Universidad Politécnica de Cataluña
Facultad de Matemáticas y Estadística
MAMMEE
Hamiltonian Systems

Twist Maps and Aubry-Mather Sets

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- We will be working with **area-preserving monotone twist maps** preserving the boundary components

$$\varphi : S^1 \times [0,1] \longrightarrow S^1 \times [0,1]$$

- The **Mather sets** will be a particular φ-invariant subsets of the cylinder.
- We want to find φ-invariant closed curves which separates $S^1 \times \{0\}$ and $S^1 \times \{1\}$. They are related with the stability of the system $\{\varphi^n\}_{n\in\mathbb{N}}$.

Curves and stability of $\{\varphi^n\}_{n\in\mathbb{N}}$

Relationship with KAM theory

- KAM-Theory for this kind of maps shows that for this kind of maps which are sufficiently $\mathcal{C}^k$-close to an integrable one, then many of the invariant curves persists. The invariant curves are destroyed when we go too far away from the integrable situation and the Mather sets, M_α re the most important remmants of the invariant curves of irrational rotation number α.
- The Aubry-Mather theory appears to explain what happend with perturbed systems and its main contribution is to explain what happend with the orbits in the cases that are not covered by KAM theory.

- **Trajectory**: $x = (x_i)_{i\in\mathbb{Z}} \in \mathbb{R}^{\mathbb{Z}}$.
- **Convergence** is the obvious notion (with the product topology) for each i.
- Given a function $H : \mathbb{R}^2 \to \mathbb{R}$ we can extend it to trajectories (or finite segments) by:

$$H(x_j, \cdots, x_k) = \sum_{i=j}^{k-1} H(x_i, x_{i+1})$$

Bibliography

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