Stochastic Signal Processing

Lecture 1 - Introduction

August, 2025

Course's Team

Instructor: Naveed ur Rehman
email: naveed.rehman@ece.au.dk
phone: 60202663

Course information

This is a 5 ECTS course with a duration of 14 weeks.

Contact days:

Tuesday (12:00 - 16:00): 2h of lecture followed by 2h of exercises

Problem Sets:

- You will work on the *book exercises* and *programming exercises* (Matlab, Python) during the exercise sessions

Assessment:

- **Oral Exam**: Individual 20-minute presentation on a pre-selected topic. Questions will be related to the presentation as well as the overall course content
- **Mandatory Assignments**: You MUST submit and pass 3 out of the total 4 assignments to become eligible to appear in the oral exam.
- You will get **detailed feedback** ONLY on those assignments which you have NOT passed; you will have the opportunity to resubmit it after improving based on my feedback. This applies only if you have submitted the assignment before the first deadline.

Brightspace

- Familiarize yourself with the course's page in Brightspace
- You will lecture material, curriculum and other information

brightspace.au.dk

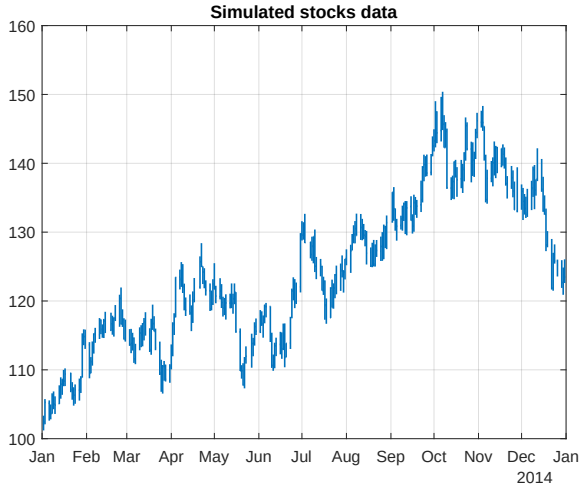
Stochastic Signal Processing

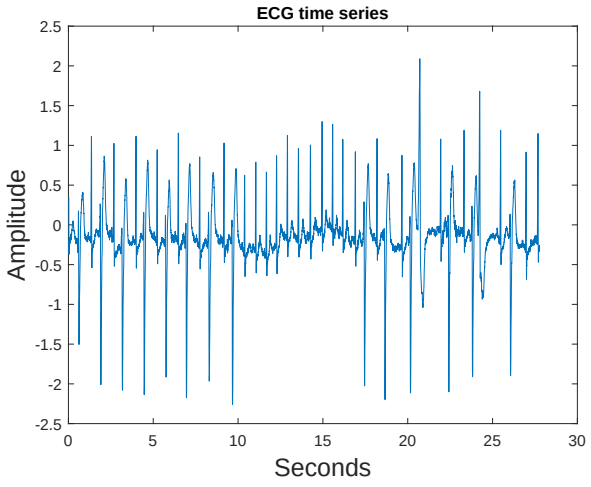
Signal here refers to observations (from some process or experiment) in the form of data.

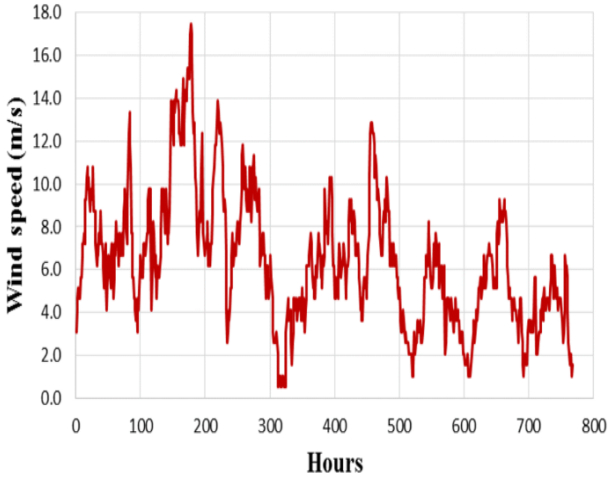
- examples: speech, radar/communication signals, electrocardiogram (ECG), daily temperature, satellite images
- mathematical representation: function of time or other variables e.g., space, frequency
- signal classification: discrete, continuous, analog, digital
- signal classification: univariate, multivariate, multidimensional (single function of multiple independent variables e.g. images)

Orderliness in the data (e.g., time series data) will be the key! Our modelling strategies will rely on ordered data.

Examples of Signals (Time-series)







A time-series may exhibit different features, including:

- nonstationarity (e.g. trend in the stock market index data, random walk)
- apparent periodicity (e.g. periodic spikes in the ECG data)
- seasonality (e.g., in wind data)
- nonlinearity (e.g. EEG data; not within the scope of this course)

