GTU
Electronics Engineering

ELEC 331
Electronic Circuits 2

Fall Semester

Instructor: Assist. Prof. Önder Şuvak

HW 1
Questions and Answers

Updated October 20, 2017 - 13:33

Assigned:

Due:

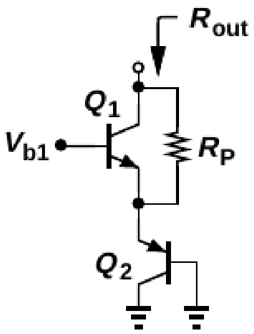
Answers Out:

Late Due:

Contents

BJT Cascode Active Load **Razavi 9.11 d**

11. Determine the output impedance of each circuit shown in Fig. 9.46. Assume $\beta \gg 1$. Explain which ones are considered cascode stages.

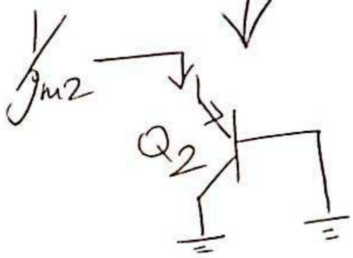


(d)

Necessary Knowledge and Skills: Output impedance calculation, BJT cascode stage properties, relatively high impedance

Razavi
9.11d

$$R_{out} = \left(1 + g_{m1}\left[r_{o1} // R_p\right]\right)\left[r_{\pi 1} // \frac{1}{g_{m2}}\right] + \left[r_{o1} // R_p\right]$$



This is not a cascode stage.

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Active-Loaded MOS Amplifier **Razavi 9.68**

68. The common-gate stage of Fig. 9.83 employs the current source M_3 as the load to achieve

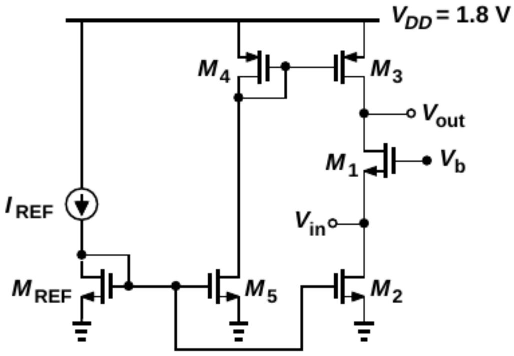

Figure 9.83

a high voltage gain. For simplicity, neglect channel-length modulation in M_1. Assuming $(W/L)_3 = 40/0.18, \lambda_n = 0.1\ \mathrm{V}^{-1}$, and $\lambda_p = 0.2\ \mathrm{V}^{-1}$, design the circuit for a voltage gain of 20, an input impedance of 50 Ω, and a power budget of 13 mW. (You may not need all of the power budget.)

Necessary Knowledge and Skills: Current mirrors, DC bias computation, common-gate amplifier design, voltage gain and input impedance computations, power budget considerations

Razavi 9.68

note that $I_{D,1} = I_{D,2} = I_{D,3}$

$I_{D,4} = I_{D,5}$

$M_3 \leftarrow \frac{1}{\lambda_p I_{D,1}}$ (resistance seen looking into the drain of M_3 in SS)

(1) $\frac{v_{out}}{v_{in}} \cong g_{m1} r_{o3}$

$$= \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D,1}} \; \frac{1}{\lambda_p I_{D,1}} = 20 \text{ specif}$$

(2) $r_{in} \cong \frac{1}{g_{m1}} \parallel r_{o2} \approx \frac{1}{g_{m1}} = 50\Omega$ specif

if $\frac{1}{g_{m1}} \gg r_{o2}$

$\Rightarrow$ check $r_{o2} = \frac{1}{\lambda_n I_{D,2}}$ (check this later (3))

$$\frac{v_{out}}{v_{in}} = \frac{1}{50\Omega} \left(\frac{1}{\lambda_p I_{D,1}}\right) = 20 \quad \text{(see (1) and (2) above)}$$

