COURCE 1

<question> Which of the following are possible elements of robots in this specialization? Select all that apply

<variantright>Rigid bodies.

<variant>Soft, flexible bodies.

<variantright>Joints.

<question>The number of degrees of freedom of a robot is (select all that apply):

<variantright>the dimension of its configuration space.

<variantright>the number of real numbers needed to specify its configuration.

<variant>the number of points on the robot.

<variant>the number of joints of the robot.

<variant>the number of bodies comprising the robot.

<variantright>the number of freedoms of the bodies minus the number of independent constraints between the bodies.

<question>The number of degrees of freedom of a planar rigid body is

<variantright> 3

<question>The number of degrees of freedom of a spatial rigid body is

<variantright>6

<question>A rigid body in nn-dimensional space has mm total degrees of freedom. How many of these mm degrees of freedom are angular (not linear)? Select all that apply. **(This is consistently one of the most incorrectly answered questions in this course, so think about it carefully!)**

<variantright> m−n

<variantright> n(n−1)/2

<variant> Neither of the above.

<question>Consider a joint between two rigid bodies. Each rigid body has mm degrees of freedom (m=3m=3 for a planar rigid body and m=6m=6 for a spatial rigid body) in the absence of any constraints. The joint has ff degrees of freedom (e.g., f=1f=1 for a revolute joint or f=3f=3 for a spherical joint). How many constraints does the joint place on the motion of one rigid body relative to the other? Write your answer as a mathematical expression in terms of mm and ff.

<variantright> −f+m

<question>Consider a mechanism consisting of three spatial rigid bodies (including ground, N=4N=4) and four joints: one revolute, one prismatic, one universal, and one spherical. According to Grubler's formula, how many degrees of freedom does the mechanism have?

<variantright>1

<question>A mechanism that is incapable of motion has zero degrees of freedom. In some circumstances, Grubler's formula indicates that the number of degrees of freedom of a mechanism is negative. How should that result be interpreted?

<variantright>The constraints implied by the joints must not be independent.

<variant>The number of joints, the degrees of freedom of those joints, or the number of rigid bodies must have been counted incorrectly.

<question>Using the methods for determining the number of degrees of freedom of a rigid body in 3-dimensional space from the book and the video, find the number of degrees of freedom of a rigid body in a conceptual 4-dimensional space. Your answer should be an integer.

<variantright> 10

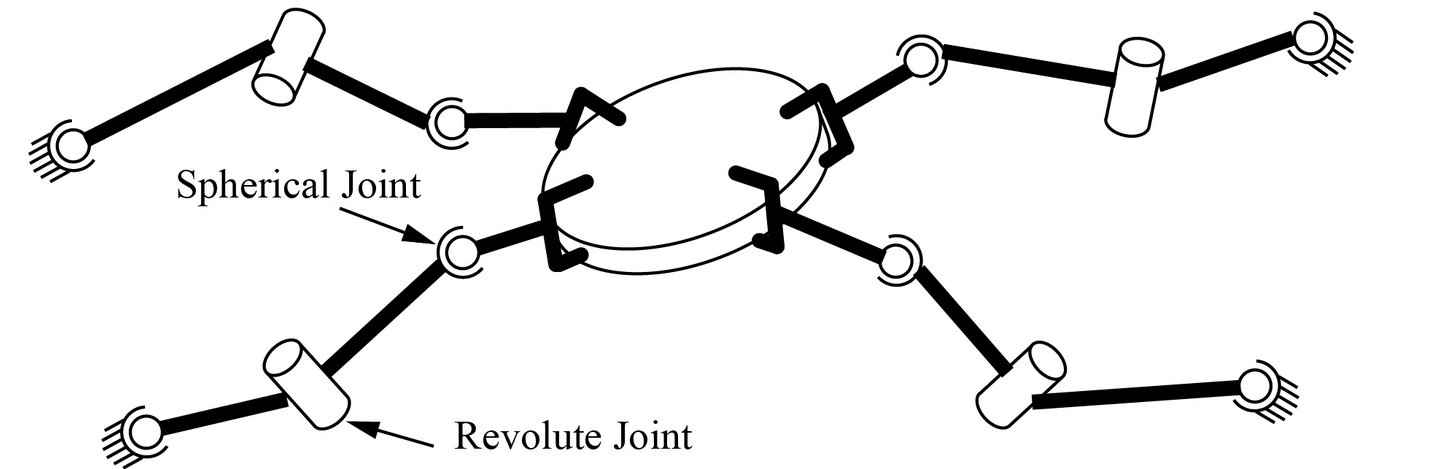
<question>Referring back to Question 1, indicate how many of the total degrees of freedom are angular (rotational). Your answer should be an integer.