Stochastic Signal Processing

Stochastic = **Randomness or Uncertainty**

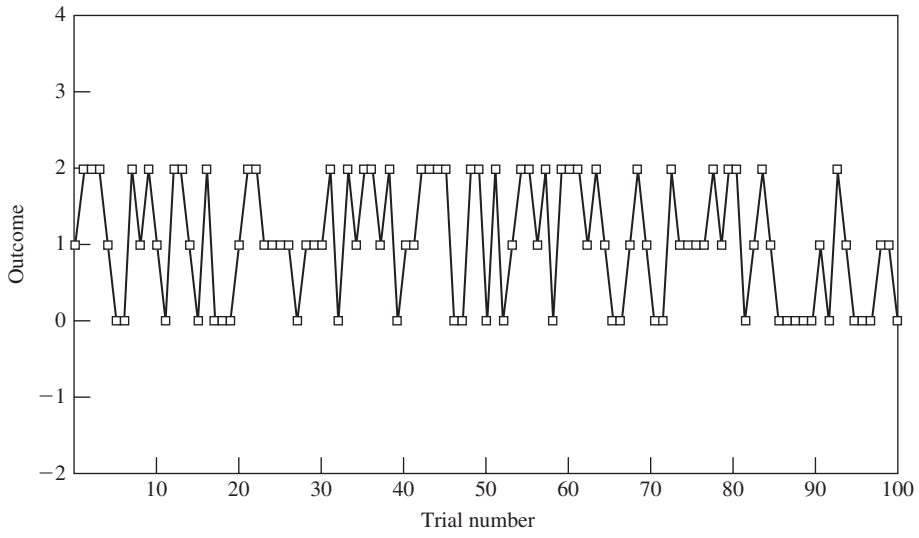
- Randomness implies lack of precise (deterministic) mathematical function to describe a process (or accompanying signals) e.g., EEG/wind/radar as output of brain/atmosphere/channel
- As a result, the outcome of such process varies unpredictably under same conditions OR the observations of that process cannot be predicted accurately.
- Examples:
  1. Tossing a coin and observing the output
  2. Toss two dice and take the sum of the numbers that land up
  3. current through a resistor R, provided a given input voltage of say, 5 volts
  4. selection of a ball from urn containing 3 identical balls, numbered 0, 1 and 2;
  5. prediction of rainfall in Aarhus

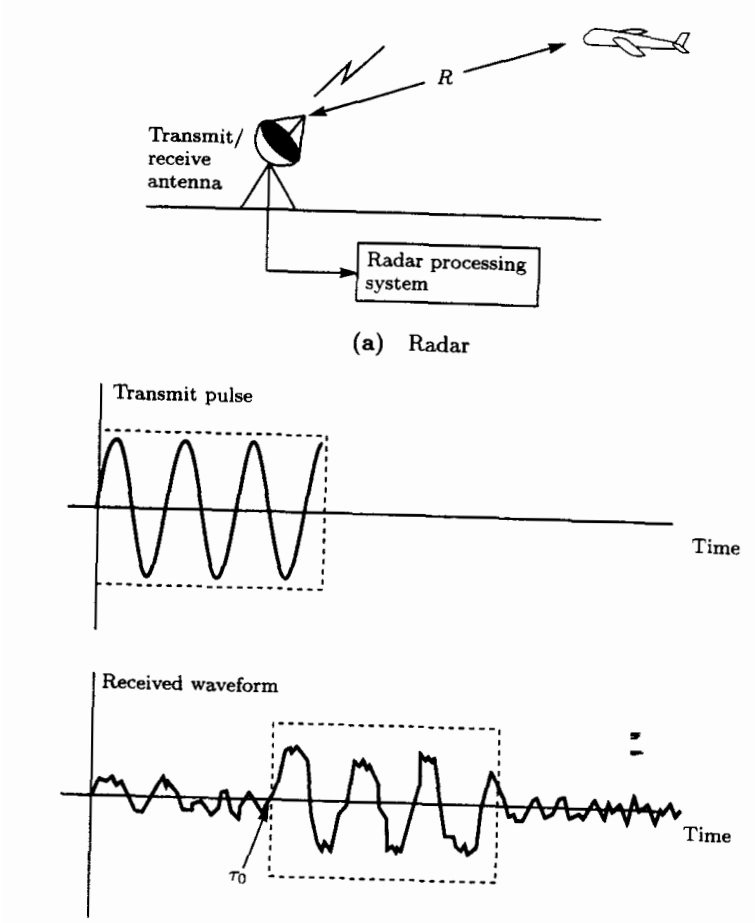
Can you identify which of the above processes are random and which are (predominantly) deterministic?

Sources of Uncertainty or Randomness

1. Uncertainty in process knowledge:
   - Lack of information about the process in hand e.g., atmospheric processes (wind speed), neurological processes (EEG);
   - The fact that we resort to time-series (stochastic signal) processing is because the prior process knowledge is lacking to develop a (deterministic) model hence the data-driven approach!
2. Uncertainty in measurement:
   - Sensor imperfections;
   - Sensor properties are not fully understood and hence are not modelled accurately.

