Welcome to Physics 212 – Fall 2024!

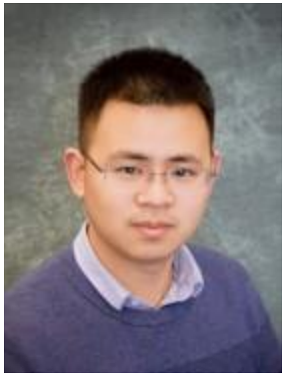
Electricity & Magnetism
Lecture 1

- Coulomb's Law
- electric fields
- Gauss' Law
- electric potential
- capacitance
- circuits
- magnetic forces and fields
- Ampere's law
- induction
- electromagnetic waves
- polarization
- geometrical optics.

Instructor

Prof. Chao Qian

chaoq@intl.zju.edu.cn



Prof. Ruisheng Diao

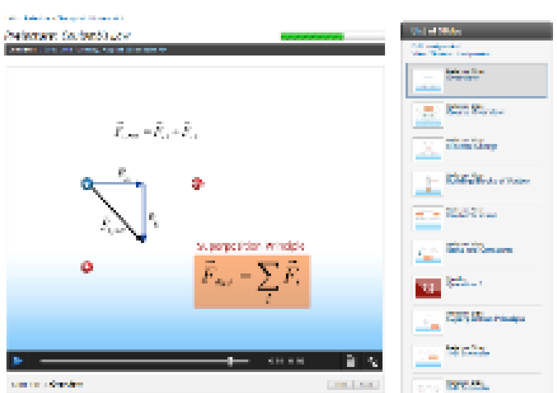
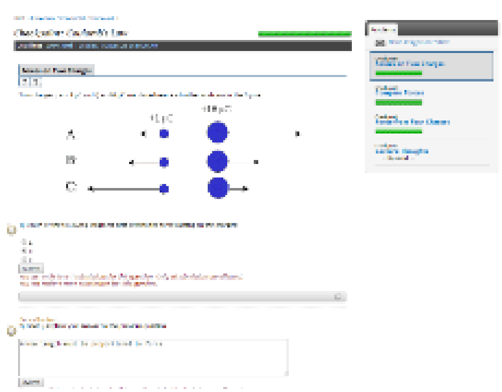
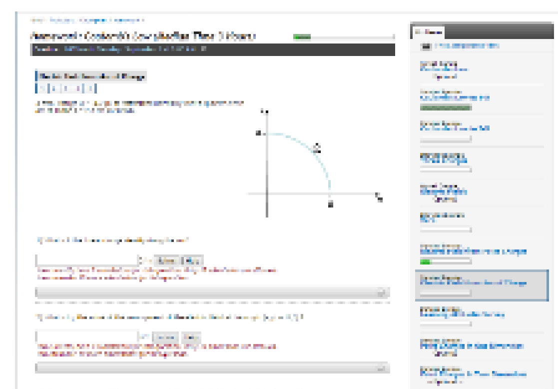
ruishengdiao@intl.zju.edu.cn

Course Structure

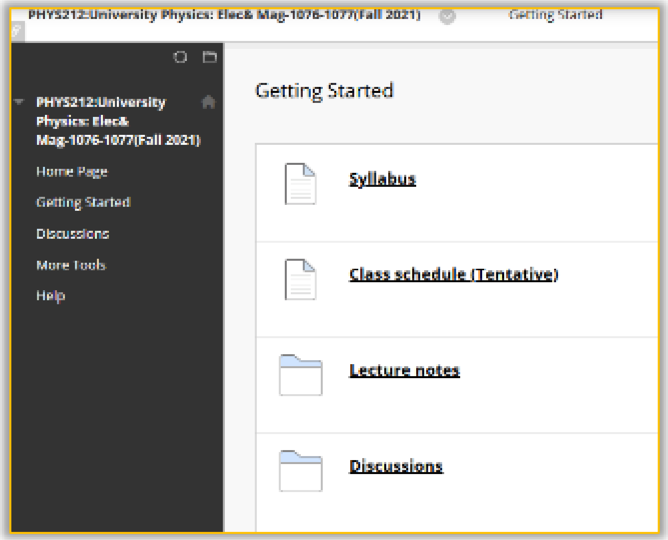
There are several parts, all are important:

- **Online** Prelectures (animated textbook, *before lecture*)
- **Online** CheckPoints (check knowledge, *before lecture*)
- Lectures – interactive, address issues found by checkpoints.
- **Online** Homework (first deadline next week)
- Discussion Sections (start this week)
- Exams (don't worry about that yet!)

Go to the right one !
Don't be late!

Syllabus…



PHYS 212 Fall 2024

University Physics: Electricity & Magnetism

Instructor: **Prof. Chao Qian & Prof. Ruisheng Diao**

Contact Information

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Login the online system to find your pre-lectures and homework:

https://byteshelf.physics.illinois.edu/

PHYS 212 Class Schedule

Fall 2024

Week	Date	Event	Reading	Notes
1	**Monday** 9/9/2024	**Lecture 1:** Introduction and Coulomb's Law	OpenStax Vol. 2 5.1-5.3	
	Monday 9/9/2024	Discussion 1-8		Quiz 1
	Thursday 9/12/2024	**Lecture 2:** Electric Fields	5.4-5.5	
2	**Monday** 9/16/2024	**No Class & No Discussion**		Mid-Autumn Festival
	Thursday 9/19/2024	**Lecture 3:** Electric Fields and Electric Flux	5.6, 5.7, 6.1	
3	**Monday** 9/23/2024	**Lecture 4:** Gauss's Law	6.2-6.4	
	Monday 9/23/2024	Discussion 1-8		Quiz 2
	Thursday 9/26/2024	**Lecture 5:** Electric Potential	7.1	

Grading…

14 weeks in this semester (except holiday)

Your final grade for Physics 212 will be based upon your total score on all the components of the course. The total possible score is 1000 points.

Course component	Number of assignments	Number dropped per semester	Maximum points per semester
Prelectures	28	3	25
Checkpoints	28	3	25
Lecture (Participation)	28	3	50
Homework	14	1	130
Discussion quizzes	14	1	130
Labs	9	1	120
Hour exams	3	0	3 X 80= 240
Final exam	1	0	280

Bonus Points: 25 points

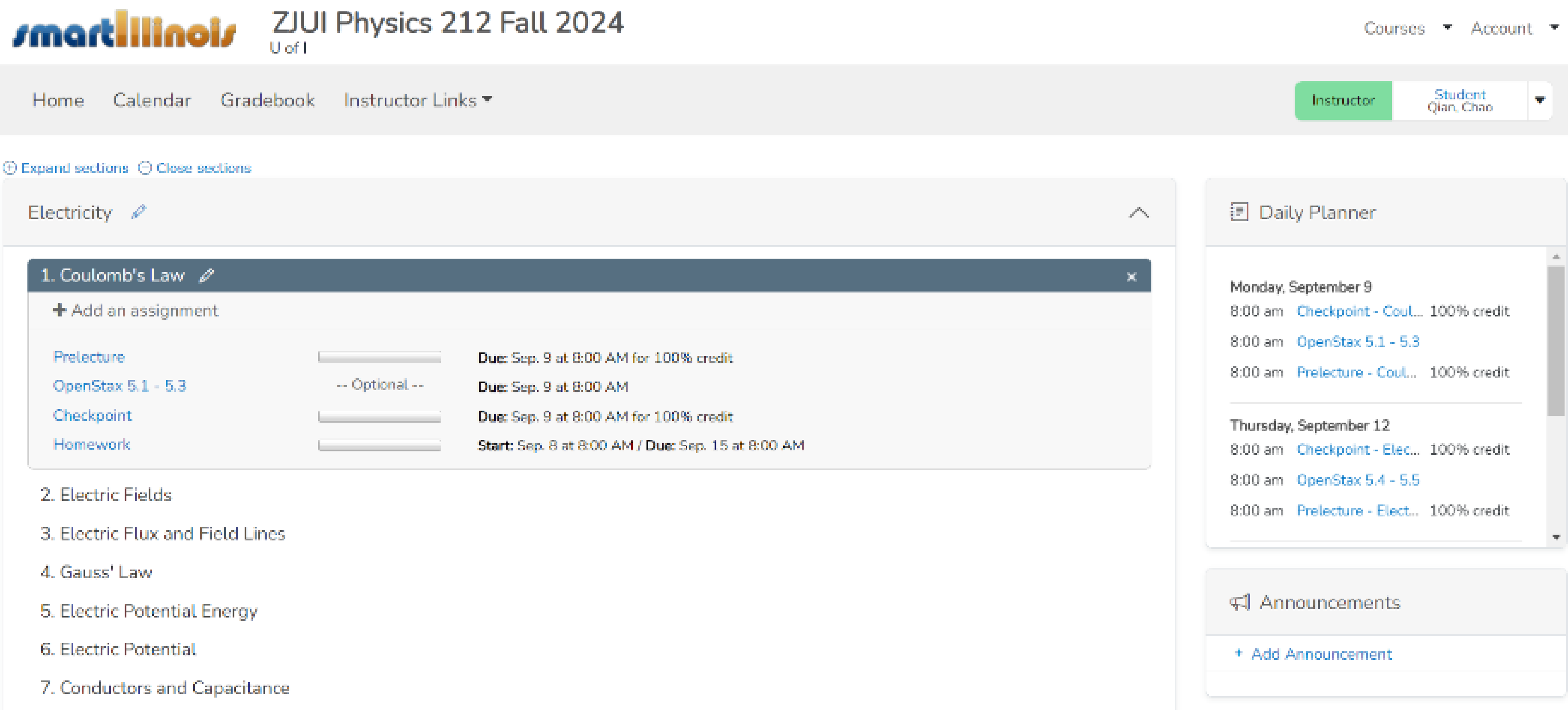
You will have an opportunity to earn up to 25 bonus points via the following activities.

- answering *Online Quiz* correctly in lecture (15 points)
- participating in activity correctly in discussion (10 points)

The bonus points will be distributed among any assignment scores except exam scores when calculating your final score at the end of the semester.

Final Grade	Minimum Points
A+	950
A	920
A-	900
B+	880
B	860
B-	835
C+	810
C	780
C-	750
D+	720
D	690
D-	610
F	<610

Smart Physics



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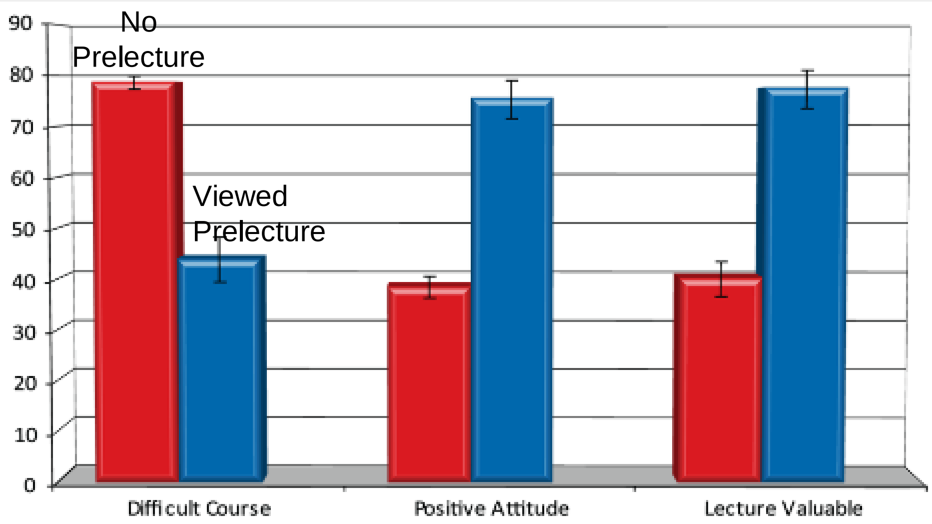
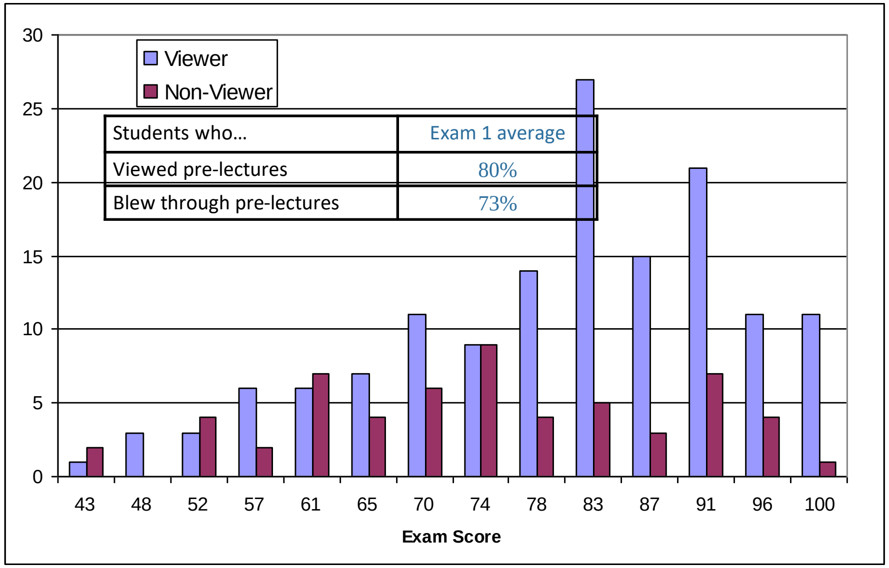
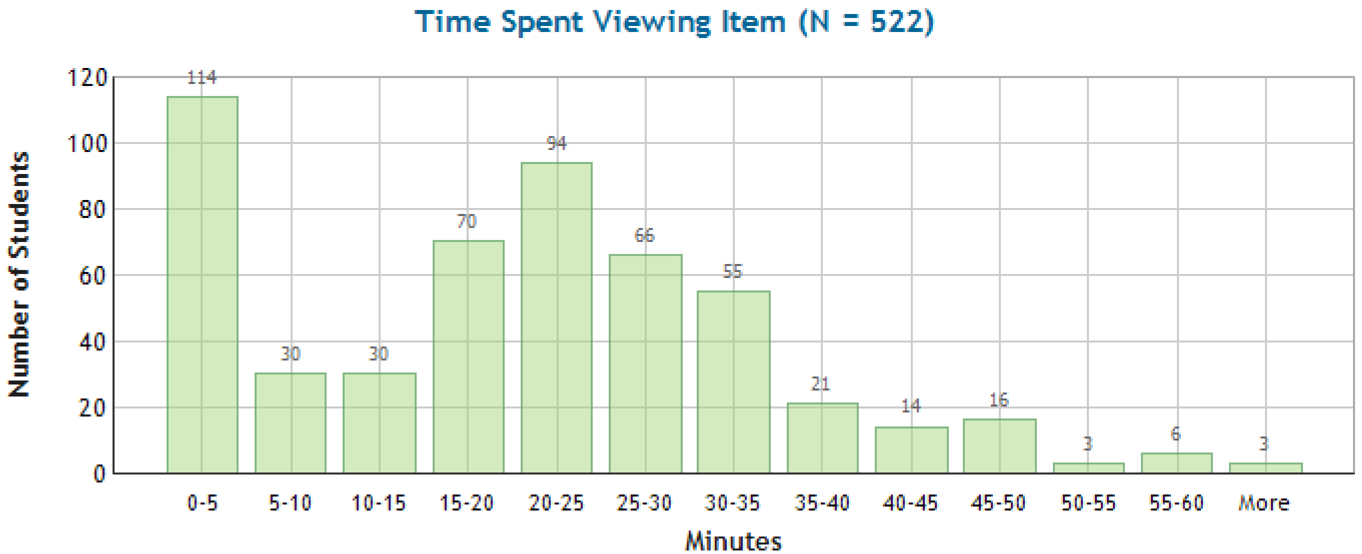
Prelectures: Just Do It





Students who…	Exam 1 average
Viewed pre-lectures	80%
Blew through pre-lectures	73%



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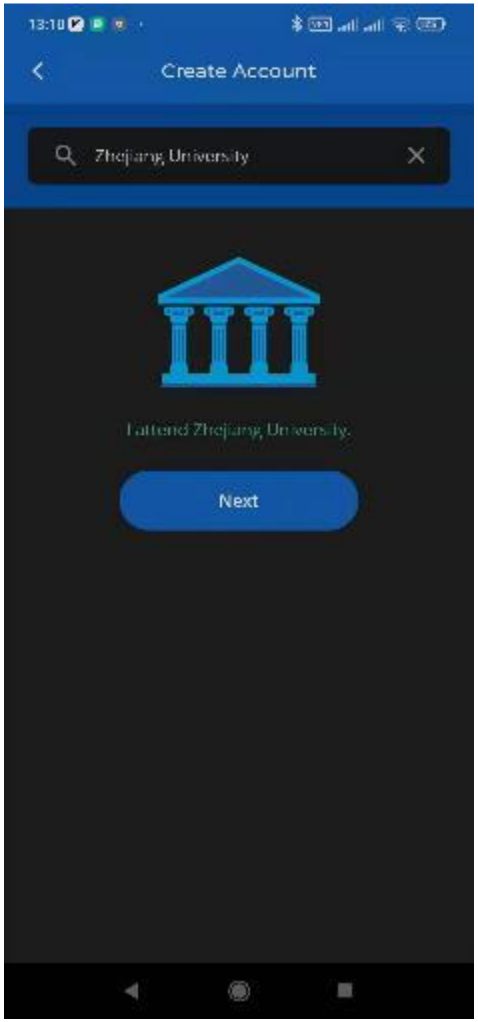

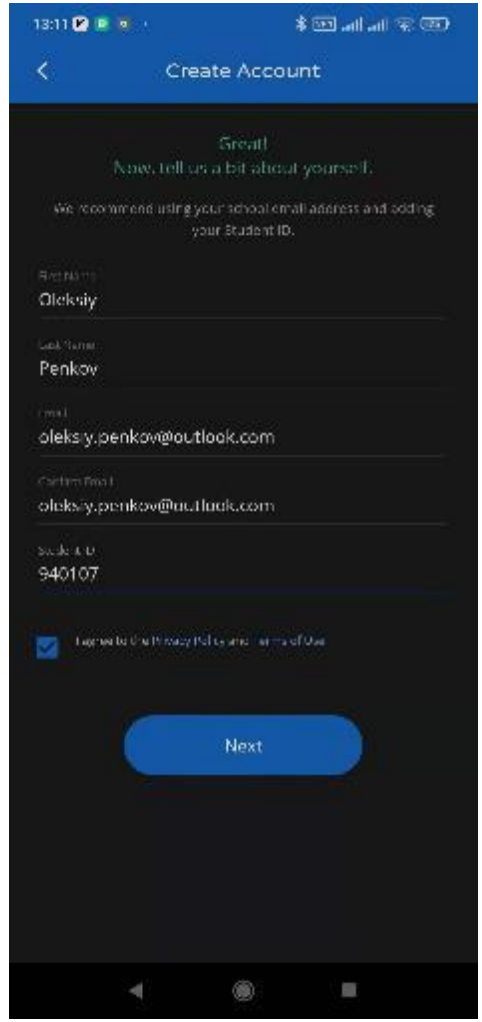

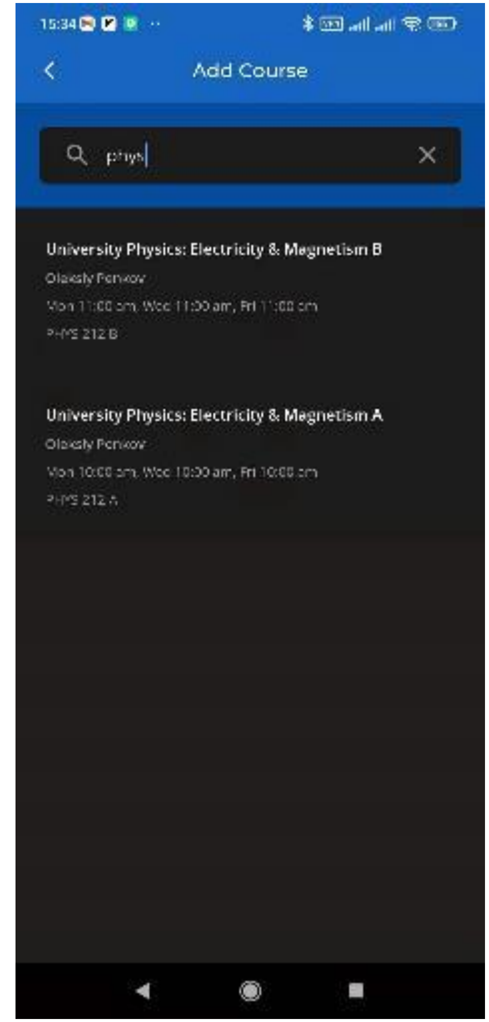

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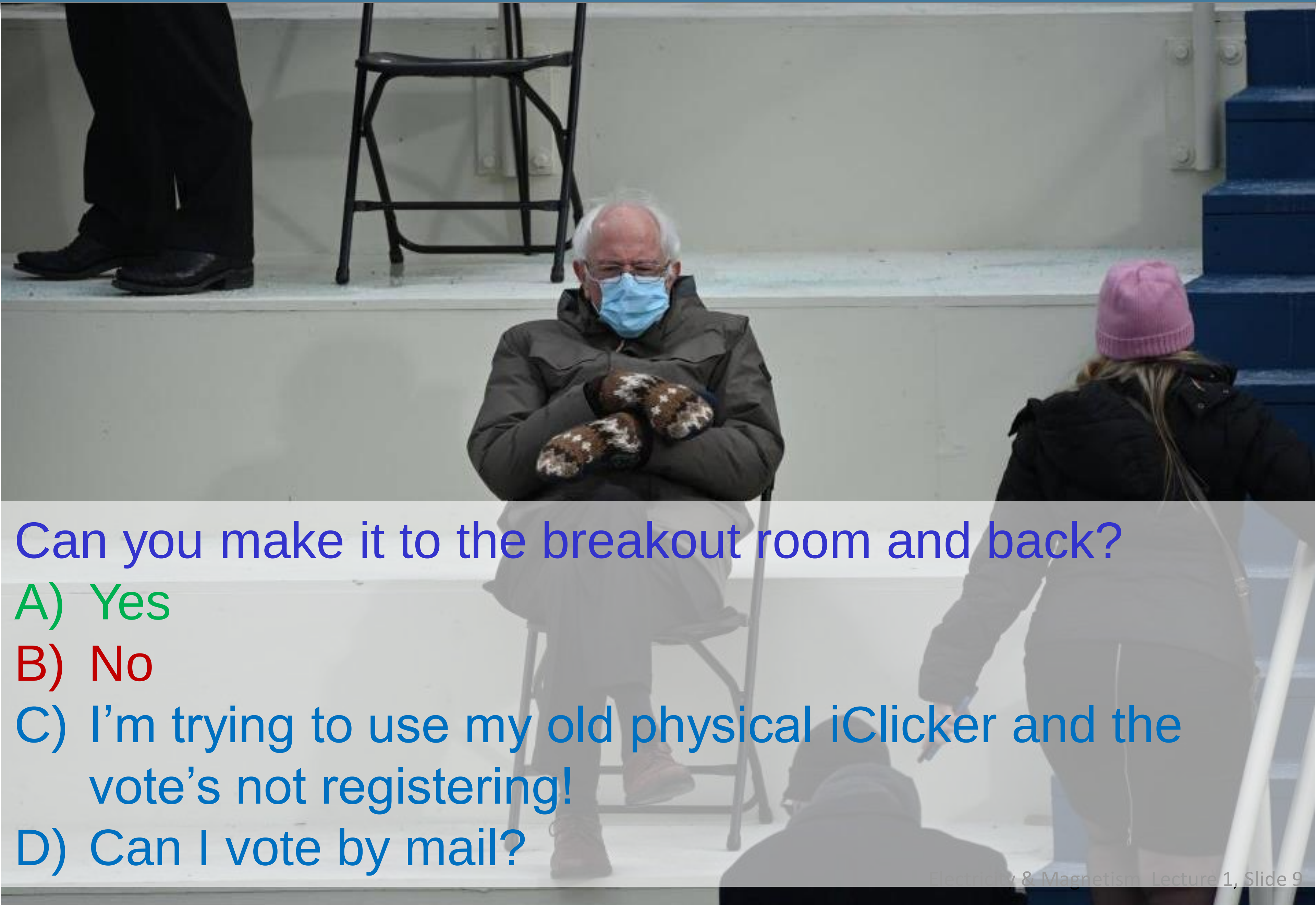
Pop Quiz



Can you make it to the breakout room and back?

A) Yes
B) No
C) I’m trying to use my old physical iClicker and the vote’s not registering!
D) Can I vote by mail?

About me…

Electricity & Magnetism
Lecture 1

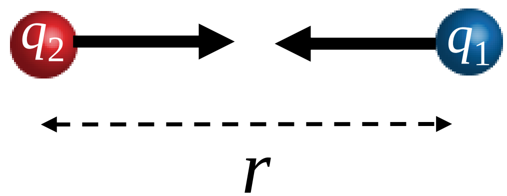
Today's Concepts:

A) Coulomb's Law

B) Superposition

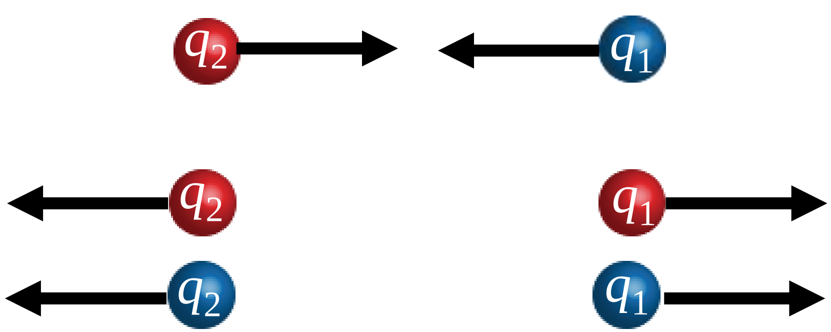
Coulomb's Law:

The force on a charge due to another charge is proportional to the product of the charges and inversely proportional to the separation squared.



$$F \propto \frac{q_1 q_2}{r^2}$$

The force is always parallel to a line connecting the charges, but the direction depends on the signs of the charges:

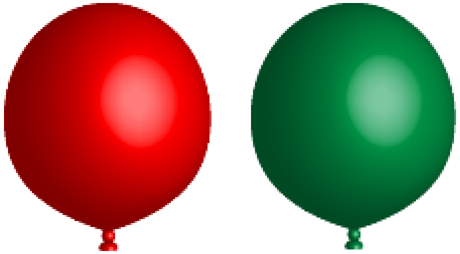


Opposite signs attract

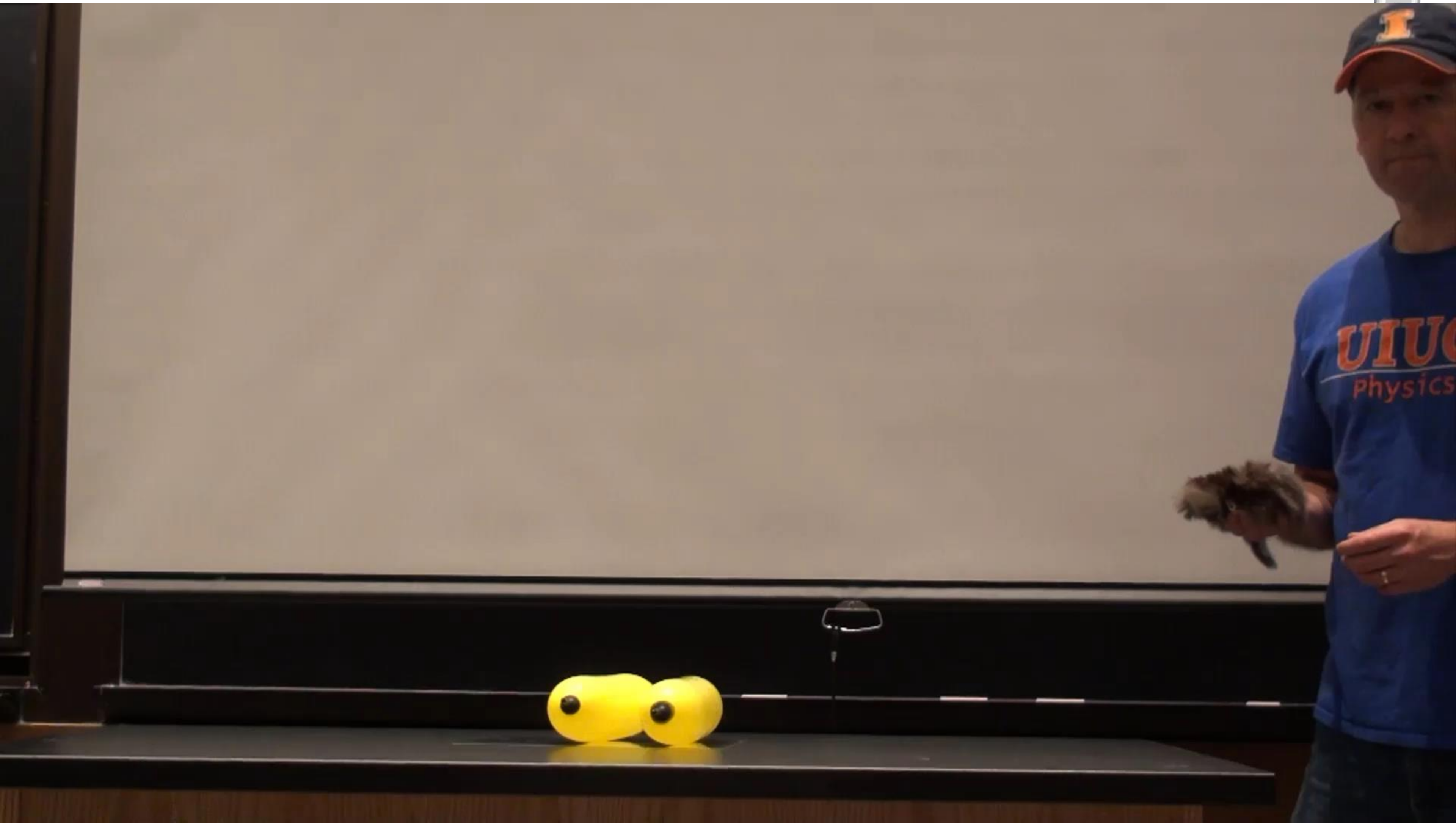
Like signs repel

Balloons

Take two balloons and rub them
both with a piece of cloth.
After you rub them they will:



A) Attract each-other
B) Repel each-other
C) Either – it depends on the material of the cloth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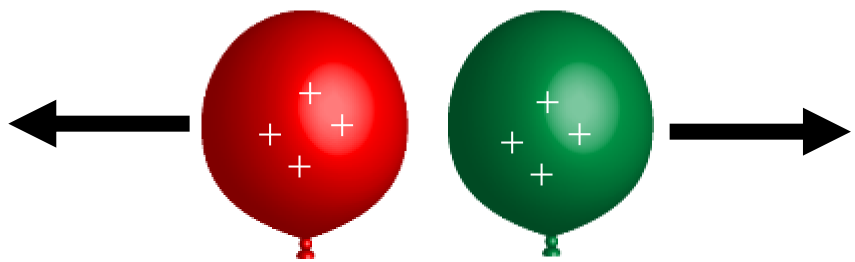
Physics

Balloons

If the same thing is done to both balloons they will acquire the same sign charge.

They will repel!