<variantright> 6

<question>Assume your arm, from your shoulder to your palm, has 7 degrees of freedom. You are carrying a tray like a waiter, and you must keep the tray horizontal to avoid spilling drinks on the tray. How many degrees of freedom does your arm have while satisfying the constraint that the tray stays horizontal? Your answer should be an integer.

<variantright>5

<question>Four identical SRS arms are grasping a common object as shown below.



<variantright> 10

<question>Referring back to Question 4, suppose there are now a total of nn such arms grasping the object. What is the number of degrees of freedom of this system? Your answer should be a mathematical expression including nn. Examples of mathematical expressions including nn are 4∗n−74∗n−7 or n/3n/3.

<variantright>n+6

<question>Referring back to Question 4 and 5, suppose the revolute joint in each of the nn arms is now replaced by a universal joint. What is the number of degrees of freedom of the overall system? Your answer should be a mathematical expression including nn. Examples of mathematical expressions including nn are 4∗n−74∗n−7 or n/3n/3.

<variantright> 2\*n + 6

<question>Use the planar version of Grubler's formula to determine the number of degrees of freedom of the mechanism shown below. Your answer should be an integer. (Remember that a single joint can only connect two rigid bodies, so if you see more than two connecting at a single point, there must be more than one joint there. Also, the two blocks in the channels are only allowed to move prismatically in those channels, and one of the joints is labeled "P" for prismatic. You will need to identify all the other joints, and links.)

<variantright> 3

<question>To deform one nn-dimensional space into another topologically equivalent space, which operations are you allowed to use? Select all that apply.

<variantright>Stretching

<variant>Cutting.

<variant>Gluing.

<question>True or false? An nn-dimensional space can be topologically equivalent to an mm-dimensional space, where m≠nm​=n.

<variant>True.

<variantright>False

<question>True or false? An explicit parametrization uses fewer numbers to represent a configuration than an implicit representation.

<variantright>True.

<variant>False

<question>A kk-dimensional space is represented by 7 coordinates subject to 3 independent constraints. What is kk?

<variantright>4

<question>True or false? A nonholonomic constraint implies a configuration constraint.

<variant>True.

<variantright>False

<question>True or false? A Pfaffian velocity constraint is necessarily nonholonomic.

<variant>True.

<variantright>False

<question>A wheel moving in free space has the six degrees of freedom of a rigid body. If we constrain it to be upright on a plane (no "leaning") and to roll without slipping, how many holonomic and nonholonomic constraints is the wheel subject to?

<variantright>Two holonomic constraints and two nonholonomic constraints.

<variant>Three holonomic constraints and zero nonholonomic constraints.

<variant>Zero holonomic constraints and three nonholonomic constraints.

<variant>One holonomic constraint and two nonholonomic constraints.

<question>How many degrees of freedom does the upright wheel on the plane have? (What is the minimum number of coordinates needed to describe its configuration?)

<variantright> 4

<question>If the task is to control the orientation of a spaceship simulator, but not its position, how many degrees of freedom does the task space have?

<variantright> 3

<question>True or false? The workspace depends on the robot's joint limits but the task space does not.

<variantright>True.

<variant>False

<question>The tip coordinates for the two-link planar 2R robot of figure below are given by

x=cos⁡θ1+2cos⁡(θ1+θ2)x=cosθ1​+2cos(θ1​+θ2​)

y=sin⁡θ1+2sin⁡(θ1+θ2)y=sinθ1​+2sin(θ1​+θ2​)

(In other words, link 1 has length 1 and link 2 has length 2.) The joint angles have no limits.

Which of the following best describes the shape of the robot's workspace (the set of locations the endpoint can reach)?

<variant>A circle and its interior.

<variant>A circle only (not including the interior).

<variantright>Annulus or ring (the area between two concentric bounding circles).

<question>The chassis of a mobile robot moving on a flat surface can be considered as a planar rigid body. Assume that the chassis is circular, and the mobile robot moves in a square room. Which of the following could be a mathematical description of the C-space of the chassis while it is confined to the room? (See Chapter 2.3.1 for related discussion.)

<variantright>[a,b]×[a,b]×S1

<variant> [a,b]×R1×S1[a,b]×R1×S1

<variant> [a,b]×[a,b]×R1[a,b]×[a,b]×R1

<variant>R2×S1R2×S1

<question>Which of the following is a possible mathematical description of the C-space of a rigid body in 3-dimensional space?

<variant> R3×S3

<variant> R3×T3R3×T3

<variant> R3×T2×S1R3×T2×S1

<variantright> R3×S2×S1R3×S2×S1

<question>A spacecraft is a free-flying rigid body with a 7R arm mounted on it. The joints have no joint limits. Give a mathematical description of the C-space of this system. (See Chapter 2.3.1 for related discussion.)

