Association Analysis

Example Applications in which Co-Occurrence Matters

We are often interested in co-occurrence relationships

Marketing

- 1. identify items that are bought together by sufficiently many customers
- use this information for marketing or supermarket shelf management purposes



- identify parts that are often needed together for repairs
- 2. use this information to equip your repair vehicles with the right parts

Usage Mining

- 1. identify words that frequently appear together in search queries
- 2. use this information to offer auto-completion features to the user







Outline

- 1. Correlation Analysis
- 2. Association Analysis
 - 1. Frequent Itemset Generation
 - 2. Rule Generation
 - 3. Handling Continuous and Categorical Attributes
 - 4. Interestingness Measures

1. Correlation Analysis

- Correlation analysis measures the degree of dependency between two variables
 - Continuous variables: Pearson's correlation coefficient (PCC)
 - Binary variables: Phi coefficient

$$PCC(x,y) = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum (x_i - \overline{x})^2} \sqrt{\sum (y_i - \overline{y})^2}} \qquad Phi(x,y) = \frac{f_{11}f_{00} - f_{01}f_{10}}{\sqrt{f_{1+}f_{+1}f_{0+}f_{+0}}}$$

- Value range [-1,1]
 - 1 : positive correlation
 - 0 : variables independent
 - -1 : negative correlation

$$Phi(x,y) = \frac{f_{11}f_{00} - f_{01}f_{10}}{\sqrt{f_{1+}f_{+1}f_{0+}f_{+0}}}$$

	Attribute 1		
${\bf Attribute}\ 2$	Yes	No	
Yes No	$a \\ c$	d	

If a, b, c, and d represent the frequencies of observation, then ϕ is determined by the relationship

$$\phi = \frac{ad - bc}{\sqrt{\{(a+b)(c+d)(a+c)(b+d)\}}}$$

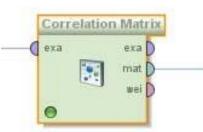
Correlations between Products in Shopping Baskets

	P1	P2	Р3	P4	P5
Basket 1	1	1	0	1	1
Basket 2	1	0	0	1	1
Basket 3	1	0	0	0	1

1 : always bought together

0 : sometimes bought together

-1: never bought together



Correl	Correlation Matrix (Correlation Matrix)									
Table Vie	Table View Pairwise Table Plot View Annotations									
Attributes	ThinkPad X	Asus EeePC	HP Laserjet	2 GB DDR3	8 GB DDR3	Lenovo Tab	Netbook-Sc	HP CE50 T	LT Laser M	LT Minimaus
ThinkPad X2	1	-1	0.356	-0.816	0.612	0.583	-0.667	0.356	0.167	-0.408
Asus EeePC	-1	1	-0.356	0.816	-0.612	-0.583	0.667	-0.356	-0.167	0.408
HP Laserjet	0.356	-0.356	1	-0.218	-0.327	0.356	-0.535	1	-0.089	-0.655
2 GB DDR3	-0.816	0.816	-0.218	1	-0.500	-0.816	0.816	-0.218	0	0.200
8 GB DDR3	0.612	-0.612	-0.327	-0.500	1	0.102	-0.408	-0.327	0.102	0
Lenovo Tabl	0.583	-0.583	0.356	-0.816	0.102	1	-0.667	0.356	-0.250	0
Netbook-Sch	-0.667	0.667	-0.535	0.816	-0.408	-0.667	1	-0.535	0.167	0.408
HP CE50 To	0.356	-0.356	1	-0.218	-0.327	0.356	-0.535	1	-0.089	-0.655
LT Laser Ma	0.167	-0.167	-0.089	0	0.102	-0.250	0.167	-0.089	1	-0.408
LT Minimaus	-0.408	0.408	-0.655	0.200	0	0	0.408	-0.655	-0.408	1

Shortcoming: Measures correlation only between two items but not between multiple items, e.g. {ThinkPad, Cover} → {Minimaus}

2. Association Analysis

- Association analysis can find multiple item co-occurrence relationships (descriptive method)
- focuses on occurring items, not absent items
- first algorithms developed in the early 90s at IBM by Agrawal & Srikant
- initially used for shopping basket analysis to find how items purchased by customers are related
- later extended to more complex data structures
 - sequential patterns
 - subgraph patterns
- and other application domains
 - web usage mining, social science, life science

Association Analysis

Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction.

Shopping Transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Examples of Association Rules

```
{Diaper} \rightarrow {Beer}
{Beer, Bread} \rightarrow {Milk}
{Milk, Bread} \rightarrow {Eggs, Coke}
```

Implication means co-occurrence, not causality!

Definition: Support and Frequent Itemset

Itemset

- collection of one or more items
- example: {Milk, Bread, Diaper}
- k-itemset: An itemset that contains k items

Support count (σ)

- frequency of occurrence of an itemset
- e.g. $\sigma(\{Milk, Bread, Diaper\}) = 2$

Support (s)

- fraction of transactions that contain an itemset
- e.g. s({Milk, Bread, Diaper}) = 2/5 = 0.4

Frequent Itemset

 an itemset whose support is greater than or equal to a minimal support (minsup) threshold specified by the user

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

Association Rule

- an implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
- an association rule states that when X occurs, Y occurs with certain probability.
- Example: $\{Milk, Diaper\} \rightarrow \{Beer\}$ Condition Consequent

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Rule Evaluation Metrics

- Support (s) fraction of transactions that contain both X and Y
- Confidence (c) measures how often items in Y appear in transactions that contain X

$$s(X \to Y) = \frac{|X \cup Y|}{|T|}$$

$$s(X \to Y) = \frac{|X \cup Y|}{|T|}$$
 $s = \frac{o \text{ (Milk, Diaper, Beer)}}{|T|} = \frac{2}{5} = 0.4$

$$c(X \to Y) = \frac{\sigma(X \cup Y)}{\sigma(X)}$$

$$c(X \to Y) = \frac{\sigma(X \cup Y)}{o(X)}$$
 $c = \frac{\sigma(\text{Milk, Diaper, Beer})}{o(\text{Milk, Diaper})} = \frac{2}{3} = 0.67$

