

Kelompok 9

- Malvin Leonardo Hartanto (5025221033)
- Mohammad Hanif Furqan Aufa Putra (5025221161)
- Muhammad Alif Satriadhi (5025221188)

Tugas 6

Komputasi Numerik D

1. Carilah $\int f(x) dx$ dari data-data berikut dengan batas $x = 1$ sampai $x = 7$ menggunakan integrasi Trapezoida, Simpson 1/3, Simpson 3/8, dan Kuadratur jika diketahui data-data berikut:

x	1,0	2,0	3,0	4,0	5,0	6,0	7,0
f(x)	1,8287	5,6575	11,4862	19,3149	29,1437	40,9724	54,8011

x	1,0	2,0	3,0	4,0	5,0	6,0	7,0
f(x)	2,1353	6,2707	12,4060	20,5413	30,6767	42,8120	56,9473

x	1,0	2,0	3,0	4,0	5,0	6,0	7,0
f(x)	1,8419	5,6838	11,5257	19,3676	29,2095	41,0514	54,8933

Jawaban

1. Berikut ini cara mencari $\int f(x) dx$ dari data-data di soal dengan batas $x = 1$ sampai $x = 7$ menggunakan integrasi yang diminta.

- Integrasi Trapezoida

Karena data bersifat *equispaced*, maka $\Delta x = 1$.

• Tabel data pertama

$$\int_1^7 f(x) dx = \frac{\Delta x}{2} [f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n)]$$

$$\begin{aligned} \int_1^7 f(x) dx &= \frac{1}{2} [1,8287 + 2(5,6575 + 11,4862 + 19,3149 + 29,1437 \\ &\quad + 40,9724) + 54,8011] \\ &= 134,8896 \end{aligned}$$

• Tabel data kedua

$$\int_1^7 f(x) dx = \frac{\Delta x}{2} [f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n)]$$

$$\begin{aligned} \int_1^7 f(x) dx &= \frac{1}{2} [2,1353 + 2(6,2707 + 12,4060 + 20,5413 + 30,6767 \\ &\quad + 42,8120) + 56,9473] \\ &= 142,248 \end{aligned}$$

• Tabel data ketiga

$$\int_1^7 f(x) dx = \frac{\Delta x}{2} [f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n)]$$

$$\begin{aligned} \int_1^7 f(x) dx &= \frac{1}{2} [1,8419 + 2(5,6838 + 11,5257 + 19,3676 + 29,2095 \\ &\quad + 41,0514) + 54,8933] \\ &= 135,2056 \end{aligned}$$

- Integrasi Simpson 1/3

Karena data bersifat *equispaced*, maka $\Delta x = 1$.

- Tabel data Pertama

$$\int_1^7 f(x) dx = \frac{\Delta x}{3} [f(x_0) + 4 \sum_{ganjil} f(x_i) + 2 \sum_{genap} f(x_i) + f(x_n)]$$

$$\int_1^7 f(x) dx = \frac{1}{3} [1,8287 + 4(1,8287 + 11,4862 + 29,1437) + 2(5,6575 + 19,3149 + 40,9724) + 54,8011]$$

$$\int_1^7 f(x) dx = 133,89$$

- Tabel data Kedua

$$\int_1^7 f(x) dx = \frac{\Delta x}{3} [f(x_0) + 4 \sum_{ganjil} f(x_i) + 2 \sum_{genap} f(x_i) + f(x_n)]$$

$$\int_1^7 f(x) dx = \frac{1}{3} [2,1353 + 4(2,1353 + 12,4060 + 30,6767) + 2(6,2707 + 20,5413 + 42,8120) + 56,9473]$$

$$= 141,248$$

- Tabel data Ketiga

$$\int_1^7 f(x) dx = \frac{\Delta x}{3} [f(x_0) + 4 \sum_{ganjil} f(x_i) + 2 \sum_{genap} f(x_i) + f(x_n)]$$

$$\int_1^7 f(x) dx = \frac{1}{3} [1,8419 + 4(1,8419 + 11,5257 + 29,2095) + 2(5,6838 + 19,3676 + 41,0514) + 54,8933]$$

