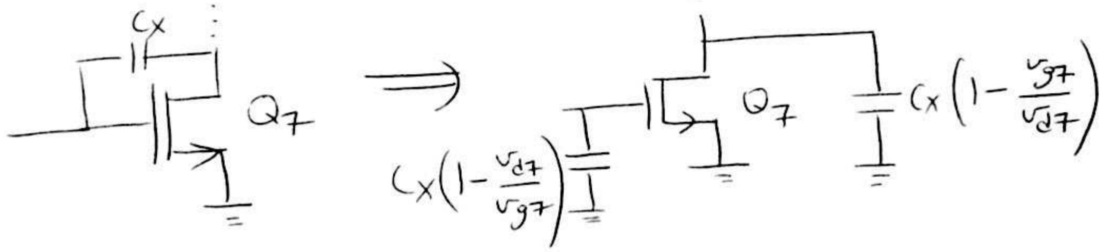
Rashid 14-19 continued

part (d)



See the derivations to confirm: $A_{vo} \cdot w_C = w_u$
(for this single dominant pole system)

Use the Miller's thm to transform C_X



$C_X\left(1-\frac{v_{d7}}{v_{g7}}\right)$ corresponds to the time constant that will dominate in the OCTC calculation for w_C.

Now use the OCTC method:

$$w_C \cong \frac{1}{\underbrace{C_X\left(1-\frac{v_{d7}}{v_{g7}}\right)} \cdot \underbrace{\left(r_{o2} \,//\, r_{o4}\right)}_{\text{Thevenin Resistance for this cap.}}}$$

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Rashid
14-19
contin-

part(d)
contin-

$$w_u = 1\ \text{MHz} \quad (\text{given})$$

$$\cong A_{vo}\, w_c$$

$$= A_{vo} \frac{1}{C_x\left(1 - \frac{v_{d7}}{v_{g7}}\right)(r_{o2} // r_{o4})}$$

$$= g_{m1}(r_{o2}//r_{o4})\, g_{m7}(r_{o8}//r_{o7}) \frac{1}{C_x\left(1 + g_{m7}(r_{o8}//r_{o7})\right)(r_{o2}//r_{o4})}$$

$$\simeq g_{m1}\cancel{(r_{o2}//r_{o4})}\,\cancel{g_{m7}}\cancel{(r_{o8}//r_{o7})} \frac{1}{C_x\, \cancel{g_{m7}}\cancel{(r_{o8}//r_{o7})}\cancel{(r_{o2}//r_{o4})}}$$

$$= \frac{g_{m1} \longrightarrow \text{computed}}{C_x \longrightarrow \text{compute numerically now}}$$

$$\text{Slew rate} = \frac{I_Q}{C_x} = \frac{50\,\mu A}{C_x} \longrightarrow \text{computed above}$$

(see the related derivations)

Now compute the slew rate.

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Rashid
14.19
continued

$V_{SD6} \geq V_{SG6} - |V_t|$

for Q_6 to remain in saturation. part (f)

$V_{SG1} = V_{SG2}$ (identical transistors)

then the maximum common-mode voltage

$$V_{CM,max} = \underbrace{V_{DD}}_{\text{given}} - \left[\underbrace{V_{SG6}}_{\text{see part(a)}} - \underbrace{|V_t|}_{\text{given}}\right] - \underbrace{V_{SG,1}}_{\text{see part(a)}}$$

$V_{SD,1} \geq V_{SG,1} - |V_t|$ for Q_1 to remain in saturation

$|V_t| \geq V_{SG,1} - V_{SD,1} = V_{DG,1}$

$V_{GD,1} \geq -|V_t|$

$V_{G,1} - V_{D,1} \geq -|V_t|$

$V_{G,1} \geq -|V_t| + V_{D,1}$

$$V_{G,1} \geq -\underbrace{|V_t|}_{\text{given}} + -\underbrace{V_{SS}}_{\text{given}} + \underbrace{V_{GS,3}}_{\text{see part(a)}}$$

then the minimum common-mode voltage
$v_{CM,min} = -|V_t| - V_{SS} + V_{GS,3}$

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Rashid
14-19
contin.