Coulomb's Law

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$

Our notation:

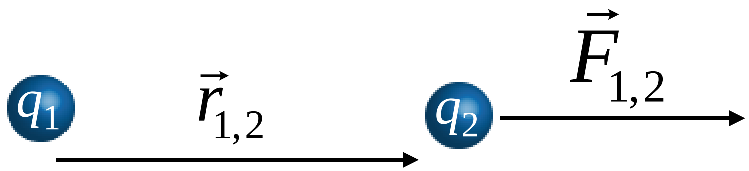
$\vec{F}_{1,2}$ is the force by 1 *on* 2 (think "*by-on* ")

$\hat{r}_{12}$ is the unit vector that points *from* 1 *to* 2.

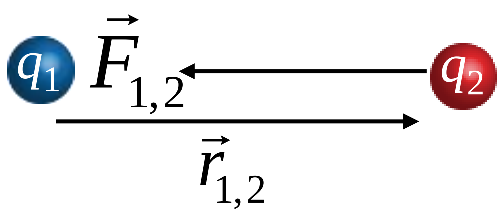
what is 'k'?

Examples:

If the charges have the same sign, the force **by** charge 1 on charge 2 would be in the direction of r_{12} (to the right).



If the charges have opposite sign, the force **by** charge 1 on charge 2 would be opposite the direction of r_{12} (left).



Example: Coulomb Force

Two iron paperclips are separated by 3 meters. Then you remove 1 electron from each atom on the first paperclip and place it on the second one.

$$\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$k = 9 \times 10^9$ N m^2 / C^2

electron charge $= 1.6 \times 10^{-19}$ Coulombs

$N_A = 6.02 \times 10^{23}$

Iron molar mass $\sim$ 56 grams/mol

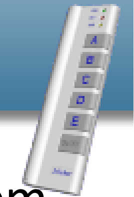
mass of paper clip $\sim$ 1 grams

What will the direction of the force be?

A) Attractive

B) Repulsive

Example: Coulomb Force



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electron charge = 1.6×10^{-19} Coulombs

$N_A = 6.02 \times 10^{23}$

Iron molar mass ~ 56 grams/mol

mass of paper clip ~ 1 grams

Which weight is closest to the approximate force between those paperclips (recall that weight = mg, g = 9.8 m/s^2)?

A) Paperclip (1 grams x g)

B) Text book (1 kg x g)

C) Truck (10^4 kg x g)

D) Aircraft carrier (10^8 kg x g)

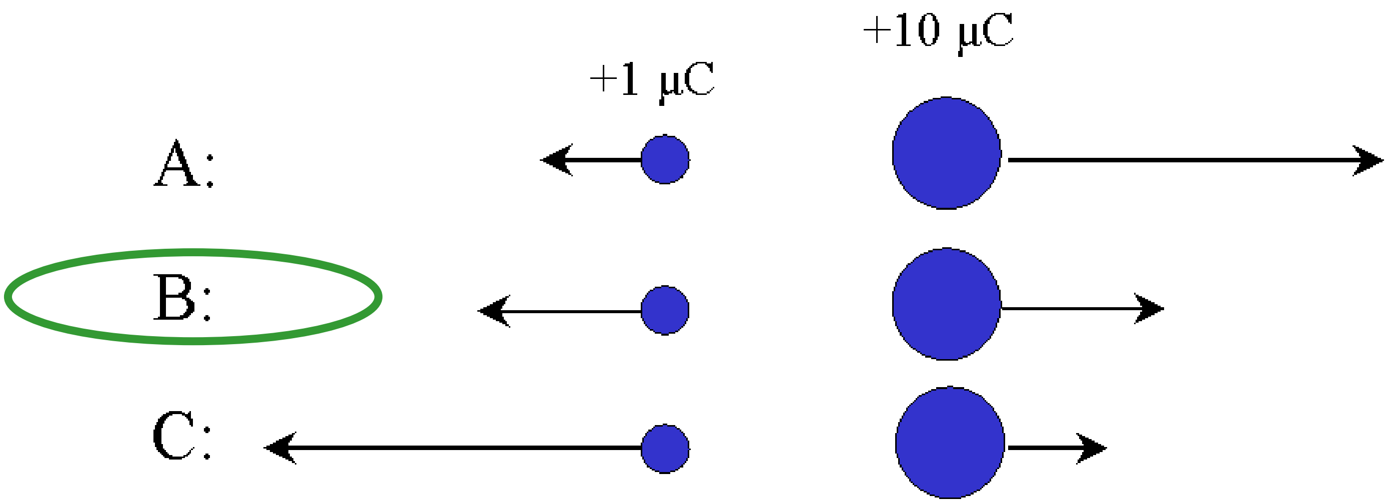
E) Mt. Everest (10^{14} kg x g)

F = 9e9 x (1.6e-19 * 1e22)^2 / 10 = 2 e 15 Newtons

Physics 212 >>> Physics 211

Check Point 1

1) Two charges $q = +1\ \mu C$ and $Q = +10\ \mu C$ are placed near each other as shown in the figure. Which of the following diagrams depicts the forces acting on the charges:



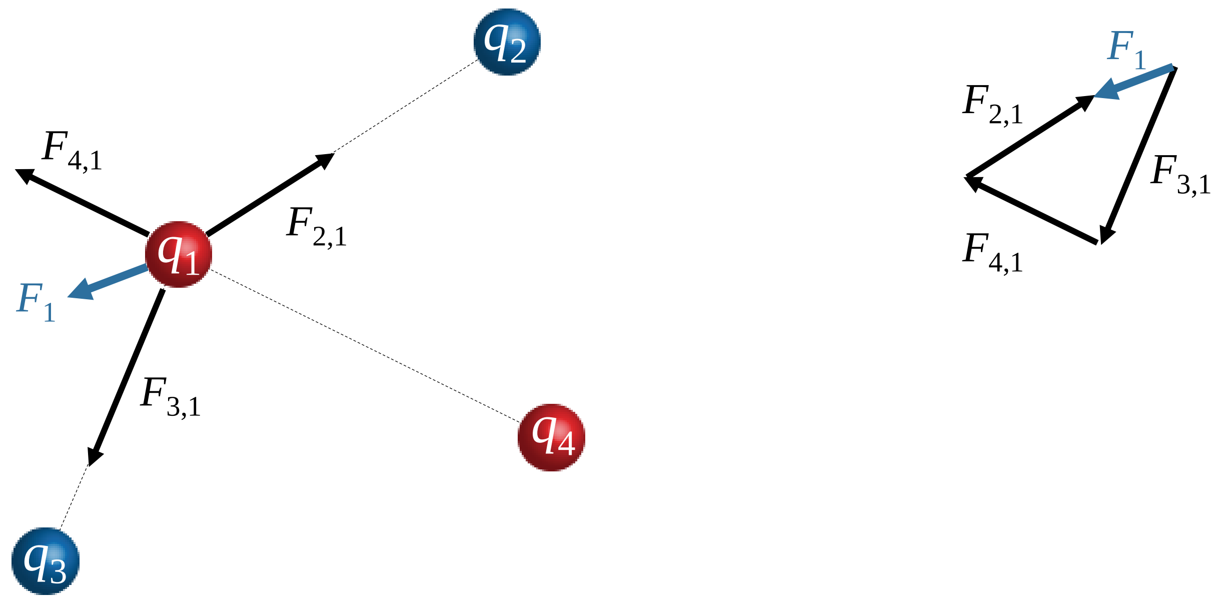
A) bigger charge experiences a bigger force

B) Newton's 3rd law says the forces must be equal and opposite.

C) Bigger charge creates a bigger force

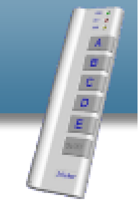
Superposition:

If there are more than two charges present, the total force on any given charge is just the vector sum of the forces due to each of the other charges:



$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1} + \ldots$$

Check Point 2



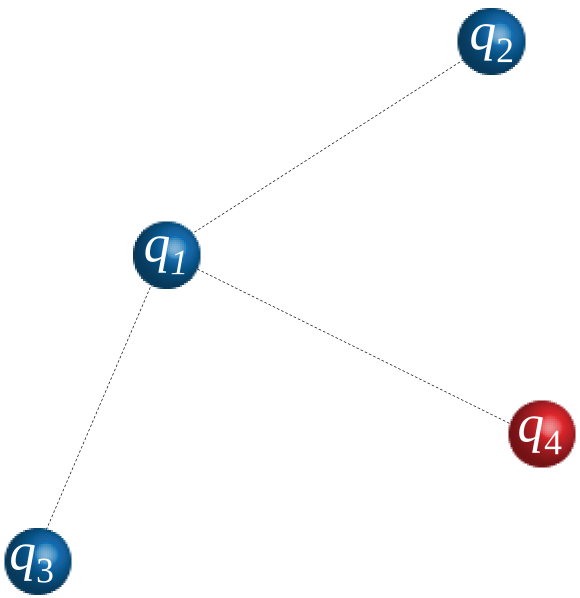
What happens to the magnitude of the Force on q_1 if its sign is changed from negative to positive?

A) $|F_1|$ increases

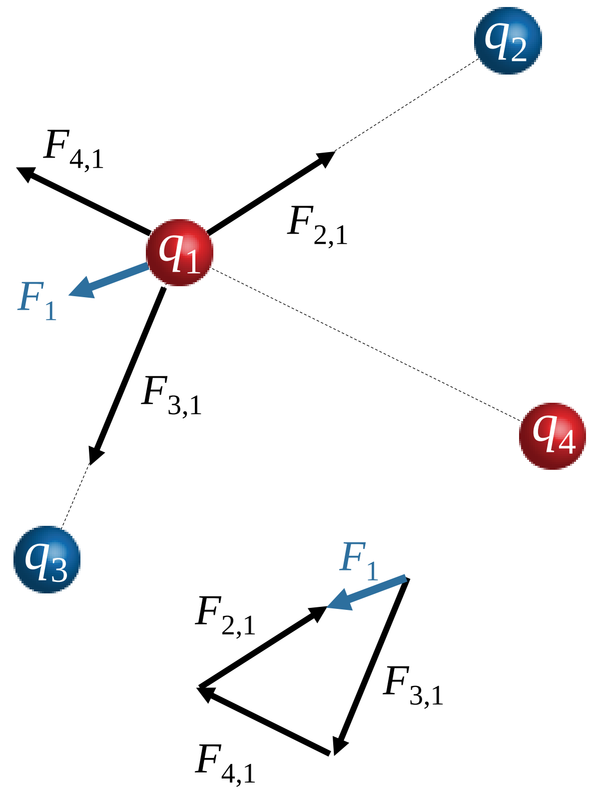
B) $|F_1|$ remains the same

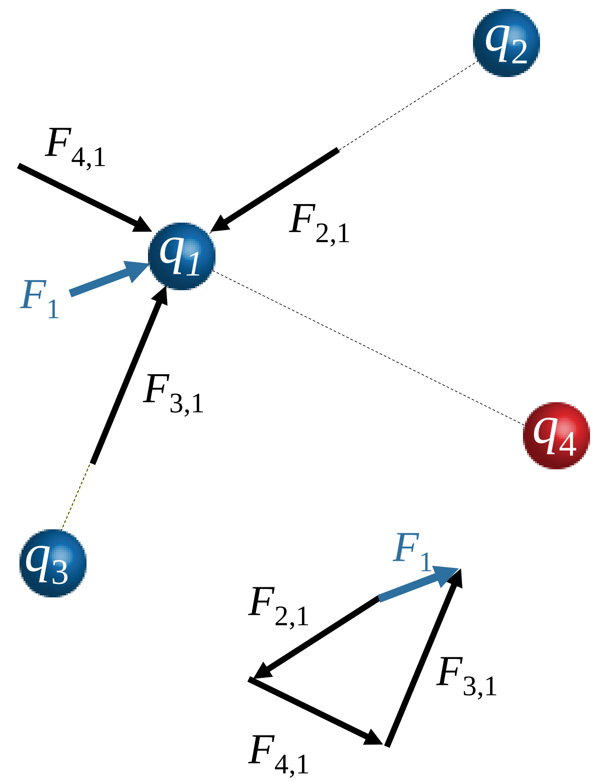
C) $|F_1|$ decreases

D) Need more information to determine



The direction of all forces changes by 180° – the magnitudes stay the same:



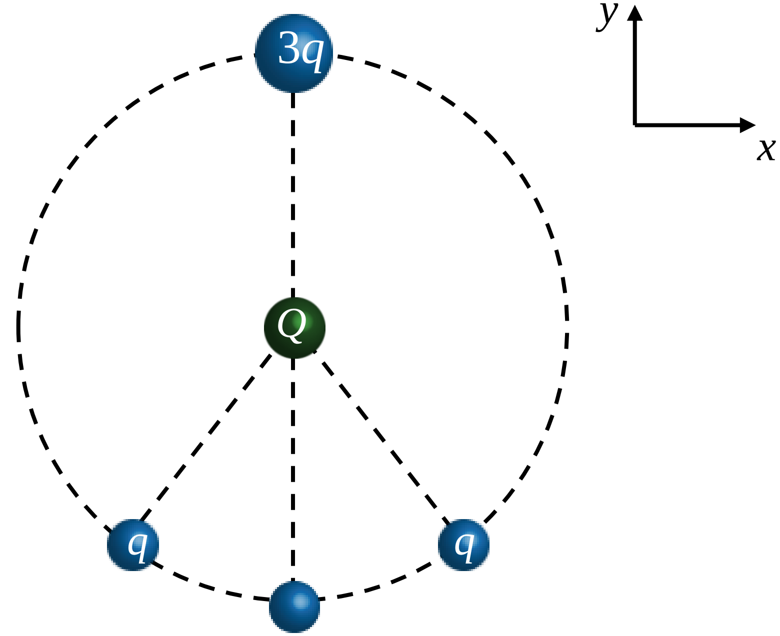


$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1} + \ldots$$

$$-\vec{F}_1 = -\vec{F}_{2,1} - \vec{F}_{3,1} - \vec{F}_{4,1} - \ldots$$

Check Point 3

Four positively charged particles are placed on a circular ring with radius 3 m as shown below. A particle with positive charge Q is placed in the center of the ring



What is vertical force on Q?

A) $F_y > 0$ B) $F_y = 0$ C) $F_y < 0$

A) Part of the charges of the 3 q objects go towards a force in the x direction so the total force of the 3 in the y direction will be less than 3q leaving the net force of Q to point in the direction of the 3q object which is positive.
B) The magnitude of the charge on top is equal to the 3 charges on bottom so they cancel each other out.
C) The 3q charge exerts a downward force on the Q charge. The bottom three q charges exert a upward force on the Q charge. Because those bottom two q charges are exerting a diagonal force, the force in the y-direction is less than the actual force exerted. The magnitude of the downward force would be greater than the sum of the upward forces.

Takeaways!

Course logistics

Should be able to apply Coulomb's law

- Determine direction and magnitude of electric force

Should be able to apply superposition of forces

- Determine direction and magnetic of net electric force

Thursday: electric fields!

See you Thursday!

Discussion Sections meet this week!

Be sure to complete prelecture 1 and checkpoint 1.

Start on Homework 1 now!

Electricity & Magnetism Lecture 2

Today's Concepts:

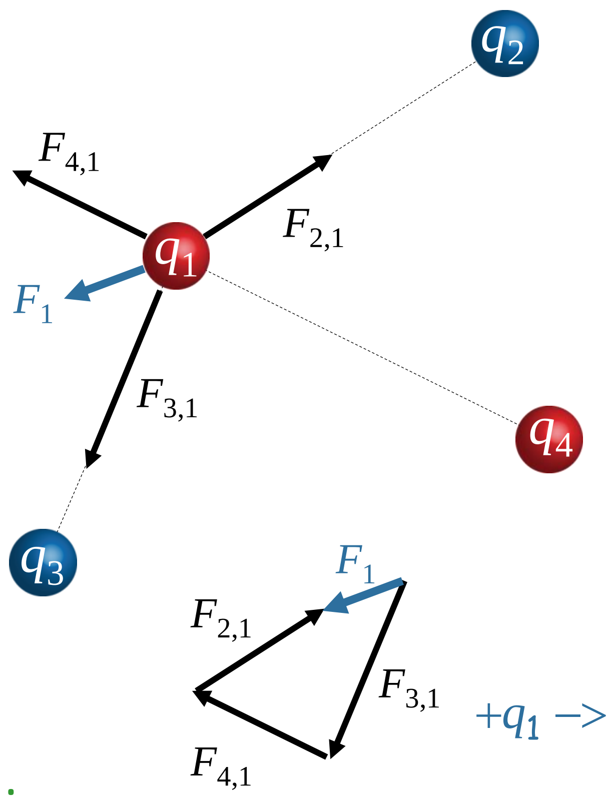
A) The Electric Field

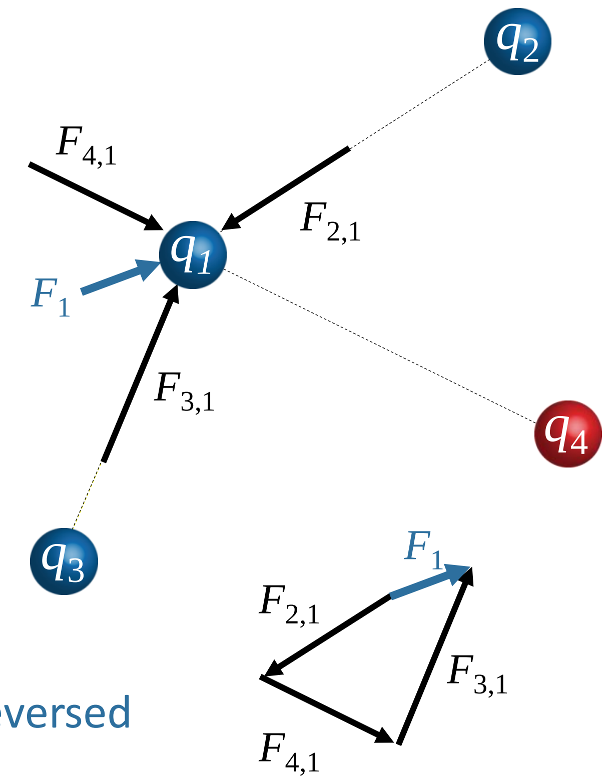
B) Continuous Charge Distributions

Reading: Ch. 5.4-5.5

Coulomb's Law & superposition!

If there are more than two charges present, the total force on any given charge is just the vector sum of the forces due to each of the other charges:





$+q_1 \rightarrow -q_1$ -> direction reversed

MATH:

$$\vec{F}_1 = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_1q_3}{r_{13}^2}\hat{r}_{13} + \frac{kq_1q_4}{r_{14}^2}\hat{r}_{14} \longrightarrow \vec{E} = \frac{\vec{F}_1}{q_1} = \frac{kq_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_3}{r_{13}^2}\hat{r}_{13} + \frac{kq_4}{r_{14}^2}\hat{r}_{14}$$

Electric Field

The electric field E at a point in space is simply the force per unit charge at that point.

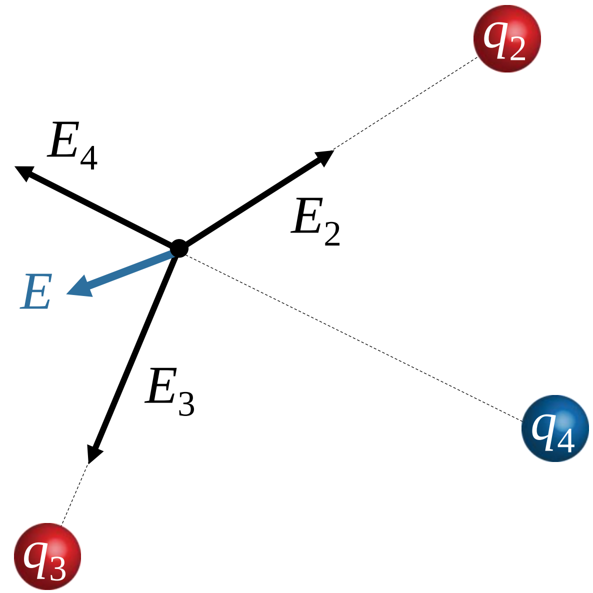
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

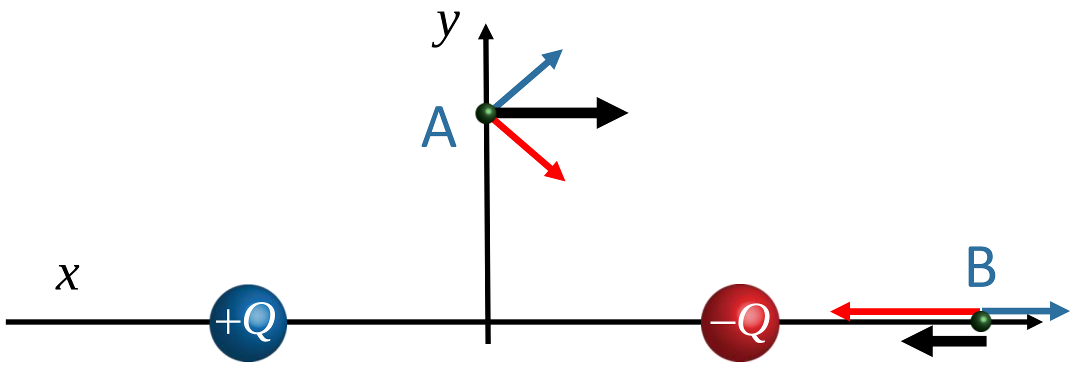
Superposition

$$\vec{E} = \sum_i k\frac{Q_i}{r_i^2}\hat{r}_i$$



Field points away from positive charges.
Field points toward negative charges.

Check Point 1



Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is direction at point A

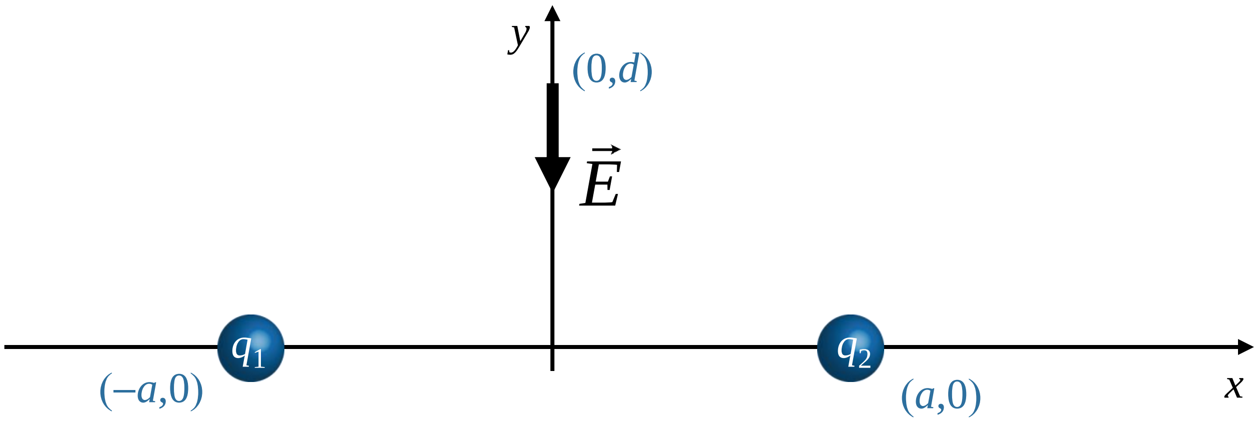
a) Up b) down c) Left d) Right e) zero

What is direction at point B

a) Up b) down c) Left d) Right e) zero

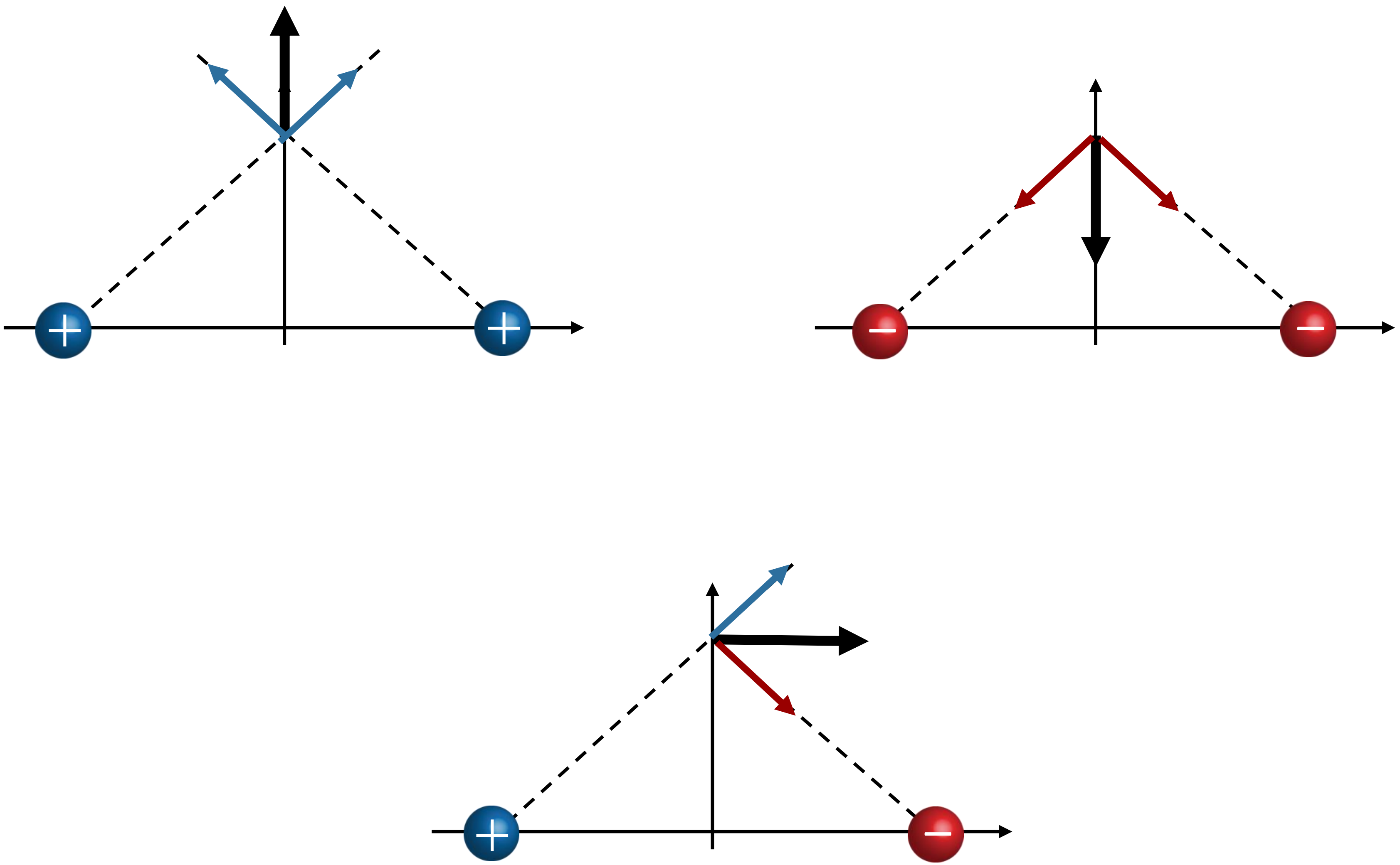
Two Charges

Two charges q_1 and q_2 are fixed at points $(-a,0)$ and $(a,0)$ as shown. Together they produce an electric field at point $(0,d)$ which is directed along the negative *y*-axis.

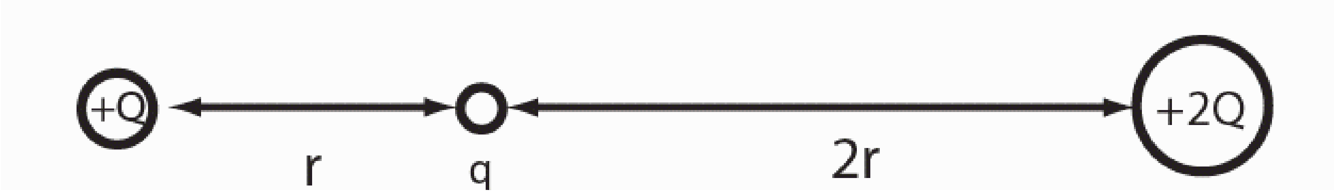


Which of the following statements is true:

A) Both charges are negative
B) Both charges are positive
C) The charges are opposite
D) There is not enough information to tell how the charges are related



Check Point 2



A positive test charge q is released from rest at distance r away from a charge of +Q and a distance 2r away from a charge of +2Q.
How will the charge q accelerate immediately after it is released?

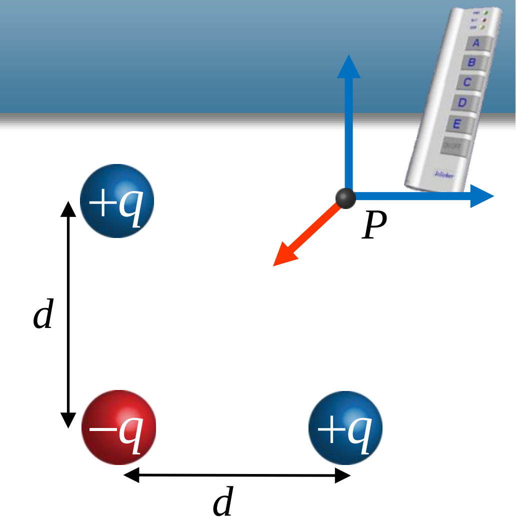
Left	Right	Still	Other
A	B	C	D

(A Left) According to coulomb's law, distance is inverse squared while the charge is linear so the force enacted by the 2Q charge is less than the one from Q charge.

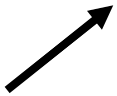
(B Right) The electric field of +Q is greater than that of +2Q as the electric field is related to the inverse SQUARE of distance and only directly related to charge.

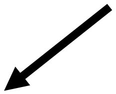
(C Still) I would say it will stay still since it is closer to the q charge but then the 2q+ charge will have about the same force but opposite direction.

Example



What is the direction of the electric field at point P, the unoccupied corner of the square?

A) 

B) 

C) $E = 0$

D) Need to know d

E) Need to know d & q

Calculate E at point P.

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

$$E_x = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2} \cos\frac{\pi}{4} \right)$$

$$E_y = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2} \sin\frac{\pi}{4} \right)$$

Charge Density

Linear $(\lambda = Q/L)$ Coulombs/meter

Surface $(\sigma = Q/A)$ Coulombs/meter^2

Volume $(\rho = Q/V)$ Coulombs/meter^3

Some Geometry

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \tfrac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

What has more net charge?.

A) A sphere w/ radius 4 meters and volume charge density $\rho = 2$ C/m^3

B) A sphere w/ radius 4 meters and surface charge density $\sigma = 2$ C/m^2

C) Both A) and B) have the same net charge.

$$Q_A = \rho V = \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi 4^3 (2)$$

$$Q_B = \sigma A = 4\pi R^2 \sigma = 4\pi 4^2 (2) = \pi 4^3 (2)$$

Continuous Charge Distributions

Summation becomes an integral (be careful with vector nature)

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i \quad \longrightarrow \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

WHAT DOES THIS MEAN ?