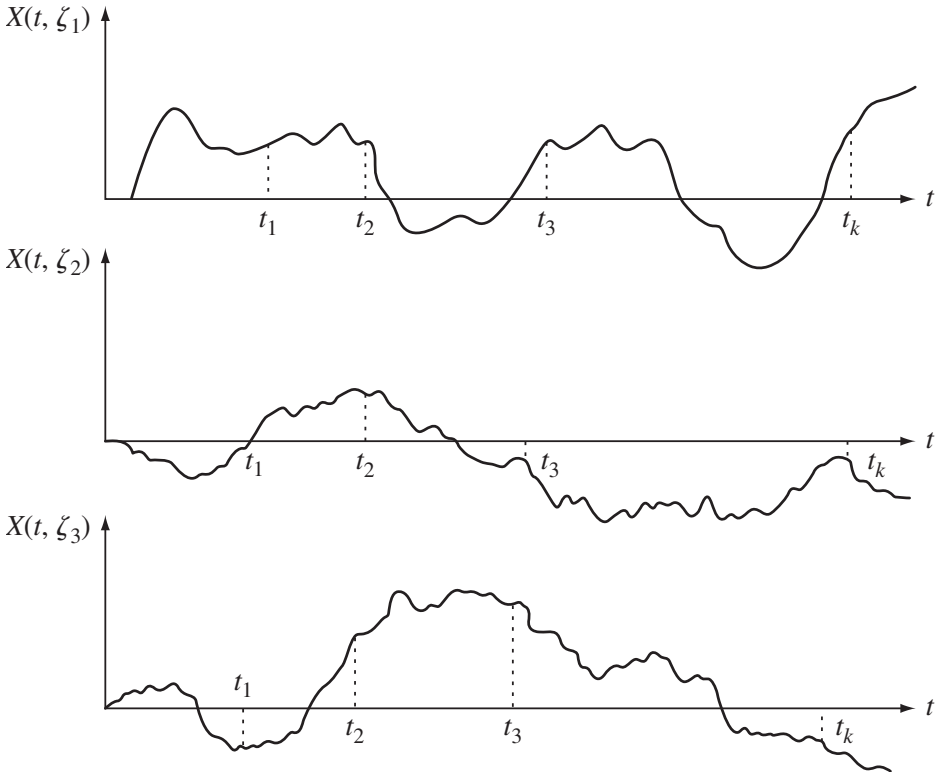
Stochastic Signal Processing





Stochastic signals corresponding to the urn experiment (left) and radar receiver signal (right)

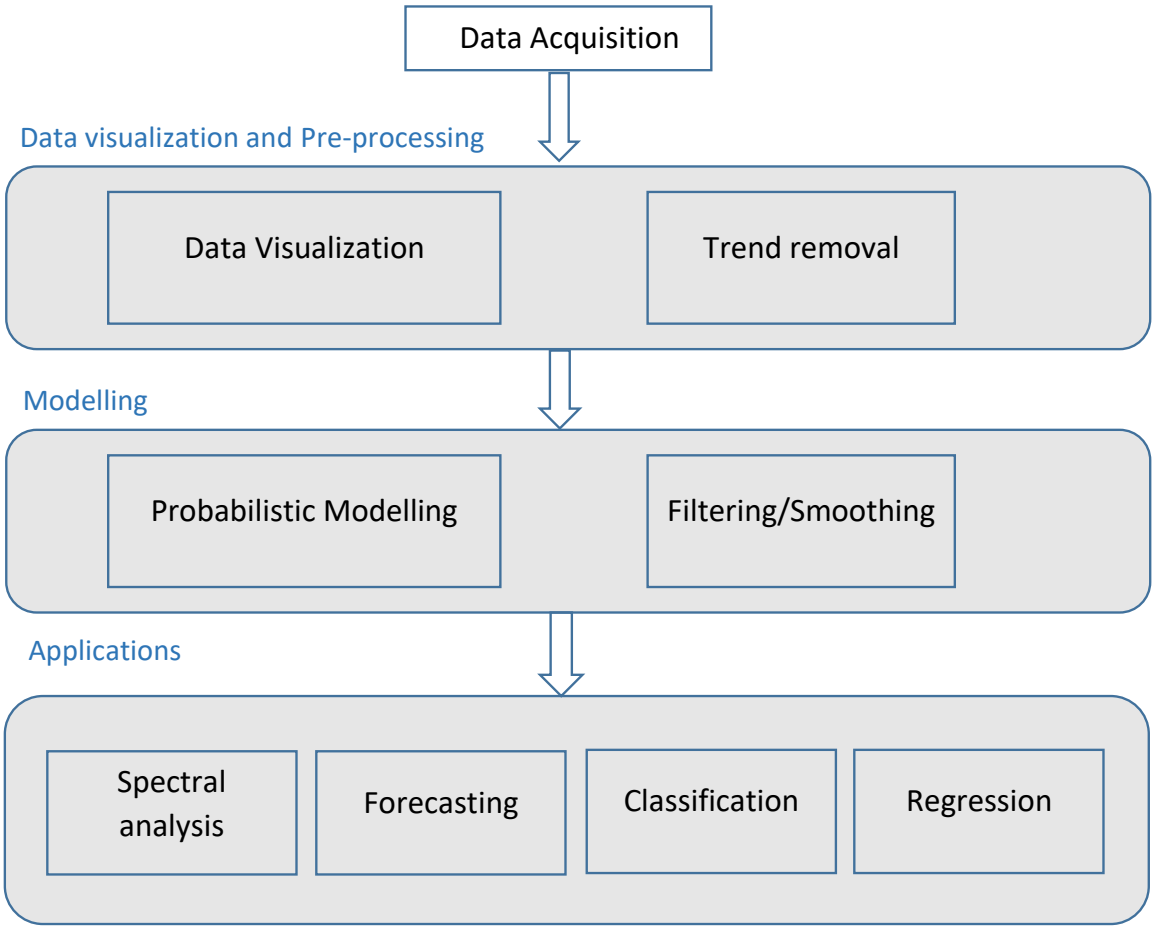
Stochastic process (ensemble) and single realization (time series)



