GIT Electronics Engineering
ELEC 232 and ELEC 331

Some Derivations for the Analysis of Simple Electronic Circuits

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Abstract

In English: The contents of this document are derivations that are frequently used and mostly common to the topics of the courses ELEC 232 and ELEC 331 (Electronic Circuits 1 and 2, respectively).

In Turkish: Bu dokümanın içeriğinde ELEC 232 ve ELEC 331 (Elektronik Devreler 1 ve 2) derslerinde sıkça kullanılan ve her iki dersin konularında ortak olan çıkarımlar bulunmaktadır.

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Contents

Title Page **1**

Contents **1**

1 Introduction and Essentials **4**

1.1 Resistances . . . 4

Significance of Equivalent Parallel Resistances . . . 5

1.2 Dividers . . . 10

Voltage Divider . . . 11

Current Divider . . . 12

1.3 AC and DC Coupling . . . 14

Explanations and Definitions on Coupling . . . 15

Treatment of a Periodic Pulse Waveform . . . 17

1.4 RC Low-Pass Filter . . . 21

Analysis of Magnitude and Phase due to a Sinusoidal Input . . . 22

2 Device Models **27**

2.1 Transitions between Regimes . . . 27

Transition Boundary between Triode and Saturation for a MOS Transistor . . . 28

2.2 Small-Signal Equivalent Circuits . . . 32

Small-Signal Model for Diodes . . . 33

Small-Signal Model for BJT . . . 40

Small-Signal Model for MOS . . . 46

3 Diode Applications **58**

3.1 Clamping Circuits . . . 58

Diode Clamper Example - Q . . . 59

Diode Clamper Example - A . . . 60

4 Amplifiers **62**

4.1 Types . . . 62

Properties of Voltage Amplifiers . . . 63

Properties of Current Amplifiers . . . 65

Properties of Trans-Conductance Amplifiers . . . 67

Properties of Trans-Resistance Amplifiers . . . 70

4.2 Modeling Examples . . . 72

Modeling the Common Emitter Amplifier as a Trans-Conductance Amplifier . . . 73

4.3 Design Considerations . . . 79

How to Design an Amplifier with Large Gain and Large Input Impedance? . . . 80

5 Small-Signal Impedances **84**

5.1 Impedances into the Three Nodes of a BJT 84
Impedance Looking into the Base of a BJT 85
Impedance Looking into the Emitter of a BJT 90
Impedance Looking into the Collector of a BJT 95

6 Power Amplifiers **98**
6.1 Miscellaneous Analyses 98
Effects of Load Resistor Values on the Performance of Class A Power Amplifiers . 99
Efficiency Calculation of Class B Power Amplifiers 104

7 Differential Amplifiers **109**
7.1 Fundamentals 109
Simple Differential Amplifier Analysis 110

8 Frequency Response **114**
8.1 Fundamentals 114
Gain-Bandwidth Product 115
Open-Circuit Time Constants Method for Computing High Frequency Response . 117
8.2 Intrinsic Responses of Transistors 121
BJT Frequency Response 122

9 Feedback **127**
9.1 Benefits 127
Effects of Feedback on Low Frequency Response 128
Effects of Feedback on High Frequency Response 131
Effects of Feedback on Sensitivities 134
Example on Effects of Feedback on Distortion - Sedra & Smith 08.020 - Q 136
Example on Effects of Feedback on Distortion - Sedra & Smith 08.020 - A 137
9.2 Polarity Determination 139
Negative Feedback Example - Q 140
Negative Feedback Example - A 141
Positive Feedback Example - Q 142
Positive Feedback Example - A 143
Positive Feedback in a CE-CC Cascaded Voltage Amplifier 144
9.3 Types 147
Voltage-Series Feedback on Voltage Amplifiers 148
Voltage-Shunt Feedback on Trans-Resistance Amplifiers 152
Current-Series Feedback on Trans-Conductance Amplifiers 155
9.4 Modeling of Feedback Networks 158
Voltage-Series Feedback Networks 159
Current-Series Feedback Networks 161
Voltage-Shunt Feedback Networks 163
Current-Shunt Feedback Networks 165

9.5 Auxiliaries 167
Analysis of a Feedback Network of Small Capacitors 168
9.6 Stability 171
Example on Adjusting the Feedback Factor to Maintain a Specified Phase Margin - Sedra 08.073 - Q 172
Example on Adjusting the Feedback Factor to Maintain a Specified Phase Margin - Sedra 08.073 - A 173
Example on Adjusting the Real-Valued Feedback Factor for a Two-Pole System to Maintain a Specified Phase Margin - Rashid 10.056 - Q 176
Example on Adjusting the Real-Valued Feedback Factor for a Two-Pole System to Maintain a Specified Phase Margin - Rashid 10.056 - A 177
9.7 Compensation 182
Bandwidth Limiting Compensation Example on a CE-CE-CC Voltage Amplifier with Voltage-Series Feedback 183
9.8 Analysis in Practical Circuits 188
Voltage-Shunt Feedback on a Practical Trans-Resistance Amplifier 189
Voltage-Series Feedback Example - Razavi 12.25 - Q 195
Voltage-Series Feedback Example - Razavi 12.25 - A 196
Analysis of an Emitter-Degenerated Common Emitter Amplifier as a TCA with Current-Series Feedback 199
Voltage-Series Feedback Example - Rashid 10.013 - Q 203
Voltage-Series Feedback Example - Rashid 10.013 - A 204
Current-Series Feedback Example - Rashid 10.028 - Q 208
Current-Series Feedback Example - Rashid 10.028 - A 209
Analysis of Several Feedback Amplifiers with OpAmps - Sedra 08.026 - Q 212
Analysis of Several Feedback Amplifiers with OpAmps - Sedra 08.026 a - A - *Voltage-Series Feedback* 213
Analysis of Several Feedback Amplifiers with OpAmps - Sedra 08.026 b - A - *Voltage-Shunt Feedback* 215
Analysis of Several Feedback Amplifiers with OpAmps - Sedra 08.026 c - A - *Current-Series Feedback* 219
Analysis of Several Feedback Amplifiers with OpAmps - Sedra 08.026 d - A - *Current-Shunt Feedback* 223

10 Oscillators **227**
10.1 Auxiliaries 227
Analysis of LC Networks 228
Analysis of an RC Feedback Network - Sedra 13.013 - Q 233
Analysis of an RC Feedback Network - Sedra 13.013 - A 234

1. Introduction and Essentials

1.1 Resistances

Significance of
Equivalent
Parallel Resistances

Worked on this on:
January 15, 2016/Fri

Parallel resistance $R_p = R_1 // R_2$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_p}$$

$$\frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2} = \frac{1}{R_p}$$

$$\frac{R_1 + R_2}{R_1 R_2} = \frac{1}{R_p}$$

then $\boxed{R_p = \frac{R_1 R_2}{R_1 + R_2}}$

Note the identity

$\boxed{\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_1 // R_2}}$ where $R_1 // R_2 = R_p$

Also note the identity

$\boxed{R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}} \Longrightarrow \frac{1}{\frac{1}{R_1 // R_2}} = R_1 // R_2 = R_p$

(1)

Note

$$R_p = \frac{R_1 R_2}{R_1+R_2} = R_1 \cdot \frac{R_2}{R_1+R_2} < R_1$$

since $\frac{R_2}{R_1+R_2} < 1$

Assuming $R_1, R_2 > 0$

Also

$$R_p = R_2 \frac{R_1}{R_1+R_2} < R_2$$

since $\frac{R_1}{R_1+R_2} < 1$

Then $R_p = R_1 // R_2 < \max(R_1, R_2)$

and $R_p = R_1 // R_2 < \min(R_1, R_2)$

We have

$$R_p < \min(R_1, R_2) \leq \max(R_1, R_2)$$

$\Downarrow$

since it may be that $R_1 = R_2$

2

Now assume that for some $\alpha > 0$

$$R_2 = \alpha R_1$$

$$R_p = R_1 /\!/ R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{\alpha R_1^2}{R_1 + \alpha R_1} = \frac{\alpha R_1^2}{(1+\alpha) R_1}$$

$$= \frac{\alpha}{1+\alpha} R_1$$

For an α that is very big $(\alpha > 10)$

$$R_p = \frac{\alpha}{1+\alpha} R_1 \approx R_1 \quad \text{since } \lim_{\alpha \to +\infty} \frac{\alpha}{1+\alpha} = 1$$

Note that for $\alpha > 10$

$$R_p < \min(R_1, R_2) = \min(R_1, \alpha R_1) = \underbrace{R_1}_{\substack{\text{since} \\ \alpha R_1 > R_1}}$$

Then

$$R_p < R_1$$

But

$$R_p \approx R_1$$

For two resistances that are in parallel, if there is a big difference between the two, the equivalent parallel resistance will almost be equal to the minimum of the two.

Now assume

$$R_2 = \alpha R_1 \quad \text{but } 0 < \alpha \ll 1$$

$$R_p = R_1 // R_2 = \frac{\alpha R_1^2}{(1+\alpha)R_1} = \frac{\alpha}{1+\alpha} R_1 \approx \alpha R_1$$

$$R_p < \min(R_1, R_2) = \min(R_1, \alpha R_1) = \underbrace{\alpha R_1}_{\substack{\text{since} \\ \alpha R_1 < R_1}}$$

And

$$R_p < \alpha R_1$$

But

$$R_p \approx \alpha R_1$$

Read the note at the end of page 3.

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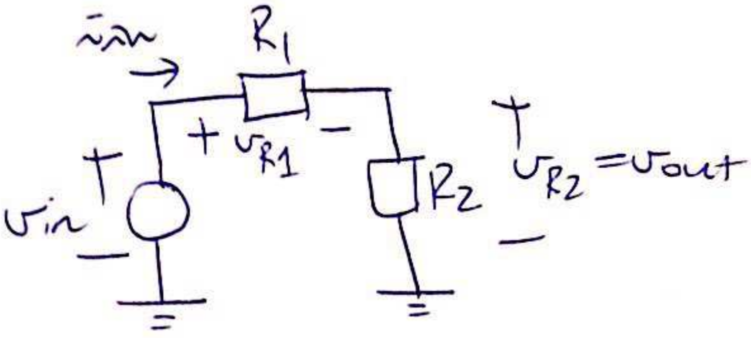
(4)

1.2 Dividers

Voltage divider



Compute v_{out} in terms of v_{in}.

KVL

$$+v_{in} - v_{R1} - v_{R_2} = 0$$

Ohm's law

$$v_{R1} = R_1 i_{in}$$
$$v_{R2} = R_2 i_{in}$$

Then

$$+v_{in} - R_1 i_{in} - R_2 i_{in} = 0$$
$$v_{in} = (R_1 + R_2) i_{in}$$
$$\frac{v_{in}}{i_{in}} = R_1 + R_2$$

And

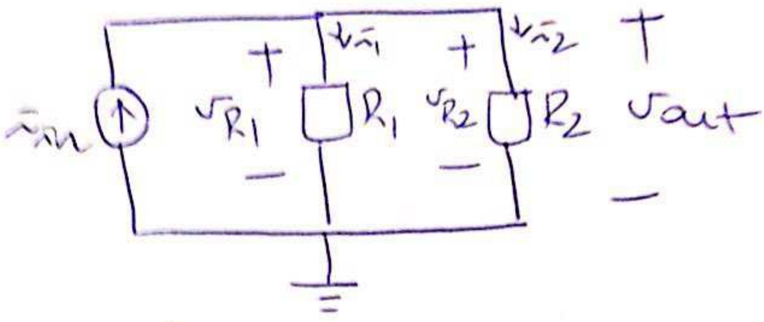
$$v_{out} = v_{R_2} = R_2 i_{in}$$
$$\frac{v_{out}}{i_{in}} = R_2$$

Note that

$$\frac{\frac{v_{out}}{i_{in}}}{\frac{v_{in}}{i_{in}}} = \frac{v_{out}}{v_{in}}$$

$$\boxed{\frac{R_2}{R_1 + R_2} = \frac{v_{out}}{v_{in}}}$$

Current divider



Note that

$$v_{R_1} = v_{R_2} = v_{out}$$

$$v_{R_1} = i_1 R_1$$

$$v_{R_2} = i_2 R_2$$

$$i_{in} = i_1 + i_2 \quad \underline{\text{by KCL}}$$

Then

$$i_1 R_1 = i_2 R_2 \Rightarrow i_2 = i_1 \frac{R_1}{R_2}$$

$$i_1 + i_1 \frac{R_1}{R_2} = i_{in}$$

$$i_1 \left(1 + \frac{R_1}{R_2}\right) = i_{in}$$

$$\left.\frac{i_1}{i_{in}} = \frac{1}{1+\frac{R_1}{R_2}} = \frac{R_2}{R_1+R_2}\right\} \text{current division factor}$$

Note also that

$$\frac{i_1}{i_{in}} + \frac{i_2}{i_{in}} = \frac{i_1+i_2}{i_{in}} = \frac{i_{in}}{i_{in}} \Rightarrow \left.\frac{i_2}{i_{in}} = \frac{R_1}{R_1+R_2}\right\} \text{Another current division factor}$$

1

Current divider

$$\frac{v_{out}}{i_{in}} = ?$$

$$v_{out} = v_{R_1} = R_1 \cdot \frac{R_2}{R_1+R_2} i_{in}$$

$$\frac{v_{out}}{i_{in}} = R_1 \frac{R_2}{R_1+R_2} = \frac{R_1 R_2}{R_1+R_2} = R_1 /\!/ R_2$$

Could also compute this as in:

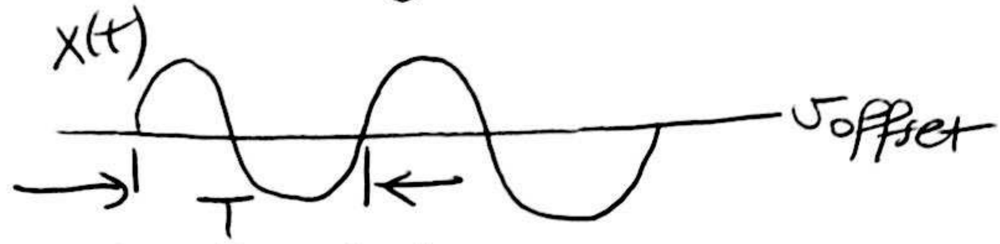
$$v_{out} = v_{R_2} = R_2 \frac{R_1}{R_1+R_2} i_{in}$$

$$\frac{v_{out}}{i_{in}} = \frac{R_1 R_2}{R_1+R_2} = R_1 /\!/ R_2$$

1.3 AC and DC Coupling

AC and DC coupling

For a sinusoidal signal with offset



$$x(t) = A\sin(\omega t) + v_{offset}$$

On the oscilloscope, the DC coupled waveform is $x(t)$ itself.

The DC component of $x(t)$ is computed as in

$$x_{DC} = \frac{1}{T}\int_0^T x(t)\,dt$$

where T is the period of the signal.

The DC component is invariant under time-shifts in the signal, i.e.

$$x_{DC} = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt \quad \text{for } t_0 \in \mathbb{R}$$

1

For this signal $x(t)$ note that

$$T = \left(\frac{w}{2\pi}\right)^{-1} = \left(\frac{\cancel{2\pi} f}{\cancel{2\pi}}\right)^{-1} = \frac{1}{f}$$

and

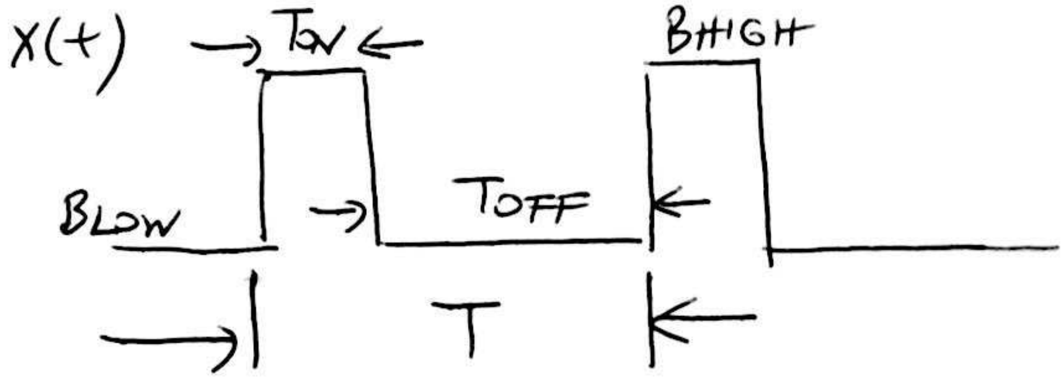
$$X_{DC} = \frac{1}{T}\int_{t_0}^{t_0+T}\left[A\sin(wt) + v_{offset}\right]dt$$

$$= \underbrace{\frac{A}{T}\int_{t_0}^{t_0+T}\sin(wt)\,dt}_{=0} + \underbrace{\frac{1}{T}\int_{t_0}^{t_0+T} v_{offset}\,dt}_{\frac{1}{\cancel{T}}\cancel{T}\,v_{offset}}$$

$$X_{DC} = v_{offset}$$

(2)

AC and DC coupled waveforms for a periodic pulse signal



Note that $\underbrace{T}_{\text{period}} = T_{ON} + T_{OFF}$

DC coupled waveform is $x(t)$ itself.

DC component of $x(t)$ is

$$X_{DC} = \frac{1}{T}\int_0^T x(t)\,dt$$

$$= \frac{1}{T}\left[T_{ON}B_{HIGH} + T_{OFF}B_{LOW}\right]$$

1

Note now that

$$x_{DC,coupled}(t) = x(t)$$

$$x_{DC} = \text{ as computed on pg } 1$$

$$x_{AC,coupled}(t) = x(t) - x_{DC}$$

Q: When do we have

$$x_{DC,coupled}(t) = x_{AC,coupled}(t) \; ?$$

A: When $x_{DC} = 0$

$$\frac{1}{T}\left[T_{ON} B_{HIGH} + T_{OFF} B_{LOW}\right] = 0$$

$$\frac{T_{ON}}{T} B_{HIGH} + \frac{T - T_{ON}}{T} B_{LOW} = 0$$

$$\frac{T_{ON}}{T}\left[B_{HIGH} - B_{LOW}\right] = -B_{LOW}$$

$$\frac{T_{ON}}{T} = -\frac{B_{LOW}}{B_{HIGH} - B_{LOW}}$$

2

The duty cycle of a periodic pulse waveform is

$$\text{duty cycle} = \frac{T_{ON}}{T} \times 100\%$$

Note that on pg 1 we have computed

$$X_{DC} = \frac{T_{ON}}{T} B_{HIGH} + \frac{T - T_{ON}}{T} B_{LOW}$$

$$= \frac{T_{ON}}{T}\left[B_{HIGH} - B_{LOW}\right] + B_{LOW}$$

then

$$\text{duty cycle} = \frac{T_{ON}}{T} \times 100\% = \frac{X_{DC} - B_{LOW}}{B_{HIGH} - B_{LOW}} \times 100\%$$

Also note that

X_{DC} changes if B_{HIGH} and B_{LOW} changes but the duty cycle remains constant

~~changes~~.

2

This means since X_{DC} changes with the duty cycle, $X_{AC\text{-}coupled}(t)$ will change with the duty cycle aswell.

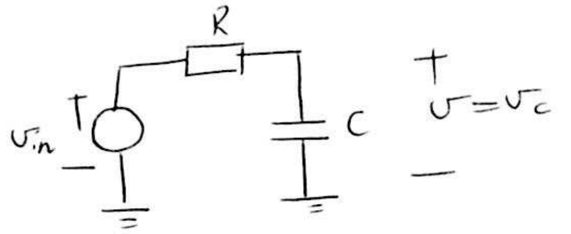
Observe this with a signal generator and oscilloscope.

(4)

1.4 RC Low-Pass Filter

Response of an RC
Low-Pass Filter to
a sinusoidal input

Worked on this on: Dec. 24, 2015/Th



$v_{in}(t) = A \sin(wt)$

$$\frac{v - v_{in}}{R} + C\frac{dv}{dt} = 0$$

$$\frac{v - v_{in}}{RC} + \frac{dv}{dt} = 0$$

$$\frac{dv}{dt} + \frac{1}{RC} v = \frac{1}{RC} v_{in}(t)$$

$$sV(s) - v(0^-) + \frac{1}{RC} V(s) = \frac{1}{RC} V_{in}(s)$$

$$= \frac{A}{RC} \frac{w}{s^2 + w^2}$$

$$\left(s + \frac{1}{RC}\right) V(s) = v(0^-) + \frac{A}{RC} \frac{w}{s^2 + w^2}$$

$$V(s) = \frac{v(0^-)}{s + \frac{1}{RC}} + \frac{A}{RC} \frac{w}{(s^2 + w^2)(s + \frac{1}{RC})}$$

Should now compute the periodic part of the soln $v(t)$ in the time domain.

Take care of

$$\frac{1}{(s^2+w^2)\left(s+\frac{1}{RC}\right)} = \frac{Fs+G}{s^2+w^2} + \frac{H}{s+\frac{1}{RC}}$$

$$(Fs+G)\left(s+\frac{1}{RC}\right) + H(s^2+w^2) = 1$$

$$\underline{\underline{Fs^2}} + \frac{G}{RC} + \underline{Gs} + \underline{\frac{F}{RC}s} + \underline{\underline{Hs^2}} + Hw^2 = 1$$

$$F+H=0 \Longrightarrow F=-H$$

$$G+\frac{F}{RC}=0 \Longrightarrow G-\frac{H}{RC}=0 \Rightarrow G=\frac{H}{RC}$$

$$\underbrace{\frac{G}{RC}}+Hw^2=1$$

$$\frac{H}{(RC)^2}+Hw^2=1$$

$$H\left[1+(RCw)^2\right]=(RC)^2$$

$$\boxed{H=\frac{(RC)^2}{1+(RCw)^2}}$$

$$\boxed{G=\frac{H}{RC}=\frac{RC}{1+(RCw)^2}}$$

$$\boxed{F=-\frac{(RC)^2}{1+(RCw)^2}}$$

(2)

Note that the periodic part has the Laplace Transform

$$V_{per}(s) = \frac{Aw}{RC}\left[\frac{Fs+G}{s^2+w^2}\right]$$

$$= \frac{AFw}{RC}\frac{s}{s^2+w^2} + \frac{AG}{RC}\frac{w}{s^2+w^2}$$

$$v_{per}(t) = \frac{AFw}{RC}\cos(wt) + \frac{AG}{RC}\sin(wt)$$

$$K\cos\phi = \frac{AG}{RC}$$

$$K\sin\phi = \frac{AFw}{RC}$$

Then

$$\tan\phi = \frac{\cancel{A}Fw}{\cancel{RC}}\frac{\cancel{RC}}{\cancel{A}G} = \frac{Fw}{G}$$

$$= -\frac{(RC)^2}{\cancel{1+(RCw)^2}}\,w\,\frac{\cancel{1+(RCw)^2}}{\cancel{RC}}$$

$$\tan\phi = -RCw$$

and

$$K^2 = \left(\frac{AFw}{RC}\right)^2 + \left(\frac{AG}{RC}\right)^2$$

$$= \left(\frac{A}{RC}\right)^2\left[(Fw)^2 + G^2\right]$$

$$= \left(\frac{A}{RC}\right)^2\left[\frac{(RC)^4w^2}{\left[1+(RCw)^2\right]^2} + \frac{(RC)^2}{\left[1+(RCw)^2\right]^2}\right]$$

3

$$K^2 = \left(\frac{A}{RC}\right)^2 \left[(RC)^2\right] \left[\frac{\cancel{1+(RC w)^2}}{\left[1+(RC w)^2\right]^{\cancel{2}}}\right]$$

$$= \frac{A^2}{1+(RC w)^2}$$

$$K = \frac{A}{\sqrt{1+(RC w)^2}}$$

We have

$$v_{per}(t) = K \sin(wt + \phi) = \frac{A}{\sqrt{1+(RC w)^2}} \sin\left(wt + \arctan(-RC w)\right)$$

at $w = w_0 = \frac{1}{RC}$

$$K = \frac{A}{\sqrt{1+\left(RC\frac{1}{RC}\right)^2}} = \frac{A}{\sqrt{2}}$$

$$\phi = \arctan\left(-RC\frac{1}{RC}\right) = \arctan(-1)$$

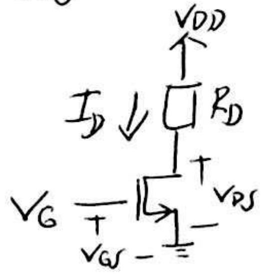
$$= -\frac{\pi}{4}$$

(4)

2. Device Models

2.1 Transitions between Regimes

Solution for the Boundary of Triode and Saturation Regimes in a CS type stage



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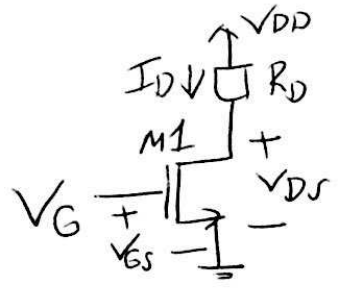


What is the value of $V_G = V_{GS}$ for which

⇒ M1 runs just into triode?

⇒ M1 resides just at the boundary of triode and saturation?

} equivalent questions

$$V_G = V_G - 0V$$
$$= V_G - V_S$$
$$= V_{GS}$$

$$V_D = V_D - 0V$$
$$= V_D - V_S$$
$$= V_{DS}$$
$$= V_{DD} - R_D I_D$$

The boundary between triode and saturation determined by

$$V_{DS} = V_{GS} - V_{th}$$
$$= V_{ov}$$
$$= V_{overdrive}$$

$$V_{DS} = V_{DD} - R_D I_D \qquad (1)$$
$$= V_{GS} - V_{th}$$

At The boundary between the triode and saturation

$\Rightarrow I_D$ is still equal to

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2$$

Then we have to solve from (1)

$$V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2 = V_{GS} - V_{th} \quad (2)$$

plug in $K_n = \frac{1}{2} \mu_n C_{ox} \frac{W}{L}$

and $V_{ov} = V_{GS} - V_{th}$

Then (2) becomes

$$V_{DD} - R_D K_n V_{ov}^2 = V_{ov}$$
$$\Rightarrow R_D K_n V_{ov}^2 + V_{ov} - V_{DD} = 0 \qquad (3)$$

In (3), check the units

V_{ov}: Volts $\qquad R_n : \Omega$

V_{DD} : Volts $\qquad K_n = \frac{A}{V^2}$

 2

Then $R_D K_n V_{ov}^2 \Rightarrow$ unit: $\Omega \frac{A}{V^2} \cdot V^2 = V$

units check!

Solve (3).

$$V_{ov} = \frac{-1 \pm \sqrt{1+4R_D K_n V_{DD}}}{2R_D K_n}$$

Check the units:

$R_D K_n V_{DD} \Rightarrow$ unit: $\Omega \frac{A}{V^2} V \Rightarrow$ unitless

$R_D K_n \Rightarrow$ unit: $\Omega \frac{A}{V^2} \Rightarrow \frac{1}{V} = V^{-1}$

Units again check!

Note that

$$\sqrt{1+4R_D K_n V_{DD}} > 1$$

Then definitely

$$-1 - \sqrt{1+4R_D K_n V_{DD}} < 0$$

$$-1 + \sqrt{1+4R_D K_n V_{DD}} > 0$$

$$V_{ov} = \frac{-1 + \sqrt{1+4R_D K_n V_{DD}}}{2R_D K_n}$$ in units of Volts

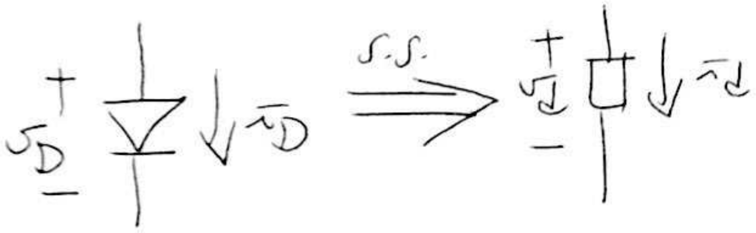
is the physically valid soln.

$$V_{GS} = V_{ov} + V_{th} = V_{th} + \frac{-1 + \sqrt{1+4R_D K_n V_{DD}}}{2R_D K_n}$$

3

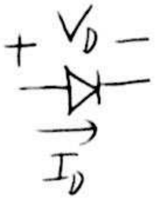
2.2 Small-Signal Equivalent Circuits

Small-Signal Model Derivation for the Diode around a D.C. Operating Point



<u>Worked on this on</u>: Oct 19, 2015/Mon

Small Signal Model Derivation
For a Diode



Diode model

$$I_D = I_S\left(\exp\left(\frac{V_D}{nV_T}\right) - 1\right) \qquad (*)$$

I_S : saturation current

n : emission coef.

$$V_T = \frac{kT}{q}$$

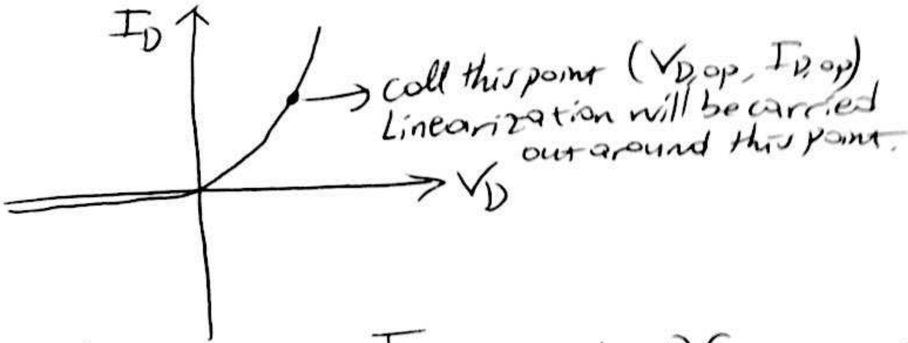
T → Temperature in Kelvins

k → Boltzman constant

q → charge of a single charge carrier

V_T → Thermal voltage

I_D vs V_D plot (DC transfer characteristics)



Around $(V_{D,op}, I_{D,op})$ [called the DC operating point, Q-point, or simply DC O.P.]

$(*)$ can be approximated as

$$I_D \triangleq I_S \exp\left(\frac{V_D}{nV_T}\right)$$

Some Definitions

V_D : DC voltage across the diode

v_d : small perturbation on the DC voltage

$v_D = V_D + v_d \Rightarrow$ sum of the two

I_D : DC current through the diode

i_d : small perturbation on the DC current

$i_D = I_D + i_d$

Note that

$$I_D = I_S \exp\left(\frac{V_D}{nV_T}\right) \text{ implies } \underbrace{i_D = I_S \exp\left(\frac{v_D}{nV_T}\right)}_{(**)}$$

Now Taylor-expand both sides of the equation in $(**)$.

From (**):

$$(***) \quad \underbrace{I_{D,op} + i_d}_{\text{The expansion is around } I_{D,op}} = \underbrace{\sum_{k=0}^{+\infty} \frac{1}{k!} \left.\frac{\partial^k f(x)}{\partial x^k}\right|_{x=V_{D,op}} \cdot \left(v_D - V_{D,op}\right)^k}_{\text{The expansion is around } V_{D,op}}$$

The whole expansion above is around the DC operating point $\left(V_{D,op}, I_{D,op}\right)$.

First note in (***) that

$$v_D - V_{D,op} = \underbrace{v_d}_{\text{small signal perturbation}}$$

Second note that

$$f(x) = I_S \exp\left(\frac{x}{nV_T}\right)$$

Third express the sum in the RHS of (***) as two terms as in:

$$(****) \quad I_{D,op} + i_d = \sum_{k=0}^{1} \frac{1}{k!} \left.\frac{\partial^k f(x)}{\partial x^k}\right|_{x=V_{D,op}} v_d^k$$

$$+ \sum_{k=2}^{+\infty} \frac{1}{k!} \left.\frac{\partial^k f(x)}{\partial x^k}\right|_{x=V_{D,op}} v_d^k$$

 3

In (****), the terms with $\sum_{k=2}^{+\infty}$

correspond to higher order terms
and can be omitted if v_d is small, which it is.

Then from (****) we have:

$$(\ast\ast\ast\ast\ast) \quad I_{D,op} + i_d \cong \frac{1}{0!} \left.\frac{\partial^0 f(x)}{\partial x^0}\right|_{x=V_{D,op}} v_d^0 + \frac{1}{1!} \left.\frac{\partial^1 f(x)}{\partial x^1}\right|_{x=V_{D,op}} v_d^1$$

Above $v_d^0 = 1$ $\quad v_d^1 = v_d$

$0! = 1$ $\quad 1! = 1$

$\frac{\partial^0 f(x)}{\partial x^0} = f(x) = I_S \exp\left(\frac{x}{nV_T}\right)$ $\quad \frac{\partial f(x)}{\partial x} = \frac{I_S}{nV_T} \exp\left(\frac{x}{nV_T}\right)$

Then

$$(\ast\ast\ast\ast\ast\ast) \quad I_{D,op} + i_d = \left. I_S \exp\left(\frac{x}{nV_T}\right)\right|_{x=V_{D,op}} + \left.\frac{I_S}{nV_T} \exp\left(\frac{x}{nV_T}\right)\right|_{x=V_{D,op}} \cdot v_d$$

$I_S \exp\left(\frac{V_{D,op}}{nV_T}\right) = I_{D,op}$

$I_{D,op}$

We have

$$\cancel{I_{D,op}} + i_d = \cancel{I_{D,op}} + \frac{I_{D,op}}{nV_T} \cdot v_d$$

 4

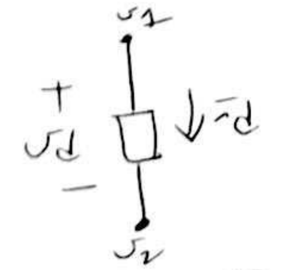
Therefore the fundamental relation for a diode in small-signal around an operating point $(V_{D,op}, I_{D,op})$ is:

$$i_d = \frac{I_{D,op}}{nV_T} \cdot v_d$$

manipulate this to obtain

$$(\ast\ast\ast\ast\ast\ast\ast) \qquad \frac{v_d}{i_d} = \frac{nV_T}{I_{D,op}}$$

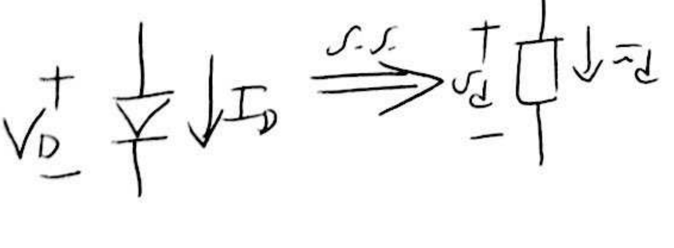
This equation defines the small-signal model for a diode:



$v_1 - v_2 = v_d$

The current for this passive element flows in between the nodes (1) and (2) whose difference defines v_d. v_d and i_d are related by a constant factor (depending on $I_{D,op}$). See $(\ast\ast\ast\ast\ast\ast\ast)$.

Then the small signal model is indeed a resistance with value $r_d = \frac{v_d}{i_d} = \frac{nV_T}{I_{D,op}}$.



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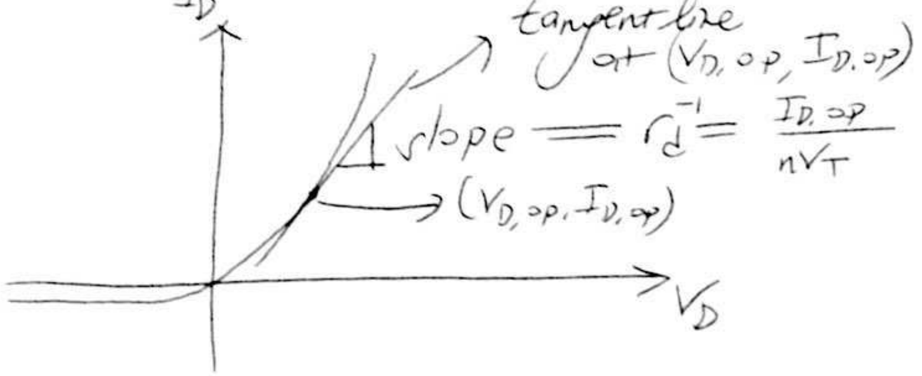
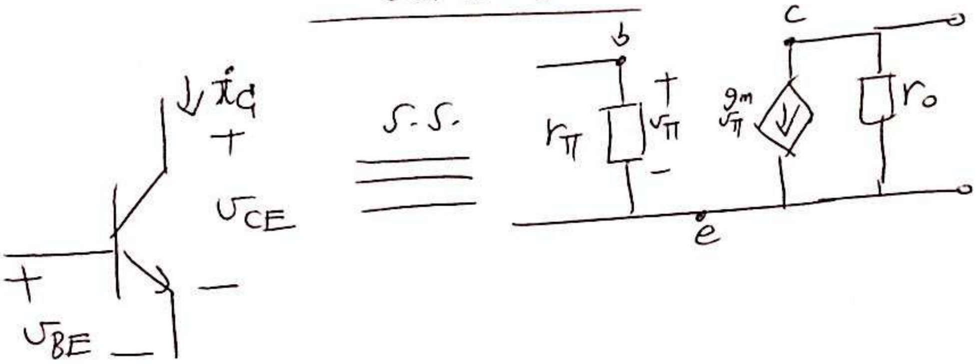
How does this resistance relate to the I_D vs V_D plot (the DC transfer characteristic)?



