GTU
Electronics Engineering

ELEC 331
Electronic Circuits 2

Fall Semester

Instructor: Assist. Prof. Önder Şuvak

HW 10
Questions and Answers

Updated November 17, 2017 - 13:06

Assigned:

Due:

Answers Out:

Late Due:

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Stability Analysis through the Phase Margin **Rashid 10.56**

10.56 If the phase margin of an amplifier is PM = 40° and the magnitude of the open-loop gain is $|A(j\omega)| = 50$, find the magnitude of the closed-loop gain $|A_f(j\omega)|$.

Notes: Assume a two-pole system for modeling the high-frequency response of the open-loop amplifier. Note that the ratio between these two poles is not given.

Additional Tasks: Observe and conclude that if the ratio between the two poles of the system had been given, there would be no missing information and you could have solved for the feedback factor.

Necessary Knowledge and Skills: Transfer function in terms of poles and midband gain, magnitude and phase computation through the transfer function, open-loop transfer function assembly, loop gain calculation for analyzing stability, phase margin calculation through the loop gain, Bode plot illustrations of the phase margin.

$\Rightarrow$ assume that the open-loop amplifier is a two-pole system and β is real.

Rashid 10.56

Then

$$A(jw) = \frac{A}{\left(1+\frac{jw}{w_1}\right)\left(1+\frac{jw}{w_2}\right)}$$

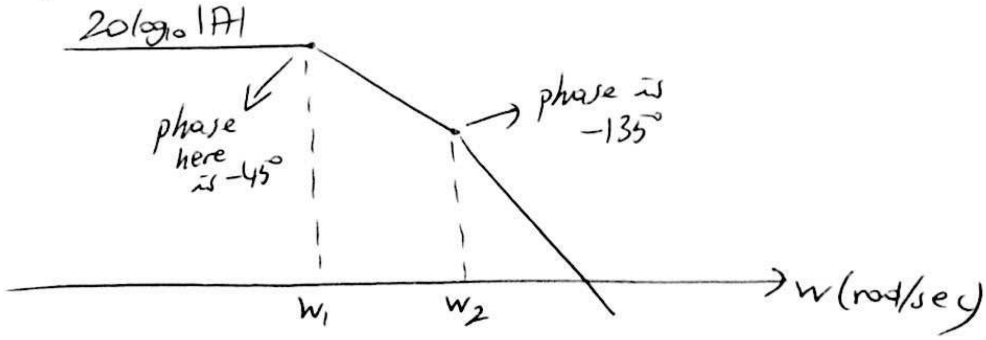
$A \Rightarrow$ given as 5^{∞}.

we are assuming also that $w_1 \ll w_2$.

$\Rightarrow$ Note that

$$20\log_{10}\left|A(jw)\beta\right| = 20\log_{10}\left|A(jw)\right| - 20\log_{10}\left|\frac{1}{\beta}\right|$$

$\Rightarrow$ Bode plot of $A(jw)$ is:
(asymptotic magnitude)



note here that

$$A(jw_1) \simeq \frac{A}{\left(1+\frac{jw_1}{w_1}\right)(1)} \Rightarrow \text{phase is } -45^\circ$$

$$A(jw_2) \simeq \frac{A}{\left(\frac{jw_2}{w_1}\right)\left(1+\frac{jw_2}{w_2}\right)} \Rightarrow \text{phase is } -135^\circ$$

 1

Rashid
1056
contin.

We would like to set the phase margin to 40° ($PM = 40°$) by designing β.

Phase margin is $\Rightarrow PM = 180° - \left| \underbrace{\angle A(jw_u)\beta}_{\text{phase of } A(jw_u)\beta} \right|$

at w_u: unity gain frequency for $\underbrace{T_L}_{\text{loopgain}}(jw) = A(jw)\beta$

Note that there is no contribution to $T_L(jw) = A(jw)\beta$ in terms of phase by β since β is positive and real.

Therefore, we take into consideration the phase of $A(jw)$ only. But β contributes to $T_L(jw)$ in terms of magnitude.

Remember

$$20\log_{10}|A(jw)\beta| = 20\log_{10}|A(jw)| - 20\log_{10}\left|\frac{1}{\beta}\right|$$

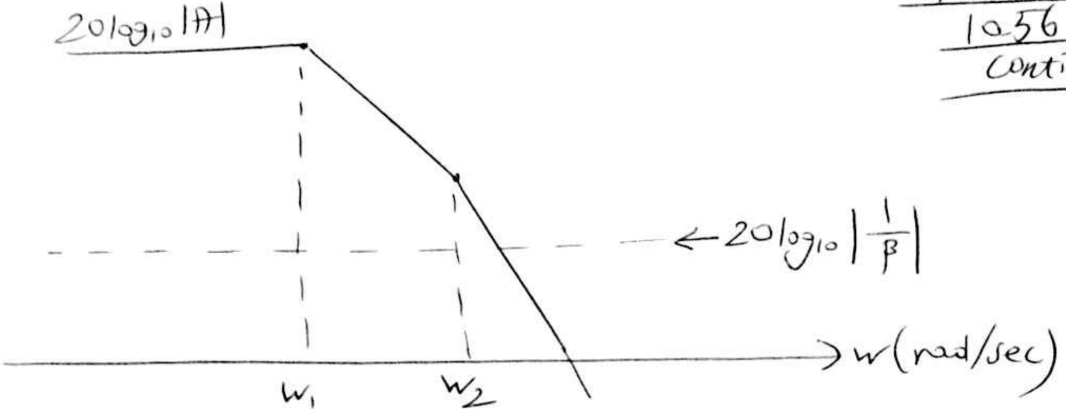
Therefore the difference of the two magnitude Bode plots is equal to $20\log_{10}|T_L(jw)|$

