Exercises

Week 4

Programming Exercises:

Problem O4.1:

Generate and plot the scattergram of 1000 pairs of random variables **Y**=(Y_1, Y_2), having the covariance matrix $K_1 = [2,\ 1;\ 1,\ 4]$, using the linear transformation of vector **X**=(X_1, X_2) where

a) X_1 and X_2 are independent random variables that are each uniform in the unit interval;
b) X_1 and X_2 are independent zero-mean, unit-variance Gaussian random variables

Repeat the above exercise with the covariance matrix $K_2 = [4,\ 1;\ 1,\ 4]$

Problem O4.2:

Let **X** be the jointly Gaussian random variables with mean = [1, 0, 2] and covariance matrix $K_1 = [3/2,\ 0,\ 1/2;\ 0,\ 1,\ 0;\ 1/2,\ 0,\ 3/2]$

(a) Find a linear transformation **A** that diagonalizes the covariance matrix.
(b) Generate 1000 triplets of **Y=AX** and plot the scattergrams in Matlab or Python for Y_1 and Y_2, Y_1 and Y_3, Y_2 and Y_3. Confirm that the scattergrams are as expected.

Problem O4.3:

Let $X_1, X_2, \ldots X_n$ be independent zero mean Gaussian random variables. Let $Y_k = (X_k + X_{k-1})/2$, that is, Y_k is the moving average of pairs of values of X. Assume $X_{-1} = 0$.

(a) Find the covariance matrix of the random variables Y_k
(b) Use Matlab or Python to generate a sequence of 1000 samples $Y_1, Y_2, \ldots Y_n$. How would you check that the Y_k have the correct covariance?

Repeat the above problem with $Y_k = X_k - X_{k-1}$.

Miscellaneous Problems:

Let $X_1, X_2, \ldots, X_n$ be jointly Gaussian RVs with joint PDF specified by mean **m** and covariance matrix **K.** Show that $Z = a_1X_1 + a_2X_2 + \ldots + a_nX_n$ is a Gaussian RV.

Problem set from Textbook[1]:

Problems 6.33, 6.50 (a,b), 6.54, 6.55, 6.80, 6.86

[1] Textbook: A. Leon-Garcia, *Probability, Statistics and Random Processes for Electrical Engineering*, 2008, 3rd Ed. Prentice Hall