<variant>R3×T10R3×T10

<variantright>R3×S2×T8R3×S2×T8

<variant>R3×S3×T7R3×S3×T7

<variant>R4×S2×T7R4×S2×T7

<question>A mobile robot is moving on an infinite plane with an RPR robot arm mounted on it. The prismatic joint has joint limits, but the revolute joints do not. Give a mathematical description of the C-space of the chassis (which can rotate and translate in the plane) plus the robot arm. (See Chapter 2.3.1 for related discussion.)

<variant>R2×S2×S1×[a,b]R2×S2×S1×[a,b]

<variant>R2×S3×[a,b]R2×S3×[a,b]

<variantright>R2×T3×[a,b]R2×T3×[a,b]

<variant>R3×T3R3×T3

<question>Determine whether the following differential constraint is holonomic or not (nonholonomic). See the example in Chapter 2.4.

(1+cos⁡q1)q˙1+(2+sin⁡q2)q˙2+(cos⁡q1+sin⁡q2+3)q˙3=0.(1+cosq1​)q˙​1​+(2+sinq2​)q˙​2​+(cosq1​+sinq2​+3)q˙​3​=0.

<variant>Holonomic

<variantright>Nonholonomic

<question>The task is to carry a waiter's tray so that it is always horizontal (orthogonal to the gravity vector), but otherwise free to move in any other direction. How many degrees of freedom does the task space (the C-space of a horizontal tray) have? (Enter an integer number.)

<variantright> 4

<question>Which do we typically use to represent the C-space of a rigid body?

<variant> Explicit parametrization (minimum number of coordinates).

<variantright> Implicit representation.

<question> By the right-hand rule, which fingers of your right hand correspond to the x, y, and z axes of a coordinate frame, respectively?

<variant> Thumb, index, middle

<variant> Middle, index, thumb

<variantright> Index, middle, thumb

<question>When your thumb points along an axis of rotation, positive rotation about the axis is defined by the direction your fingers curl if you use which thumb?

<variant> Right thumb

<variantright> Left thumb

<question>When we refer to a frame attached to a moving body, we always consider a stationary frame {b}, because

<variant> the motion of all other frames is expressed relative to {b}.

<variantright> {b} is the stationary frame that is coincident (at a particular instant) with the frame attached to the moving body.

<question>For the rotation matrix *Rba*​ representing the frame {a} relative to {b},  
<variant> the rows are the x, y, z axes of {a} written in {b} coordinates.

<variantright> the columns are the x, y, z axes of {a} written in {b} coordinates.

<variant> the rows are the x, y, z axes of {b} written in {a} coordinates.

<variant> the columns are the x, y, z axes of {b} written in {a} coordinates.

<question>The 3×3 rotation matrix is an implicit representation of spatial orientations consisting of 9 numbers subject to how many independent constraints?

<variantright> 6

<question>The inverse of a rotation matrix *Rab*​, i.e., *Rab*−1​, is (select all that apply):

<variant> −*Rab*​

<variantright> *Rab*T​

<variant> *R*−*I*

<variantright> *Rba*

<question>Multiplication of ��(3)*SO*(3) rotation matrices is (select all that apply):

<variantright> associative.

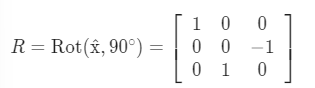
<variant> commutative.

<question>Which of the following is equivalent to *Rac*​, the representation of the orientation of the {c} frame relative to the {a} frame? Select all that apply.

<variantright> *Rab*​*Rbc*​  
<variantright> *Rab*​*Rcb*T​  
<variantright> (*Rbc*T​*Rab*T​)T

<variantright> *Rad*​*Rdb*​*Rbc*

<question>The matrix



represents the orientation *Rsa*​ of a frame {a} that has been achieved by rotating the {s} frame by 90 degrees about its x^x^-axis. Now, given a matrix *Rsb*​ representing the orientation of {b} relative to {s}, which of the following represents the orientation of a frame (relative to {s}) that was initially aligned with {b}, but then rotated about the {b}-frame's x^x^-axis by 90 degrees?

<variantright> *Rsb*​*R*  
<variant> *RRsb*​

<question>The matrix

Изображение выглядит как текст, Шрифт, типография, дизайн

Автоматически созданное описание

represents the orientation *Rsa*​ of a frame {a} that has been achieved by rotating the {s} frame by 90 degrees about its x^x^-axis. Now, given a matrix *Rsb*​ representing the orientation of {b} relative to {s}, which of the following represents the orientation of a frame (relative to {s}) that was initially aligned with {b}, but then rotated about the {s}-frame's x^x^-axis by 90 degrees?