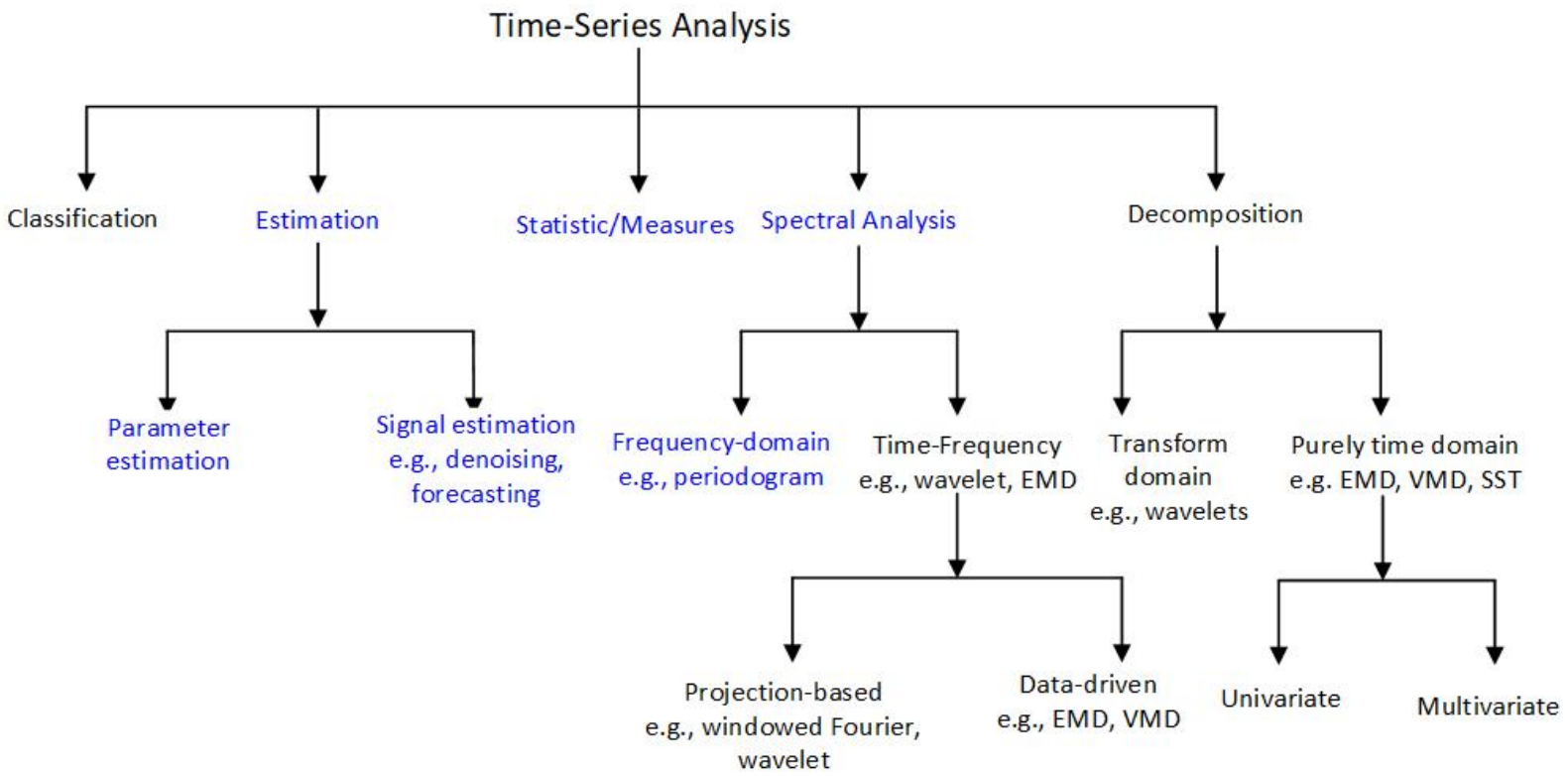
Stochastic Signal Processing

1. Processing implies manipulation of signals to extract useful information from those e.g. detecting cardiovascular disease from observing ECG, target detection after processing radar signal
2. Key Challenge: Uncertainty in data
   - To make inferences about a process which can produce many outcomes, from a single record of observation (that is maybe one in millions)
3. Examples of processing:
   - *Estimation* of unknown parameters from given (uncertain) observations e.g., density estimation or model fitting.
   - Classification
   - Regression
   - Spectral analysis