$$V_{SD,8} \geq V_{SG,8} - |V_t|$$

for Q_8 to remain in saturation part (g)

$$V_{DS,7} \geq V_{GS,7} - V_t$$ for Q_7 to remain in saturation

$$V_{o,max} = +V_{DD} - \left(V_{SG,8} - |V_t|\right)$$

$$V_{o,min} = -V_{SS} + \left(V_{GS,7} - V_t\right)$$

all the values above were given or computed in part (a)

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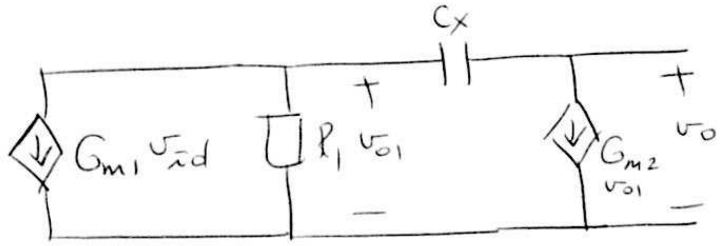
Rashid 14.19 Continued

part e)

See the discussion on pg 929–930 of Rashid.



By KCL

$$G_{m2} v_{o1} = (v_{o1} - v_o)\, w C_X$$ magnitude wise

for w_z we should have $v_o = 0$

$\Downarrow$

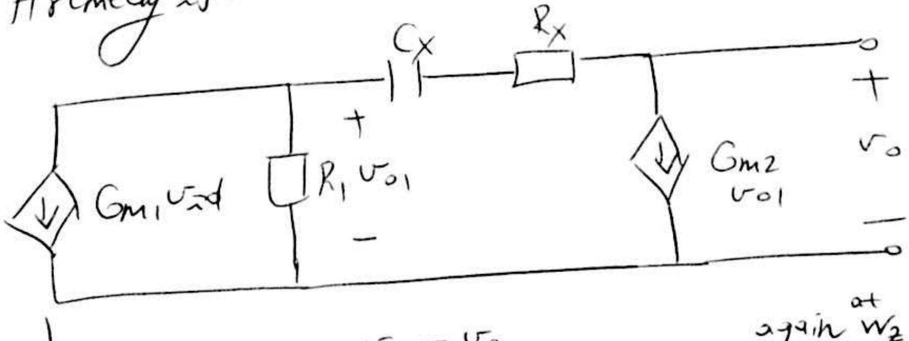
zero location

$$G_{m2} \cancel{v_{o1}} = \cancel{v_{o1}}\, w_z C_X$$

$$w_z = \frac{G_{m2}}{C_X}$$

this freq may be close to the unity gain freq and cause phase shift.

A remedy is:



(By KCL) $$G_{m2} v_{o1} = \frac{v_{o1} - v_o}{R_X + \frac{1}{j w C_X}}$$

again at w_z we have $v_o \approx 0$

$$G_{m2} \cancel{v_{o1}} = \frac{\cancel{v_{o1}}\, j w_z C_X}{1 + j w_z R_X C_X}$$

$$G_{m2} + j w_z R_X C_X G_{m2} = j w_z C_X \Rightarrow G_{m2} + j w_z C_X (R_X G_{m2} - 1) = 0$$

(8)

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Rashid
14-19
continued

part (e)
contin.

Then we obtain a term as

$$G_{m2}\left[1 + j\,\frac{w_z}{\dfrac{1}{C_X\left(R_X - \dfrac{1}{G_{m2}}\right)}}\right]$$

The zero freq $\left[C_X\left(R_X - \dfrac{1}{G_{m2}}\right)\right]^{-1}$ is $+\infty$

if $R_X = \dfrac{1}{G_{m2}}$

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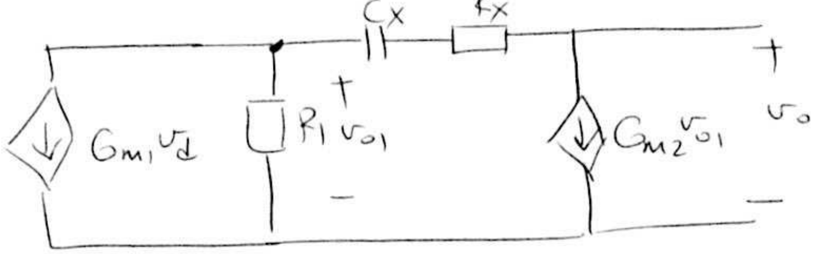
Rashid
14.19
Contin
part(e)
Contin