For higher values of $I_{D,op}$, I_D vs V_D plot is steeper, r_d^{-1} is higher, r_d is lower. For lower values of $I_{D,op}$, vice versa.

Most of the applications with diodes utilize the large signal model of diodes, for these are nonlinear circuits. For the Nonlinear circuit applications of diodes, the above derivation is most of the useless, since DC op changes with the applied inputs (with a large enough range).

BJT small-signal equivalent model derivation



Worked on this on: Nov 5, 2014/ Wed

BJT small-signal model derivation

Fundamental equations: $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$

eqn (1)

collector current

saturation current

Thermal voltage

base-emitter voltage

eqn (2) $I_C = \beta I_B$

collector current

current gain

base current

Defns:

$$v_{BE} = V_{BE} + v_{be} \qquad (3)$$

DC value

small signal value

$$i_C = I_C + i_c \qquad (4)$$

DC value

small signal value

$$i_B = I_B + i_b \qquad (5)$$

DC value

small signal value

1

Apply (3) and (4) on (1) $\Rightarrow$ perturb the variables of (1) and obtain a relation between the small signal parameters

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$$

$$I_C + i_c = I_S \exp\left(\frac{V_{BE} + v_{be}}{V_T}\right)$$

$$\underset{\text{Taylor expansions}}{\overset{\text{apply}}{=}} \left[I_S \exp\left(\frac{v}{V_T}\right)\right]\Bigg|_{v=V_{BE}}$$

$$+ \frac{d}{dv}\left[I_S \exp\left(\frac{v}{V_T}\right)\right]\Bigg|_{v=V_{BE}} \cdot \left(v_{BE} - V_{BE}\right)$$

+ Higher Order Terms (H.O.T.)

Note that $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$

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2

Now we have (cancelling equal terms and H.O.T.)

$$i_c \cong \underbrace{\frac{I_s}{V_T} \exp\left(\frac{V_{BE}}{V_T}\right)}_{\frac{I_C}{V_T}} \cdot v_{be}$$

$I_C \Rightarrow$ DC value

call the transconductance $\quad g_m \triangleq \frac{i_c}{v_{be}} = \frac{I_C}{V_T} \quad (6)$

Apply (4) and (5) on (2).

$$i_C = \beta\, i_B$$

$$I_C + i_c = \beta\left(I_B + i_b\right)$$

(Taylor expansion is not necessary, this is a linear relation)

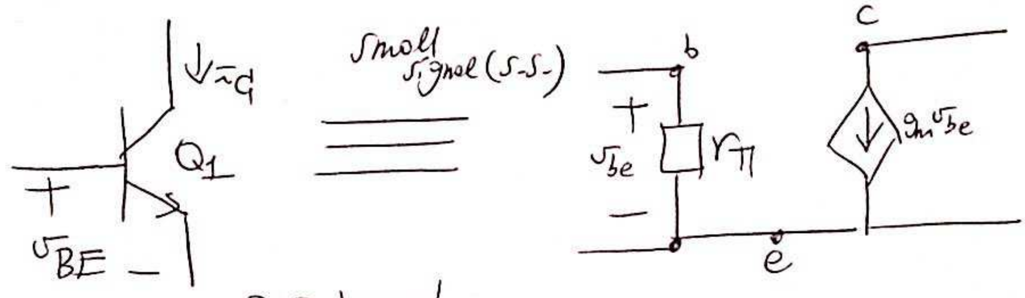
Cancel equal terms $\quad i_c = \beta\, i_b \quad (7)$

call the input impedance $\quad r_\pi = \frac{v_{be}}{i_b} = \frac{\frac{i_c}{g_m} \Rightarrow \text{From } (6)}{\frac{i_c}{\beta} \Rightarrow \text{from } (7)}$

$$= \frac{\beta}{g_m} \quad (8)$$

3

The small-signal equivalent model thus far is



Transistor Q_1 is DC-biased in forward active mode.

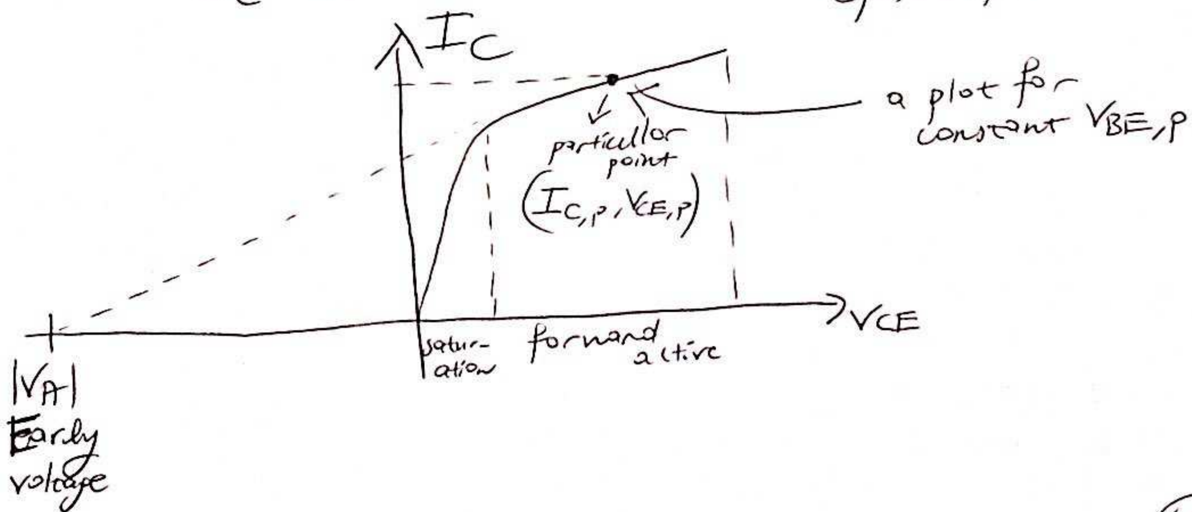
We will next account for the output impedance.

r_o appears due to base-width modulation (otherwise called the Early effect).

Due to this effect, the collector current I_C continues to increase with increasing V_{CE}.

r_o is the parameter that expresses this effect in the small signal model.

$I_C - V_{CE}$ (DC transfer characteristics) plot of the npn BJT



4

The slope in the $I_C - V_{CE}$ plot (forward active mode)

is given by

$$\text{slope} = \frac{I_{C,p}}{|V_A| + V_{CE,p}}$$

The slope is almost constant in this regime (forward-active).

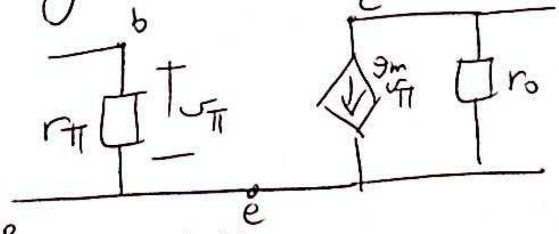
Typically $|V_A| \gg V_{CE,p}$.

Therefore

$$\text{slope} \simeq \frac{I_{C,p}}{|V_A|}$$ at a particular DC oper-point

The s.s. output impedance $$r_o = \frac{1}{\text{slope}} = \frac{|V_A|}{I_{C,p}}$$ at the DC o.p. $(V_{BE,p}, V_{CE,p}, I_{C,p})$

The small signal equiv model of BJT is then



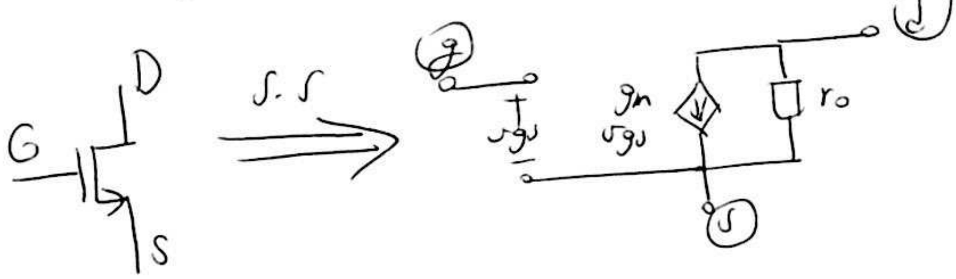
where $v_\pi = v_{be}$

$$r_\pi = \frac{\beta}{g_m} \quad r_o = \frac{|V_A|}{I_C}$$

$$g_m = \frac{I_C}{V_T}$$

(5)

Derivation of the small signal model for MOS



Worked on this on:
April 9, 2015 / Thursday

Small-signal model derivation for NMOS

The large-signal model can be approximated as in

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2$$

Some definitions:

I_D : DC value of the drain current

i_d : small perturbation on the drain current

$i_D = I_D + i_d$

V_{GS} : DC value of the gate-source voltage

v_{gs} : small perturbation on the gate-source voltage

$v_{GS} = V_{GS} + v_{gs}$

Define function f:

$$I_D = f(V_{GS})$$

see here

It is also true that

$$i_D = f(v_{GS})$$

1

Then we have

$$I_D + i_d = f(V_{GS} + v_{gs})$$

Expand the RHS around V_{GS} (the DC value)

$$I_D + i_d = \sum_{k=0}^{+\infty} \frac{1}{k!} \left.\frac{\partial^k f(x)}{\partial x^k}\right|_{x=V_{GS}} \cdot (v_{GS} - V_{GS})^k$$

We will keep the first two terms on the RHS and omit the rest (the higher order terms or H.O.T.)

$$I_D + i_d \cong \sum_{k=0}^{1} \frac{1}{k!} \left.\frac{\partial^k f(x)}{\partial x^k}\right|_{x=V_{GS}} \cdot (v_{GS} - V_{GS})^k$$

$$= \frac{1}{0!} \left. f(x)\right|_{x=V_{GS}} \cdot (v_{GS} - V_{GS})^0$$

$$+ \frac{1}{1!} \left.\frac{\partial f(x)}{\partial x}\right|_{x=V_{GS}} \cdot \underbrace{(v_{GS} - V_{GS})}_{v_{gs}}^1$$ (see the defns on pg 1)

$$= f(V_{GS})$$

$$+ \left.\frac{\partial f(x)}{\partial x}\right|_{x=V_{GS}} \cdot v_{gs}$$

In all we have

$$\cancel{I_D} + i_d = \cancel{f(V_{GS})} + \left.\frac{\partial f(x)}{\partial x}\right|_{x=V_{GS}} \cdot v_{gs}$$

$\longrightarrow$ see pg 1 for how these cancel

We now have a relation between i_d and v_{gs} (the small-signal variables)

$$i_d = \left.\frac{\partial f(x)}{\partial x}\right|_{x=V_{GS}} \cdot v_{gs}$$

In this equation

$$\left.\frac{\partial f(x)}{\partial x}\right|_{x=V_{GS}} = \left.\frac{\partial}{\partial x}\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(x-V_t)^2\right]\right|_{x=V_{GS}}$$

$$= \left.\left[\mu_n C_{ox}\frac{W}{L}(x-V_t)\right]\right|_{x=V_{GS}}$$

the 1st formula for the transconductance of MOS $\quad \boxed{g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)}$

$$i_d = \underbrace{g_m}_{\substack{\text{this is the}\\ \text{proportionality}\\ \text{constant between}\\ i_d \text{ and } v_{gs}}} v_{gs}$$

Note that

$$\left[2\mu_n C_{ox}\frac{W}{L}\right]\overbrace{\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2\right]}^{I_D} =$$

$$\left[\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)\right]^2 = g_m^2$$

3

Therefore the <u>second</u> formula for the transconductance of MOS is

$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

Also note that

$$\frac{2 \overbrace{\left[\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2\right]}^{I_D}}{(V_{GS} - V_t)} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)$$

therefore the <u>third</u> formula for the transconductance of MOS is

$$g_m = \frac{2 I_D}{V_{GS} - V_t}$$

Now compare the g_m's of BJT and MOS (using the 3rd formula for the g_m of MOS)

	BJT	MOS
g_m	$\frac{I_C}{V_T}$	$\frac{I_D}{(V_{GS} - V_t)/2}$

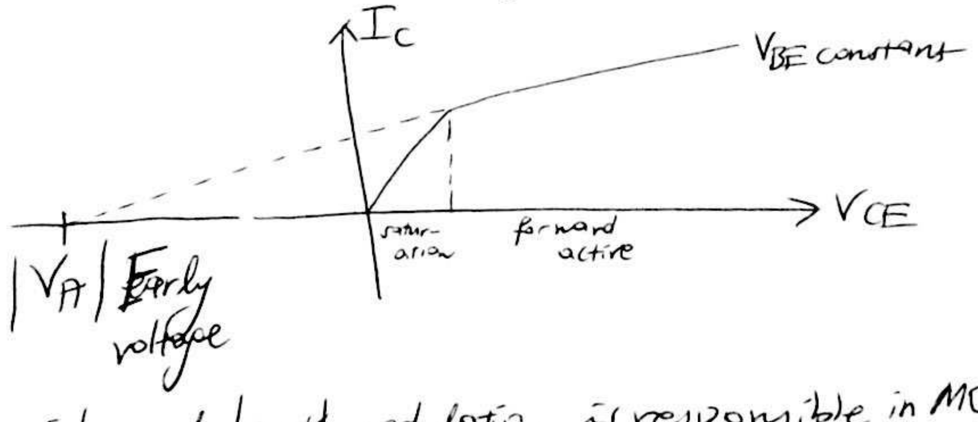
4

V_T is 25-30 mV.

$\frac{V_{GS}-V_t}{2}$ is on the order of 100's of milli Volts.

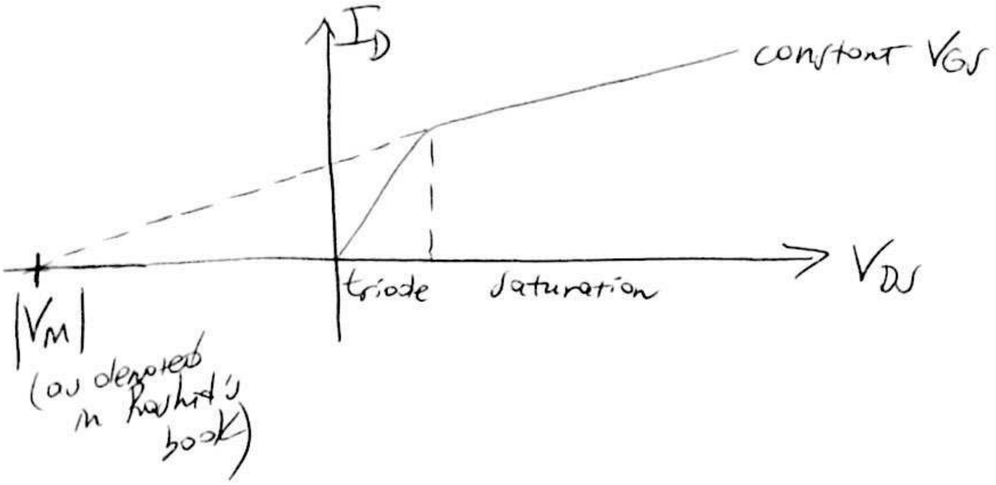
Therefore the transconductance/current ratio of BJTs is higher than that of MOS.
BJT is said to be stronger than MOS for this reason.

Recall that base-width modulation is responsible in BJTs for the slight increase of I_C with increasing V_{CE} in the forward active regime. Another name for this phenomenon is the Early effect.

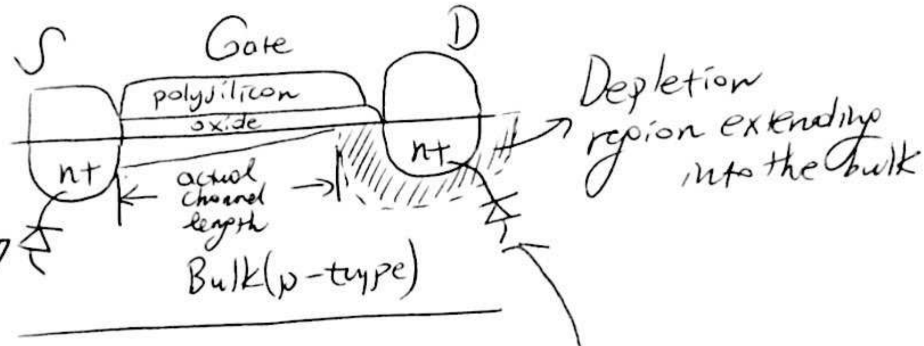


Channel length modulation is responsible in MOS for the slight increase of I_D with increasing V_{DS} in the saturation regime.

5



The structure of the NMOS,



Recall that we must have

$$V_{SB} \geq 0$$

for reversed biased diodes

$V_D > V_S$ makes the junction we call the drain.

This condition ensures

$$V_{DB} \geq 0$$

since we have this.

The B–D pn-junction is more reverse-biased than the B–S pn-junction. The depletion region into the bulk of B–D junction widens into the channel, shortening the effective channel length.

The equation for I_D is

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$$

In this eqn, shorter L means larger current.

Therefore, by this effect (channel length modulation) we have the I_D current slightly increasing with V_{DS}.

The I_D equation can be remodeled as

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 [1 + \lambda V_{DS}]$$

where λ (V^{-1}) models the rise in I_D with V_{DS}.

Now I_D is a function of both V_{GS} and V_{DS}

$$I_D = f(V_{GS}, V_{DS})$$

7

It is also true that

$$i_D = f(v_{GS}, v_{DS})$$

$$i_D \to I_D + i_d \qquad v_{GS} \to V_{GS} + v_{gs} \qquad v_{DS} \to V_{DS} + v_{ds}$$

The first few terms of the Taylor expansion for the RHS reads:

$$I_D + i_d \doteq f(V_{GS}, V_{DS}) + \left.\frac{\partial f(x,y)}{\partial x}\right|_{\substack{x=V_{GS}\\ y=V_{DS}}} \cdot \underbrace{(v_{GS} - V_{GS})}_{v_{gs}} + \left.\frac{\partial f(x,y)}{\partial y}\right|_{\substack{x=V_{GS}\\ y=V_{DS}}} \cdot \underbrace{(v_{DS} - V_{DS})}_{v_{ds}}$$

$$\left.\frac{\partial f(x,y)}{\partial x}\right|_{\substack{x=V_{GS}\\ y=V_{DS}}} \doteq g_m =$$ the three formulas as found before on pp 3–4

8

$$\left.\frac{\partial f(x,y)}{\partial y}\right|_{\substack{x=V_{GS}\\ y=V_{DS}}} = \frac{\partial}{\partial y}\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(x-V_t)^2\left[1+\lambda y\right]\right]\Bigg|_{\substack{x=V_{GS}\\ y=V_{DS}}}$$

$$= \lambda \underbrace{\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2\right]}_{\approx I_D}$$

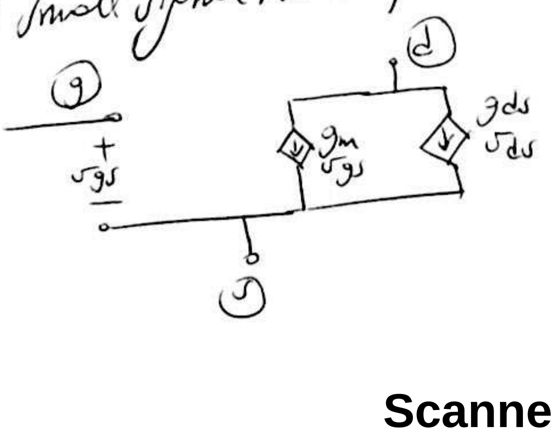
$$\simeq \lambda I_D$$

In all

$$\cancel{I_D} + i_d = \cancel{f(V_{GS}, V_{DS})} + g_m v_{gs} + \underbrace{\lambda I_D}_{\text{call this } g_{ds}} v_{ds}$$

these two cancel since they are the same

In the small signal model of MOS



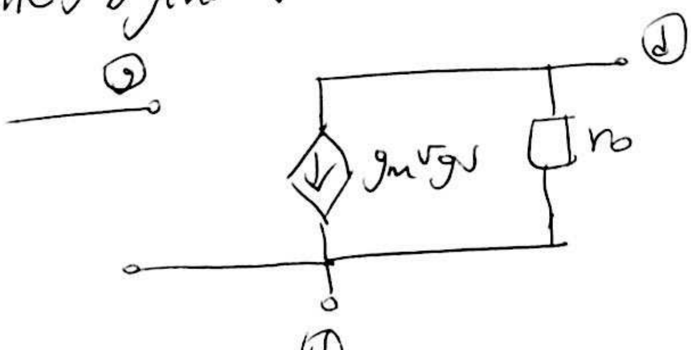
The element 

has its current controlled by the voltage of the nodes it is connected to. This can be modelled by a linear resistor since it is linear.

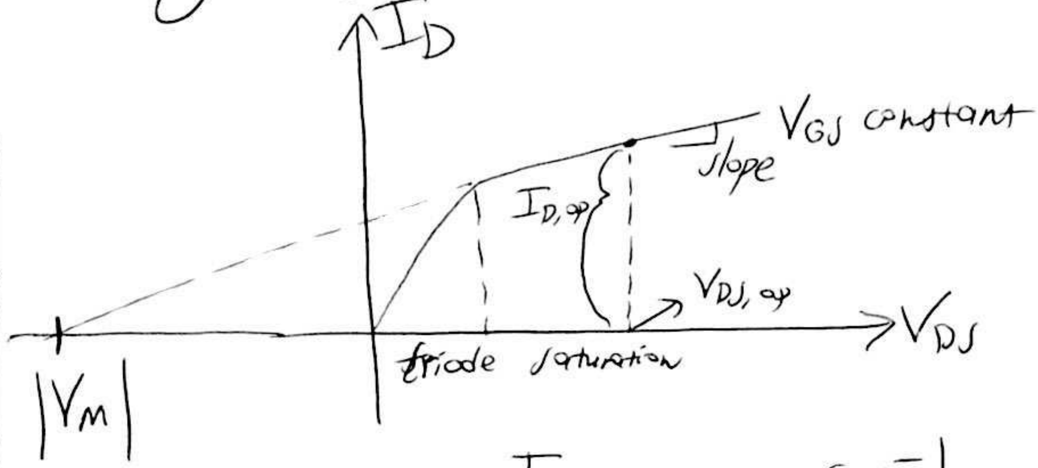
The resistance value r_o is

$$r_o = \frac{1}{g_{ds}} = \frac{1}{\underbrace{\lambda}_{(1/V)} \underbrace{I_D}_{(A)}} \Rightarrow \left(\frac{V}{A}\right) \text{ in units}$$

Therefore the simplified small signal model of MOS is given as:



In some books λ is not given, but the channel length modulation is modelled with a voltage that corresponds to the Early voltage in BJTs.



$$\text{slope} = \frac{I_{D,op}}{|V_M| + V_{DS,op}} = (r_o)^{-1}$$

$$r_o = \frac{|V_M| + V_{DS,op}}{I_{D,op}}$$

Since usually $|V_M| \gg V_{DS,op}$

$$r_o \cong \frac{|V_M|}{I_{D,op}}$$

11

3. Diode Applications

3.1 Clamping Circuits

Diode Applications

3.52 Figure P3.52 shows a clamping circuit and its input voltage waveform.

(a) Carefully sketch the output waveform if the diode is ideal.

(b) Sketch the output waveform if the diode has an offset of 0.6 V.

(c) Repeat part (a) after reversing the diode orientation.

(d) Repeat part (b) after reversing the diode orientation.

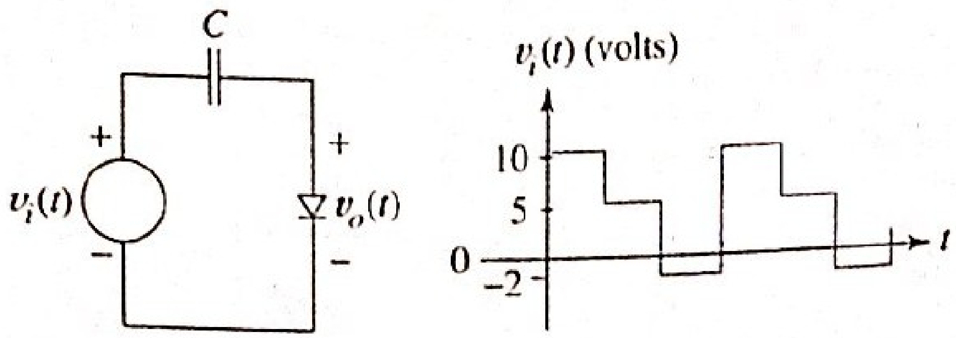


Figure P3.52

Malik 3.52

a)

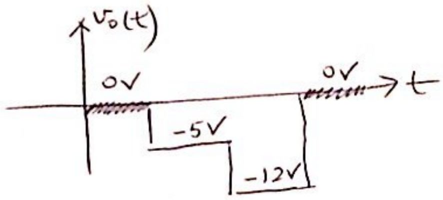
capacitor voltage charges to

$V_{\hat{i}}$, peak (positive peak)

$= 10V$

After reaching the first positive peak in the periodic waveform $\Rightarrow v_o(t) = v_{\hat{i}}(t) - v_{\hat{i}}$, peak

$= v_{\hat{i}}(t) - 10V$

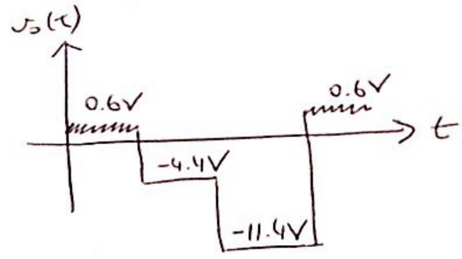


b)

The cap charges to $v_{\hat{i}}$, peak $- V_{D,ON}$

$\underbrace{v_{\hat{i}}, \text{peak}}_{10V} - 0.6V = 9.4V$

After the init. transient $\Rightarrow v_o(t) = v_{\hat{i}}(t) - 9.4V$



c)

In the initial case $\Rightarrow$ cap has no charge on it

$\rightarrow$ the diode turns ON when $v_{\hat{i}}(t) = 0$

$\rightarrow$ then the cap voltage is charged to $-2V$

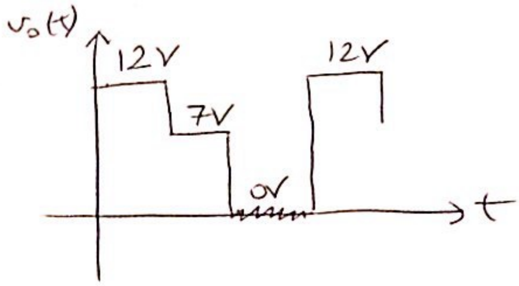
$\rightarrow$ the diode turns off and never turns ON again

$\Rightarrow$ after the init transient $v_o(t) = v_{\hat{i}}(t) + 2V$

1

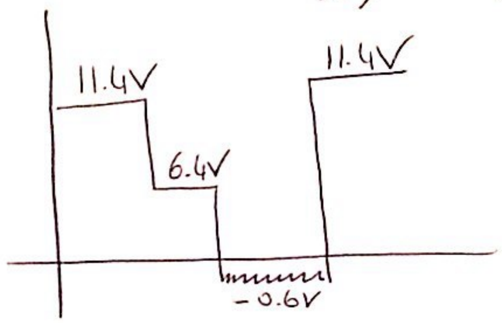
Molik 3.52
Cont.

c)
Cont-



d)

Similar to (c) but the diode turns ON
when $v_i(t) = -0.6V$
the cap charges to $-1.4V$
after the init. transient
$v_o(t) = v_i(t) + 1.4V$



(62)

4. Amplifiers

4.1 Types

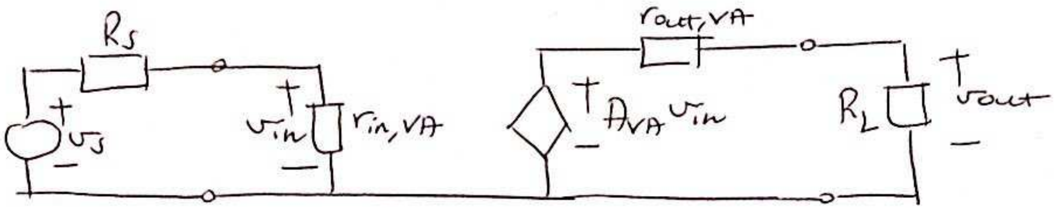
Properties of Voltage Amplifiers (VA)

⇒ Input/output impedances (for an ideal VA)

⇒ Load/source impedances (for an ideal VA)

Worked on this on: Oct 30, 2014/Th

Voltage amplifiers – properties



Realized voltage gain

$$A_{VA,realized} = \frac{r_{in,VA}}{r_{in,VA} + R_S} A_{VA} \frac{R_L}{R_L + r_{out,VA}}$$

if $\left.\begin{array}{l} r_{out,VA} \to 0 \\ r_{in,VA} \to +\infty \end{array}\right\} \Longrightarrow A_{VA,realized} \to A_{VA}$

$\Downarrow$

These are the properties of an ideal voltage amplifier with finite gain (A_{VA}).

if $\left.\begin{array}{l} R_L \to +\infty \\ R_S \to 0 \end{array}\right\} \Longrightarrow A_{VA,realized} \to A_{VA}$

These are the ideal load and ideal source for a voltage ampli.

In the case that the VA is loaded and sourced by these quantities, $r_{out,VA}$ and $r_{in,VA}$ will have no effect. The VA will act as if it is ideal with finite gain (A_{VA}).

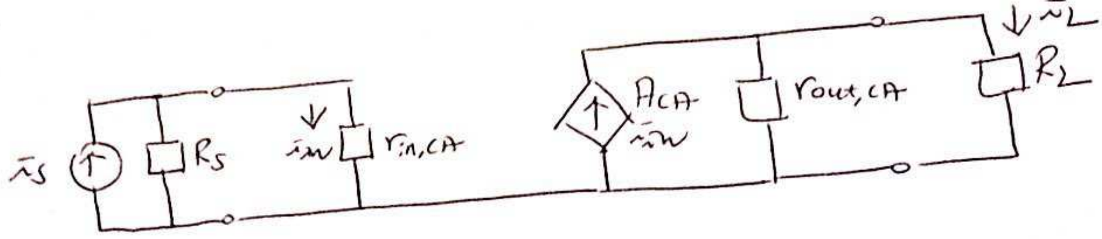
Properties of Current Amplifiers

⇒ Input/Output impedances for an ideal CA

⇒ Ideal source and load impedances for any CA

Worked on this on: Oct 30, 2014/Th

Current amplifiers – properties



Realized current gain

$$A_{CA,realized} = \frac{R_S}{R_S + r_{in,CA}} A_{CA} \frac{r_{out,CA}}{r_{out,CA} + R_L}$$

$$\text{if} \quad \underbrace{\left.\begin{matrix} r_{in,CA} \to 0 \\ r_{out,CA} \to +\infty \end{matrix}\right\}}_{\Downarrow} \Longrightarrow A_{CA,realized} = A_{CA}$$

for any source and load

properties of an ideal current amplifier with finite gain (A_{CA}).

$$\text{if} \quad \underbrace{\left.\begin{matrix} R_S \to +\infty \\ R_L \to 0 \end{matrix}\right\}}_{\Downarrow} \Longrightarrow A_{CA,realized} = A_{CA}$$

ideal source and load for current amplifiers, respectively.

If a CA is sourced and loaded by these impedances, the CA will act as an ideal CA with finite gain (A_{CA}).

Properties of the Transconductance Amplifier

⇒ properties of an ideal ampli

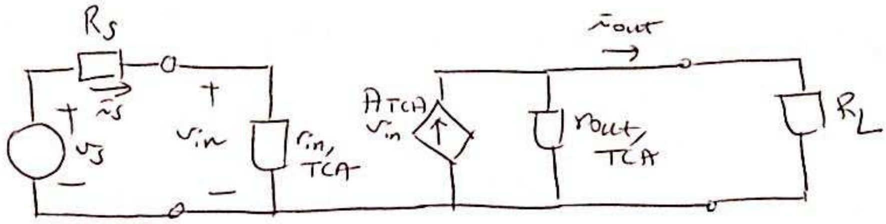
⇒ ideal input/output impedances

⇒ ideal source and load

⇒ how to compute input/output impedances and gain

Worked on this on: Oct 19, 2014/Sun

Properties of the transconductance ampli.



$$\frac{i_{out}}{v_s} = \frac{r_{in,TCA}}{r_{in,TCA}+R_s} \, A_{TCA} \, \frac{r_{out,TCA}}{r_{out,TCA}+R_L} = A_{TCA,realized}$$

if $r_{in,TCA} \to +\infty$, $r_{out,TCA} \to +\infty$ $\Rightarrow$ then the transcond ampli is ideal with finite gain, and $A_{TCA,realized} = A_{TCA}$

if $R_s = 0$, $R_L = 0$ $\Rightarrow$ $A_{TCA,realized} = A_{TCA}$

Computing A_{TCA}

$\Rightarrow$ connect v_s without R_s so that $v_s = v_{in}$

$\Rightarrow$ set R_L to short cir.

$\Rightarrow$ Compute $\frac{i_{out}}{v_s} = A_{TCA}$

Computing $r_{in,TCA}$

$\Rightarrow$ connect v_s without R_s so that $v_s = v_{in}$

$\Rightarrow$ set R_L to short cir

$\Rightarrow$ compute $\frac{v_s}{i_s} = r_{in,TCA}$

 1

Computing $r_{out,TCA}$

$\Rightarrow$ set $v_s = 0$ and connect no R_s

$\Rightarrow$ connect a v_{test} instead of R_L
(i_{test} flows out of the positive terminal of v_{test})

$\Rightarrow$ compute $\dfrac{v_{test}}{i_{test}} = r_{out,TCA}$

2

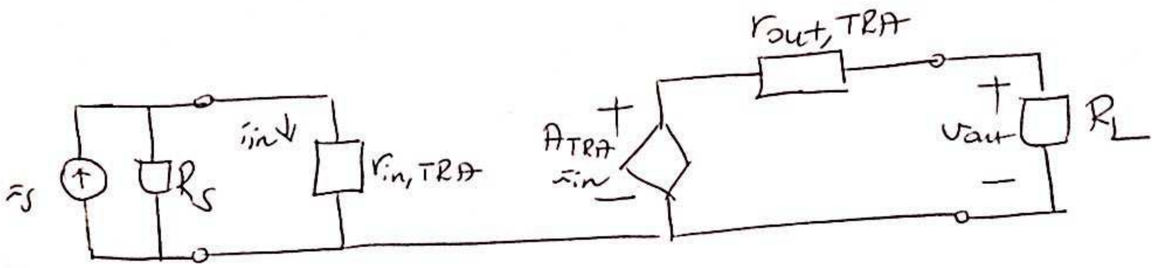
Properties of Transresistance Amplifiers

$\Rightarrow$ Input/output impedances for the ideal TRA

$\Rightarrow$ Ideal source/load impedances for any TRA

Worked on this on: Oct 30, 2014/ Th

Transresistance amplifier – properties



$A_{TRA,realized} \Rightarrow$ realized gain

$$A_{TRA,realized} = \frac{R_S}{R_S + r_{in,TRA}} \, A_{TRA} \, \frac{R_L}{R_L + r_{out,TRA}}$$

if $\left.\begin{matrix} r_{in,TRA} \to 0 \\ r_{out,TRA} \to 0 \end{matrix}\right\} \Rightarrow A_{TRA,realized} = A_{TRA}$

$\Downarrow$

ideal input/output impedances for the TRA with finite gain (A_{TRA})

if $\left.\begin{matrix} R_L \to +\infty \\ R_S \to +\infty \end{matrix}\right\} \Rightarrow A_{TRA,realized} = A_{TRA}$

ideal load/source impedances (respectively) for the TRA

if the TRA is loaded/sourced by these impedances, the TRA will act as an ideal TRA with finite gain (A_{TRA}).