$$= 20\log_{10}|A(jw)\beta|$$

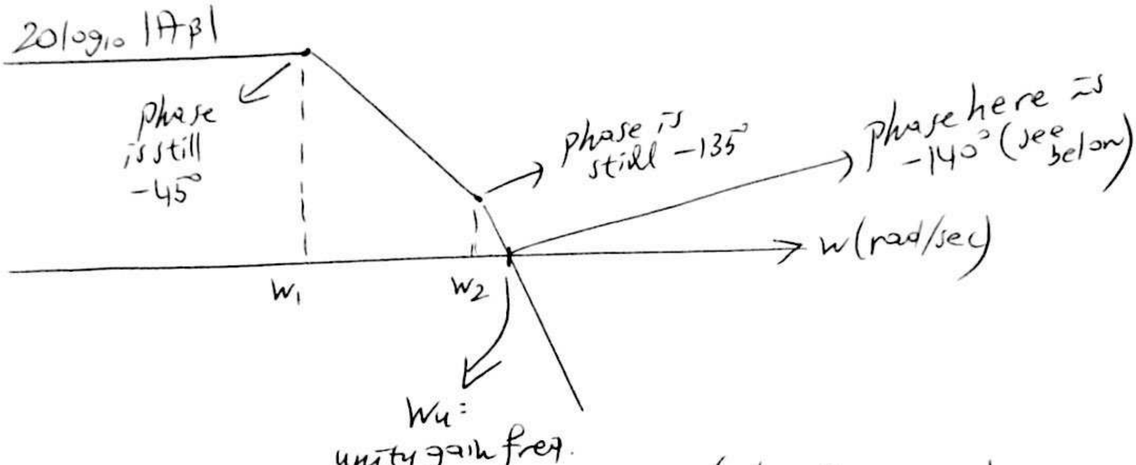
See the next page for the Bode plots.

2

Rashid
1056
Contin-



Then the difference will be



for $T_L(j\omega) = A(j\omega)\beta$ (the loop gain)

Since the phase margin PM is

$$PM = 180^\circ - |\angle A(j\omega_u)\beta| = 180^\circ - |\angle T_L(j\omega_u)|$$

$= 40^\circ \Rightarrow$ given in the question

Then $|\angle T_L(j\omega_u)| = 140^\circ$

and $\angle T_L(j\omega_u) = -140^\circ$

Rashid
10.56
contin.

Note that we have

$$T_L(j\omega_u) = \frac{A\beta}{\left(1+\frac{j\omega_u}{\omega_1}\right)\left(1+\frac{j\omega_u}{\omega_2}\right)}$$

$$\simeq \frac{A\beta}{\left(\frac{j\omega_u}{\omega_1}\right)\left(1+\frac{j\omega_u}{\omega_2}\right)}$$

Since $\omega_u \gg \omega_1$
but $\omega_u > \omega_2$ only.

Then

$$\angle T_L(j\omega_u) = -90^\circ - \arctan\left(\frac{\omega_u}{\omega_2}\right)$$

$$= -140^\circ$$

and $\arctan\left(\frac{\omega_u}{\omega_2}\right) = 50^\circ$

$$\frac{\omega_u}{\omega_2} \simeq \cancel{1.551}\ 1.19 \Longrightarrow \underbrace{\omega_u = \cancel{1.551}\ 1.19\,\omega_2}$$

And by definition we have

$$|T_L(j\omega_u)| = 1$$

$$\simeq \frac{A\beta}{\underbrace{\left(\frac{\omega_u}{\omega_1}\right)}_{\text{unknown}}\underbrace{\sqrt{1+\frac{\omega_u^2}{\omega_2^2}}}_{\text{can be computed from here}}}$$

β → given as 50

A → to be designed

(4)

Rashid
10.56
contin.

Then

$$\beta \simeq \frac{\left(\frac{w_u}{w_1}\right)\overbrace{\sqrt{1+\frac{w_u^2}{w_2^2}}}^{1.55}}{\underset{50}{\underbrace{A}}}$$

if we had been given $\frac{w_2}{w_1}$

we could compute

$$\frac{w_u}{w_1} = \frac{w_2}{w_1}\left(\frac{w_u}{w_2}\right) \rightarrow 1.19$$

Then we could obtain the design value for β.

And the closed-loop gain would be (at low frequencies)

$$A_f = \frac{A}{1+\beta A}$$

5

Stability Analysis through Phase and Gain Margins **Rashid 10.57**

10.57 The open-loop gain of an amplifier has break frequencies at $f_{p1} = 10$ kHz, $f_{p2} = 100$ kHz, and $f_{p3} = 1$ MHz. The low-frequency gain is $A_o = 250$, and the feedback factor is $\beta = 0.9$. Calculate the gain margin GM and the phase margin PM.