Steps in Stochastic Signal Processing



Overview of Time Series Analysis Tasks



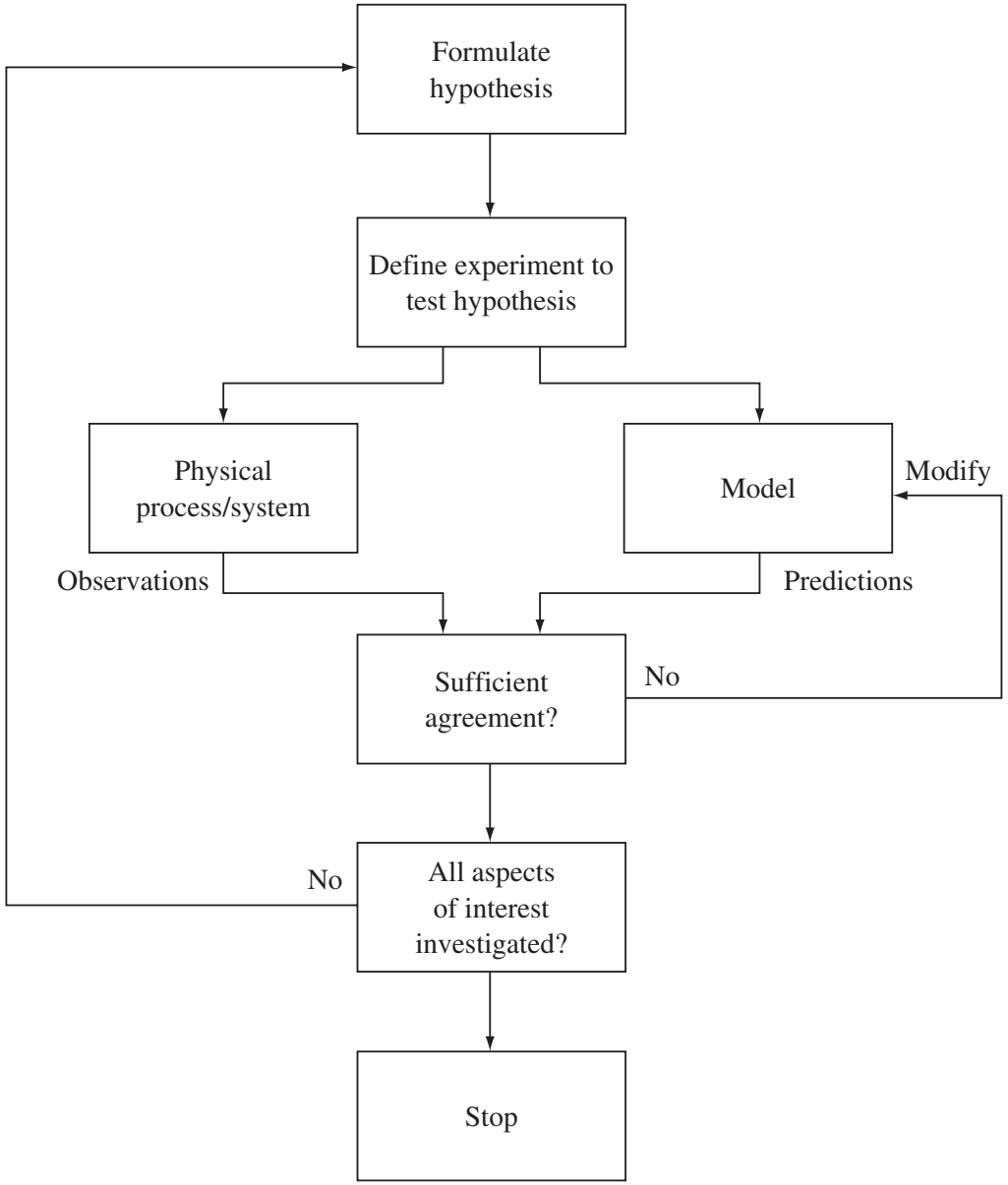
Mathematical Models as Tools in Analysis and Design

Model: approximate representation of a physical system that attempts to explain its observed behaviour;
Mathematical Model: consists of a set of assumptions/rules about how a system/process works. Those assumptions are stated in the form of mathematical relations.
Advantages of mathematical models:

- Allow performance evaluation of complex systems without physically working on those systems;
- Enable simulation of complex scenarios which may not be easily implemented (and hence investigated) on actual systems;
- Reduce time, effort and cost.

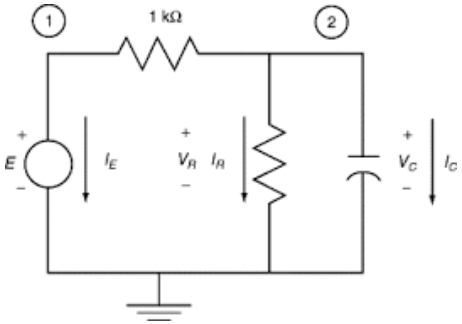
Examples: resistor, capacitor, inductor models

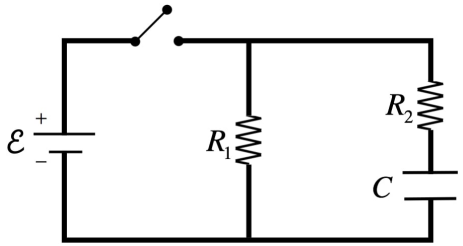
The Modelling process

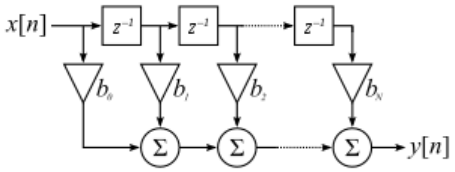


Deterministic vs Probability Models

Deterministic Models: give an exact outcome of a process or experiment, e.g., linear circuits; higher-order (nonlinear) circuits involving capacitors, transistors etc; FIR and IIR filters.