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4.2 Modeling Examples

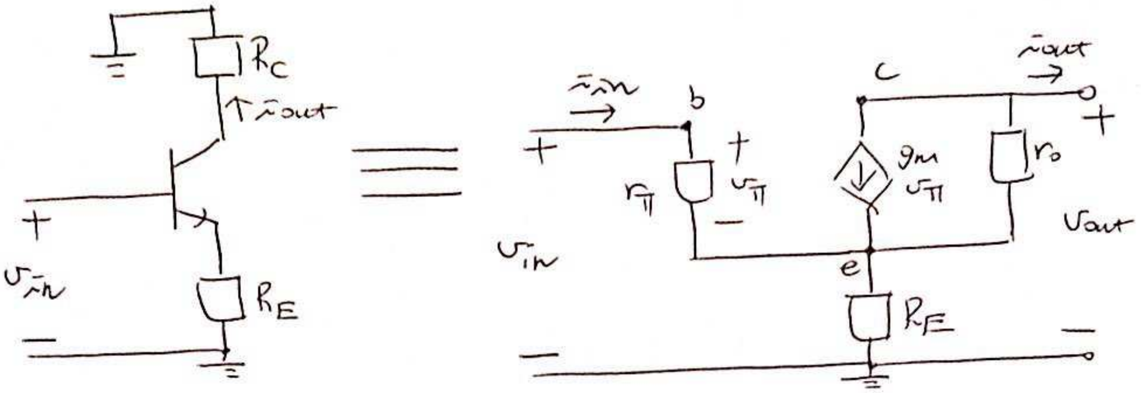
Modeling the Common Emitter Amplifier as a Transconductance Amplifier

$\Rightarrow$ A_{TCA}, $r_{in,TCA}$, $r_{out,TCA}$

exact and approximate results

Worked on this on: Oct 31, 2014/ F

Analysis of the common emitter amplifier as a transconductance amplifier



⟹ Computation of

$$A_{TCA} = \left.\frac{i_{out}}{v_{in}}\right|_{v_{out}=0}$$

$$v_e = v_{in} - v_\pi$$

KCL at (e)

$$\frac{\cancel{v_{in}} - v_\pi - \cancel{v_{in}}}{r_\pi} + \frac{v_{in}-v_\pi}{R_E} - g_m v_\pi + \underbrace{\frac{v_{in}-v_\pi}{r_o}}_{\text{ignore this term assuming } r_o \text{ is big}} = 0 \qquad (1)$$

$$-g_m v_\pi = i_{out} \quad (\text{assuming } r_o \text{ is big}) \qquad (2)$$

From (1) we have

$$\frac{v_{in}}{R_E} = \frac{v_\pi}{r_\pi /\!/ R_E /\!/ \frac{1}{g_m}} \overset{\text{From (2)}}{=} \frac{-\frac{i_{out}}{g_m}}{r_\pi /\!/ R_E /\!/ \frac{1}{g_m}} \qquad (3)$$

From (3), we have

$$\frac{v_{out}}{v_{in}} = -\frac{g_m}{R_E}\left[r_\pi // \underbrace{R_E // \frac{1}{g_m}}_{\dfrac{R_E \frac{1}{g_m}}{R_E + \frac{1}{g_m}}} \right]$$

$$= -\frac{\cancel{g_m}}{\cancel{R_E}}\left[\frac{r_\pi \dfrac{\cancel{R_E/g_m}}{R_E + 1/g_m}}{r_\pi + \dfrac{R_E/g_m}{R_E + 1/g_m}} \right]$$

$$= -\left[\frac{r_\pi \dfrac{g_m}{1+g_m R_E}}{r_\pi + \dfrac{R_E}{1+g_m R_E}} \right]$$

$$A_{TCA} = -\left[\frac{r_\pi g_m}{R_E + (1+g_m R_E) r_\pi} \right] \quad (4)$$ exact answer (for $r_o \to +\infty$)

$$A_{TCA} = -\left[\frac{\beta}{r_\pi + (1+\beta) R_E} \right] \quad (5)$$ another exact answer (for $r_o \to +\infty$)

From (4) we have

$$A_{TCA} \cong -\frac{g_m}{1+g_m R_E} \quad (6)$$ approximate answer

 2

$\Longrightarrow$ computation of $\left.\frac{v_{in}}{i_{in}}\right|_{v_{out}=0} = r_{in,TCA}$

Note that

$$i_{in} = \frac{v_\pi}{r_\pi} \qquad \frac{v_{in}}{i_{in}} = \frac{v_{in}}{v_\pi} r_\pi = -\frac{v_{in}}{i_{out}} \underbrace{g_m r_\pi}_{\beta} \quad (7)$$

From (2) $\Rightarrow v_\pi = -\frac{i_{out}}{g_m}$

From (7) and (5)

$$r_{in,TCA} = \frac{v_{in}}{i_{in}} = -\left[-\frac{r_\pi + (1+\beta)R_E}{\cancel{\beta}}\right]\cancel{\beta} = r_\pi + (1+\beta)R_E \quad (8)$$

exact answer (for $r_o \to +\infty$)

From (7) and (6)

$$r_{in,TCA} = \frac{v_{in}}{i_{in}} \cong -\left[-\frac{1+g_m R_E}{\cancel{g_m}}\right]\cancel{g_m} r_\pi$$

$$= r_\pi\left[1 + g_m R_E\right] \quad (9)$$

approximate answer (for $r_o \to +\infty$)

$\Rightarrow$ Computation of

$$r_{out,TCA} = -\left.\frac{v_{out}}{i_{out}}\right|_{v_{in}=0}$$

$$v_e = -v_\pi$$

$$(KVL) \Rightarrow \quad v_{out} = \left(-i_{out} - g_m v_\pi\right) r_o - v_\pi \qquad (10)$$

$$= -i_{out} r_o - v_\pi\left(1 + g_m r_o\right)$$

and we have

$$-i_{out} \frac{R_E}{R_E + r_\pi} \cdot r_\pi = -v_\pi \qquad (11)$$

$$i_{out}\left(r_\pi // R_E\right) = v_\pi$$

From (10) and (11)

$$v_{out} = -i_{out}\left[r_o + \left(r_\pi // R_E\right)\left(1 + g_m r_o\right)\right]$$

$$r_{out,TCA} = -\left.\frac{v_{out}}{i_{out}}\right|_{v_{in}=0} = r_o + \left(r_\pi // R_E\right)\left(1 + g_m r_o\right) \qquad (12)$$

exact answer

if $r_\pi \gg R_E$

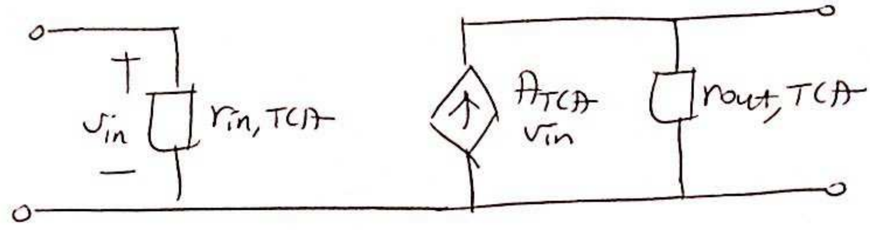
$$r_{out,TCA} \cong r_o + R_E\left(1 + g_m r_o\right)$$

$$= R_E + r_o\left(1 + g_m R_E\right) \cong r_o\left(1 + g_m R_E\right) \qquad (13)$$

approx answer

4

Transconductance amplifier model



$A_{TCA} \Rightarrow$ see (4), (5), (6)

$r_{in,TCA} \Rightarrow$ see (8), (9)

$r_{out,TCA} \Rightarrow$ see (12), (13)

5

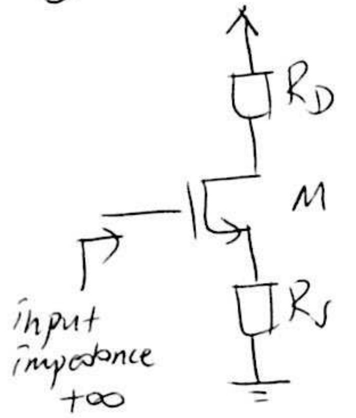
4.3 Design Considerations

How to construct an amplifier with large input impedance and big voltage gain?

CS – CE cascaded stages

Worked on this on:
April 9, 2015 / Thursday

Consider using a CS (common source) stage at the input



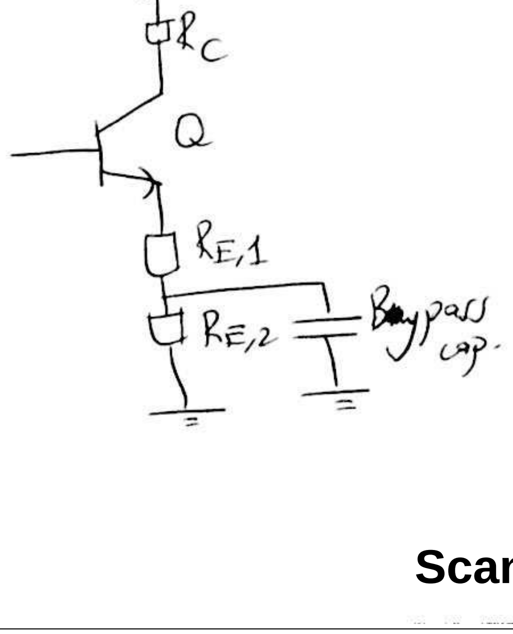
transconductance is given by

$$-\frac{g_{m,M}}{1+g_{m,M}R_S}$$

Voltage gain is given by

$$-\frac{g_{m,M}}{1+g_{m,M}R_S}\left[R_D \,//\, \left[r_{o,M}(1+g_{m,M}R_S)\right]\right]$$

Cascade this stage with a CE (common emitter)



transconductance given by

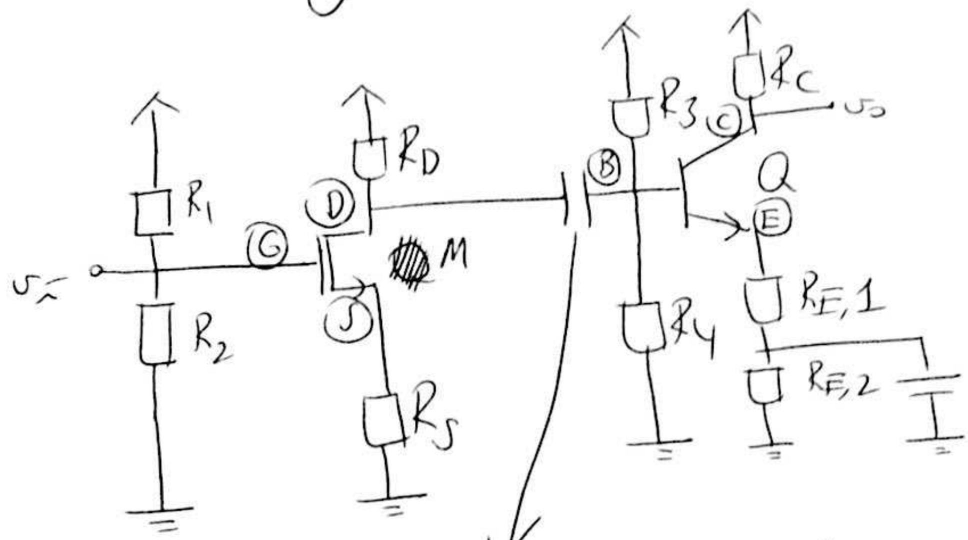
$$-\frac{g_{m,Q}}{1+g_{m,Q}R_{E,1}}$$

voltage gain given by

$$-\frac{g_{m,Q}}{1+g_{m,Q}R_{E,1}} \times \left[R_C \,//\, \left[r_{o,Q}(1+g_{m,Q}R_{E,1})\right]\right]$$

1

Two two stages cascaded with complete bias detail:



The DC bias voltage of D and B may be different, then we need this ~~bypass~~ coupling cap.

We desire $R_{E,1}+R_{E,2}$ to establish Bias point stability. $R_{E,1}$ is better designed as small since we do not want to lose much of the gain of the second stage, though feedback is retained in DC as well.

But then $R_{E,1}$ cannot be too small since the small signal resistance

$$R_{in,base,Q} \cong r_{\pi,Q}(1+g_{m,Q}R_{E,1})$$

comes in parallel with $R_3 // R_4 // R_D // [r_{o,M}(1+g_{m,M}R_S)]$

 2

Having $R_{in,base,Q}$ too small due to small $R_{E,1}$ would kill the voltage gain of the first stage.

The overall expression for the voltage gain is

$$\frac{v_o}{v_i} = \left[-\frac{g_{m,M}}{1+g_{m,M}R_S}\right]\left[\left[r_{\pi,Q}(1+g_{m,Q}R_{E,1})\right] // R_3 // R_4 // R_D // \left[r_{o,M}(1+g_{m,M}R_S)\right]\right]$$

$$\left[-\frac{g_{m,Q}}{1+g_{m,Q}R_{E,1}}\right]\left[R_C // \left[r_{o,Q}(1+g_{m,Q}R_{E,1})\right]\right]$$

(3)

5. Small-Signal Impedances

5.1 Impedances into the Three Nodes of a BJT

Computation of Impedance

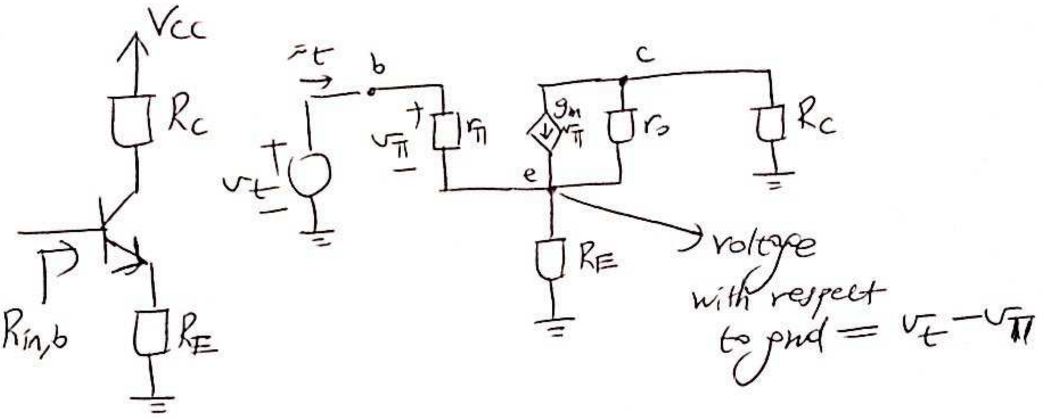
$R_{in,b}$

Looking into the base of an NPN BJT

worked on this on:

Oct 19, 2014 / Sun

Computation of R looking into the base



KCL at (e)

$$(1) \quad \frac{v_t - v_\pi}{R_E} + \frac{\cancel{v_t} - v_\pi - \cancel{v_t}}{r_\pi} - g_m v_\pi + \frac{v_t - v_\pi - v_c}{r_o} = 0$$

KCL at (c)

$$(2) \quad \frac{v_c}{R_C} + \frac{v_c - v_t - v_\pi}{r_o} + g_m v_\pi = 0$$

and we have

$$i_t r_\pi = v_\pi$$

From (1) and (2)

$$(3) \quad \frac{v_t}{R_E // r_o} = \frac{v_\pi}{R_E // r_\pi // \frac{1}{g_m} // r_o} + \frac{v_c}{r_o}$$

$$(4) \quad \frac{v_c}{R_C // r_o} + \frac{v_\pi}{r_o // \frac{1}{g_m}} = \frac{v_t}{r_o}$$

1

⇒ In (3) and (4), solve for v_C:

$$(5)\quad \left[\frac{v_t}{R_E /\!/ r_o} - \frac{v_\pi}{R_E /\!/ r_\pi /\!/ \frac{r_o}{1+g_m r_o}}\right] r_o$$

$$= \left[\frac{v_t}{r_o} - \frac{v_\pi}{\frac{r_o}{1+g_m r_o}}\right](R_C /\!/ r_o) = \left[\frac{v_t}{\cancel{r_o}} - \frac{(1+g_m r_o)v_\pi}{\cancel{r_o}}\right]\frac{R_C \cancel{r_o}}{R_C + r_o}$$

$$v_t \frac{R_E + r_o}{\cancel{r_o} R_E} \cdot \cancel{r_o} - v_t \frac{R_C}{R_C + r_o}$$

$$= v_t\left[1 + \frac{r_o}{R_E} - \frac{R_C}{R_C + r_o}\right] = v_t\left[\frac{r_o}{R_E} + \frac{r_o}{R_C + r_o}\right]$$

$$= v_t\left[\frac{r_o}{R_E /\!/ (R_C + r_o)}\right]$$

Note that

$$r_o \frac{1}{(R_E /\!/ r_\pi) /\!/ \left(\frac{r_o}{1+g_m r_o}\right)} = \frac{(1+g_m r_o)[R_E /\!/ r_\pi] + r_o}{(R_E /\!/ r_\pi)\cancel{r_o}} \cancel{r_o}$$

$$= (1+g_m r_o) + \frac{r_o}{R_E /\!/ r_\pi}$$

$$v_\pi\left[(1+g_m r_o) + \frac{r_o}{(R_E /\!/ r_\pi)} - (1+g_m r_o)\frac{R_C}{R_C + r_o}\right]$$

$$= v_\pi\left[(1+g_m r_o)\frac{r_o}{(R_C + r_o)} + \frac{r_o}{(R_E /\!/ r_\pi)}\right] = v_\pi \frac{r_o}{\left(\frac{R_C + r_o}{1+g_m r_o}\right) /\!/ (R_E /\!/ r_\pi)}$$

2

Therefore from (5), we have

$$\frac{v_t}{(R_C+r_o)//R_E} = \frac{i_t r_\pi}{\left(\dfrac{R_C+r_o}{1+g_m r_o}\right)//\left(R_E//r_\pi\right)}$$

$$R_{in,b} = \frac{v_t}{i_t} = \frac{\dfrac{\cancel{(R_C+r_o)}R_E}{R_C+r_o+R_E}\cancel{r_\pi}}{\dfrac{\dfrac{\cancel{(R_C+r_o)}}{1+g_m r_o}\cdot\dfrac{\cancel{R_E r_\pi}}{R_E+r_\pi}}{\dfrac{R_C+r_o}{1+g_m r_o}+\dfrac{R_E r_\pi}{R_E+r_\pi}}}$$

$$= \frac{(R_C+r_o)(R_E+r_\pi)+R_E r_\pi(1+g_m r_o)}{R_C+r_o+R_E}$$

$$= \frac{r_\pi(R_C+r_o+R_E)+R_E(R_C+r_o)+g_m r_o R_E r_\pi}{R_C+r_o+R_E}$$

$$= r_\pi + \frac{\tilde{R}}{R_C+r_o+R_E}$$

where $\boxed{\tilde{R} = R_E R_C + r_o R_E(1+g_m r_\pi)}$

if we let $r_o \to +\infty$

(6) $$R_{in,b} \cong r_\pi + R_E(1+\overbrace{g_m r_\pi}^{\beta}) \to \approx R_E(1+\beta)$$
$$= R_E + (1+g_m R_E)r_\pi \to \approx r_\pi(1+g_m R_E)$$

3

Note That we could have gotten the same approx result if we had let $r_0 \to +\infty$ in (3) and (4).

then (3) and (4) become uncoupled eqns.

From the approximate form of (3), we have

$$\frac{v_t}{R_E} \cong \frac{v_\pi = i_t r_\pi}{R_E /\!/ r_\pi /\!/ \frac{1}{g_m}}$$

$$\frac{v_t}{i_t} = \frac{R_E r_\pi}{r_\pi /\!/ \frac{R_E}{1+g_m R_E}} = \frac{\cancel{R_E r_\pi}}{\cancel{R_E r_\pi}}\left[R_E + (1+g_m R_E) r_\pi\right]$$

From which follow the results of (6).

(4)

$R_{in,e}$ derivation

Impedance (small signal) looking into the emitter of an NPN BJT

$\Rightarrow$ (9) is the exact formula

$\Rightarrow$ (10) $R_{in,e} \cong \dfrac{R_B + r_\pi}{1+\beta}$ (the usual approx formula for BJTs)

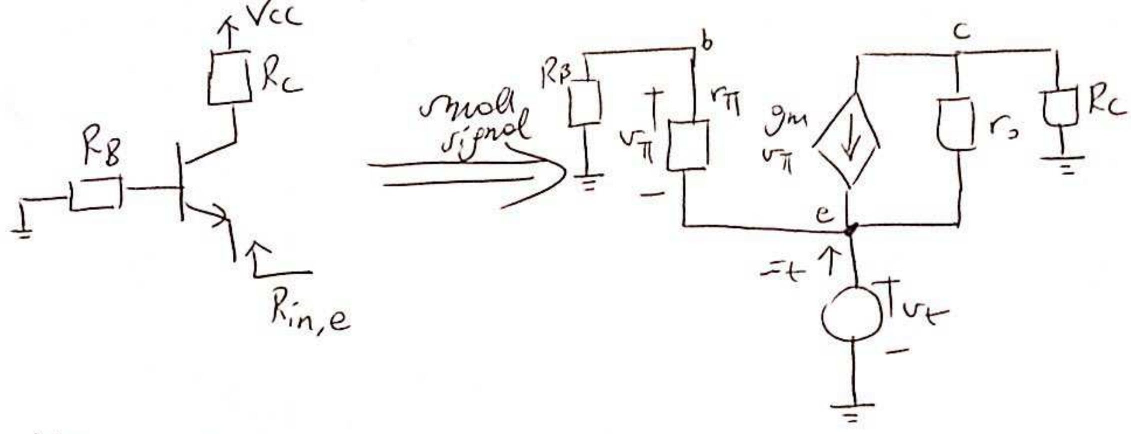
$\Rightarrow$ (11) $R_{in,e} \cong \dfrac{1}{g_m}$ (a more approximate formula in the case $r_\pi >> R_B$ or $R_B = 0$)

$\Rightarrow$ (13) $R_{in,e} \cong \dfrac{R_C + r_o}{1 + g_m r_o}$ (approx formula for MOS)

$\Rightarrow$ (14) $R_{in,e} \cong \dfrac{1}{g_m}$ (a more approx formula for MOS in the case $r_o >> R_C$ or $R_C = 0$)

Worked on this on: Oct 19, 2014/Sun

$R_{in,e}$: impedance looking into the emitter of an NPN BJT



KCL at (e)

$$\frac{v_t}{R_B + r_\pi} - i_t - g_m v_\pi + \frac{v_t - v_c}{r_o} = 0 \qquad (1)$$

KCL at (c)

$$g_m v_\pi + \frac{v_c - v_t}{r_o} + \frac{v_c}{R_C} = 0 \qquad (2)$$

and we have

$$v_\pi = v_t \left[\frac{-r_\pi}{r_\pi + R_B} \right] \qquad (3)$$

From (1) and (3)

$$\frac{v_t}{R_B + r_\pi} + \frac{g_m r_\pi v_t}{R_B + r_\pi} - i_t + \frac{v_t}{r_o} = \frac{v_c}{r_o} \qquad (4)$$

From (2) and (3)

$$\frac{v_c}{R_C // r_o} = \frac{v_t}{r_o} + \frac{g_m r_\pi v_t}{r_\pi + R_B} \qquad (5)$$

 (1)

From (4) and (5)

$$\cancel{r_o}\left[\frac{v_t}{\frac{R_B+r_\pi}{\beta}//r_o} + \frac{v_t}{R_B+r_\pi} - i_t\right] = \left[\frac{v_t}{\frac{R_B+r_\pi}{\beta}//r_o}\right]\frac{R_C \cancel{r_o}}{R_C+r_o} \quad (6)$$

From (6)

$$\frac{v_t}{\frac{R_B+r_\pi}{\beta}//r_o}\,\frac{r_o}{R_C+r_o} + \frac{v_t}{R_B+r_\pi} = i_t \quad (7)$$

From (7)

$$\frac{\beta v_t}{R_B+r_\pi}\,\frac{r_o}{R_C+r_o} + \frac{v_t}{\cancel{r_o}}\,\frac{\cancel{r_o}}{R_C+r_o} + \frac{v_t}{R_B+r_\pi} = i_t \quad (8)$$

From (8)

$$R_{in,e} = \frac{v_t}{i_t} = \frac{(R_B+r_\pi)(R_C+r_o)}{\beta r_o + R_C + r_o + R_B + r_\pi}$$

$$= \frac{(R_B+r_\pi)R_C + (R_B+r_\pi)r_o}{R_C + R_B + r_\pi + r_o(1+\beta)} \quad (9)$$

From (9), with $r_o \to +\infty$

$$R_{in,e} \cong \frac{v_t}{i_t} = \frac{R_B + r_\pi}{1+\beta} \quad \left(\begin{array}{c}\text{the usual approximate}\\ \text{formula for BJTs,}\\ \text{it is justified}\\ \text{since } r_o \text{ is}\\ \text{in reality big}\end{array}\right) \qquad (10)$$

From (10), if further we have

$R_B = 0$ or $r_\pi \gg R_B$ (and with $\beta \gg 1$)

$$R_{in,e} \cong \frac{r_\pi}{1+\beta} \cong \frac{r_\pi}{\beta} = \frac{r_\pi}{g_m r_\pi} = \frac{1}{g_m} \qquad (11)$$

Reorganize (9)

$$R_{in,e} = \frac{r_\pi (R_C + r_o) + R_B (R_C + r_o)}{r_\pi (1 + g_m r_o) + r_o + R_C + R_B} \qquad (12)$$

Let $r_\pi \to +\infty$ (as for MOS)

$$R_{in,e} \cong \frac{R_C + r_o}{1 + g_m r_o} \quad \left(\begin{array}{c}\text{the usual}\\ \text{formula for MOS}\end{array}\right) \qquad (13)$$

Note that R_C and r_o may, in cases, be comparable, especially with an active load instead of R_C.

If $R_c = 0$ or $r_o \gg R_c$ (and with $g_m r_o \gg 1$, as the intrinsic gain is much bigger than 1)

From (13)

$$R_{in,e} \cong \frac{r_o}{1 + g_m r_o} \cong \frac{r_o}{g_m r_o} = \frac{1}{g_m} \quad (14)$$

(4)

$R_{in,C}$ calculation:

impedance into the collector of a BJT

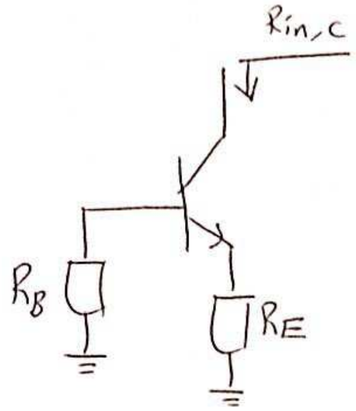
$\Rightarrow$ (1) exact formula

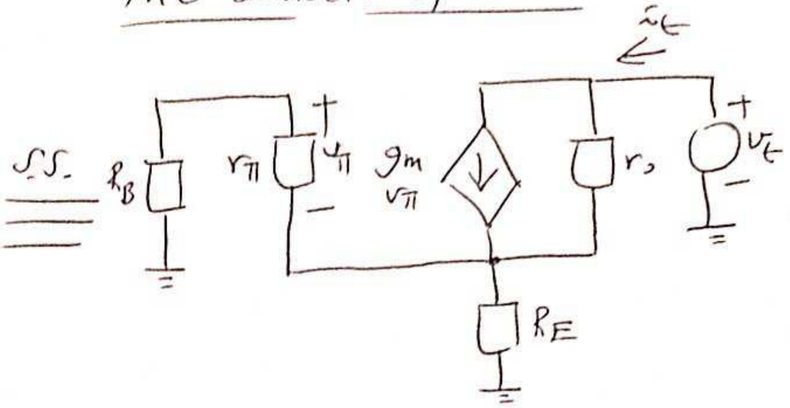
$\Rightarrow$ (3) when $R_B = 0$ or $R_B \ll r_\pi$

$\Rightarrow$ (4) when $r_\pi \to +\infty$ (the case for MOS)

Worked on this on: Oct 22, 2014/Wed

Calculation of $R_{in,C}$: impedance looking into the collector of a BJT





$$R_{in,C} = \frac{v_t}{i_t}$$

(KVL) $$v_t = \left(i_t - g_m v_\pi\right) r_o + i_t \left[R_E // (r_\pi + R_B)\right]$$

$$v_\pi = -i_t \frac{R_E}{R_E + r_\pi + R_B} \cdot r_\pi$$

(1) $$\frac{v_t}{i_t} = r_o \left[1 + \frac{g_m R_E r_\pi}{R_E + r_\pi + R_B}\right] + R_E // \left(r_\pi + R_B\right)$$

Also can be written as

$$= r_o + (1 + g_m r_o) \frac{R_E r_\pi}{R_E + r_\pi + R_B} + \frac{R_E R_B}{R_E + r_\pi + R_B}$$

(2) $$= \left(1 + g_m r_o\right) \frac{R_E r_\pi}{R_E + r_\pi + R_B} + r_o + \frac{R_E R_B}{R_E + r_\pi + R_B}$$

 1

From (1), if $R_B = 0$:

$$\frac{v_t}{i_t} = r_o\left[1 + \frac{g_m R_E r_\pi}{R_E + r_\pi}\right] + (R_E // r_\pi)$$

$$= r_o\left[1 + g_m (R_E // r_\pi)\right] + (R_E // r_\pi)$$

$$(3) \qquad = (R_E // r_\pi)\left[1 + g_m r_o\right] + r_o$$

Note that (3) is also approximately correct if $R_B \ll r_\pi$.

If $r_\pi \to +\infty$, there will be no current on R_B, then $v_b \to 0$ and $v_\pi \to -v_e$.
In this case, we will have: (from (1) and (3))

$$\frac{v_t}{i_t} \cong r_o\left[1 + g_m R_E\right] + R_E$$

$$(4) \qquad = R_E\left[1 + g_m r_o\right] + r_o$$

(4) is $R_{in,C}$ when we have a MOS transistor instead of a BJT.

2

6. Power Amplifiers

6.1 Miscellaneous Analyses

Analysis of the Implications of the Load Resistor Value on the performance of a Class-A Power amplifier

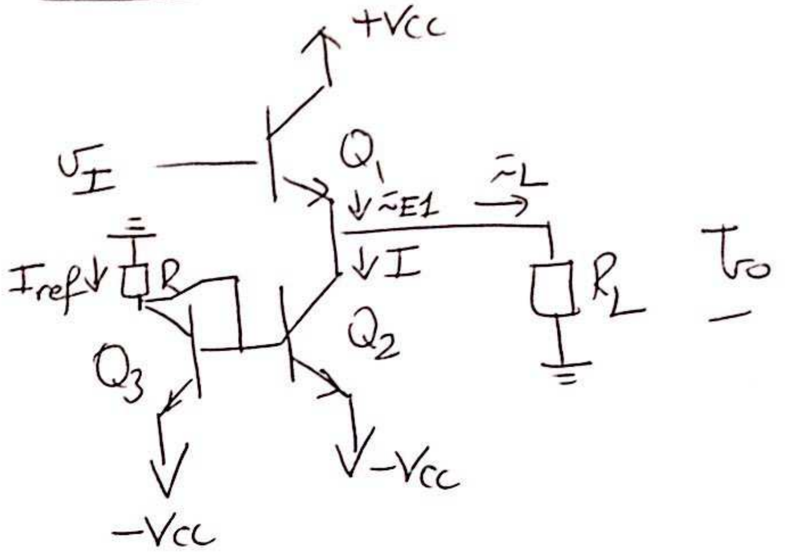
η_{max} : max efficiency

$R_{L,\eta max}$: the load resistor value which yields the max efficiency

R_L : actual load resistor value

Worked on this on: May 14, 2015/Th

Class-A Power Amplifier



Calculations show that:
(if Q_2 and Q_3 are matched and $\beta \to +\infty$)

$$I = I_{ref} = \frac{0 - (-V_{CC} + V_{BE})}{R}$$

$$I = \frac{V_{CC} - V_{BE}}{R}$$

Also

$R_{L,\eta max}$: the value for R_L which yields the maximum value for the efficiency η ($\eta_{max} = 25\%$)

$$R_{L,\eta max} = \frac{V_{CC} - V_{CE,sat}}{I}$$

1

Note that

$$R = \frac{V_{CC} - V_{BE}}{I}$$

so that

$$\frac{R_{L,\eta_{max}}}{R} = \frac{V_{CC} - V_{CE,sat}}{V_{CC} - V_{BE}}$$

Consider again the two possibilities

Q$_1$ cuts off before Q$_2$ runs into saturation

$$i_{E1} = 0 = I + i_L$$

$$i_L = -I$$

$$v_{out} = -IR_L$$

(*) For this assumption to be true we must have $V_{CE,2} > V_{CE,sat}$ at the moment Q_1 cuts off

Q$_2$ runs into saturation before Q$_1$ cuts off

$$v_{out} = -V_{CC} + V_{CE,sat}$$

$$i_L = \frac{v_{out}}{R_L} = -\frac{V_{CC} - V_{CE,sat}}{R_L}$$

(**) For this assumption to be true, we must have $i_{E1} = I + i_L > 0$ at the moment Q_2 runs into saturation.

2

Analyze (*) on pp2

$$V_{CE,2} = v_{out} - (-V_{CC})$$

$$= -IR_L - (-V_{CC})$$

$$= -R_L \frac{V_{CC} - V_{BE}}{R} + V_{CC} > V_{CE,sat}$$

$$\Rightarrow \quad V_{CC} - V_{CE,sat} > \frac{R_L}{R}(V_{CC} - V_{BE})$$

$$\Rightarrow \quad \frac{R_L}{R} < \frac{V_{CC} - V_{CE,sat}}{V_{CC} - V_{BE}} = \frac{R_{L,\eta max}}{R}$$

$$\Rightarrow \quad R_L < R_{L,\eta max}$$

Analyze (**) on pp2:

$$I + i_L > 0$$

$$\frac{V_{CC} - V_{BE}}{R} - \frac{V_{CC} - V_{CE,sat}}{R_L} > 0$$

$$\Rightarrow \quad \frac{R_L}{R} > \frac{V_{CC} - V_{CE,sat}}{V_{CC} - V_{BE}} = \frac{R_{L,\eta max}}{R}$$

$$\Rightarrow \quad R_L > R_{L,\eta max}$$

3

Therefore

$R_L < R_{L,\eta max} \Rightarrow$ Q_1 cuts off before Q_2 runs into saturation and Q_2 consequently never runs into saturation

$R_L > R_{L,\eta max} \Rightarrow$ Q_2 runs into saturation before Q_1 cuts off

Then

$R_L = R_{L,\eta max} \Rightarrow$ Q_1 cuts off and Q_2 runs into saturation at the same moment

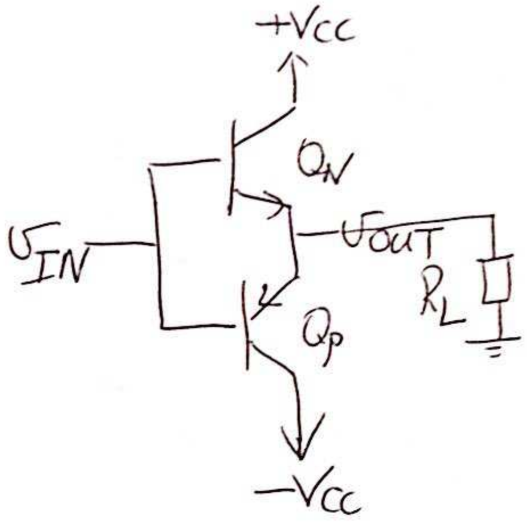
More over, $\eta = \eta_{max} = 25\%$ for the class A power amplifier.