Notes: Do not make use of approximations in the transfer function while computing the magnitude and phase.

Additional Tasks: None.

Necessary Knowledge and Skills: Transfer function in terms of poles and midband gain, magnitude and phase computation through the transfer function, open-loop transfer function assembly, loop gain calculation for analyzing stability, phase and gain margin calculation through the loop gain, Bode plot illustrations of phase and gain margins.

Rashid
10.57

$$A(j\underset{j2\pi f}{\underbrace{w}}) = \frac{A_o}{\left(1+\frac{jf}{f_1}\right)\left(1+\frac{jf}{f_2}\right)\left(1+\frac{jf}{f_3}\right)}$$

$$= \frac{250}{\left(1+\frac{jf}{10KHz}\right)\left(1+\frac{jf}{100KHz}\right)\left(1+\frac{jf}{1MHz}\right)}$$

$$\beta = 0.9$$

$$A(jf)\beta = \frac{225}{\left(1+\frac{jf}{10kHz}\right)\left(1+\frac{jf}{100KHz}\right)\left(1+\frac{jf}{1MHz}\right)}$$

To compute the phase margin, compute f_u where

$$|A(jf_u)\beta| = 1$$

$$= \frac{225}{\left|1+\frac{jf_u}{10K}\right|\left|1+\frac{jf_u}{100K}\right|\left|1+\frac{jf_u}{1MHz}\right|}$$

$$= \frac{225}{\sqrt{1+\frac{f_u^2}{(10K)^2}}\sqrt{1+\frac{f_u^2}{(100K)^2}}\sqrt{1+\frac{f_u^2}{(1MHz)^2}}}$$

1

Rashid
10.57

$f_u \cong 10^{5.651} \approx 448\ kHz$
(between f_2 and f_3)

$\angle A(jf_u)\beta \cong -190.25 < -180°$

Therefore the phase margin
$PM = -10.25° \Rightarrow$ unstable

To compute the gain margin, compute f_0 where

$\angle A(jf_0)\beta = -180°$

$f_0 \cong 10^{5.52} \cong 331\ KHz$
(between f_2 and f_3
and smaller than f_u)

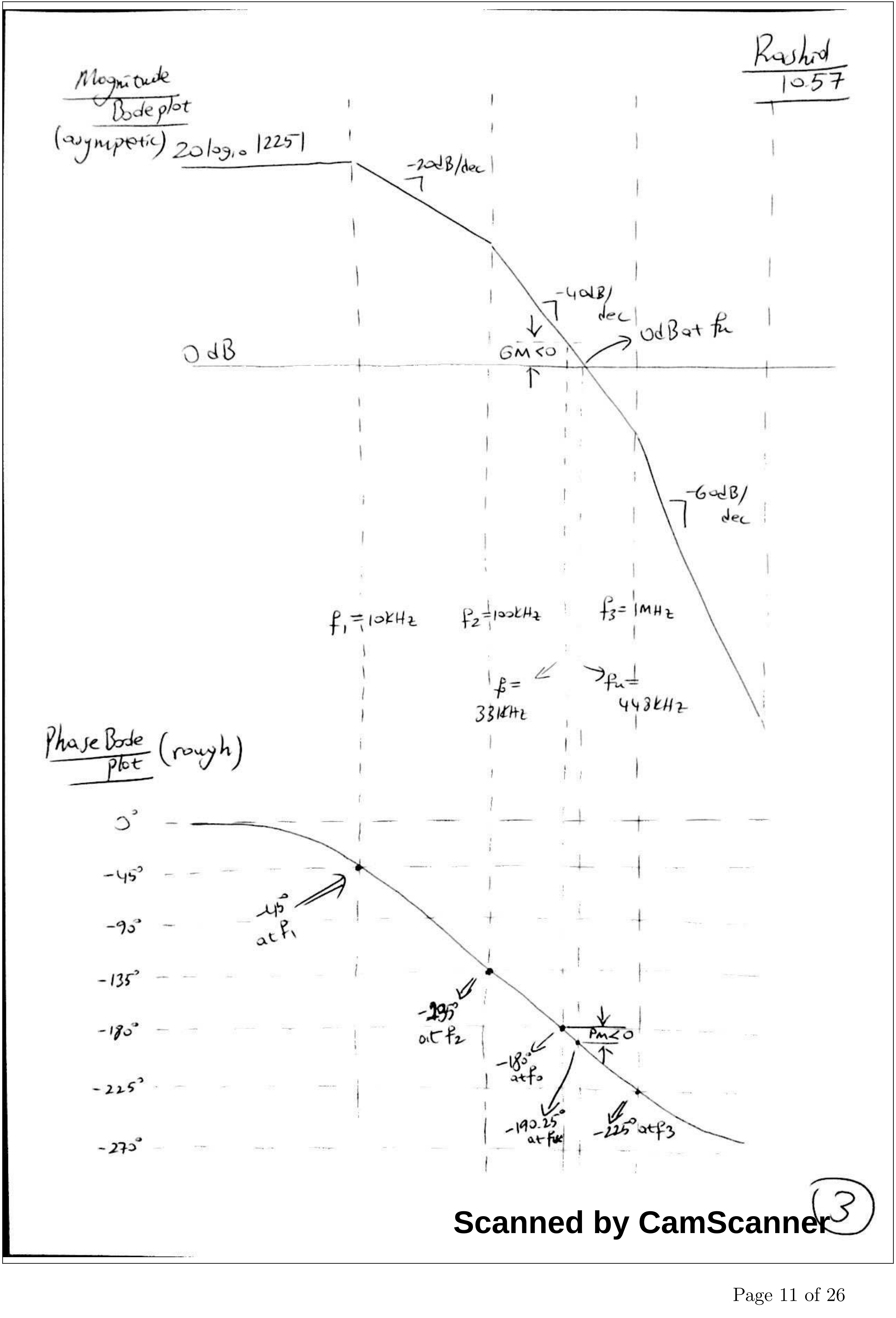
Compute the gain at f_0:

$$|A(jf_0)\beta| = \frac{225}{\sqrt{1+\left(\frac{331\,KHz}{10\,KHz}\right)^2}\sqrt{1+\left(\frac{331\,KHz}{100\,KHz}\right)^2}\sqrt{1+\left(\frac{331\,KHz}{1000\,KHz}\right)^2}}$$

$\cong 1.86$

gain margin $GM = 20\log_{10}|1| - 20\log_{10}|1.86|$
$\cong -5.39\ dB$

2

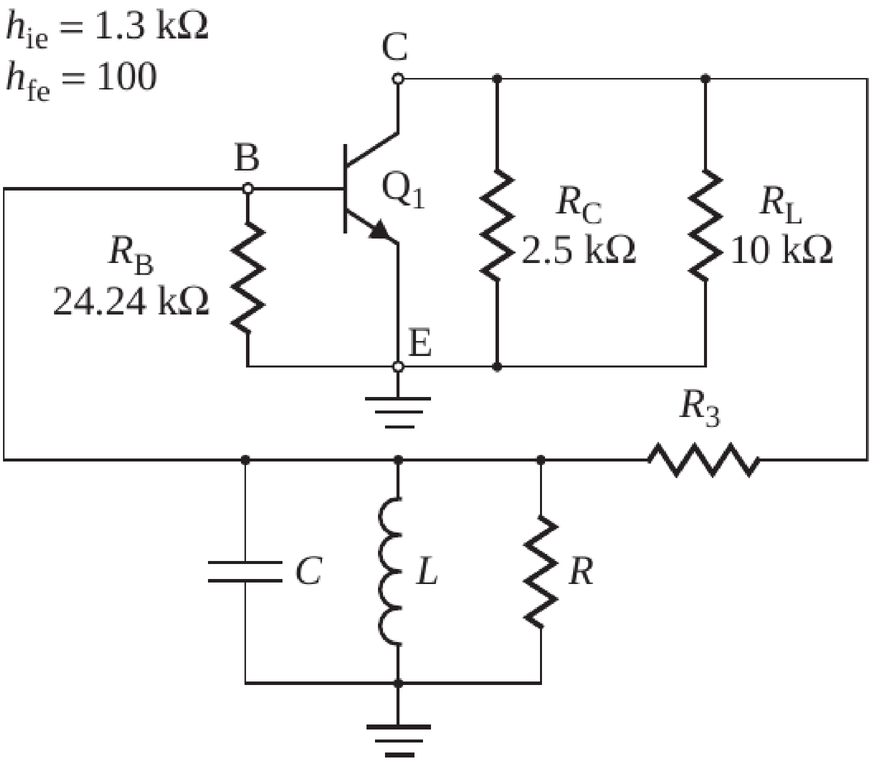

Rashid
1057
Magnitude Bode plot (asymptotic)
20log10 |225|
-20dB/dec
-40dB/dec
-60dB/dec
0 dB
GM<0
0dB at fu
f1 = 10KHz
f2 = 100kHz
f3 = 1MHz
f0 = 331kHz
fu = 448kHz
Phase Bode plot (rough)
0°
-45°
-90°
-135°
-180°
-225°
-270°
-45° at f1
-135° at f2
-180° at f0
PM<0
-190.25° at fu
-225° at f3
3

Audio Oscillator Analysis **Rashid 13.6**

13.6 Find the values of R, R_3, C, and L for the phase-shift oscillator in Fig. P13.6 so that the oscillation frequency is $f_o = 5$ kHz.

D

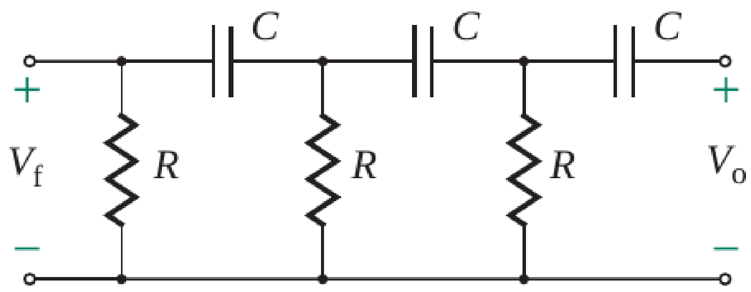
FIGURE P13.6



Notes: The oscillation condition will not be met.

Additional Tasks: Observe how the oscillation condition will not be met and then replace the feedback network with the following one and analyze.