$$= 134,2506$$

- Integrasi Simpson 3/8

Karena data bersifat *equispaced*, maka $\Delta x = 1$

• Tabel data Pertama

$$\int_1^7 f(x) dx = \frac{3\Delta x}{8} [f(x_0) + 3 \sum_{i=1, i \neq 3k}^{n-1} f(x_i) + 2 \sum_{k=1}^{\frac{n}{3}-1} f(x_i) + f(x_n)]$$

$$\int_1^7 f(x) dx = \frac{3}{8} [1,8287 + 3(5,6575 + 19,3149 + 29,1437) + 2(11,4862 + 40,9724) + 54,8011]$$

$$\int_1^7 f(x) dx = 133,89$$

• Tabel data Kedua

$$\int_1^7 f(x) dx = \frac{3\Delta x}{8} [f(x_0) + 3 \sum_{i=1, i \neq 3k}^{n-1} f(x_i) + 2 \sum_{k=1}^{\frac{n}{3}-1} f(x_i) + f(x_n)]$$

$$\int_1^7 f(x) dx = \frac{3}{8} [2,1353 + 3(6,2707 + 20,5413 + 30,6767) + 2(12,4060 + 42,8120) + 56,9473]$$

$$= 141,248$$

• Tabel data Ketiga

$$\int_1^7 f(x) dx = \frac{3\Delta x}{8} [f(x_0) + 3 \sum_{i=1, i \neq 3k}^{n-1} f(x_i) + 2 \sum_{k=1}^{\frac{n}{3}-1} f(x_i) + f(x_n)]$$

$$\int_1^7 f(x) dx = \frac{3}{8} [1,8419 + 3(5,6838 + 19,3676 + 29,2095) + 2(11,5257 + 41,0514) + 54,8933]$$

$$= 134,2506$$

- Integrasi Kuadratur

Titik x_i dan bobot w_i untuk $n = 7$ adalah:

$w_0 = 0,1294849661688697$	$x_0 = -0,9491079123427585$
$w_1 = 0,2797053914892766$	$x_1 = -0,7415311855993945$
$w_2 = 0,3818300505051189$	$x_2 = -0,4058451513773972$
$w_3 = 0,4179591836734694$	$x_3 = 0,0000000000000000$
$w_4 = 0,3818300505051189$	$x_4 = 0,4058451513773972$
$w_5 = 0,2797053914892766$	$x_5 = 0,7415311855993945$
$w_6 = 0,1294849661688697$	$x_6 = 0,9491079123427585$

Kemudian, transformasikan titik dan bobot ke interval $[1,7]$.

$$a = 1 \quad b = 7$$

$$x'_i = \frac{(b+a) + (b-a)x_i}{2} = \frac{(7+1) + (7-1)x_i}{2} = 4 + 3x_i$$

$$w'_i = \frac{b-a}{2} w_i = \frac{7-1}{2} w_i = 3w_i$$

$$x'_0 = 4 + 3x_0 = 4 + 3(-0,9491079123427585) = 1,1526762629717245$$

$$w'_0 = 3w_0 = 3(0,1294849661688697) = 0,3884548985066091$$

$$x'_1 = 4 + 3x_1 = 4 + 3(-0,7415311855993945) = 1,7754064432018165$$

$$w'_1 = 3w_1 = 3(0,2797053914892766) = 0,8391161744678298$$

$$x'_2 = 4 + 3x_2 = 4 + 3(-0,4058451513773972) = 2,7824645458678084$$

$$w'_2 = 3w_2 = 3(0,3818300505051189) = 1,1454901515153567$$

$$x'_3 = 4 + 3x_3 = 4 + 3(0,0000000000000000) = 4,0000000000000000$$

$$w'_3 = 3w_3 = 3(0,4179591836734694) = 1,2538775510204082$$

$$x'_4 = 4 + 3x_4 = 4 + 3(0,4058451513773972) = 5,2175354541321916$$

$$w'_4 = 3w_4 = 3(0,3818300505051189) = 1,1454901515153567$$

$$x'_5 = 4 + 3x_5 = 4 + 3(0,7415311855993945) = 6,2245935567981835$$

$$w'_5 = 3w_5 = 3(0,2797053914892766) = 0,8391161744678298$$

$$x'_6 = 4 + 3x_6 = 4 + 3(0,9491079123427585) = 6,8473237370282755$$

$$w'_6 = 3w_6 = 3(0,1294849661688697) = 0,3884548985066091$$

- Tabel data pertama

➤ Evaluasi fungsi $f(x'_i)$.

