```
(*1 (a)*)
```

$$\prod_{i=2}^{100}\sum_{j=1}^{i}\frac{1}{j} \text{ // N}$$

$2.47832\times10^{61}$

```
Sum[1 / j, {i, 1, 100}, {j, 1, i}]
```

$$\frac{118224876331270555852423808661226806199161 1}{278881500918849908658135235741249214227 2}$$

```
(*1 (b) i *)
```

$$\text{Sum}\left[\frac{r\text{^}2}{(r+1)*(r+2)}*4\text{^}(r),\ \{r,\ 100\}\right] \text{ // N}$$

$2.07957\times10^{60}$

```
(*1 (b) ii *)
```

$$\text{Sum}\left[\text{ArcTan}\left[\frac{2\text{^}(r-1)}{1+2\text{^}(r-1)*2\text{^}r}\right],\ \{r,\ 1,\ \text{Infinity}\}\right] \text{ // N}$$

$0.785398$

```
(*2 (a)*)
p = 2 x^4 - 15 x^3 + 39 x^2 - 40 x + 12;
q = 4 x^4 - 24 x^3 + 45 x^2 - 29 x + 6;
gcd = PolynomialGCD[p, q]
lcm = PolynomialLCM[p, q]
Expand[gcd * lcm] == Expand[p * q]
```

$-6 + 17 x - 11 x^2 + 2 x^3$

$(-2 + x)\,(6 - 29 x + 45 x^2 - 24 x^3 + 4 x^4)$

True

```
(*2 (b)*)
x1 = 3; y1 = 8; z1 = 3; x2 = -3; y2 = -7; z2 = 6;
a1 = 3; b1 = -1; c1 = -1; a2 = -3; b2 = 2; c2 = 4;
θ = ArcSin[√((a1 * b2 - a2 * b1) ^2 + (b1 * c2 - b2 * c1) ^2 + (c1 * a2 - c2 * a1) ^2)];
l = (b1 * c2 - b2 * c1) / Sin[θ];
m = (c1 * a2 - c2 * a1) / Sin[θ];
n = (a1 * b2 - a2 * b1) / Sin[θ];
sd = l (x2 - x1) + m (y2 - y1) + n (z2 - z1) // Simplify
```

$78 \sqrt{\frac{2}{47}}$

```
(*3 (a) i *)
f[x_, y_] = x^2 * y + y^2 * x + x^3 + y^3;
D[f[x, y], x]
```

$3x^2 + 2xy + y^2$

```
D[f[x, y], y]
```

$x^2 + 2xy + 3y^2$

```
D[f[x, y], {x, 2}]
```

$6x + 2y$

```
D[f[x, y], {y, 2}]
```

$2x + 6y$

```
D[f[x, y], x, y]
```

$2x + 2y$

```
(* 3 (a) ii *)
f[x_] := Abs[x + 1] + Abs[x];
lhs = Limit[f[x], x → -1, Direction → -1]
rhs = Limit[f[x], x → -1, Direction → 1]
f1 = f[-1]
If[lhs == rhs == f1, Print["f(x) is continous at the point =-1"], Print["f(x) is not continous at the point=-1"]]
```

1

1

1

```
f(x) is continous at the point =-1
```

```
(*3 (b)*)
list1 = Range[99];
list2 = Range[7, 99, 7];
list3 = Complement[list1, list2];
TableForm[Partition[list3, 5]]
```