Audio Oscillator Analysis **Rashid 13.6**



(b) Feedback network

Necessary Knowledge and Skills: Oscillators, Barkhausen criteria, analysis of feedback networks, oscillation frequency, phase condition, amplitude condition.

Rashid
13.6

analyze the amplifier

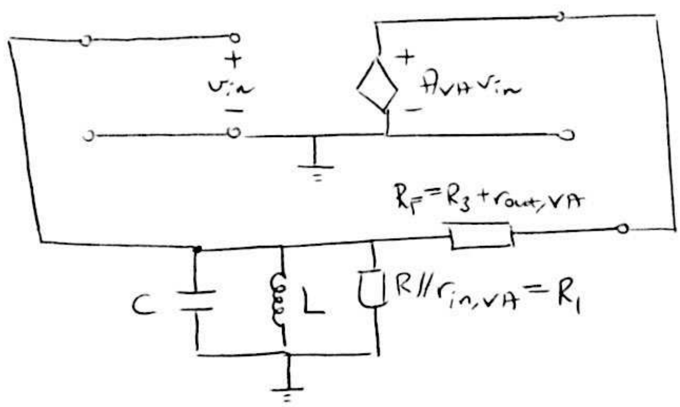
$r_{in,vA} = R_B // r_\pi$

$r_{out,vA} = R_C // R_L // r_o \cong R_C // R_L$ since $r_o \to +\infty$

r_π and β given $\Rightarrow g_m = \frac{\beta}{r_\pi} = \frac{100}{1.3k\Omega}$

$A_{vA} = -g_m (R_C // R_L)$

Redraw the schematic with the model of the amplifier



Analyze the feedback network

$$\beta(j\omega) = \frac{v_f}{v_o} = \frac{j\omega L // \frac{1}{j\omega C} // R_1}{j\omega L // \frac{1}{j\omega C} // R_1 + R_F}$$

$$\frac{j\omega L \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$

1

Rashid
13.6
contin.

$$\frac{\dfrac{j\omega L}{1-\omega^2 LC} R_1}{\dfrac{j\omega L}{1-\omega^2 LC} + R_1} = \frac{j\omega L R_1}{j\omega L + R_1(1-\omega^2 LC)}$$

$$\beta(j\omega) = \frac{\dfrac{j\omega L R_1}{j\omega L + (1-\omega^2 LC)R_1}}{\dfrac{j\omega L R_1}{j\omega L + (1-\omega^2 LC)R_1} + R_F}$$

$$= \frac{j\omega L R_1}{j\omega L R_1 + j\omega L R_F + (1-\omega^2 LC)R_1 R_F}$$

$$= \frac{j\omega L}{j\omega L\left(1 + \frac{R_F}{R_1}\right) + R_F(1-\omega^2 LC)}$$

and the loop gain is

$$A_{VA}\,\beta(j\omega) = \left[-g_m(R_C /\!/ R_L)\right]\beta(j\omega)$$

in an attempt to meet the phase condition
set $1-\omega^2 LC = 0$ to obtain

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (\text{osc. freq.})$$

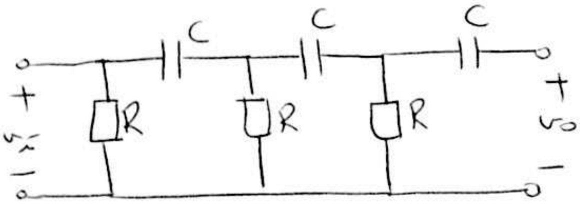
But $$A_{VA}\,\beta(j\omega)\Big|_{\omega=\omega_0} = \frac{\cancel{j\omega L}\left(-g_m(R_C /\!/ R_L)\right)}{\cancel{j\omega L}\left(1+\frac{R_F}{R_1}\right)} < 0$$

and the phase of the loop gain at $\omega = \omega_0$ is 180°
The phase condition cannot indeed be met.

 2

Instead, replace the feedback network with the following:

Rashid 13.6 contin.



$$\beta(jw) = \frac{v_f}{v_o} = \frac{1}{1 - j\left(\frac{6}{wRC}\right) - \frac{5}{w^2R^2C^2} + j\frac{1}{w^3R^3C^3}}$$

(see the soln to Rashid 13.3 for the derivation)

With this feedback network, the loop gain is

$$A_{vA}\,\beta(jw) = \left[-g_m\,(R_C // R_L)\right]\beta(jw)$$

Apply the Barkhausen criterion:

To meet the phase condition set

$$\frac{1}{w^3R^3C^3} - \frac{6}{wRC} = 0$$

$$1 - 6w^2R^2C^2 = 0$$

$$w_o = \frac{1}{\sqrt{6}\,RC} \quad (\text{osc freq})$$

The amplitude condition says: (Note that the phase condition can be met with positive R_c and R_L)

$$A_{vA}\,\beta(jw)\Big|_{w=w_o} = +1$$

$$= \frac{-g_m\,(R_c // R_L)}{1-30} = +1$$

3

Rashid
13.6
contin.