$\Downarrow$

$1k\Omega$

$$I_{D,1} = \frac{1}{(\lambda_p)(1k\Omega)} = \frac{1}{(0.2V^{-1})(1k\Omega)} = \frac{5V}{1k\Omega} = 5mA$$

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Razavi 9.68 continued

now check (3)

$$\frac{1}{\lambda_n I_{D,2}} = \frac{1}{\lambda_n I_{D,1}} = \frac{1}{(0.1\,V^{-1})(5mA)}$$

$$= \frac{10V}{5mA} = 2k\Omega$$

Observe that $2k\Omega \gg 50\Omega$

then $\frac{1}{g_{m1}} \,//\, r_{o2} \approx \frac{1}{g_{m1}}$ is justified.

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D,1}} = \frac{1}{50\Omega} = 20mS$$

$$\left(\frac{W}{L}\right)_1 \cdot 2 \cdot \underset{5\times10^{-3}A}{\cancel{5mA}} \cdot \underset{100\times10^{-6}A}{\cancel{100\mu A}}/V^2 = 400\times10^{-6}\,S^2$$

$$\left(\frac{W}{L}\right)_1 = 400 = \left(\frac{72}{0.18}\right)$$

Remember the parameters

$\mu_n C_{ox} = 100\mu A/V^2$ $\quad V_{th,n} = 0.4V$

$\mu_p C_{ox} = 50\mu A/V^2$ $\quad V_{th,p} = -0.5V$

(2)

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Razavi
9.68
cont-

$$\frac{\text{pow. budget}}{V_{DD}} = \frac{13mW}{1.8V} \simeq 7.2mA$$

5mA expended in the final branch

Let the other branches have 1mA each flow through them.

then $I_{ref} = 1mA$

$I_{D,4} = I_{D,5} = 1mA$

Mref transistor

$$\underbrace{I_{ref}}_{1mA} = \frac{1}{2}\underbrace{\mu_n C_{ox}}_{100\mu A/V^2}\underbrace{\left(\frac{W}{L}\right)_{ref}}_{?}(\underbrace{V_{GS,ref}}_{\text{set } 0.6V} - \underbrace{V_{th,n}}_{0.4V})^2$$

$$\frac{2\times 10^{-3}}{100\times 10^{-6}}\cancel{\frac{A}{A}}\cancel{V^2}\frac{1}{0.04\cancel{V^2}} = \left(\frac{W}{L}\right)_{ref}$$

$= 500$

$= \frac{90}{0.18}$

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Razavi 9.68 cont.

Current mirror of $M_{ref} - M_5$
$(1mA)$ $(1mA)$

M_{ref} and M_5 should be of the same size.

$$\left(\frac{W}{L}\right)_{ref} = \left(\frac{W}{L}\right)_5 = 500 = \left(\frac{90}{0.18}\right)$$

Current mirror of $M_4 - M_3$
$(1mA)$ $(5mA)$

$$\left(\frac{W}{L}\right)_3 = \frac{40}{0.18} \quad \left(\frac{W}{L}\right)_4 = \frac{8}{0.18}$$

$\underline{\underline{M_4}}$ (V_{SG4} computation)

$$\underbrace{I_{D,4}}_{1mA} = \frac{1}{2} \underbrace{\mu_p C_{ox}}_{50\mu A/V^2} \underbrace{\left(\frac{W}{L}\right)_4}_{\frac{8}{0.18}} \left(V_{SG} - \underbrace{|V_{th,P}|}_{0.5V}\right)^2$$

$$\frac{\cancel{2}^{\,405} \times \cancel{10^{-3}}^{\,1000}}{50 \times 10^{-6}} \frac{A}{A} V^2 \left(\frac{0.18}{8}\right) = \left(V_{SG} - 0.5\right)^2 \; V^2$$

$$0.9 = \left(V_{SG} - 0.5\right)^2$$

$$0.95 + 0.5 = V_{SG,4} = 1.45\,V$$

$$V_{SG,4} = V_{SD,4} = 1.45\,V$$

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(4)

Razavi 9-68 cont-

$\underline{\underline{M_3}}$ mode check

note $V_{DS} > V_{GS} - V_{th,n}$ (NMOS) for satur.

and $V_{SD} > V_{SG} - |V_{th,p}|$ (PMOS) for satur.