Integrate over all charges (dq)

r is vector from dq to the point at which E is being calculated

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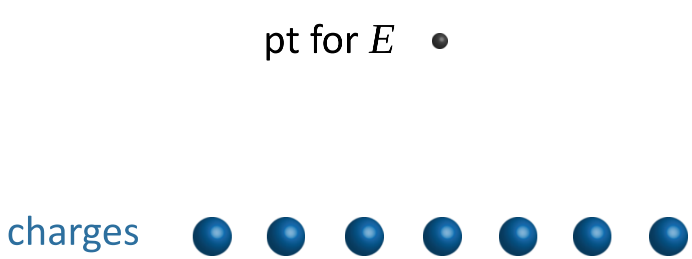
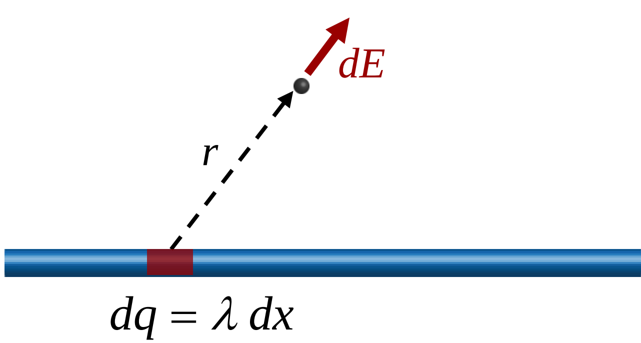
Linear Example:

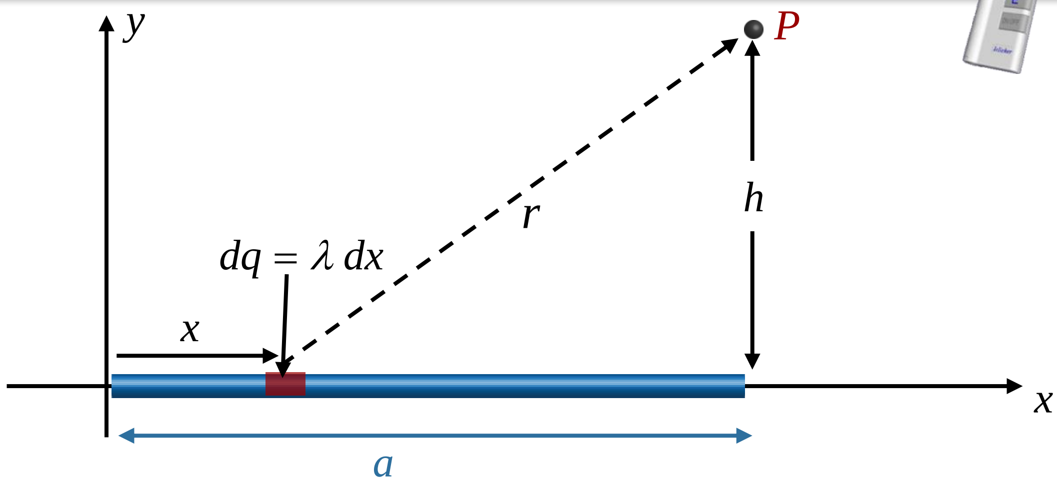
$$\lambda = Q/L$$





Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

What is $\dfrac{dq}{r^2}$?

A) $\dfrac{dx}{x^2}$

B) $\dfrac{dx}{a^2 + h^2}$

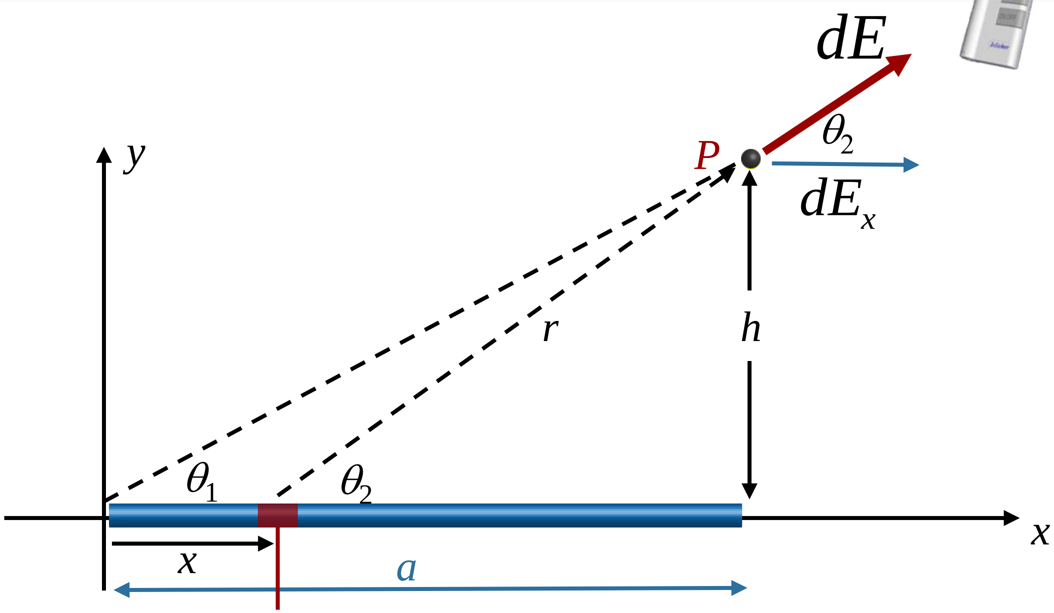
C) $\dfrac{\lambda dx}{a^2 + h^2}$

D) $\dfrac{\lambda dx}{(a - x)^2 + h^2}$

E) $\dfrac{\lambda dx}{x^2}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2}\hat{r} \qquad \frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

We want:

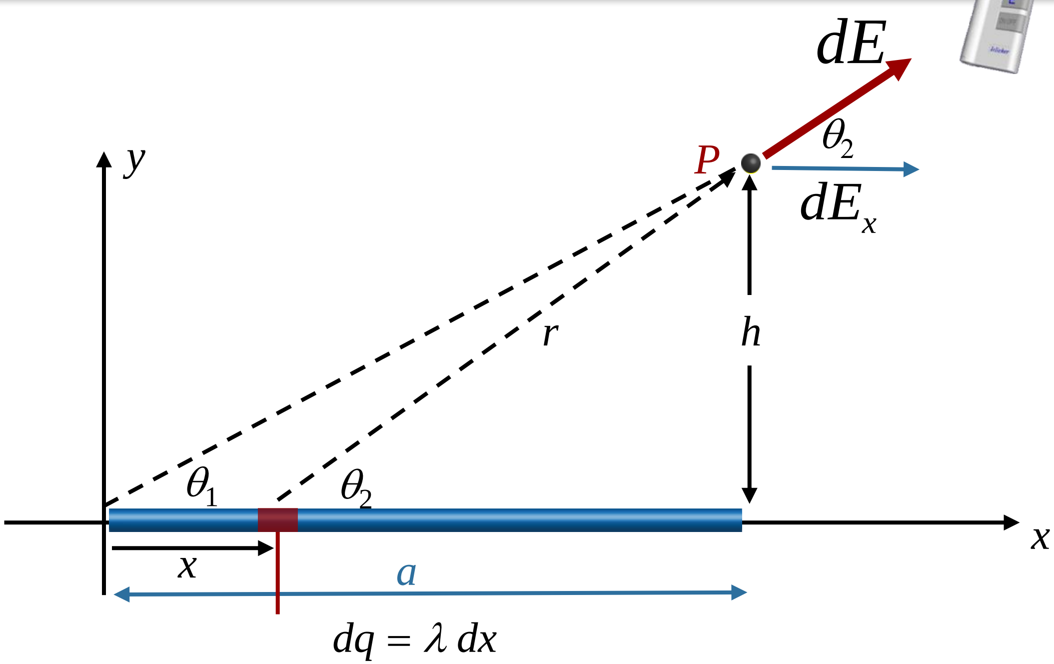
$$E_x = \int dE_x$$

What is correct expression for E_x ?

A) $\displaystyle\int \frac{\lambda k \cos\theta_1 dx}{(a-x)^2 + h^2}$ B) $\displaystyle\int \frac{\lambda k \cos\theta_2 dx}{(a-x)^2 + h^2}$ C) $\displaystyle\int \frac{\lambda k \sin\theta_1 dx}{(a-x)^2 + h^2}$ D) $\displaystyle\int \frac{\lambda k \sin\theta_2 dx}{(a-x)^2 + h^2}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2}\hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int \frac{\lambda k \cos\theta_2 dx}{(a-x)^2 + h^2}$$

What is E_x ?

A) $\displaystyle\int_0^a \frac{\lambda k \cos\theta_2 dx}{(a-x)^2 + h^2}$

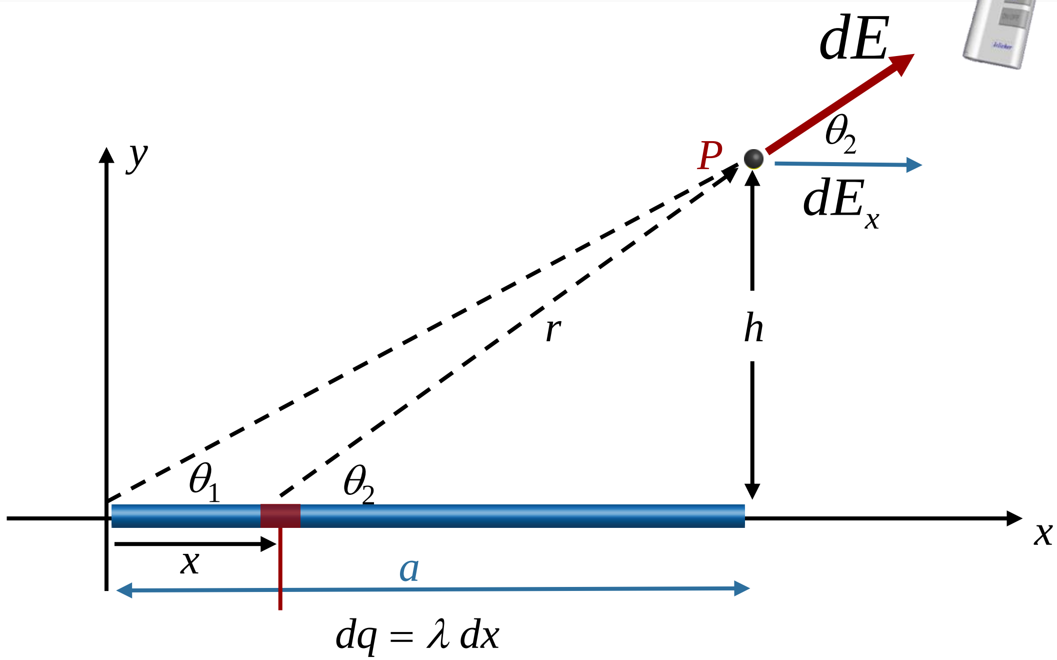
B) $\displaystyle\lambda k \cos\theta_2 \int_0^a \frac{dx}{h^2 + (x-a)^2}$

C) A and B are both OK

$\cos\theta_2$ DEPENDS ON x!

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

We know:

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE \cos\theta_2$$

What is $\cos\theta_2$?

A) $\dfrac{x}{\sqrt{a^2 + h^2}}$

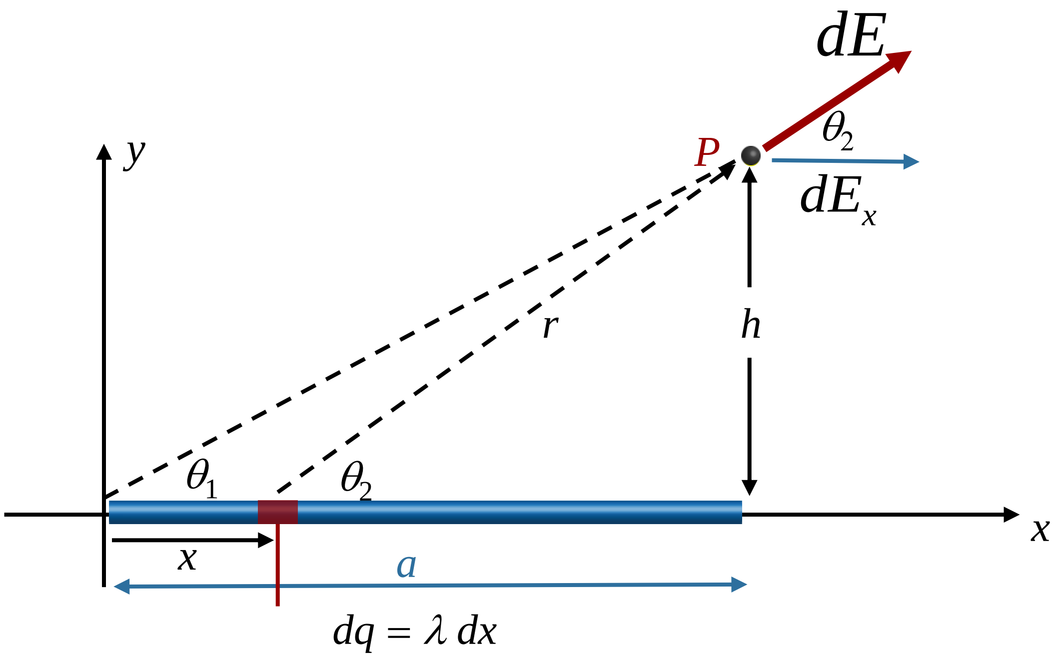
B) $\dfrac{a-x}{\sqrt{(a-x)^2 + h^2}}$

C) $\dfrac{a}{\sqrt{a^2 + h^2}}$

D) $\dfrac{a}{\sqrt{(a-x)^2 + h^2}}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know: $$\vec{E} = \int k \frac{dq}{r^2}\hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2} \qquad E_x = \int dE \cos\theta_2 \qquad \cos\theta_2 = \frac{a-x}{\sqrt{(a-x)^2 + h^2}}$$

Putting it all together

$$E_x(P) = \lambda k \int_0^a \frac{a-x}{\left((a-x)^2 + h^2\right)^{3/2}} dx \quad \longrightarrow \quad E_x(P) = \frac{\lambda k}{h}\left(1 - \frac{h}{\sqrt{h^2 + a^2}}\right)$$

Takeaways

Electric field

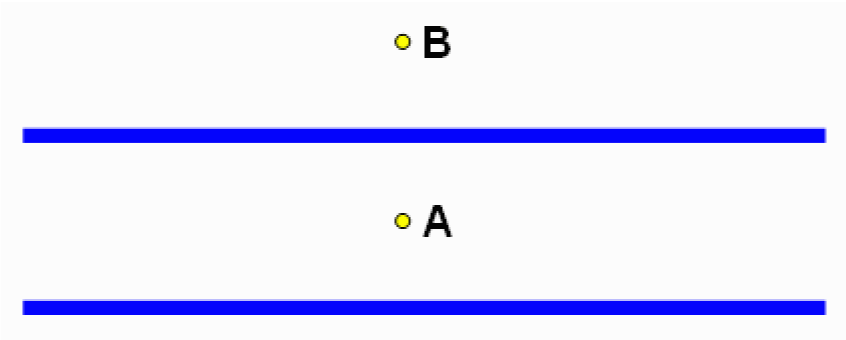
- Definition
- How to add electric fields due to multiple charges?
- Separating Ex and Ey

Concept of charge density

- Discrete --> continuous charge density
- Setting up integral for determining net electric field

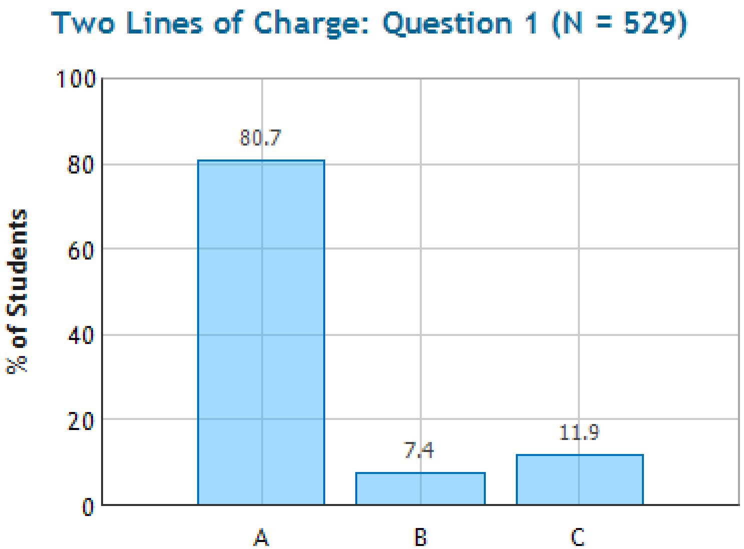
CheckPoint

Two infinite lines of charge are shown below.



Both lines have identical charge densities $+\lambda$ C/m. Point A is equidistant from both lines and Point B is located a above the top line as shown. How does E_A, the magnitude of the electric field at point A, compare to E_B, the magnitude of the electric field at point B?

- $E_A < E_B$
- $E_A = E_B$
- $E_A > E_B$



Electric Field at point A cancels out to be zero and electric field at point B experiences E field from both line to move upward.

Electricity & Magnetism
Lecture 3

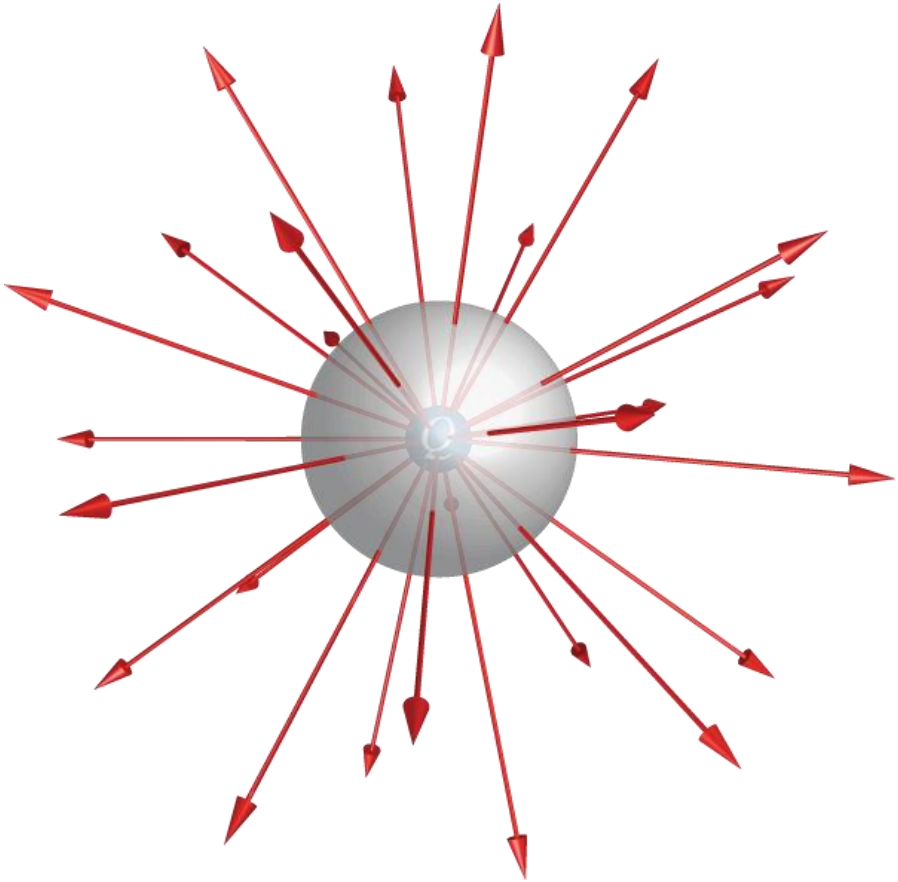
Today's Concepts:

A) Electric Flux

B) Field Lines

→ Gauss' Law

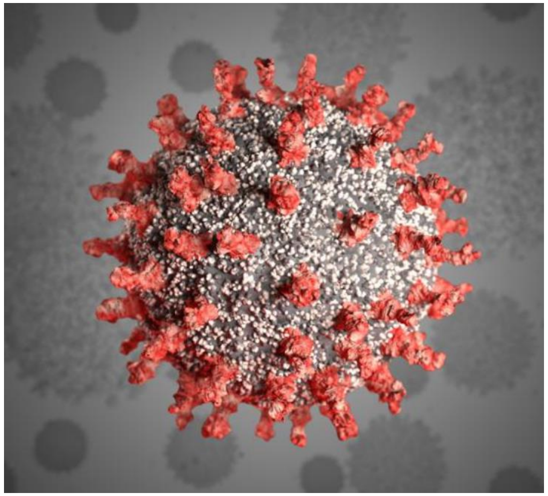
Electric Field Lines

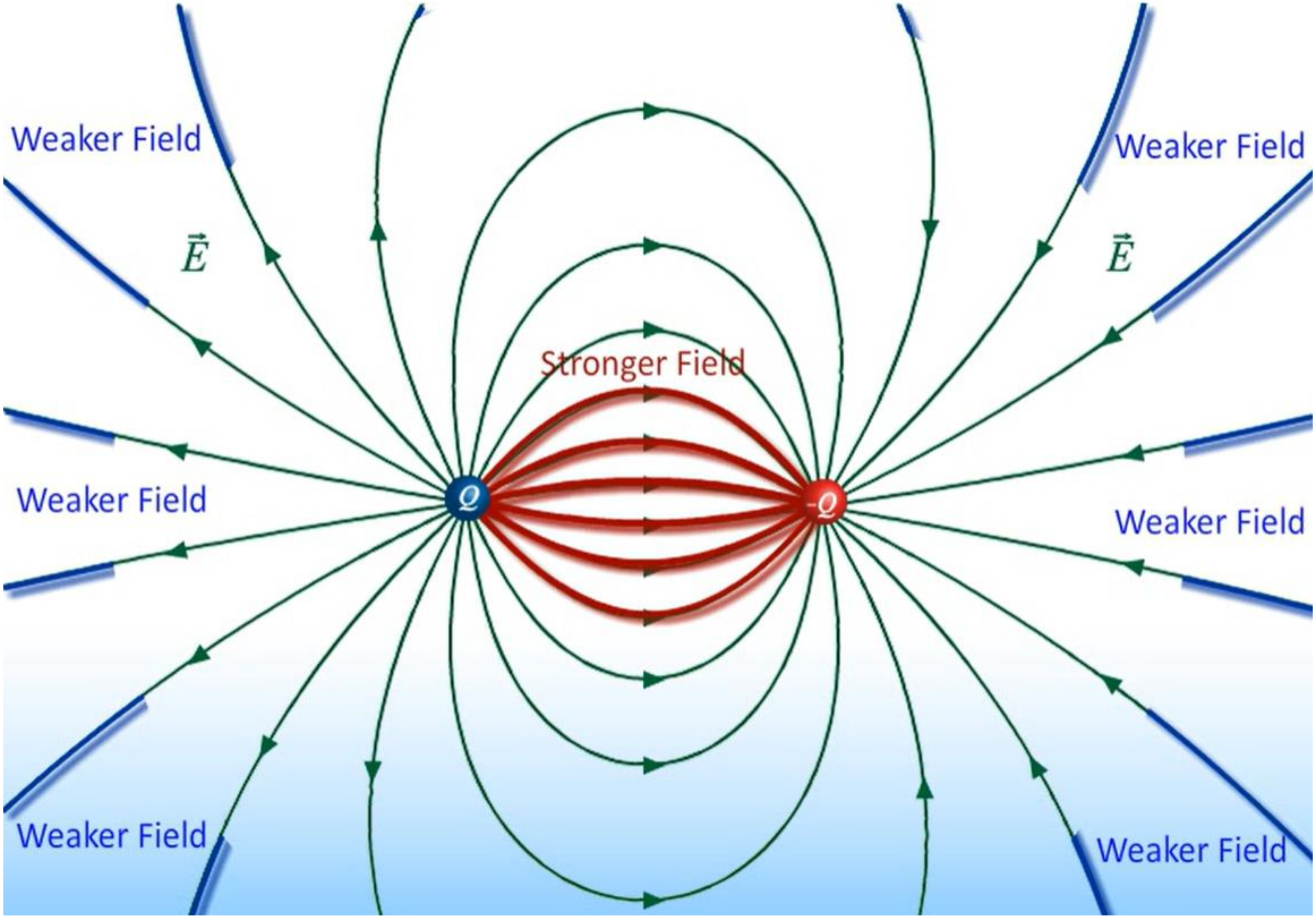


Direction & Density of Lines
represent
Direction & Magnitude of E

Point Charge:
Direction is radial
Density $\propto 1/R^2$

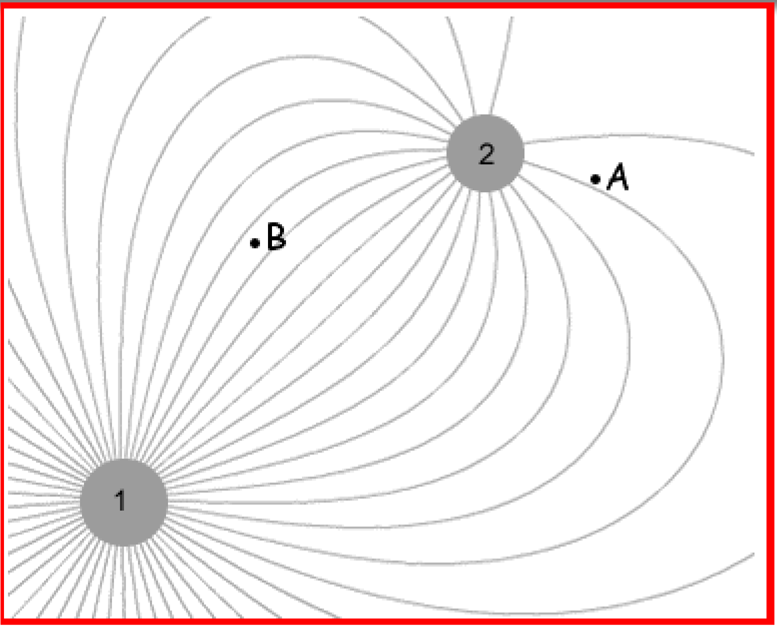
Why does this look familiar?





Weaker Field
Weaker Field
$\vec{E}$
$\vec{E}$
Stronger Field
Q
$-Q$
Weaker Field
Weaker Field
Weaker Field
Weaker Field

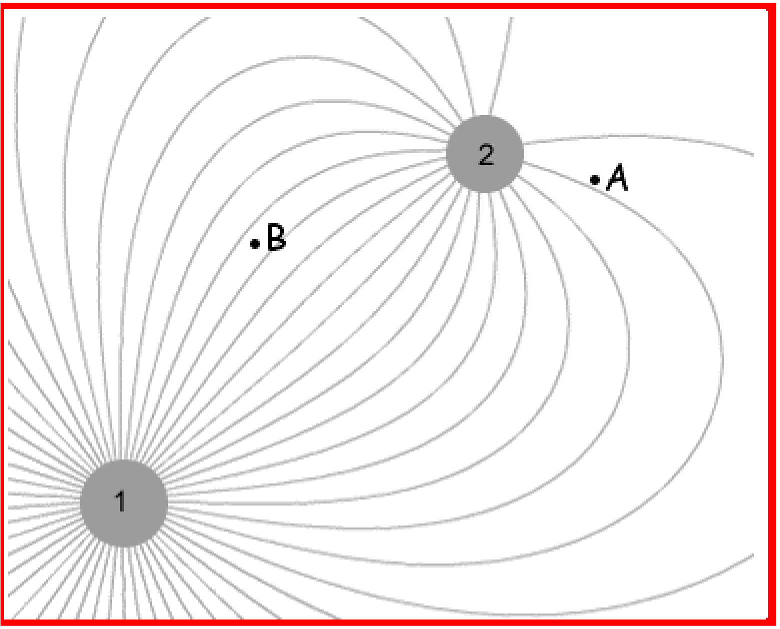
Check Point 1



A. Q_1 and Q_2 have the same sign
B. Q_1 and Q_2 have opposite signs
C. Not enough info

"They are connected by field lines and that can only happen if the charges are opposites."

Check Point 2

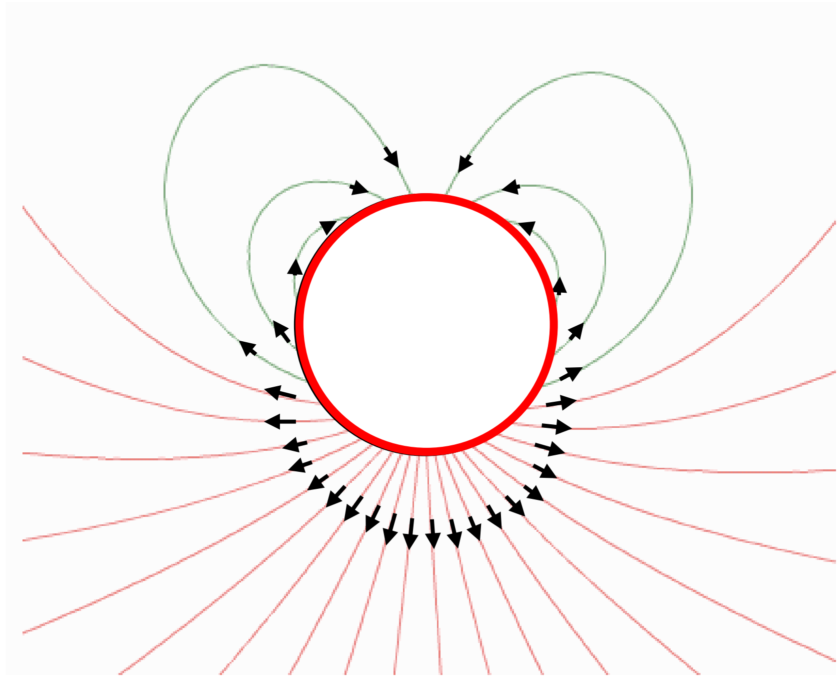


A. $|E_A| > |E_B|$
B. $|E_A| = |E_B|$
C. $|E_A| < |E_B|$
D. Not enough info

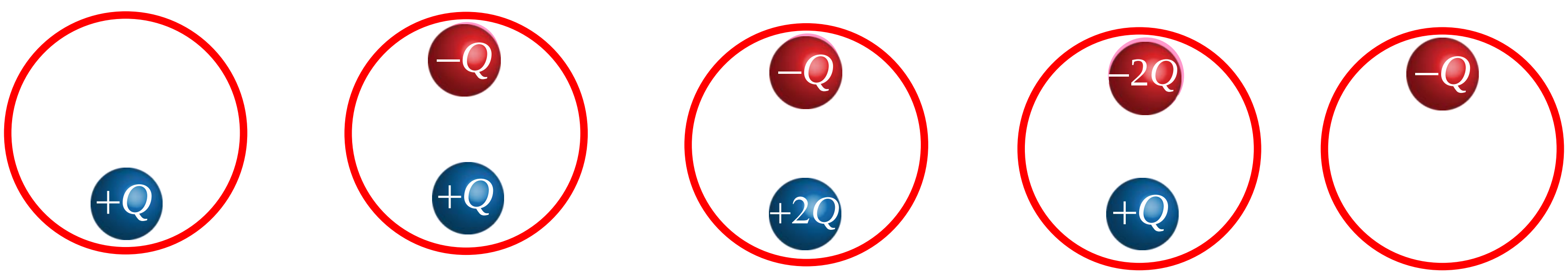
"The density of field lines is greater at point B which means the magnitude of the field is greater.."

Point Charges

Which configuration of charges inside the red circle match electric field pattern show?