Main Challenges concerning Association Analysis

- 1. Mining associations from large amounts of data can be computationally expensive
 - algorithms need to apply smart pruning strategies
- 2. Algorithms often discover a large number of associations
 - many of them are uninteresting or redundant
 - the user needs to select the subset of the associations that is relevant given her task at hand

The Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
 - support ≥ minsup threshold
 - 2. confidence \geq minconf threshold
- minsup and minconf are provided by the user.
- Brute Force Approach:
 - 1. list all possible association rules
 - 2. compute the support and confidence for each rule
 - 3. remove rules that fail the *minsup* and *minconf* thresholds
 - ⇒ Computationally prohibitive due to large number of candidates!

Mining Association Rules

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example rules:

```
\{Milk, Diaper\} \rightarrow \{Beer\} \ (s=0.4, c=0.67) \ \{Milk, Beer\} \rightarrow \{Diaper\} \ (s=0.4, c=1.0) \ \{Diaper, Beer\} \rightarrow \{Milk\} \ (s=0.4, c=0.67) \ \{Beer\} \rightarrow \{Milk, Diaper\} \ (s=0.4, c=0.67) \ \{Diaper\} \rightarrow \{Milk, Beer\} \ (s=0.4, c=0.5) \ \{Milk\} \rightarrow \{Diaper, Beer\} \ (s=0.4, c=0.5)
```

Observations:

- All the above rules are binary partitions of the same itemset: {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence.
- Thus, we may decouple the support and confidence requirements.

Mining Association Rules

– Two-step approach:

1. Frequent Itemset Generation

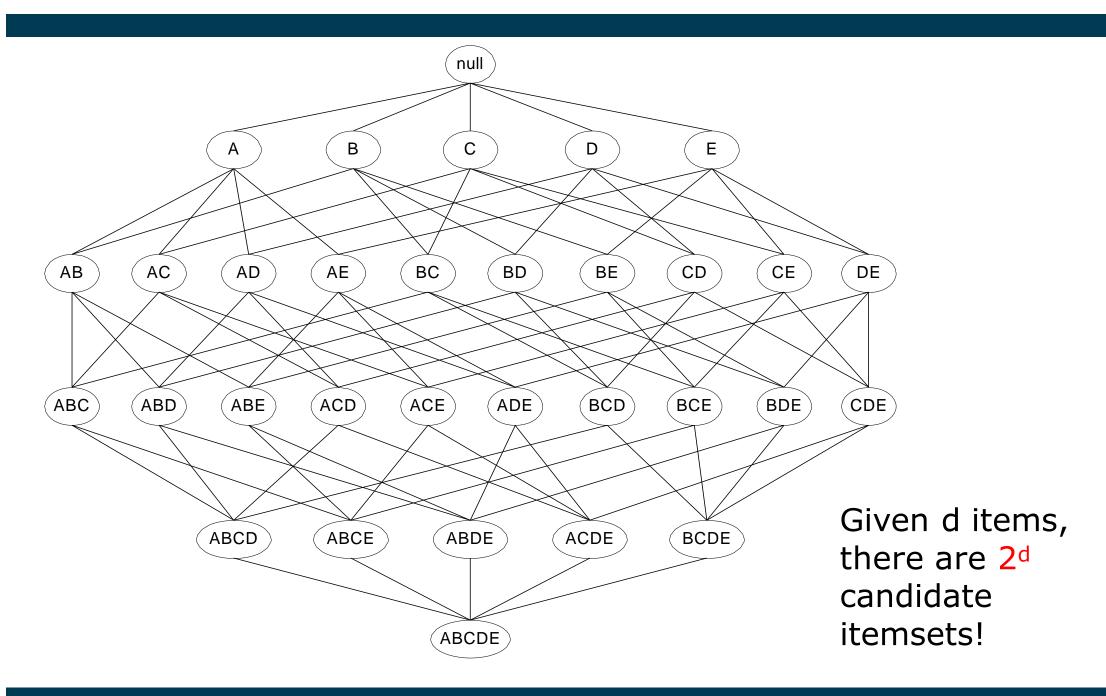
generate all itemsets whose support ≥ minsup

2. Rule Generation

generate high confidence rules from each frequent itemset,
 where each rule is a binary partitioning of a frequent itemset

Frequent itemset generation is still computationally expensive

2.1 Frequent Itemset Generation



2.1 Frequent Itemset Generation

Given d unique items:

Total number of itemsets = 2d

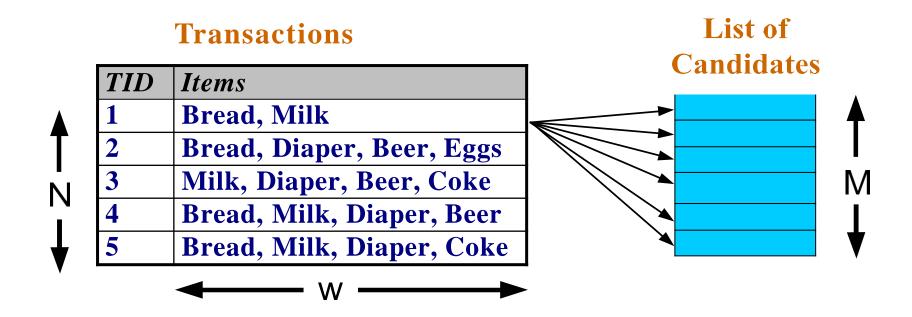
More specifically, the total number of possible rules extracted from a data set that contains d items is:

$$R = \sum_{k=1}^{d-1} \left[\binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \right]$$
$$= 3^{d} - 2^{d+1} + 1$$

If d=6, R=602 rules

Brute Force Approach

- Each itemset in the lattice is a candidate frequent itemset
- Count the support of each candidate by scanning the database
- Match each transaction against every candidate



- Complexity ~ O(NMw) → Expensive since M = 2^d!!!
- A smarter algorithm is required

Example: Brute Force Approach

- Example:
 - Amazon has 10 million books (i.e., Amazon Germany, as of 2011)
- That is 2^{10.000.000} possible itemsets
- As a number:
 - $9.04981... \times 10^{3.010.299}$
 - that is: a number with 3 million digits!