Transfer function analysis



KCL

$$G_{m1} v_d + \frac{v_{o1}}{R_1} + \underbrace{\frac{v_{o1} - v_o}{Z_x}}_{G_{m2} v_{o1}} = 0$$

$$G_{m1} v_d + \frac{v_{o1}}{R_1 \, // \, \frac{1}{G_{m2}}} = 0 \qquad (*)$$

and we have

$$v_o = -G_{m2} v_{o1} \left[Z_x \right] + v_{o1}$$

$$= v_{o1} \left[1 - G_{m2} Z_x \right]$$

$$\frac{v_o}{v_d} = \frac{v_{o1}}{v_d} \left[1 - G_{m2} Z_x \right]$$

Compute $\frac{v_{o1}}{v_d}$ from $(*)$

$$\frac{v_{o1}}{v_d} = -G_{m1} \left[R_1 \, // \, \frac{1}{G_{m2}} \right] = -G_{m1} \frac{R_1 \frac{1}{G_{m2}}}{R_1 + \frac{1}{G_{m2}}} = \frac{-G_{m1} R_1}{1 + G_{m2} R_1}$$

then

$$\frac{v_o}{v_d} = -\frac{G_{m1} R_1}{1 + G_{m2} R_1} \left[1 - G_{m2} Z_x \right]$$

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$$\frac{v_o}{v_d} = -\frac{G_{m1}R_1}{1+G_{m2}R_1}\left[1 - G_{m2}\left[R_x + \frac{1}{jwC_x}\right]\right]$$

Rashid 14-19

Contin.

part (e) cont-

$$= \frac{G_{m1}R_1}{1+G_{m2}R_1}\left[\left(G_{m2}R_x - 1\right) + \frac{G_{m2}}{jwC_x}\right]$$

$$= \frac{G_{m1}R_1}{1+G_{m2}R_1}\left[\frac{1 + jwC_x\left(R_x - \frac{1}{G_{m2}}\right)}{jwC_x}\right] G_{m2}$$