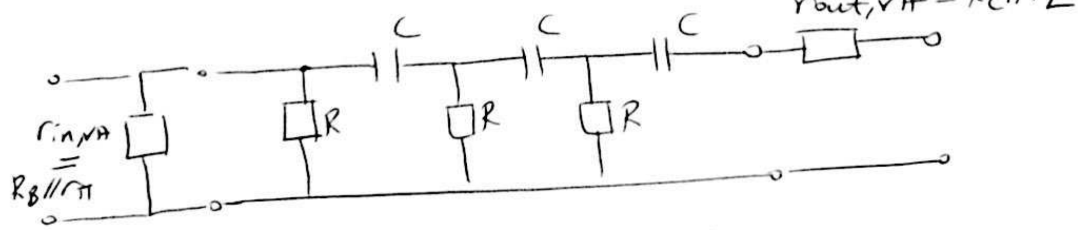
For self starting oscillations we must have:

$$\frac{-g_m(R_C // R_L)}{-29} \geq +1$$

$$g_m(R_C // R_L) \geq 29$$ (the gain of the CE amplifier must, in absolute value, be larger than 29)

There are also other conditions to meet for this condition to be accurate: (design)

Observe the load on the feedback network due to the forward amplifier:



$$\left| \frac{1}{j w_o C} \right| \geq \underbrace{10}_{\text{at least } 10} \cdot (R_C // R_L)$$ at $w_o = \frac{1}{\sqrt{6} RC}$

and $$r_{in,vA} = R_B // r_\pi \geq \underbrace{10}_{\text{at least } 10} \cdot R$$

The load on the feedback network will then have negligible effect.

4

Audio Oscillator Analysis **Sedra 13.13**

13.13 For the circuit in Fig. P13.13 find $L(s)$, $L(j\omega)$, the frequency for zero loop phase, and R_2/R_1 for oscillation.

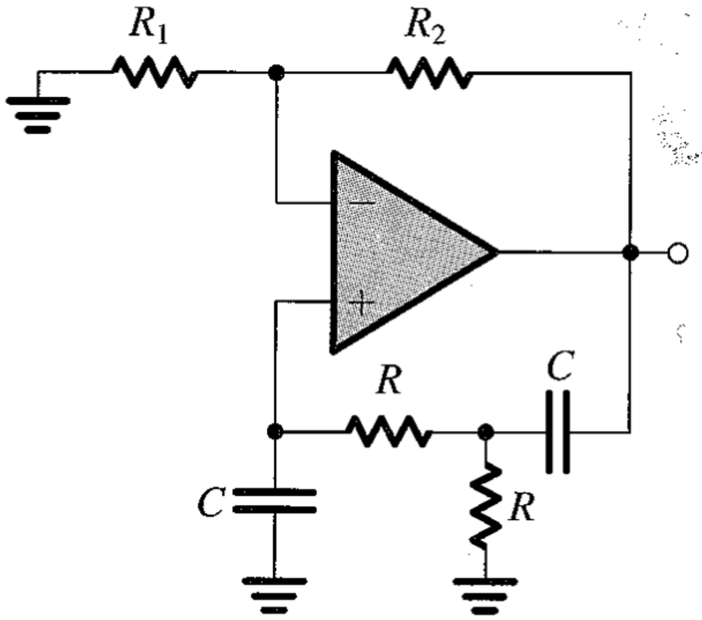


FIGURE P13.13

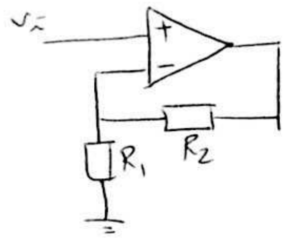
Notes: None.

Additional Tasks: None.

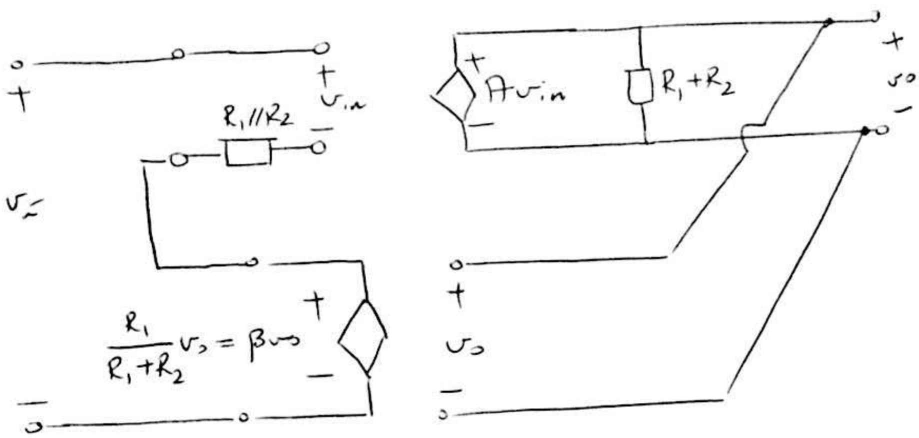
Necessary Knowledge and Skills: Oscillators, Barkhausen criteria, analysis of feedback networks, oscillation frequency, phase condition, amplitude condition.

