GYTE
Electronics Engineering

ELEC 331
Electronic Circuits 2

Fall 2014

Instructor: Assist. Prof. Önder Şuvak

HW 13
Questions and Answers

Updated January 2, 2015 - 19:50

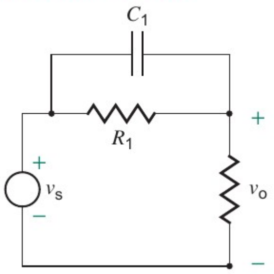
Assigned:	20141222
Due:	20141229
Answers Out:	20141230
Late Due:	20150105

Contents

First Order Filter **Rashid 12.2**

12.2 Determine **(a)** the transfer function of the network shown in Fig. P12.2 and **(b)** its poles and zeros.

FIGURE P12.2

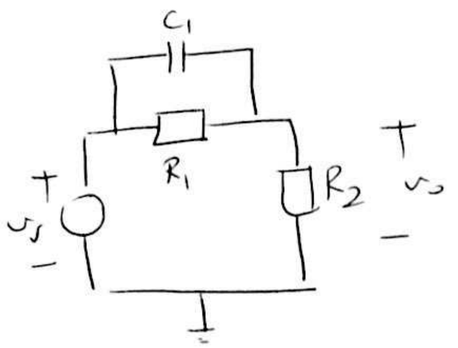


Notes: None.

Additional Tasks: Sketch the magnitude and phase Bode plots.

Necessary Knowledge and Skills: Laplace transforms, transfer functions, poles and zeros, gain, filtering operation.

Rashid
12.2



$$\frac{v_o}{v_s} = \frac{R_2}{R_2 + R_1 // \frac{1}{sC_1}} = \frac{R_2}{R_2 + \frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}}}$$

$$= \frac{R_2}{R_2 + \frac{R_1}{1 + sC_1R_1}}$$

$$= \frac{R_2\left(1 + R_1C_1s\right)}{R_1 + R_2 + R_1R_2C_1s}$$

$$= \frac{R_2}{R_1 + R_2} \frac{1 + \frac{s}{1/R_1C_1}}{1 + \frac{s}{\frac{1}{C_1(R_1 // R_2)}}}$$

low frequency gain: $\frac{R_2}{R_1 + R_2}$

zero : $1/R_1C_1$

pole : $\frac{1}{(R_1 // R_2)C_1}$

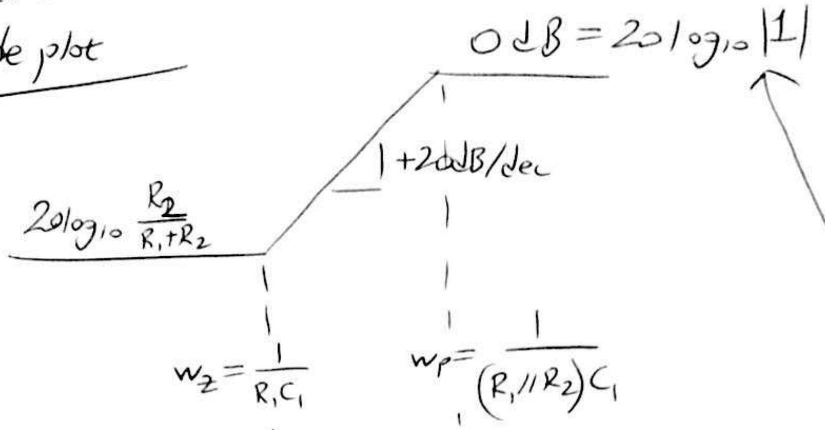
zero > pole

zero < pole ⇐ this is correct

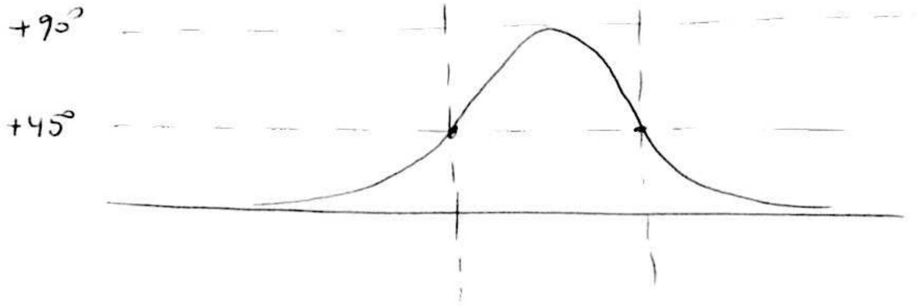
1

Rashid
12.2
contin.