– However:

- most itemsets will not be important at all, e.g., books on Chinese calligraphy, Inuit cooking, and data mining bought together
- thus, smarter algorithms should be possible
- intuition for the algorithm: All itemsets containing Inuit cooking are likely infrequent

Reducing the Number of Candidates

Apriori Principle

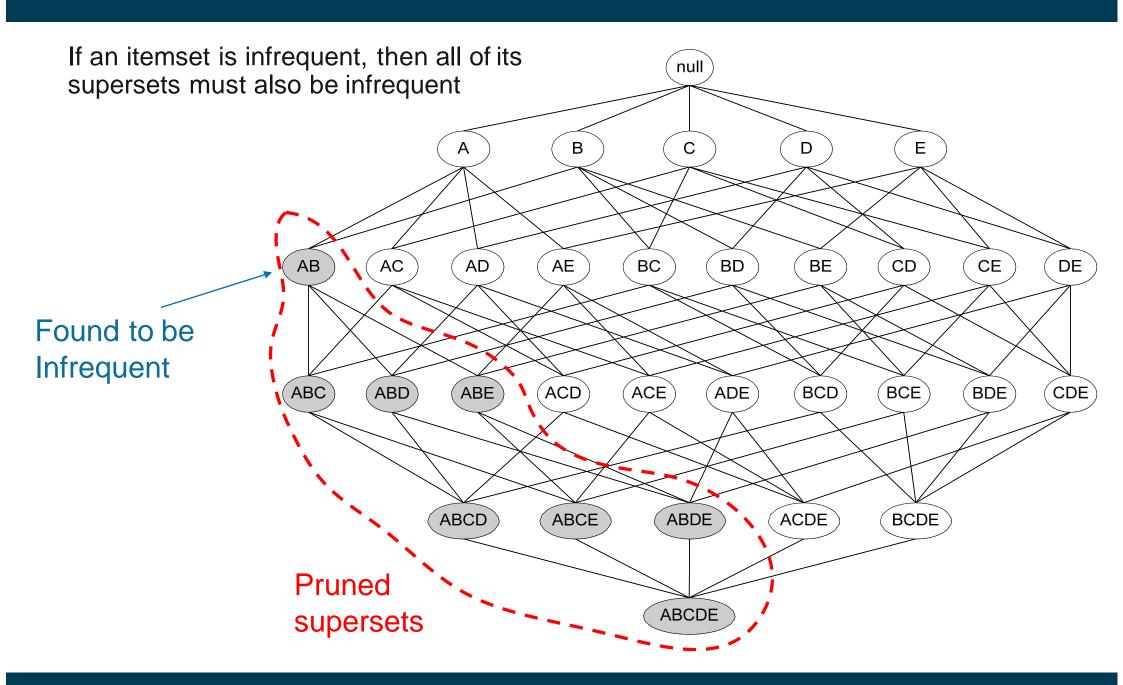
If an itemset is frequent, then all of its subsets must also be frequent.

 Apriori principle holds due to the following property of the support measure:

$$\forall X, Y: (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

- support of an itemset never exceeds the support of its subsets
- this is known as the anti-monotone property of support

Using the Apriori Principle for Pruning



Example: Using the Apriori Principle for Pruning

TID	Items
1	Bread, Milk, Diaper
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example: Using the Apriori Principle for Pruning

Minimum Support Count = 3

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

No need to generate

candidates involving

Coke or Eggs



Items (1-itemsets)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

TID	Items
1	Bread, Milk, Diaper
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke



Triplets (3-itemsets)

Item s e t	Count	
{Bread, Milk, Diaper}	3	

No need to generate candidate {Milk, Diaper, Beer} as count {Milk, Beer} = 2

If every subset is considered, ${}^{6}C_{1} + {}^{6}C_{2} + {}^{6}C_{3} = 41$ With support-based pruning, 6 + 6 + 1 = 13

The Apriori Algorithm

- 1. Let k=1
- 2. Generate frequent itemsets of length 1
- 3. Repeat until no new frequent itemsets are identified
 - 1. Generate length (k+1) candidate itemsets from length k frequent itemsets
 - 2. Prune candidate itemsets that can not be frequent because they contain subsets of length k that are infrequent (Apriori Principle)
 - 3. Count the support of each candidate by scanning the DB
 - Eliminate candidates that are infrequent, leaving only those that are frequent

Example: Apriori Algorithm

itemset:count

minsup=2

Dataset T

1. scan T

- \rightarrow Cand₁: {1}:2, {2}:3, {3}:3, {4}:1, {5}:3
- → Frequ₁: {1}:2, {2}:3, {3}:3,
- {5}:3
- \rightarrow Cand₂: {1,2}, {1,3}, {1,5}, {2,3}, {2,5}, {3,5}

TID	Items	
T100	1, 3, 4	
T200	2, 3, 5	
T300	1, 2, 3, 5	
T400	2, 5	

2. scan T

- \rightarrow Cand₂: {1,2}:1, {1,3}:2, {1,5}:1, {2,3}:2, {2,5}:3, {3,5}:2
- → Frequ₂:
- {1,3}:2,
- {2,3}:2, {2,5}:3, {3,5}:2