Observe $V_{SG,4} = V_{SG,3}$ ($M_4 - M_3$ is a cur. mirror)

$$\left.\begin{aligned} V_{SD,3} &> 1.45 - 0.5 \\ &> 0.95\,V \end{aligned}\right\}$$ for M_3 to remain in saturation

$\underline{\underline{M_5}}$ - mode check

note ~~$V_{DS,r}$~~ $V_{GS,ref} = V_{GS,5}$

then $V_{DS,5} > V_{GS,5} - V_{th,n}$

$V_{DS,5} > 0.6 - 0.4 = 0.2\,V$

But we have $\underbrace{V_{SD,4}}_{1.45\,V} + \underbrace{V_{DS,5}}_{0.35\,V} = V_{DD} = 1.8\,V$

(see the previous page) $\quad 0.35\,V > 0.2\,V$

then M_5 will be in saturation.

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Razavi 9.68 cont.

$\underline{M_2}$-size computation

note $M_{ref} - M_2$ is a current mirror
$(1mA)$ $(5mA)$

$$M_2\text{-size } \left(\frac{W}{L}\right)_2 = 5\left(\frac{W}{L}\right)_{ref} = \frac{450}{1.8}$$

Recall

$$V_{SD,3} > 0.95V \text{ for sat}$$
$$V_{DS,2} > 0.2V \text{ " "}$$

$V_{GS,1}$ computation

$$g_{m,1} = \frac{1}{50\Omega} = \frac{2\, I_{D,1}}{V_{GS,1} - V_{th,n}}$$
$$= \frac{2 \cdot 5mA}{V_{GS,1} - 0.4V}$$
$$\Rightarrow V_{GS,1} - 0.4V = 0.5V$$
$$V_{GS,1} = 0.9V$$

→ need to set a V_b value to place M_3, M_1, M_2 in satur.

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Razavi
9.68
cont.

set $V_{S1} = 0.25V$

$\Rightarrow V_{DS,2} = 0.25V > 0.2V$
(M_2 in sat.)

$V_b = \underbrace{V_{S1}}_{0.25V} + \underbrace{V_{GS,1}}_{0.9V} = 1.15V$

$\Rightarrow V_{DS,1} > \underbrace{0.5V}_{V_{GS,1} - V_{th,n}}$ for satur.

$\Rightarrow V_{D,1} > \underbrace{0.75V}_{(V_{S,1} + V_{DS,1})}$

$\Rightarrow V_{SD,3} < 1.05\,V$
since $V_{DD} = 1.8V = V_{SD,3} + V_{D,1}$

$\Rightarrow$ We needed to have $V_{SD,3} > 0.95V$
then O.K.

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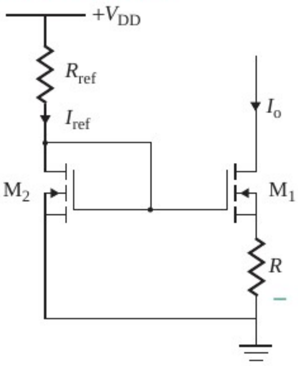
MOS Widlar Current Source **Rashid 9.6**

9.6 D The Widlar current source shown in Fig. P9.6 has $I_{ref} = 50\ \mu A$, $R = 2\ k\Omega$, and $V_{DD} = 12$ V. The MOS parameters are $K_n = 100\ \mu A/V^2$, $V_t = 1$ V, $|V_M| = 100$ V, and $(W/L)_1 = (W/L)_2 = 20$. Determine **(a)** the output current I_o, **(b)** the output resistance r_{o2}, and **(c)** the value of R_{ref}.