Magnitude Bode plot



phase Bode plot



$$\frac{R_2}{R_1+R_2}\cdot\frac{R_1\cancel{C_1}}{(R_1//R_2)\cancel{C_1}} = \frac{R_2}{R_1+R_2}\cdot R_1\cdot\frac{R_1+R_2}{R_1R_2}$$

$$= 1$$

Biquadratic Function **Rashid 12.5**

12.5 Determine **(a)** the pole and zero quality factors Q_p and Q_z, **(b)** the pole and zero resonant frequencies ω_p and ω_z, **(c)** the pole factor β_p, and **(d)** the pole angle ϕ_p. The transfer function has the general form as given by

$$H(s) = \frac{5s^2 + 15s + 100}{s^2 + 20s + 200}$$

Notes: See Section 12.5 of Rashid.

Additional Tasks: Switch to Fourier domain and use Matlab to sketch the magnitude and phase Bode plots.

Necessary Knowledge and Skills: Biquadratic function, quality factors, resonant frequencies.

Rashid
12-5

$$H(s) = 5\,\frac{s^2+3s+20}{s^2+20s+200}$$

$$= A\,\frac{s^2+2\delta_z w_{o,z} s + w_{o,z}^2}{s^2+2\delta_p w_{o,p} s + w_{o,p}^2}$$

see other questions for the derivations of BW and Q.

$$BW_z = 2\delta_z w_{o,z} \qquad Q_z = \frac{w_{o,z}}{BW_z} = \frac{\sqrt{20}}{3} = \frac{2\sqrt{5}}{3}$$

$$BW_p = 2\delta_p w_{o,p} \qquad Q_p = \frac{w_{o,p}}{BW_p} = \frac{\sqrt{200}}{20} = \frac{10\sqrt{2}}{20} = \frac{\sqrt{2}}{2}$$

Examine the denominator of $H(s)$:

$$s^2+20s+200 = (s+\alpha_p+j\beta_p)(s+\alpha_p-j\beta_p)$$
$$= s^2+2\delta_p w_{o,p} s + w_{o,p}^2$$

$$\alpha_p^2+\beta_p^2 = w_{o,p}^2 = 200$$
$$2\alpha_p = 2\delta w_{o,p} = 20 \Rightarrow \alpha_p = 10$$
$$\beta_p = \sqrt{200-(10)^2} = 10$$

1

poles

Rashid
12.5
contin.

$$p_1 = -\alpha_p - j\beta_p$$

$$p_2 = -\alpha_p + j\beta_p$$

pole angle given as

$$\phi_p = \cos^{-1}\left(\frac{\alpha_p}{\sqrt{\alpha_p^2 + \beta_p^2}}\right) = \cos^{-1}\left(\frac{\alpha_p}{w_p}\right)$$

compute!

2

```
!cl .

close all
clear classes
clear all

f = logspace(-16,16,10000);
w = 2*pi*f;

H = ...
    ( 5*(1j*w).^2 + 15*(1j*w) + 100 ) ...
    ./ ...
    ( 1*(1j*w).^2 + 20*(1j*w) + 200 );

% plots
myLineWidth = 4;
myFontSize  = 24;

figure(101);
h1 = ...
    semilogx ...
    ( ...
    f , 20*log10(abs(H)) , ...
    'b' , ...
    'LineWidth' , myLineWidth ...
    );

grid on;

set(gca,'units','normalized')
set(gca,'Box','on','FontName','Arial',...
    'FontSize',myFontSize,'FontWeight','bold','LineWidth',4)

xlabel('Frequency (Hz)');
ylabel('Magnitude (dB)');
title('Magnitude Bode Plot')

legend(...
    h1 , ...
    'Bode Plot' , ...
    'Orientation' , 'Horizontal' , ...
    'Location'    , 'South' ...
    );

figure(102);
h2 = ...
    semilogx ...
    ( ...
    f , 180 / pi * phase(H) , ...
    'r' , ...
    'LineWidth' , myLineWidth ...
    );

grid on;

axis([ 1e-20 1e+20 -20 120 ])

set(gca,'units','normalized')
set(gca,'Box','on','FontName','Arial',...
    'FontSize',myFontSize,'FontWeight','bold','LineWidth',4)

xlabel('Frequency (Hz)');
ylabel('Phase (degrees)');
title('Phase Bode Plot')

legend(...
    h2 , ...
    'Bode Plot' , ...
    'Orientation' , 'Horizontal' , ...
    'Location'    , 'South' ...
```