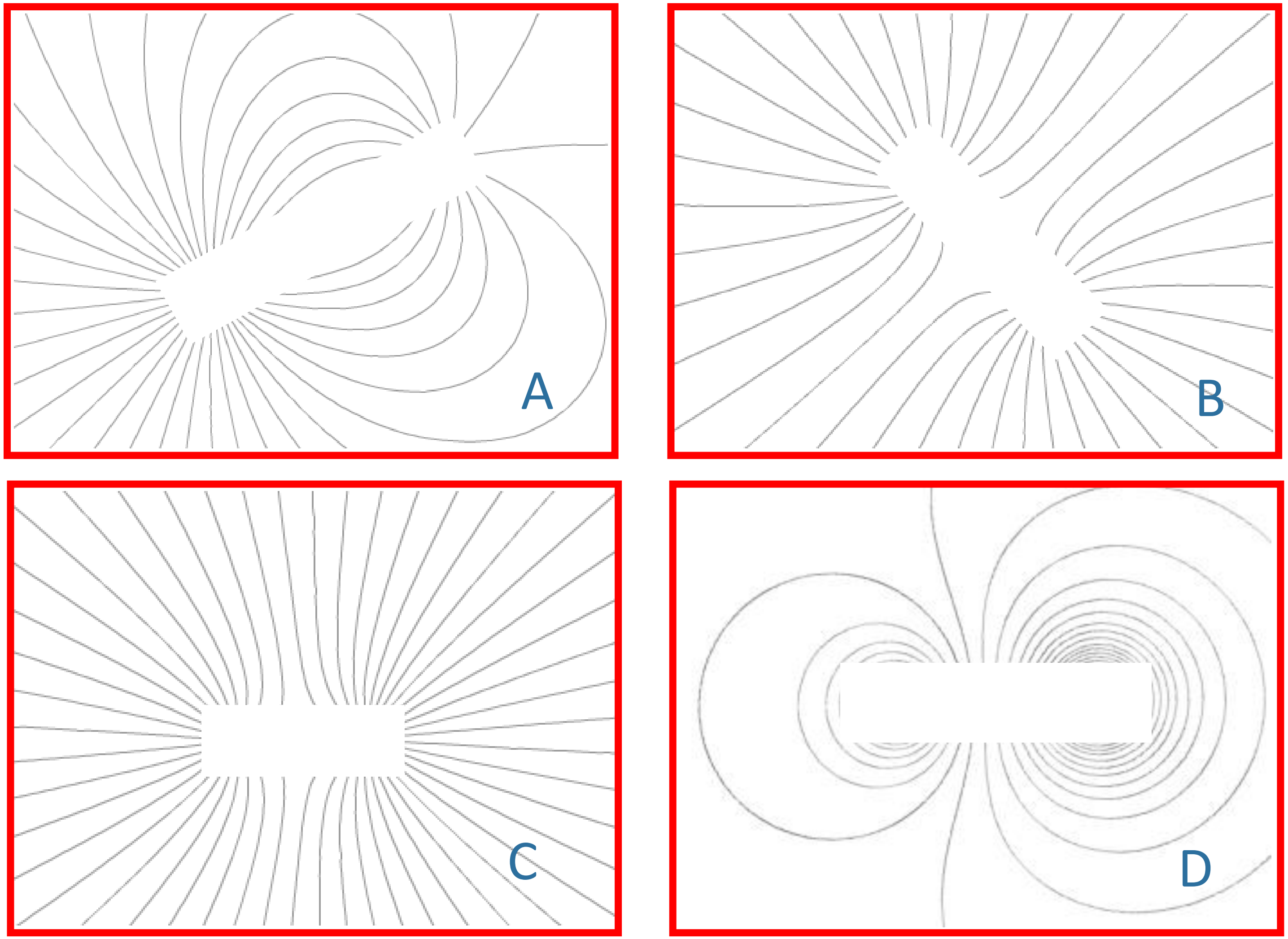
What charges are inside the red circle?



A B C D E

Electric Field lines

Which of the following field line pictures best represents the electric field from two charges that have the same sign but different magnitudes?



Electric Flux "Counts Field Lines"

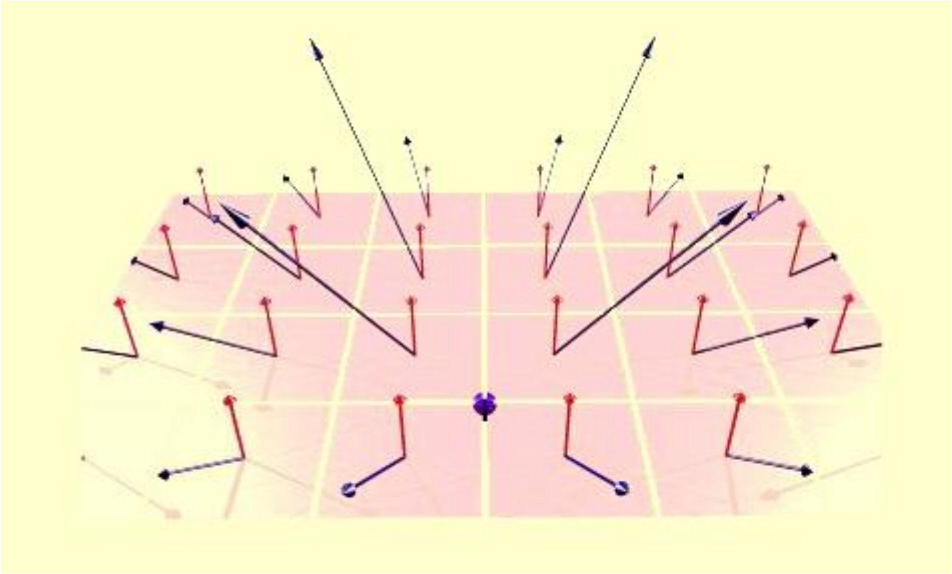
Can you give us a clear, simple definition of what flux is?

$$\Phi_S \equiv \underbrace{\int_S \vec{E} \cdot d\vec{A}}_{\text{Integral of } \vec{E} \cdot d\vec{A} \text{ on surface } S}$$

Flux through surface S

Integral of $\vec{E} \cdot d\vec{A}$ on surface S

Representing the area of a surface as a vector in order to take the dot product.



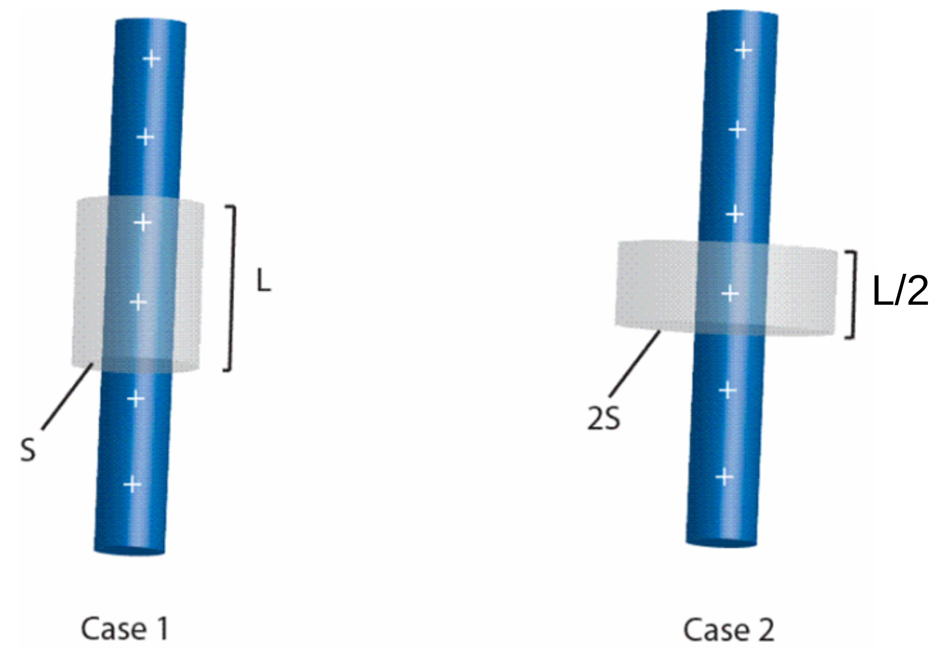
Check Point 3

A) The field lines travel horizontally through the rod, so more of them will pass through a taller cylinder.

B) There is twice the charge, but half of the surface area so they are equal

C) Twice the radius means 4x the Surface area of the base. 1/2 L turns this into 2x the flux in case 2.

An infinitely long charged rod has uniform charge density λ and passes through a cylinder (gray). The cylinder in Case 2 has twice the radius and half the length compared with the cylinder in Case 1.



$\Phi_1 = 2\Phi_2$	$\Phi_1 = \Phi_2$	$\Phi_1 = 1/2\Phi_2$	none
(A)	(B)	(C)	(D)

Check Point (Hard way shown in prelecture)

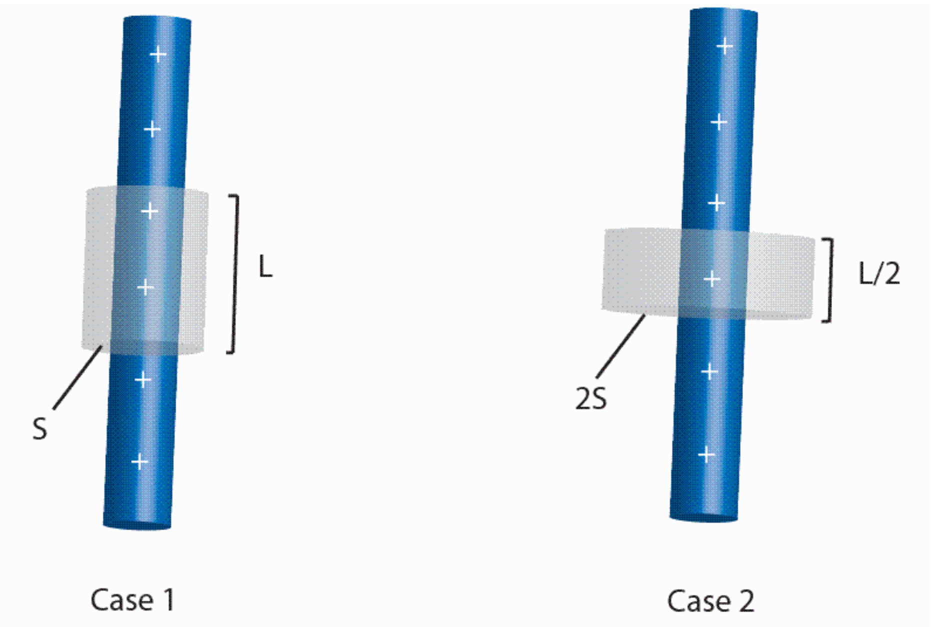
Definition of Flux:

$$\Phi_S \equiv \int_S \vec{E} \cdot d\vec{A}$$

E constant on barrel of cylinder
E perpendicular to barrel surface
(E parallel to dA)

$$\Phi_S = E \int_{barrel} d\vec{A}$$

$$= E A_{barrel}$$



$\Phi_1 = 2\Phi_2$	$\Phi_1 = \Phi_2$	$\Phi_1 = 1/2\Phi_2$	none
(A)	(B)	(C)	(D)

RESULT: GAUSS' LAW
Φ proportional to charge enclosed !

Case 1

$$A_{barrel} = 2\pi L s$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 s}$$

$$\boxed{\Phi_1 = \frac{\lambda L}{\varepsilon_0}}$$

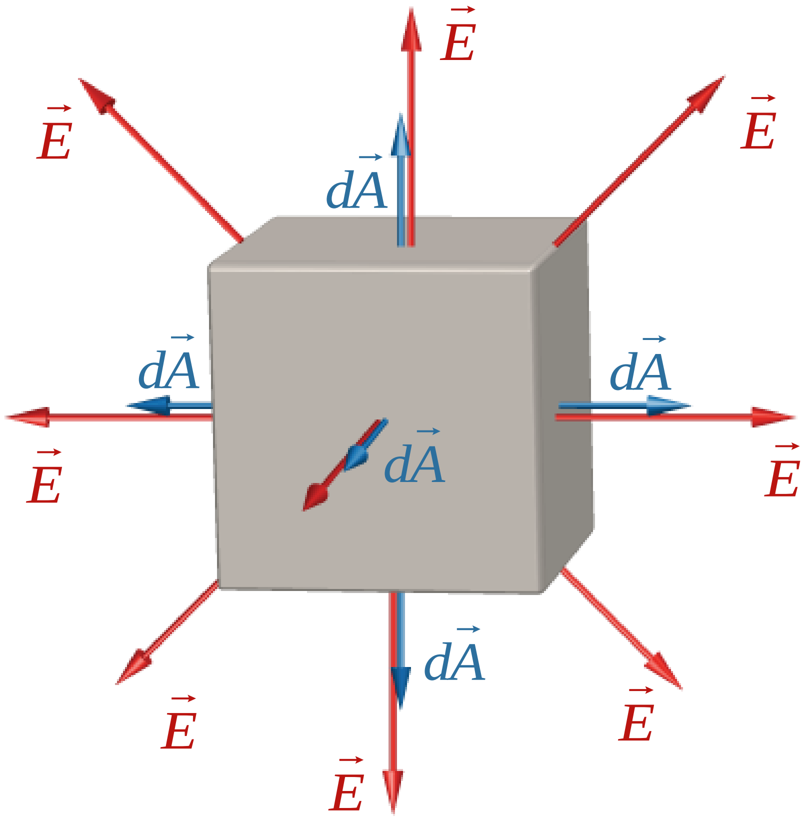
Case 2

$$A_2 = 2\pi(2s)\frac{L}{2}$$

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 2s}$$

$$\boxed{\Phi_2 = \frac{\lambda \frac{L}{2}}{\varepsilon_0}}$$

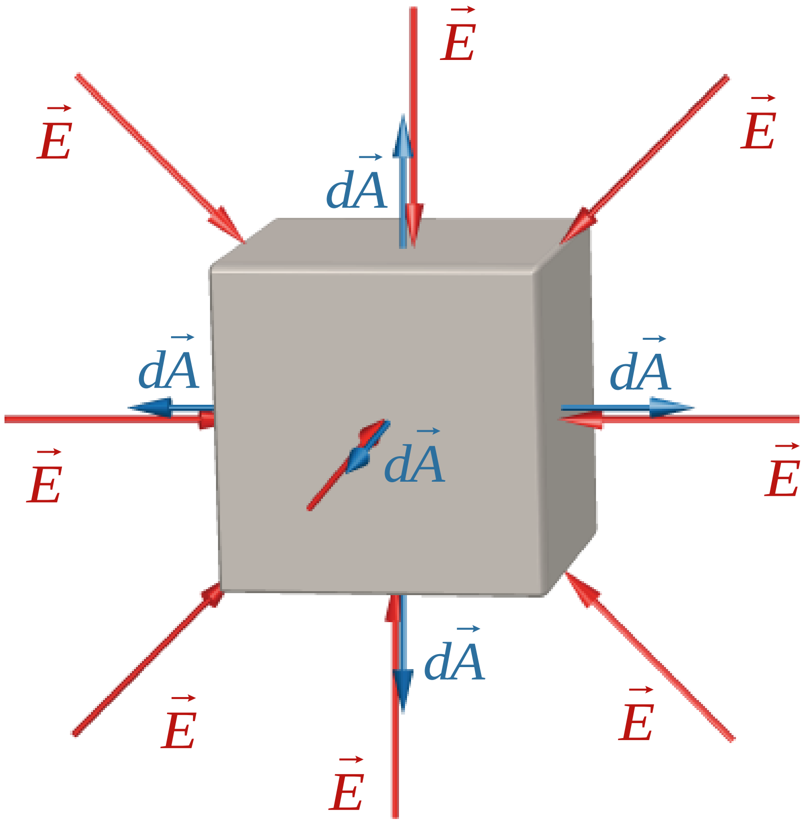
Direction Matters:



For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} > 0$$

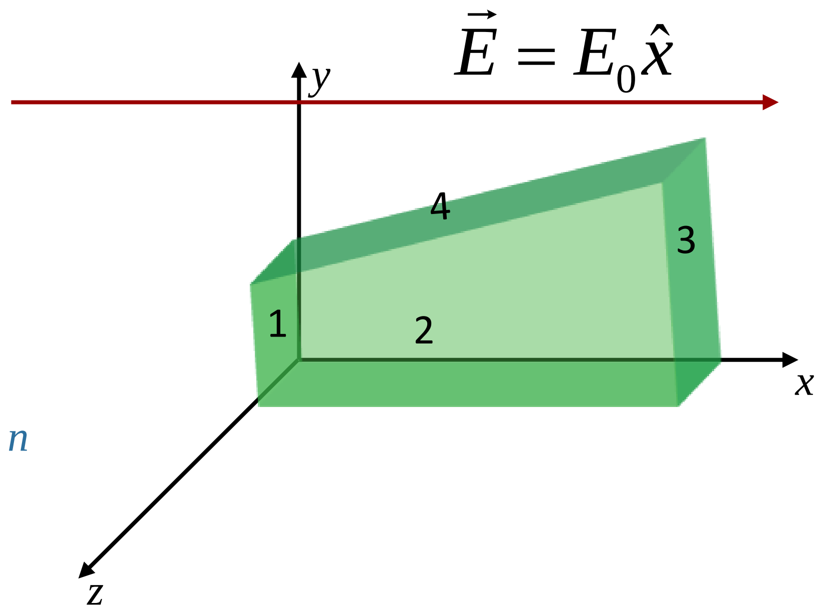
Direction Matters:



For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} < 0$$

Trapezoid in Constant Field



Label faces:

1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted

Define $\Phi_n =$ Flux through Face n

Q1

A) $\Phi_1 < 0$

B) $\Phi_1 = 0$

C) $\Phi_1 > 0$

Q2

A) $\Phi_2 < 0$

B) $\Phi_2 = 0$

C) $\Phi_2 > 0$

Q3

A) $\Phi_3 < 0$

B) $\Phi_3 = 0$

C) $\Phi_3 > 0$

Q4

A) $\Phi_4 < 0$

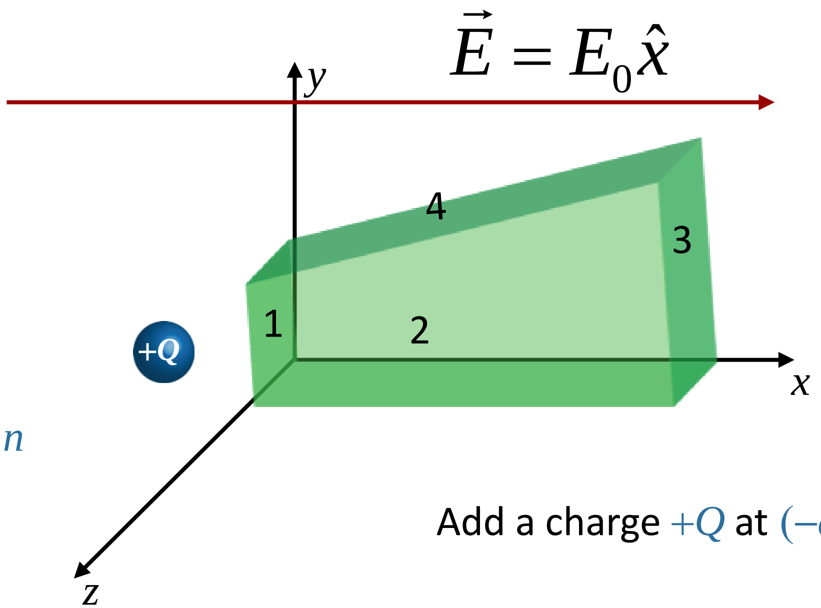
B) $\Phi_4 = 0$

C) $\Phi_4 > 0$

Trapezoid in Constant Field + Q

Label faces:

1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted



Define Φ_n = Flux through Face n
Φ = Flux through Trapezoid

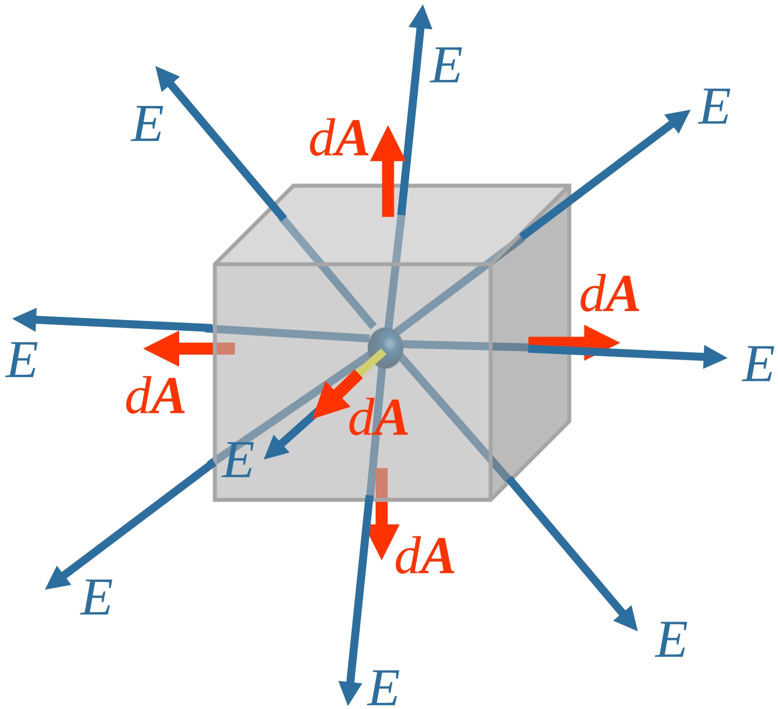
Add a charge $+Q$ at $(-a, a/2, a/2)$

How does Flux change?
Note (-6 < -4) sign matters

A) Φ_1 increases
B) Φ_1 decreases
C) Φ_1 remains same

A) Φ_3 increases
B) Φ_3 decreases
C) Φ_3 remains same

Gauss Law

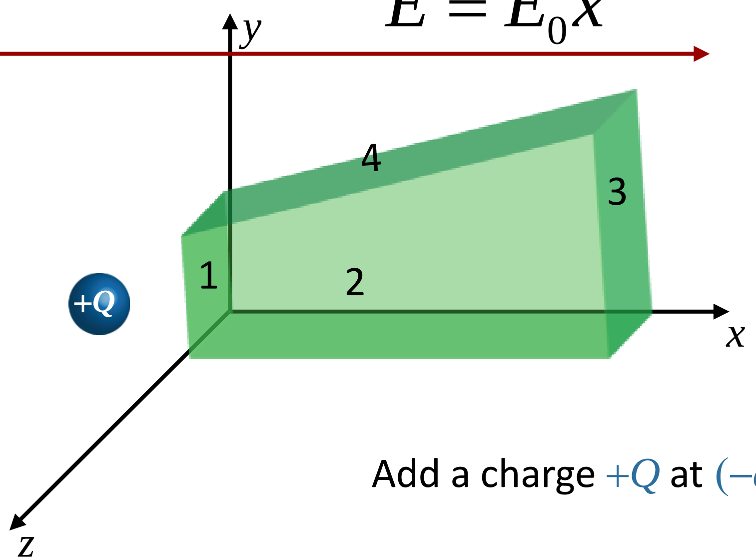


$$\int_{closed-surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Trapezoid in Constant Field + Q

$$\vec{E} = E_0 \hat{x}$$

Label faces:
1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted



Define Φ_n = Flux through Face n
Φ = Flux through Trapezoid

Add a charge $+Q$ at $(-a, a/2, a/2)$

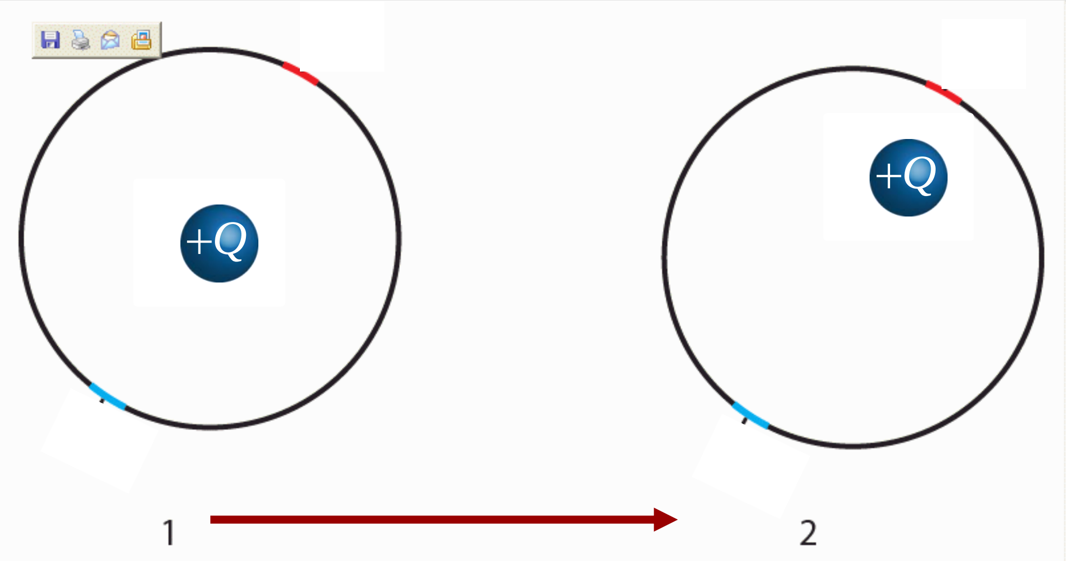
How does Flux change?
Note (-6 < -4) sign matters

A) Φ_1 increases
B) Φ_1 decreases
C) Φ_1 remains same

A) Φ_3 increases
B) Φ_3 decreases
C) Φ_3 remains same

A) Φ increases
B) Φ decreases
C) Φ remains same

Check Point 4



How does flux through entire surface change as charge is moved from center toward edge?

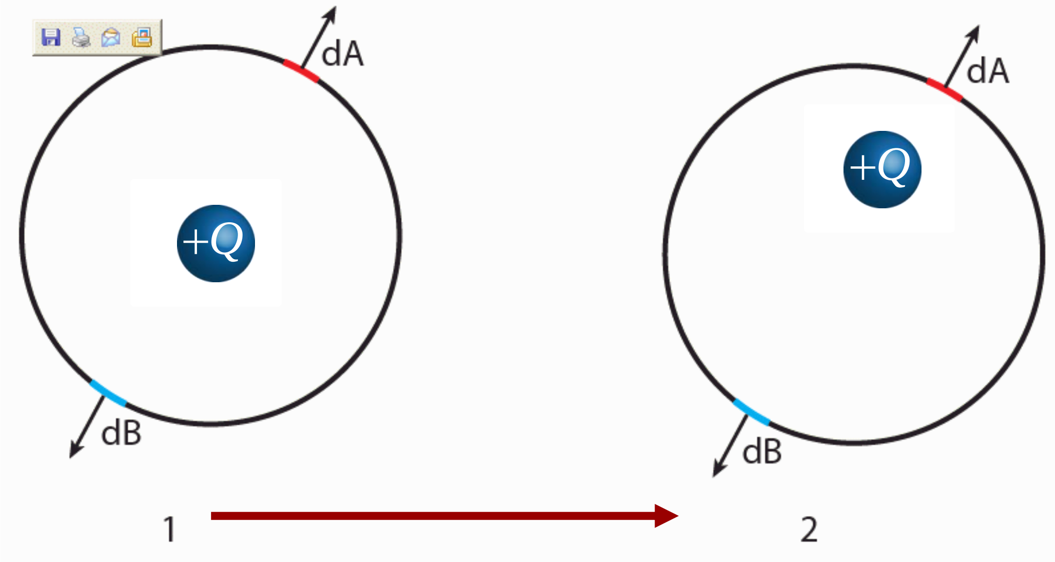
A
Φ_E increases

B
Φ_E decreases

C
Φ_E stays same

"The same number of lines exit the surface."

Check Point 5

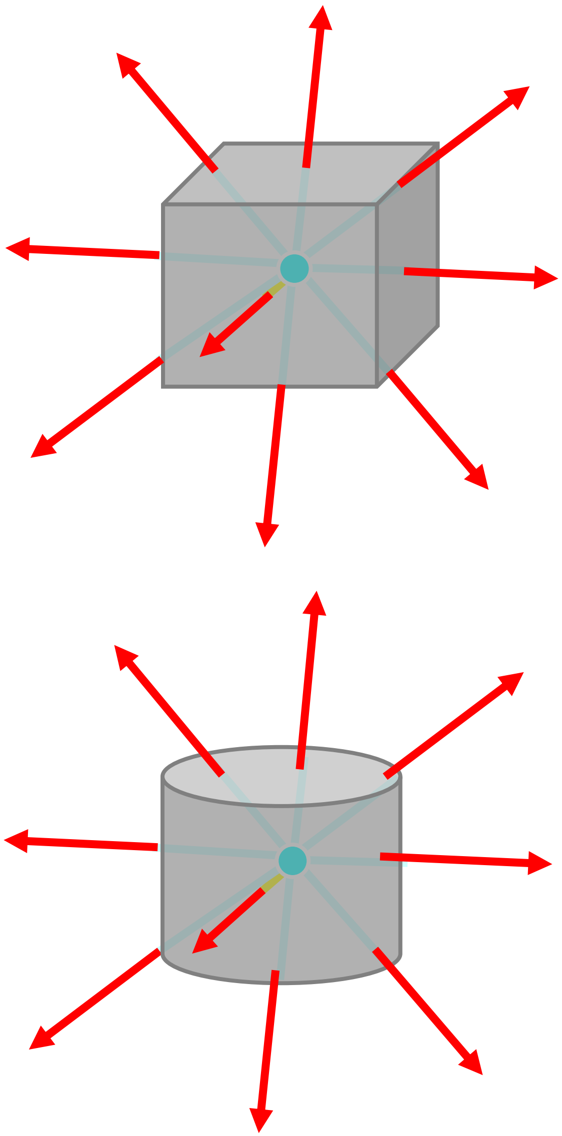


How does flux through red surface element dA change as charge is moved from center toward edge?

A	B	C
$d\Phi_A$ increases	$d\Phi_A$ decreases	$d\Phi_A$ stays same
$d\Phi_B$ decreases	$d\Phi_B$ increases	$d\Phi_B$ stays same

"Since the area is the same, as the charge moves closer, the electric fields becomes stronger. Meaning larger electric flux for the point. "

Things to notice about Gauss Law



$$\Phi_{closed-surface} = \int_{closed-surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

If $Q_{enclosed}$ is the same, the flux has to be the same, which means that the integral must yield the same result for any surface.

Things to notice about Gauss Law

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

In cases of high symmetry, it may be possible to bring E outside the integral. In these cases, we can solve Gauss Law for E

$$E \int d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

So - if we can figure out $Q_{enclosed}$ and the area of the surface A, then we know E!

This is the topic of the next lecture.

Takeaways

Electric field lines

- Direction and density
- Field distribution due to combination of charges

Concept of electric flux

- Definition, + or – ve
- Total flux through a closed surface

Gauss' law introduction

- Integral becomes simple for symmetrical distributions!

Electricity & Magnetism
Lecture 4

Today's Concepts:

A) Conductors

B) Using Gauss' Law

Gauss (not just a good idea, it's the law!)

"What exactly is Gauss's law used to find? It's confusing what exactly it's used to find or how it can be applied?"

$$\int_{closed-surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

ALWAYS TRUE!

Two uses

1) If know E everywhere on surface can calculate Q_{enc}

   (e.g. in metal E = 0)

2) In cases of high symmetry can pull E outside the integral and solve

$$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

Conductors and Insulators

Conductors = charges free to move

e.g. metals

Insulators = charges fixed

e.g. glass (air is insulator for this class)

Define: Conductors = Charges Free to Move

I didn't understand, why the electric field inside a conductor is zero and why the charge lies at the surface.

Claim: $E = 0$ inside any conductor at equilibrium

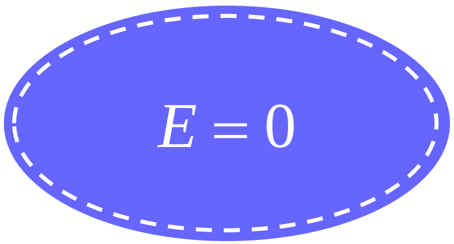
Charges in conductor move to make E field zero inside. (Induced charge distribution).

If $E \neq 0$, then charge feels force and moves!