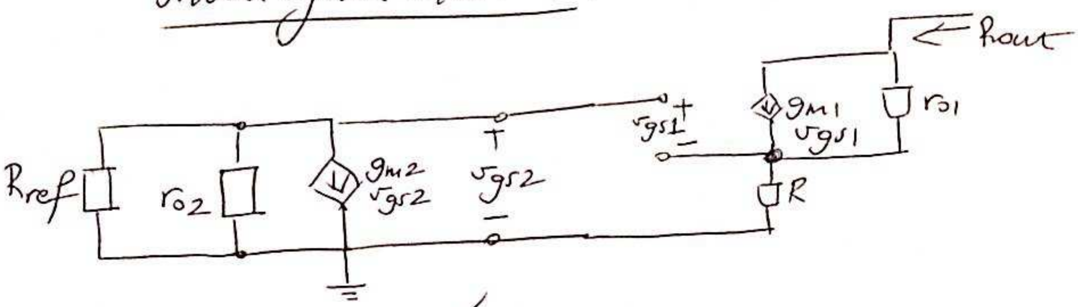
FIGURE P9.6



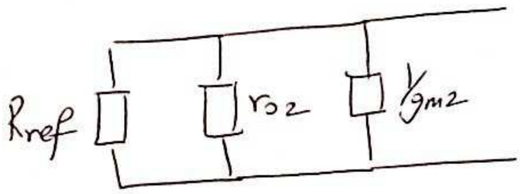
Necessary Knowledge and Skills: Widlar current source, DC bias computation, small-signal model and approximations, output impedance calculation

Rashid 9.6

Small signal equivalent



equivalent to



no source
no current
no voltage

$\Rightarrow v_{gs2} = 0$

$\Rightarrow V_{g1} = v_{g2} = 0$

$$R_{out} = (1 + g_{m1} r_{o1}) R + r_{o1}$$

check $\Rightarrow$ $v_X = (i_X - g_{m1} v_{gs1}) r_{o1} - v_{gs1}$

$$v_{gs1} = -i_X R$$

$$v_X = i_X [1 + g_{m1} R] r_{o1} + R i_X$$

$$\frac{v_X}{i_X} = R_{out} = (1 + g_{m1} R) r_{o1} + R$$

$$= (1 + g_{m1} r_{o1}) R + r_{o1}$$

$$I_D = K_n (V_{GS} - V_t)^2$$

$$\Downarrow$$

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L}$$

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Rashid 9.6
continued

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2K_n(V_{GS} - V_t)$$

DC analysis of M_2

$$KVL \Rightarrow V_{DD} = R_{ref}I_{ref} + V_{DS2}$$

$$= R_{ref}\underbrace{I_{ref}}_{= I_{D2}} + V_{GS2}$$

(DC no current into the gates)

$$V_{DD} = R_{ref}\left[K_n(V_{GS2} - V_t)^2\right] + V_{GS2} \Rightarrow$$ quadratic eqn solve for V_{GS2}

Note that the $I_{D2} - V_{GS2}$ saturation current eqn assumption is valid since M_2 will always be in satur.

$$g_{m2} = 2K_n(V_{GS2} - V_t)$$

$$r_{o2} = \frac{|V_m|}{I_{D2}}$$

DC analysis of M_1 (assuming M_1 will be in satur.)

$$\frac{V_{GS2} - V_{GS1}}{R} = \underbrace{I_{D1}}_{I_o} = K_n(V_{GS1} - V_t)^2$$

solve for V_{GS1} and then $I_{D1} = I_o$

assumption that must hold for M_1 to be in satur $\Rightarrow$ $V_{DS1} \geq V_{GS1} - V_t$

$V_{D1} - RI_o \geq V_{GS1} - V_t$ (equivalently

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(2)

Rashid 9.6
continued

equivalently for M_1 satur. $\Longrightarrow$ $V_{D1} - RI_o \geq (V_{GS2} - RI_o) - V_t$

$$V_{D1} \geq V_{GS2} - V_t$$

Note that numerical values are left to be computed.