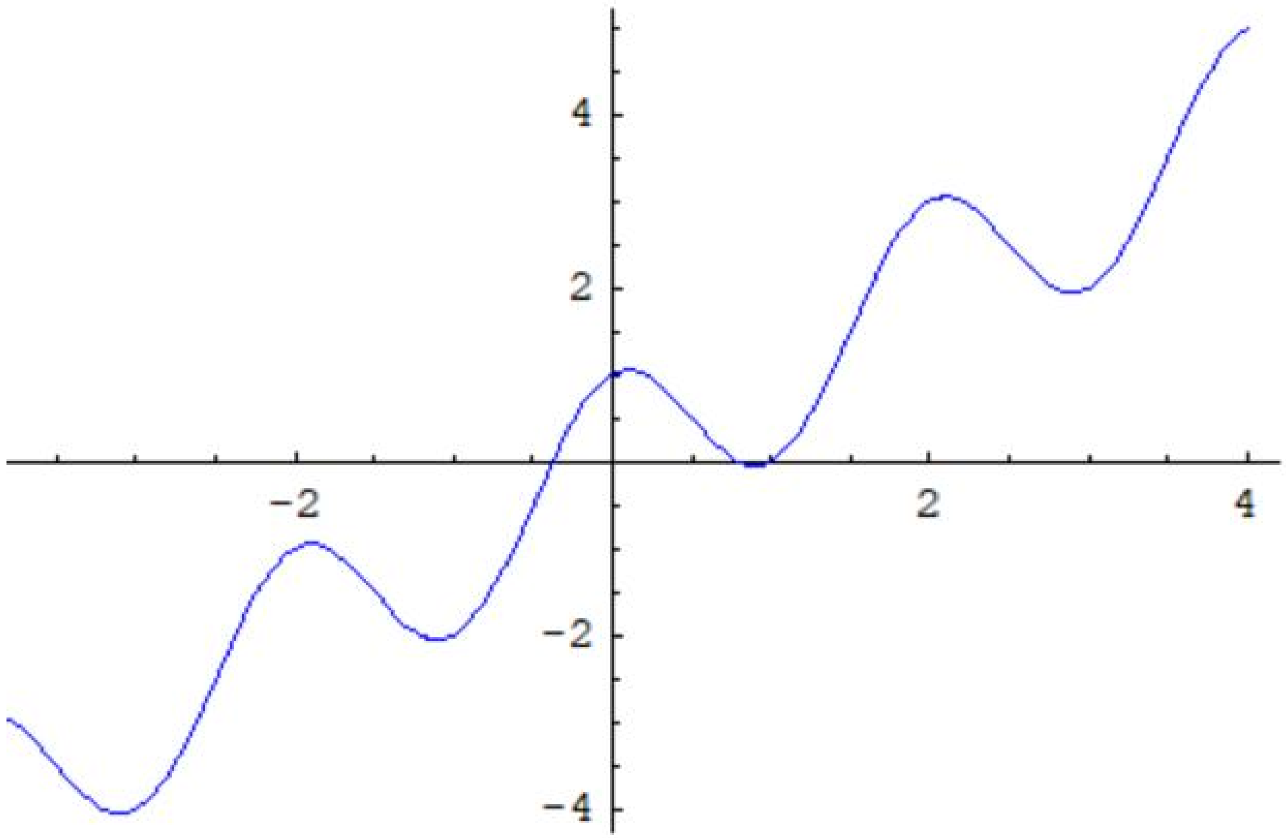
ableForm=

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 6 | 8 | 9 | 10 | 11 |
| 12 | 13 | 15 | 16 | 17 |
| 18 | 19 | 20 | 22 | 23 |
| 24 | 25 | 26 | 27 | 29 |
| 30 | 31 | 32 | 33 | 34 |
| 36 | 37 | 38 | 39 | 40 |
| 41 | 43 | 44 | 45 | 46 |
| 47 | 48 | 50 | 51 | 52 |
| 53 | 54 | 55 | 57 | 58 |
| 59 | 60 | 61 | 62 | 64 |
| 65 | 66 | 67 | 68 | 69 |
| 71 | 72 | 73 | 74 | 75 |
| 76 | 78 | 79 | 80 | 81 |
| 82 | 83 | 85 | 86 | 87 |
| 88 | 89 | 90 | 92 | 93 |
| 94 | 95 | 96 | 97 | 99 |

```
(* 4 (a) i *)
Timing[Prime[10000]]
```

```
{0. Second, 104729}
```

```
(* 4 (a) ii *)
g[x_] = Cos[π * x] + x;
Plot[g[x], {x, -4, 4}, PlotStyle → {RGBColor[0, 0, 1]}]
```



```
(* 4 (b) *)
```

$$A = \begin{pmatrix} 5 & 3 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 2 \end{pmatrix}; \; Ax = \begin{pmatrix} -1 & 3 & -3 \\ -1 & 2 & -2 \\ 8 & -1 & 2 \end{pmatrix};$$

$$Ay = \begin{pmatrix} 5 & -1 & -3 \\ 3 & -1 & -2 \\ 2 & 8 & 2 \end{pmatrix}; \; Az = \begin{pmatrix} 5 & 3 & -1 \\ 3 & 2 & -1 \\ 2 & -1 & 8 \end{pmatrix};$$

$$x = \frac{\text{Det[Ax]}}{\text{Det[A]}}$$

$$y = \frac{\text{Det[Ay]}}{\text{Det[A]}}$$

$$z = \frac{\text{Det[Az]}}{\text{Det[A]}}$$

1

2

4

```
(* 5 (a) *)
```

$$a = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 1 \end{pmatrix};$$

```
Eigenvalues[a] // N
```

```
{5.30278, 3., 1.69722}
```

```
Eigenvectors[a] // N
```

```
{{1.43426, 2.86852, 1.}, {-1., 1., 0.}, {0.232408, 0.464816, 1.}}
```

```
CharacteristicPolynomial[a, λ]
```

$$27 - 30\lambda + 10\lambda^2 - \lambda^3$$

```
ch = 27 IdentityMatrix[3] - 30 a + 10 MatrixPower[a, 2] - MatrixPower[a, 3];
If[ch == {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, Print["Cayley-Hamilton Theorem is verified"]]
```

```
Cayley-Hamilton Theorem is verified
```

```
(* 5 (b) i *)
```