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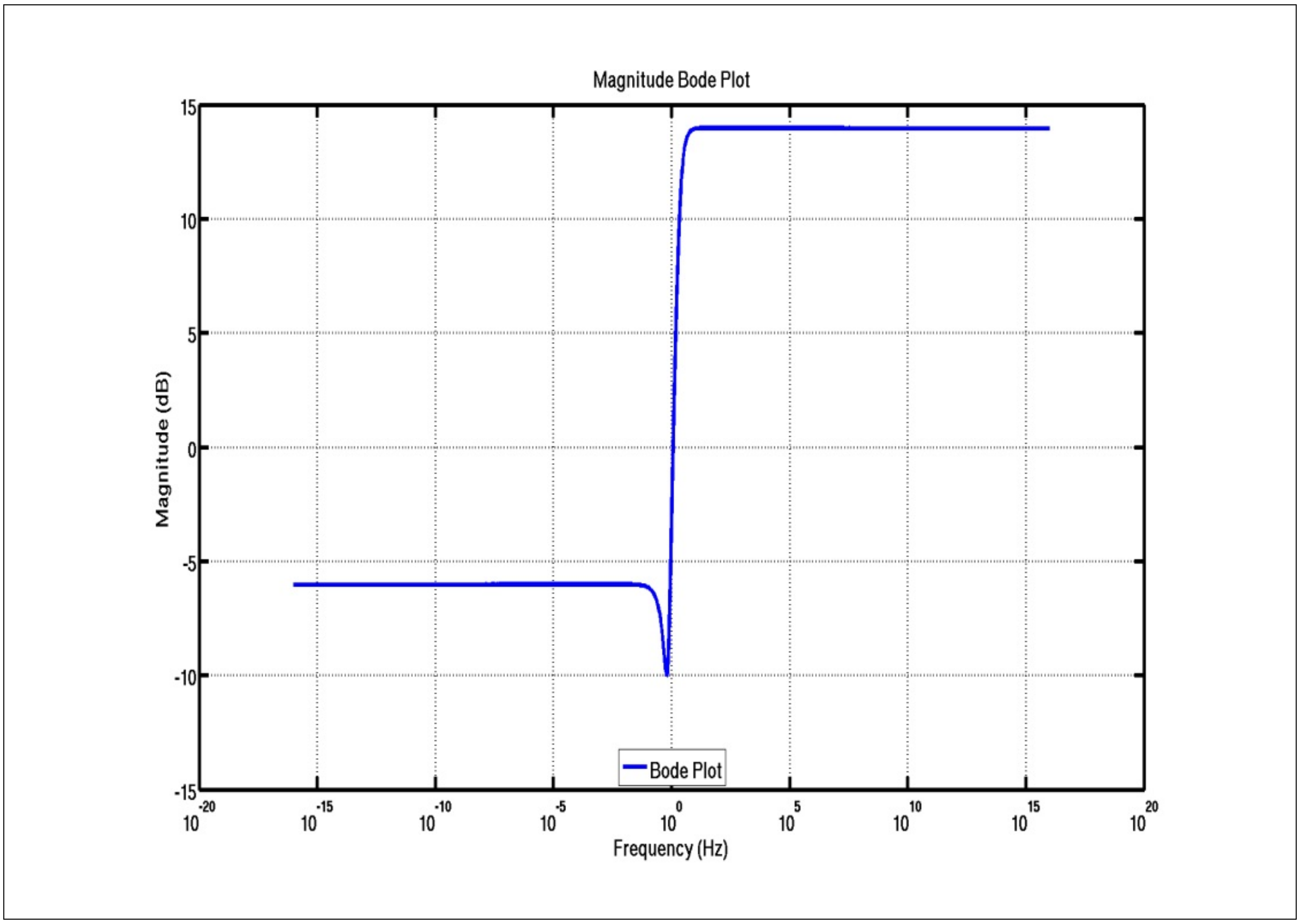
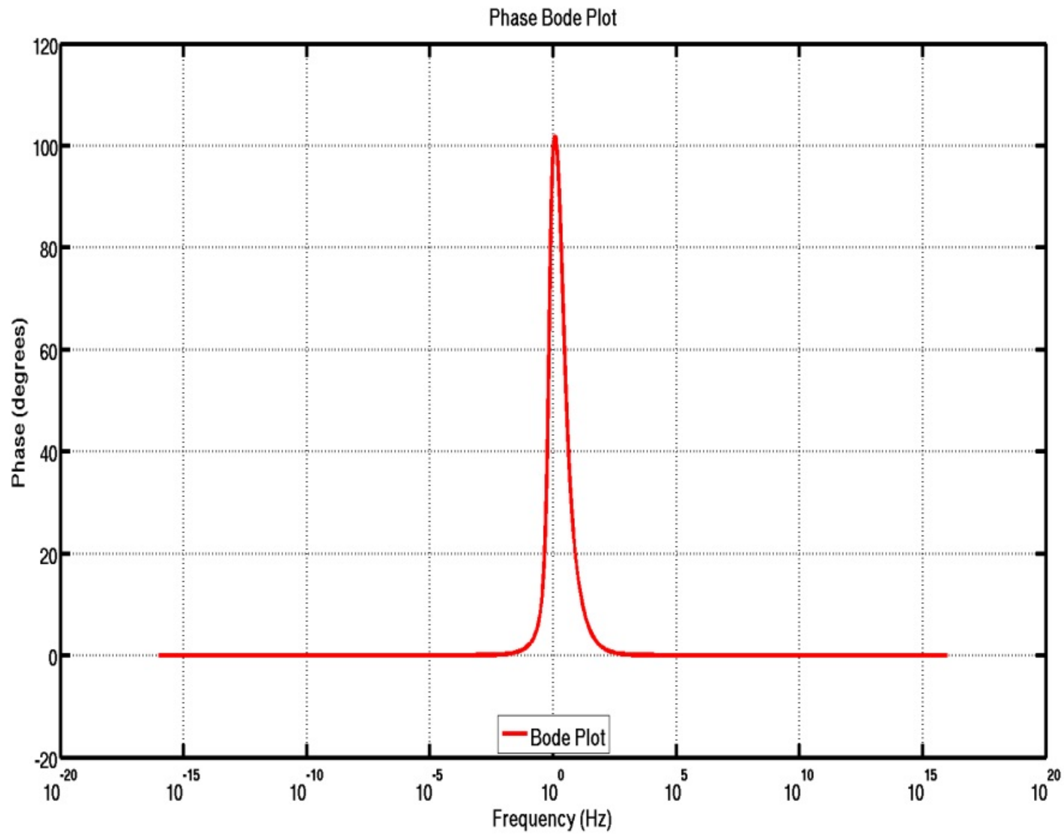
    );
```


Magnitude Bode Plot
Magnitude (dB)
15
10
5
0
-5
-10
-15
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}
10^{10}
10^{15}
10^{20}
Frequency (Hz)
Bode Plot


Phase Bode Plot
Phase (degrees)
Frequency (Hz)
Bode Plot
120
100
80
60
40
20
0
-20
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}
10^{10}
10^{15}
10^{20}

LC Resonator **Sedra 12.73**

12.73 A coil having an inductance of 10 μH is intended for applications around 1-MHz frequency. Its Q is specified to be 200. Find the equivalent parallel resistance R_p. What is the value of the capacitor required to produce resonance at 1 MHz? What additional parallel resistance is required to produce a 3-dB bandwidth of 10 kHz?

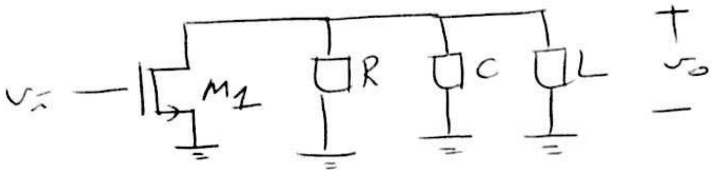
Notes: None.

Additional Tasks: None.

Necessary Knowledge and Skills: LC resonators, bandwidth, quality factor Q, resonance frequency.

See pg 1141-1142 of Sedra and Smith.
Consider the following tuned amplifier.

Sedra 12.73



