Twist Maps and Aubry-Mather Sets

Manuel Gijón Agudo

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1 Introduction

Caso 2 dimensional

Se trataría de dar una introducción y comparar con la teoría KAM explicada por Marcel, o sea que él te podrá dar más detalles. También en el último capítulo de Meyer Offin hay una aproximación variacional.

Our objetive in this work is give an introduccion to the Aubry-Mathers theory, explain its importance in the context it is on and expose the fundamental resoutls of the topic. In order to arrive to this point, we first present resoults and definitions in the next section.

2 First definitions and basic resoutls

bla bla bla bla

3 Main resoults

Definition: a monotone **twist map** is an orientation preserving $\mathcal{C}^1$-diffeomorphismm $\varphi : S^1 \times [0,1] \longrightarrow S^1 \times [0,1]$ of an annulus which admits a lift $\bar{\varphi} = (f, g) : \mathbb{R} \times [0,1] \longrightarrow \mathbb{R} \times [0,1]$ whit the following properties:

(a) $\bar{\varphi}$ preserves (Lebesgue) area.

(b) **Twist condiction**: $D_2 f > 0$.

(c) $g(\xi, 0) = 0,\ g(\xi, 1) = 1$.

Notes:

- Instead of (a) we could require $det(\bar{\varphi}') = 1$.
- Condiction (c) means that φ does not commute the boundary components.

A fundamental property of the monotone twist maps is that it can be globally described by a generating function:

$$H : D \to \mathbb{R}, \quad \text{where} \quad D := \{(\xi, \eta) \in \mathbb{R}^2 | f(\xi, 0) \leq \eta \leq f(\xi, 1)\}$$

up to an additive constant, H is uniquely determined by:

$$\bar{\varphi}(x_0, y_0) = (x_1, y_1) \Leftrightarrow \begin{cases} -D_1 H(x_0, y_0) & = y_0 \\ D_2 H(x_0, y_0) & = y_1 \end{cases} \tag{1}$$

To construct H let $a, b : D \to \mathbb{R}$ be defined by:

$$a(\xi, f(\xi, y)) := y, \qquad b(\xi, f(\xi, \eta)) := g(\xi, a(\xi, \eta))$$

Then, 1 is equivalent to:

$$dH = -a d\xi + b d\eta$$

4 Conclusions

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References

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