Claim: Excess charge on conductor only on surface at equilibrium

Why?

- Apply Gauss' Law
  - Take Gaussian surface to be just inside conductor surface
  - $E = 0$ everywhere inside conductor $\longrightarrow$ $\oint \vec{E} \cdot d\vec{A} = 0$
  - Gauss' Law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{enc}}{\varepsilon_0}$ $\longrightarrow$ $\boxed{Q_{enc} = 0}$



Gauss' Law + Conductors + Induced Charges

Could we go over how when there is a placed charge within a hollow conducting sphere the electric field is still zero with in that sphere. Wouldn't Gauss' Law say that because we are containing a charge there would have to be an electric field?

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

ALWAYS TRUE!

If choose a Gaussian surface that is entirely in metal, then $E = 0$ so $Q_{enclosed}$ must also be zero!

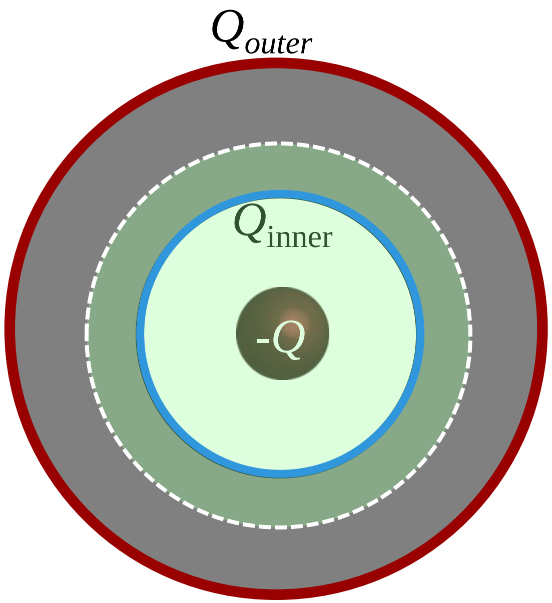
How Does This Work?

Charges in conductor move to surfaces to make $Q_{enclosed} = 0$.

We say charge is induced on the surfaces of conductors

Charge in Cavity of Conductor

A particle with charge $-Q$ is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?



A) inner = $-Q$, outer = $+Q$

B) inner = $-Q/2$, outer = $+Q/2$

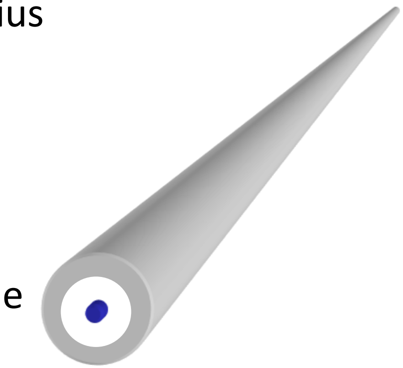
C) inner = 0, outer = 0

D) inner = $+Q/2$, outer = $-Q/2$

E) inner = $+Q$, outer = $-Q$

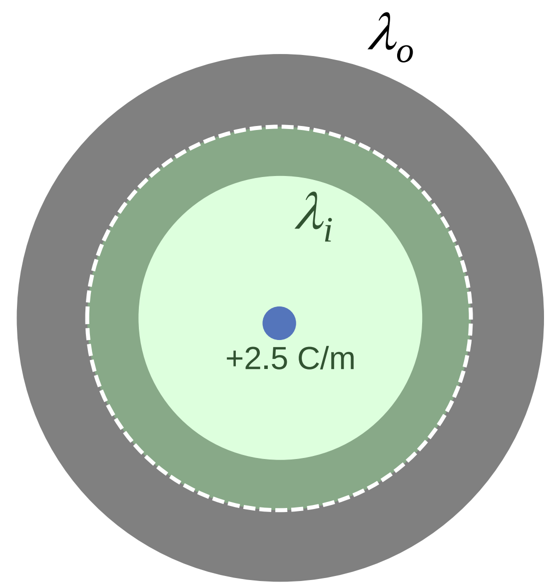
Infinite Cylinders

A long thin wire has a uniform positive charge density of 2.5 C/m. Concentric with the wire is a long thick conducting cylinder, with inner radius 3 cm, and outer radius 5 cm. The conducting cylinder has a net linear charge density of -4C/m.



What is the linear charge density of the induced charge on the inner surface of the conducting cylinder (λ_i) and on the outer surface (λ_o)?

	A	B	C	D	E
λ_i:	+2.5 C/m	–4 C/m	0	–2.5 C/m	–2.5 C/m
λ_o:	–6.5 C/m	0	-4 C/m	+2.5 C/m	–1.5 C/m



Using Gauss' Law to determine E

How do you choose the Gaussian surface???

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $E = \frac{Q_{enclosed}}{A\varepsilon_0}$

In General, integral to calculate flux is difficult…. and not useful!

To use Gauss' Law to calculate E, need to choose surface carefully!

1) Want E to be constant and equal to value at location of interest

OR

2) Want E dot $A = 0$ so doesn't add to integral

Gauss' Law Symmetries

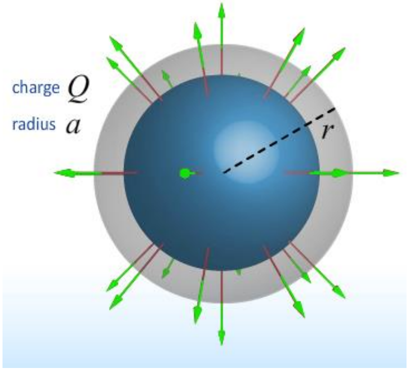
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get

$$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

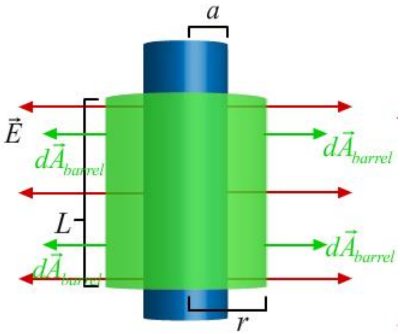
Spherical



$$A = 4\pi r^2$$

$$E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}$$

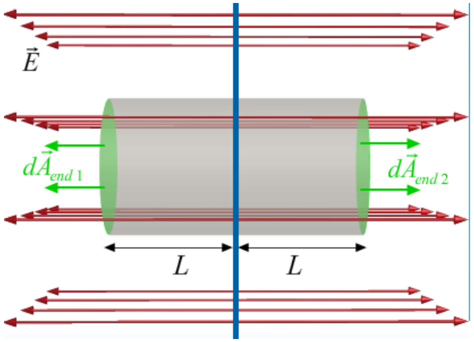
Cylindrical



$$A = 2\pi r L$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Planar

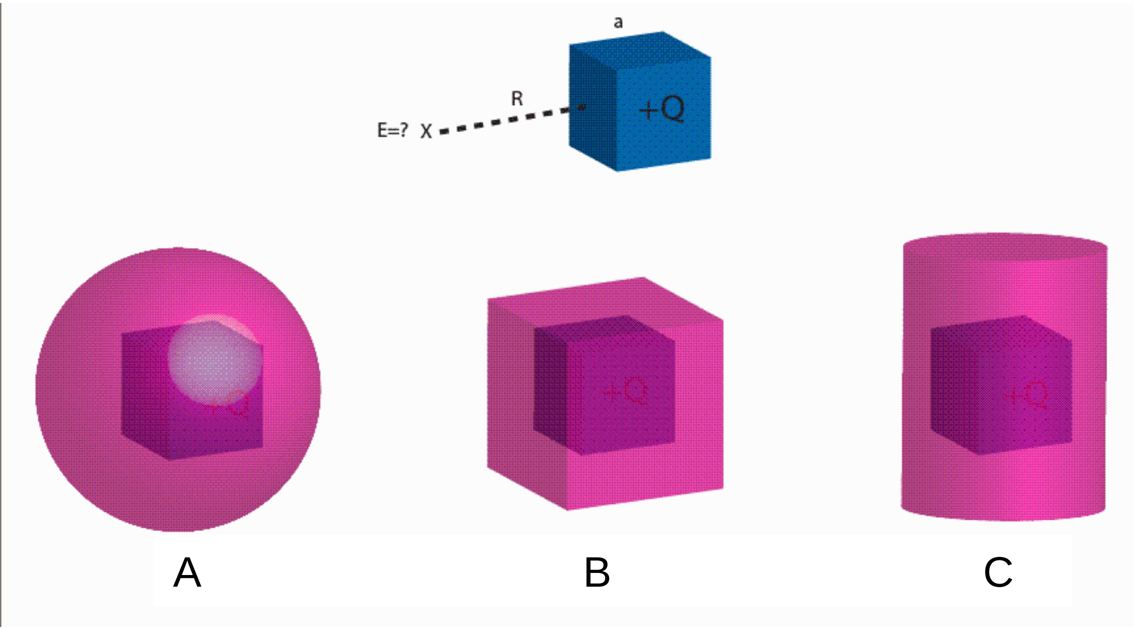


$$A = 2\pi r^2$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Check Point 1

Which Gaussian Surface would you use to calculate E due to cube of charge?



D) The field cannot be calculated using Gauss’ law for the drawn surfaces
E) None of the above

Cube is NOT one of 3 symmetries that works because

THE FIELD AT THE FACE OF THE CUBE

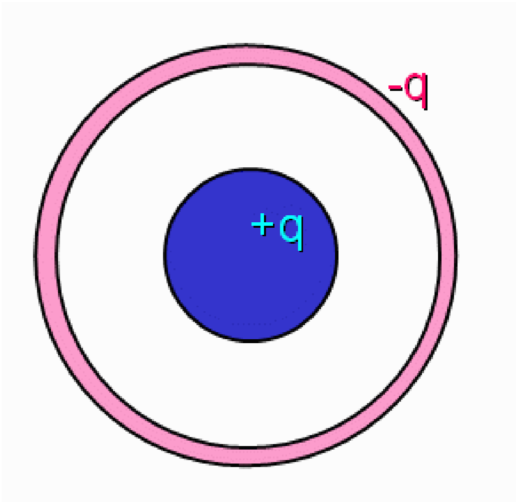
IS NOT

PERPENDICULAR OR PARALLEL

3D	POINT	-	SPHERICAL
2D	LINE	-	CYLINDRICAL
1D	PLANE	-	PLANAR

Check Point 2

A positively charged solid conducting sphere (blue) is inside a negatively charged conducting shell (red).



What is direction of field between blue and red spheres?

A) Outward B) Inward C) Zero

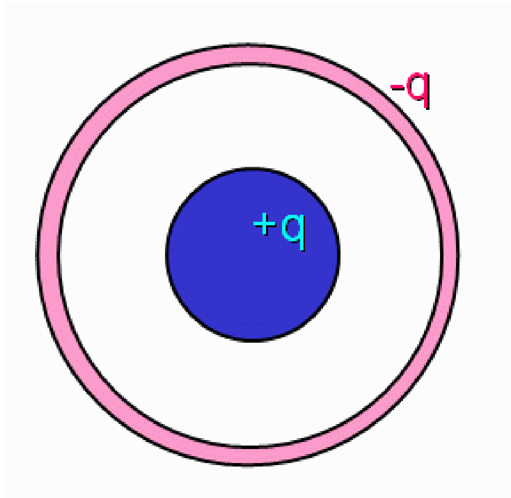
Careful: what does inside mean?
This is always true for a solid conductor (within the material of the conductor)
Here we have a charge "inside"

A) "Gauss's law, the region between the spheres encloses a positive charge, and thus the field must point outward."

C) "Within the boundaries of a conductor, the electric field will be 0."

Check Point 3

A positively charged solid conducting sphere (blue) is inside a negatively charged conducting shell (red).

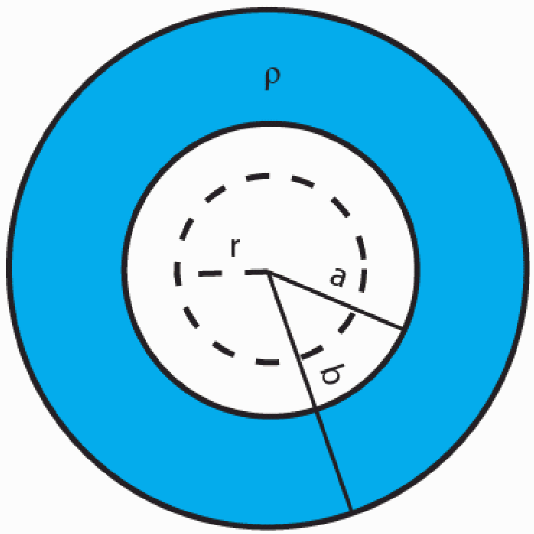


What is direction of field OUTSIDE the red sphere?

A) Outward B) Inward C) Zero

Check Point 4

A spherical insulating shell has inner radius a, and outer radius b, and uniform charge density ρ



What is magnitude of E at dashed line (r)?

A) $\dfrac{\rho}{\varepsilon_0}$

B) Zero

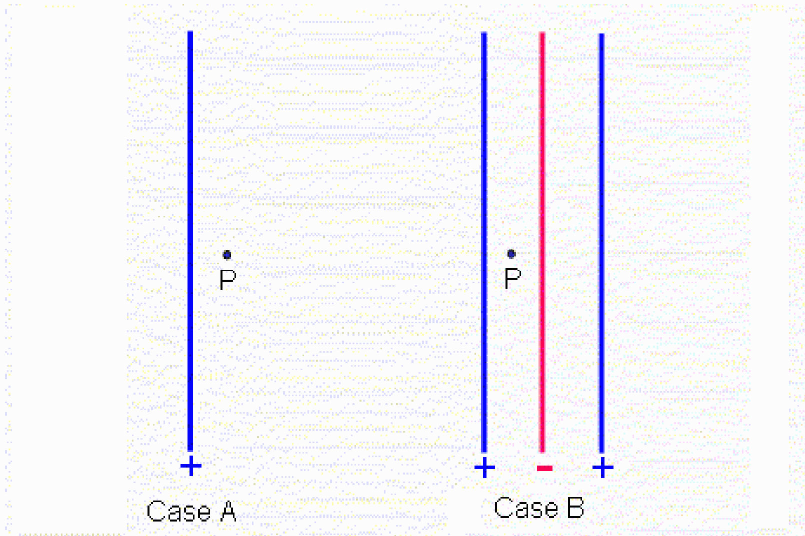
C) $\dfrac{\rho(b^2 - a^2)}{3\varepsilon_0 r^2}$

D) None of above

"Since the charge enclosed by r<a = 0, the electric field must also be 0 by Gauss' Law."

Check Point 5

10) In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes. The magnitudes of the charge per unit area on each plane is the same.



In which case is E at point P the biggest?

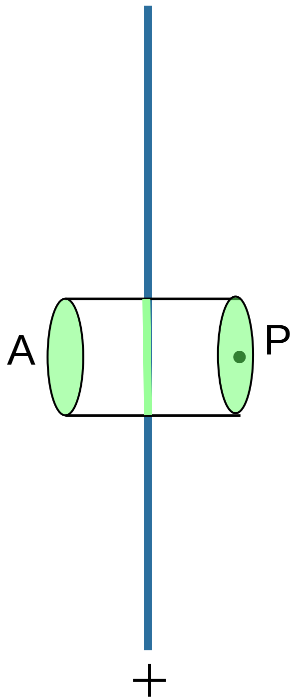
A) A B) B C) the same

B) In case B, the surrounding planes both emit a field in the same direction, so they "add together," and as such, the field experienced at P is stronger in case B.

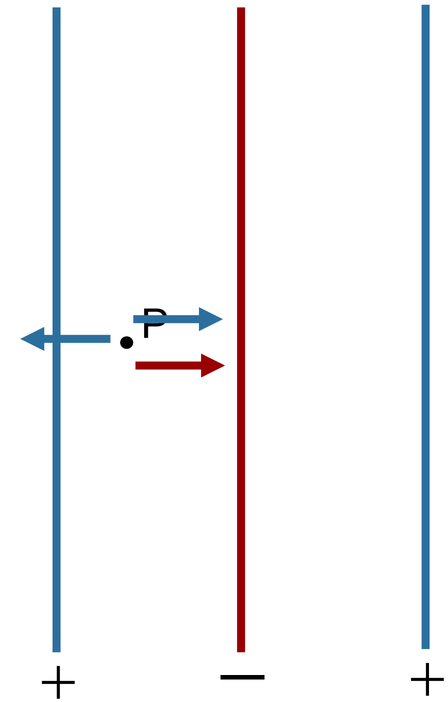
C) “The two positive planes in case B have a net 0 effect because they are on opposite sides of point P, so they can be ignored. Both cases can be thought of as having only one plane.”

Gauss's Law and Superposition

Lets do calculation!



Case A



Case B

Superposition

Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2A) = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2A) = \frac{\sigma A}{\varepsilon_0}$$

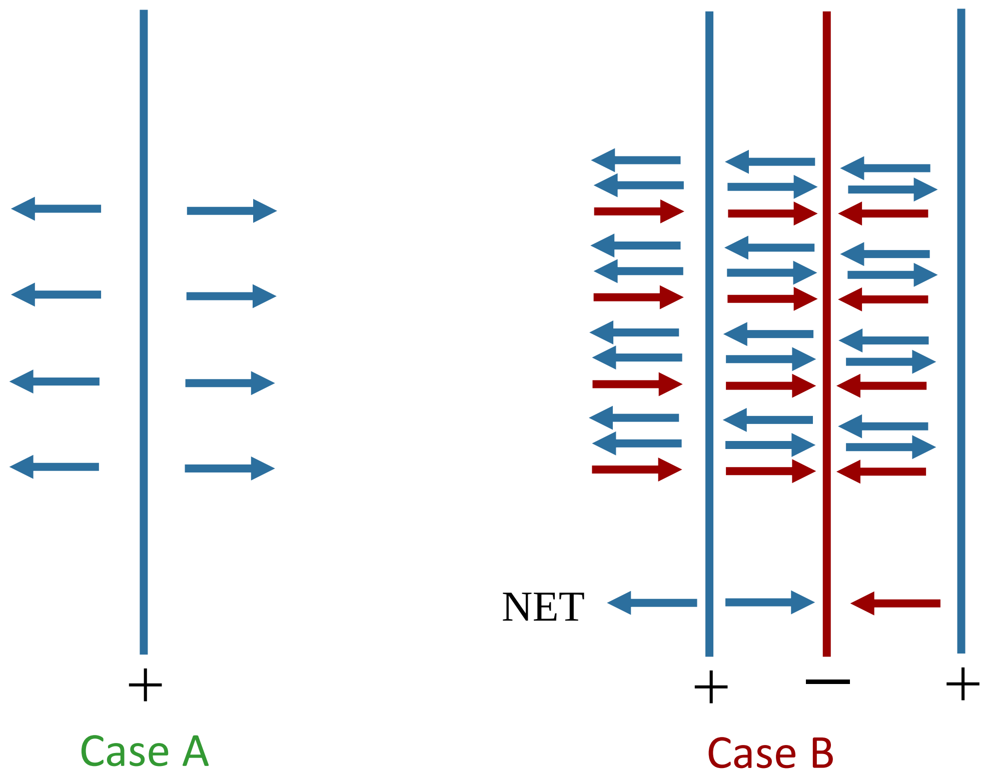
$$E = \frac{\sigma}{2\varepsilon_0}$$

$$E = +\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

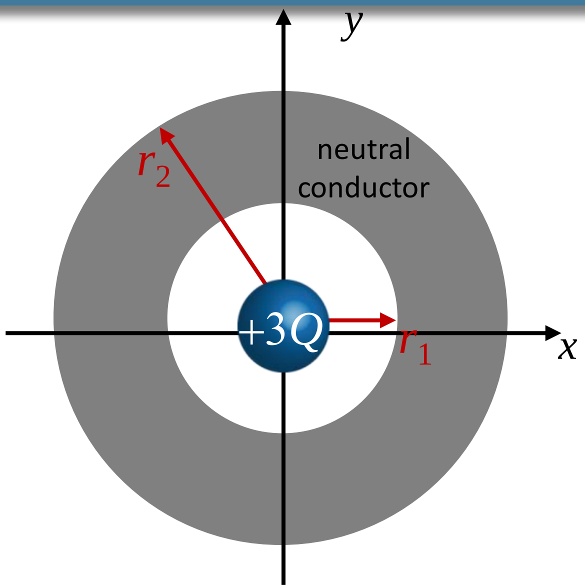
$$E = \frac{\sigma}{2\varepsilon_0}$$

Superposition:

Can you explain about the infinite sheets of charge problem? Like draw out the directions to where they are going during lecture?



Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

a) What is E everywhere?

First question: Do we have enough symmetry to use Gauss' Law to determine E?

Yes, Spherical Symmetry (what does this mean???)

Magnitude of E depends only on R

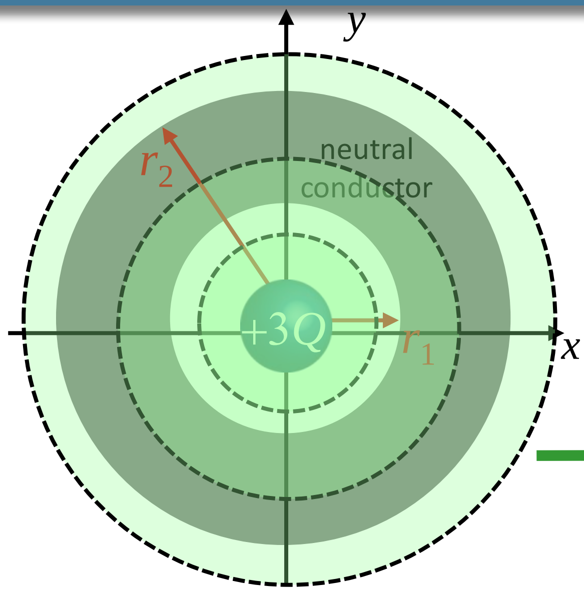
A) Direction of E is along $\hat{x}$

B) Direction of E is along $\hat{y}$

C) Direction of E is along $\hat{r}$

D) None of the above

Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

A) What is E everywhere?

We know:

magnitude of E is *fcn* of r

direction of E is along $\hat{r}$

We can use Gauss' Law to determine E

Use Gaussian surface = sphere centered on origin

$r < r_1$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{3Q}{\varepsilon_0}$$

$$E = \frac{3Q}{4\pi r^2 \varepsilon_0}$$

$r_1 < r < r_2$

A) $E = \dfrac{3Q}{4\pi r^2 \varepsilon_0}$

B) $E = \dfrac{3Q}{4\pi r_1^2 \varepsilon_0}$

C) $E = 0$

$r > r_2$

A) $E = \dfrac{3Q}{4\pi r^2 \varepsilon_0}$

B) $E = \dfrac{3Q}{4\pi (r - r_2)^2 \varepsilon_0}$

C) $E = 0$

Physics 212
Lecture 5

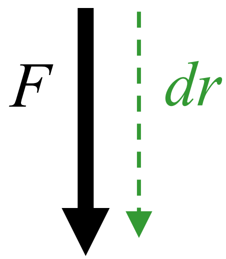
Today's Concept:

Electric Potential Energy

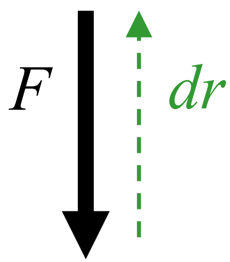
Work (Mechanics Review)

Recall from physics 211:

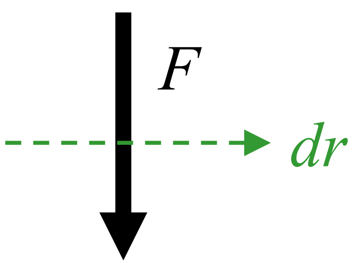
$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \qquad W_{TOT} = \Delta K$$



$W > 0$ (e.g. $W_{gravity}$ on object dropped)



$W < 0$ (e.g. $W_{gravity}$ on ball going up)



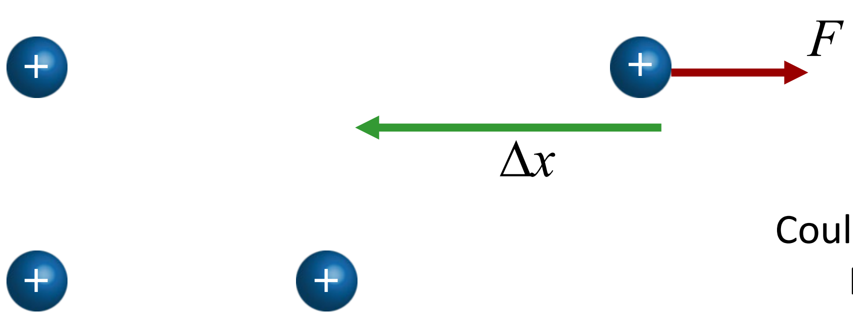
$W = 0$ (e.g. $W_{gravity}$ on moving horizontally)

Potential Energy

$$\Delta U \equiv -W_{conservative}$$

If gravity does negative work, potential energy increases!

Same idea for Coulomb force… if Coulomb force does negative work, potential energy increases.



Coulomb force does negative work
Potential energy increases

Check Point 1

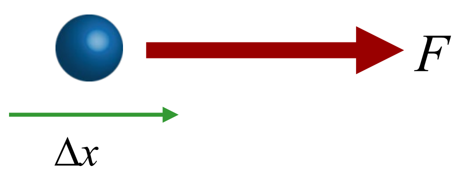
A charge is released from rest in a region of electric field. The charge will start to move

A) In a direction that makes its potential energy increase.

B) In a direction that makes its potential energy decrease.

C) Along a path of constant potential energy.

"It will move in the same direction as F, Work done by force is positive, so the potential energy decreases.."



It will move in the same direction as F

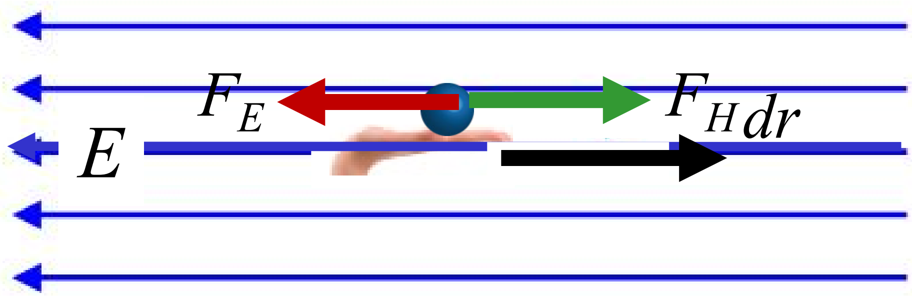
Work done by force is positive

$\Delta U = -$Work, so change in pot. Energy is negative

Nature wants things to move in such a way that PE decreases

Question

You hold a positively charged ball and walk due east in a region that contains an electric field directed due west.



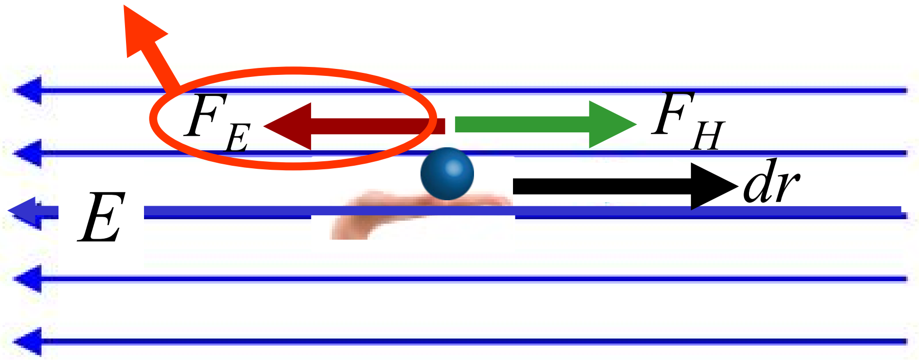
W_H is the work done by the hand on the ball
W_E is the work done by the electric field on the ball

Which of the following statements is true:

A) $W_H > 0$ and $W_E > 0$
B) $W_H > 0$ and $W_E < 0$
C) $W_H < 0$ and $W_E < 0$
D) $W_H < 0$ and $W_E > 0$

Question

Conservative force: $\Delta U = -W_E$



B) $W_H > 0$ and $W_E < 0$

Is ΔU positive, negative or zero?

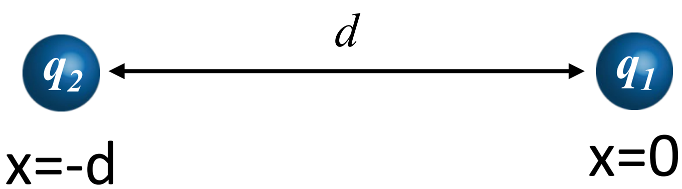
A) Positive

B) Negative

C) Zero

Example: Two Point Charges

Calculate the change in potential energy for two point charges originally very far apart moved to a separation of "d"



$$\Delta U \equiv -\int_i^f \vec{F} \cdot d\vec{r}$$

$$= -\int_{-\infty}^{-d} F dx$$

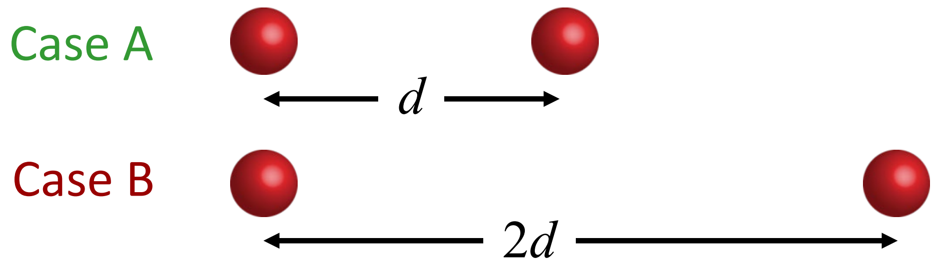
$$= \int_{-\infty}^{-d} k \frac{q_1 q_2}{x^2} dx$$

$$= -kq_1 q_2 \left[-\frac{1}{d} - \left(-\frac{1}{\infty} \right) \right] = k \frac{q_1 q_2}{d} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{d}$$

Charged particles with the same sign have an increase in potential energy when brought closer together.

For point charges often choose $r =$ infinity as "zero" potential energy.

Question



In case A two negative charges which are equal in magnitude are separated by a distance d. In case B the same charges are separated by a distance $2d$. Which configuration has the highest potential energy?