The zero frequency w_z above is clearly

$$w_z = \frac{1}{C_x\left(R_x - \frac{1}{G_{m2}}\right)}$$

and $R_x = \dfrac{1}{G_{m2}}$ will send $w_z \to +\infty$

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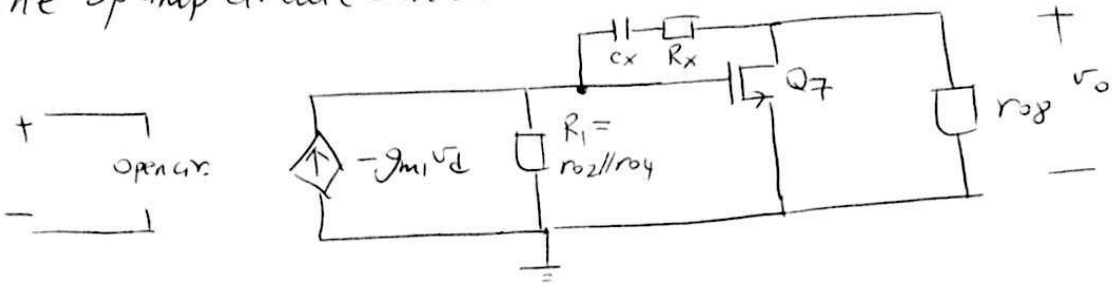
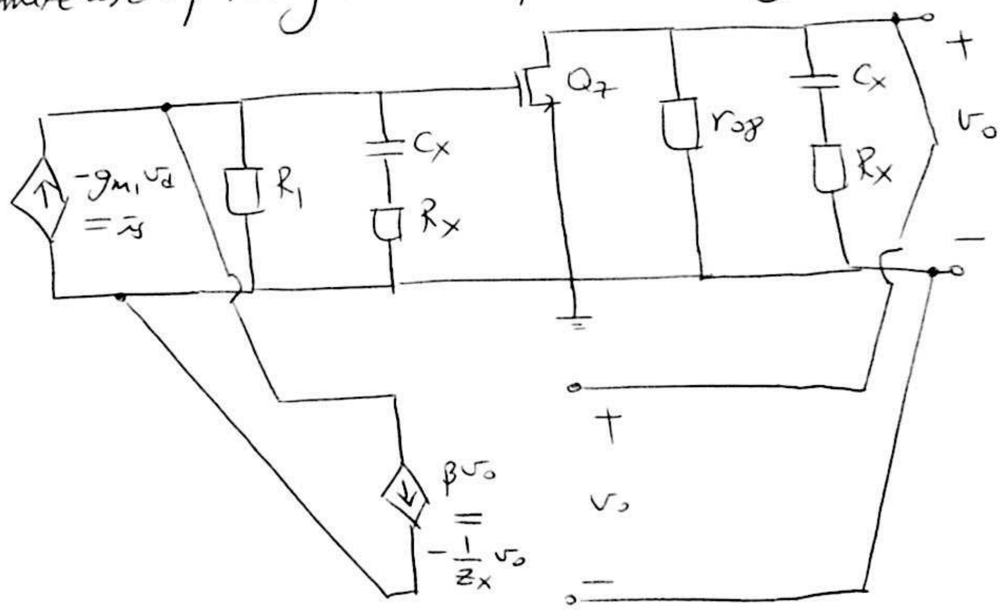
Rashid 14-19 part (e) continued

A more insightful analysis of the circuit is through feedback concepts and some approximations.

The opamp circuit can be modeled as in



Now make use of voltage-shunt feedback analysis.



where

$$Z_x = R_x + \frac{1}{jwC_x}$$

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Now kill β and compute A_{TRA} for the transresistance amplifier.

Rashid 14-19 part (e) continued

$$A_{TRA} = \frac{v_o}{i_s} = \left[R_1 // Z_x\right]\left[-g_{m7}\right]\left[R_2 // Z_x\right]$$

and compute the gain with feedback

$$A_f = \frac{A_{TRA}}{1+\beta A_{TRA}} = \frac{v_o}{i_s}$$

$$= \frac{-g_{m7}\left[R_1 // Z_x\right]\left[R_2 // Z_x\right]}{1 + g_{m7}\left[R_1 // Z_x\right]\left[R_2 // Z_x\right]\frac{1}{Z_x}}$$

Now recall that $i_s = -g_{m1} v_d$

$$\frac{v_o}{i_s} = \frac{v_o}{-g_{m1} v_d}$$

then the voltage gain of the opamp i_s

$$\frac{v_o}{v_d} = \frac{g_{m1} g_{m7}\left[R_1 // Z_x\right]\left[R_2 // Z_x\right]}{1 + g_{m7}\left[R_1 // Z_x\right]\left[R_2 // Z_x\right]\frac{1}{Z_x}}$$

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(13)

Rashid 14.19 part (e) contin

play with the gain expression:

$$\frac{v_o}{v_d} = \frac{g_{m1} g_{m7} \frac{R_1 Z_x}{R_1 + Z_x} \frac{R_2 Z_x}{R_2 + Z_x}}{1 + g_{m7} \frac{R_1 Z_x}{R_1 + Z_x} \frac{R_2 \cancel{Z_x}}{R_2 + Z_x} \frac{1}{\cancel{Z_x}}}$$

$$= \frac{g_{m1} g_{m7} R_1 R_2 Z_x^2}{(R_1 + Z_x)(R_2 + Z_x) + g_{m7} R_1 R_2 Z_x}$$

$$= \frac{g_{m1} g_{m7} R_1 R_2 Z_x^2}{R_1 R_2 + Z_x (\underbrace{R_1 + R_2 + g_{m7} R_1 R_2}) + Z_x^2}$$

Note that $g_{m7} R_1 R_2 \gg R_1 + R_2$

$$\approx \frac{g_{m1} g_{m7} R_1 R_2 Z_x^2}{R_1 R_2 + Z_x (g_{m7} R_1 R_2) + Z_x^2}$$