Analyze the amplifier

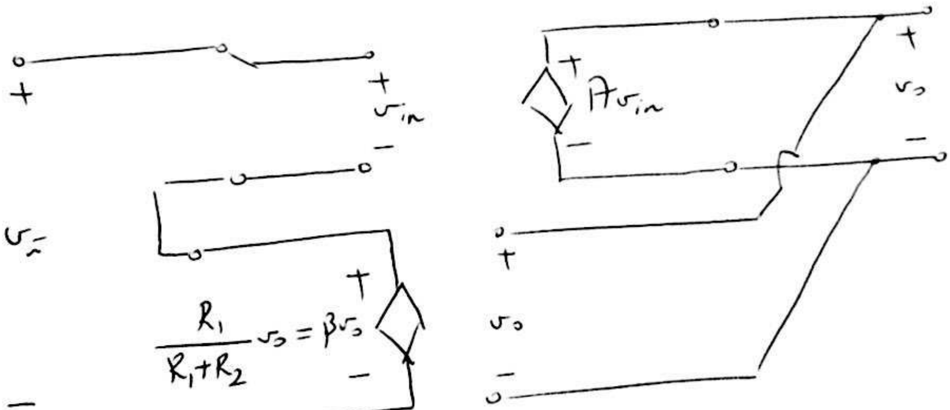
Sedra 13.13



This is a voltage amplifier with voltage series feedback



This is equivalent to



$$\text{Then } \frac{v_o}{v_i} = \frac{A}{1 + A\,\dfrac{R_1}{R_1+R_2}} \simeq \frac{R_1+R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

with $A \to +\infty$

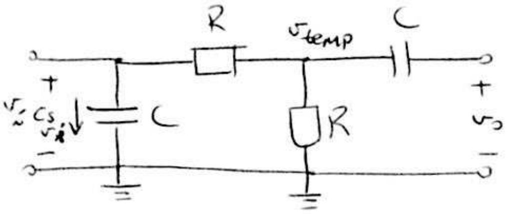
and $r_{in,f} \to +\infty$

$r_{out,f} \to 0$

1

Analyze the feedback network

Sedra 13-13 contin.



$$\frac{v_f}{v_o} = \frac{\left(R+\frac{1}{Cs}\right)//R}{\left(R+\frac{1}{Cs}\right)//R+\frac{1}{Cs}} \cdot \frac{1/Cs}{1/Cs+R}$$

OR do it like this

$$\frac{v_{temp}-v_f}{R} = \frac{v_f}{1/Cs} = Cs\, v_f$$

$$v_{temp} = v_f + RCs\, v_f$$

$$= (1+RCs)\, v_f$$

and KCL at the node with v_{temp}

$$\frac{v_{temp}}{R} + Cs\, v_f + Cs\,(v_{temp}-v_o) = 0$$

$$v_{temp} + RCs\, v_f + RCs\,(v_{temp}-v_o) = 0$$

$$v_{temp}(1+RCs) + RCs\, v_f = RCs\, v_o$$

$$v_f(1+RCs)^2 + RCs\, v_f = RCs\, v_o$$

$$v_f\left[R^2C^2s^2 + 3RCs + 1\right] = RCs\, v_o$$

$$\frac{v_f}{v_o} = \frac{RCs}{R^2C^2s^2+3RCs+1}$$

$$\frac{v_f}{v_o} = \frac{1}{3+\frac{1}{RCs}+RCs}$$

2

Sedra 13.13 contin-

Therefore

$$\beta(j\omega) = \frac{v_-(j\omega)}{v_o(j\omega)} = \frac{1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)}$$

Recall that A_{VA} of the forward amplifier is

$$A_{VA} = 1 + \frac{R_2}{R_1}$$

$$A_{VA}\,\beta(j\omega) = \frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)}$$

Now apply the Barkhausen criterion:
Zero phase condition requires

$$\omega CR - \frac{1}{\omega CR} = 0$$

$$\Rightarrow \omega_o = \frac{1}{RC} \quad \text{(osc frequency)}$$

unity magnitude condition requires (at the osc freq. ω_o)

$$\left| A_{VA}\,\beta(j\omega_o) \right| = 1$$

$$\frac{1 + \frac{R_2}{R_1}}{3} = 1$$

and for self-starting osc

$$1 + \frac{R_2}{R_1} \geq 3$$

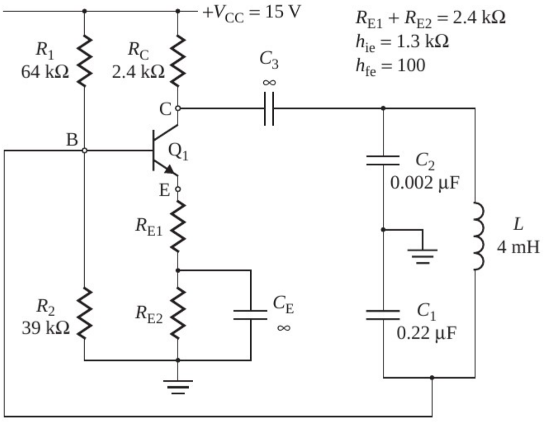
$$\frac{R_2}{R_1} \geq 2$$

3

RF Oscillator Analysis — Rashid 13.17

13.17 A Colpitts BJT oscillator is shown in Fig. P13.17. Calculate the frequency of oscillation f_o and the value of R_{E1} required to sustain the oscillation.

FIGURE P13.17



Notes: None.