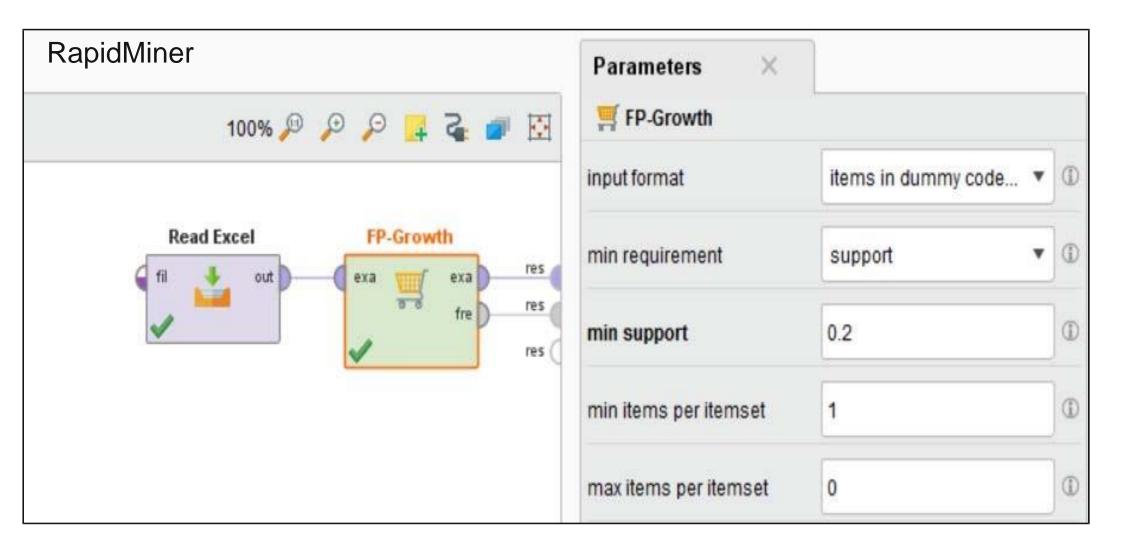
→ Cand₃:

 $\{2, 3, 5\}$

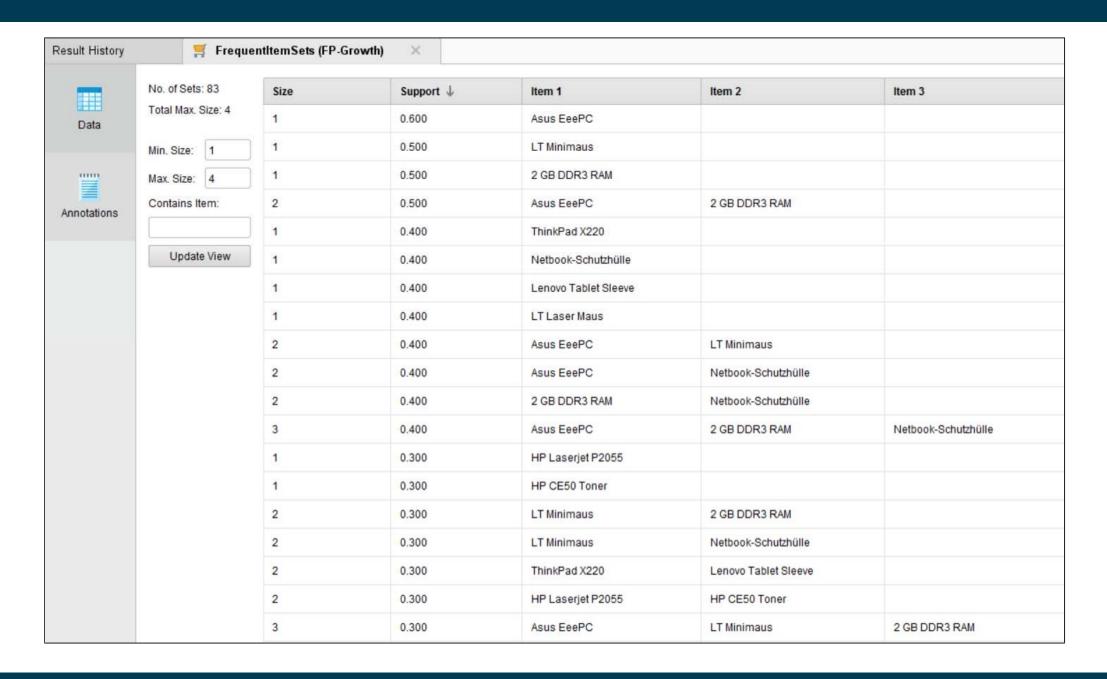
3. scan T

- \rightarrow C₃: {2, 3, 5}:2
- \rightarrow $F_{3:}\{2, 3, 5\}$

Frequent Itemset Generation in Rapidminer



Frequent Itemsets in Rapidminer



Example Application of Frequent Itemsets

- 1. Take top-k frequent itemsets of size 2 containing item A
- 2. Rank second item according to
 - profit made by selling item
 - whether you want to reduce number of items B in stock
 - knowledge about customer preferences
- 3. Offer special price for combination with top-ranked second item



Wird oft zusammen gekauft



Preis für beide: EUR 138,00

Beides in den Einkaufswagen

Verfügbarkeit und Versanddetails anzeigen

- Dieser Artikel: Introduction to Data Mining von Pang-Ning Tan Taschenbuch EUR 85,05
- Data Mining: Concepts and Techniques (Morgan Kaufmann Series in Data Management Systems)

2.2 Rule Generation

Given a frequent itemset L, find all non-empty subsets f ⊂ L such that f → L − f satisfies the minimum confidence requirement.