<variant> *Rsb*​*R*  
<variantright> *RRsb*​

<question> Our representation of the three-dimensional orientation uses an implicit representation (a 3x3 SO(3) matrix with 9 numbers), but our usual representation of the angular velocity uses only three numbers, i.e., an explicit parametrization of the three-dimensional velocity space. Why do we use an implicit representation of the orientation but an explicit parametrization of the angular velocity?

<variant> There is no natural implicit representation of an angular velocity.

<variantright> The space of angular velocities can be equated to a "flat" 3d space (a linear vector space) tangent to the curved 3d surface of orientations at any given time, so it can be globally represented by 3 numbers without singularities. The space of orientations, on the other hand, is not flat, and cannot be globally represented by 3 numbers without a singularity.

<question>A rotation matrix is an element of which space?

<variant> R3  
<variantright> *SO*(3)  
<variant> *so*(3)

<question>An angular velocity is an element of which space?

<variantright> R3  
<variant> *SO*(3)  
<variant> *so*(3)

<question>The 3x3 skew-symmetric matrix representation of an angular velocity is an element of which space?

<variant> R3  
<variant> *SO*(3)  
<variantright> *so*(3)

<question>If an angular velocity is represented as *ωb*​ in the body frame {b}, what is the representation of the same angular velocity in the space frame {s}?

<variantright>*Rsb*​*ωb*​  
<variant>*Rbs*​*ωb*​  
<variant> *ωb*​*Rsb*  
<variant> *ωb*​*Rbs*​

<question>The cross-product *ω*×*p* can be written [*ω*]*p*, where [*ω*] is

<variant> the (3)*SO*(3) representation of *ω*.  
<variantright> the skew-symmetric *so*(3) representation of *ω*.

<question>The orientation of a frame {d} relative to a frame {c} can be represented by a unit rotation axis *ω*^ and the distance *θ* rotated about the axis. If we rotate the frame {c} by *θ* about the axis *ω*^ expressed in the {c} frame, we end up at {d}. The vector *ω*^ has 3 numbers and *θ* is 1 number, but we only need 3 numbers, the exponential coordinates *ω*^*θ*, to represent {d} relative to {c}, because

<variantright> though we use 3 numbers to represent *ω*^, *ω*^ actually only represents a point in a 2-dimensional space, the 2-dimensional sphere of unit 3-vectors.  
<variant> the choice of *θ* is not independent of *ω*^.

<question>One reason we use 3x3 rotation matrices (an implicit representation) to represent orientation is because it is a good global representation: there is a unique orientation for each rotation matrix, and vice-versa, and there are no singularities in the representation. In what way does the 3-vector of exponential coordinates fail these conditions? Select all that apply.

<variantright> There could be more than one set of exponential coordinates representing the same orientation.  
<variant> Some orientations cannot be represented by exponential coordinates.

<question>The vector linear differential equation *x*˙(*t*)=*Bx*(*t*), where *x* is a vector and *B* is a constant square matrix, is solved as *x*(*t*)=*eBtx*(0), where the matrix exponential *eBt* is defined as

<variantright> the sum of an infinite series of matrices of the form (*Bt*)0+*Bt*+(*Bt*)2/2!+(*Bt*)3/3!…  
<variant> the sum of an infinite series of matrices of the form *Bt*+*Bt*/2+*Bt*/3+….

<question>The solution to the differential equation *p*˙​(*t*)=*ω*^×*p*(*t*)=[*ω*^]*p*(*t*) is *p*(*t*)=*e*[*ω*^*θ*]*p*(0), where *p*(0) is the initial vector and *p*(*t*) is the vector after it has been rotated at the angular velocity *ω*^ for time *t*=*θ* (where *ω*^*θ* are the exponential coordinates). You can think of *R*=*e*[*ω*^*θ*] as the rotation operation that moves *p*(0) to *p*(*t*)=*p*(*θ*).

<variantright> *Rsb*′​=*Rsb*​*e*[*ω*^*θ*] represents the orientation of a new frame {b'} relative to {s} after the frame {b} has been rotated by *θ* about an axis w represented in the {b} frame as *ω*^  
<variant> *Rsb*′​=*Rsb*​*e*[*ω*^*θ*] represents the orientation of a new frame {b'} relative to {s} after the frame {b} has been rotated by *θ* about an axis w represented in the {s} frame as *ω*^.  
<variant> *Rsb*′​=*e*[*ω*^*θ*]*Rsb*​ represents the orientation of a new frame {b'} relative to {s} after the frame {b} has been rotated by *θ* about an axis w represented in the {b} frame as *ω*^.  
<variantright> *Rsb*′​=*e*[*ω*^*θ*]*Rsb*​ represents the orientation of a new frame {b'} relative to {s} after the frame {b} has been rotated by *θ* about an axis w represented in the {s} frame as *ω*^.

<question>The simple closed-form solution to the infinite series for the matrix exponential when the matrix is an element of *so*(3) (a skew-symmetric 3x3 matrix) is called what?