Additional Tasks: None.

Necessary Knowledge and Skills: Oscillators, Barkhausen criteria, analysis of feedback networks, oscillation frequency, phase condition, amplitude condition.

Rashid
13.17

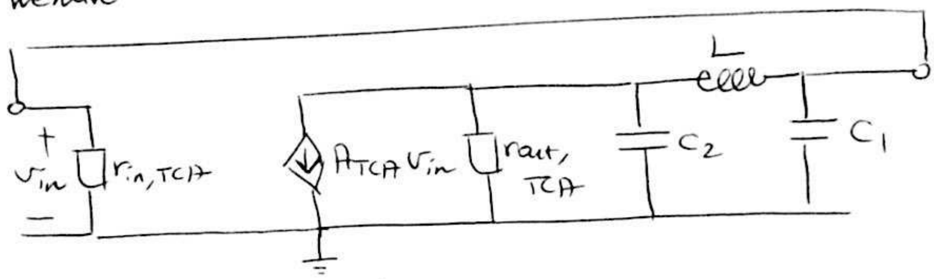
Model the CE amplifier as a TCA

$$r_{in,TCA} \cong R_1 // R_2 // \left[r_{\pi 1} (1 + g_{m1} R_{E1}) \right]$$

$$r_{out,TCA} \cong R_C // \left[(1 + g_{m1} r_{o1}) \frac{R_{E1} r_{\pi 1}}{R_{E1} + r_{\pi 1} + R_1 // R_2} \right] \Rightarrow$$ see the related document

$$A_{TCA} \cong + \frac{g_{m1}}{1 + g_{m1} R_{E1}}$$

Therefore we have



For a Collpitts osc we have

$$w_0 = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}} \quad \text{(osc freq)}$$

For self starting osc in Colpitts:

$$A_{TCA} \, r_{out,TCA} \geq \frac{C_1}{C_2}$$

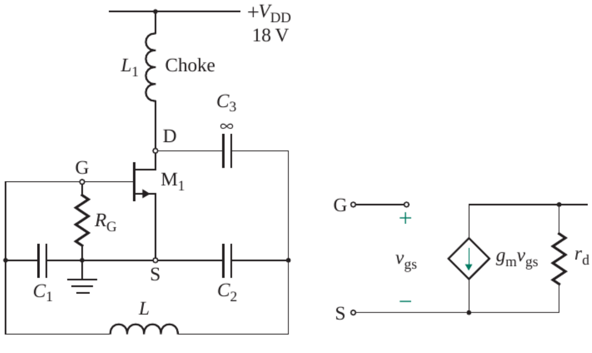
Note that we must also have:

$$\left| \frac{1}{j w_0 C_1} \right| \ll r_{in,TCA}$$ for w_0 given as above.

RF Oscillator Analysis **Rashid 13.20**

13.20 Determine the frequency of oscillation for the Colpitts MOSFET oscillator in Fig. P13.20(a). The MOSFET can be replaced by its transconductance model, shown in Fig. P13.20(b). The parameters are $r_d = 25\ \text{k}\Omega$, $g_m = 5$ mA/V, $R_G = 1\ \text{M}\Omega$, $L = 1.5$ mH, $C_1 = 10$ nF, and $C_2 = 10$ nF. Calculate the frequency of oscillation and check to make sure the condition for oscillation is satisfied.

FIGURE P13.20



(a) Circuit (b) JFET model

Notes: None.

Additional Tasks: None.

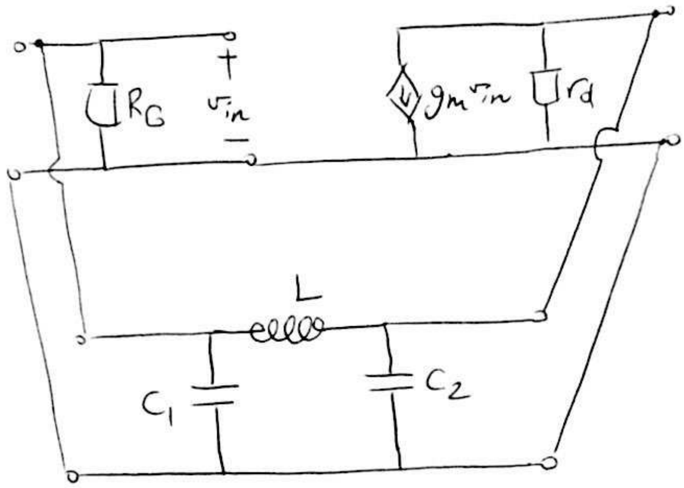
Necessary Knowledge and Skills: Oscillators, Barkhausen criteria, analysis of feedback networks, oscillation frequency, phase condition, amplitude condition.

Rashid
13.20

RF Choke : short cir. at DC
open cir for higher frequencies

Therefore intrinsic gain of the MOS applies.

The MOS is already modelled as a TCA.



Colpitts osc phase condition ⟶ osc freq derived

$$w_o = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}} \quad (\text{osc freq})$$

Condition for self-starting osc:

$$g_m r_d \geq \frac{C_1}{C_2}$$

Also we must have:

$$\left| \frac{1}{j w_o C_1} \right| \ll R_G \quad \text{at the osc freq}$$