Example Frequent Itemset:

{Milk, Diaper, Beer}

Example Rule:

 $\{Milk, Diaper\} \Rightarrow Beer$

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

$$c = \frac{\sigma \text{ (Milk, Diaper, Beer)}}{o \text{ (Milk, Diaper)}} = \frac{2}{3} = 0.67$$

Challenge: Large Number of Candidate Rules

– If {A,B,C,D} is a frequent itemset, then the candidate rules are:

$ABC \rightarrow D$,	$ABD \rightarrow C$,	$ACD \rightarrow B$,	$BCD \rightarrow A$,
$A \rightarrow BCD$,	$B \to ACD$,	$C \rightarrow ABD$,	$D \rightarrow ABC$
$AB \rightarrow CD$,	$AC \rightarrow BD$,	$AD \rightarrow BC$,	$BC \rightarrow AD$,
$BD \rightarrow AC$.	$CD \! o \! AB$		

- If |L| = k, then there are $2^k - 2$ candidate association rules (ignoring $L \to \emptyset$ and $\emptyset \to L$)

Rule Generation

- How to efficiently generate rules from frequent itemsets?
 - In general, confidence does not have an anti-monotone property
 c(ABC →D) can be larger or smaller than c(AB →D)
 - But confidence of rules generated from the same itemset has an anti-monotone property
 - e.g., L = {A,B,C,D}: $c(ABC \rightarrow D) \ge c(AB \rightarrow CD) \ge c(A \rightarrow BCD)$
 - Confidence is anti-monotone with respect to the number of items on the right hand side of the rule

Explanation

Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

i.e., "moving elements from left to right" cannot increase confidence

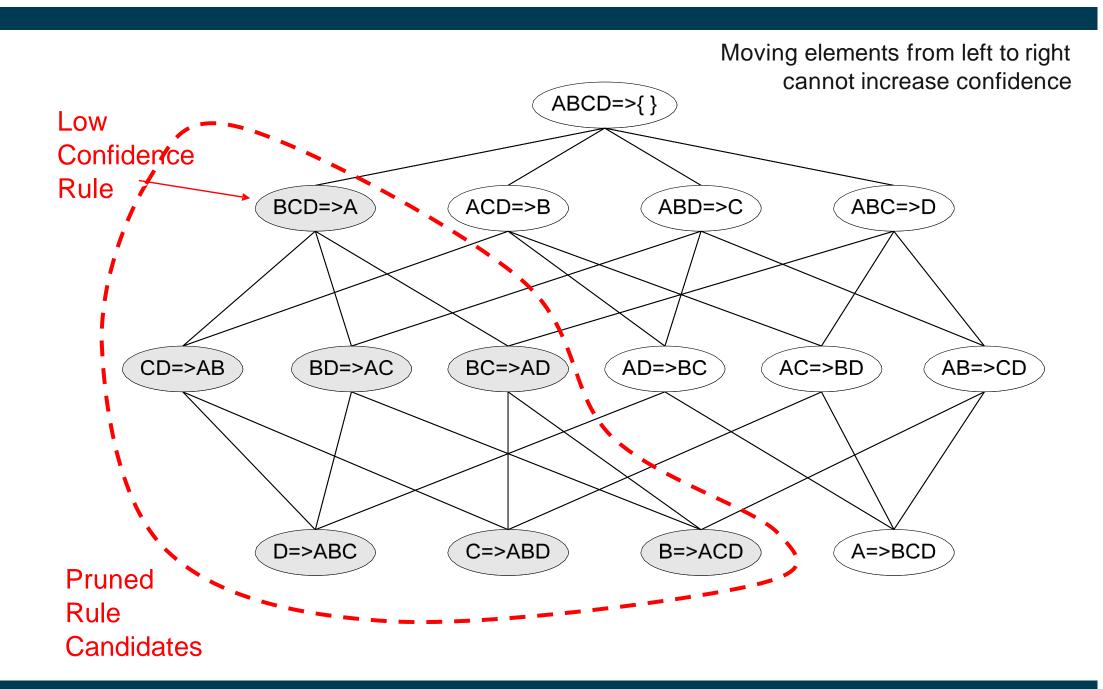
Reason:

$$c(AB \to C) := \frac{s(ABC)}{s(AB)} \qquad c(A \to BC) := \frac{s(ABC)}{s(A)}$$

- Due to anti-monotone property of support, we know
 s(AB) ≤ s(A)
- Hence

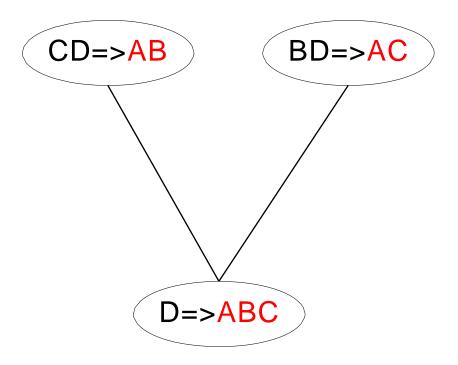
$$c(AB \to C) \ge C(A \to BC)$$

Candidate Rule Pruning



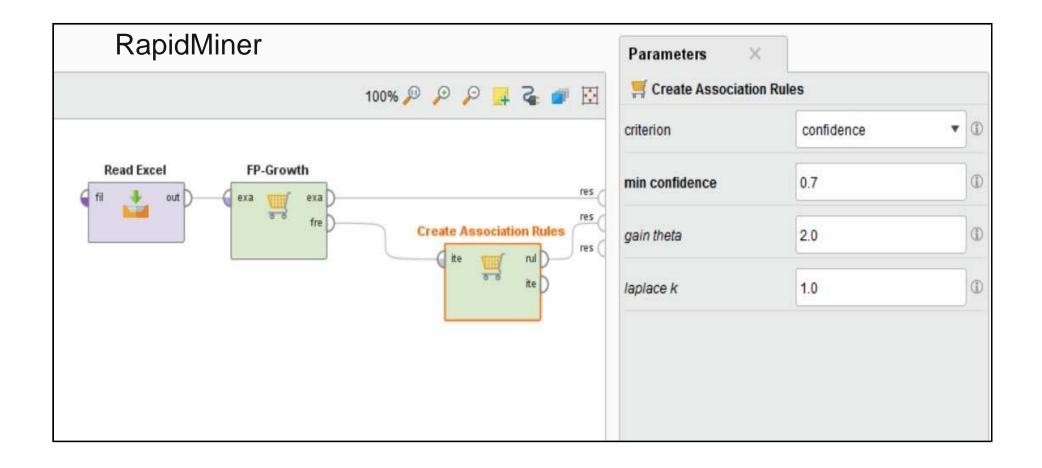
Candidate Rule Generation within Apriori Algorithm