4

Derivation of the
Power Efficiency
for Class B
Power Amplifier

Worked on this on: May 18, 2015/M

Power efficiency of a Class B Amplifier



$v_{OUT,max} = +V_{CC} - V_{CE,sat}$

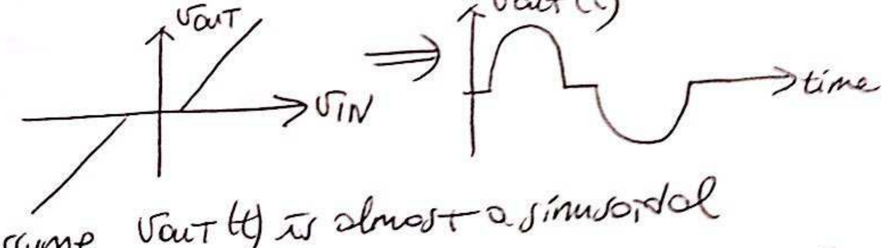
$v_{out,min} = -V_{CC} + V_{CE,sat}$

assuming $|V_{CC}| \gg |V_{CE,sat}|$

and $V_{CE,sat} = V_{CE,N,sat} = V_{EC,P,sat}$

max peak to peak value for v_{OUT} $\quad v_{out,pp} = v_{out,max} - v_{out,min}$

Neglecting the crossover distortion effect



We may assume $v_{OUT}(t)$ is almost a sinusoidal given by $v_{OUT}(t) = \hat{V}_o \sin(wt)$ $\quad w$: angular freq. $\quad \hat{V}_o$: magnitude

Note that $\hat{V}_o = \dfrac{v_{OUT,pp}}{2} \simeq V_{CC}$

1

power efficiency $\eta = \dfrac{\text{average power delivered to the load}}{\text{average power supplied by the DC voltage sources}}$

average power delivered to the load

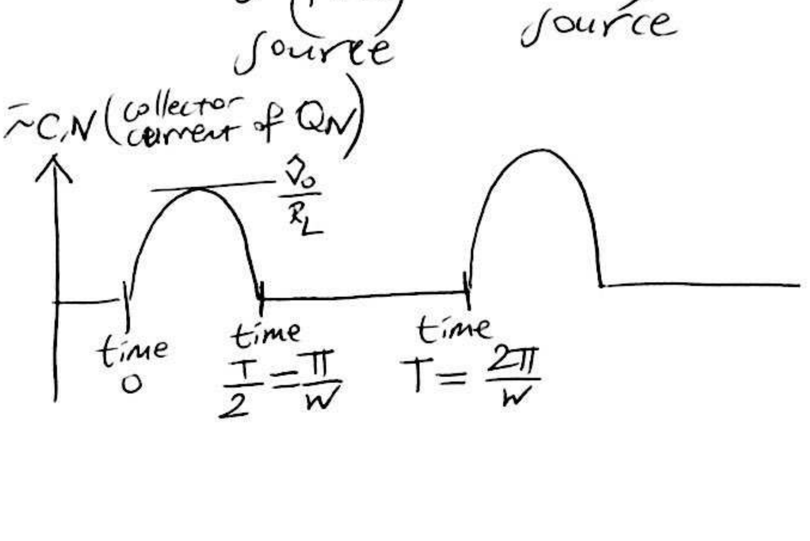
$$P_{L,av} = \left(\frac{\hat{V}_o}{\sqrt{2}}\right)^2 \frac{1}{R_L}$$

$$= \left(\begin{array}{c}\text{rms value}\\ \text{of the output}\\ \text{voltage}\end{array}\right)^2 \frac{1}{R_L}$$

average power supplied by the voltage sources

$$P_S = \underbrace{P_{S+}}_{\text{by the } (+V_{CC}) \text{ source}} + \underbrace{P_{S-}}_{\text{by the } (-V_{CC}) \text{ source}}$$

Plot of the current that comes out of the $(+V_{CC})$ source [neglecting the crossover distortion]



$$P_{S+} = \underbrace{(+V_{CC})}_{\text{DC voltage value}} \underbrace{\frac{1}{T}\int_0^{T/2} \underbrace{\frac{\hat{V}_o}{R_L}\sin(wt)}_{\text{instantaneous current}} dt}_{\text{average current}}$$

Note that Q_N is OFF during

$$\frac{T}{2} = \frac{\pi}{w} \leq t < \frac{2\pi}{w} = T$$

Compute

$$\int_0^{\frac{T}{2}=\frac{\pi}{w}} \frac{\hat{V}_o}{R_L}\sin(wt)\,dt = \frac{\hat{V}_o}{R_L}\frac{1}{w}\Big[-\cos(wt)\Big]\Big|_0^{\pi/w}$$

$$= \frac{\hat{V}_o}{R_L}\frac{T}{2\pi}\underbrace{\Big[-\cos(\pi)+\cos(0)\Big]}_{2}$$

$$= \frac{\hat{V}_o}{R_L}\frac{T}{\pi}$$

$$P_{S+} = (+V_{CC})\frac{1}{\cancel{T}}\frac{\hat{V}_o}{R_L}\frac{\cancel{T}}{\pi} = \frac{V_{CC}\hat{V}_o}{\pi R_L}$$

it happens that $P_{S+} = P_{S-}$

3

The total average supplied power is

$$P_s = P_{s+} + P_{s-} = \frac{2V_{CC}\hat{V}_o}{\pi R_L}$$

Therefore the efficiency

$$\eta = \frac{P_{L,av}}{P_s} = \frac{\hat{V}_o^{2}\frac{1}{2R_L}}{\frac{2V_{CC}\hat{V}_o}{\pi R_L}} = \frac{\hat{V}_o}{V_{CC}}\frac{\pi}{4}$$

See the bottom of pg (1)

max value for $\hat{V}_o \simeq V_{CC}$

Then $\eta_{max} = \frac{\pi}{4} \simeq 78.5\%$

(4)

7. Differential Amplifiers

7.1 Fundamentals

Simple Differential Amplifier Analysis

⇒ Differential mode half circuit
(virtual gnd concept in symmetrical diff. amp.)

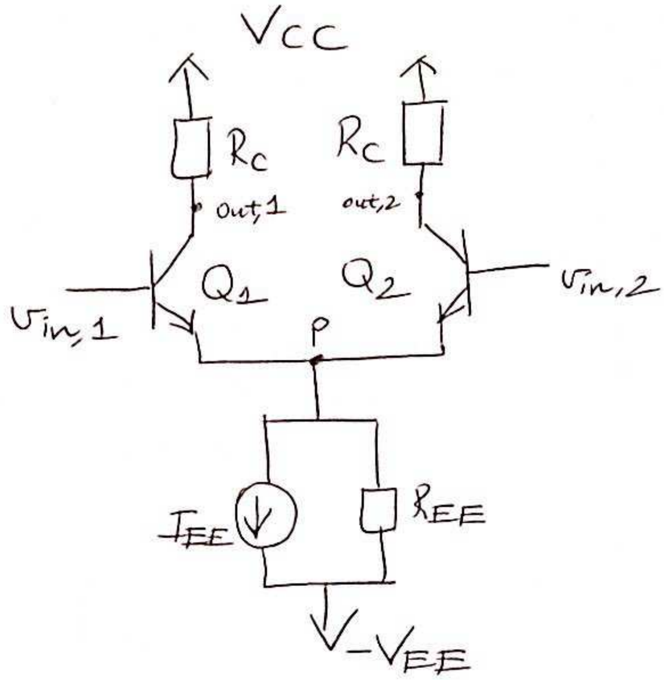
⇒ Common mode half circuit

⇒ Gain, I/O impedances

⇒ CMRR

Worked on this on: NOV 5, 2014/ Wed

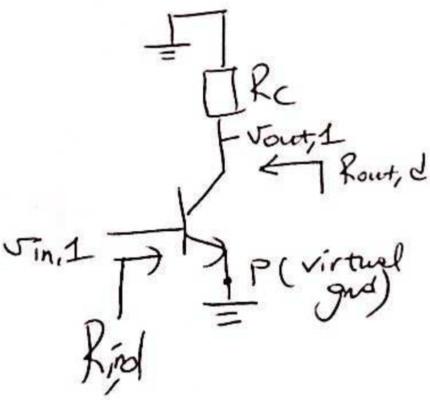
Simple Differential Amplifier Analysis



Q_1 and Q_2 are identical (the small signal parameters are the same since DC o.p. is the same)

Differential mode half circuit

node P can be proved to be virtual gnd. (in small signal)



$$\frac{v_{out,1}}{v_{in,1}} = -g_{m1}(R_C // r_{o,1})$$

similarly we have

$$\frac{v_{out,2}}{v_{in,2}} = -g_{m2}(R_C // r_{o2})$$

Note that $g_{m1} = g_{m2} \triangleq g_m$

$r_{o1} = r_{o2} \triangleq r_o$

$r_{\pi 1} = r_{\pi 2} \triangleq r_\pi$

1

observe also that

diff. mode gain $A_d \triangleq -g_m(R_C // r_o)$

$v_{out,1} = A_d\, v_{in,1}$

$v_{out,2} = A_d\, v_{in,2}$

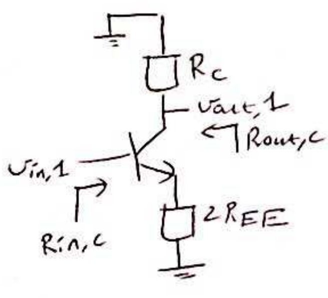
$v_{out,1} - v_{out,2} = A_d(v_{in,1} - v_{in,2})$

Then $\dfrac{v_{out,1} - v_{out,2}}{v_{in,1} - v_{in,2}} = A_d$

diff mode input imp $\Longrightarrow R_{in,d} = r_\pi$

diff mode output imp $\Longrightarrow R_{out,d} = R_C // r_o$

Common mode half circuit



common-mode voltage gain $A_c \cong -g_m \dfrac{\left[R_C // r_o\left[1+g_m 2R_{EE}\right]\right]}{1+g_m 2R_{EE}}$

common-mode input imp $R_{in,c} \cong r_\pi\left[1+g_m 2R_{EE}\right]$

common-mode output imp. $R_{out,c} \cong R_C // \left[\left[1+g_m 2R_{EE}\right] r_o\right]$

2

Common-mode Rejection Ratio (CMRR)

$$\frac{|A_d|}{|A_c|} = CMRR = \frac{|-g_m(R_C // r_o)|}{\left|-\frac{g_m}{1+g_m 2R_{EE}}\left[R_C // \left[r_o\left[1+g_m 2R_{EE}\right]\right]\right]\right|}$$

$$= (1+g_m 2R_{EE}) \frac{R_C // r_o}{R_C // \left[r_o\left[1+g_m 2R_{EE}\right]\right]}$$

if $r_o \gg R_C \cong (1+g_m 2R_{EE})$

CMRR is expected/designed to be much greater than 1.

8. Frequency Response

8.1 Fundamentals

A Note on the Gain-Bandwidth Product of $A(jw) = \dfrac{A}{1 + \dfrac{jw}{w_b}}$

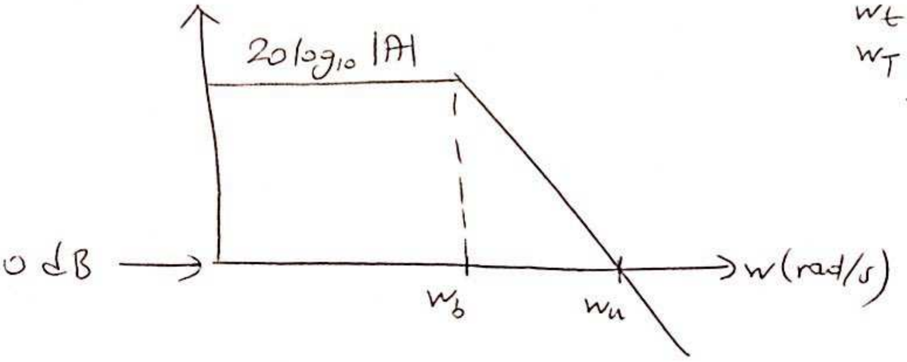
Worked on this on: Oct 23, 2014/Th

Gain-Bandwidth product (see Sedra & Smith pp 90-93)

Consider $A(j\omega) = \dfrac{A}{1 + \dfrac{j\omega}{\omega_b}}$

with the Bode plot

$\left.\begin{matrix}\omega_u \\ \omega_t \\ \omega_T\end{matrix}\right\}$ the same thing unity-gain frequency



Note that for $\omega \gg \omega_b$

$$A(j\omega) \simeq \frac{A}{\frac{j\omega}{\omega_b}}$$

then $20\log_{10} \underbrace{\left|A(j\omega_u)\right|} = 0\,\text{dB}$

$$\left|A(j\omega_u)\right| = 1 = \left| \frac{A}{\frac{j\omega_u}{\omega_b}} \right| \quad \text{since } \omega_u \gg \omega_b$$

$\Longrightarrow A\omega_b = \omega_u \Longrightarrow$ the unity gain freq is equal to the gain-bandwidth product.

A → gain (midband)
ω_b → bandwidth

OCTC - Open Circuits
Time Constants
Method

$$\frac{1}{\left(1+\frac{j\omega}{\omega_1}\right)\left(1+\frac{j\omega}{\omega_2}\right)} \xrightarrow{\text{OCTC}} \frac{1}{1+\frac{j\omega}{\omega_0}}$$

Worked on this on: Oct 27, 2015 / Tues

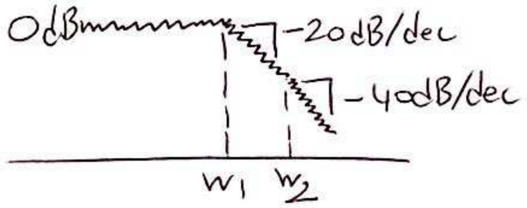
Open Circuit Time Constant Method (OCTC) for computing the high frequency cut-off of an amplifier circuit

OCTC is an approximation. In some cases it is applied after Miller's approximation.

An example of a high-frequency response:

$$A(jw) = \frac{1}{\left(1+\frac{jw}{w_1}\right)\left(1+\frac{jw}{w_2}\right)}$$

The corresponding magnitude Bode Plot is



How do we approximate this frequency response so that there is only one pole w_0 to account for?

Consider $$A(jw) = \frac{1}{1+\frac{jw}{w_1}+\frac{jw}{w_2}+\underbrace{\left(\frac{jw}{w_1}\right)\left(\frac{jw}{w_2}\right)}}$$

Now omit this term

①

Then approximately

$$H(jw) \cong \frac{1}{1+\frac{jw}{w_0}}$$

Note that $\frac{1}{w_0} = \frac{1}{w_1} + \frac{1}{w_2}$

w_1 and w_2 correspond to time constants τ_1 and τ_2, respectively.

$$w_1 = \frac{1}{\tau_1} \qquad w_2 = \frac{1}{\tau_2}$$

Then $\frac{1}{w_0} = \tau_1 + \tau_2$

$$w_0 = \frac{1}{\tau_1 + \tau_2}$$

Note that the time constants may be given as

$$\tau_1 = R_1 C_1 \quad \tau_2 = R_2 C_2$$

In order to make $\tau_2 = 0$ when computing τ_1, we let $C_2 = 0$. This corresponds to $\frac{1}{jwC_2} \to +\infty$.

Effectively an open circuit.

The procedure goes like this:

→ set $C_2 = 0$, make C_2 an open circuit
compute the resistance R_1 that C_1 "sees"
compute $\tau_1 = R_1 C_1$

→ set $C_1 = 0$, make C_1 an open circuit
compute the resistance R_2 that C_2 "sees"
compute $\tau_2 = R_2 C_2$

2

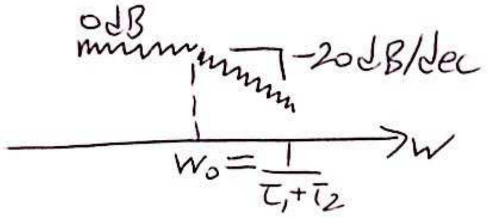
Then compute

$$w_0 = \frac{1}{\tau_1 + \tau_2}$$

$H(jw)$ can be modelled (approximated) as

$$H(jw) \approx \frac{1}{1 + \frac{jw}{w_0}}$$

The corresponding Bode plot is:



 3

8.2 Intrinsic Responses of Transistors

BJT Frequency Response

$\Rightarrow$ $\beta(j\omega) = \dfrac{\tilde{i}_c(j\omega)}{\tilde{i}_b(j\omega)}$ modeling

$\Rightarrow$ Bode plot for $\beta(j\omega)$

$\Rightarrow$ ω_β = -3 dB freq for $\beta(j\omega)$

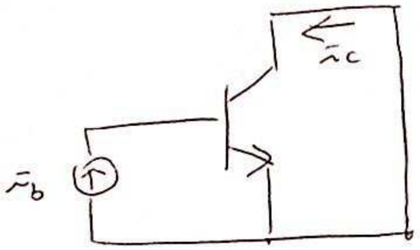
$\Rightarrow$ ω_T or $f_T = \dfrac{\omega_T}{2\pi}$ is called the transition freq

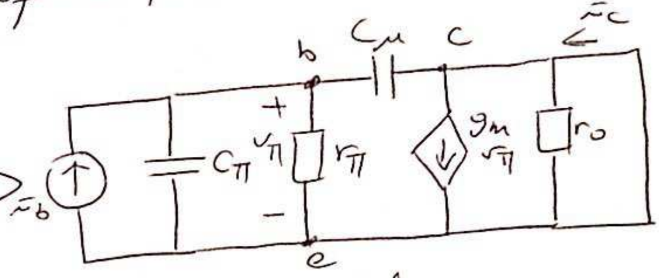
$$\omega_T = \frac{g_m}{C_\mu + C_\pi}$$

Worked on this on: Oct 23, 2014 / Th

BJT Frequeny Response

See Rashid pp 504-506,
excerpting some useful facts to be utilized in the soln of hw questions.





BJT s.s. model
(i_b and i_c are smallsignal currents, there are no DC sources in the schem.)

BJT s.s model replaced by the high freq model
(Note that there is no current through r_o, and C_π and C_μ come in parallel)

$$v_\pi = i_b \left[r_\pi \,//\, \underbrace{\frac{1}{sC_\pi} \,//\, \frac{1}{sC_\mu}}_{\frac{1}{s(C_\pi + C_\mu)}} \right]$$

(capacitors in parallel added)

KCL at node $\underline{\underline{b}}$ ⟹ yields the above formula

KCL at node $\underline{\underline{c}}$ ⟹ $\dfrac{v_\pi}{\frac{1}{sC_\mu}} + i_c = g_m v_\pi$

$$\Rightarrow sC_\mu v_\pi + i_c = g_m v_\pi$$

$$\Rightarrow i_c = v_\pi (g_m - sC_\mu)$$

1

calculate

$$\left(\frac{\tilde{i}_b(s)}{\tilde{i}_c(s)}\right)^{-1} = \frac{\tilde{i}_c(s)}{\tilde{i}_b(s)} = \frac{v_\pi\left(g_m - sC_\mu\right)}{\dfrac{v_\pi}{r_\pi // \dfrac{1}{s(C_\mu + C_\pi)}}}$$

$$= \frac{\left(g_m - sC_\mu\right)\dfrac{r_\pi}{s(C_\mu + C_\pi)}}{r_\pi + \dfrac{1}{s(C_\mu + C_\pi)}}$$

$$= \frac{\left(g_m - sC_\mu\right) r_\pi}{1 + s\left(C_\mu + C_\pi\right) r_\pi}$$

Now we are given $g_m \gg sC_\mu$ at the frequencies of interest and we know $\beta = g_m r_\pi$.

Then

$$\beta(jw) = \frac{\tilde{i}_c(jw)}{\tilde{i}_b(jw)} \cong \frac{g_m r_\pi}{1 + jw\left(C_\mu + C_\pi\right)\dfrac{\beta}{g_m}} = \frac{\beta}{1 + \dfrac{jw}{\dfrac{g_m}{\beta(C_\mu + C_\pi)}}}$$

This is the frequency model for the current gain β of the BJT.

2

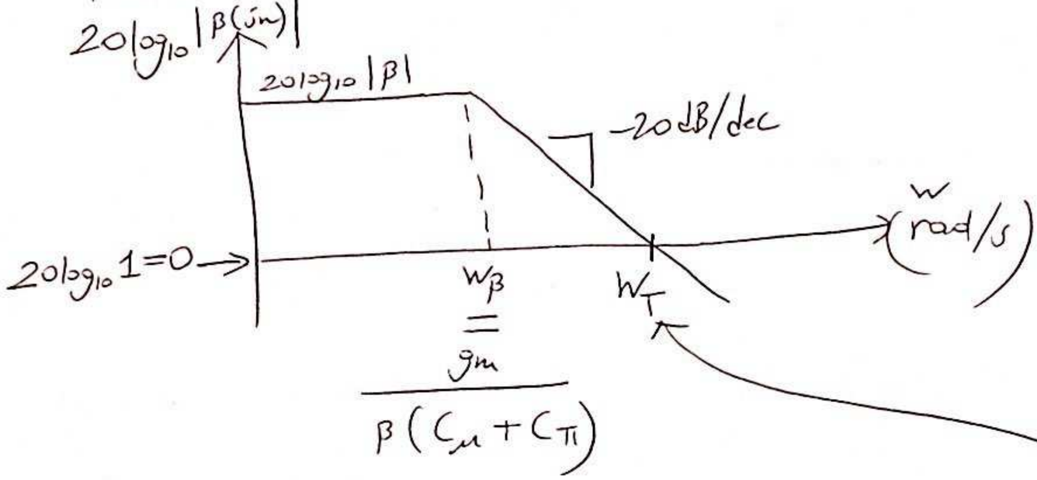
Bode plot for $\beta(jw)$

Note that we can write

$$\beta(jw) = \frac{\beta}{1 + \frac{jw}{w_\beta}}$$

where w_β is the -3dB frequency for $\beta(jw)$

Then we have



Note that for $\overline{w \gg w_\beta} \Rightarrow \beta(jw) \cong \frac{\beta}{\frac{jw}{w_\beta}}$

(this is an asymptotic approximation)

Now $20\log_{10}\left|\beta(jw)\right| = 0$ when $w = w_T$

then

3

then we need to have

$$\left| \frac{\beta}{\frac{j w_T}{w_\beta}} \right| = 1$$

$$\Rightarrow \frac{\beta w_\beta}{w_T} = 1 \Rightarrow w_T = \beta w_\beta$$

$$w_T = \beta w_\beta = \cancel{\beta} \frac{g_m}{\cancel{\beta}(C_\mu + C_\pi)} = \frac{g_m}{C_\mu + C_\pi}$$

$\Downarrow$

called transition frequency

for $w > w_T$, $|\beta(jw)| < 1$ then the BJT is not amplifying i_b anymore, it is attenuating i_b.

- Note that you will be given either w_T or $f_T = \frac{W_T}{2\pi}$ in the questions.

(4)

9. Feedback

9.1 Benefits

Effect of feedback on low frequency response

$\Rightarrow$ w_L lowered $\longrightarrow$ By the same factor

$\Rightarrow$ A_M (midband gain) lowered $\longrightarrow$

Worked on this on: Oct 27, 2014/Mon

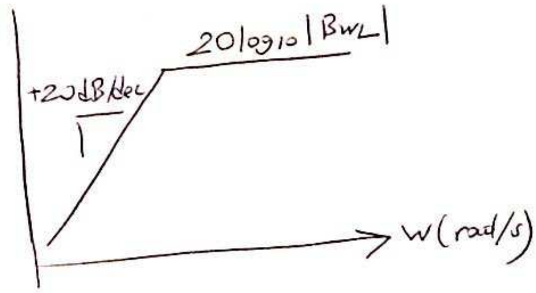
Effect of feedback on low-freq response

T.F. model $\Rightarrow$ $$A(jw) = B \frac{jw}{1 + \frac{jw}{w_L}}$$

for $w \gg w_L \Rightarrow$ $$A(jw) \cong B \frac{jw}{\frac{jw}{w_L}}$$

$$A_M = |A(jw)| = |Bw_L| \quad \left(\begin{matrix}\text{midband} \\ \text{gain}\end{matrix}\right)$$

Magnitude Bode plot $\Rightarrow$



T.F. model with feedback (where β is the feedback factor)

$$A_f(jw) = \frac{A(jw)}{1 + \beta A(jw)}$$

$$= \frac{B \frac{jw}{1+\frac{jw}{w_L}}}{1 + \beta B \frac{jw}{1+\frac{jw}{w_L}}}$$

$$= \frac{Bjw}{1 + \frac{jw}{w_L} + \frac{jw}{\frac{1}{\beta B}}} = \frac{Bjw}{1 + \frac{jw}{\frac{w_L \frac{1}{\beta B}}{w_L + \frac{1}{\beta B}}}}$$

$$= B \frac{jw}{1 + \frac{jw}{\frac{w_L}{1+\beta B w_L}}}$$

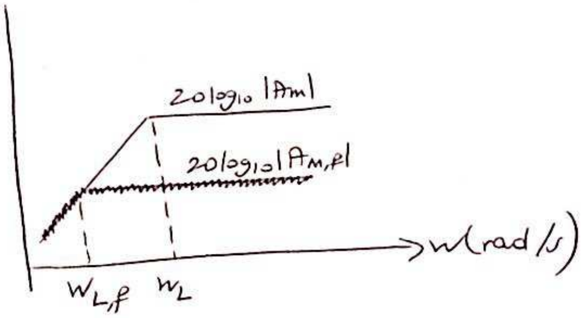
(1)

Then $w_{L,f} = \dfrac{w_L}{1+\beta B w_L} \Rightarrow$ lowered by this factor

low freq cutoff with feedback

new midband gain with feedback $\Rightarrow \dfrac{B w_L}{1+\beta B w_L} = A_{M,f}$

Bode plots



Note that $w_L > 0$.
Then the signs of B and β must be the same for negative feedback.

2

Effects of feedback on High-Frequency Cut-off

$\Rightarrow$ w_T (~~[illegible]~~ GBW gain-bandwidth product) remains the same

$\Rightarrow$ bandwidth increased
gain reduced

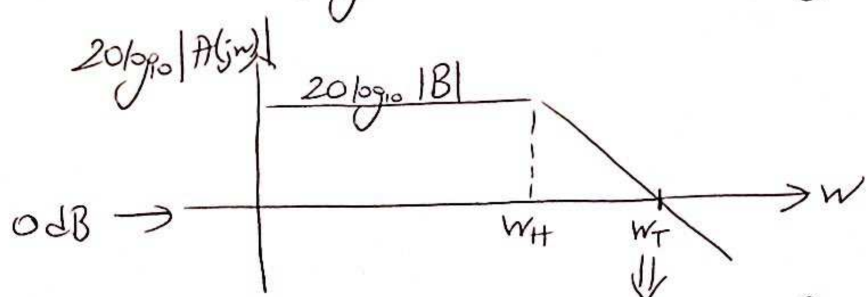
Worked on this on: Nov 5, 2014/ Wed

Effects of feedback on high-frequency cut-off

Our model is $A(jw) = \dfrac{B}{1 + \frac{jw}{w_H}}$

Feedback factor is β

Bode Plot (magnitude) of $A(jw)$ in asymptotic form



unity-gain freq.
or transition freq.

$W_T = B \cdot W_H$ (Gain-Bandwidth product)

System transfer function with feedback

$$A_f(jw) = \frac{A(jw)}{1 + \beta A(jw)}$$

 1

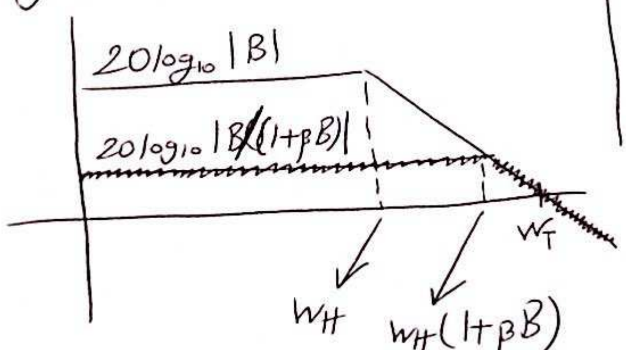
$$A_f(jw) = \frac{\dfrac{B}{1+\dfrac{jw}{w_H}}}{1+\beta\dfrac{B}{1+\dfrac{jw}{w_H}}}$$

$$= \frac{B}{1+\dfrac{jw}{w_H}+\beta B}$$

$$= \frac{\dfrac{B}{1+\beta B}}{1+\dfrac{jw}{w_H(1+\beta B)}}$$

Bode plots of $A(jw)$ and $A_f(jw)$
(Magnitude-asymptotic)

$A(jw) \to A_f(jw)$
Bandwidth increased but gain reduced



w_T (GBW) for both plots is the same

$$w_T = B\cdot w_H = \frac{B}{1+\beta B}\cdot w_H(1+\beta B)$$

2

Feedback decreases the sensitivity with respect to a parameter

Worked on this on: Nov 6, 2014/ Thu

Given an amplifier with gain A and feedback factor β, we examine sensitivities with respect to a parameter p (β is assumed to be independent of p).

$$S_p^A = \frac{\partial A}{\partial p}\frac{p}{A}$$

$$A_f = \frac{A}{1+\beta A}$$

How is $S_p^{A_f}$ related to S_p^A?

$$S_p^{A_f} = \frac{\partial A_f}{\partial p}\frac{p}{A_f} = \frac{p}{A_f}\frac{\partial}{\partial p}\left[\frac{A}{1+\beta A}\right]$$

$$= \frac{p}{A_f}\,\frac{\frac{\partial A}{\partial p}(1+\beta A) - A\left[\beta\frac{\partial A}{\partial p}\right]}{(1+\beta A)^2}$$

$$= \frac{p}{A_f}\,\frac{\frac{\partial A}{\partial p}}{(1+\beta A)^2}$$

$$= \frac{p}{A}\,\frac{1+\beta A}{(1+\beta A)^2}\,\frac{\partial A}{\partial p}$$

$$= \frac{S_p^A}{1+\beta A}$$

Therefore negative feedback decreased the sensitivity with respect to a parameter, by the improvement factor $(1+\beta A)$.

Effects of Feedback on Distortion

D8.20 A particular amplifier has a nonlinear transfer characteristic that can be approximated as follows:

(a) For small input signals, $|v_I| \le 10$ mV, $v_O/v_I = 10^3$
(b) For intermediate input signals, 10 mV $\le |v_I| \le 50$ mV, $v_O/v_I = 10^2$
(c) For large input signals, $|v_I| \ge 50$ mV, the output saturates

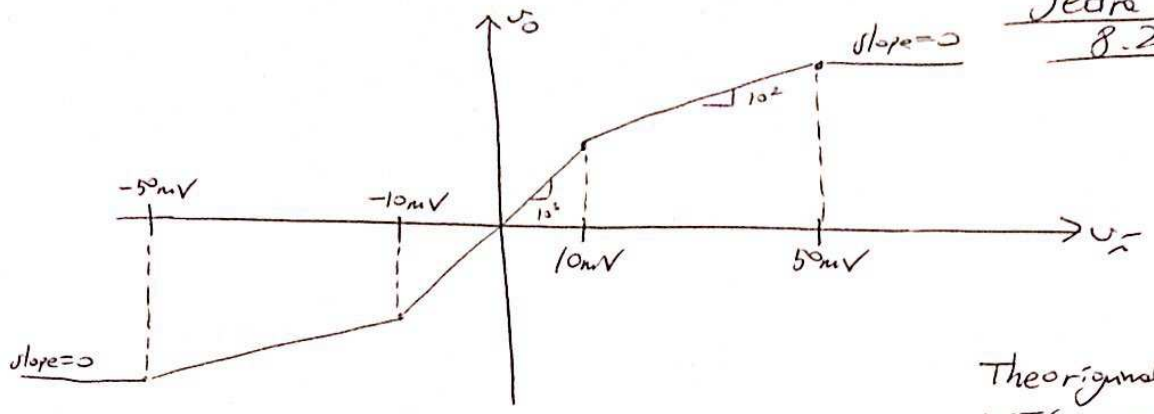
If the amplifier is connected in a negative-feedback loop, find the feedback factor β that reduces the factor-of-10 change in gain (occurring at $|v_I| = 10$ mV) to only a 10% change. What is the transfer characteristic of the amplifier with feedback?

Notes: None.

Additional Tasks: None.

Necessary Knowledge and Skills: Gain calculations for a feedback amplifier.

Sedra
8.20



The original VTC curve is highly nonlinear

$$\text{Let } A_1 = 10^3$$
$$A_2 = 10^2$$

with feedback

$$A_{1,f} = \frac{A_1}{1+A_1\beta}$$

$$A_{2,f} = \frac{A_2}{1+A_2\beta}$$

the same feedback factor β needs to be used.

With feedback, we will still have

$$A_{1,f} > A_{2,f}$$

It is required that we have

$$A_{1,f} = A_{2,f}\,(1 + 10\%)$$

$$\frac{\cancel{A_1}^{10^3}}{1+\underset{10^3}{\cancel{A_1}}\beta} = \frac{\cancel{A_2}^{10^2}}{1+\underset{10^2}{\cancel{A_2}}\beta}(1.1) \quad \Rightarrow \text{solve for } \beta$$

1

Sedra 8.20 contin-

$$10(1+100\beta) = (1+1000\beta)1.1$$

$$10+1000\beta = 1.1 + 1100\beta$$

$$8.9 = 100\beta$$

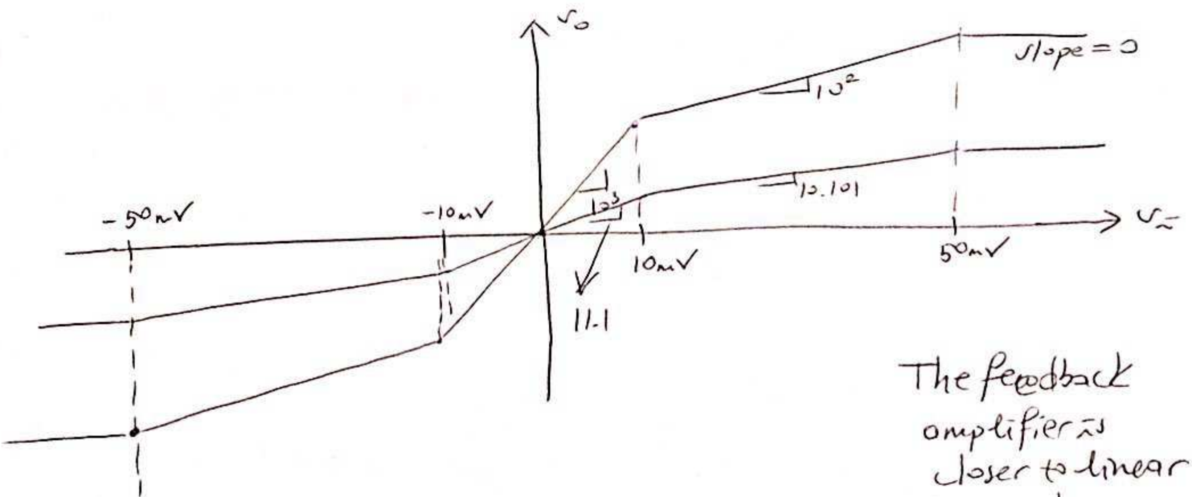
$$\beta = \frac{8.9}{100}$$

then $$A_{1,f} = \frac{10^3}{1+10^3\frac{8.9}{10^2}} = \frac{1000}{90} = 11.1$$

10% difference

$$A_{2,f} = \frac{10^2}{1+10^2\frac{8.9}{10^2}} = \frac{100}{9.9} \simeq 10.101$$

The new (and old) VTC



The feedback amplifier is closer to linear (almost has a uniform gain at all ranges of input)

Note: Such VTC graphs as above need to be interpreted as in:

→ The component of the input ≤ 10mV gets A_1 ($A_{1,f}$) gain

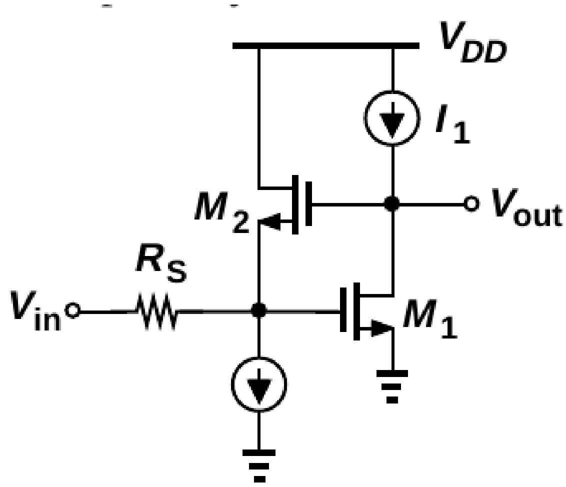
→ The component of the input in excess of 10mV but ≤ 50mV obtains A_2 ($A_{2,f}$) gain

→ The component of the input in excess of 50mV obtains no gain.

9.2 Polarity Determination

Polarity of Feedback

22. Determine the polarity of feedback in each of the stages illustrated in Fig. 12.87.



(a)

Notes: None.

Additional Tasks: None.

Necessary Knowledge and Skills: KCL, KVL, amplifier types, feedback polarity determination.

Razavi
12.22
a)

Assume

(*) $\Rightarrow v_{g1}$ inc.

$\Rightarrow v_{out}$ dec.

(**) $\Rightarrow v_{s2} = v_{g1}$ dec. (source follower)

(negative feedback)

Alternatively.

$\Rightarrow v_{g1}$ inc.

$\Rightarrow \frac{v_{in} - v_{g1}}{R_s}$ dec.

(*) $\Rightarrow i_{d2}$ ~~~~ inc. (due to the const. cur. source at the bottom)

$\Rightarrow v_{out}$ dec.

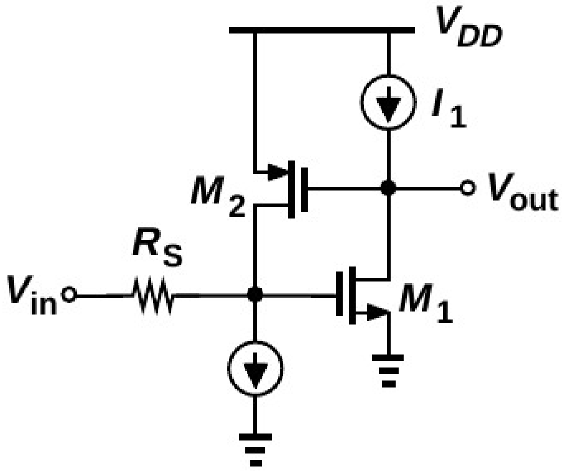
$\Rightarrow v_{gs2}$ dec.