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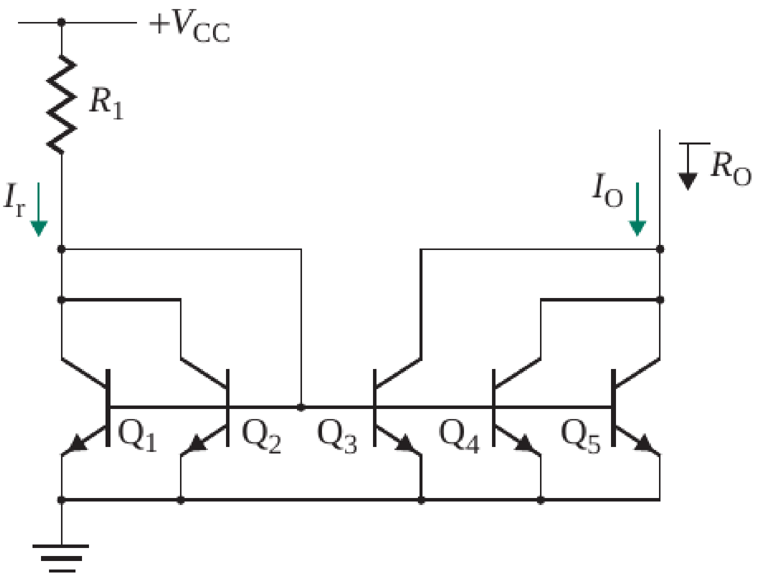
BJT Current Mirrors **Rashid 9.26**

9.26 The multiple transistors of the current source in Fig. P9.26 have $\beta_F = 150$, $R_1 = 10\ \text{k}\Omega$, $V_{CC} = 15$ V, and $V_A = 100$ V. The B-E voltages are equal, $V_{BE} = 0.7$ V. Calculate **(a)** the output current I_O, **(b)** the output resistance R_o, **(c)** Thevenin's equivalent voltage V_{Th}, and **(d)** the collector current ratio if $V_{CE2} = 15$ V.

FIGURE P9.26



Necessary Knowledge and Skills: Current Mirrors, small signal equiv. of BJT, output impedance computation, current assembly, Early voltage and its graphical interpretation

Rashid
9.26

Assuming β is too big:
base currents are nearly
zero, they can all be neglected.

$$\frac{V_{CC} - V_{BE}}{R_1} = I_r = \frac{15 - 0.7}{10k}$$

$$= \frac{14.3\,V}{10k\Omega} = 1.43\,mA$$

$$I_o = I_r \frac{3}{2} = 1.43\,\frac{3}{2} = 2.16\,mA$$

$$R_o = \frac{V_A}{I_o/3} \,//\, \frac{V_A}{I_o/3} \,//\, \frac{V_A}{I_o/3}$$

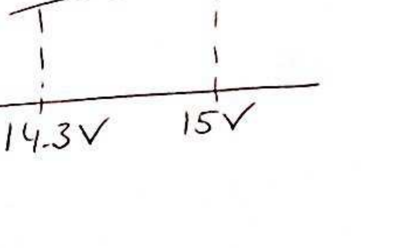
$$= \frac{V_A}{I_o} = \frac{100\,V}{2.16\,mA} \quad \text{(compute)}$$

Vth (Thevenin's equiv voltage)

$$= I_o R_o = V_A = 100\,V$$

collector current ratio computation

$$= \frac{I_o + \Delta I_C}{I_r} = \frac{3}{2} + \frac{\Delta I_C}{I_r}$$



$$\frac{\Delta I_C}{15 - 14.3} = \frac{1}{R_o}$$

$$\Delta I_C = \frac{0.7}{R_o}$$

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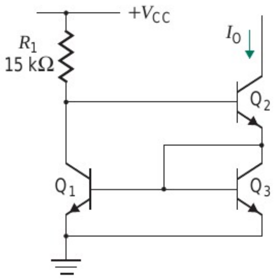
Wilson Current Source **Rashid 9.31**

9.31 For the Wilson current source in Fig. P9.31, determine the output current I_O and the output resistance R_o. Assume $V_{CC} = 20$ V, $V_{BE} = 0.7$ V, $V_T = 26$ mV, $V_A = 150$ V, and $\beta_F = 150$.