<variant> Ramirez's formula.  
<variantright> Rodrigues' formula.  
<variant> Robertson's formula.

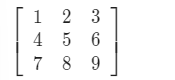
<question>The matrix exponential and the matrix log relate a rotation matrix (an element of *SO*(3)) and the skew-symmetric representation of the exponential coordinates (elements of *so*(3)), which can also be thought of as the *so*(3) representation of the angular velocity followed for unit time. Which of the following statements is correct? Select all that apply.

<variantright> exp: *so*(3)→*SO*(3)  
<variant> exp: (3)*SO*(3)→*so*(3)  
<variant> log: (3)*so*(3)→*SO*(3)  
<variantright> log: *SO*(3)→*so*(3)

<question>In terms of the *x*^s​, *y*^​s​, *z*^s​ coordinates of a fixed space frame {s}, the frame {a} has its *x*^a​-axis pointing in the direction (0,0,1)(0,0,1) and its *y*^​a​-axis pointing in the direction (1,0,0)(1,0,0), and the frame {b} has its *x*^b​-axis pointing in the direction (1,0,0)(1,0,0) and its *y*^​b​-axis pointing in the direction (0,0,−1)(0,0,−1). Draw the {s}, {a}, and {b} frames, similar to examples in the book and videos (e.g., Figure 3.7 in the book), for easy reference in this question and later questions.

Write the rotation matrix *Rsa*​. All elements of this matrix should be integers.

**If your answer is**



**for example, you should just type**

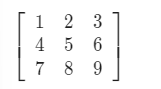
[[1,2,3],[4,5,6],[7,8,9]]

**in the answer box below. (You can just modify the matrix that is currently written there.) Then click "Run." You will not get any immediate feedback; the grade will be given when you submit the whole quiz.**

<variantright> [[0,1,0],[0,0,1],[1,0,0]]

<question>Referring to your drawing from Question 1, write *Rsb*−1​. All elements of this matrix should be integers

**If your answer is**



**you should just type**

[[1,2,3],[4,5,6],[7,8,9]]

**in the answer box below. (You can just modify the matrix that is currently written there.) Then click "Run." You will not get any immediate feedback; the grade will be given when you submit the whole quiz.**

<variantright> [[1,0,0],[0,0,-1],[0,1,0]]

<question>Referring to your drawing from Question 1, write *Rab*​. All elements of this matrix should be integers

**Write your matrix in the answer box below, using the format mentioned in questions 1 and 2, and click "Run."**

<variantright> [[0,-1,0],[1,0,0],[0,0,1]]

<question>Referring back to Question 1, let *R*=*Rsb*​ be considered as a transformation operator consisting of a rotation about *x*^ by −90∘−90∘. Calculate *R*1​=*Rsa*​*R*, and think of *Rsa*​ as the representation of the initial orientation of {a} relative to {s}, *R* as a rotation operation, and *R*1​ as the new orientation of {a} after performing the rotation. The new orientation *R*1​ corresponds to the orientation of the new {a} frame relative to {s} after rotating the original {a} frame by −90∘−90∘ about which axis?

<variantright> The *x*^a​-axis of the {a} frame.  
<variant> *x*^s​-axis of the {s} frame.

<question>Referring back to Question 1, use *Rsb*​ to change the representation of the point *pb*​=(1,2,3)⊺ (in {b} coordinates) to {s} coordinates. All elements of this vector should be integers.

**If your answer is**



**you should enter**

[1,2,3]

**in the text box below and click "Run."**

<variantright> [1,3,-2]

<question>Referring back to Question 1, choose a point p represented by *ps*​=(1,2,3)⊺ in {s} coordinates. Calculate *q*=*Rsb*⊺​*ps*​. Is *q* a representation of p in {b} coordinates?

<variantright> Yes  
<variant> No

<question>Referring back to Question 1, an angular velocity *w* is represented in {s} as *ωs*​=(3,2,1)⊺. What is its representation *ωa*​? All elements of this vector should be integers.

**If your answer is**



**you should enter**

[1,2,3]

**in the text box below and click "Run."**

<variantright> [1,3,2]

<question>Referring back to Question 1, calculate the matrix logarithm [*ω*^]*θ* of *Rsa*​ by hand. (You may verify your answer with software.) Extract and enter the rotation amount *θ* in radians with at least two decimal places.

<variantright> 2.09

<question>alculate the matrix exponential corresponding to the exponential coordinates of rotation *ω*^*θ*=(1,2,0)⊺. The maximum allowable error for any matrix element is 0.01, so give enough decimal places where necessary.