(**) $\Rightarrow i_{d2}$ dec.

(again negative feedback)

Polarity of Feedback

22. Determine the polarity of feedback in each of the stages illustrated in Fig. 12.87.



(b)

Notes: None.

Additional Tasks: None.

Necessary Knowledge and Skills: KCL, KVL, amplifier types, feedback polarity determination.

Razavi
12.22
b)

Assume

$\Rightarrow v_{g1}$ incr.

$\Rightarrow \dfrac{v_{in} - v_{g1}}{R_S}$ dec.

(*) $\Rightarrow$ i_{d2} incr. (due to the constant cur. source at the bottom)

$\Rightarrow v_{out}$ dec.

$\Rightarrow v_{g2}$ inc

(**) $\Rightarrow$ i_{d2} incr.

(positive feedback)

Generated by CamScanner

Positive feedback example

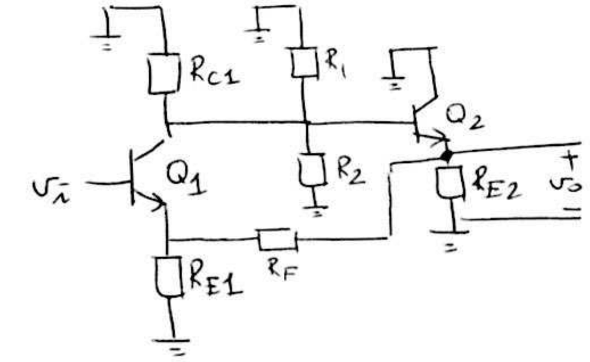
$\Rightarrow$ Voltage amplifier with CE – CC cascaded stages

$\Rightarrow$ Compute AV A, fl and β

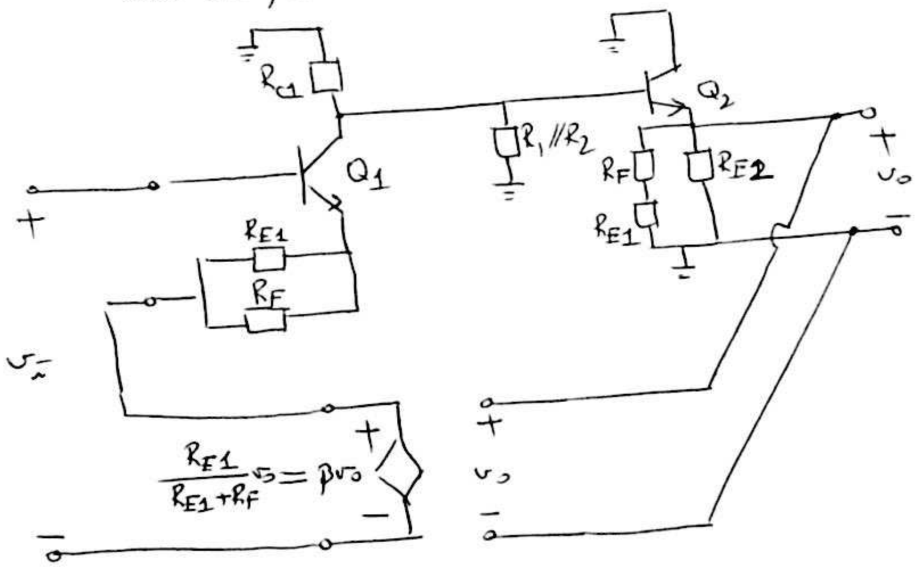
Worked on this on: Nov 18, 2014 / Tues

A positive feedback example

Consider the following CE−CC ampli:



Above is a voltage amplifier with voltage series feedback. Analyze the VA,fl (feedback-loaded voltage amplifier) and the feedback network:



Kill β ($\beta = 0$) above and compute the gain of the VA,fl:

1

$$A_{VA,fe} \cong \left[\frac{-g_{m1}}{1+g_{m1}(R_{E1}//R_F)}\right] \cdot \left[R_{C1} // R_1 // R_2 // \left[r_{\pi 2}(1+g_{m2}(R_{E2}//(R_{E1}+R_F)))\right]\right]$$

$$\cdot \frac{R_{E2}//(R_F+R_{E1})}{\frac{1}{g_{m2}} + R_{E2}//(R_F+R_{E1})}$$

Note that $A_{VA,fe} < 0$

but $\beta = \frac{R_{E1}}{R_{E1}+R_F} > 0$

Therefore $A_{VA,fe}\,\beta < 0$

so there is positive feedback in this circuit.

Instead consider constructing a CE-CE-CC stage of VA.

2

9.3 Types

Voltage Series Feedback

on

Voltage Amplifiers

⇒ Gain

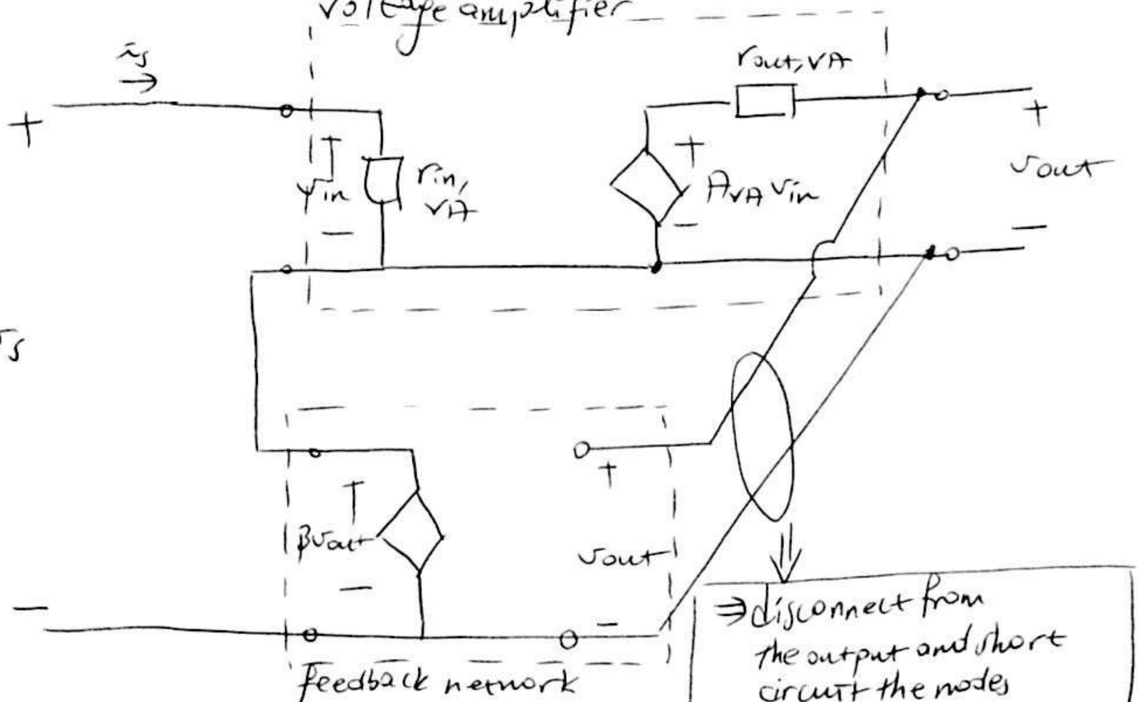
⇒ Input impedance

⇒ Output impedance

Calculations. Comments on the effect of feedback.

Worked on this on: Oct 20, 2014/Mon

Voltage series feedback on voltage amplifiers



⇒ disconnect from the output and short circuit the nodes
⇒ $v_{out} = 0$
⇒ $\beta v_{out} = 0$
⇒ we then have only the original voltage amplifier

Compute the gain $\overset{\circ\!-\!\circ}{A_f}$ of the feedback amplifier

$$\left.\begin{aligned} v_{in} &= v_s - \beta v_{out} \\ A_{VA} v_{in} &= v_{out} \end{aligned}\right\} \quad v_{in} + A_{VA} \beta v_{in} = v_s$$

$$\frac{v_{in}}{v_s} = \frac{1}{1 + A_{VA} \beta}$$

$$\Rightarrow \frac{v_{out}}{v_s} = \frac{A_{VA}}{1 + A_{VA} \beta}$$

Compute the input impedance of the feedback amplifier

$$r_{in,f} = \frac{v_s}{i_s} = ?$$

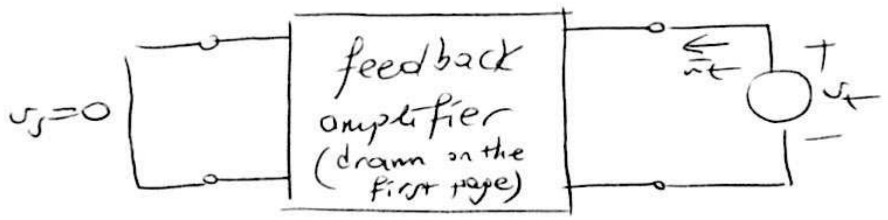
(output is open cir.)

$$\frac{v_s - \beta v_{out}}{r_{in,VA}} = i_s$$

$$\frac{v_s - \beta \dfrac{A_{VA}}{1 + A_{VA}\beta} v_s}{r_{in,VA}} = i_s$$

$$\frac{v_s}{i_s} = r_{in,VA}\left(1 + A_{VA}\beta\right)$$

Compute the output impedance of the feedback amplifier



$$\frac{v_t}{i_t} = r_{out,f}$$

$$\frac{v_t - A_{VA} v_{in}}{r_{out,VA}} = i_t$$

$$v_{out} = v_t$$

$$-v_{in} = \beta v_{out} = \beta v_t$$

$$v_{in} = -\beta v_t$$

$$\frac{v_t - A_{VA}(-\beta v_t)}{r_{out,VA}} = i_t \Rightarrow r_{out,f} = \frac{v_t}{i_t} = \frac{r_{out,VA}}{(1 + A_{VA}\beta)}$$

2

	without feedback	with feedback	change
Gain	A_{VA}	$\frac{A_{VA}}{1+A_{VA}\beta}$	decreased
Input Impedance	$r_{in,VA}$	$r_{in,VA}(1+A_{VA}\beta)$	increased
Output Impedance	$r_{out,VA}$	$\frac{r_{out,VA}}{1+A_{VA}\beta}$	decreased

All by the same factor. $(1+A_{VA}\beta)$

$\Rightarrow$ The voltage amplifier with feedback has properties that are closer to those of an ideal voltage ampli. (high input impedance, low output impedance)

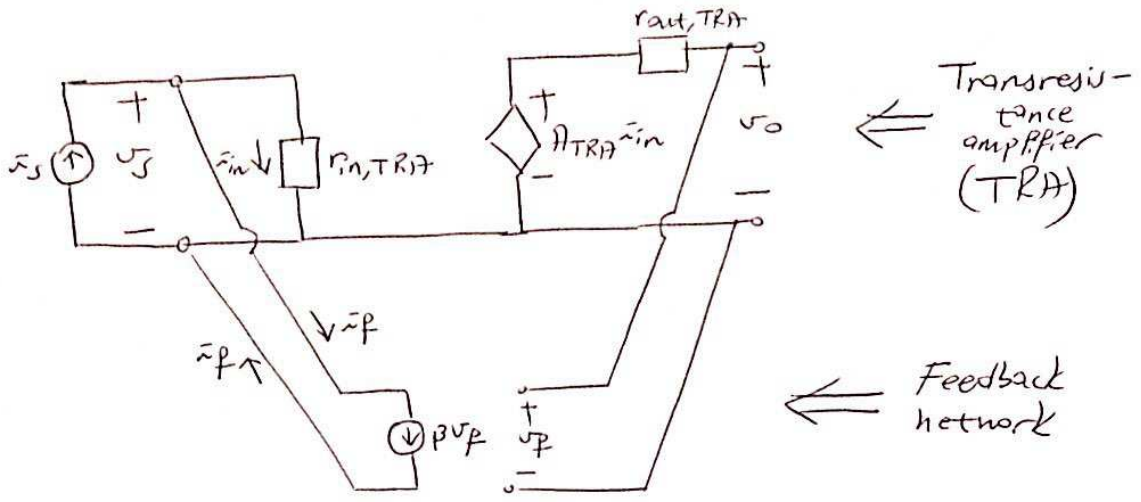
3

Voltage-Shunt
Feedback
on a
Trans-Resistance amplifier

⇒ Gain, input/output impedance calculations

Worked on this on: Oct 27, 2014/Mon

Voltage shunt feedback on a transresistance amplifier



equations

$$i_s = i_f + i_{in}$$

$$v_f = v_o$$

$$A_{TRA}\, i_{in} = v_o$$

$$i_f = \beta v_o$$

Gain of the feedback amplifier (A_f)

$$A_f = ? \qquad A_f = \frac{v_o}{i_s}$$

$$i_s = \beta \underbrace{v_o}_{A_{TRA} i_{in}} + i_{in}$$

$$i_s = \underbrace{i_{in}}_{\frac{v_o}{A_{TRA}}} \left(1 + \beta A_{TRA}\right)$$

$$A_f = \frac{v_o}{i_s} = \frac{A_{TRA}}{1 + \beta A_{TRA}}$$

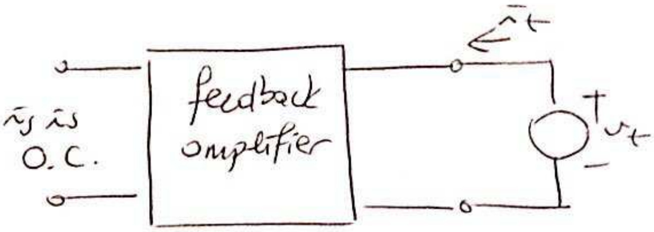
input resistance of the feedback amplifier ($r_{in,f}$)

$$r_{in,f} = \frac{v_s}{i_s}$$

$$\left.\begin{aligned} v_s &= i_{in}\, r_{in,TRA} \\ i_{in} &= \frac{i_s}{1 + \beta A_{TRA}} \end{aligned}\right\} \quad r_{in,f} = \frac{v_s}{i_s} = \frac{r_{in,TRA}}{1 + \beta A_{TRA}}$$

1

$r_{out,f}$ (Output resistance of the feedback amplifier)



$$r_{out,f} = \frac{v_t}{i_t}$$

$$i_f + i_{in} = 0$$

$$i_f = \beta v_t \Rightarrow i_{in} = -\beta v_t$$

$$\frac{v_t - A_{TRA}\, i_{in}}{r_{out,TRA}} = i_t \Longrightarrow r_{out,f} = \frac{v_t}{i_t} = \frac{r_{out,TRA}}{1+\beta A_{TRA}}$$

	TRA	Feedback amplifier	
Gain	A_{TRA}	$\frac{A_{TRA}}{1+\beta A_{TRA}}$	decreased
r_{in}	$r_{in,TRA}$	$\frac{r_{in,TRA}}{1+\beta A_{TRA}}$	decreased
r_{out}	$r_{out,TRA}$	$\frac{r_{out,TRA}}{1+\beta A_{TRA}}$	decreased

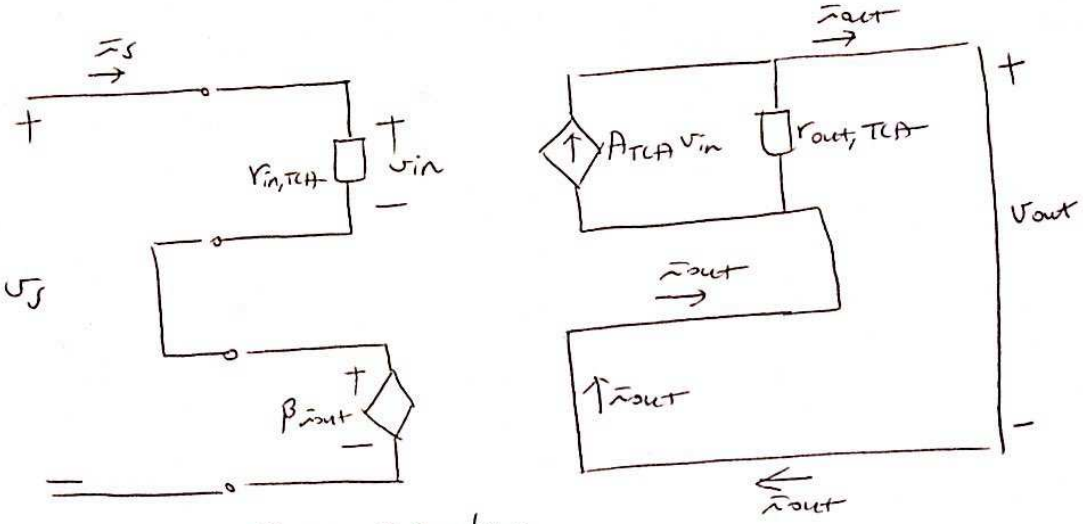
A transresistance amplifier should have low input/output impedances. Feedback decreases both input and output impedances.

Current–Series Feedback
on a Trans-Conductance Amplifier

⇒ Gain, I/O impedances computation

Worked on this on: Nov 5, 2014/Wed

Current-Series Feedback on TCA



Note: β in ohms
A_{TCA} is Siemens

Equations

$$\left.\begin{aligned} v_s &= v_{in} + \beta i_{out} \\ A_{TCA} v_{in} &= i_{out} \end{aligned}\right\} \quad \begin{aligned} v_s &= v_{in}(1+\beta A_{TCA}) \\ v_{in} &= \frac{i_{out}}{A_{TCA}} \end{aligned}$$

$$\Downarrow$$

$$v_s = \frac{i_{out}}{A_{TCA}}(1+\beta A_{TCA})$$

$$\Rightarrow A_f = \frac{i_{out}}{v_s} = \frac{A_{TCA}}{1+\beta A_{TCA}} \quad \left(\begin{array}{c}\text{gain of} \\ \text{the} \\ \text{feedback} \\ \text{ampli}\end{array}\right)$$

$$r_{in,f} = \frac{v_s}{i_s} = ?$$

$$v_{in} = i_s \cdot r_{in,TCA} \Rightarrow v_s = i_s \cdot r_{in,TCA}(1+\beta A_{TCA})$$

$$r_{in,f} = \frac{v_s}{i_s} = r_{in,TCA}(1+\beta A_{TCA})$$

 1

$r_{out,f}$ computation

$$r_{out,f} = -\frac{v_{out}}{i_{out}}$$ when $v_s = 0$ and v_{act} is connected at the output port.

$$\left.\begin{aligned} (i_{out} - A_{TCA} v_{in})\, r_{out,TCA} &= -v_{out} \\ v_{in} &= -\beta i_{out} \end{aligned}\right\} \quad i_{out}\left[1+\beta A\right] r_{out,TCA} = -v_{out}$$

$$r_{out,f} = -\frac{v_{out}}{i_{out}} = r_{out,TCA}(1+\beta A)$$

	Without feedback	With feedback
Gain	A_{TCA}	$\frac{A_{TCA}}{1+\beta A_{TCA}}$
Input impedance	$r_{in,TCA}$	$r_{in,TCA}(1+\beta A_{TCA})$
Output impedance	$r_{out,TCA}$	$r_{out,TCA}(1+\beta A_{TCA})$

Gain decreased by the improvement factor but I/O impedances multiplied by the same factor. We have obtained a better TCA.

2

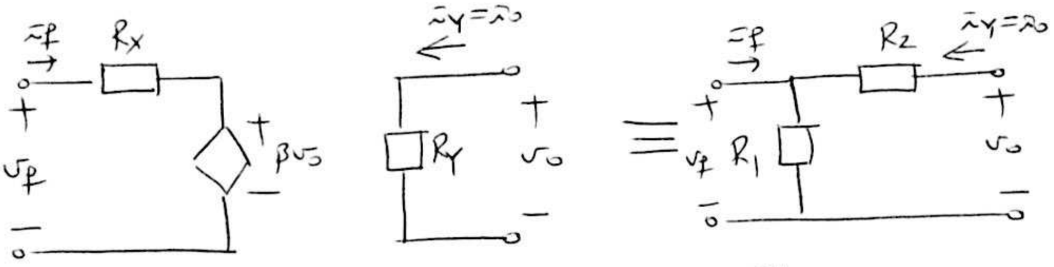
9.4 Modeling of Feedback Networks

Modelling a voltage series feedback network

used in voltage amplifiers

Worked on this on: Nov 19, 2014/Wed

Modeling a voltage series feedback network



Model　　　　　　　　Example

$$\beta = \frac{v_f}{v_o}\bigg|_{i_f=0} = \frac{R_1}{R_1+R_2}$$

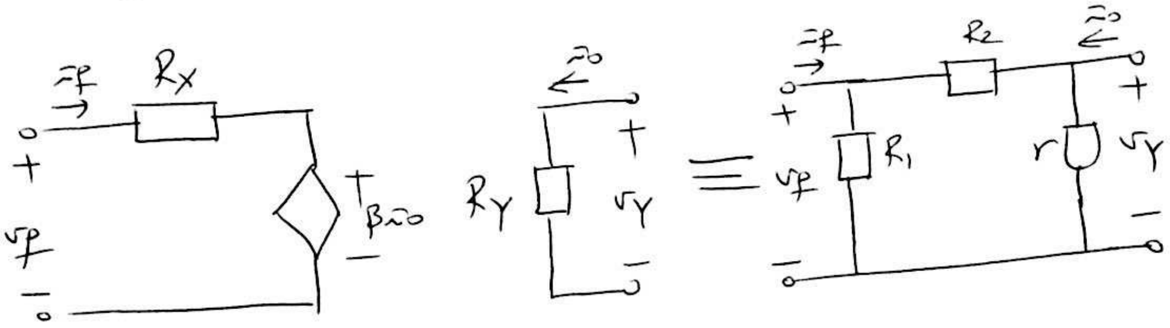
$$R_Y = \frac{v_o}{i_Y}\bigg|_{i_f=0} = R_1+R_2$$

$$R_x = \frac{v_f}{i_f}\bigg|_{v_o=0} = R_1 // R_2$$

Modelling a current-series feedback network (used in transconductance (TCA) amplifiers)

Worked on this on: Nov 19, 2014/Wed

Modeling a current series feedback network



Model   Example

$$\beta = \left.\frac{v_f}{i_o}\right|_{i_f=0} = \frac{r}{r+R_1+R_2}\cdot R_1$$

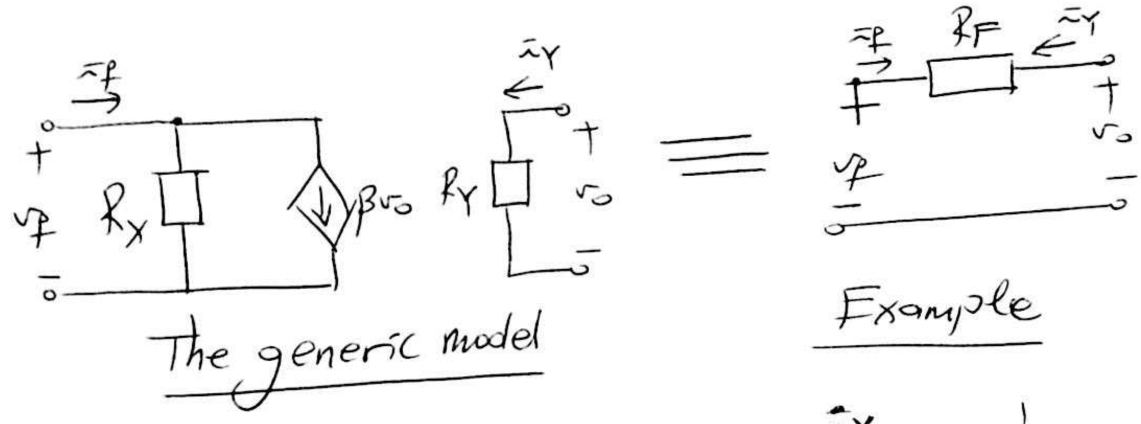
$$R_X = \left.\frac{v_f}{i_f}\right|_{i_o=0} = R_1 /\!/ (R_2+r)$$

$$R_Y = \left.\frac{v_Y}{i_o}\right|_{i_f=0} = r /\!/ (R_1+R_2)$$

Voltage-Shunt
Feedback Network
Modelling

Worked on this on: Nov 19, 2014/Wed

Analysis of the feedback networks used in voltage-shunt feedback circuits



$$\beta = \frac{i_f}{v_o}\bigg|_{v_f=0} \qquad = -\frac{i_Y}{v_o} = -\frac{1}{R_2}$$

$$R_Y = \frac{v_o}{i_Y}\bigg|_{v_f=0} \qquad = R_2$$

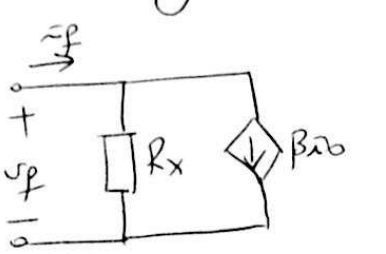
$$R_X = \frac{v_f}{i_f}\bigg|_{v_o=0} \qquad = R_2$$

Modeling a current-shunt feedback network

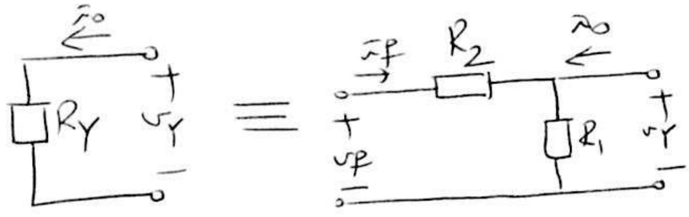
used in current amplifiers

Worked on this on: Nov 19, 2014 / Wed

Modeling a current-shunt feedback network



Model



Example

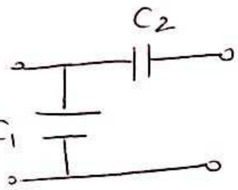
$$R_X = \left.\frac{v_f}{i_f}\right|_{i_o=0} = R_2 + R_1$$

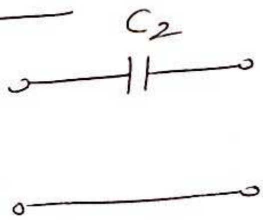
$$R_Y = \left.\frac{v_Y}{i_o}\right|_{v_f=0} = R_2 // R_1$$

$$\beta = \left.\frac{i_f}{i_o}\right|_{v_f=0} = -\frac{R_1}{R_1+R_2}$$

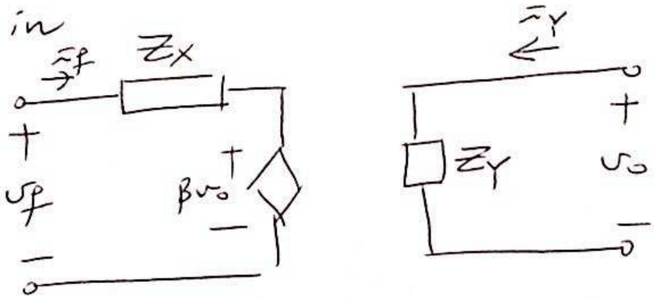
9.5 Auxiliaries

Analysis of capacitor feedback networks typically used in voltage-series (or series-shunt) feedback ~~net~~ circuits

$\Rightarrow$ Model 

and 

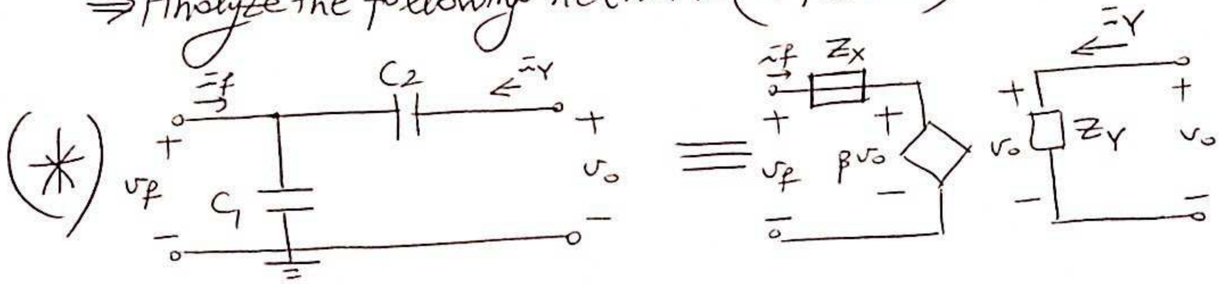
as in



Worked on this on: Nov 10, 2014 / Mon

Analysis of capacitor feedback networks

$\Rightarrow$ Analyze the following network (in feedback)



$$\beta = \left.\frac{v_f}{v_o}\right|_{i_f=0} = \frac{\frac{1}{C_1 s}}{\frac{1}{C_1 s} + \frac{1}{C_2 s}}$$

$$= \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{1/C_1}{\frac{C_1 + C_2}{C_1 C_2}}$$

$$= \frac{C_2}{C_1 + C_2}$$

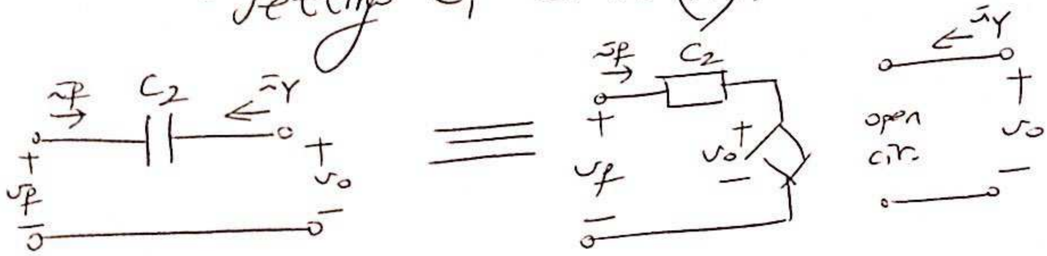
$$Z_Y = \left.\frac{v_o}{i_Y}\right|_{i_f=0} = \frac{1}{C_1 s} + \frac{1}{C_2 s} = \frac{1}{s} \frac{1}{\left(\frac{C_1 C_2}{C_1 + C_2}\right)}$$

$$Z_X = \left.\frac{v_f}{i_f}\right|_{v_o=0} = \frac{\frac{1}{C_1 s}\,\frac{1}{C_2 s}}{\frac{1}{C_1 s} + \frac{1}{C_2 s}} = \frac{1}{s}\,\frac{\frac{1}{C_1 C_2}}{\frac{C_1 + C_2}{C_1 C_2}}$$

$$= \frac{1}{s}\,\frac{1}{(C_1 + C_2)}$$

1

The following network is obtained by setting $C_1 = 0$ in (*):



$$\beta = \left.\frac{C_2}{C_1 + C_2}\right|_{C_1=0} = 1$$

$$Z_Y = \left.\frac{1}{s}\,\frac{1}{\frac{C_1 C_2}{C_1 + C_2}}\right|_{C_1=0} = +\infty$$

$$Z_X = \left.\frac{1}{s(C_1 + C_2)}\right|_{C_1=0} = \frac{1}{sC_2}$$

 2

9.6 Stability

Phase Margin as a Measure of Stability in Feedback Systems

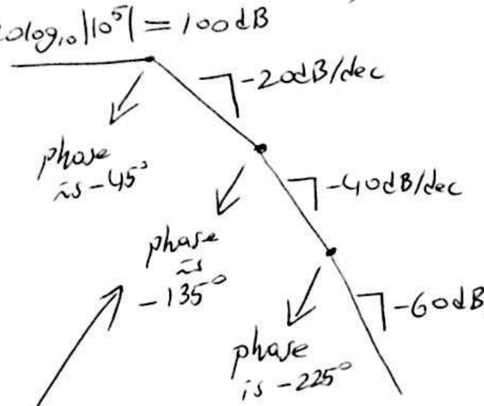
8.73 An amplifier has a dc gain of 10^5 and poles at 10^5 Hz, 3.16×10^5 Hz, and 10^6 Hz. Find the value of β, and the corresponding closed-loop gain, for which a phase margin of 45° is obtained.

Notes: None.

Additional Tasks: None.

Necessary Knowledge and Skills: Open-loop and closed loop gains in feedback, feedback factor, stability, phase margin as a stability measure, transfer functions, magnitude and phase Bode plots, phases at the poles of a transfer function when the poles are far apart.

Magnitude Bode plot for $A(jf)$



Sedra 8.73

The asymptotic Bode plot and the phase values are accurate if the poles of $A(jf)$ are far apart.

→ β is given to be indep of frequency, positive and real.

→ Note that

$$20\log_{10}|A(jf)\beta| = 20\log_{10}|A(jf)| - 20\log_{10}\left|\frac{1}{\beta}\right|$$

$$\angle A(jf)\beta = \angle A(jf) \quad \text{since } \beta \text{ is real and positive.}$$

→ Phase margin to be 45°

then $|A(jf_u)\beta| = 1$ and $\angle A(jf_u)\beta = -(180^\circ - 45^\circ) = -135^\circ$

Since β is real and positive, the unity gain frequency f_u (for $A(jf)\beta$) needs to be at the second pole of $A(jf)$. See here.

→ Note that

$$A(jf) = \frac{10^5}{\left(1 + \frac{jf}{10^5}\right)\left(1 + \frac{jf}{3.16\times10^5}\right)\left(1 + \frac{jf}{10^6}\right)}$$

1

Sedra 8.73 contin.

Now solve for β in

$$\left|A(jf_u)\beta\right| = \left|\frac{10^5\beta}{\left(1+\frac{jf_u}{10^5}\right)\left(1+\frac{jf_u}{3.16\times10^5}\right)\left(1+\frac{jf_u}{10^6}\right)}\right| = 1$$

where $f_u = 3.16\times10^5$ Hz (the second pole)

then

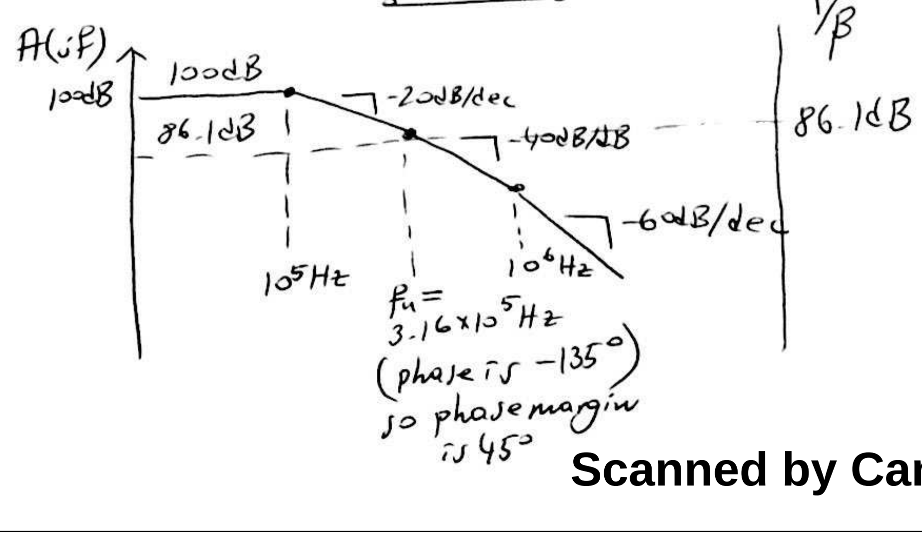
$$1 = \frac{10^5\beta}{\sqrt{1+\left(\frac{3.16\times10^5}{10^5}\right)^2}\sqrt{1+\left(\frac{3.16\times10^5}{3.16\times10^5}\right)^2}\sqrt{1+\left(\frac{3.16\times10^5}{10^6}\right)^2}}$$

$$= \frac{10^5\beta}{\sqrt{1+(3.16)^2}\ \sqrt{2}\ \sqrt{1+(0.316)^2}}$$

$\beta = 4.9\times10^{-5}$

then $\frac{1}{\beta} = 2.03\times10^4$

and $20\log_{10}\left|\frac{1}{\beta}\right| = 86.1$ dB



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②

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8.73
contin.

Midband gain with feedback

$$A_f = \frac{10^5}{1+10^5(4.9\times10^{-5})} \simeq 16.9\times10^3$$

$$= \frac{A}{1+A\beta}$$

(3)

Stability Analysis through the Phase Margin

10.56 If the phase margin of an amplifier is PM $= 40°$ and the magnitude of the open-loop gain is $|A(j\omega)| = 50$, find the magnitude of the closed-loop gain $|A_f(j\omega)|$.

Notes: Assume a two-pole system for modeling the high-frequency response of the open-loop amplifier. Note that the ratio between these two poles is not given.

Additional Tasks: Observe and conclude that if the ratio between the two poles of the system had been given, there would be no missing information and you could have solved for the feedback factor.