FIGURE P9.31



Necessary Knowledge and Skills: Wilson current source analysis, BJT large and small signal analysis, output impedance computation

Rashid
9.31

compute first I_R (reference current)
that flows over $R_1 = 15k\Omega$

$$I_R = \frac{V_{CC} - (V_{BE1} + V_{BE2})}{R_1}$$

note that $V_{BE1} = V_{BE2} = 0.7V$

$$I_R = \frac{20 - (0.7 + 0.7)}{15k\Omega} = \frac{18.6V}{15k\Omega}$$

⇒ See pages 596-597 in Rashid for the derivation of the output current for the Wilson current source.

- All the transistors are identical.

$$I_{E2} = I_{C2}\,\frac{\beta+1}{\beta} = \left(1 + \frac{2}{\beta}\right)\underbrace{I_{C3}}_{I_{C1}}$$

$$I_R = I_{C1} + I_{B2}$$

$$= I_{C2}\,\frac{\beta+1}{\beta}\,\frac{1}{\left(1+\frac{2}{\beta}\right)} + I_{C2}\,\frac{1}{\beta}$$

$$I_R = I_{C2}\left[\frac{\beta+1}{\beta+2} + \frac{1}{\beta}\right] \Rightarrow \frac{I_{C2}}{I_R} = \frac{\beta(\beta+2)}{\beta^2 + 2\beta + 2}$$

$$= 1 - \frac{2}{\beta^2 + 2\beta + 2}$$

⇒ I_R computed above, β given, compute $I_o = I_{C2}$

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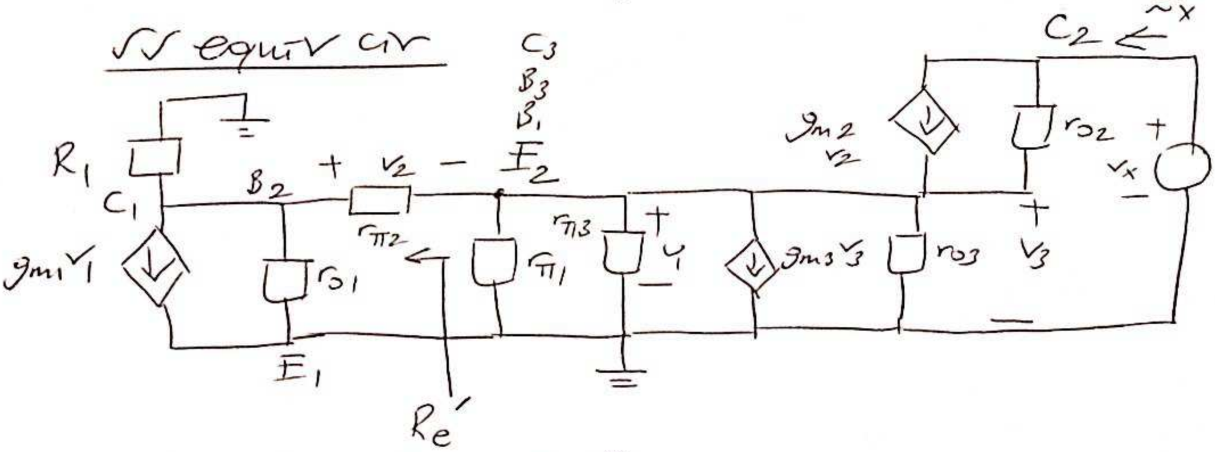
For R_o (output impedance) calculation, see pp 597–598 of Rashid