A) Case A

B) Case B

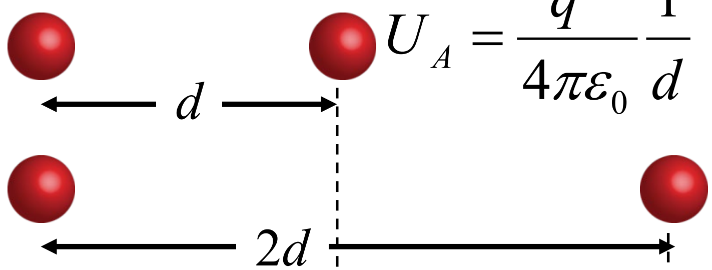
As usual, choose $U = 0$ to be at infinity:

$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}$$

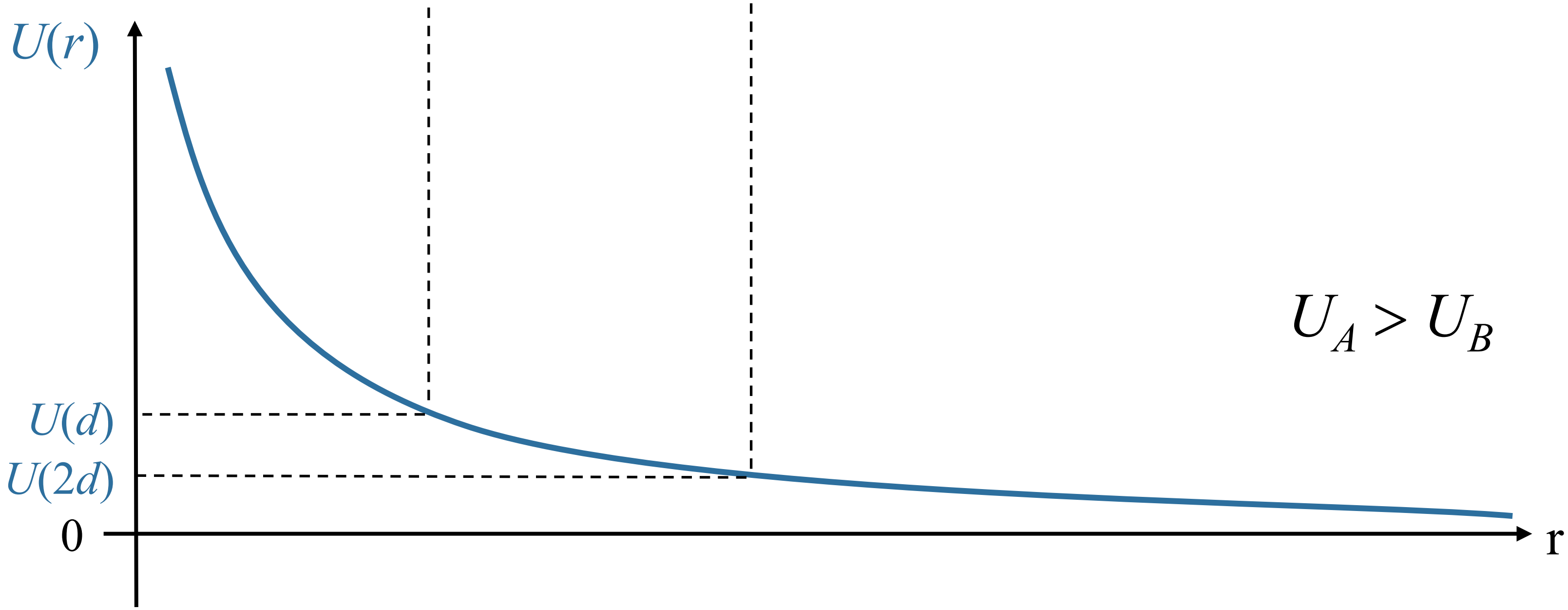
Case A

$$U_A = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{d}$$

Case B



$$U_B = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{2d}$$

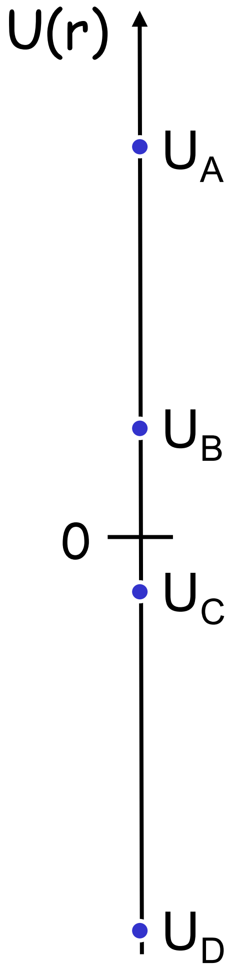


$$U_A > U_B$$

And Remember

U is just a number (not a vector)

- U DOES have a sign



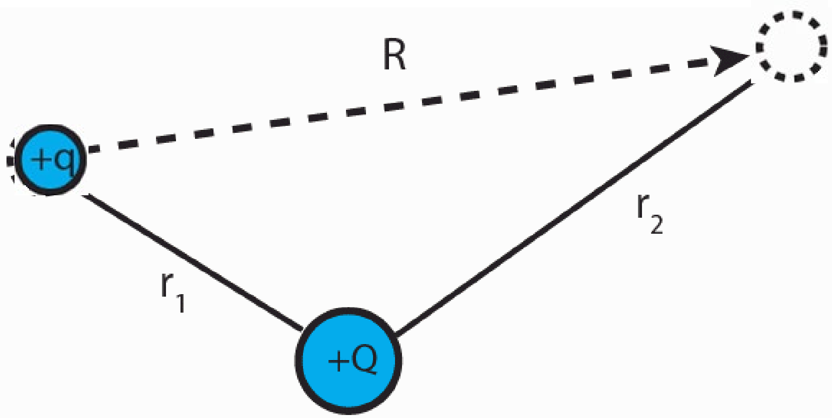
$U_A > U_B$

$U_B > U_C$

$U_C > U_D$

Check Point 2

A charge +q is moved from position 1 to position 2,
What is the change in potential energy?



A $\dfrac{kQq}{R}$ B $\dfrac{kQqR}{r_1^2}$ C $\dfrac{kQqR}{r_2^2}$

D $kQq\left(\dfrac{1}{r_2}-\dfrac{1}{r_1}\right)$ E $kQq\left(\dfrac{1}{r_1}-\dfrac{1}{r_2}\right)$

"The electric potential at 2 is kQq/r2^2 and the electric potential energy at 1 is kQq/r1^2. The change in the potential energy should be U2-U1 no matter what path q takes."

$$U_1 = \frac{kQq}{r_1} \qquad U_2 = \frac{kQq}{r_2}$$

$$\Delta U \equiv U_2 - U_1 = kQq\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

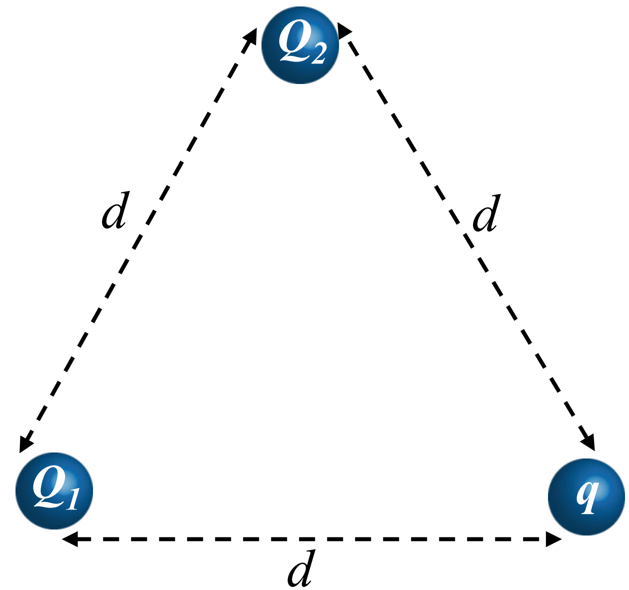
Note: $+q$ moves AWAY from $+Q$.
Its Potential energy MUST DECREASE
$$\Delta U < 0$$

Potential Energy of Many Charges

Two charges are separated by a distance d. What is the change in potential energy when a third charge q is brought from far away to a distance d from the original two charges?

$$\Delta U = \frac{qQ_1}{4\pi\varepsilon_0}\frac{1}{d} + \frac{qQ_2}{4\pi\varepsilon_0}\frac{1}{d}$$

(superposition)



"Can you go over in further depth, what electric potential energy actually means for a system of charges?"

Potential Energy of Many Charges

What is the change in potential energy when we bring in three identical charges, from infinitely far away, to the points on an equilateral triangle shown.

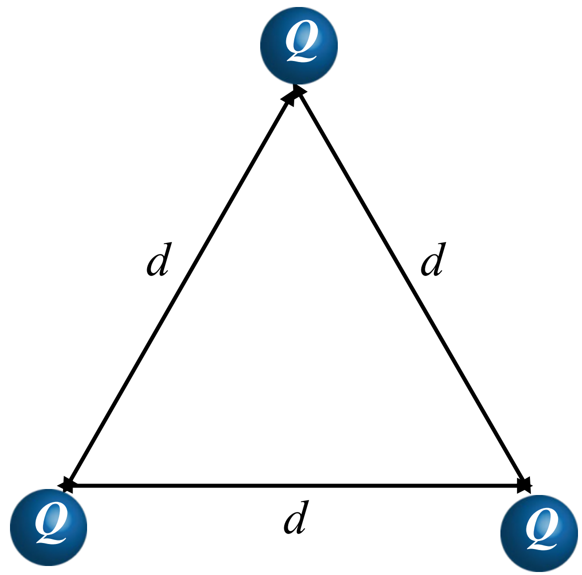
A) 0

B) $\Delta U = \dfrac{Q^2}{4\pi\varepsilon_0}\dfrac{1}{d}$

C) $\Delta U = 2\dfrac{Q^2}{4\pi\varepsilon_0}\dfrac{1}{d}$

D) $\Delta U = 3\dfrac{Q^2}{4\pi\varepsilon_0}\dfrac{1}{d}$

E) $\Delta U = 6\dfrac{Q^2}{4\pi\varepsilon_0}\dfrac{1}{d}$



$$W = \sum W_i = -\frac{3}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

Work by E to bring in first charge: $W_1 = 0$

Work by E to bring in second charge : $W_2 = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{d}$

Work by E to bring in third charge : $W_3 = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{d} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{d} = -\dfrac{2}{4\pi\varepsilon_0}\dfrac{Q^2}{d}$

Potential Energy of Many Charges

Suppose one of the charges is negative. Now what is the change in potential energy when we bring the three charges in from infinitely far away?

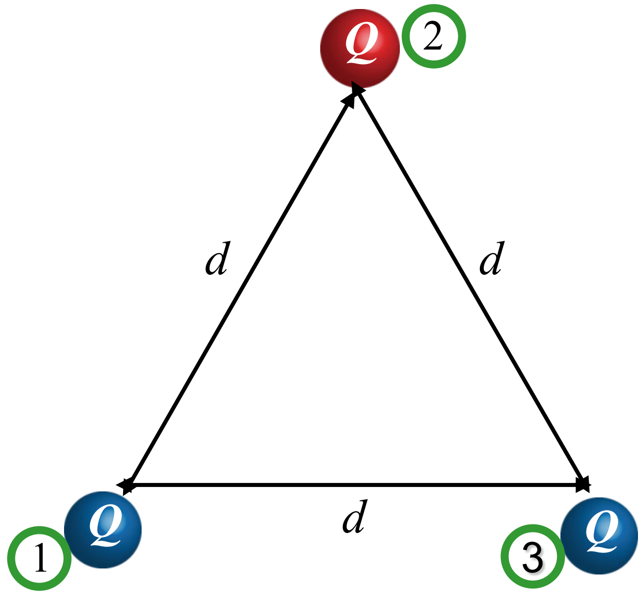
A) 0

B) $\Delta U = +1\dfrac{Q^2}{4\pi\varepsilon_0}\dfrac{1}{d}$

C) $\Delta U = -1\dfrac{Q^2}{4\pi\varepsilon_0}\dfrac{1}{d}$

D) $\Delta U = +2\dfrac{Q^2}{4\pi\varepsilon_0}\dfrac{1}{d}$

E) $\Delta U = -2\dfrac{Q^2}{4\pi\varepsilon_0}\dfrac{1}{d}$



$$W = \sum W_i = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

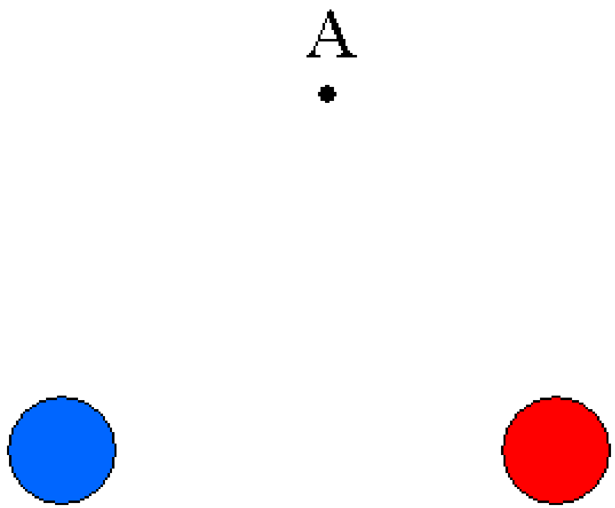
Work by E to bring in first charge: $W_1 = 0$

Work by E to bring in second charge : $W_2 = +\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{d}$

Work by E to bring in third charge : $W_3 = +\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{d} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{d} = 0$

Check Point 3

Two charges with equal magnitude but opposite sign are located at equal distances from the point labeled A



If a third charge is brought in from far away to point A, how does the potential energy of the collection of charges change?

Increases	Decreases	Same	Depends on sign of charge
A	B	C	D

"Because the signs are oppositely charged, newly added kqq/d and the -kqq/d will cancel out"

Check Point 4

A positive charge is placed on the left side of a negative charge. The magnitude of the negative charge is twice that of the positive charge.

Is there any (finite) location that a third charge can be placed such that the total potential energy of the system does not change?

- YES, as long as the third charge is positive
- YES, as long as the third charge is negative
- YES, no matter what the third charge is
- NO

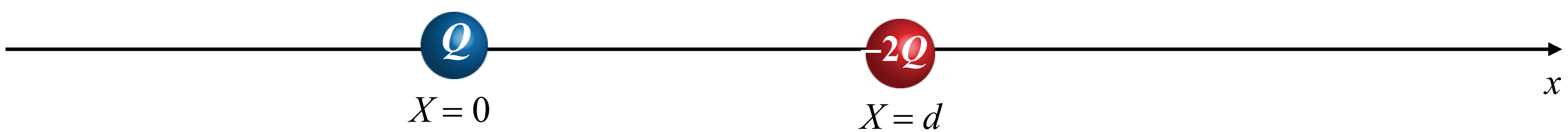
C) "As long as the third charge is twice as far from the larger negative charge as it is the smaller positive charge, the total potential energy of the system will be unaffected."

D) "A third charge cannot cancel out because the magnitudes of the two initial charges are not equal.."

LET' S DO THE CALCULATION!

Example

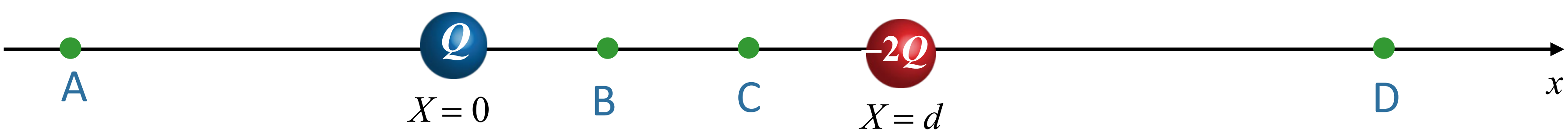
A positive charge q is placed at $x = 0$ and a negative charge $-2q$ is placed at $x = d$. At how many different places along the x axis could another positive charge be placed without changing the total potential energy of the system?



A 0
B) 1
C) 2
D) 3

Example

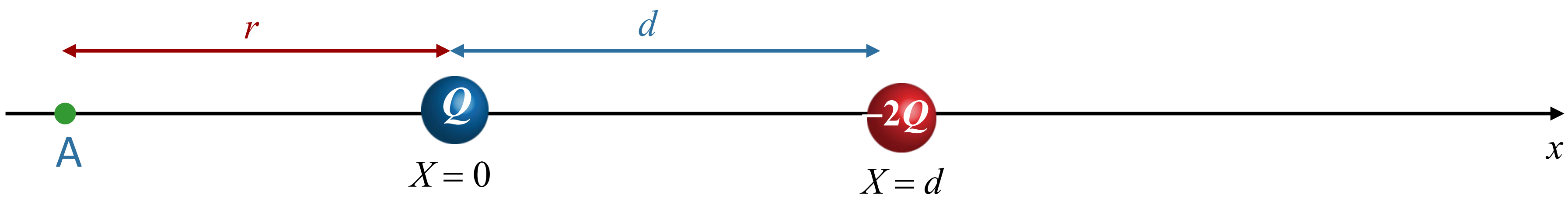
At which two places can a positive charge be placed without changing the total potential energy of the system?



A) A & B
B) A & C
C) B & C
D) B & D
E) A & D

Let's calculate the positions of A and B

Lets work out where A is



$$\Delta U = +\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} - \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{r+d}$$

Set $\Delta U = 0$

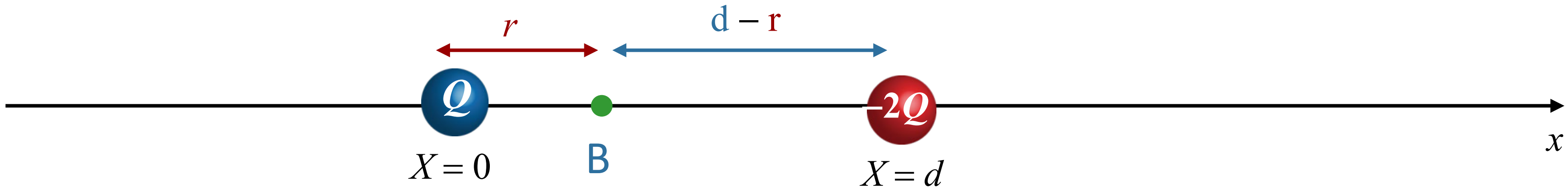
$$\frac{1}{r} = \frac{2}{r+d}$$

$$\boxed{r = d}$$

Makes Sense!

q is twice as far from $-2Q$ as it is from $+Q$

Lets work out where B is



Setting $\Delta U = 0$ $\rightarrow$ $$\frac{1}{r} = \frac{2}{d-r}$$

$$2r = d - r$$

$$\boxed{r = \frac{d}{3}}$$

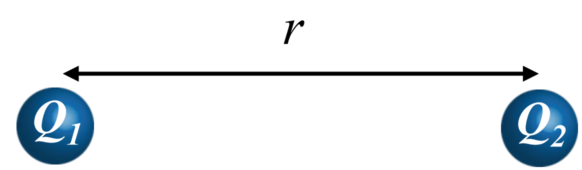
Makes Sense!

q is twice as far from -2*Q* as it is from + *Q*

Takeaways

For a pair of charges:

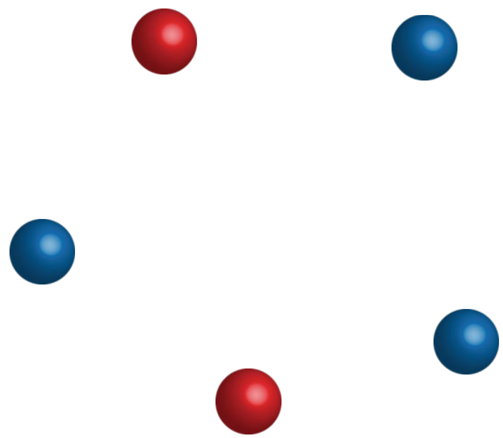
$$U = k\frac{q_1 q_2}{r}$$



(We usually choose $U = 0$ to be where the charges are far apart)

For a collection of charges:

Sum up $U = k\frac{q_1 q_2}{r}$ **for all pairs**



Next: electric potential

Physics 212
Lecture 6

Today's Concept:

Electric Potential

(Defined in terms of Path Integral of Electric Field)

Big Idea

Last time we defined the electric potential energy of charge q in an electric field:

$$\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{l} = -\int_a^b q\vec{E}\cdot d\vec{l}$$

The only mention of the particle was through its charge q.

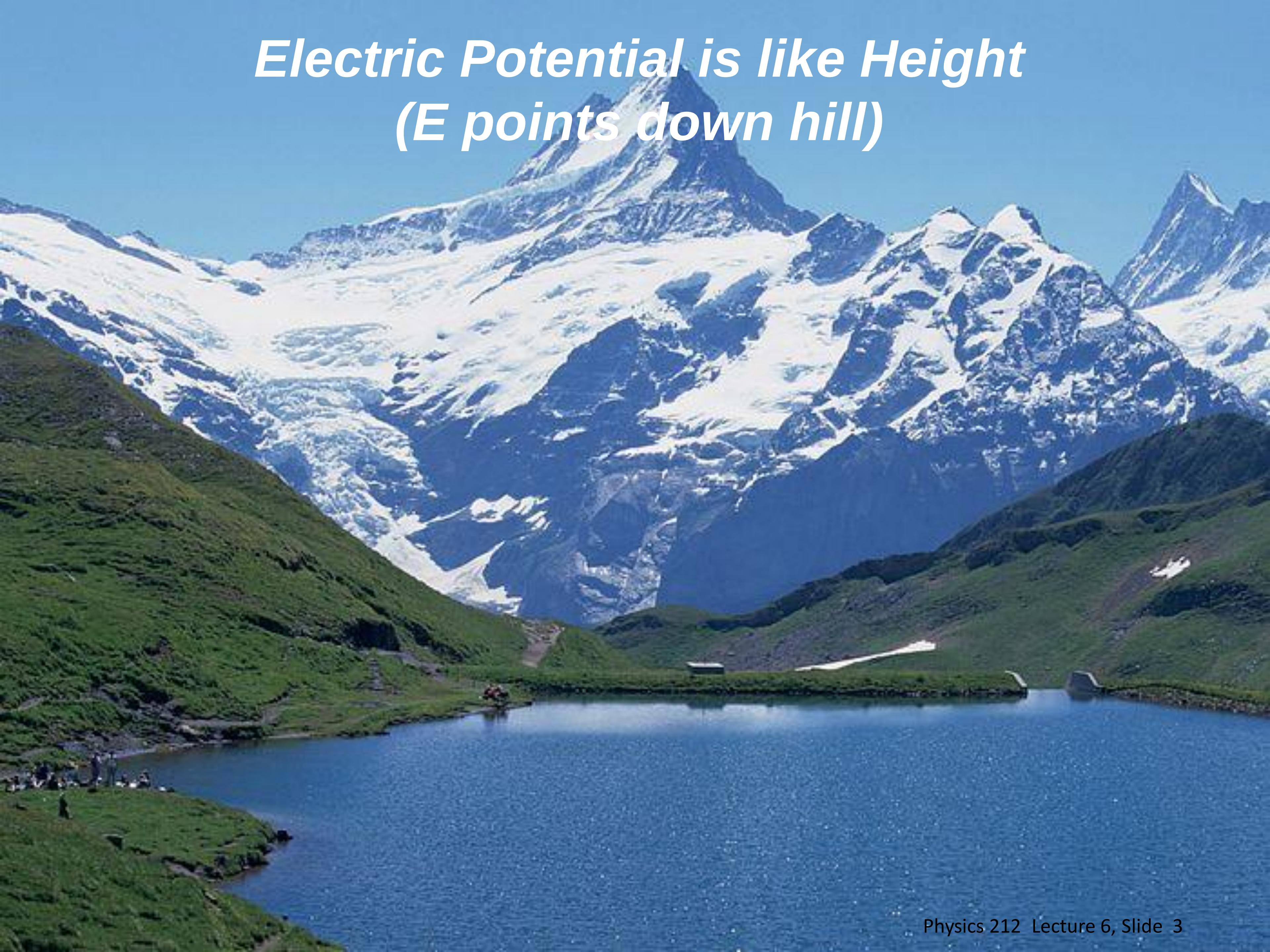
We can obtain a new quantity, the electric potential, which is a PROPERTY OF THE SPACE, as the potential energy per unit charge.

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Note the similarity to the definition of another quantity which is also a PROPERTY OF THE SPACE, the electric field.

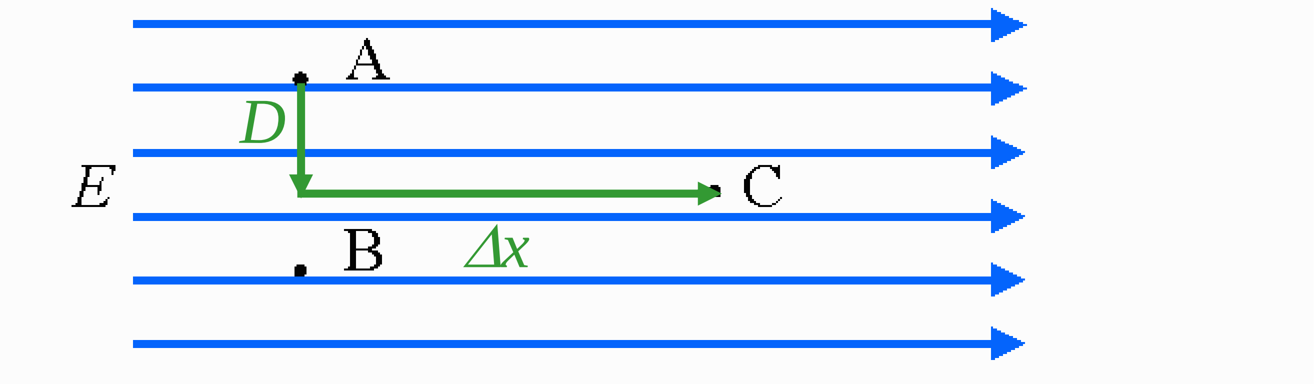
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric Potential is like Height (E points down hill)



Electric Potential from E field

Consider the three points A, B, and C located in a region of constant electric field as shown.



What is the sign of $\Delta V_{AC} = V_C - V_A$?

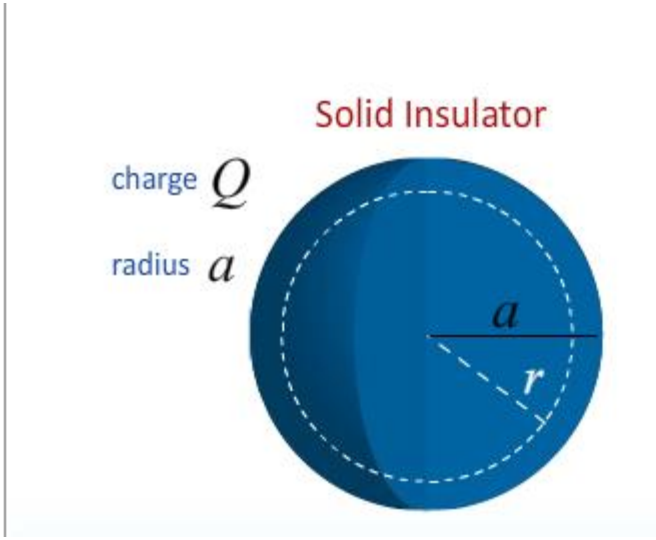
A) $\Delta V_{AC} < 0$ B) $\Delta V_{AC} = 0$ C) $\Delta V_{AC} > 0$

E points down hill

Remember the definition: $\Delta V_{A\to C} = -\int_A^C \vec{E}\cdot d\vec{l}$

Charged Spherical Insulator

I didn't understand the voltage calculation within an insulating sphere.



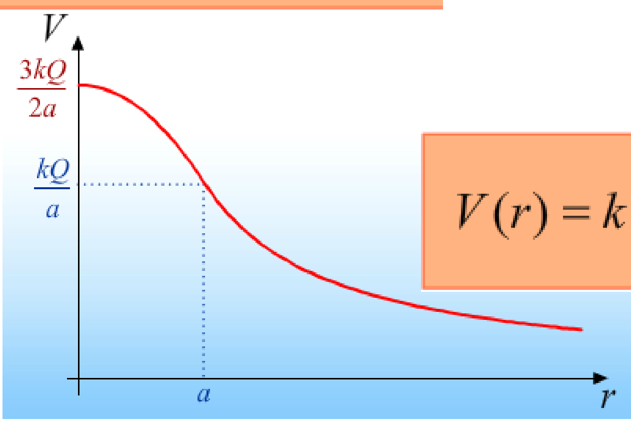
$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l}$$ For $r < a$

$$V(r) = -\int_{\infty}^{a} E\, dr \; - \int_{a}^{r} E\, dr$$

$$V(r) = -\int_{\infty}^{a} k\frac{Q}{r^2}\, dr \; - \int_{a}^{r} k\frac{Q}{a^3}\, r\, dr$$

$$V(r) = k\frac{Q}{a} + k\frac{Q}{2a^3}(a^2 - r^2)$$

$$\boxed{V(r) = k\frac{Q}{2a^3}(3a^2 - r^2)}$$ For $r < a$



$$\boxed{V(r) = k\frac{Q}{r}}$$ For $r > a$

CheckPoint 2

If the electric field is zero in a region of space, what does that tell you about the electric potential in that region, (which statement is always true)?

A) The electric potential is zero everywhere in this region.
B) The electric potential is zero at at least one point in this region.
C) The electric potential is constant everywhere in this region.

Remember the definition

$$\Delta V_{A\rightarrow B} = -\int_A^B \vec{E}\cdot d\vec{l}$$

E from V

If we can get the potential by integrating the electric field:

$$\Delta V_{a\to b} = -\int_a^b \vec{E}\cdot d\vec{l}$$

We should be able to get the electric field by differentiating the potential

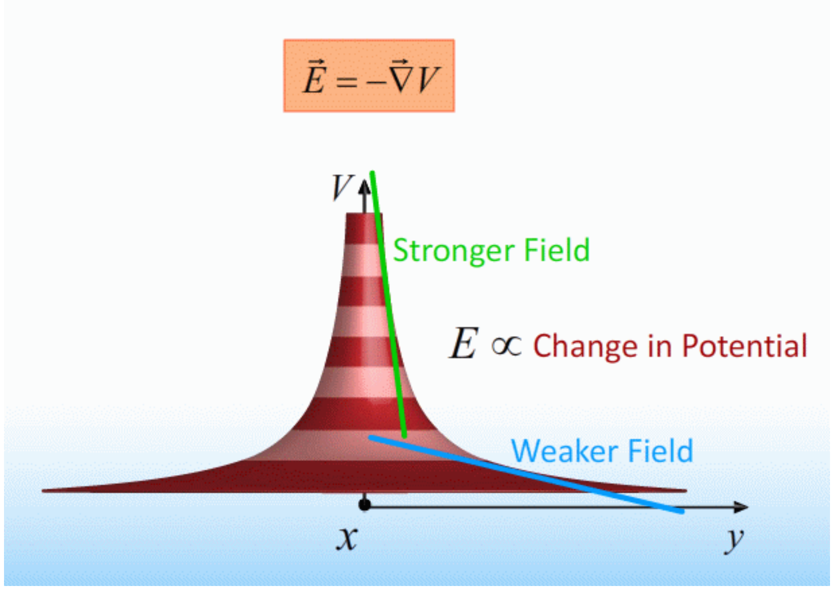
$$\vec{E} = -\vec{\nabla} V$$

In Cartesian coordinates:

$$E_x = -\frac{\partial V}{dx}$$

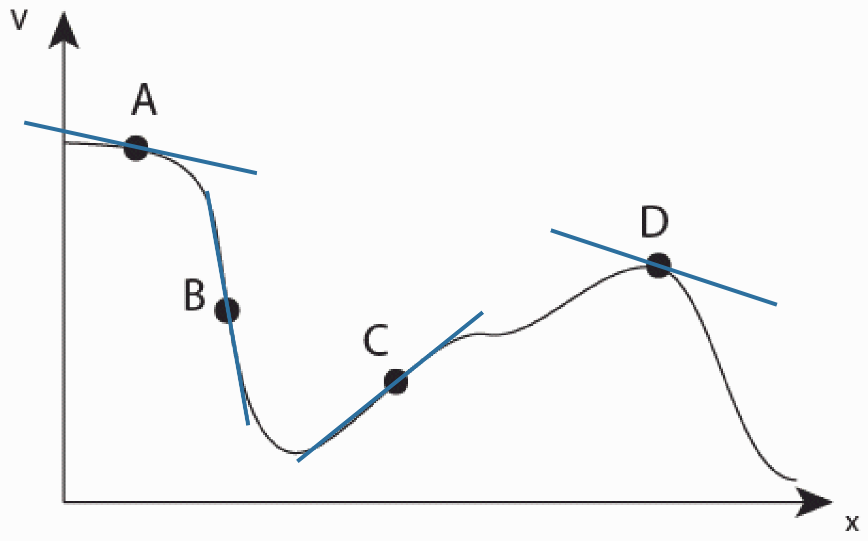
$$E_y = -\frac{\partial V}{dy}$$

$$E_z = -\frac{\partial V}{dz}$$



CheckPoint 1a

2) The electric potential in a certain region is plotted in the following graph



At which point is the magnitude of the electric field greatest?