Necessary Knowledge and Skills: Transfer function in terms of poles and midband gain, magnitude and phase computation through the transfer function, open-loop transfer function assembly, loop gain calculation for analyzing stability, phase margin calculation through the loop gain, Bode plot illustrations of the phase margin.

$\Rightarrow$ assume that the open-loop amplifier is a two-pole system and β is real.

Rashid 10.56

Then

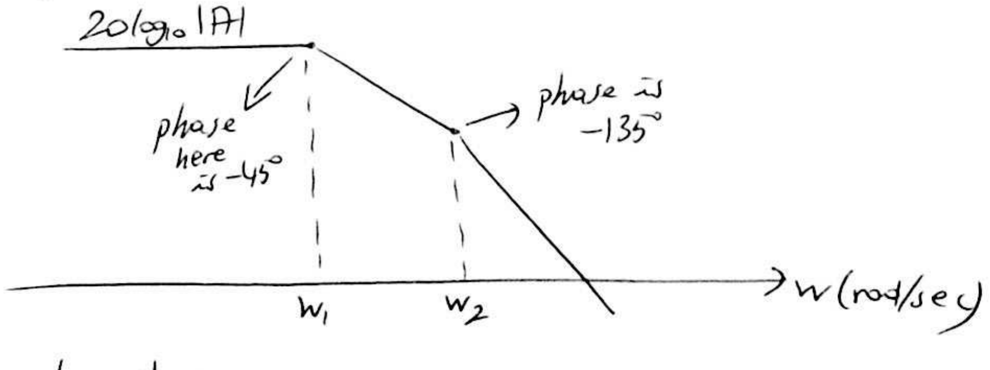
$$A(jw) = \frac{A^{\Rightarrow \text{given as } 50.}}{\left(1+\frac{jw}{w_1}\right)\left(1+\frac{jw}{w_2}\right)}$$

we are assuming also that $w_1 << w_2$.

$\Rightarrow$ Note that

$$20\log_{10}|A(jw)\beta| = 20\log_{10}|A(jw)| - 20\log_{10}\left|\frac{1}{\beta}\right|$$

$\Rightarrow$ Bode plot of $A(jw)$ is:
(asymptotic magnitude)



note here that

$$A(jw_1) \simeq \frac{A}{\left(1+\frac{jw_1}{w_1}\right)(1)} \Rightarrow \text{phase is } -45^\circ$$

$$A(jw_2) \simeq \frac{A}{\left(\frac{jw_2}{w_1}\right)\left(1+\frac{jw_2}{w_2}\right)} \Rightarrow \text{phase is } -135^\circ$$

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Rashid
1056
contin.

We would like to set the phase margin to 40° ($PM = 40^\circ$) by designing β.

Phase margin is $\Rightarrow PM = 180^\circ - \left| \underbrace{\angle A(j w_u)\beta}_{\text{phase of } A(jw_u)\beta} \right|$

phase of $A(jw_u)\beta$ at w_u: unity gain frequency for $\underbrace{T_L}_{\text{loopgain}}(jw) = A(jw)\beta$

Note that there is no contribution to $T_L(jw) = A(jw)\beta$ in terms of phase by β since β is positive and real.

Therefore, we take into consideration the phase of $A(jw)$ only. But β contributes to $T_L(jw)$ in terms of magnitude.

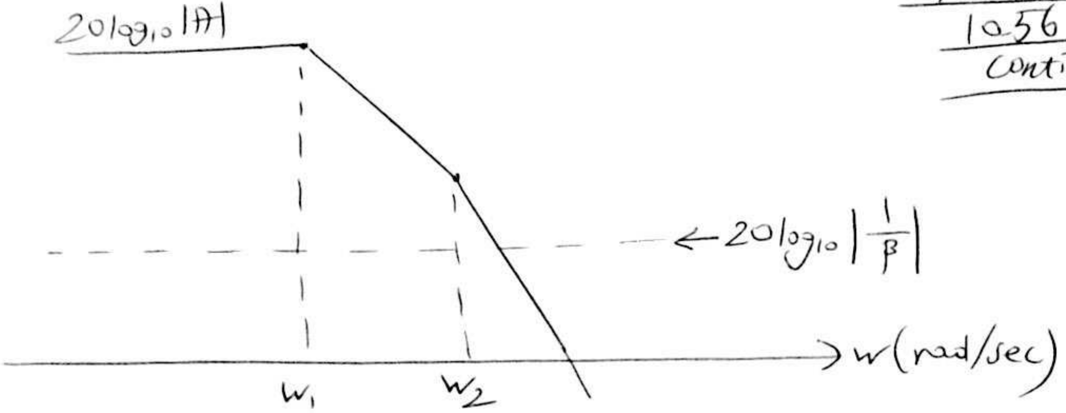
Remember
$$20\log_{10}|A(jw)\beta| = 20\log_{10}|A(jw)| - 20\log_{10}\left|\frac{1}{\beta}\right|$$

Therefore the difference of the two magnitude Bode plots is equal to $20\log_{10}|T_L(jw)|$

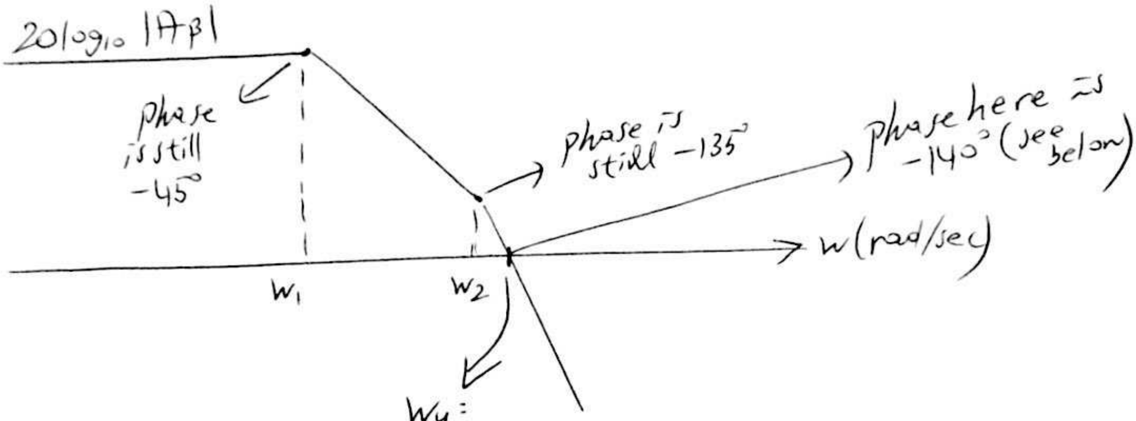
$$= 20\log_{10}|A(jw)\beta|$$

See the next page for the Bode plots.

Rashid
1056
Contin-



Then the difference will be



Since the phase margin PM is

$$PM = 180° - \left|\angle A(jw_u)\beta\right| = 180° - \left|\angle T_L(jw_u)\right|$$

$$= 40° \Rightarrow \text{given in the question}$$

Then $\left|\angle T_L(jw_u)\right| = 140°$

and $\angle T_L(jw_u) = -140°$

(3)

Rashid
10.56
contin.

Note that we have

$$T_L(j w_u) = \frac{A\beta}{\left(1+\frac{j w_u}{w_1}\right)\left(1+\frac{j w_u}{w_2}\right)}$$

$$\simeq \frac{A\beta}{\left(\frac{j w_u}{w_1}\right)\left(1+\frac{j w_u}{w_2}\right)}$$

Since $w_u \gg w_1$
but $w_u > w_2$ only.

Then

$$\angle T_L(j w_u) = -90^\circ - \arctan\left(\frac{w_u}{w_2}\right)$$

$$= -140^\circ$$

and $\arctan\left(\frac{w_u}{w_2}\right) = 50^\circ$

$$\frac{w_u}{w_2} \simeq \cancel{1.551}\ 1.19 \Longrightarrow \underbrace{w_u = \cancel{1.551}\ 1.19\, w_2}$$

And by definition we have

$$|T_L(j w_u)| = 1$$

$$\simeq \frac{A\beta}{\underbrace{\left(\frac{w_u}{w_1}\right)}_{\text{unknown}}\underbrace{\sqrt{1+\frac{w_u^2}{w_2^2}}}_{\text{can be computed from here}}}$$

$A\beta \to$ given as 50
$\to$ to be designed

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4

Then

$$\beta \cong \frac{\overbrace{\left(\frac{w_u}{w_1}\right)\sqrt{1+\frac{w_u^2}{w_2^2}}}^{\cancel{1.855}\ 1.55}}{\underbrace{A}_{50}}$$

Rashid
10.56
contin.

if we had been given $\frac{w_2}{w_1}$

we could compute

$$\frac{w_u}{w_1} = \frac{w_2}{w_1}\left(\frac{w_u}{w_2}\right) \rightarrow \cancel{1.551}\ 1.19$$

Then we could obtain the design value for β.

And the closed-loop gain would be (at low frequencies)

$$A_f = \frac{A}{1+\beta A}$$

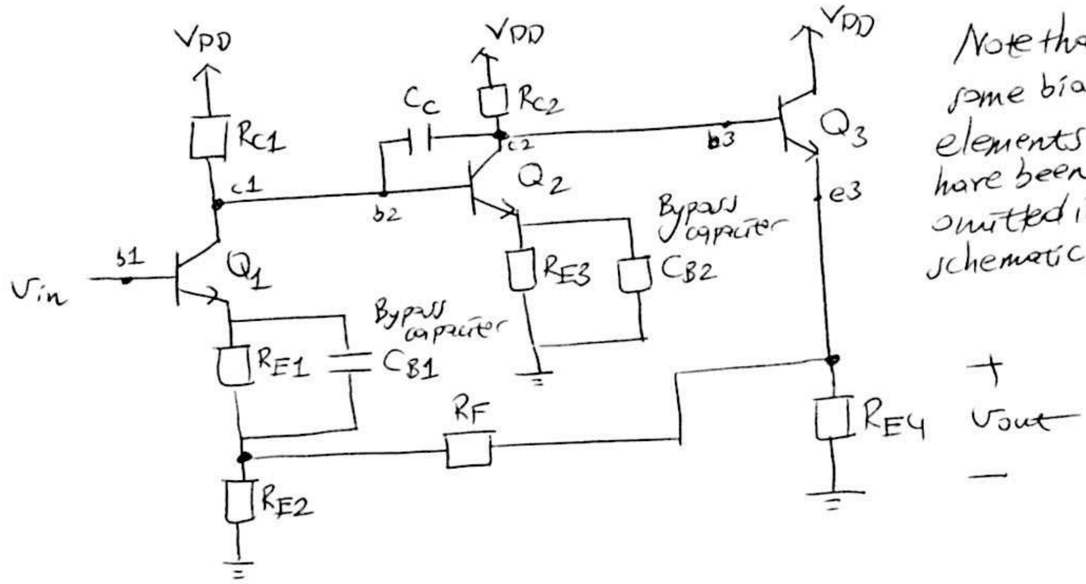
(5)

9.7 Compensation

Example of a bandwidth-limiting compensation on a voltage amplifier with voltage-series feedback

Worked on this on: Nov 26, 2014/Wed

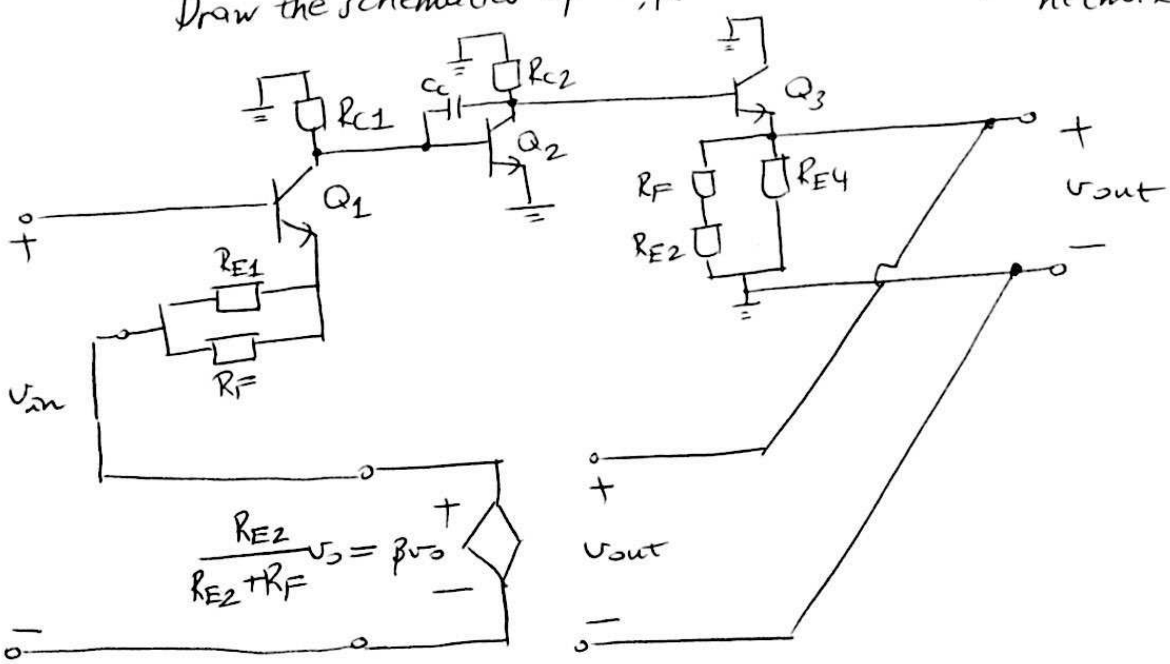
Example of a bandwidth limiting compensation on a voltage amplifier with voltage-series feedback



Note that some biasing elements have been omitted in this schematic.

Above C_c is a small compensating capacitance.
There is negative feed back in the circuit above,
and it is a voltage amplifier with voltage series feedback.

Draw the schematics of VA_{FL} and the idealized feedback network. (small-signal)



 1

Analyze VA, FL by killing β (β=0).

$$A_{VA,FL} = \left.\frac{v_{out}}{v_{in}}\right|_{\beta=0} = \frac{v_{c1}}{v_{b1}}\underbrace{\frac{v_{c2}}{v_{b2}}}\frac{v_{e3}}{v_{b3}}$$

this the gain stage (CE with no source degeneration in small-signal)

Note $\Rightarrow$ Therefore Miller multiplication of the compensating cap. is to be considered.

$$A_{VA,FL} \cong \left[\frac{-g_{m1}}{\left[1+g_{m1}\left[R_{E1}//R_F\right]\right]}\right]\left[R_{C1}//\left[r_{o1}\left(1+g_{m1}\left(R_{E1}//R_F\right)\right)\right]//r_{\pi 2}\right]$$

$$\cdot\left[-g_{m2}\right]\left[R_{C2}//r_{o2}//\left[r_{\pi 3}\left(1+g_{m3}\left(R_{E4}//\left(R_F+R_{E2}\right)\right)\right)\right]\right]$$

$$\cdot\frac{R_{E4}//\left[R_F+R_{E2}\right]}{\frac{1}{g_{m3}}+R_{E4}//\left[R_F+R_{E2}\right]}$$

 2

C_c will be designed such that its time constant in OCTC analysis will dominate all the others

therefore

$$W_{H,VA,fe} \cong \frac{1}{\tau_{C_c}}$$

where $\tau_{C_c} = R_{C_c} C_c$

$$R_{C_c} = \left(1 - \frac{v_{c2}}{v_{b2}}\right) C_c \cdot R_{C1} // \left[r_{o1}\left(1 + g_{m1}\left(R_{E1} // R_F\right)\right) \right] // r_{\pi 2}$$

recall that $\frac{v_{c2}}{v_{b2}} = \left[-g_{m2}\right]\left[R_{C2} // r_{o2} // \left[r_{\pi 3}\left(1 + g_{m3}\left(R_{E4} // \left(R_F + R_{E2}\right)\right)\right)\right]\right]$

Therefore we have

$$A_{VA,fe}(j\omega) \cong \frac{A_{VA,fe}}{1 + \frac{j\omega}{\omega_{H,VA,fe}}}$$

with $\omega_{H,VA,fe}$ having become the dominant pole. C_c limits the bandwidth but also is used to adjust the gain and phase margins of the loop gain

$$T_L(j\omega) = A_{VA,fe}(j\omega)\,\beta$$

 3

Calculate the input and output impedances of VA, FL

$$r_{in,VA,FL} \cong r_{\pi 1}\left[1 + g_{m1}\left(R_{E1} // R_F\right)\right]$$

$$r_{out,VA,FL} \cong R_{E4} // \left(R_F + R_{E2}\right) // \frac{r_{\pi 3} + \left(R_{C2} // r_{o2}\right)}{1 + \beta_3}$$

Analysis of the feedback amplifier (VA, FL and the idealized feedback network)

$$A_f = \frac{A_{VA,FL}}{1 + \beta A_{VA,FL}} \Longrightarrow A_f(jw) = \frac{\dfrac{A_{VA,FL}}{1 + \beta A_{VA,FL}}}{1 + \dfrac{jw}{w_{H,VA,FL}\left(1 + \beta A_{VA,FL}\right)}}$$

$$w_{H,f} = w_{H,VA,FL}\left(1 + \beta A_{VA,FL}\right)$$

$$r_{in,f} = r_{in,VA,FL}\left(1 + \beta A_{VA,FL}\right)$$

$$r_{out,f} = \frac{r_{out,VA,FL}}{1 + \beta A_{VA,FL}}$$

Due to the properties of voltage-series feedback

4

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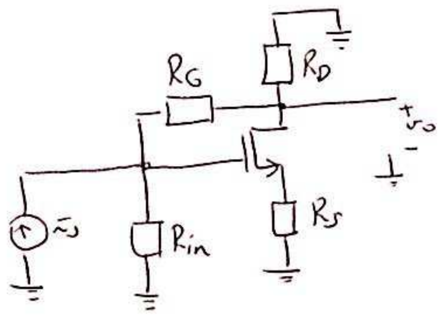
9.8 Analysis in Practical Circuits

Application of Voltage shunt feedback on a transresistance amplifier and related derivations

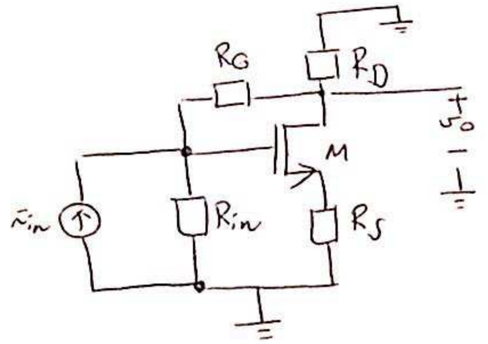
$\Rightarrow$ Applied the analysis techniques on



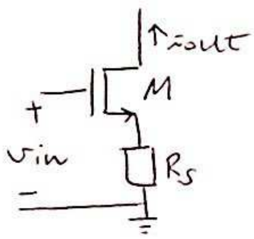
With both of $\rightarrow R_{in} \rightarrow +\infty$ and $\rightarrow R_{in} < +\infty$

Worked on this on: Oct 27, 2014/Mon

⇒ Analysis of the following amplifier as a transresistance amplifier with voltage-shunt feedback



→ Note that the small signal model for the transistor M should be used in the schematic.

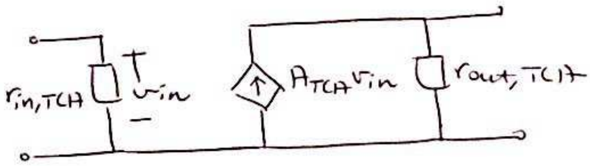


→ We are already acquainted with this circuit as a transconductance amplifier.

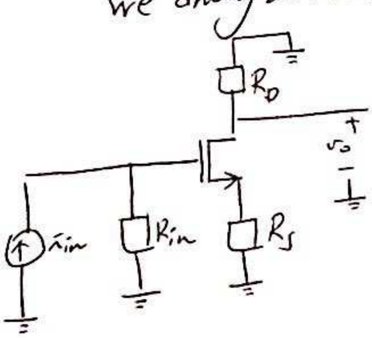
$$r_{in,TCA} = +\infty$$

$$A_{TCA} \cong -\frac{g_m}{1+g_m R_S}$$

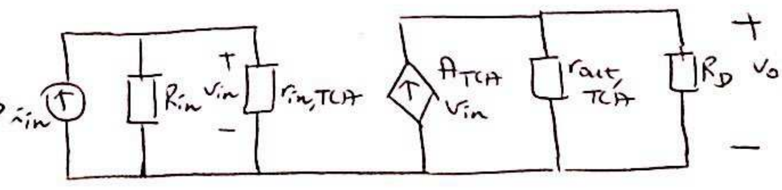
$$r_{out,TCA} \cong (1+g_m R_S) r_o$$



To the TCA above, with the addition of R_{in} and R_D, we analyze the resulting amplifier as a transresistance amplifier.



⇒



1

$v_{in} = i_{in} R_{in}$ (since $r_{in,TCA} = +\infty$)

$v_{out} = A_{TCA}\, i_{in} R_{in} \left(r_{out,TCA} // R_D\right)$

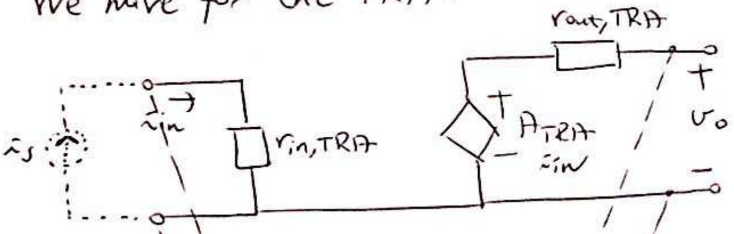
Then the following parameter values can easily be calculated for the transresistance amplifier (TRA).

$$A_{TRA} = -\frac{g_m}{1+g_m R_s} R_{in} \underbrace{\left[r_{out,TCA} // R_D \right]}_{r_{out,TRA}} = \frac{v_{out}}{i_{in}}$$

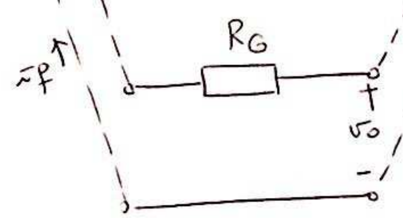
$r_{in,TRA} = R_{in}$

$r_{out,TRA} = r_{out,TCA} // R_D$ ← see

We have for the TRA:



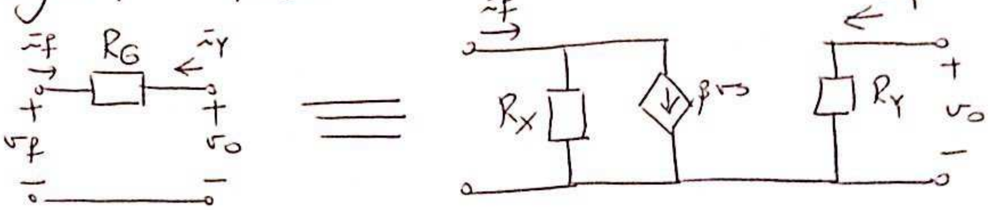
And we have for the feedback network:



⇒ Dashed (- - -) lines are the connections between the TRA and the feedback network.

⇒ The circuit is fully operational with the inclusion of the (dotted ···) source.

⇒ Analysis of the feedback network:



$$R_X = \frac{v_f}{i_f}\Big|_{v_o=0} = R_G$$

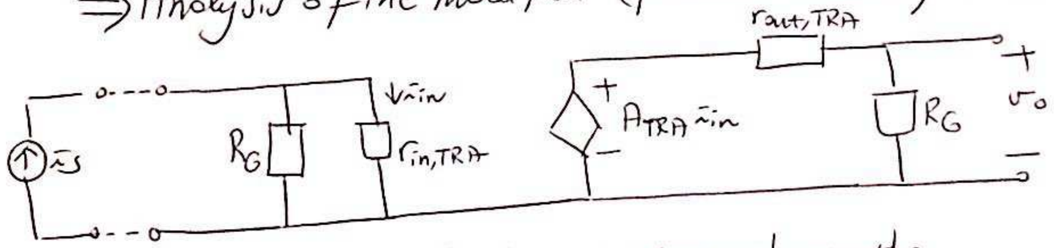
$$R_Y = \frac{v_o}{i_Y}\Big|_{v_f=0} = R_G$$

$$\beta = \frac{i_f}{v_o}\Big|_{v_f=0} \Longrightarrow -i_f R_G = v_o$$

therefore

$$\beta = \frac{i_f}{v_o} = -\frac{1}{R_G}$$

⇒ Analysis of the modified (feedback-loaded) TRA:



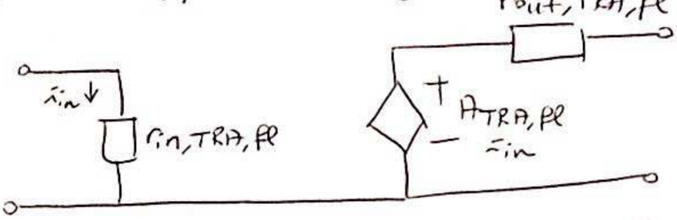
· Note that above, the shunt resistances due to the feedback network now load the forward TRA amplifier. Analyzing the new amplifier (TRA, fl) ⇒ fl for feedback-loaded network.

$$A_{TRA,fl} = \frac{R_G}{R_G + r_{in,TRA}}\, A_{TRA}\, \frac{R_G}{R_G + r_{out,TRA}}$$

$$r_{in,TRA,fl} = R_G \,//\, r_{in,TRA}$$

$$r_{out,TRA,fl} = R_G \,//\, r_{out,TRA}$$

$\Rightarrow$ Before analyzing the feedback amplifier (TRA,fe + feedback network), analyze TRA,fe more closely:



• Let $R_{in} \to +\infty$. Note that $r_{in,TRA} = R_{in}$.

then

$$r_{in,TRA,fe} = R_G \,//\, r_{in,TRA} = R_G \,//\, +\infty = R_G$$

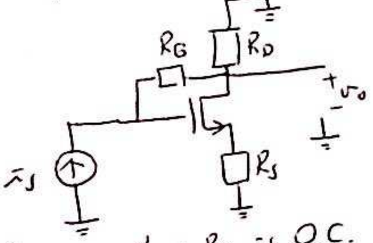
• Analyze $A_{TRA,fe}$ as well for $R_{in} \to +\infty$

$$A_{TRA,fe} = \frac{R_G}{R_G + R_{in}}\left[\frac{-g_m}{1+g_m R_S}\right] R_{in}\, r_{out,TRA}\, \frac{R_G}{R_G + r_{out,TRA}}$$

$$= \left[\frac{R_G(-g_m)}{1+g_m R_S}\right]\left[\frac{R_G\, r_{out,TRA}}{R_G + r_{out,TRA}}\right]\left[\frac{R_{in}}{R_G + R_{in}}\right]$$

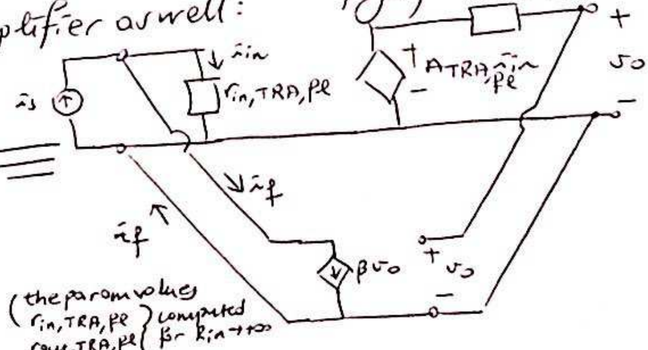
$$\lim_{R_{in}\to+\infty} A_{TRA,fe} = \left[\frac{-g_m R_G}{1+g_m R_S}\right]\left[R_G \,//\, r_{out,TRA}\right]$$

Therefore our analysis will be valid (with the parameter values given on this page) for the following amplifier as well:



(Notice that R_{in} is O.C. here. $R_{in} \to +\infty$)

$\equiv$



(the param values $r_{in,TRA,fe}$, $r_{out,TRA,fe}$, $A_{TRA,fe}$ computed for $R_{in} \to +\infty$)

(4)

⇒ In both cases (with $R_{in} \to +\infty$ or not) the analysis of the feedback amplifier (~~A~~ TRA_{fe} + feedback network (nonidealized)) proceeds as follows:

- Refer to the figure at the bottom of pg 4 (feedback ampli.)

- $$\beta = -\frac{1}{R_G}$$

-

	$R_{in} \to +\infty$	$R_{in} < +\infty$
$A_{TRA,fe}$ $r_{in,TRA,fe}$ $r_{out,TRA,fe}$	see pg 4	see pg 3

- See the note (a separate document) on voltage-shunt feedback on transresistance amplifiers. OR read Rashid pg 677-686.

We have of the feedback amplifier:

$$A_f = \frac{A_{TRA,fe}}{1+\beta A_{TRA,fe}}$$

$$r_{in,f} = \frac{r_{in,TRA,fe}}{1+\beta A_{TRA,fe}}$$

$$r_{out,f} = \frac{r_{out,TRA,fe}}{1+\beta A_{TRA,fe}}$$

Note that $A_{TRA,fe} < 0$ and $\beta < 0$, but $1+\beta A_{TRA,fe} > 0$ therefore negative feedback applies.

Voltage-Series Feedback Example

25. Consider the feedback circuit shown in Fig. 12.88, where $R_1 + R_2 \gg R_D$. Compute the

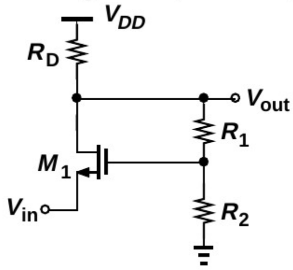


Figure 12.88

closed-loop gain and I/O impedances of the circuit. Assume $\lambda \neq 0$.

Notes: None.

Additional Tasks: None.

Necessary Knowledge and Skills: Voltage-Series Feedback, MOS small signal model, input/output impedance and voltage gain computation.

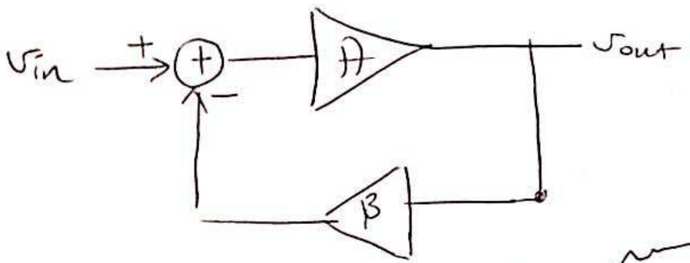
Razavi
12.25

With the condition

$$R_1 + R_2 \gg R_D$$

the current through R_1 and R_2 will be much less than that through R_D.

The circuit can then be accurately modeled by



where $A = g_m R_D$

$$\beta = \frac{R_2}{R_1 + R_2}$$

$$\Delta I_{D1} = \Delta V \cdot g_{m1}$$

$$-R_D \Delta I_{D1} = V_{out}$$

$$\Delta V = V_{out} \frac{R_2}{R_1 + R_2} - V_{in}$$

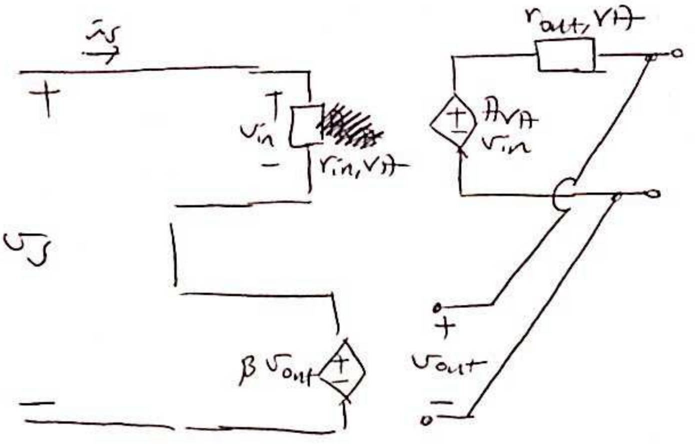
then

$$-R_D \left[V_{out} \frac{R_2}{R_1 + R_2} - V_{in} \right] g_{m1} = V_{out}$$

$$g_{m1} R_D V_{in} = V_{out} \left[1 + g_{m1} R_D \frac{R_2}{R_1 + R_2} \right]$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} R_D}{1 + g_{m1} R_D \frac{R_2}{R_1 + R_2}} = \frac{A}{1 + A\beta}$$ (see the model above)

Razavi
12.25
cont.



$$v_S - \beta v_{out} = v_{in}$$

$$A_{VA} v_{in} = v_{out} \Rightarrow A_{VA}(v_S - \beta v_{out}) = v_{out}$$

$$A_{VA} v_S = (A_{VA}\beta + 1) v_{out}$$

$$\frac{v_{out}}{v_S} = \frac{A_{VA}}{1 + A_{VA}\beta}$$

compute $\dfrac{v_S}{i_S} = r_{in,f}$ (r_{in} with feedback)

$$\frac{v_S - \beta v_{out}}{r_{in,VA}} = i_S = \frac{v_S - \beta \dfrac{A_{VA}}{1 + A_{VA}\beta} v_S}{r_{in,VA}} = i_S$$

$$\Rightarrow \frac{v_S}{r_{in,VA}(1 + A_{VA}\beta)} = i_S$$

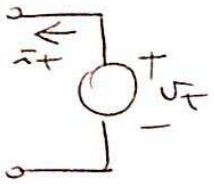
$$\Rightarrow \frac{v_S}{i_S} = r_{in,f} = r_{in,VA}(1 + A_{VA}\beta)$$

input imp multiplied by a large factor

2

Razavi 12.25 cont-

Compute $\frac{v_t}{i_t}$



(connect the source at the output and set $v_s = 0$)

$$\beta v_{out} = \beta v_t = -v_{in}$$

$$v_t - i_t\, r_{out,vA} = A_{vA} \underbrace{v_{in}}_{-\beta v_t}$$

$$v_t(1 + A_{vA}\beta) = i_t\, r_{out,vA}$$

$$\frac{v_t}{i_t} = \frac{r_{out,vA}}{1 + A_{vA}\beta} \Rightarrow \text{this is } r_{out,f}$$

(output imp. with feedback) decreased by the same factor with respect to $r_{out,vA}$

This config. is called voltage series feedback

$$A_{vA} \rightarrow \frac{A_{vA}}{1 + A_{vA}\beta}$$

$$r_{out,vA} \rightarrow \frac{r_{out,vA}}{1 + A_{vA}\beta}$$

$$r_{in,vA} \rightarrow (1 + A_{vA}\beta)\, r_{in,vA}$$

Note that $r_{in,vA} \cong \frac{1}{g_{m1}}$ $\quad r_{out,vA} \cong R_D$ $\quad A_{vA} \cong g_{m1} R_D$

$$\beta = \frac{R_2}{R_1 + R_2}$$

3

Worked on this on: Nov 5, 2014 / Wed

Analysis of the Emitter-Degenerated Common Emitter Configuration as a Trans-Conductance Amplifier with Current-Series (or Series-Series) Feedback

⇒ Schematics

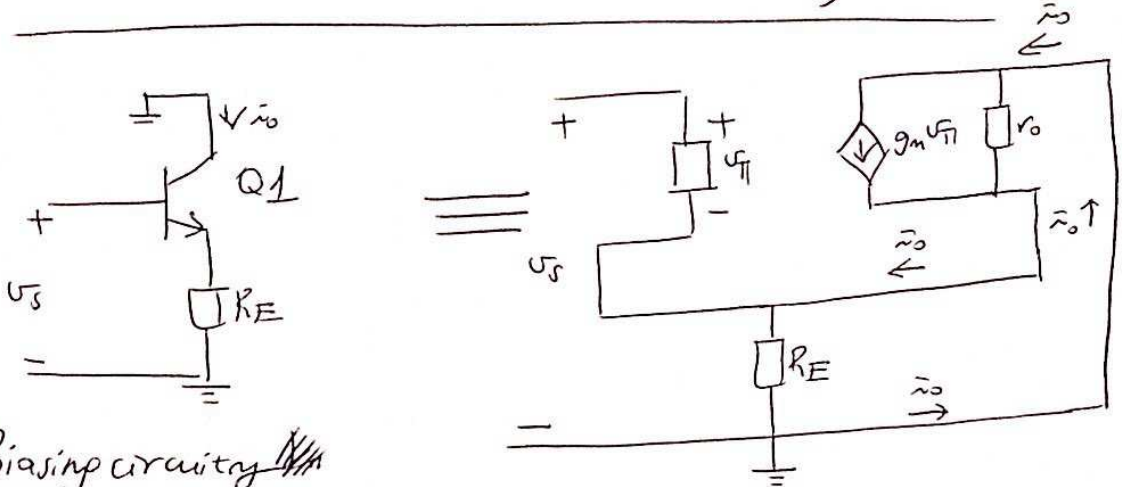
⇒ Identification of the feedback network