- Candidate rule is generated by merging two rules that share the same prefix in the rule consequent (right hand side of rule)
- join(CD → AB, BD → AC)
 would produce the candidate
 rule D → ABC
- Prune rule D → ABC if one of its parent rules does not have high confidence (e.g. AD → BC)

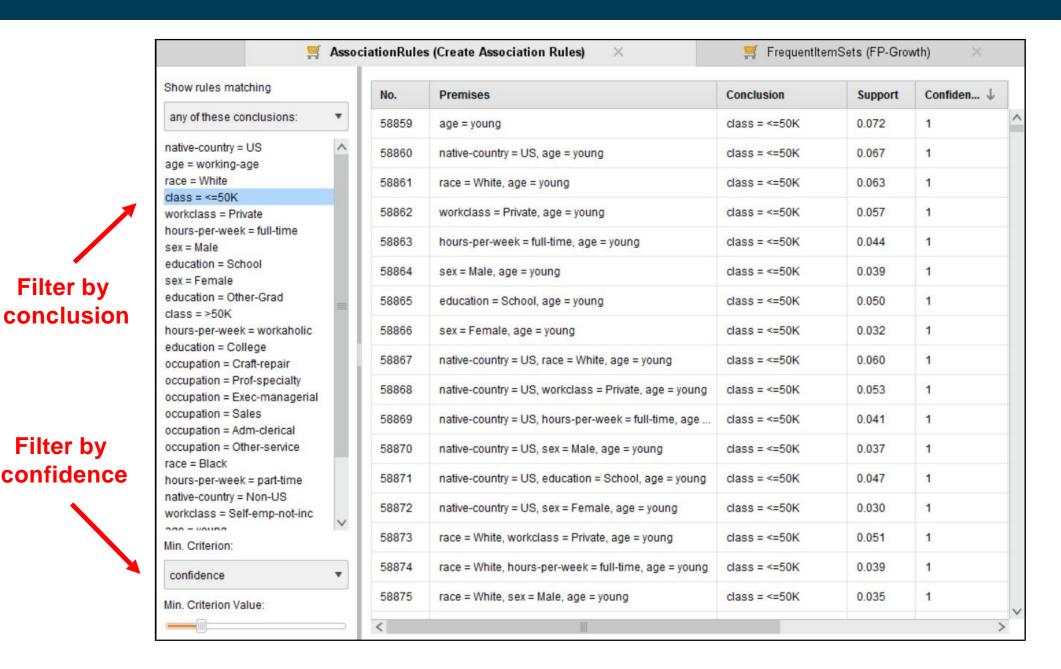


- All the required information for confidence computation has already been recorded in itemset generation.
- Thus, there is no need to scan the transaction data T any more