Now compute the root of the denom.

$$\text{roots} = \frac{-g_{m7} R_1 R_2 \pm \sqrt{(g_{m7} R_1 R_2)^2 - 4 R_1 R_2}}{2} \quad (**)$$

Note that $(g_{m7} R_1 R_2)^2 \gg 4 R_1 R_2$

approximate $\sqrt{(g_{m7} R_1 R_2)^2 - 4 R_1 R_2} \approx + g_{m7} R_1 R_2 - 2 r_{sm}$

For some small resistance r_{sm}.

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Rashid
14.19
part (e)
contin

Then the roots are

$$root_1 = -g_{m7} R_1 R_2 + r_{sm}$$
$$\approx -g_{m7} R_1 R_2$$
$$root_2 = -r_{sm}$$

The gain expression now becomes

$$\frac{v_o}{v_d} = \frac{g_{m1} g_{m7} R_1 R_2 Z_X^2}{(Z_X + r_{sm})(Z_X + g_{m7} R_1 R_2)}$$

Recall that $Z_X = R_X + \frac{1}{j\omega C_X}$

Then

$$\frac{Z_X^2}{(Z_X + r_{sm})(Z_X + g_{m7} R_1 R_2)} = \frac{(1 + j\omega C_X R_X)^2}{[1 + j\omega C_X(R_X + r_{sm})][1 + j\omega C_X(R_X + g_{m7} R_1 R_2)]}$$

and we have

$$\frac{(1 + j\omega C_X R_X)^2}{1 + j\omega C_X(R_X + r_{sm})} \approx \frac{(1 + j\omega 2C_X R_X)(1 - j\omega C_X(R_X + r_{sm}))}{1 + \underbrace{\omega^2 [C_X(R_X + r_{sm})]^2}_{\text{depends on } \omega^2\text{, omit this}}}$$

$$\approx \frac{1 + j\omega 2C_X R_X - j\omega C_X R_X - j\omega C_X r_{sm}}{1}$$

$$\cong 1 + j\omega C_X (R_X - r_{sm})$$

(15)

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Rashid 14.19 part (e) contin

Then the gain expression becomes

$$\frac{v_o}{v_d} = \underbrace{g_{m1} g_{m7} R_1 R_2}\, \frac{1 + j\omega C_x (R_x - r_{sm})}{1 + j\omega C_x (R_x + g_{m7} R_1 R_2)}$$

zero freq $\quad w_z = \dfrac{1}{C_x (R_x - r_{sm})}$

pole freq $\quad w_p = \dfrac{1}{C_x (R_x + g_{m7} R_1 R_2)}$

Note that $\underbrace{g_{m1} g_{m7} R_1 R_2}$ is the midband gain

And with R_x a small value

$$w_p \approx \frac{1}{\underbrace{C_x (1 + g_{m7} \underset{r_{o8} \| r_{o7}}{R_2})}_{\text{Miller multiplied cap}} \cdot \underset{r_{o2} \| r_{o4}}{R_1}}$$

Therenin Resis. of this

Note that
$w_z \to +\infty$ if $R_x \approx r_{sm}$

We should find an expression for r_{sm}
from (**) on pg 14.
through binomial expansions.
This will yield an answer close to $\frac{1}{g_{m7}}$.

(16)

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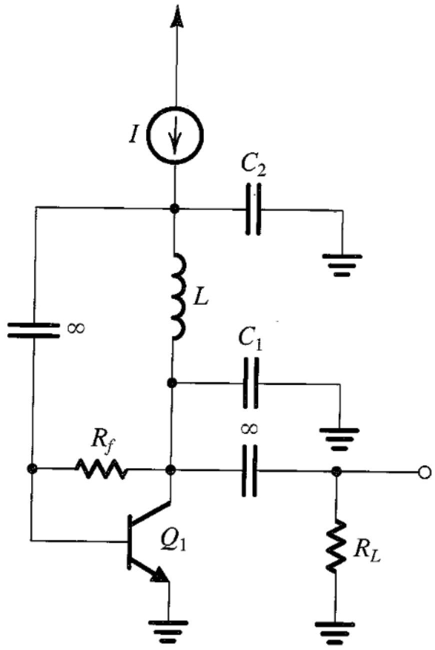
Colpitts Oscillator **Sedra 13.21**

****13.21** Figure P13.21 shows four oscillator circuits of the Colpitts type, complete with bias detail. For each circuit, derive an equation governing circuit operation, and find the frequency of oscillation and the gain condition that ensures that oscillations start.