⇒ Modelling (computing the parameters) of the feedback network

⇒ Analysis of the feedback-loaded TCA (TCA,fl)
(This is a non-feedback or forward amplifier)

⇒ Applying the idealized feedback network to the TCA,fl and computing the parameters of the feedback amplifier

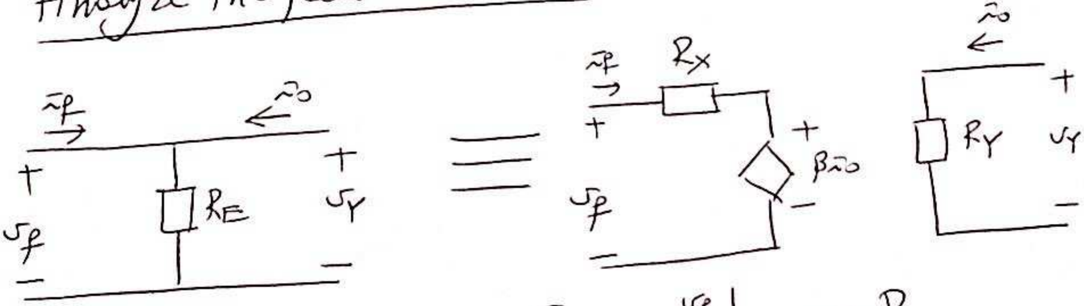
Analysis of the emitter-degenerated common-emitter configuration as a Trans-Conductance amplifier with current-series (or series-series feedback)



Biasing circuitry not shown - already in small-signal

TCA with the feedback network

Analyze the feedback network



See Rashid section 10.7 for similar derivations

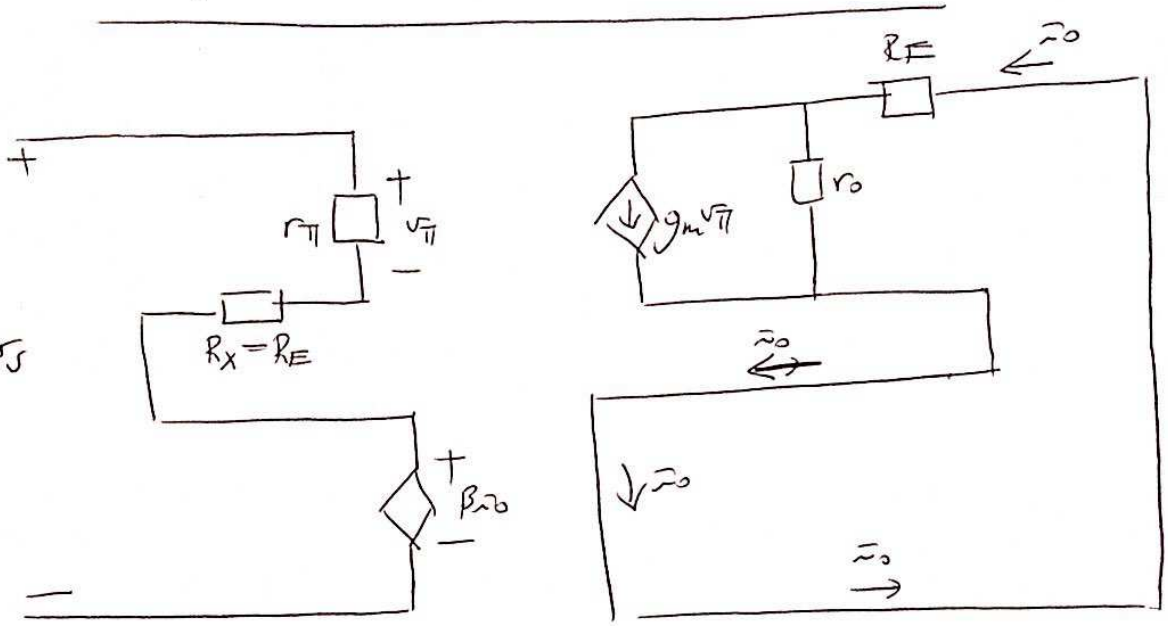
$$\beta = \left.\frac{v_f}{i_o}\right|_{i_f=0} = R_E$$

$$R_X = \left.\frac{v_f}{i_f}\right|_{i_o=0} = R_E$$

$$R_Y = \left.\frac{v_Y}{i_o}\right|_{i_f=0} = R_E$$

1

Draw the feedback-loaded TCA with the idealized feedback network



In the above schematic, set $\beta = 0$ and compute

(gain of the feedback loaded TCA) $$A_{TCA,fl} = \frac{i_o}{v_s} = \frac{r_\pi}{r_\pi + R_E}\, g_m \,\frac{r_o}{r_o + R_E}$$

$$r_{in,TCA,fl} = r_\pi + R_E$$

$$r_{out,TCA,fl} = r_o + R_E$$

Note that above we assume $r_\pi \gg R_E$, $r_o \gg R_E$

then $A_{TCA,fl} \cong g_m$

$r_{in,TCA,fl} \cong r_\pi$

$r_{out,TCA,fl} \cong r_o$

2

The improvement factor of the current series feedback

$$1 + \beta A_{TCA,fe} \cong 1 + g_m R_E$$

Current-series feedback properties:

gain	$A_{TCA,fe} \cong g_m \implies A_f = \dfrac{A_{TCA,fe}}{1 + A_{TCA,fe}\beta} \cong \dfrac{g_m}{1+g_m R_E}$
input impedance	$r_{in,TCA,fe} \cong r_\pi \implies r_{in,f} = r_{in,TCA,fe}(1 + A_{TCA,fe}\beta) \cong r_\pi(1+g_m R_E)$
output impedance	$r_{out,TCA,fe} \cong r_o \implies r_{out,f} = r_{out,TCA,fe}(1 + A_{TCA,fe}\beta) \cong r_o(1+g_m R_E)$

Gain is decreased by the improvement factor, but both i/o impedances are multiplied by the same factor. We have obtained a better TCA at the expense of some gain.

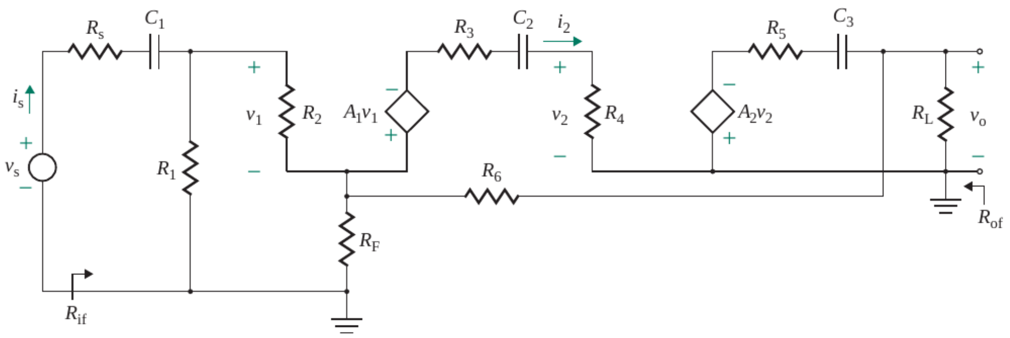
③

Voltage Series Feedback on a Voltage Amplifier

10.13 The feedback amplifier in Fig. P10.13 has $A_1 = 50$, $A_2 = 60$, $R_s = 500\ \Omega$, $R_1 = 15\ \text{k}\Omega$, $R_2 = 1.5\ \text{k}\Omega$, $R_3 = 250\ \Omega$, $R_4 = 1.5\ \text{k}\Omega$, $R_5 = 250\ \Omega$, $R_6 = 2\ \text{k}\Omega$, $R_L = 4.7\ \text{k}\Omega$, $R_F = 500\ \Omega$, $C_1 = C_2 = C_3 = 0.1\ \mu\text{F}$, and $v_s = 100$ mV. Determine **(a)** the input resistance $R_{if} = v_s/i_s$, **(b)** the output resistance R_{of}, and **(c)** the overall voltage gain $A_f = v_o/v_s$. Assume C_1, C_2, and C_3 are shorted at the operating frequency.

P

FIGURE P10.13



Notes: None.

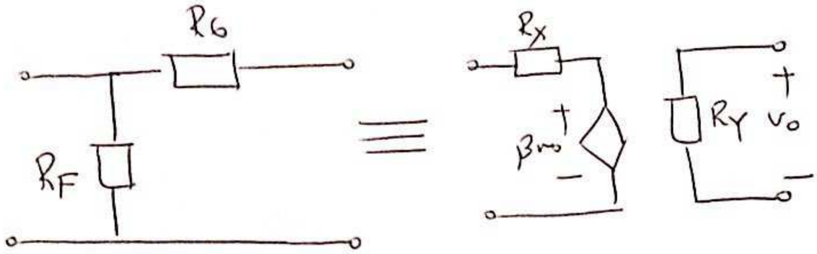
Additional Tasks: Analyze this circuit as a voltage amplifier with voltage series feedback. Numerical values of the required quantities should be computed as the last task.

Necessary Knowledge and Skills: Modeling a given circuit as a voltage amplifier, input/output impedance and gain calculations, feedback network modeling, idealizing the feedback network by carrying its impedance loads into the non-feedback amplifier, remodeling the non-feedback amplifier, analyzing the feedback amplifier (non-feedback amplifier and the feedback network combined) to compute the input/output impedances and gain.

Model the feedback network

Rashid 10.13

(*)



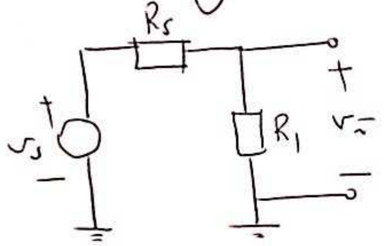
(This is a voltage amplifier with voltage-series feedback.)

$$R_X = R_F // R_G$$

$$R_Y = R_F + R_G$$

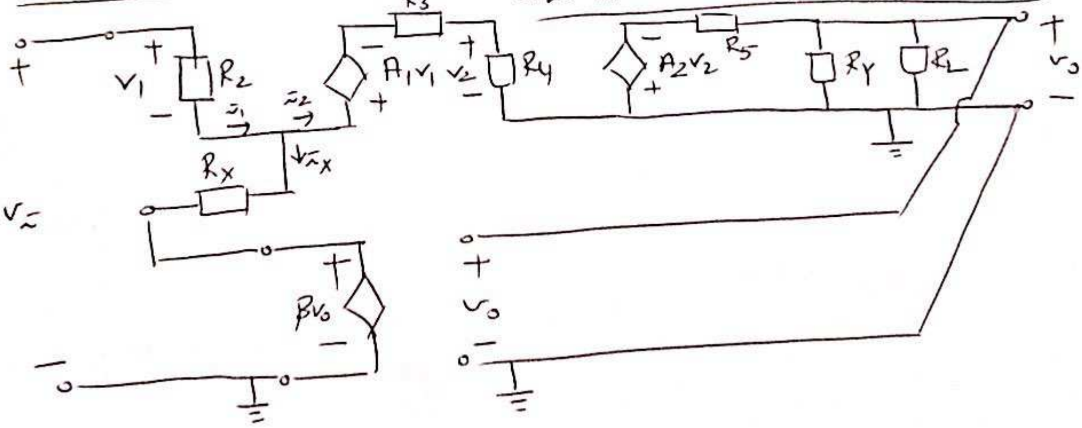
$$\beta = \frac{R_F}{R_F + R_G}$$

(**) Leave the following part until later.



Schematics of the (VA, FL) Feedback-Loaded Voltage Amplifier and the idealized feedback network

(***)



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Kill β ($\beta = 0$) and analyze VA, FL

Rashid 10.13 contin.

Compute $\frac{v_1}{v_i}$, $\frac{v_2}{v_i}$, $\frac{v_i}{i_1}$ (Refer to (***))

(KVL) $+v_i - v_1 - A_1 v_1 - (R_3 + R_4) i_2 = 0$

(KVL) $+v_i - v_1 - i_x R_x = 0$

(KCL) $i_1 = i_2 + i_x$

$$v_1 = i_1 R_2$$

$$v_2 = i_2 R_4$$

$$v_i = (1 + A_1) v_1 + (R_3 + R_4) i_2$$

$$\frac{v_1}{R_2} = i_2 + i_x$$

$$\frac{v_i - v_1}{R_x} = i_x$$

$$v_i = (1 + A_1) v_1 + (R_3 + R_4) \left[\frac{v_1}{R_2} - \frac{v_i - v_1}{R_x} \right]$$

$$v_i \left[1 + \frac{R_3 + R_4}{R_x} \right] = v_1 \left[1 + A_1 + \frac{R_3 + R_4}{R_2 /\!/ R_x} \right]$$

$$\frac{v_1}{v_i} = \frac{\left[1 + \frac{R_3 + R_4}{R_x} \right]}{\left[(1 + A_1) + \left(\frac{R_3 + R_4}{R_2 /\!/ R_x} \right) \right]}$$

2

Rashid
10.13
Contin.

Note that

$$\frac{v_1}{v_i} = \frac{i_1 R_2}{v_i} = R_2 \left(\frac{v_i}{i_1}\right)^{-1}$$

$$r_{in,VA,fl} = \frac{v_i}{i_1} = R_2 \underbrace{\left(\frac{v_1}{v_i}\right)^{-1}}_{\text{see the bottom of pg 2}}$$

$$\frac{v_2}{v_i} = \frac{R_4 i_2}{v_i} = \frac{R_4}{v_i}\left[\frac{v_1}{R_2} - \frac{v_i - v_1}{R_x}\right]$$

$$= \frac{R_4}{v_i}\left[\frac{v_1}{R_2 // R_x} - \frac{v_i}{R_x}\right]$$

$$= \frac{R_4}{R_2 // R_x} \underbrace{\frac{v_1}{v_i}}_{\substack{\text{see}\\ \text{the bottom}\\ \text{of pg 2}}} - \frac{R_4}{R_x} \underbrace{<}_{\substack{\Uparrow\\ \text{check this}\\ \text{numerically}}} 0$$

Refer to (***)

$$\frac{v_o}{v_2} = (-A_2)\,\frac{R_Y // R_L}{R_Y // R_L + R_5} < 0 \quad \text{if } A_2 > 0$$

then

$$\frac{v_o}{v_i} = \underbrace{\frac{v_2}{v_i}}_{\text{see pg 3}}\ \underbrace{\frac{v_o}{v_2}}_{\text{see above}} = A_{VA,fl}$$

$$r_{out,VA,fl} = R_5 // R_Y // R_L$$

3

Rashid
10.13
contin.

Now apply feedback theory to compute:

$$A_f = \frac{A_{vA,fe}}{1 + \beta A_{vA,fe}}$$

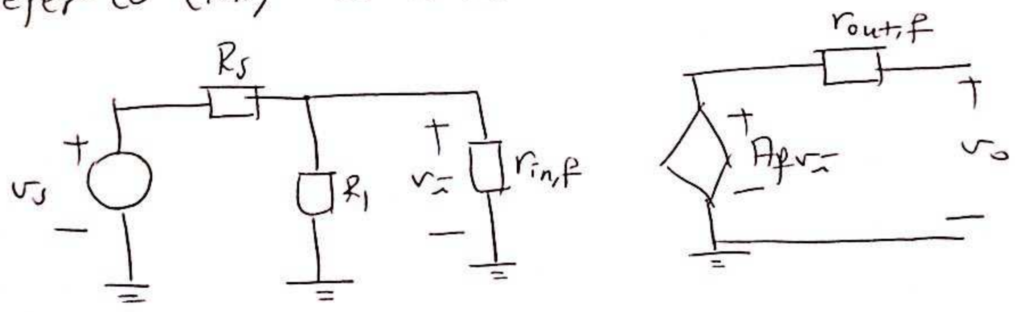
$$r_{in,f} = \left(1 + \beta A_{vA,fe}\right) r_{in,vA,fe}$$

$$r_{out,f} = \frac{r_{out,vA,fe}}{1 + \beta A_{vA,fe}}$$

Analysis of the feedback amplifier

Now analyze the whole circuit:

Refer to (**) and above



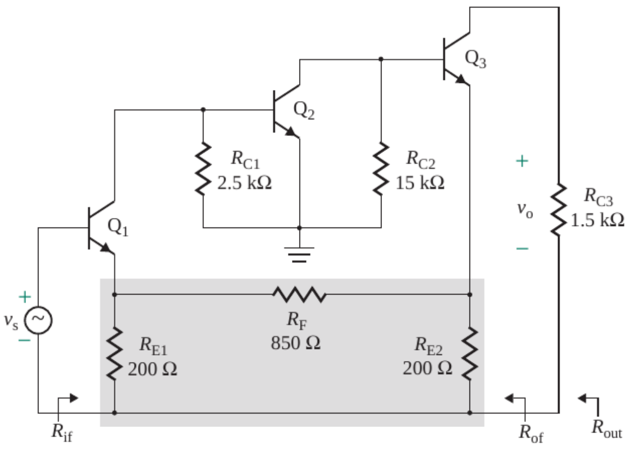
$$\frac{v_o}{v_s} = \frac{r_{in,f} // R_1}{r_{in,f} // R_1 + R_s} A_f$$

(4)

Current-Series Feedback on a TCA

10.28 The AC equivalent circuit of a feedback amplifier is shown in Fig. P10.28. The circuit values are R_{C1} = 2.5 kΩ, R_{C2} = 5 kΩ, R_{C3} = 1.5 kΩ, R_{E1} = 100 Ω, R_{E2} = 100 Ω, R_F = 750 Ω, and R_s = 0. The transistor parameters are h_{fe} = 100, r_π = 2.5 kΩ, r_o = 25 kΩ, and $r_\mu = \infty$. Use the techniques of feedback analysis to calculate **(a)** the input resistance R_{if}, **(b)** the output resistance R_{of}, and **(c)** the closed-loop voltage gain A_f.

FIGURE P10.28



Notes: None.

Additional Tasks: None.

Necessary Knowledge and Skills: Modeling the CE stage as a transconductance amplifier, current-series feedback network modeling in a practical amplifier, gain and i/o impedance computations, feedback-loaded TCA analysis, effect of feedback on gain and i/o impedances.

There is current–series feedback

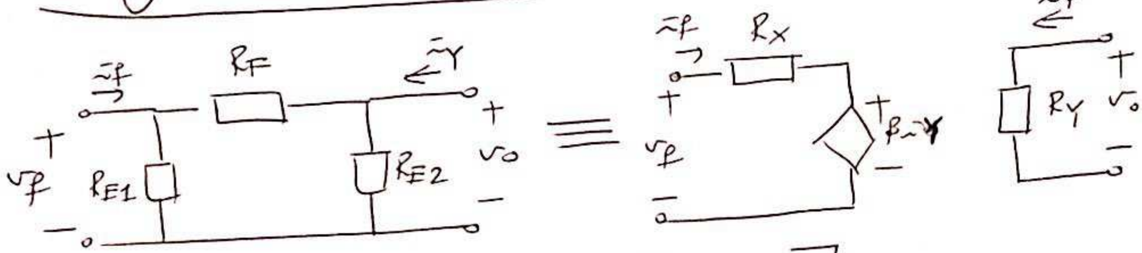
Rashid 10.28

series series

involved in this circuit.

The whole amplifier identifying the feedback network has already been drawn.

Analyze the feedback network (see pg 668 of Rashid)



$$R_x = \frac{v_f}{i_f}\Big|_{i_Y=0} = R_{E1} // \left[R_F + R_{E2}\right]$$

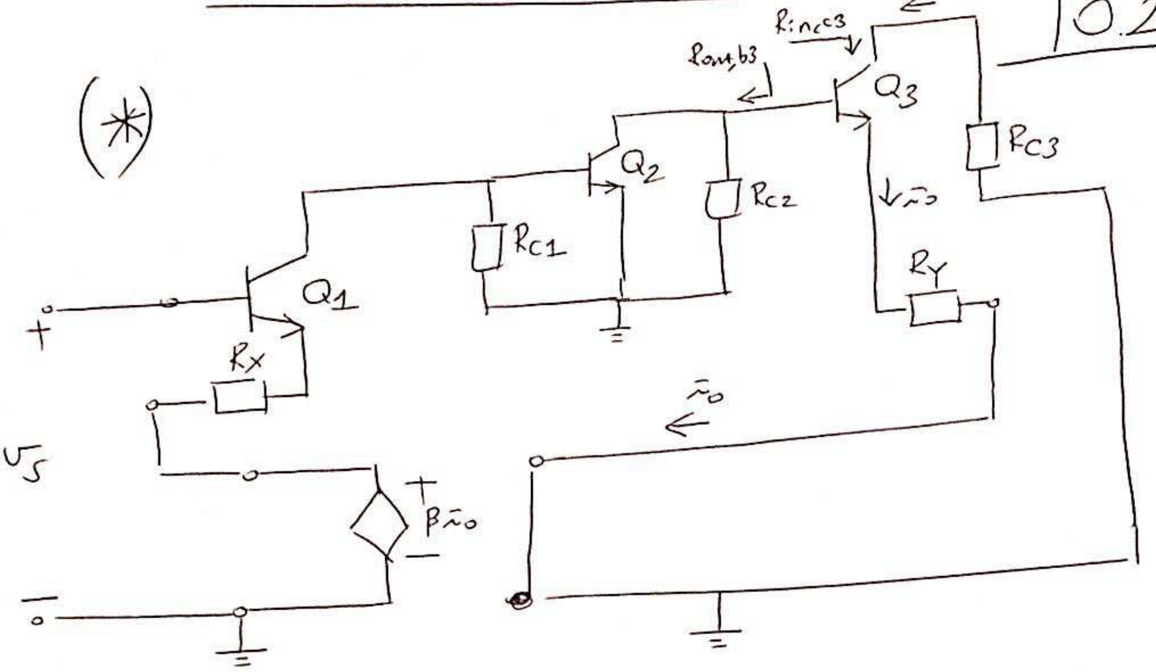
$$R_Y = \frac{v_o}{i_Y}\Big|_{i_f=0} = R_{E2} // \left[R_{E1} + R_F\right]$$

$$\beta = \frac{v_f}{i_Y}\Big|_{i_f=0} = \frac{R_{E2}}{R_{E2} + R_{E1} + R_F} \cdot R_{E1}$$

Next $\Rightarrow$ Draw the schematic of TCA, R_l and the current-series feedback network.

TCA, fl and the feedback network

Rashid
10.28



Above in (*), set $\beta = 0$, Analyze TCA, fl

$$A_{TCA,fl} \cong \left[\frac{-g_{m1}}{1+g_{m1}R_X}\right]\left[\left[(1+g_{m1}R_X)r_{o1}\right] // R_{C1} // r_{\pi 2}\right]$$

$$\cdot \left[-g_{m2}\right] \cdot \left[r_{o2} // R_{C2} // \left[r_{\pi 3}(1+g_{m3}R_Y)\right]\right]$$

$$\cdot \left[\frac{g_{m3}}{1+g_{m3}R_Y}\right]$$

2

Rashid
10.28
contin.

$$r_{in,TCA,fl} \cong r_{\pi 1}\left(1+g_{m1} R_X\right)$$

$$r_{out,TCA,fl} \cong R_{C3} + R_{in,c3}$$

note that $R_{out,b3} = R_{C2} // r_{o2}$

then $R_{in,c3} = \left(1+g_{m3} r_{o3}\right) \dfrac{R_Y r_{\pi 3}}{R_Y + r_{\pi 3} + R_{out,b3}} + r_o$

(see related document on the derivation)

$$+ \frac{R_Y R_{out,b3}}{R_Y + r_{\pi 3} + R_{out,b3}}$$

if the trailing two terms can be neglected $\approx \left(1+g_{m3} r_{o3}\right)\left[\dfrac{R_Y r_{\pi 3}}{R_Y + r_{\pi 3} + R_{out,b3}}\right]$

Now analyze the whole amplifier:

$$A_f = \frac{A_{TCA,fl}}{1+\beta A_{TCA,fl}}$$

$$r_{in,f} = r_{in,TCA,fl}\left(1+\beta A_{TCA,fl}\right)$$

$$r_{out,f} = r_{out,TCA,fl}\left(1+\beta A_{TCA,fl}\right)$$

$$\frac{v_o}{v_s} = -A_f\, r_{out,f}$$

Analysis of Feedback in OpAmp Circuits

***8.26** For each of the op-amp circuits shown in Fig. P8.26, identify the feedback topology and indicate the output variable being sampled and the feedback signal. In each case, assuming the op amp to be ideal, find an expression for β, and hence find A_f.

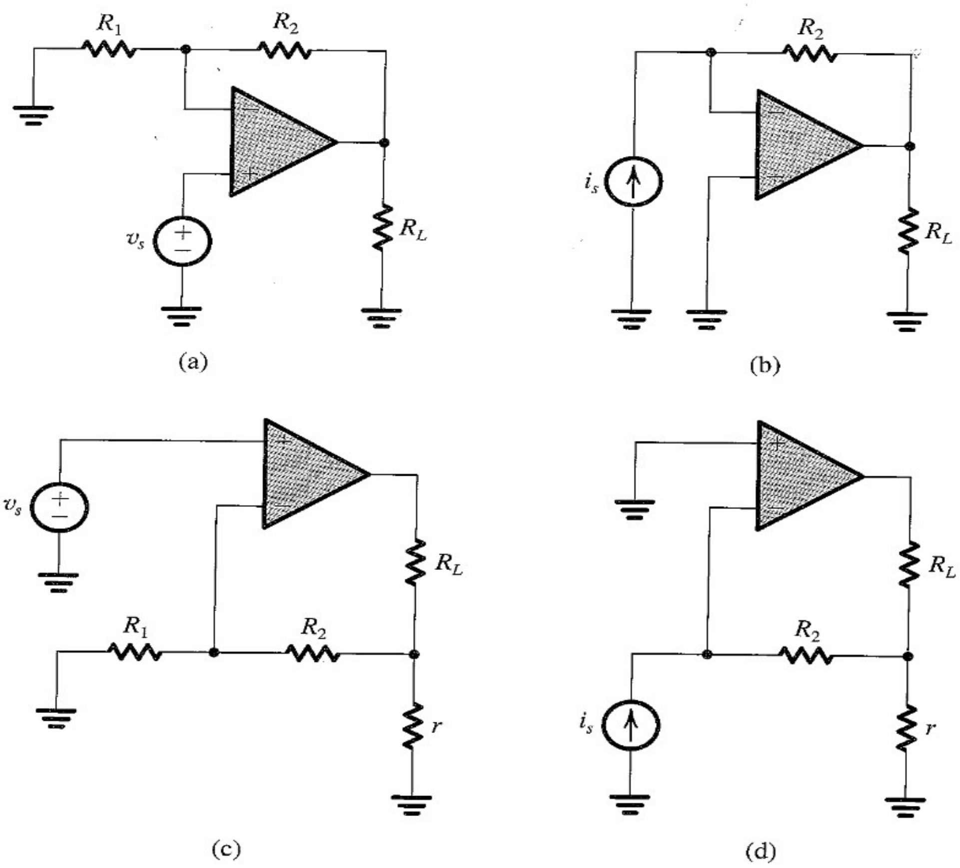


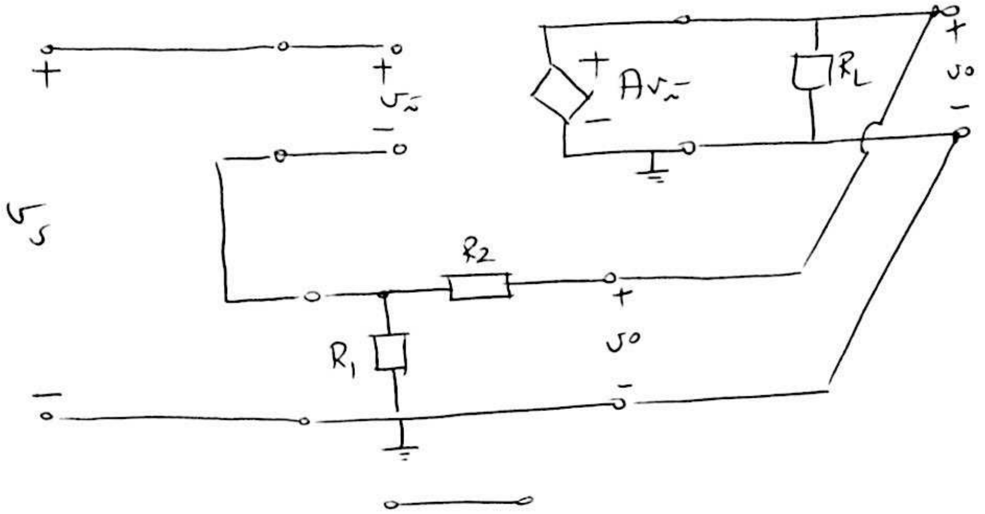
FIGURE P8.26

Notes: None.
Additional Tasks: None.
Necessary Knowledge and Skills: The four types of feedback, identification and modeling of feedback networks, feedback-loaded amplifier analysis, gain and input/output imp. computation.

This is a voltage amplifier with voltage series feedback.

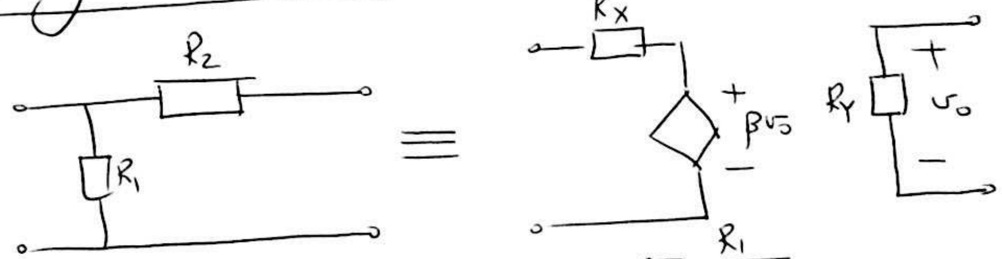
Sedra 8.260)

(*)



Analyze the feedback network:

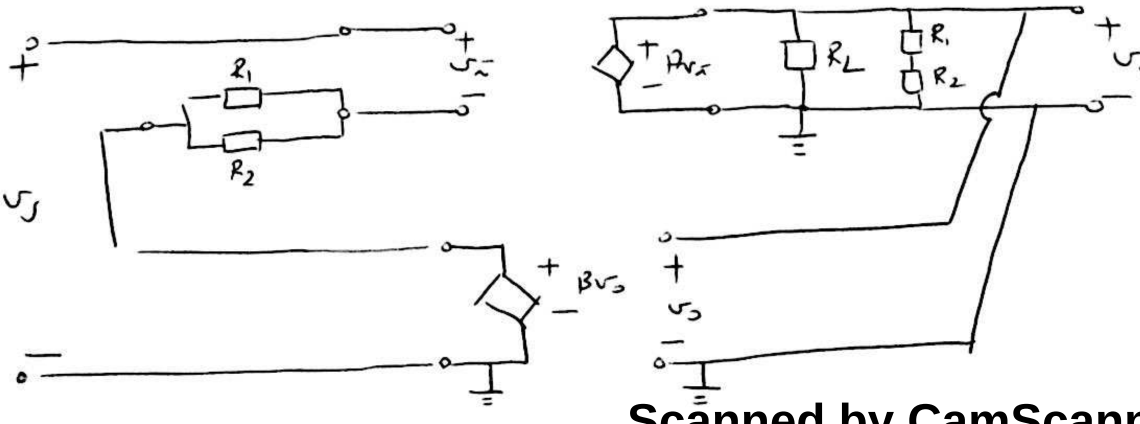
(**)



$$\beta = \frac{R_1}{R_1+R_2}$$

$$R_X = R_1 // R_2$$
$$R_Y = R_1 + R_2$$

(***) Schematic of VA, FR and the feedback network (idealized)



(1)

Sedra 8.26a) contin.

$\Rightarrow$ Kill β ($\beta = 0$)

$\Rightarrow$ Analyze VA, FL

$r_{in,VA,FL} = +\infty$ (would be $r_{in} + R_1 // R_2$ if the op amp were nonideal)

$r_{out,VA,FL} = 0$ (would be $r_{out} // R_L // (R_1 + R_2)$ if the opamp were nonideal)

$A_{VA,FL} = A$ (would be $\dfrac{r_{in}}{r_{in} + R_1 // R_2} A \dfrac{R_L // (R_1 + R_2)}{R_L // R_1 + R_2) + r_{out}}$ if the opamp were nonideal)

Now analyze the whole feedback amplifier

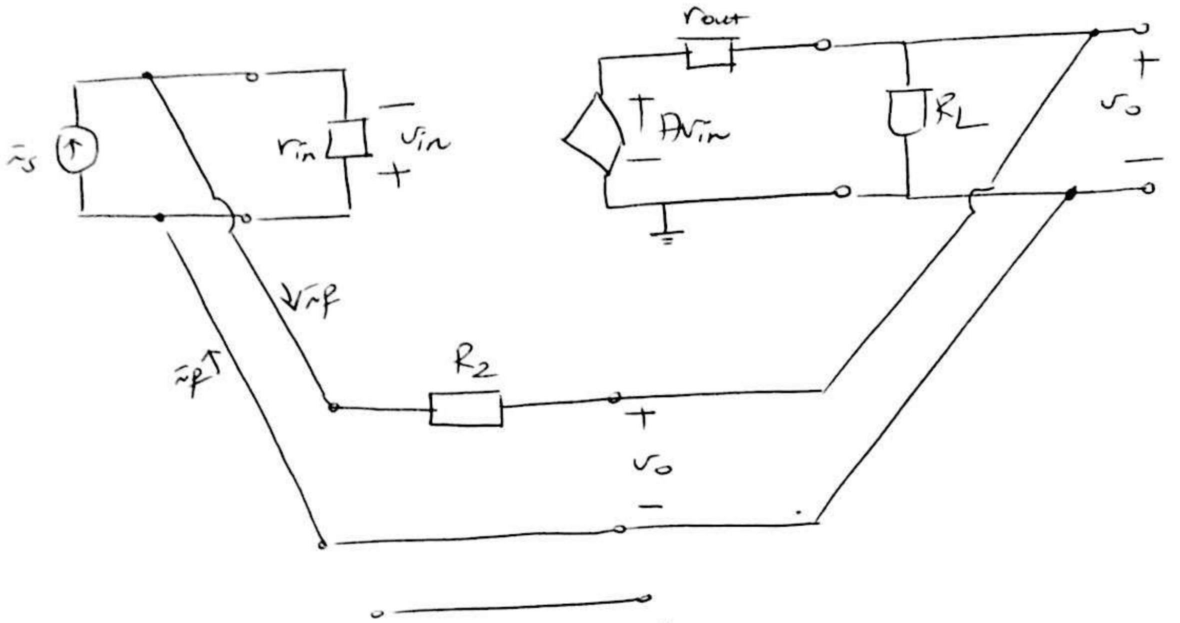
$r_{in,f} = r_{in,VA,FL} \cancel{\ } (1 + \beta A_{VA,FL}) = +\infty$ (or plug in the value above and β)

$$r_{out,f} = \frac{r_{out,VA,FL}}{1 + \beta A_{VA,FL}} = 0$$

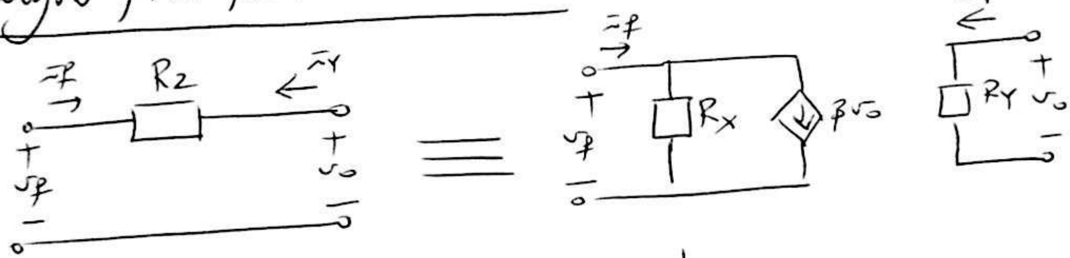
$$A_f = \frac{A_{VA,FL}}{1 + \beta A_{VA,FL}} = \frac{A}{1 + \frac{R_1}{R_1 + R_2} A} \approx \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

(if $A \gg 1$)

This is a transresistance amplifier with voltage-shunt feedback. Sedra 8.26b)



Analysis of the feedback network



$$\beta = \frac{i_f}{v_o}\bigg|_{v_f=0} = \frac{-i_Y}{v_o} = -\frac{1}{R_2}$$

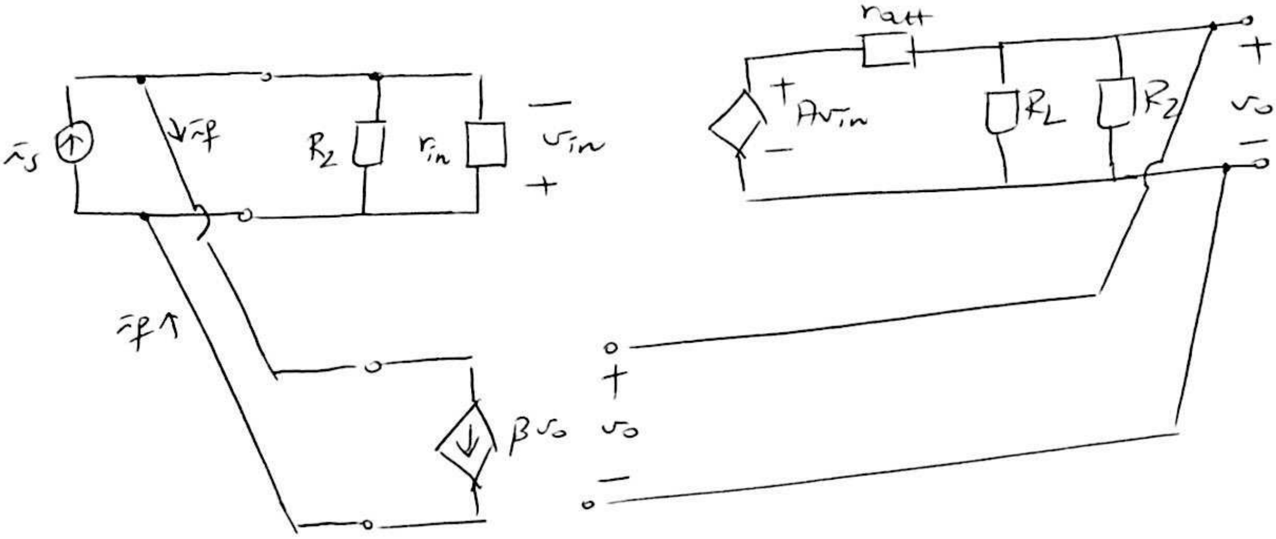
$$R_Y = \frac{v_o}{i_Y}\bigg|_{v_f=0} = R_2$$

$$R_X = \frac{v_f}{i_f}\bigg|_{v_o=0} = R_2$$

(1)

TRA, fe and the idealized feedback network

Sedra 8 26b)



Kill β ($\beta=0$) and analyze ~~the~~ TRA, fe

$$r_{in,TRA,fe} = R_2 // r_{in}$$

$$r_{out,TRA,fe} = r_{out} // R_L // R_2$$

$$A_{TRA,fe} = \left[-R_2 // r_{in}\right] A \left[\frac{R_L // R_2}{R_L // R_2 + r_{out}}\right]$$

Note that with $r_{in} = +\infty$ and $r_{out} = 0$ the above would be

$$r_{in,TRA,fe} = R_2$$

$$r_{out,TRA,fe} = 0$$

$$A_{TRA,fe} = -R_2 \cdot A$$

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2

Now analyze the whole feedback ampli.

