Exercises

Week 3

Programming Exercises:

Problem O3.1:

Use a plotting program in Matlab or Python to show the pdf for jointly Gaussian zero-mean random variables with the following parameters:

$\sigma_1 = 1;\ \sigma_2 = 1;\ \rho = 0$

$\sigma_1 = 1;\ \sigma_2 = 1;\ \rho = 0.8$

$\sigma_1 = 1;\ \sigma_2 = 0.25;\ \rho = -0.8$

$\sigma_1 = 1;\ \sigma_2 = 10;\ \rho = 0.8$

Verify that the shape of the pdf is consistent with the observations made in slide 24 of Lecture 3. For instance, in which cases the principal axes of the equal-pdf contours aligned with x- and y-axis and in which cases they are oriented along an angle?

Problem O3.2:

Assume a **joint PMF** of two random variables X and Y, taking on values i=0,1 and j=0,1 respectively, as given in the following table

	j=0	j=1
i=0	1/8	1/8
i=1	1/4	1/2

Write an Octave/Matlab program to generate a set of M=1000 realizations of this 2D random variable (X,Y). Once the realizations are available, estimate the **joint PMF** and **marginal PMFs** from those M=1000 realizations and plot those in Octave/Matlab. Are the resulting estimates close to the theoretical values obtained from the above table? (**Hint**: Use the procedure similar to the one given in *this slide*).

Problem O3.3:

A modem sends a two-dimensional signal **X**=(X_1, X_2) from the set $\{(1,1),(1,-1),(-1,1),(-1,-1)\}$The channel adds a noise signal (N_1, N_2) so the received signal is $Y = X + N$.

(a) Generate 10,000 discrete random variables (in Octave) uniformly distributed in the set $\{1,2,3,4\}$. Assign each outcome in this set to one of the signals

$\{(1,1),(1,-1),(-1,1),(-1,-1)\}$. The sequence of discrete random variables then produces a sequence of 10,000 signal points X.

(b) Generate 10,000 noise pairs **N** of independent zero-mean, unit-variance jointly Gaussian random variables.

(c) Form the sequence of 10,000 received signals

(d) Plot the scattergram of received signal vectors in Octave. Is the plot what you expected?

(e) Estimate the transmitted signal by the quadrant that Y falls in: $\widehat{X} = \{sgn(Y_1), sgn(Y_2)\}$

(f) Compare the estimates with the actually transmitted signals to estimate the probability of error.

Problem set from Textbook[1]:

Problems 5.3, 5.6, 5.11, 5.25, 5.30, 5.44, 5.66, 5.80, 5.110, 5.111

Additional question: Let Y=X+N where X (signal) and N (noise) are independent zero mean Gaussian RVs with different variances. Find the correlation coefficient between observed signal (Y) and desired signal (X).

[1] Textbook: A. Leon-Garcia, *Probability, Statistics and Random Processes for Electrical Engineering*, 2008, 3rd Ed. Prentice Hall