(Assume $R_f \gg \omega_0 L$)

(d)

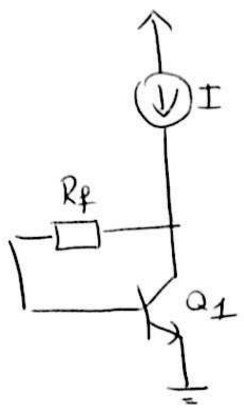
Notes: Draw the small-signal equivalent circuit, omit the output impedance of the transistor, and then analyze.

Additional Tasks: None.

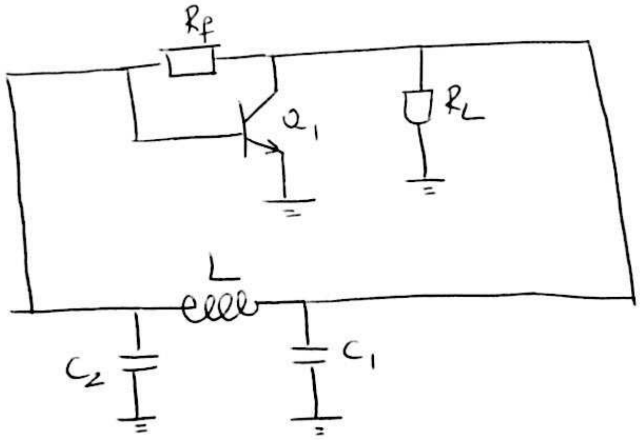
Necessary Knowledge and Skills: Colpitts oscillator, frequency, gain condition for self-starting oscillations.

Sedra 13.21

The circuit in DC (L is short cir. all cap are open cir.)



Small signal equivalent cir. (C_∞ are short cir. I (current source) is open cir.)



Note L and R_f come in parallel.

But we are given that $R_f \gg \omega_o L$

Then $R_f \,//\, (j\omega_o L) \simeq j\omega_o L$

$\Rightarrow$ R_f can be treated as open cir.

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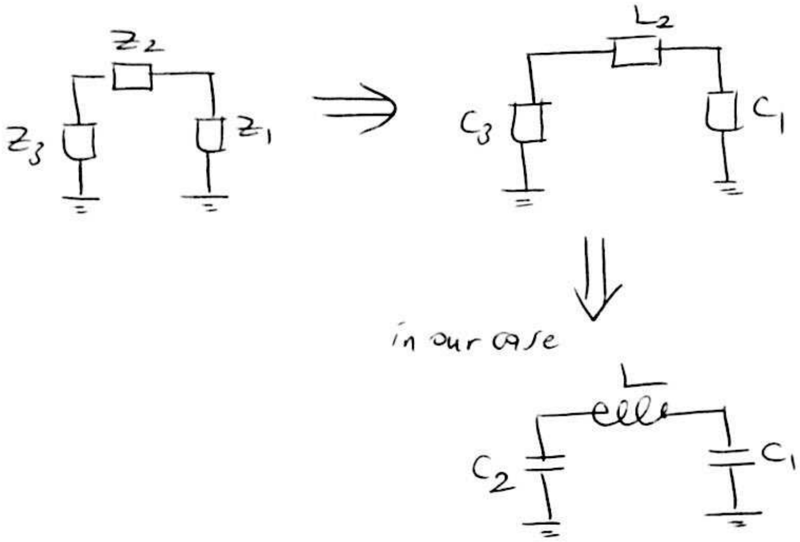
1

Sedra 13.21
contin.

See the related document for
analyzing LC networks in RF osc.



$$W_o = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}}$$

$$g_m R_L \geq \frac{C_2}{C_1}$$

gain condition
for self-starting osc.

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