**Write your matrix in the answer box below, using the format mentioned in questions 1 and 2, and click "Run."**

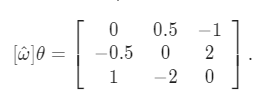
<variantright> [[-0.29,0.65,0.70],[0.65,0.68,-0.35],[-0.70,0.35,-0.62]]

<question>Write the 3×33×3 skew-symmetric matrix corresponding to *ω*=(1,2,0.5)⊺. Confirm your answer using the functionVecToso3 in the given software.

**Write your matrix in the answer box below, using the format mentioned in questions 1 and 2, and click "Run."**

<variantright> [[0,-0.5,2],[0.5,0,-1],[-2,1,0]]

<question>Use the function 3MatrixExp3 in the given software to calculate the rotation matrix *R*∈*SO*(3) corresponding to the matrix exponential of



The maximum allowable error for any matrix element is 0.01, so give enough decimal places where necessary.

**Write your matrix in the answer box below, using the format mentioned in questions 1 and 2, and click "Run."**

<variantright> [[0.60,0.79,-0.01],[0.47,-0.34,0.81],[0.64,-0.50,-0.58]]

<question>Use the function MatrixLog3 in the given software to calculate the matrix logarithm [*ω*^]*θ* *so*(3) of rotation matrix

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The maximum allowable error for any matrix element is 0.01, so give enough decimal places where necessary.

**Write your matrix in the answer box below, using the format mentioned in questions 1 and 2, and click "Run."**

**<variantright> I don’t know sorry)**

<question> A 4x4 transformation matrix (element of *SE(3))* consists of a rotation matrix, a 3-vector, and a row consisting of three zeros and a one. What is the purpose of the row of 4 constants?  
<variant> This row is a historical artifact.  
<variantright> This row allows simple matrix operations for useful calculations.

<question> Which of the following are possible uses of a transformation matrix? Select all that apply.  
<variantright> Displace (rotate and translate) a frame.  
<variantright> Displace a vector.  
<variantright> Change the frame of reference of a vector.  
<variantright> Represent the position and orientation of one frame relative to another.

<question> The representation of a point p in the {b} frame is *p­b ∈ R3*. To find the representation of this point in the {a} frame, we could write *Tab*​*pb*​, but there is a dimension mismatch; *pb*​ has only 3 components, but T*ab*​ is 4x4. How do we alter *pb*​ to allow this matrix operation?  
<variantright> Put a 1 in the last row of *pb*​, making it a 4-element column vector, and otherwise ignore the last row in your interpretation of the 4-vector.  
<variant> Put a 0 in the last row of *pb*​, making it a 4-element column vector, and otherwise ignore the last row in your interpretation of the 4-vector.

<question> Which of these is a valid calculation of *Tab*​, the configuration of the frame {b} relative to {a}? Select all that apply.  
<variantright>TacTcb<variant>TcbTac<variantright>TacTdc-1Tdb<variantright>(TbcTca)-1

<question> Any instantaneous spatial velocity of a rigid body is equivalent to the motion of the body if it were simultaneously translating along, and rotating about, a **screw** axis *S=(Sω​,Sv​)∈R6*. The screw axis is a normalized representation of the direction of motion, and ˙*θ*˙ represents how fast the body moves in that direction of motion, so that the **twist** is given by *V=Sθ˙∈R6*. The normalized screw axis for full spatial motions is analogous to the normalized (unit) angular velocity axis for pure rotations.   
 The pitch *h* of the screw axis is defined as the ratio of the linear speed over the angular speed. Which of the following is true? Select all that apply.

<variantright> If the pitch ℎ is infinite, then S*ω*​=0 and ∥S*v*​∥=1.

<variant> If the pitch ℎ is infinite, then ∥S*ω*​∥=1 and S*v*​ is arbitrary.

<variant> If the pitch ℎ is finite, then S*ω*​=0 and ∥S*v*​∥=1.  
<variantright> If the pitch ℎ is finite, then ∥S*ω*​∥=1 and S*v*​ is arbitrary.

<question> You are sitting on a horizontal rotating turntable, like a merry-go-round at an amusement park. It rotates counterclockwise when viewed from above. Your body frame {b} has an x^*b*​-axis pointing outward (away from the center of the turntable), a y^​*b*​-axis pointing in the direction the turntable is moving at your location (the direction your eyes are looking), and a z^*b*​-axis pointing upward. The turntable is rotating at 0.1 radians per second, and you are sitting 3 meters from the center of the turntable. What is the screw axis S=(S*ω*​,S*v*​) and the twist V=(*ω*,*v*) expressed in your body frame {b}? All angular velocities are in radians/second and all linear velocities are in meters/second.

<variant> *S*=(0,0,0.1,0,0.3,0), *V*=(0,0,0.01,0,0.03,0)  
<variantright> *S*=(0,0,1,0,3,0), *V*=(0,0,0.1,0,0.3,0)  
<variant> *S*=(1,0,0,0,3,0), *V*=(0.1,0,0,0,0.3,0)

<question> A twist or a screw axis can be represented in any frame. Which of the following statements are true? Select all that apply.