$$\frac{v_o}{v_i} \approx -g_m \left[R \,//\, \frac{1}{sC} \,//\, sL \right]$$

$$= -g_m \frac{1}{\frac{1}{R} + sC + \frac{1}{sL}}$$

$$= -g_m \frac{sL}{s\frac{L}{R} + s^2 LC + 1}$$

$$= -g_m \frac{s}{\frac{s}{R} + s^2 C + \frac{1}{L}}$$

$$= -\frac{g_m}{C} \frac{s}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$(\text{resonance freq})^2 = w_o^2 = \frac{1}{LC}$$

$$2\delta w_o = \frac{1}{RC}$$

$$2\delta \frac{1}{\sqrt{LC}} = \frac{1}{RC} \Rightarrow \delta = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

(1)

Sedra 12-73 Contin-

$$\frac{v_o}{v_i} = -\frac{g_m}{C}\, G(s)\, \frac{1}{2\delta w_o}$$

where $G(s) = \dfrac{2\delta w_o s}{s^2 + 2\delta w_o s + w_o^2}$

Note that the midband gain is

$$\frac{v_o}{v_i} = -\frac{g_m}{C}\frac{1}{2\delta w_o} = -\frac{g_m}{\cancel{C}} R\cancel{C} = -g_m R$$

Now will compute the bandwidth of the tuned amplifier by calculating high- and low-frequency cut-offs.

$$G(jw) = \frac{jw\, 2\delta w_o}{(w_o^2 - w^2) + j\, 2\delta w_o w}$$

$$\left| G(j w_{-3dB}) \right|^2 = \frac{(2\delta w_o)^2 w_{-3dB}^2}{(w_o^2 - w_{-3dB}^2)^2 + (2\delta w_o w_{-3dB})^2} = \frac{1}{2}$$