$$\int_0^2 \int_0^x \int_0^{x\,y} x\,y\,z \, dz \, dy \, dx$$

4

```
(* 5 (b) ii *)
```

$$\int_{-5}^5 \int_{-\sqrt{25-x^\wedge 2}}^{\sqrt{25-x^\wedge 2}} dy \, dx$$

$25\,\pi$

```
(* 6 (a) *)
f[x_, y_] := 5 x^2 + 5 y^2;
g[x_, y_] := 6 - 7 x^2 - y^2;
sol1 = Solve[f[x, y] == g[x, y], y]
```

$$\left\{\left\{y \to -\sqrt{1 - 2\,x^2}\right\},\ \left\{y \to \sqrt{1 - 2\,x^2}\right\}\right\}$$

```
sol2 = Solve[1 - 2 x^2 == 0]
```

$$\left\{\left\{x \to -\frac{1}{\sqrt{2}}\right\},\ \left\{x \to \frac{1}{\sqrt{2}}\right\}\right\}$$

$$\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{1-2\,x\verb|^|2}}^{\sqrt{1-2\,x\verb|^|2}} \int_{f[x,y]}^{g[x,y]} 1 \, dz \, dy \, dx$$

$$\frac{3\,\pi}{\sqrt{2}}$$

```
(* 6 (a) *)
f[x_, y_] := 5 x^2 + 5 y^2;
g[x_, y_] := 6 - 7 x^2 - y^2;
sol1 = Solve[f[x, y] == g[x, y], y]
```

$$\left\{\left\{y \to -\sqrt{1 - 2\,x^2}\right\}, \left\{y \to \sqrt{1 - 2\,x^2}\right\}\right\}$$

```
sol2 = Solve[1 - 2 x^2 == 0]
```

$$\left\{\left\{x \to -\frac{1}{\sqrt{2}}\right\}, \left\{x \to \frac{1}{\sqrt{2}}\right\}\right\}$$

$$\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{1-2\,x\wedge 2}}^{\sqrt{1-2\,x\wedge 2}} \int_{f[x,y]}^{g[x,y]} 1 \, dz \, dy \, dx$$

$$\frac{3\,\pi}{\sqrt{2}}$$

```
(* 6 (b) *)
a = {2, -1, 1}; b = {1, -3, -5}; c = {3, -4, -4};
If[a.b == 0 && Norm[c]^2 == Norm[a]^2 + Norm[b]^2, Print["formed a right angle triangle"]]
```

formed a right angle triangle

```
(*7 (a)*)
f[y_] := 1/8 y^2;
x1 = 2; y1 = 4;
d = D[f[y], y] /. {x → x1, y → y1}
```

1

```
tangent = (y - y1) == d (x - x1) // Simplify
```

$2 + x == y$

```
normal = (y - y1) d + (x - x1) == 0 // Simplify
```

$x + y == 6$

```
(*7 (b)*)
u = ArcSin[(x + y)/(Sqrt[x] + Sqrt[y])];
lhs = x D[u, x] + y D[u, y]
```

$$\frac{x\left(\frac{1}{\sqrt{x}+\sqrt{y}}-\frac{x+y}{2\sqrt{x}\left(\sqrt{x}+\sqrt{y}\right)^2}\right)}{\sqrt{1-\frac{(x+y)^2}{\left(\sqrt{x}+\sqrt{y}\right)^2}}}+\frac{y\left(\frac{1}{\sqrt{x}+\sqrt{y}}-\frac{x+y}{2\left(\sqrt{x}+\sqrt{y}\right)^2\sqrt{y}}\right)}{\sqrt{1-\frac{(x+y)^2}{\left(\sqrt{x}+\sqrt{y}\right)^2}}}$$

```
rhs = 1 / 2 Tan[u]
```

$$\frac{x+y}{2\left(\sqrt{x}+\sqrt{y}\right)\sqrt{1-\frac{(x+y)^2}{\left(\sqrt{x}+\sqrt{y}\right)^2}}}$$

```
lhs == rhs // Simplify
```

True

```
(*8 (a)*)
```

volume = $\int_{-1}^{1}\int_{-3\sqrt{1-y^2}}^{3\sqrt{1-y^2}} (x + 3)\, dx\, dy$

$9\pi$

```
(* 8 (b) *)
a = {a1, a2, a3};
b = {b1, b2, b3};
c = {c1, c2, c3};
lhs = Cross[a, b].Cross[Cross[b, c], Cross[c, a]] // Expand;
rhs = (a.Cross[b, c])^2 // Expand;
If[lhs == rhs, Print["The expression is true"], Print["The expression is not true"]]
```

```
The expression is true
```