"A) This is where the greatest potential is."

"B) Steepest slope here.."

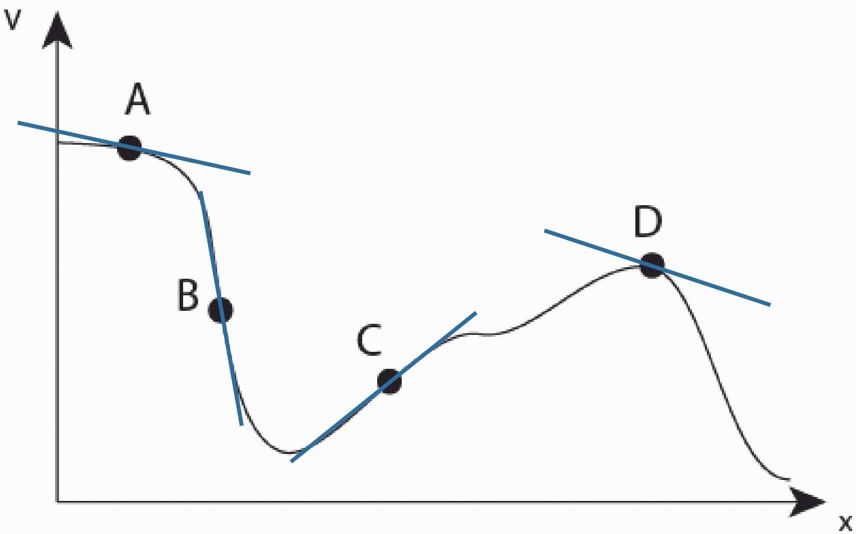
"C) At C, the change in Potential is the greatest, so the E-field is the greatest"

How do we get E from V?

$$\vec{E} = -\vec{\nabla} V \quad \longrightarrow \quad E_x = -\frac{\partial V}{dx} \quad \longrightarrow \quad \text{Look at slopes!}$$

CheckPoint 1b

2) The electric potential in a certain region is plotted in the following graph



At which point is the electric field pointing in the negative x direction?

"B) The slope is negative thus, the E-field should be pointing along the negative x-axis."

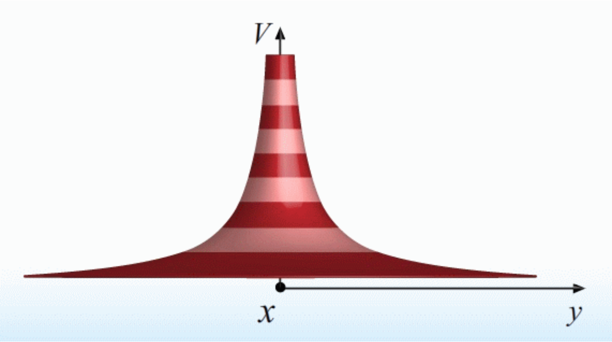
"C) E=-gradient(V). This means where the slope is positive, the E-field is negative.."

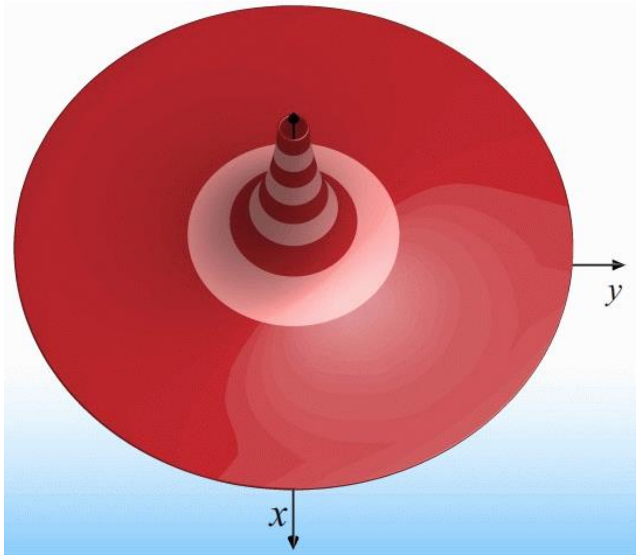
How do we get E from V?

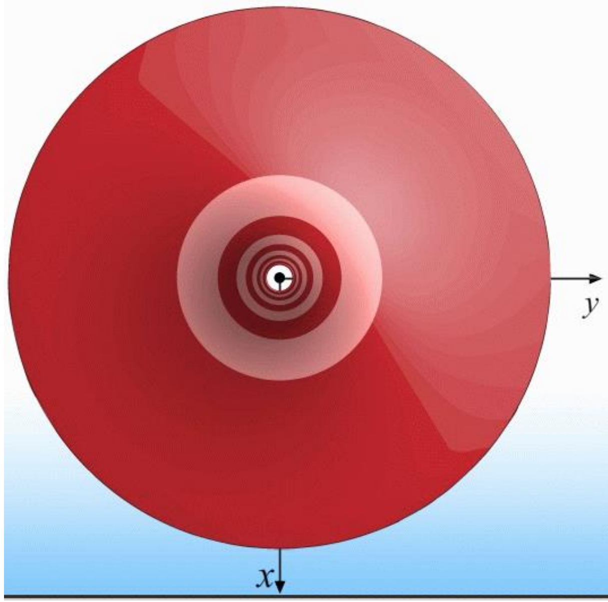
$$\vec{E} = -\vec{\nabla} V \quad \longrightarrow \quad E_x = -\frac{\partial V}{dx} \quad \longrightarrow \quad \text{Look at slopes!}$$

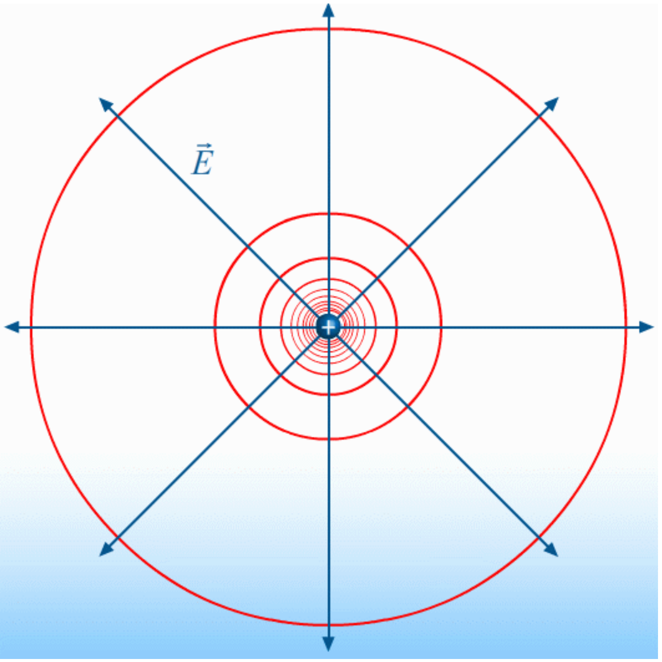
Equipotentials

Equipotentials are the locus of points having the same potential.







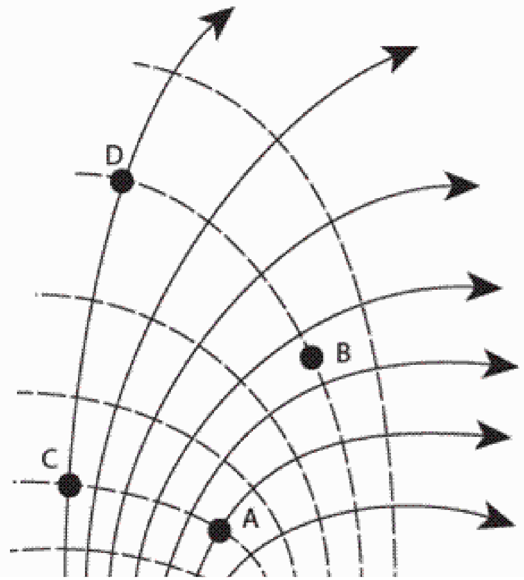


Equipotentials are
ALWAYS
perpendicular to the electric field lines.

The SPACING of the equipotentials indicates
The STRENGTH of the electric field.

CheckPoint 3b

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.

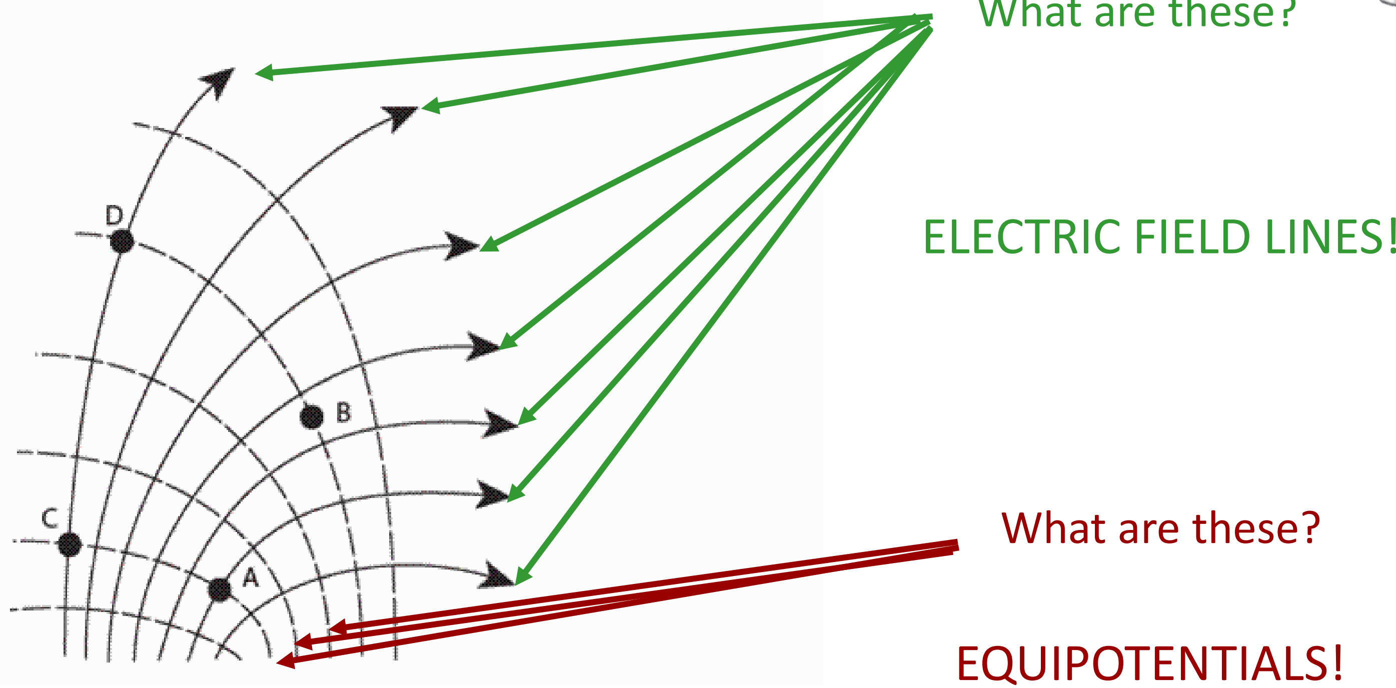


Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

A) More work from A to B
B) More work from C to D
C) Same
D) Can not determine w/o performing calculation

Hint

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.

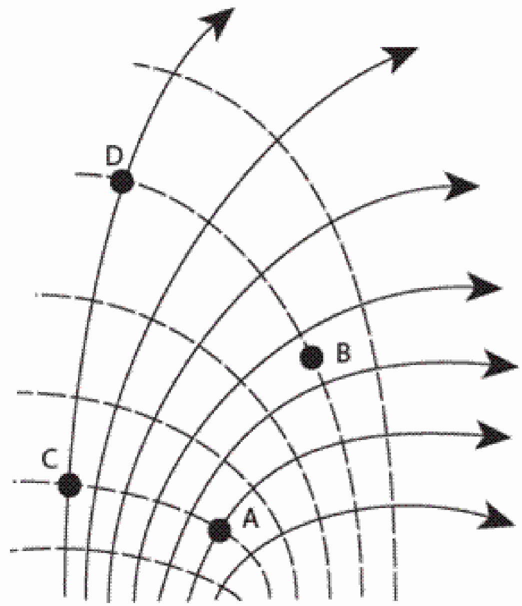


What is the sign of $W_{AC} =$ work done by E field to move negative charge from A to C ?

A) $W_{AC} < 0$ B) $W_{AC} = 0$ C) $W_{AC} > 0$

CheckPoint 3b Again?

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

A) More work from A to B — "The field is stronger in the region from A to B than from C to D."

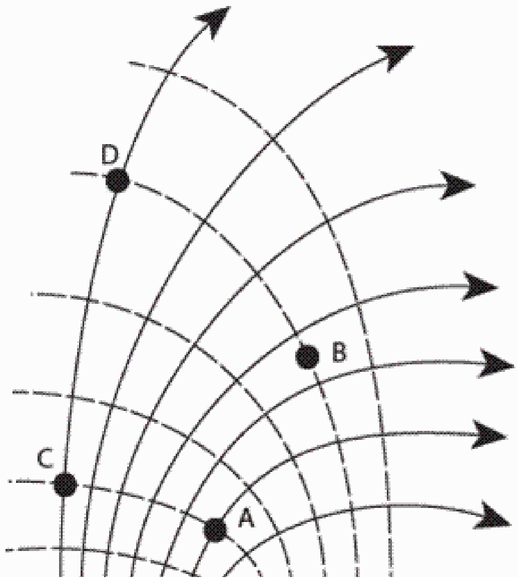
B) More work from C to D — "The distance between the two points is greater."

C) Same — "same number of equipotential jumps."

D) Can not determine w/o performing calculation

CheckPoint 3c

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from A to D

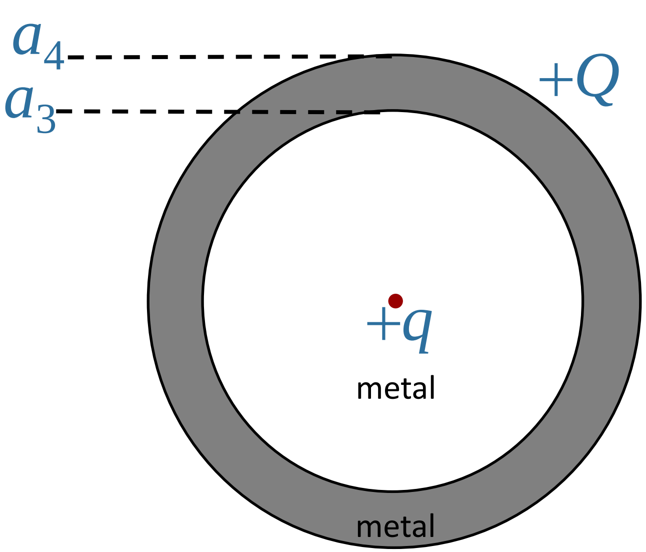
A) More work from A to B — "C and D is on the same electric field so the work is zero.."

B) More work from A to D — "The distance between the two points is greater."

C) Same — "same number of equipotential jumps."

D) Can not determine w/o performing calculation

Calculation for Potential



Point charge q at center of spherical shell of inner and outer radii a_3, and a_4. The shell carries charge Q.

What is V as a function of r?

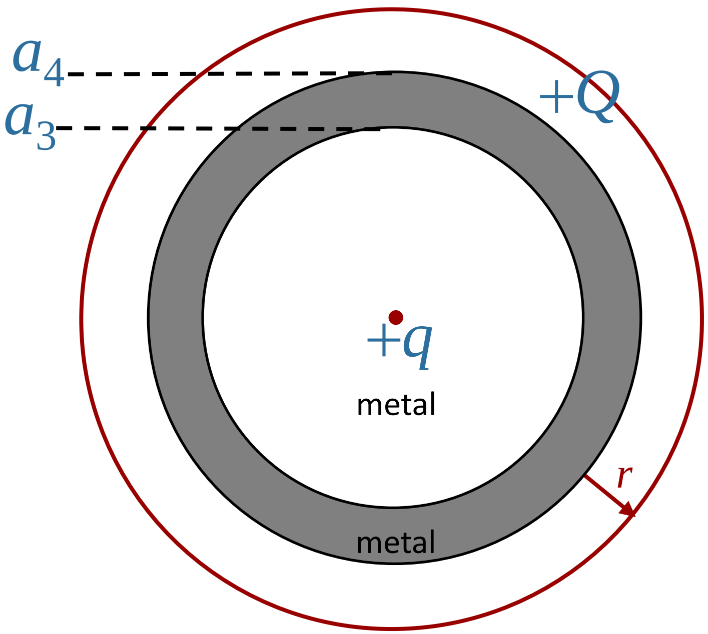
Conceptual Analysis:

- Charges q and Q will create an E field throughout space
- $V(r) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{\ell}$

Strategic Analysis:

- Spherical symmetry: Use Gauss' Law to calculate E everywhere
- Integrate E to get V

Calculation: Quantitative Analysis



$r > a_4$: What is $E(r)$ outside shell?

A) 0

B) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

C) $\dfrac{1}{2\pi\varepsilon_0}\dfrac{Q+q}{r}$

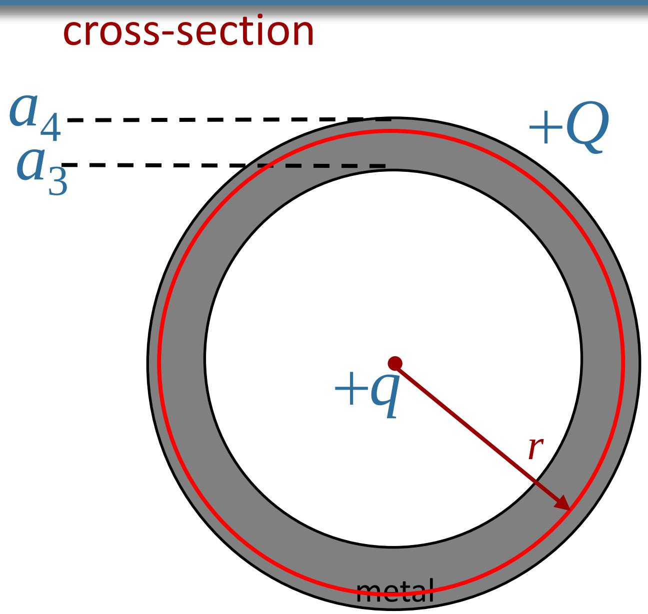
D) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r^2}$

E) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q-q}{r^2}$

Why?

Gauss' law: $$\int \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Calculation: Quantitative Analysis



$a_3 < r < a_4$: What is $E(r)$ Inside metal sphere?

A) 0 B) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ C) $\frac{1}{2\pi\varepsilon_0}\frac{q}{r}$

D) $\frac{1}{4\pi\varepsilon_0}\frac{-q}{r^2}$ E) $\frac{1}{4\pi\varepsilon_0}\frac{Q-q}{r^2}$

Applying Gauss' law, what is $Q_{enclosed}$ for red sphere shown?

A) q B) $-q$ C) 0

How is this possible?

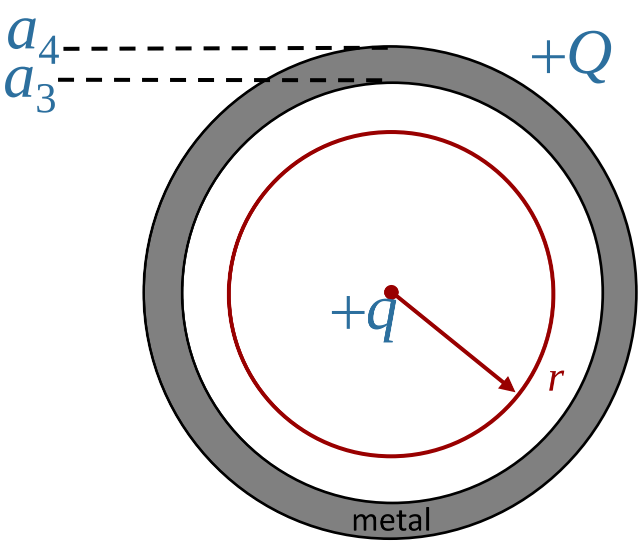
$-q$ must be induced at $r = a_3$ surface → charge at $r = a_4$ surface $= Q + q$

$$\sigma_3 = \frac{-q}{4\pi a_3^2}$$

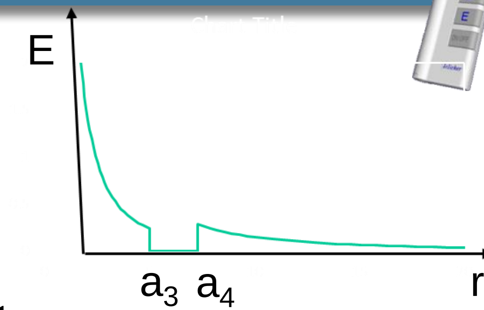
$$\sigma_4 = \frac{Q+q}{4\pi a_4^2}$$

Calculation: Quantitative Analysis

cross-section



Continue on in…



$a_3 < r < a_4$: $\quad E = 0$

$r < a_3$: $\quad E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$

To find V:

1) Choose r_0 such that $V(r_0) = 0$ (usual: r_0 = infinity)
2) Integrate!

$$V(r) = -\int_{r_0}^{r} \vec{E}\cdot d\vec{\ell}$$

$r > a_4$: $\quad V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r}$

$a_3 < r < a_4$:

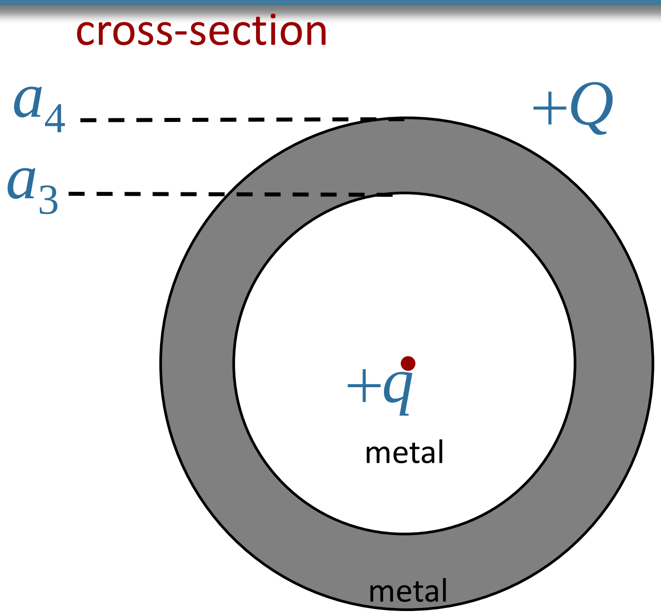
A) $V = 0$

B) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_4}$

C) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_3}$

D) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r}$

Calculation: Quantitative Analysis



To find V:

1) Choose r_0 such that $V(r_0) = 0$ (usual: r_0 = infinity)
2) Integrate!

$r > a_4$: $$V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r}$$

$a_3 < r < a_4$: $$V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{a_4}$$

$$V(r) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{\ell}$$

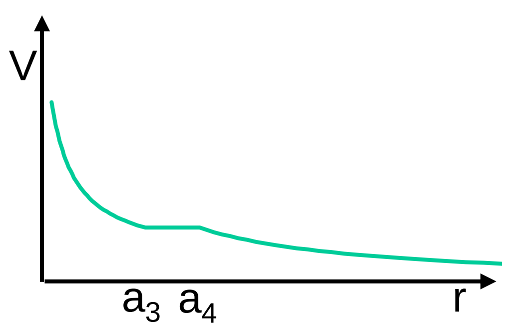
$r < a_3$:

A) $V = 0$

B) $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{r}\right)$

C) $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_3} + \frac{q}{r}\right)$

D) $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{r} - \frac{q}{a_3}\right)$



Have a Fantastic National Holiday!

Start the homework early.

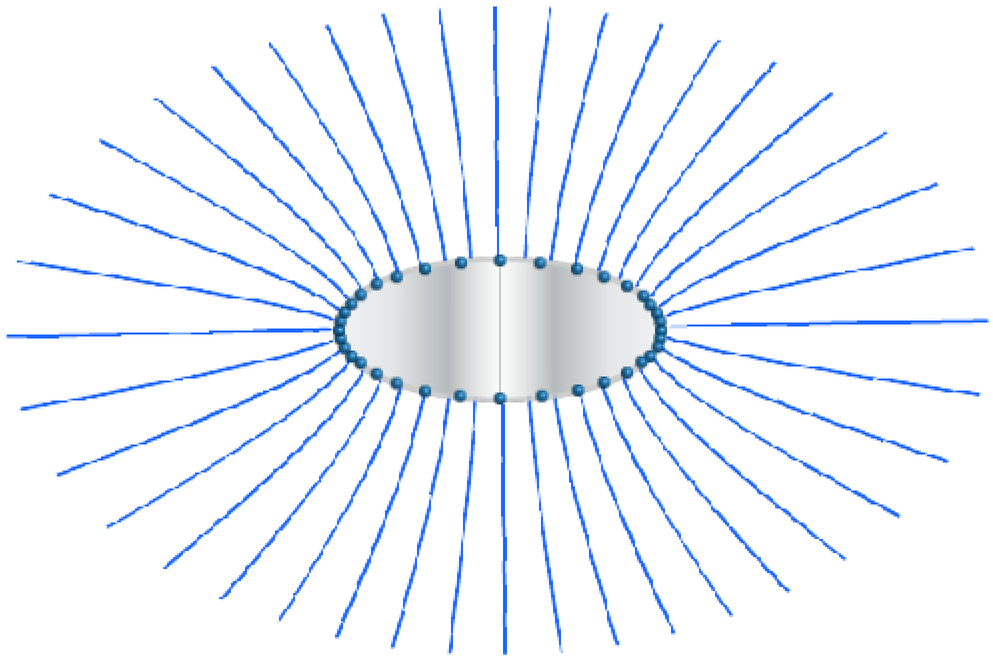
Physics 212
Lecture 7

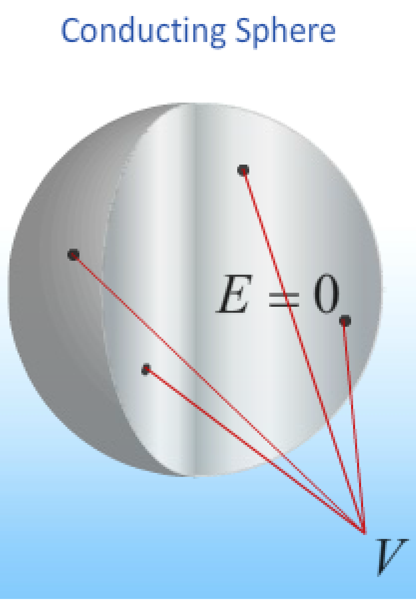
Today's Concept: (Applications of Gauss, E and V)

A) Conductors

B) Capacitance

Main Point 1: (Conductors)



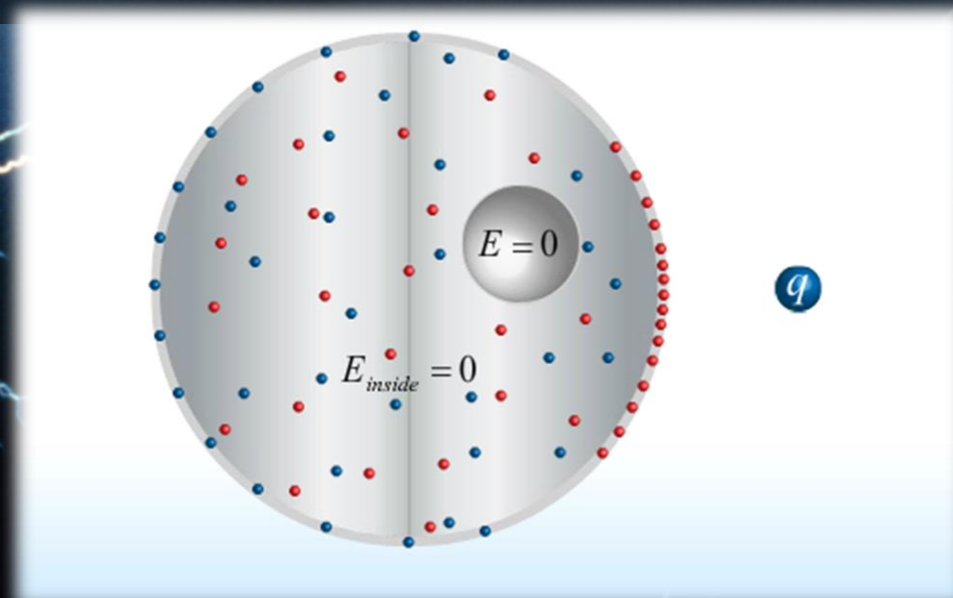


- Charges are free to move
- E = 0 in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface

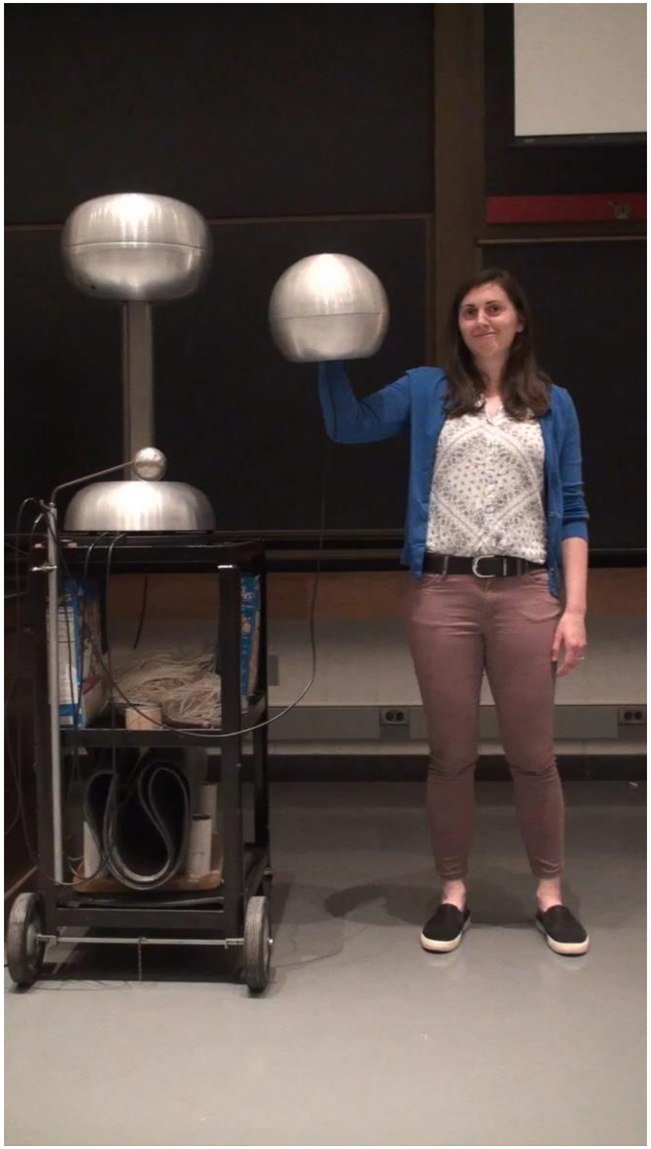
Storm Safety

You are at the park when you see lightning. You decide to take shelter in a car, which car is safer, a (mainly steel) Volkswagen with thick rubber tires, or a (mainly fiberglass) Corvette with thin rubber tires

A) Corvette because it is fiberglass
B) Corvette because it is lower to ground
C) Volkswagen because it is steel
D) Volkswagen because tires are thicker
E) Neither—social distancing!