Rashid
931
Continued

ss equiv cir



note that $R_e = R_1 // r_{o1}$

then $R_e' = \dfrac{v_{x,e_2}}{i_{x,e_2}}$ (connecting a ~~test~~ source at the E_2 node)

$$v_{x,e_2} = i_{x,e_2} \cdot r_{\pi 2} + (i_{x,e_2} - g_{m1} \underbrace{v_1}_{v_{x,e_2}}) R_e$$

$$\rightarrow v_{x,e_2}\left[1 + g_{m1} R_e\right] = i_{x,e_2}\left[r_{\pi 2} + R_e\right]$$

then $R_e' = \dfrac{r_{\pi 2} + R_e}{1 + g_{m1} R_e}$

R_e' could be $\approx \dfrac{1}{g_{m1}}$ if $\left(\begin{array}{c} r_{o1} \gg R_1 \\ R_1 \gg r_{\pi 2} \\ \text{and} \\ g_{m1} R_1 \gg 1 \end{array}\right)$

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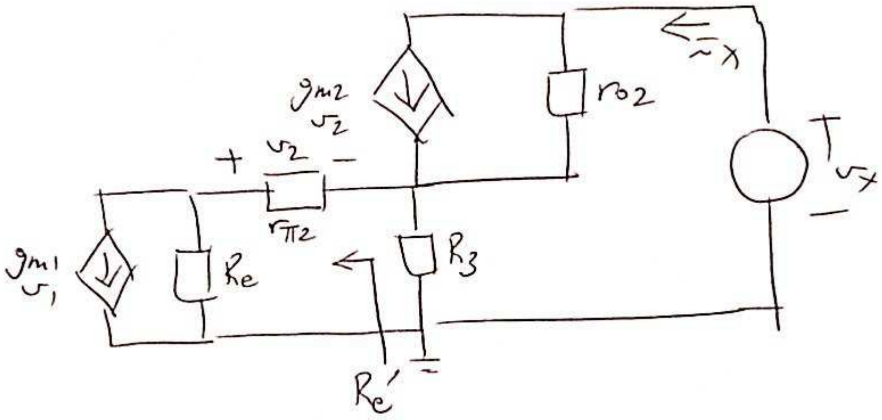
Rashid
9.31
Continued

ss equiv circuit
simplified

denote by $R_3 = r_{\pi 1} // r_{\pi 3} // \frac{1}{g_{m3}} // r_{o3} \approx \frac{1}{g_{m3}}$



(KVL) $v_x = (i_x - g_{m2} v_2) r_{o2} + i_x [R_3 // R_e']$

$$v_2 = - i_x \frac{R_3}{R_3 + R_e'} \cdot r_{\pi 2}$$

$$v_x = i_x \left[r_{o2} + R_3 // R_e' \right] - g_{m2} r_{o2} \left[- i_x \frac{R_3}{R_3 + R_e'} r_{\pi 2} \right]$$

$$R_o = \frac{v_x}{i_x} = r_{o2} + \frac{R_3 R_e'}{R_3 + R_e'} + \beta_2 \frac{R_3 r_{o2}}{R_3 + R_e'}$$

$$= r_{o2} + \frac{R_3 (R_e' + \beta_2 r_{o2})}{R_3 + R_e'}$$

3

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Razavi 9.31 Continued

$$R_o \cong r_{o2} + \underbrace{\overset{\approx \frac{1}{g_{m3}}}{R_3} // \overset{\approx 1/g_{m1}}{R_e'}}_{\text{small}} + r_{o2}\beta_2 \frac{R_e'}{R_3 + R_e'}$$

$\approx \frac{1}{2}$ if $\frac{1}{g_{m3}} \simeq \frac{1}{g_{m1}}$

then

$$R_o \cong r_{o2}\left(1 + \frac{\beta_2}{2}\right)$$

$$g_m = \frac{I_C}{V_T}$$

$$g_m r_\pi = \beta \Rightarrow r_\pi = \beta \frac{V_T}{I_C}$$

and $r_o = \frac{V_A}{I_C}$

Plug in and compute the numerical values.