If the above eqn is solved for w_{-3dB} (as done for the series RLC network)

low-freq cutoff $\quad w_L = w_o\left[-\delta + \sqrt{1+\delta^2}\right]$

high-freq cutoff $\quad w_H = w_o\left[\delta + \sqrt{1+\delta^2}\right]$

Bandwidth $BW = w_H - w_L = 2\delta w_o$

$$= \frac{1}{RC}$$

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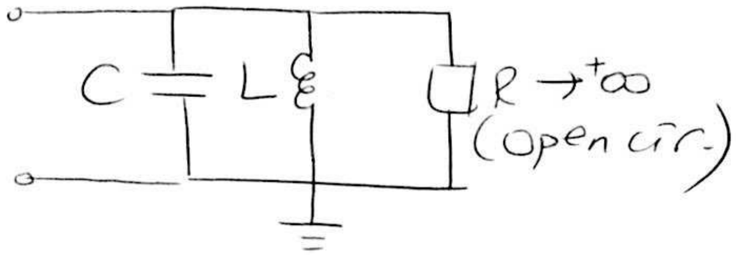
(1)

Sedra 12-73 contin.

Note that $R \to +\infty \Rightarrow BW \to 0$

The ideal LC tank is

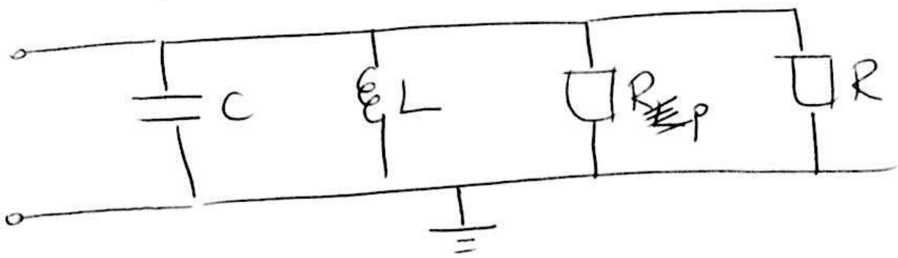


quality factor $Q = \dfrac{\omega_0}{BW} = \dfrac{1/\sqrt{LC}}{1/RC}$

$$= R\sqrt{\frac{C}{L}}$$

Note that $R \to +\infty \Rightarrow Q \to +\infty$ (ideal)

In the question we have



$L = 19\mu H$ (given)

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 2 \cdot \pi \cdot 1MHz$ (required)

compute C.

3

Sedra 12.73 contin.

(with R open cir) $Q = R_p\sqrt{\frac{C}{L}} = 200$ (given)

C computed
L known

compute R_p.

(with R not open cir)

$$BW = 2\cdot\pi\cdot 10\,KHz = \frac{1}{(R_p // R)\,C}$$

C computed.
R_p computed.
R compute ⇐

(4)