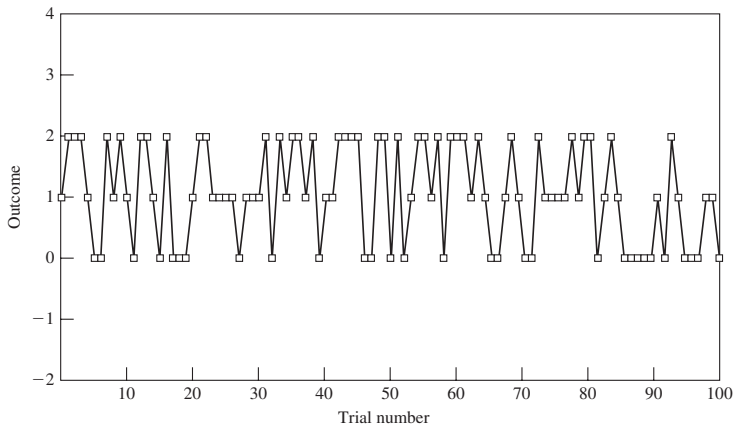




Probability Models: For random processes (signals), uncertainty in the outcome of experiment or process is modelled through random variables, probability density, and distribution functions, and their moments e.g., AWGN channel model in communication.

Probability Models

Random Experiment: whose outcome varies in unpredictable fashion when the experiment is repeated under same conditions.
Example: A ball is selected from an urn containing 3 identical balls, labelled 0, 1 and 2 respectively.
Outcome and Sample Space: is a number from a set $S = 0, 1, 2$; S that shows all possible outcomes of the experiment is called *sample space* of the experiment.



Central Idea: To deal with uncertainty in the process, we list all possible outcomes of a process/experiment and assign probabilities to those outcomes.

Probability and Statistical Regularity

1. How to quantify uncertainty of outcomes of a random experiment?

Ans: Assign a 'probability' to each outcome!

2. How to compute 'probability' values?

Ans: Statistical regularity (averages in long sequence of repetitions (trials) of random experiment consistently yield same values).

3. Example: Let $N_0(n)$, $N_1(n)$ and $N_2(n)$ denote the number of times the outcome of the urn experiment is 0, 1, and 2 respectively in n trials. The relative frequency is defined as

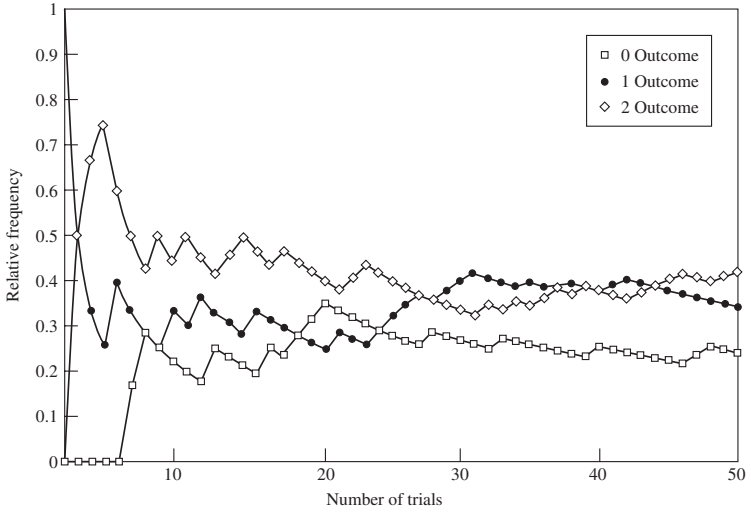
$$f_k(n) = \frac{N_k(n)}{n}$$

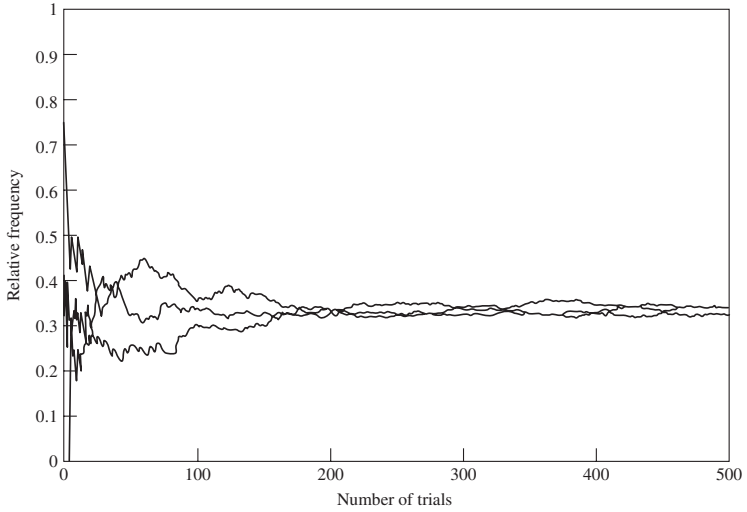
Statistical regularity implies that

$$\lim_{n \to \infty} f_k(n) = p_k$$

where p_k is a constant that denotes the probability of outcome k.

Statistical Regularity





Properties of Relative Frequency

Suppose a random experiment has K possible outcomes i.e., $S = 1, 2, , K$. Here, $N_k(n)$ and $f_k(n)$ denote the number of times the outcome is k and its relative frequency respectively. Then,

$$0 \leq f_k(n) \leq 1 \text{ for } k = 1, 2, , K$$

and

$$\sum_{k=1}^{K} f_k(n) = 1.$$

Events and relative frequencies: Event can be seen as a subset of S, e.g., "even numbered ball is selected". In this case, $N_e(n) = N_0(n) + N_2(n)$ and $f_e(n) = f_0(n) + f_2(n)$, showing that relative frequency of an 'event' is equal to the sum of the relative frequencies of its constituent outcomes.