$$f(x'_0) = f(1,1526762629717245) \approx f(1) = 1,8287$$

$$f(x'_1) = f(1,7754064432018165) \approx f(2) = 5,6575$$

$$f(x'_2) = f(2,7824645458678084) \approx f(3) = 11,4862$$

$$f(x'_3) = f(4,0000000000000000) \approx f(4) = 19,3149$$

$$f(x'_4) = f(5,2175354541321916) \approx f(5) = 29,1437$$

$$f(x'_5) = f(6,2245935567981835) \approx f(6) = 40,9724$$

$$f(x'_6) = f(6,8473237370282755) \approx f(7) = 54,8011$$

➤ Hitung integralnya.

$$\begin{aligned} \int_1^7 f(x) dx &= \sum_{i=0}^6 w'_i f(x'_i) \\ &= (0,3884548985066091 \cdot 1,8287) + (0,8391161744678298 \cdot 5,6575) \\ &\quad + (1,1454901515153567 \cdot 11,4862) + (1,2538775510204082 \cdot 19,3149) \\ &\quad + (1,1454901515153567 \cdot 29,1437) + (0,8391161744678298 \cdot 40,9724) \\ &\quad + (0,3884548985066091 \cdot 54,8011) \\ &= 131,88569633252490232971 \end{aligned}$$

- Tabel data kedua

➤ Evaluasi fungsi $f(x'_i)$.

$$f(x'_0) = f(1,1526762629717245) \approx f(1) = 2,1353$$

$$f(x'_1) = f(1,7754064432018165) \approx f(2) = 6,2707$$

$$f(x'_2) = f(2,7824645458678084) \approx f(3) = 12,4060$$

$$f(x'_3) = f(4,0000000000000000) \approx f(4) = 20,5413$$

$$f(x'_4) = f(5,2175354541321916) \approx f(5) = 30,6767$$

$$f(x'_5) = f(6,2245935567981835) \approx f(6) = 42,8120$$

$$f(x'_6) = f(6,8473237370282755) \approx f(7) = 56,9473$$

➤ Hitung integralnya.

$$\begin{aligned} \int_1^7 f(x) dx &= \sum_{i=0}^6 w'_i f(x'_i) \\ &= (0,3884548985066091 \cdot 2,1353) + (0,8391161744678298 \cdot 6,2707) \\ &\quad + (1,1454901515153567 \cdot 12,4060) + (1,2538775510204082 \cdot 20,5413) \\ &\quad + (1,1454901515153567 \cdot 30,6767) + (0,8391161744678298 \cdot 42,8120) \\ &\quad + (0,3884548985066091 \cdot 56,9473) \\ &= 139,24409633252490159387 \end{aligned}$$

- Tabel data ketiga

➤ Evaluasi fungsi $f(x'_i)$.

$$f(x'_0) = f(1,1526762629717245) \approx f(1) = 1,8419$$

$$f(x'_1) = f(1,7754064432018165) \approx f(2) = 5,6838$$

$$f(x'_2) = f(2,7824645458678084) \approx f(3) = 11,5257$$

$$f(x'_3) = f(4,0000000000000000) \approx f(4) = 19,3676$$

$$f(x'_4) = f(5,2175354541321916) \approx f(5) = 29,2095$$

$$f(x'_5) = f(6,2245935567981835) \approx f(6) = 41,0514$$

$$f(x'_6) = f(6,8473237370282755) \approx f(7) = 54,8933$$

➤ Hitung integralnya.

$$\int_1^7 f(x) dx = \sum_{i=0}^6 w'_i f(x'_i)$$

$$\begin{aligned}
&= (0,3884548985066091 \cdot 1,8419) + (0,8391161744678298 \cdot 5,6838) \\
&\quad + (1,1454901515153567 \cdot 11,5257) + (1,2538775510204082 \cdot 19,3676) \\
&\quad + (1,1454901515153567 \cdot 29,2095) + (0,8391161744678298 \cdot 41,0514) \\
&\quad + (0,3884548985066091 \cdot 54,8933) \\
&= 132,20169787189230397944
\end{aligned}$$