<variantright> A spatial twist is a representation of the twist in the space frame {s}, and it does not depend on a body frame {b}.  
<variantright> A body twist is a representation of the twist in the body frame {b}, and it does not depend on a space frame {s}.

<question> What is the dimension of the matrix adjoint representation [Ad*T*​] of a transformation matrix *T* (an element of *SE*(3))?  
<variant> 3x3  
<variant> 4x4  
<variantright> 6x6

<question> A 3-vector angular velocity *ω* can be represented in matrix form as [*ω*], an element of *so*(3), the set of 3x3 skew-symmetric matrices. Analogously, a 6-vector twist *V*=(*ω*,*v*) can be represented in matrix form as [*V*], an element of *se*(3). What is the dimension of [*V*]?  
<variant> 3x3  
<variantright> 4x4  
<variantr> 6x6

<question> Although we use six numbers to represent a screw *S=(Sω​,Sv​)*, the space of all screws is only 5-dimensional. Why?

<variant> *Sω*​ must be unit length.  
<variant> *Sv*​ must be unit length.

<variantright> Either *Sω*​ or *Sv*​ must be unit length.

<question> A transformation matrix *Tab*​, representing {b} relative to {a}, can be represented using the 6-vector exponential coordinates *Sθ*, where *S* is a screw axis (represented in {a} coordinates) and *θ* is the distance followed along the screw axis that displaces {a} to {b}. Which of the following is correct? Select all that apply.  
<variant> *Tab*​=*e*S*θ*

<variantright> *Tab*​=*e*[S]*θ*

<variant> *Tab*​=*e*[S*θ*]

<variant> *Tab*​=*e*S[*θ*]

[You didn’t select all the correct answers]

<question> The matrix representation of the exponential coordinates *Sθ∈R6* is [*Sθ*]. What space does [*Sθ*] belong to?  
<variant> *SO*(3)

<variant> *so*(3)

<variant> *SE*(3)

<variantright> *se*(3)

<question> *Tab*′​=*Tab*​*e*[S*θ*] is a representation of the new frame {b'} (relative to {a}) achieved after {b} has followed  
<variantright> the screw axis *S*, expressed in {b} coordinates, a distance *θ*.

<variant> the screw axis *S*, expressed in {a} coordinates, a distance *θ*.

<question> *Tab*′​= *e*[S*θ*]*Tab*​ is a representation of the new frame {b'} (relative to {a}) achieved after {b} has followed  
<variant> the screw axis *S*, expressed in {b} coordinates, a distance *θ*.

<variantright> the screw axis *S*, expressed in {a} coordinates, a distance *θ*.

<question> Which of the following statements is true? Select all that apply.

<variantright> The matrix exponential maps [S*θ*]∈*se*(3) to a transformation matrix *T*∈*SE*(3), where *T* is the representation of the frame (relative to {s}) that is achieved by following the screw S (expressed in {s}) a distance *θ* from the identity configuration (i.e., a frame initially coincident with {s}).

<variantright> The matrix exponential maps [*V*]∈*se*(3) to a transformation matrix *T*∈*SE*(3), where *T* is the representation of the frame (relative to {s}) that is achieved by following the twist *V* (expressed in {s}) for unit time from the identity configuration (i.e., a frame initially coincident with {s}).

If we choose *V=Sθ*, then following the twist *V* for unit time is equivalent to following the screw axis *S* a distance *θ*.

<variant> The matrix log maps an element of *se*(3) to an element of *SE*(3).

<variantright> The matrix log maps an element of *SE*(3) to an element of *se*(3).

<variantright> There is a one-to-one mapping between twists and elements of *se*(3).

<question> A wrench F*a*​ consists of a linear force *fa*​∈R3 and a moment *ma*​∈R3, both expressed in the frame {a}. How do we usually write the wrench?

<variantright> F*a*​=(*ma*​,*fa*​)

<variant> F*a*​=(*fa*​,*ma*​)

<question> We know that the power associated with a wrench and twist pair (*F,V*) does not depend on whether they are represented in the frame {a} as (*Fa​,Va*​) or the frame {b} as (*Fb​,Vb*​). Therefore, we can write *FaT​Va​=FbT​Vb*​ and then use which identity to derive the equation *Fa​=[AdTba​​]TFb*​ relating the representations *Fa​* and *Fb*​? (Also, remember the matrix identity (*AB*)T =*BTAT.*)