Axioms of Probability

- In practice, probability theory is not tied to relative frequency approach in strict sense. Why? Because an experiment cannot be performed infinite (or even finite) number of times.
- However, modern probability theory does follow the intuition behind relative frequency and therefore is built on the set of axioms, derived from the properties of relative frequency. Here, A denotes any event and A and B are mutually exclusive events.

$$
\begin{aligned}
0 \leq \quad P(A) \quad &\leq 1, && (1)\\
P(S) \quad &= \quad 1, && (2)\\
P(A \cup B) \quad &= \quad P(A) + P(B) && (3)
\end{aligned}
$$

Note the correspondence between the above axioms and the properties of relative frequency!

Building a Probability Model

Building a probability model entails:

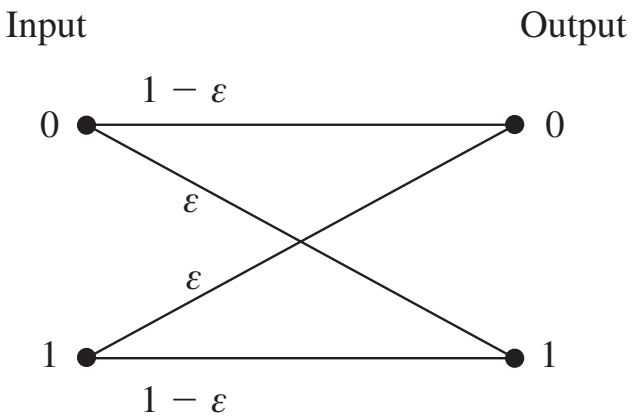
- defining a random experiment
- specifying a set of all possible outcomes
- specifying probability assignment that covers all events of interest

Example:

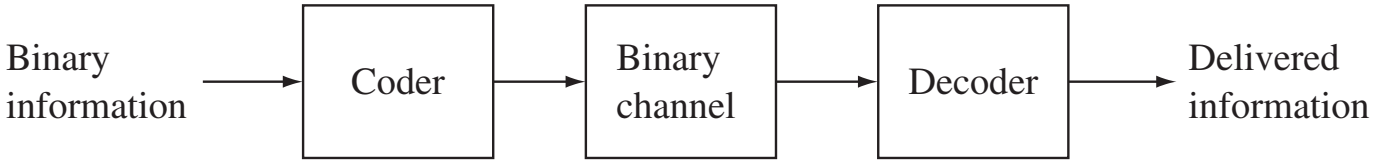
- Picking a single ball from an urn containing 3 balls, numbered (0,1,2)
- Picking two balls without replacement (ordered)
- n trials of the first experiment

Using this procedure, any assignment of probabilities that fulfills the axioms is legitimate. It is up to the design engineer to assign probabilities which make physical sense, leading to satisfactory results.

Examples: Communication over Unreliable Channels

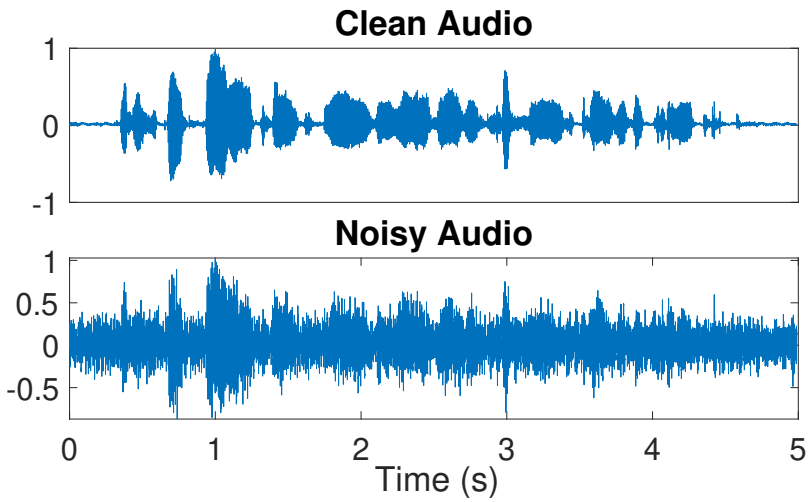


(a)

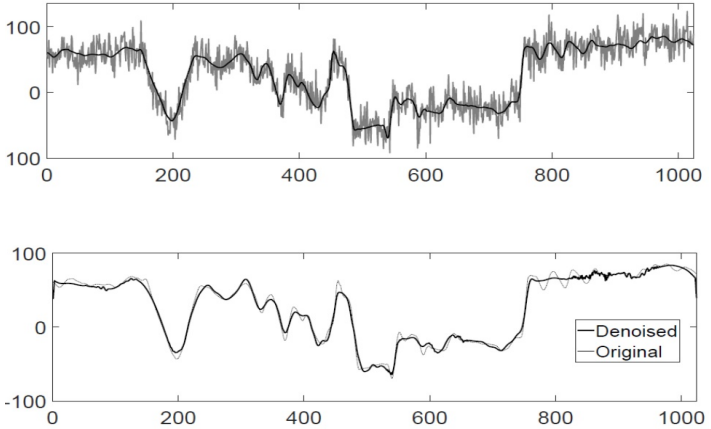


(b)