Creating Association Rules in Rapidminer

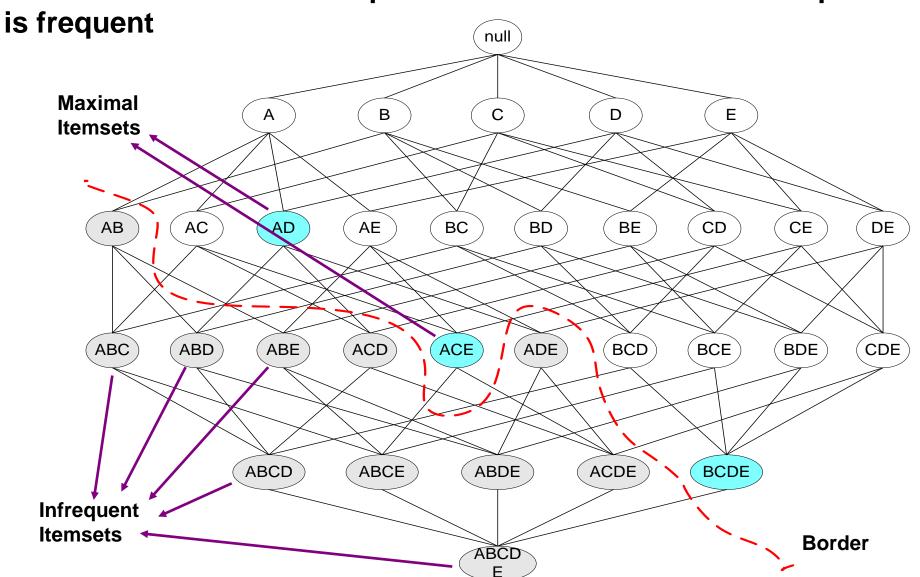


Exploring Association Rules in Rapidminer



Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets



Closed Itemset

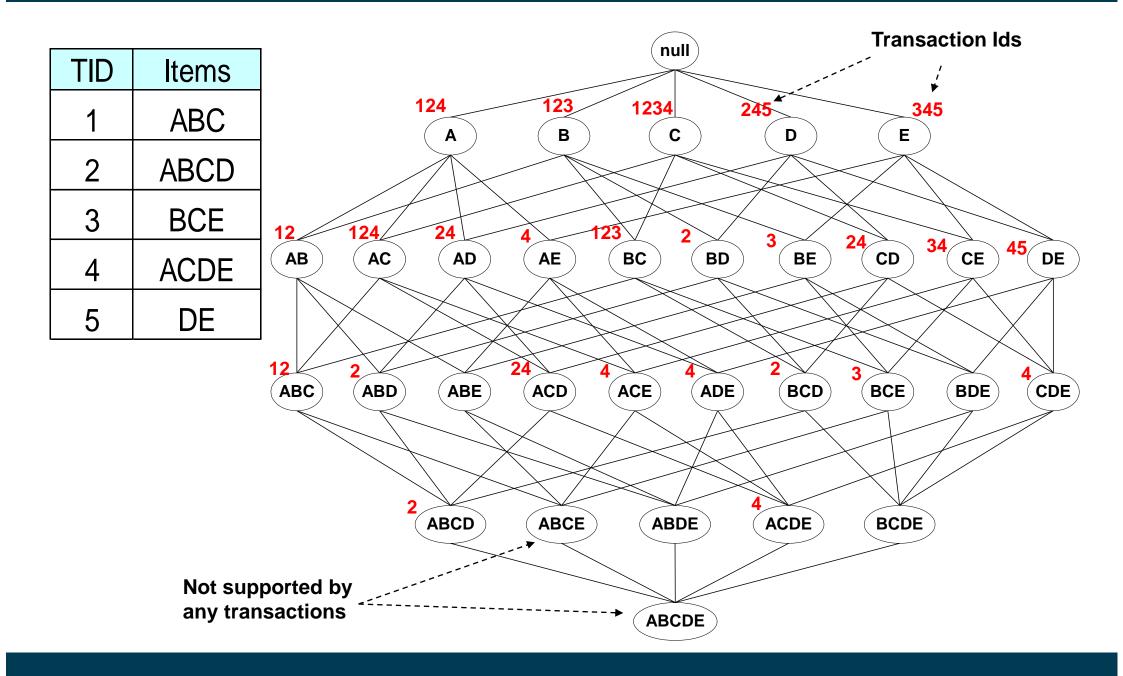
An itemset is closed if none of its immediate supersets has the same support as the itemset

TID	Items
1	{A,B}
2	$\{B,C,D\}$
3	$\{A,B,C,D\}$
4	$\{A,B,D\}$
5	$\{A,B,C,D\}$

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
$\{A,B,C\}$	2
$\{A,B,D\}$	3
$\{A,C,D\}$	2
{B,C,D}	3
$\{A,B,C,D\}$	2

Maximal vs Closed Itemsets



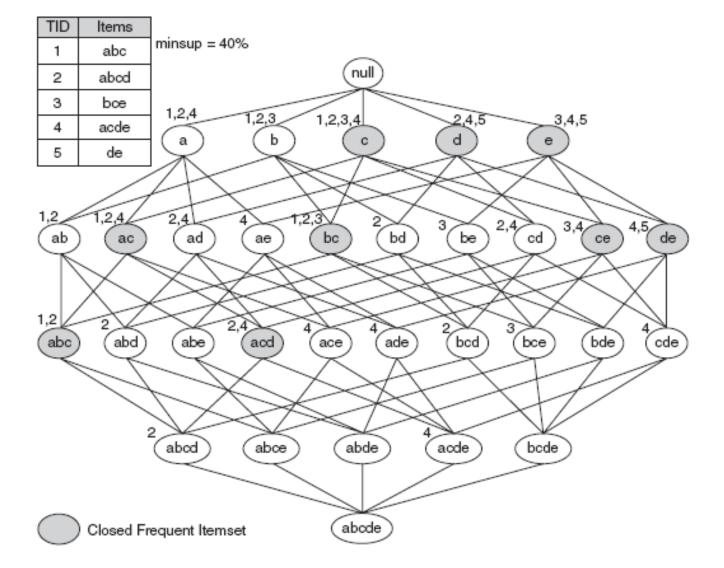
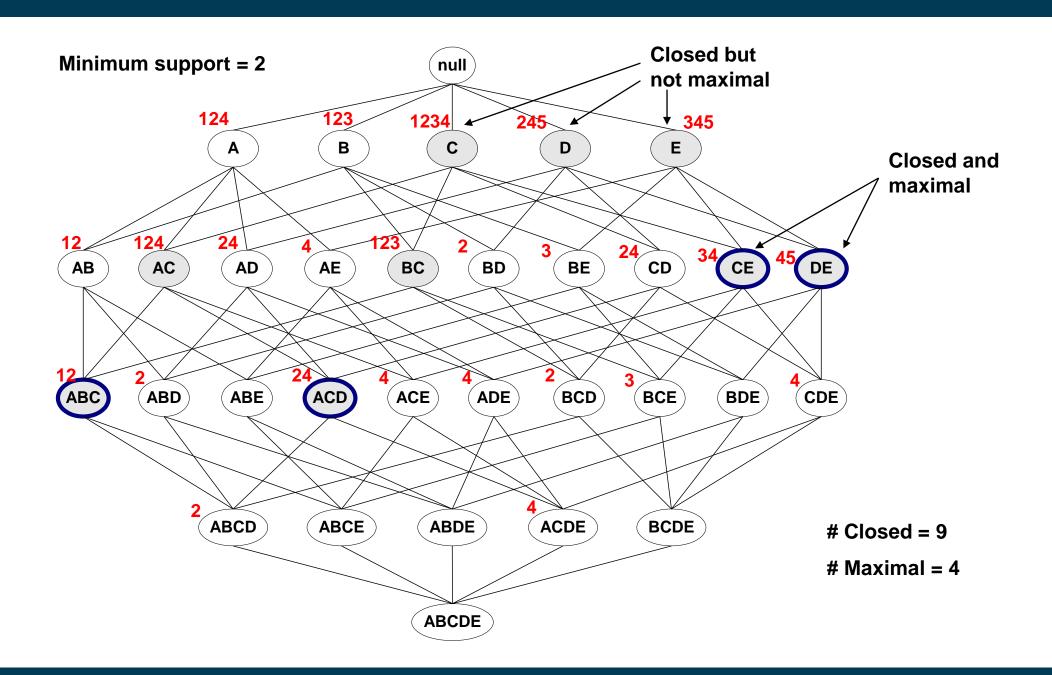


Figure 6.17. An example of the closed frequent itemsets (with minimum support count equal to 40%).

