INTEGERS

OPERATIONS

Integers support all the standard arithmetic operations:

addition	`+`
subtraction	`-`
multiplication	`*`
division	`/`
exponents	`**`

But what is the resulting type of each operation?

```
int + int    → int
int - int    → int
int * int    → int
int ** int   → int
int / int    → float
```

obviously `3 / 4` → `0.75` (float)

but, also `10 / 2` → `5` (float)

Two more operators in integer arithmetic

First, we revisit long integer division…

$\frac{155}{4}$ numerator / denominator

155 // 4

```
  38
4 | 155
    12
    ---
    35
    32
    --
     3
```

155 % 4

155 ÷ 4 = 38 with remainder 3

put another way:

155 = 4 * 38 + 3

155 = 4 * (155 // 4) + (155 % 4)

= 4 * 38 + 3

// is called floor division (div)

% is called the modulo operator (mod)

and they always satisfy: `n = d * (n // d) + (n % d)`

What is floor division exactly?

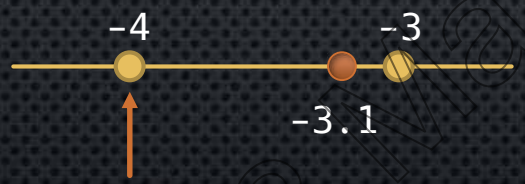
First define the floor of a (real) number

The floor of a real number a is the largest (in the standard number order) integer <= a

```
floor(3.14) → 3
floor(1.9999) → 1
floor(2) → 2
```

But watch out for negative numbers!

floor(-3.1) → -4



So, floor is not quite the same as truncation!

```
a // b = floor(a / b)
```

```
a = b * (a // b) + a % b
```

a = 135

b = 4

135 / 4 = 33.75 (33 ¾)

135 // 4 → 33

135 % 4 → 3

And, in fact: `a = b * (a // b) + a % b`

```
  4 * (135 // 4) + (135 % 4)
= 4 * 33 + 3
= 132 + 3
= 135
```

Negative Numbers

Be careful, `a//b`, is not the integer portion of `a / b`, it is the floor of `a / b`

For `a > 0` and `b > 0`, these are indeed the same thing

But beware when dealing with negative numbers!

```
a = -135

b = 4
```

```
-135 / 4 = -33.75 (-33 ¾)
```

```
-135 // 4   → -34
-135 % 4    → 1
```

```
135 // 4   → 33
135 % 4    → 3
```

And, in fact:

```
a = b * (a // b) + a % b
```

```
  4 * (-135 // 4) + (-135 % 4)
= (4 * -34) + 1
= -136 + 1
= -135 ✔
```

Expanding this further…

a = 13 b = 4	a = -13 b = 4	a = 13 b = -4	a = -13 b = -4
13 / 4 → 3.25	-13 / 4 → -3.25	13 / -4 → -3.25	-13 / -4 → 3.25
13 // 4 → 3	-13 // 4 → -4	13 // -4 → -4	-13 // -4 → 3
13 % 4 → 1	-13 % 4 → 3	13 % -4 → -3	-13 % -4 → -1

In each of these cases: a = b * (a // b) + a % b

4 * (13 // 4) + 13 % 4 = 12 + 1 = 13 ✔	4 * (-13 // 4) + -13 % 4 = -16 + 3 = -13 ✔	-4 * (13 // -4) + 13 % -4 = 16 + -3 = 13 ✔	-4 * (-13 // -4) + -13 % -4 = -12 + -1 = -13 ✔

Code