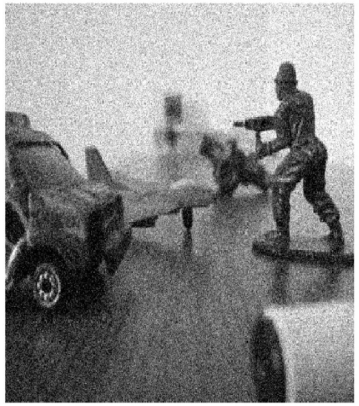
Examples: Denoising of Random Signals



(Noisy and denoised speech signal)

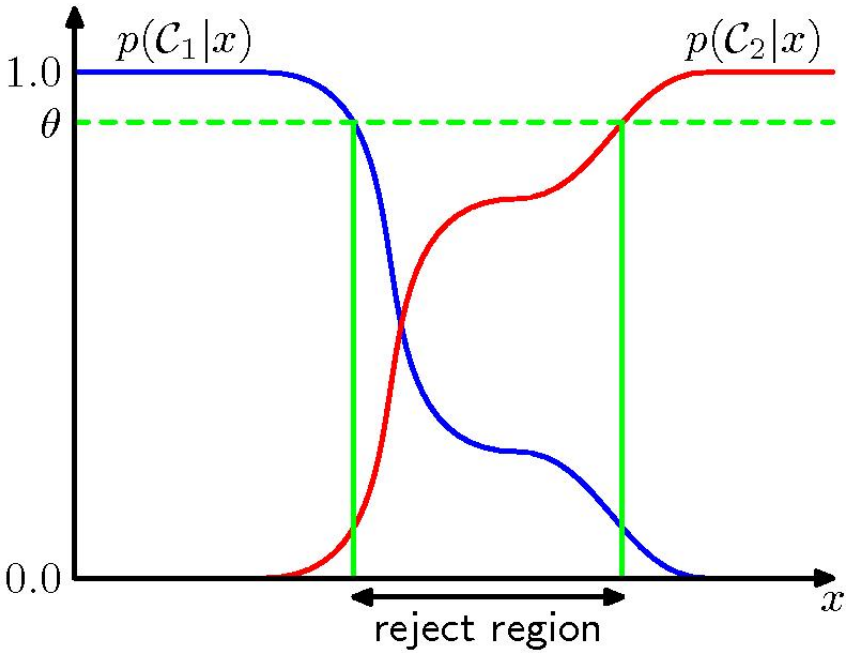


(Denoising results on wind speed signal)

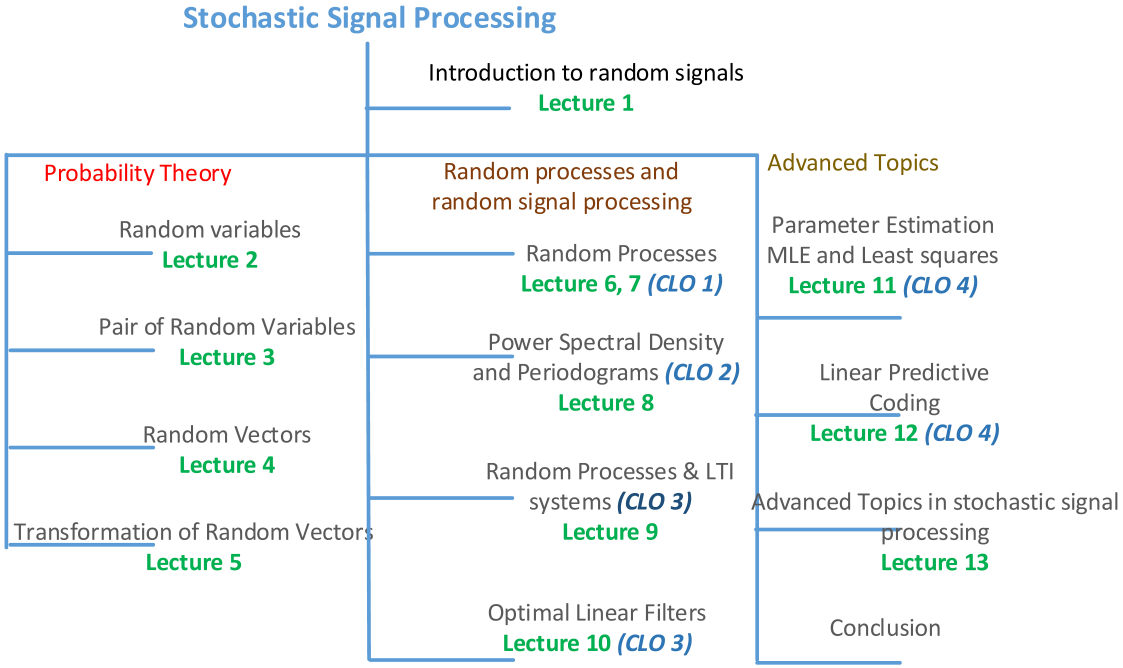


Examples: Classification

Input: x (X-ray image of patient)
Output: One of two classes, C_1 and C_2 (Cancer or No Cancer)
Goal: Finding a rule/function that assigns x to a particular class; the rule must be chosen based on probabilistic model



Course Overview



Course Learning Outcomes (CLO)

CLO 1) Analyze general mathematical models for characterizing random experiments (and signals) and extracting useful information through those models;

CLO 2) Compute power spectral density of random signals using several variants of periodograms and be able to analyze information content therein;

CLO 3) Apply optimal linear methods for the prediction and denoising of wide-sense stationary random signals;

CLO 4) Design and apply a class of estimators for the parameter estimation from random signals.