4

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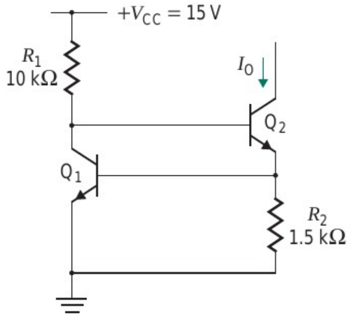
Current Source Sensitivity **Rashid 9.34**

9.34 Determine the sensitivity S of output current I_O to supply voltage V_{CC} for the circuit in Fig. P9.34. S is defined as

$$S = \frac{V_{CC}/I_O}{\delta I_O/\delta V_{CC}}$$

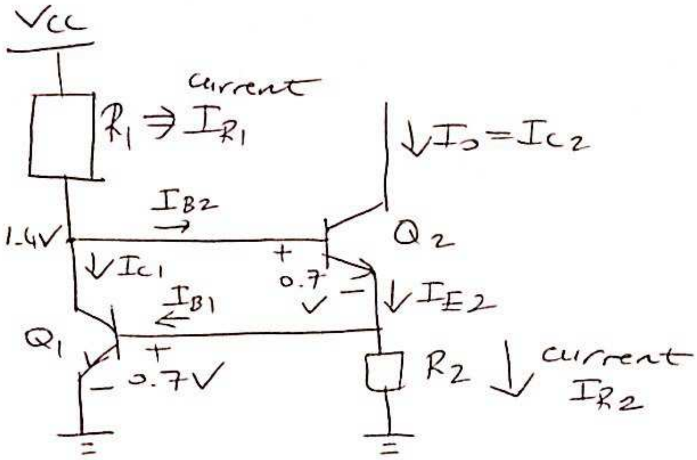
FIGURE P9.34



Necessary Knowledge and Skills: Sensitivity analysis, BJT current source/reference, BJT large and small signal analysis,

Rashid
9.34

$$S_{V_{CC}}^{I_o} = \frac{\Delta I_o / I_o}{\Delta V_{CC} / V_{CC}} = \frac{\partial I_o}{\partial V_{CC}} \frac{V_{CC}}{I_o}$$



$$I_{R_1} = \frac{V_{CC} - V_{BE1} - V_{BE2}}{R_1}$$

$$I_{R_1} = I_{B2} + I_{C1}$$

$$I_{B2} = \frac{I_o}{\beta}$$

$$I_{E2} = I_o \frac{\beta+1}{\beta}$$

$$I_{E2} = I_{B1} + \frac{V_{BE1}}{R_2}$$

$$I_{C1} = \beta I_{B1}$$

$$I_o \frac{\beta+1}{\beta} - \frac{V_{BE1}}{R_2} = I_{B1}$$

$$I_{C1} = \beta \left[\frac{\beta+1}{\beta} I_o - \frac{V_{BE1}}{R_2} \right]$$

$$= (\beta+1) I_o - \frac{\beta V_{BE}}{R_2}$$

$$V_{CC} = I_{R_1} R_1 + V_{BE1} + V_{BE2}$$

$$I_{R_1} = \frac{I_o}{\beta} + (\beta+1) I_o - \frac{\beta V_{BE1}}{R_2}$$

$$V_{CC} = R_1 \left[I_o \left(\frac{1}{\beta} + \beta + 1 \right) - \frac{\beta V_{BE1}}{R_2} \right] + V_{BE1} + V_{BE2}$$

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Rashid 9.34 cont.

$$(*)\quad I_o = \left[\frac{V_{CC} - V_{BE1} - V_{BE2}}{R_1} + \frac{\beta V_{BE1}}{R_2}\right] \cdot \left[\frac{\beta}{\beta^2+\beta+1}\right]$$

Considering that $V_{BE1} \cong V_{BE2} \approx 0.7V$

$$\frac{\partial I_o}{\partial V_{CC}} = \frac{1}{R_1} \frac{\beta}{\beta^2+\beta+1}$$

then $$S_{V_{CC}}^{I_o} = \frac{1}{R_1} \frac{\beta}{\beta^2+\beta+1} \frac{V_{CC}}{I_o}$$ → plug in $(*)$

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