For example, since the node {b, c} is associated with transaction IDs 1, 2, and 3, its support count is equal to three. From the transactions given in this diagram, notice that every transaction that contains b also contains c. Consequently, the support for {b} is identical to {b, c} and {b} should not be considered a closed itemset. Similarly, since c occurs in every transaction that contains both a and d, the itemset {a, d} is not closed. On the other hand, {b, c} is a closed itemset because it does not have the same support count as any of its supersets.

Maximal vs Closed Frequent Itemsets



2.4 Interestingness Measures

- Association rule algorithms tend to produce too many rules
 - many of them are uninteresting or redundant
 - redundant if {A,B,C} → {D} and {A,B} → {D} have same support & confidence
- Interestingness of patterns depends on application
 - one man's rubbish may be another's treasure
- Interestingness measures can be used to prune or rank the derived rules.
- In the original formulation of association rules, support & confidence were the only interestingness measures used.
- Later, various other measures have been proposed
 - See Tan/Steinbach/Kumar, Chapter 6.7
 - We will have a look at one: Lift

Drawback of Confidence

Contingency table

	Coffee	Coffee	
Tea	15	5	20
Tea	75	5	80
	90	10	100

Association Rule: Tea → Coffee

- confidence(Tea \rightarrow Coffee) = 0.75
- but support(Coffee) = 0.9
- although confidence is high, rule is misleading as the fraction of coffee drinkers is higher than the confidence of the rule
- we want confidence($X \rightarrow Y$) > support(Y)
- otherwise rule is misleading as X reduces probability of Y

Lift

- The lift of an association rule $X \rightarrow Y$ is defined as:

$$Lift = \frac{c(X \to Y)}{s(Y)}$$

Confidence normalized by support of consequent

- Interpretation
 - if lift > 1, then X and Y are positively correlated
 - if lift = 1, then X and Y are independent
 - if lift < 1, then X and Y are negatively correlated

Example: Lift

Contingency table

	Coffee	Coffee	
Tea	15	5	20
Tea	75	5	80
	90	10	100

$$Lift = \frac{c(X \to Y)}{s(Y)}$$

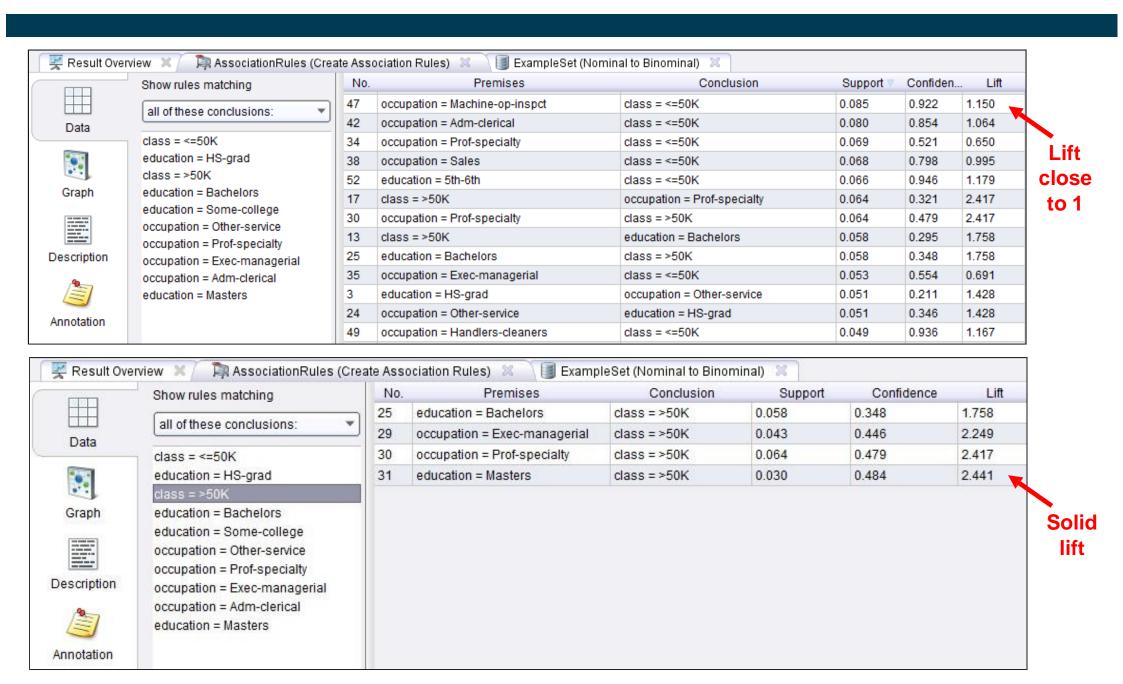
Association Rule: Tea → Coffee

- confidence(Tea \rightarrow Coffee) = 0.75
- but support(Coffee) = 0.9

Lift(Tea
$$\rightarrow$$
 Coffee) = 0.75/0.9= 0.8333

lift < 1, therefore is negatively correlated

Exploring Association Rules in RapidMiner



Conclusion

- The algorithm does the counting for you and finds patterns in the data
- You need to do the interpretation based on your knowledge about the application domain.
 - Which patterns are meaningful?
 - Which patterns are surprising?

Literature for this Slideset

Pang-Ning Tan, Michael Steinbach, Anuj Karpatne, Vipin Kumar: **Introduction to Data Mining.**2nd Edition. Pearson.

Chapter 4: Association Analysis:

Basic Concepts and Algorithms

Chapter 7: Association Analysis: Advanced Concepts