<variant> V*a*​=*Tab*​V*b*​

<variant> V*a*​=*Tba*​V*b*​

<variant> V*a*​=[Ad*Tba*​​]V*b*​

<variantright> V*a*​=[Ad*Tab*​​]V*b*​

<question> In terms of the *x*^s​, *y*^​s​, *z*^s​ coordinates of a fixed space frame {s}, the frame {a} has its *x*^a​-axis pointing in the direction (0,0,1)(0,0,1) and its *y*^​a​-axis pointing in the direction (−1,0,0)(−1,0,0), and frame {b} has its *x*^b​-axis pointing in the direction (1,0,0)(1,0,0) and its *y*^​b​-axis pointing in the direction (0,0,−1)(0,0,−1). The origin of {a} is at (0,0,1)(0,0,1) in {s} and the origin of {b} is at (0,2,0)(0,2,0). Draw the {s}, {a}, and {b} frames, similar to examples in the book and videos, for easy reference in this question and later questions. Write the transformation matrix *Tsa*​. All elements of this matrix should be integers.  
<answer> [[0,-1,0,0],[0,0,-1,0],[1,0,0,1],[0,0,0,1]]

<question> Referring back to Question 1, write *T-1sb*​. All elements of this matrix should be integers.  
 <answer> [[1,0,0,0],[0,0,-1,0],[0,1,0,-2],[0,0,0,1]]

<question> Referring back to Question 1, write *Tab*​. All elements of this matrix should be integers.  
 <answer> [[0,-1,0,-1],[-1,0,0,0],[0,0,-1,-2],[0,0,0,1]]

<question> Referring back to Question 1, let *T*=*Tsb*​ be considered as a transformation operator consisting of a rotation about *x*^ by−90∘ and a translation along *y*^​ by 2 units. Calculate *T*1​=*TTsa*​, and think of *Tsa*​ as the representation of the initial configuration of {a} relative to {s}, *T* as a transformation operation, and *T*1​ as the new configuration of {a} after performing the transformation. Are the rotation axis *x*^ and translation axis *y*^​ of the transformation *T* properly considered to be expressed in the frame {s} or the frame {a}?  
<variantright> The frame {s}.  
<variant> The frame {a}.

<question> Referring back to Question 1, use *Tsb*​ to change the representation of the point *pb*​=(1,2,3)T (in {b} coordinates) to {s} coordinates. All elements of this vector should be integers.

<answer> [1, 5, -2]

<question> Referring back to Question 1, a twist *V* is represented in {s} as *Vs*​=(3,2,1,−1,−2,−3)T. What is its representation *Va*​? All elements of this vector should be integers.

<answer> [1, -3, -2, -3, -1, 5]

<question> Referring back to Question 1, calculate the matrix logarithm [*S*]*θ* of *Tsa*​. Write the rotation amount *θ* in radians with at least 2 decimal places.  
 <answer> 2.0944

<question> Referring back to Question 1, use *Tsb*​ to change the representation of the wrench F*b*=(1,0,0,2,1,0)⊺ (in {b} coordinates) to {s} coordinates. All elements of this vector should be integers.

<answer> [-1, 0, -4, 2, 0, -1]

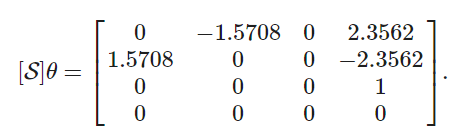
<question> Use the function TransInv in the given software to calculate the inverse of the homogeneous transformation matrix  
Изображение выглядит как диаграмма, снимок экрана, линия, дизайн

Автоматически созданное описание  
 <answer> [[0,1,0,0],[-1,0,0,3],[0,0,1,-1],[0,0,0,1]]

<question> Write the *se*(3) matrix corresponding to the twist V=(1,0,0,0,2,3)T. All elements of this matrix should be integers. Confirm your answer using the function VecTose3 in the given software.

<answer> [[0,0,0,0],[0,0,-1,2],[0,1,0,3],[0,0,0,0]]

<question> Use the function ScrewToAxis in the given software to calculate the normalized screw axis representation S of the screw described by a unit vector *s*^=(1,0,0) in the direction of the screw axis, located at the point *p*=(0,0,2), with pitch *h*=1. All elements of this vector should be integers.  
 <answer> [1,0,0,1,2,0]

<question> Use the function MatrixExp6 in the given software to calculate the homogeneous transformation matrix*T*∈*SE*(3) corresponding to the matrix exponential of  
  
 <answer> [[-0,-1,0,3],[1,-0,0,0],[0,0,1,1],[0,0,0,1]]

<question> Use the function MatrixLog6 in the given software to calculate the matrix logarithm [*S*]*θ*∈*se*(3) of the homogeneous transformation matrix  
Изображение выглядит как снимок экрана, диаграмма, линия, дизайн

Автоматически созданное описание  
 <answer> [[0,-1.5700,0,2.3562],[1.5700,0,0,-2.3562],[0,0,0,1.0000],[0,0,0,0]]