Sedra 8.26 b) contin

$$r_{in,f} = \frac{r_{in,TRA,fl}}{1+\beta A_{TRA,fl}}$$

$$= \frac{R_2 \parallel r_{in}}{1+\left(-\frac{1}{R_2}\right)\left(-R_2 \parallel r_{in}\right) A \left(\frac{R_L \parallel R_2}{R_L \parallel R_2 + r_{out}}\right)}$$

with $r_{in}=+\infty$, $r_{out}=0$

$$\cong \frac{R_2}{1+\left(-\frac{1}{R_2}\right)\left(-R_2\right) A}$$

$$= \frac{R_2}{1+A}$$

with $A \gg 1$

$$\cong 0$$

$$r_{out,f} = \frac{r_{out,TRA,fl}}{1+\beta A_{TRA,fl}}$$

with $r_{in}=+\infty$, $r_{out}=0$

$$\cong \frac{0}{1+\left(-\frac{1}{R_2}\right)\left(-R_2 \cdot A\right)}$$

$$A_f = \frac{A_{TRA,fl}}{1+\beta A_{TRA,fl}} = \frac{\left[-R_2 \parallel r_{in}\right] A \left[\frac{R_L \parallel R_2}{R_L \parallel R_2 + r_{out}}\right]}{1+\left(-\frac{1}{R_2}\right)\left[-R_2 \parallel r_{in}\right] A \left[\frac{R_L \parallel R_2}{R_L \parallel R_2 + r_{out}}\right]}$$

3

Sedra 8.26 Cont-

$$A_f \cong \frac{-R_2 A}{1 + \left(-\frac{1}{R_2}\right)\left(-R_2\right) A}$$

with $r_{in} = +\infty$
$r_{out} = 0$

$$= \frac{-R_2 A}{1 + A}$$

with $A \gg 1$ $\cong -R_2$

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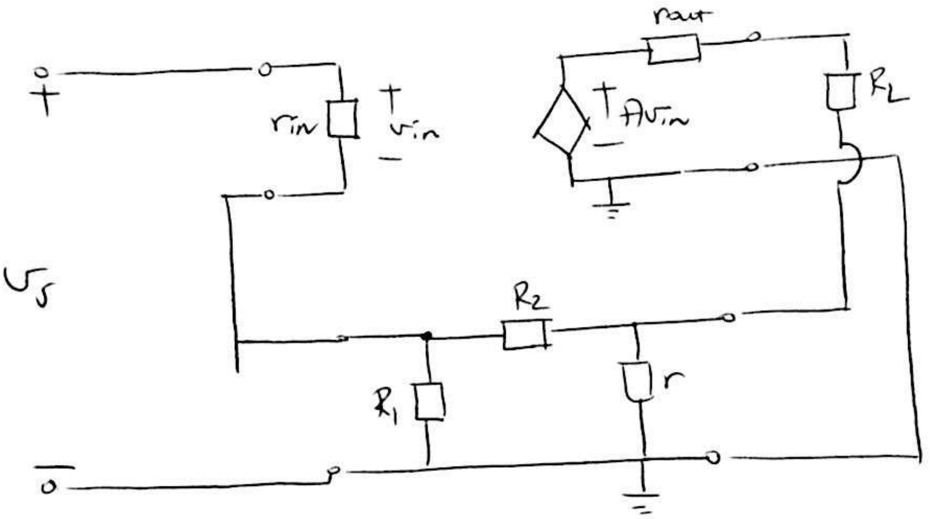
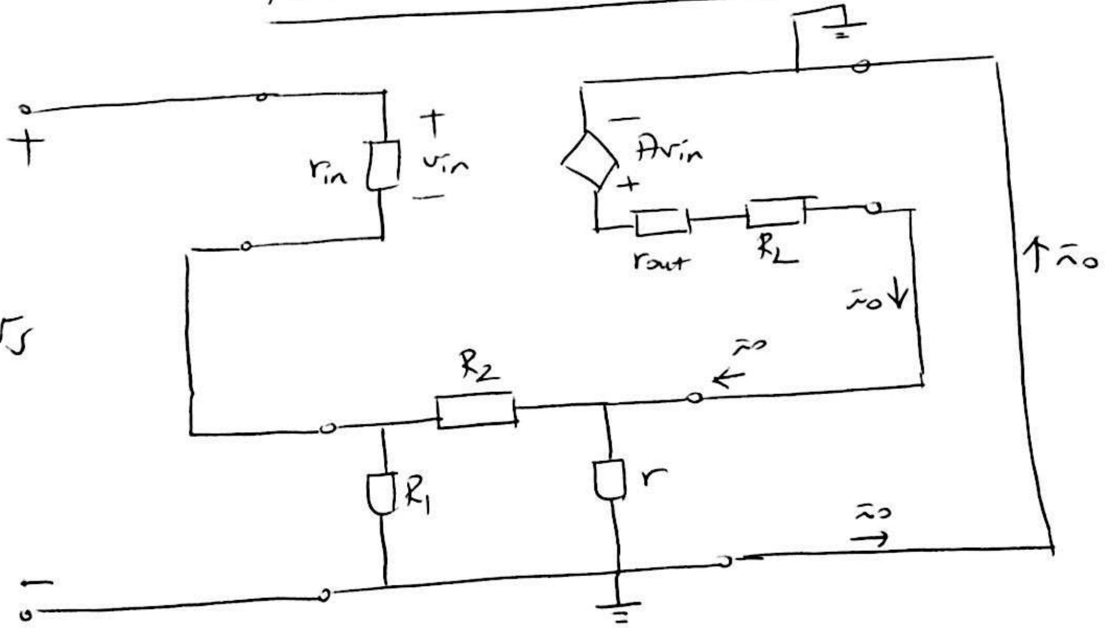
(4)

Transconductance amplifier with current-series feedback

Sedra 8.26c



Redraw the above schematics:

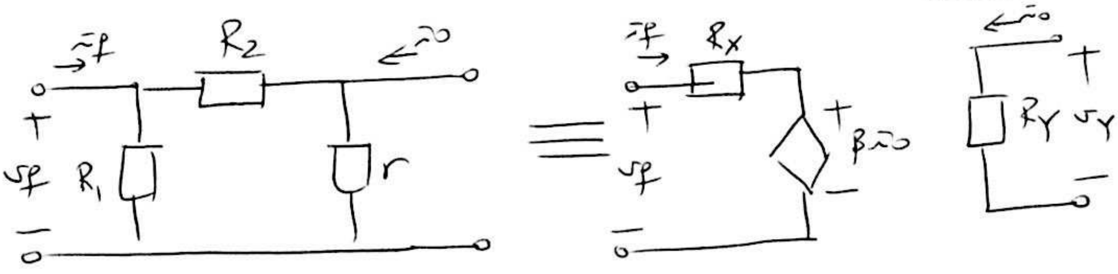


 1

Analyze the feedback network

Sedra 8.26c

cont.

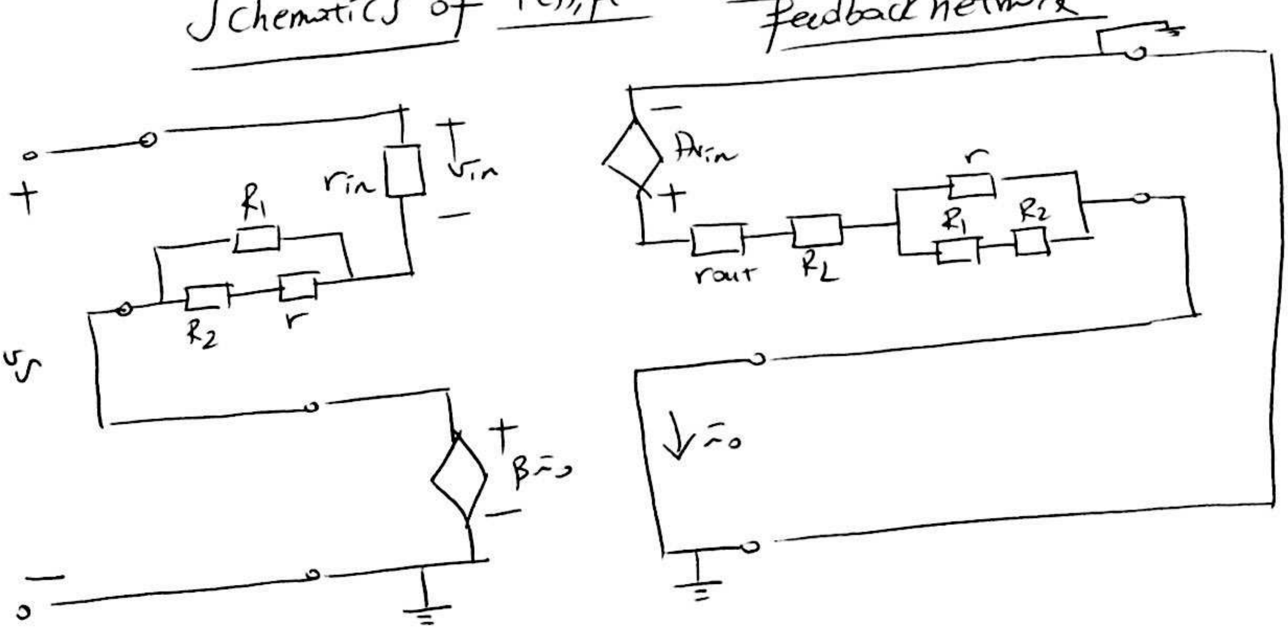


$$\beta = \frac{v_f}{i_o}\bigg|_{i_f=0} = \frac{r}{r+R_1+R_2}\cdot R_1$$

$$R_X = \frac{v_f}{i_f}\bigg|_{i_o=0} = R_1 \,//\, (R_2+r)$$

$$R_Y = \frac{v_Y}{i_o}\bigg|_{i_f=0} = r \,//\, (R_1+R_2)$$

Schematics of T(A), fl and the idealized feedback network



 2

$\Rightarrow$ Kill β ($\beta = 0$)

Sedra 8.26c) contin.

$\Rightarrow$ Analyze TCA, fl

$$r_{in,TCA,fl} = r_{in} + R_1 // (R_2 + r)$$

$$r_{out,TCA,fl} = r_{out} + R_L + r // (R_1 + R_2)$$

$$A_{TCA,fl} = \frac{r_{in}}{r_{in} + R_1 // (R_2 + r)} A \frac{1}{r_{out} + R_L + r // (R_1 + R_2)}$$

Note that with $r_{in} = +\infty$, $r_{out} = 0$

$$r_{in,TCA,fl} \cong +\infty$$

$$r_{out,TCA,fl} \cong R_L + r // (R_1 + R_2)$$

$$A_{TCA,fl} \cong A \frac{1}{R_L + r // (R_1 + R_2)}$$

 3

Now analyze the whole feedback amplifier

Sedra 8.26c contin.

$$r_{in,f} = \left(1 + \beta A_{TC(A),fl}\right) r_{in,TC(A),fl}$$

$$r_{out,f} = \left(1 + \beta A_{TC(A),fl}\right) r_{out,TC(A),fl}$$

$$A_f = \frac{A_{TC(A),fl}}{1 + \beta A_{TC(A),fl}}$$

Note that with $r_{in} = +\infty$, $r_{out} = 0$

$$\rightarrow \quad r_{in,f} = +\infty$$

$$\rightarrow \quad r_{out,f} = \left(1 + \frac{rR_1}{r+R_1+R_2} A \frac{1}{R_L + r /\!/ (R_1+R_2)}\right) \cdot \left[R_L + r /\!/ (R_1+R_2)\right]$$

$$\text{with } A \gg 1 \quad \cong \quad \frac{rR_1 A}{r+R_1+R_2}$$

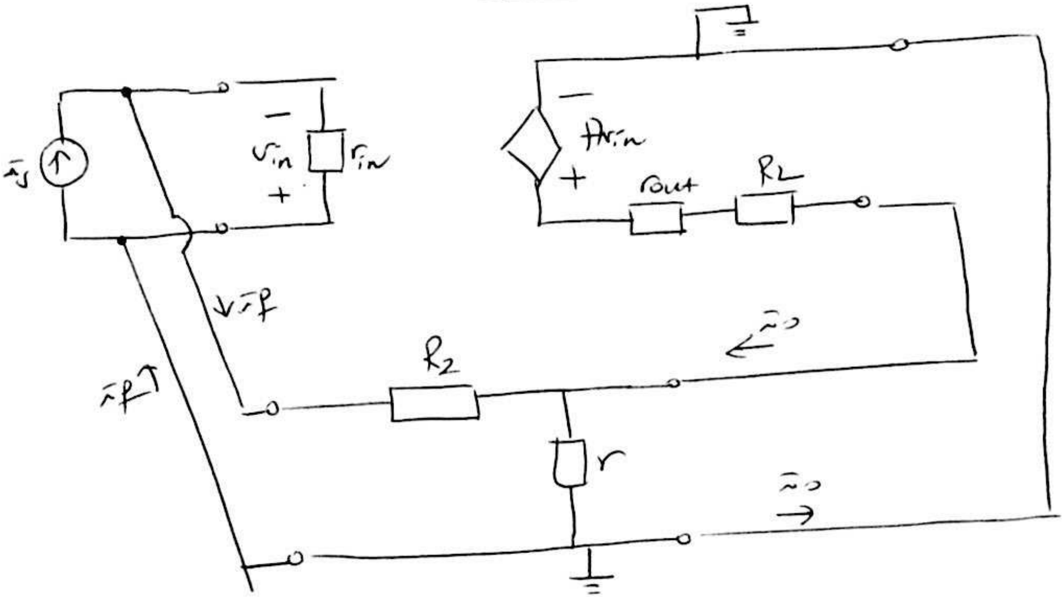
$$\rightarrow \quad A_f \cong \frac{A \dfrac{1}{R_L + r /\!/ (R_1+R_2)}}{1 + \dfrac{rR_1}{r+R_1+R_2} A \dfrac{1}{R_L + r /\!/ (R_1+R_2)}}$$

$$\text{with } A \gg 1 \quad \cong \quad \frac{r+R_1+R_2}{rR_1} = \frac{1}{r /\!/ R_1} + \frac{R_2}{rR_1}$$

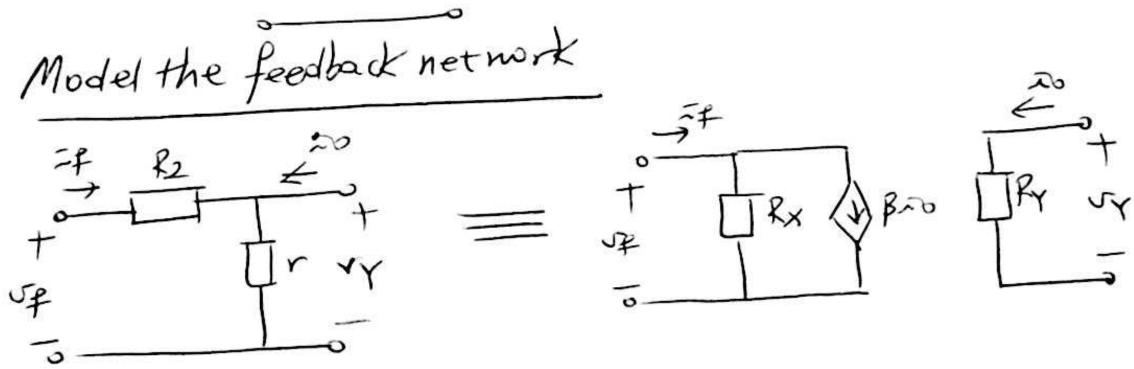
4

This is a current amplifier with current-shunt feedback

Sedra 8.26d



Model the feedback network



$$R_X = \left.\frac{v_f}{i_f}\right|_{i_o=0} = R_2 + r$$

$$R_Y = \left.\frac{v_Y}{i_o}\right|_{v_f=0} = R_2 \parallel r$$

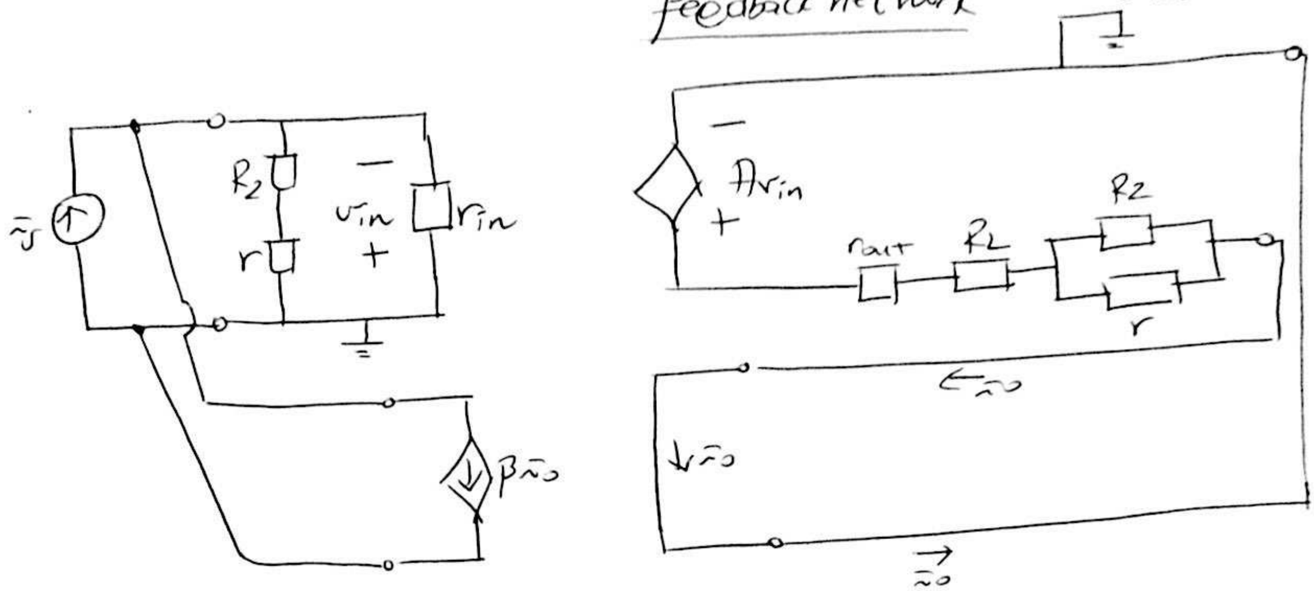
$$\beta = \left.\frac{i_f}{i_o}\right|_{v_f=0} = -\frac{r}{r+R_2}$$

1

Schematics of CA, fl and the idealized feedback network

Sedra 8.26d

cont.



$\Rightarrow$ Kill β $(\beta = 0)$

$\Rightarrow$ Analyze CA, fl

$$r_{in,CA,fl} = r_{in} \,//\, (R_2 + r)$$

$$r_{out,CA,fl} = r_{out} + R_L + R_2 // r$$

$$A_{CA,fl} = -\frac{R_2 + r}{R_2 + r + r_{in}} \cdot r_{in} A \, \frac{1}{r_{out} + R_L + R_2 // r}$$

With $r_{in} = +\infty$
$r_{out} = 0$

Sedra 8.26 d)

$$r_{in,CA,fl} \cong R_2 + r$$

$$r_{out,CA,fl} \cong R_L + R_2 // r$$

$$A_{CA,fl} \cong -A \frac{(R_2+r)}{R_L + (R_2 // r)}$$

Analyze the whole current amplifier:

$$A_f = \frac{A_{CA,fl}}{1 + \beta A_{CA,fl}}$$

with $r_{in} = +\infty$, $r_{out} = 0$

$$\cong \frac{-A \frac{(R_2+r)}{R_L + (R_2 // r)}}{1 + \left(\frac{-r}{\cancel{r+R_2}}\right)(-A)\left(\frac{\cancel{R_2+r}}{R_L + R_2 // r}\right)}$$

with $A \gg 1$

$$\cong \frac{-A \frac{(R_2+r)}{R_L + R_2 // r}}{rA \frac{1}{R_L + R_2 // r}} = - \frac{R_2 + r}{r} = -\left[1 + \frac{R_2}{r}\right]$$

$$r_{in,f} = \frac{r_{in,CA,fl}}{1 + \beta A_{CA,fl}} \cong \frac{R_2 + r}{1 + \left(\frac{-r}{\cancel{r+R_2}}\right)(-A)\left(\frac{\cancel{R_2+r}}{R_L + R_2 // r}\right)}$$

with $r_{in} = +\infty$, $r_{out} = 0$

with $A \gg 1$

$$\cong \frac{1}{rA}\left[(R_2+r)\left[R_L + \frac{R_2 r}{R_2 + r}\right]\right]$$

$$= \frac{(R_2+r) R_L + R_2 r}{rA}$$

3

Sedra 8.26 d) Contin.

$$r_{out,f} = \left(1 + \beta A_{CA,fl}\right) r_{out,CA,fl}$$

with $r_{in} = +\infty$, $r_{out} = 0$

$$\cong \left[1 + \left(\frac{-r}{\cancel{r+R_2}}\right)(-A)\left[\frac{\cancel{R_2+r}}{R_L + R_2 /\!/ r}\right]\right]\left(R_L + R_2 /\!/ r\right)$$

with $A \gg 1$

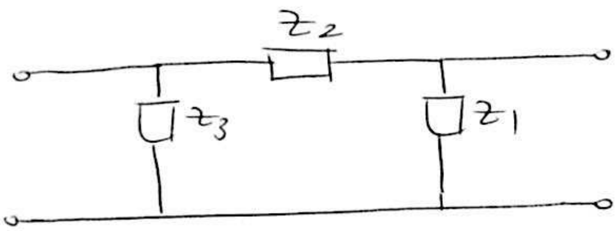
$$\cong Ar\,\frac{\cancel{R_L + R_2 /\!/ r}}{\cancel{R_L + R_2 /\!/ r}} = Ar$$

(4)

10. Oscillators

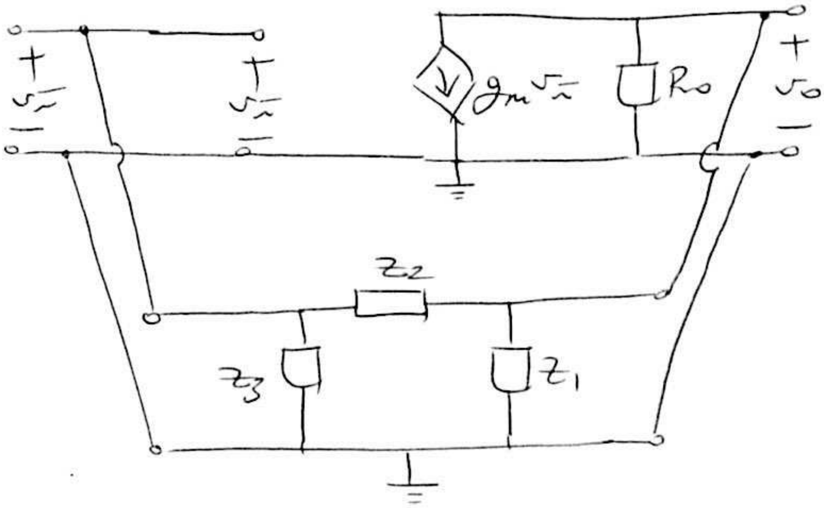
10.1 Auxiliaries

Analysis of LC networks encountered in RF oscillators



Worked on this on: Nov 27, 2014/Th

Analysis of common LC networks encountered in RF oscillators



$$A \cdot \beta = -g_m \left[R_o // Z_1 // \left[Z_3 + Z_2 \right] \right] \frac{Z_3}{Z_3 + Z_2}$$

$$= -g_m \frac{1}{\frac{1}{R_o} + \frac{1}{Z_1} + \frac{1}{Z_3 + Z_2}} \frac{Z_3}{Z_3 + Z_2}$$

$$= -g_m \frac{\cancel{Z_3 + Z_2}}{\frac{Z_3 + Z_2}{R_o} + \frac{Z_3 + Z_2}{Z_1} + 1} \frac{Z_3}{\cancel{Z_3 + Z_2}}$$

$$= \frac{-g_m R_o Z_1 Z_3}{Z_1 Z_3 + Z_1 Z_2 + R_o (Z_1 + Z_2 + Z_3)}$$

If ~~each~~ each element Z_1, Z_2, Z_3 is an inductor or a capacitor:

$$Z_i = jX_i$$

Then we have

$$A \cdot \beta = \frac{-g_m R_o(-X_1 X_3)}{-(X_1 X_3 + X_1 X_2) + jR_o(X_1 + X_2 + X_3)}$$

The phase condition requires

$$X_1 + X_2 + X_3 = 0$$

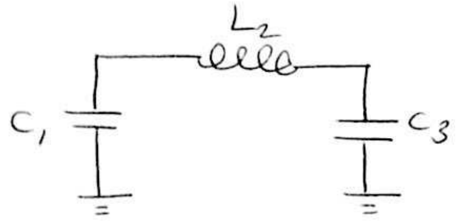
For self starting oscillations we have:

$$\frac{-g_m R_o(X_1 X_3)}{X_1(X_2 + X_3)} \geq 1$$

$$\frac{-g_m R_o X_3}{X_3 + X_2} \geq 1$$

2

Example: Colpitts Oscillator



We have for the phase condition:

$$\frac{1}{jwC_1} + jwL_2 + \frac{1}{jwC_3} = 0$$

$$\frac{-1}{wC_1} + wL_2 - \frac{1}{wC_3} = 0$$

$$wL_2 = \frac{1}{w}\frac{C_1 + C_3}{C_1 C_3}$$

then $$w_o = \frac{1}{\sqrt{L_2 \frac{C_1 C_3}{C_1 + C_3}}}$$

For self-starting oscillations:

$$\frac{g_m R_o \left(\frac{1}{w_o C_3}\right)}{-\frac{1}{w_o C_3} + w_o L_2} = \frac{g_m R_o}{w_o^2 L_2 C_3 - 1} \geq 1$$

$$= \frac{g_m R_o}{\frac{1}{\cancel{L_2}} \frac{C_1 + C_3}{C_1 \cancel{C_3}} \cancel{L_2} \cancel{C_3} - 1} \geq 1$$

3

$$g_m R_o \geq \frac{C_3}{C_1}$$ for self-starting osc in a Colpitts osc.

4

Audio Oscillator Analysis

13.13 For the circuit in Fig. P13.13 find $L(s)$, $L(j\omega)$, the frequency for zero loop phase, and R_2/R_1 for oscillation.

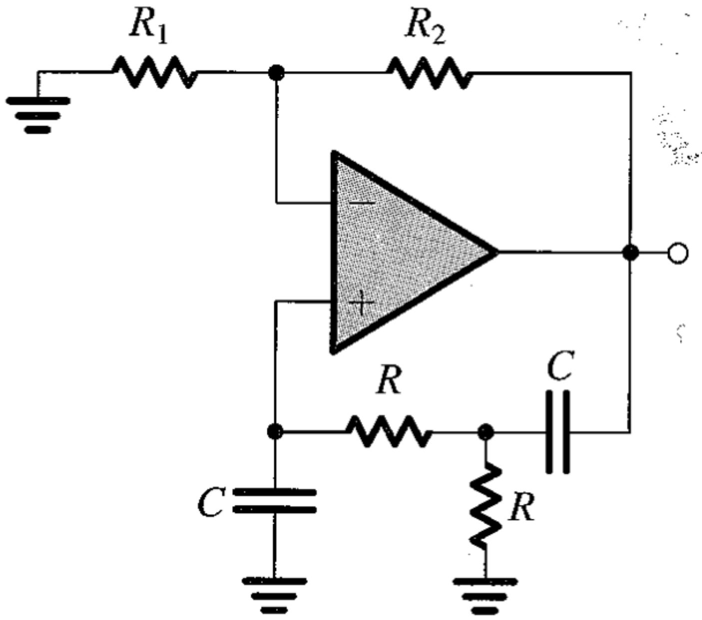


FIGURE P13.13

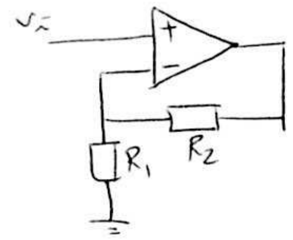
Notes: None.

Additional Tasks: None.

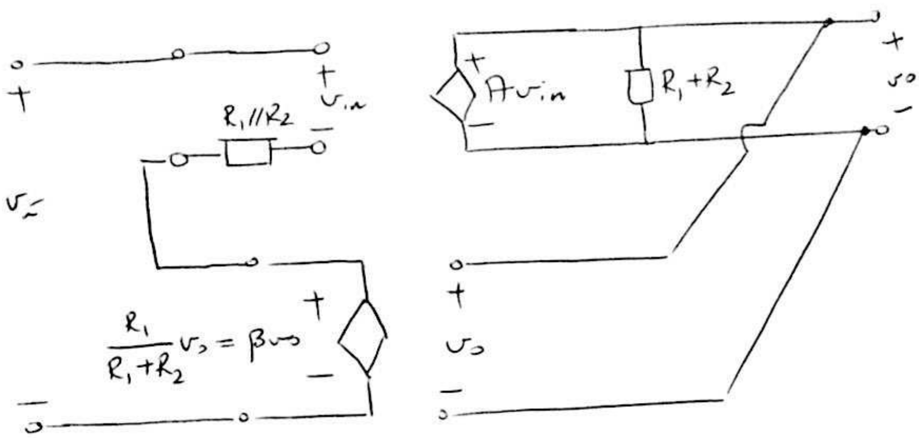
Necessary Knowledge and Skills: Oscillators, Barkhausen criteria, analysis of feedback networks, oscillation frequency, phase condition, amplitude condition.

Analyze the amplifier

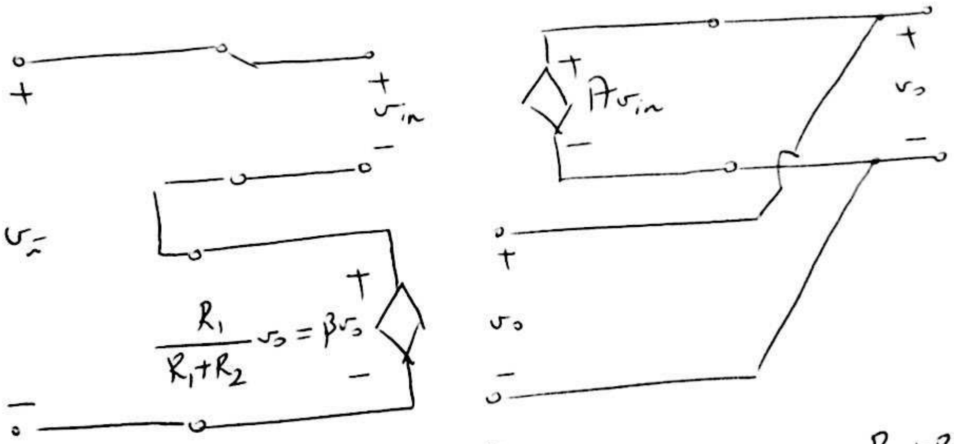
Sedra
13.13



This is a voltage amplifier with voltage series feedback



This is equivalent to



Then $\frac{v_o}{v_s} = \frac{A}{1 + A \frac{R_1}{R_1+R_2}} \simeq \frac{R_1+R_2}{R_1} = 1 + \frac{R_2}{R_1}$

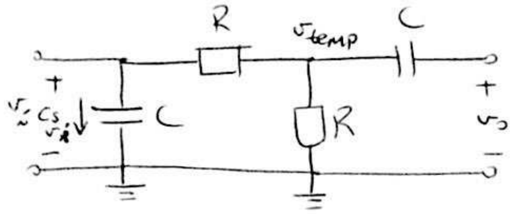
with $A \to +\infty$

and $r_{in,f} \to +\infty$

$r_{out,f} \to 0$

1

Analyze the feedback network



$$\frac{v_i}{v_o} = \frac{\left(R+\frac{1}{Cs}\right)//R}{\left(R+\frac{1}{Cs}\right)//R+\frac{1}{Cs}} \cdot \frac{1/Cs}{1/Cs+R}$$

OR do it like this

$$\frac{v_{temp}-v_i}{R} = \frac{v_i}{1/Cs} = Cs\,v_i$$

$$v_{temp} = v_i + RCs\,v_i$$

$$= (1+RCs)\,v_i$$

and KCL at the node with v_{temp}

$$\frac{v_{temp}}{R} + Cs\,v_i + Cs\,(v_{temp}-v_o) = 0$$

$$v_{temp} + RCs\,v_i + RCs\,(v_{temp}-v_o) = 0$$

$$v_{temp}(1+RCs) + RCs\,v_i = RCs\,v_o$$

$$v_i(1+RCs)^2 + RCs\,v_i = RCs\,v_o$$

$$v_i\left[R^2C^2s^2 + 3RCs + 1\right] = RCs\,v_o$$

$$\frac{v_i}{v_o} = \frac{RCs}{R^2C^2s^2+3RCs+1}$$

$$\frac{v_i}{v_o} = \frac{1}{3+\frac{1}{RCs}+RCs}$$

2

Sedra 13.13 contin-

Therefore

$$\beta(jw) = \frac{v_+(jw)}{v_o(jw)} = \frac{1}{3 + j\left(wCR - \frac{1}{wCR}\right)}$$

Recall that A_{VA} of the forward amplifier is

$$A_{VA} = 1 + \frac{R_2}{R_1}$$

$$A_{VA}\,\beta(jw) = \frac{1 + \frac{R_2}{R_1}}{3 + j\left(wCR - \frac{1}{wCR}\right)}$$

Now apply the Barkhawen criterion:
Zero phase condition requires

$$wCR - \frac{1}{wCR} = 0$$

$$\Rightarrow w_0 = \frac{1}{RC} \quad \text{(osc frequency)}$$

unity magnitude condition requires (at the osc freq. w_0)

$$\left| A_{VA}\,\beta(jw_0) \right| = 1$$

$$\frac{1 + \frac{R_2}{R_1}}{3} = 1$$

and for self-starting osc

$$1 + \frac{R_2}{R_1} \geq 3$$

$$\frac{R_2}{R_1} \geq 2$$

3