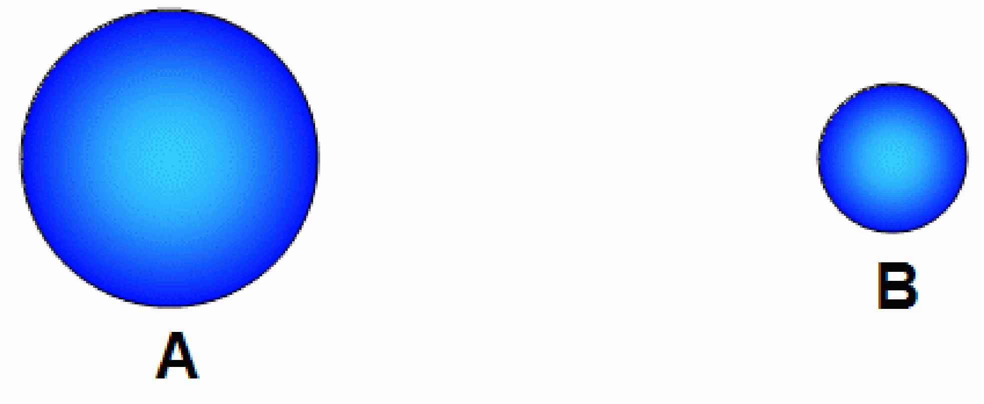


Shocking!



Check Point 1

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B

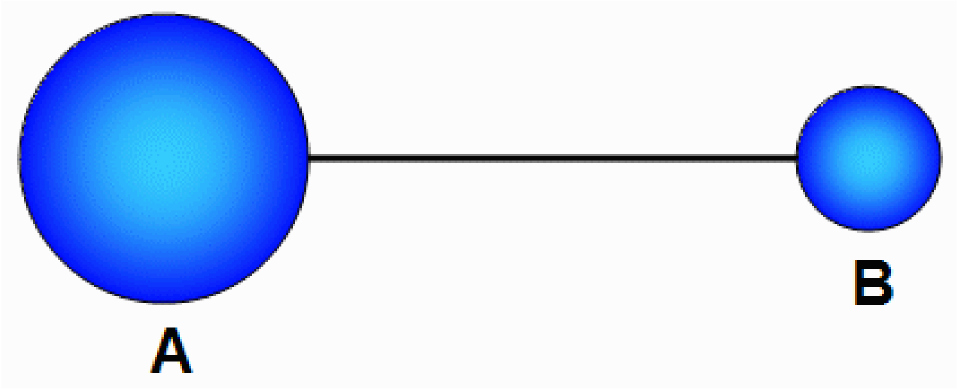


Compare the potential on surface A with the potential on surface B

A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$

Check Point 2

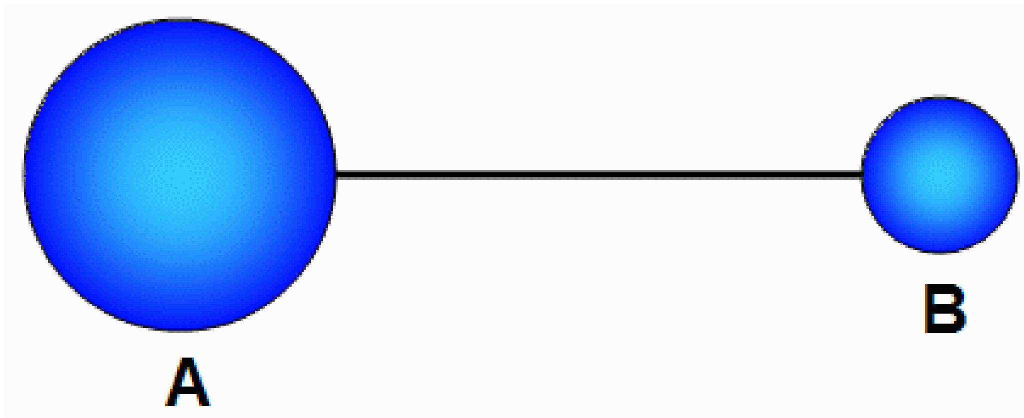
The two conductors are now attached by a conducting wire.



Compare the potential on surface A with the potential on surface B

A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$

Check Point 3



What happens to the charge on sphere A when the wire is attached

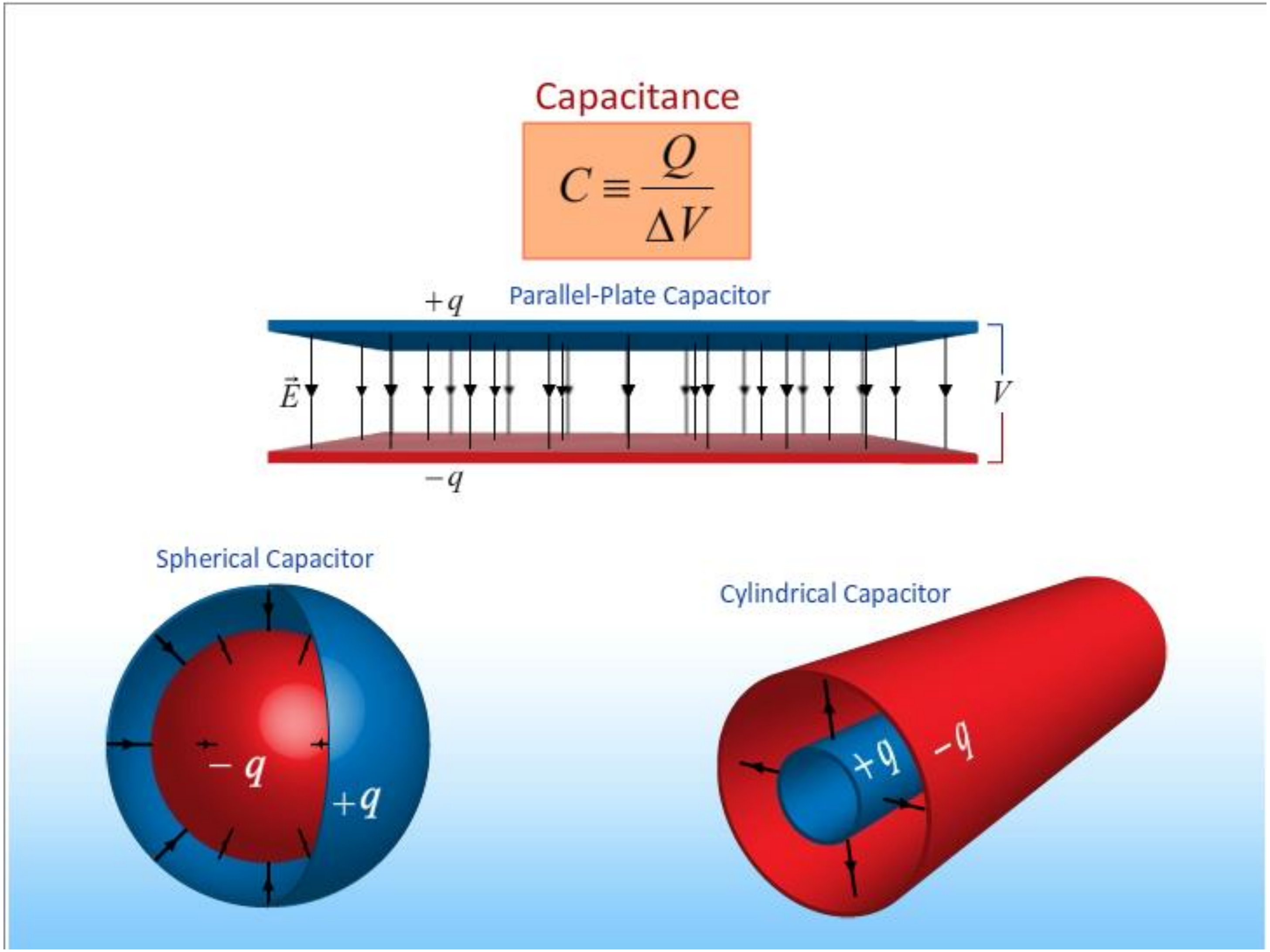
A) Q_A increases B) Q_A decreases C) Q_A does not change

$$\frac{kQ_A}{R_A} = \frac{kQ_B}{R_B}$$

$$\Rightarrow \quad Q_A = Q_B \frac{R_A}{R_B}$$

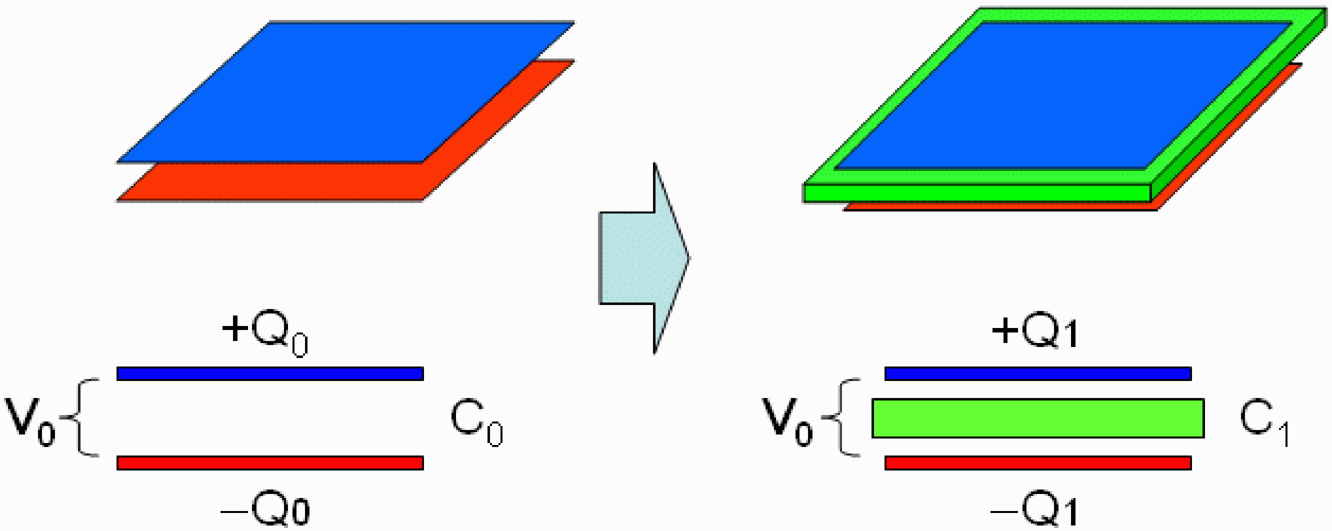
"Since the potential is greater on sphere B, the charge will flow from B to A and will increase the charge on sphere A."

Main Point 2: Capacitance = Q/V



Parallel Plate Capacitor

Two parallel plates of area carry equal and opposite charge Q_0. The potential difference between the two plates is measured to be V_0. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one.The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same as before.



THE CAPACITOR QUESTIONS WERE TOUGH!

THE PLAN:

We’ll work through the example in the prelecture and then do the checkpoint questions.

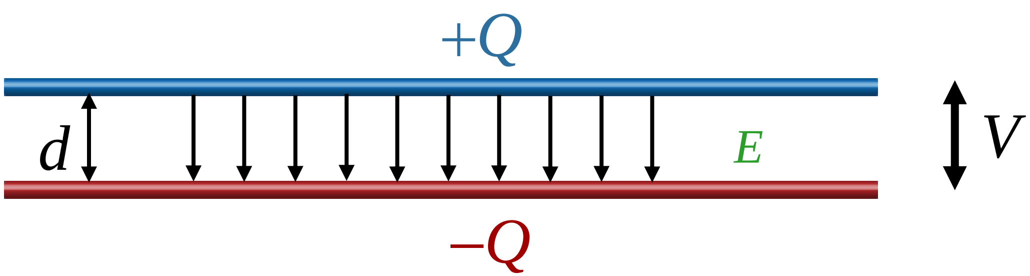
Capacitance

Capacitance is defined for any pair of spatially separated conductors.

$$C \equiv \frac{Q}{V}$$

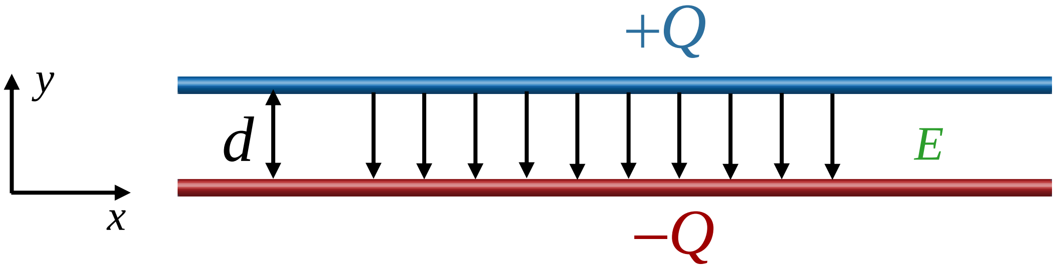
How do we understand this definition ?

- Consider two conductors, one with excess charge = $+Q$ and the other with excess charge = $-Q$



- These charges create an electric field in the space between them
- We can integrate the electric field between them to find the potential difference between the conductor
- This potential difference should be proportional to Q !
  - The ratio of Q to the potential difference is the capacitance and only depends on the geometry of the conductors

Example (done in Prelecture 7)



What is σ ?

$$E = \frac{\sigma}{\varepsilon_o} \qquad \sigma = \frac{Q}{A}$$

A = area of plate

Second, integrate E to find the potential difference V

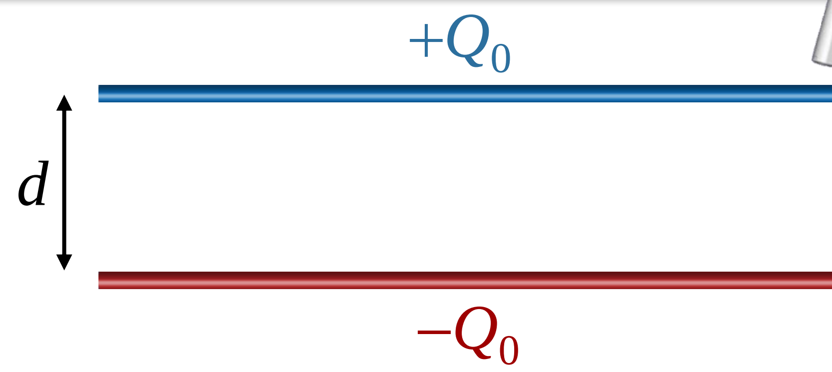
$$V = -\int_0^d \vec{E} \cdot d\vec{y} \longrightarrow V = -\int_0^d (-E dy) = E\int_0^d dy = \frac{Q}{\varepsilon_o A} d$$

As promised, V is proportional to Q !

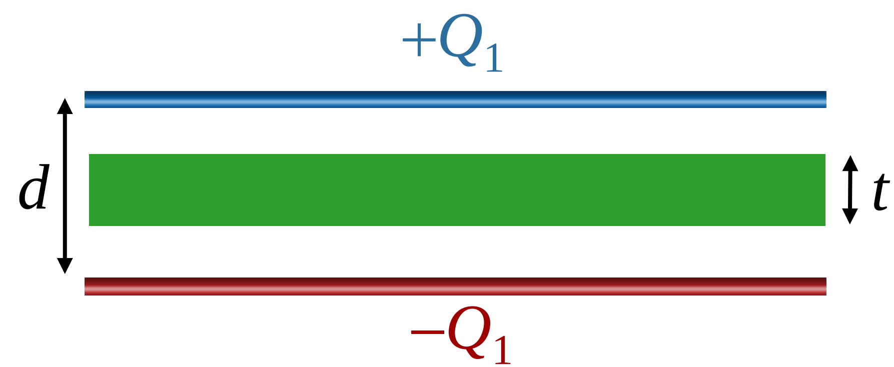
$$C \equiv \frac{Q}{V} = \frac{\cancel{Q}}{\cancel{Q}d / \varepsilon_o A} \longrightarrow C = \frac{\varepsilon_0 A}{d}$$

C determined by geometry !

Question Related to CheckPoint



Initial charge on capacitor $= Q_0$



Insert uncharged conductor

Charge on capacitor now $= Q_1$

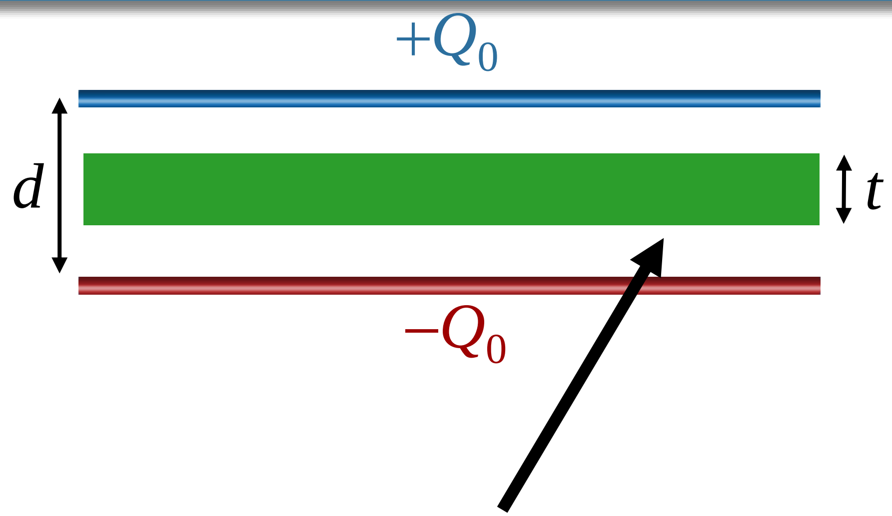
How is Q_1 related to Q_0 ?

A) $Q_1 < Q_0$
B) $Q_1 = Q_0$
C) $Q_1 > Q_0$

Plates not connected to anything

CHARGE CANNOT CHANGE !

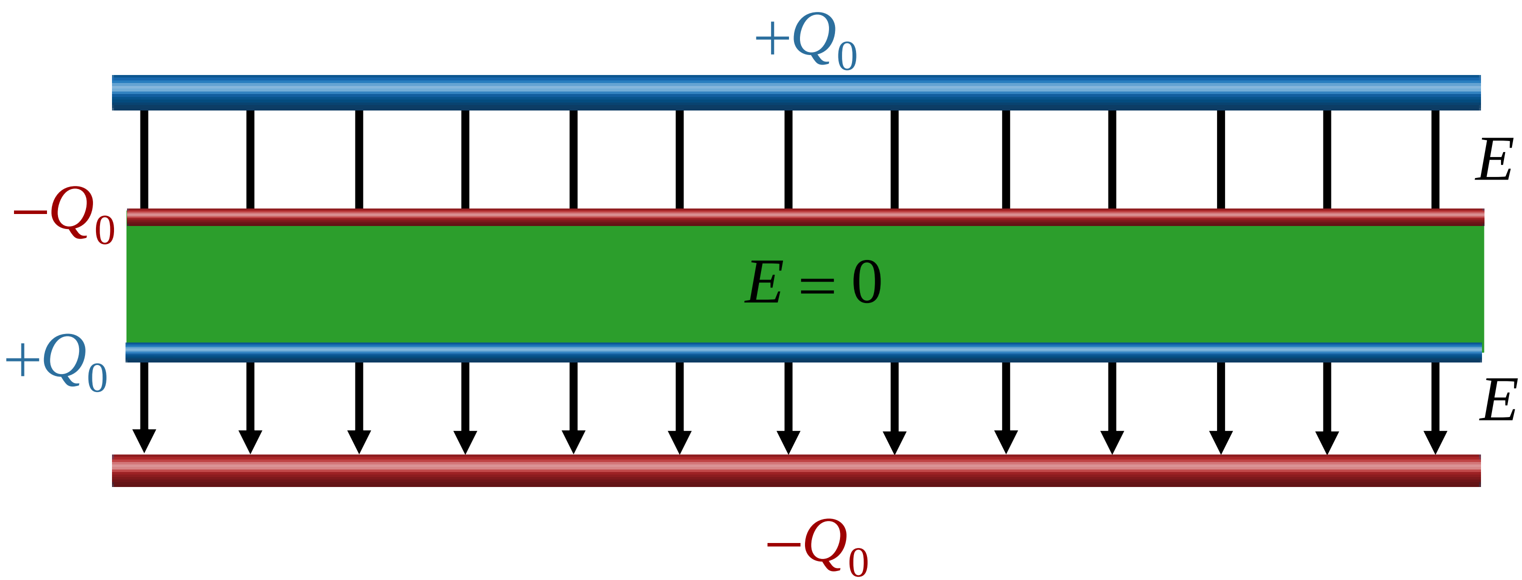
Where to Start ?



What is the total charge induced on the bottom surface of the conductor?

A) $+Q_0$
B) $+Q_0/2$
C) 0
D) $-Q_0/2$
E) $-Q_0$

Why ?



WHAT DO WE KNOW ?

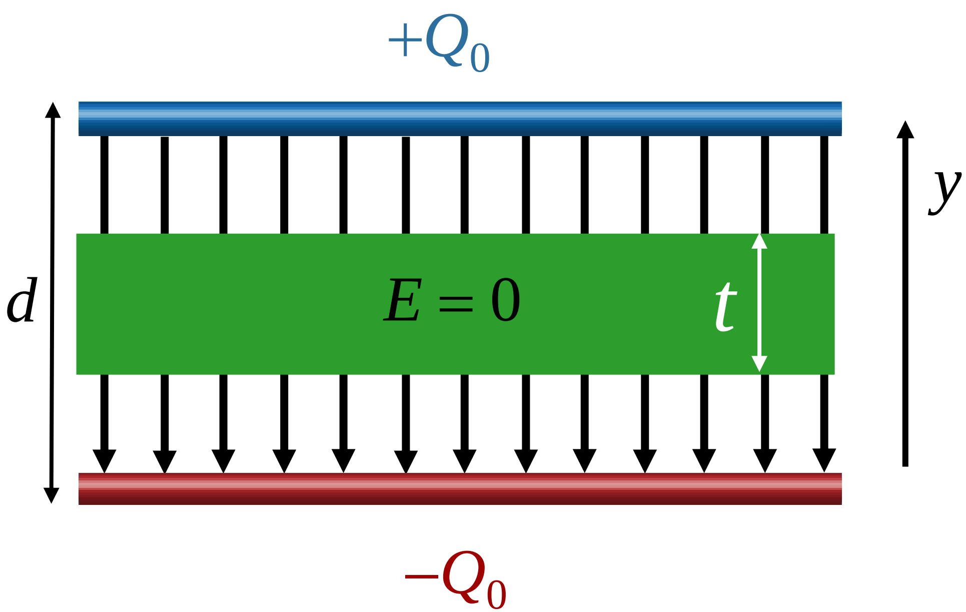
E must be $= 0$ in conductor !

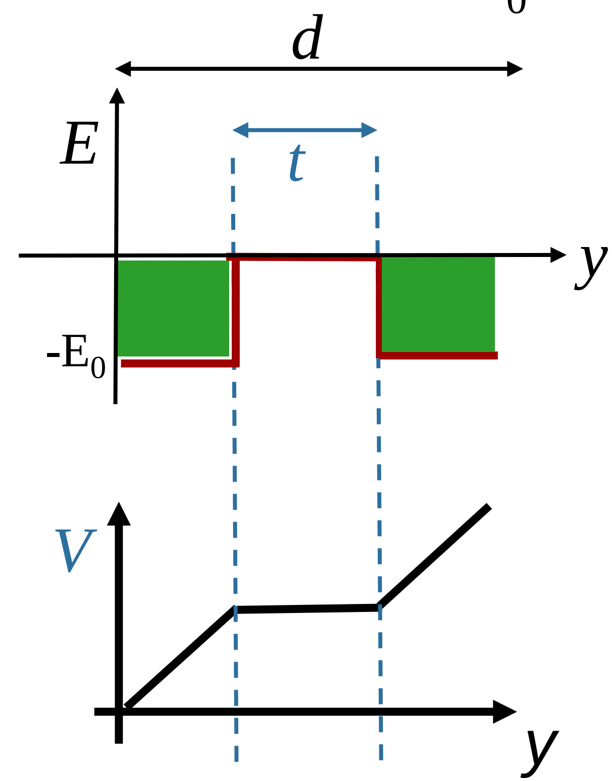
Charges inside conductor move to cancel E field from top & bottom plates.

Calculate V

Now calculate V as a function of distance from the bottom conductor.

$$V(y) = -\int_0^y \vec{E} \cdot d\vec{y}$$





What is ΔV ?

A) $\Delta V = E_0 d$

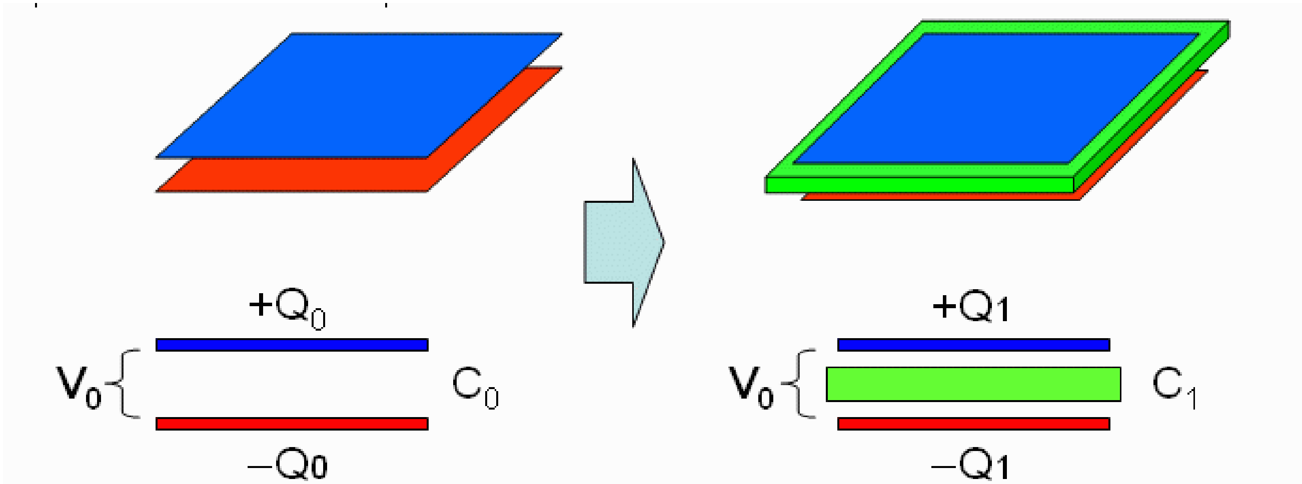
B) $\Delta V = E_0(d - t)$

C) $\Delta V = E_0(d + t)$

The integral = area under the curve

Check Point 4

Two parallel plates are given a charge Q_0 such that the potential difference between the plates is V_0. If a conductor is slid between plates, does C change?



A) $C_1 > C_o$ B) $C_1 = C_o$ C) $C_1 < C_o$

We can determine C from either case

same V (preflight)

same Q (lecture)

C depends only on geometry !

$$E_0 = Q_0 / \varepsilon_0 A$$

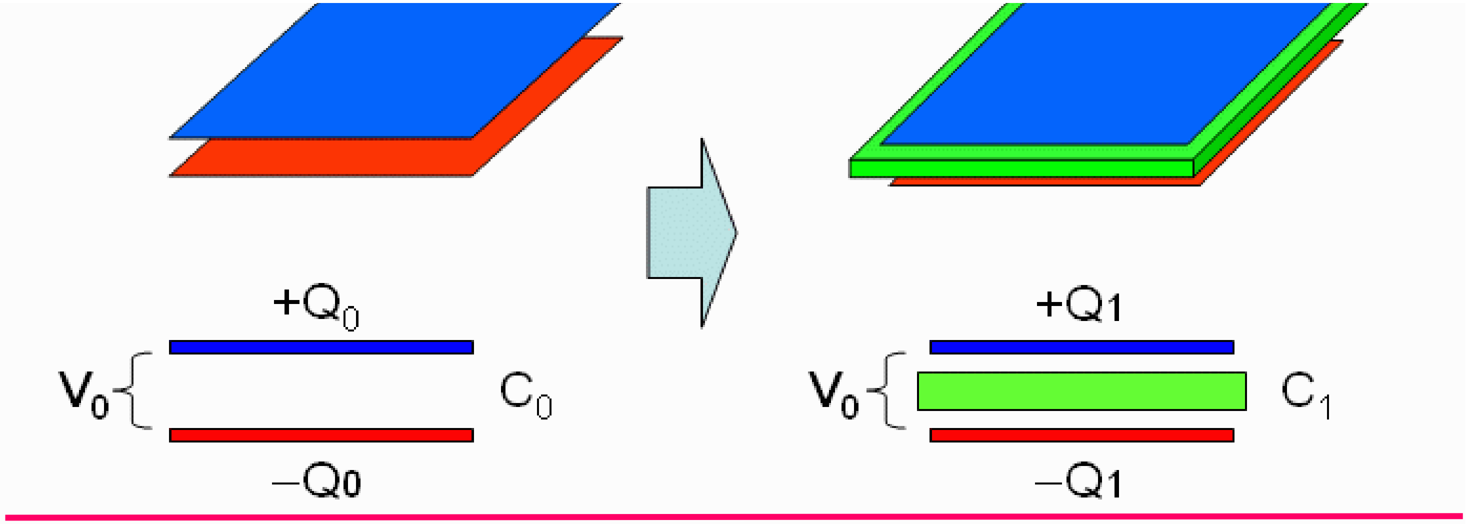
Same Q:

$$V_0 = E_0 d \qquad C_0 = Q_0 / E_0 d \qquad C_0 = \varepsilon_0 A / d$$

$$V_1 = E_0 (d - t) \qquad C_1 = Q_0 / (E_0 (d - t)) \qquad C_1 = \varepsilon_0 A / (d - t)$$

Back to Check Point 4

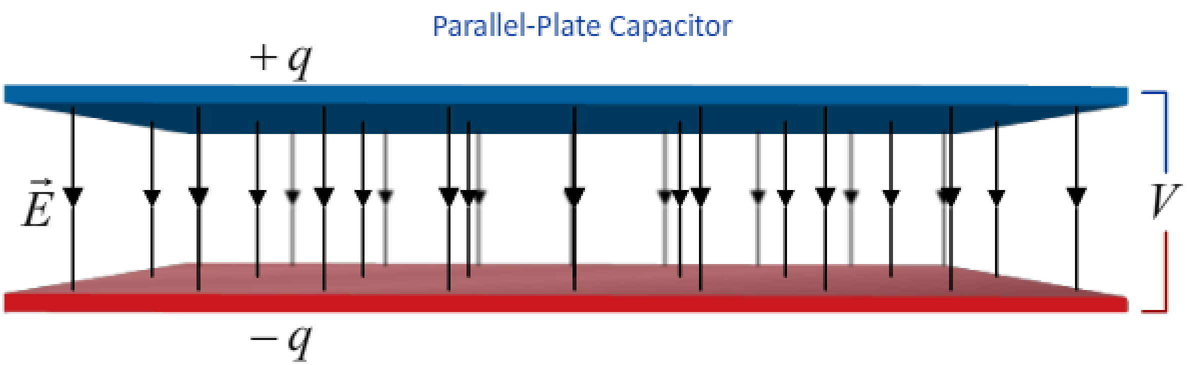
Two parallel plates are given a charge Q_0 such that the potential difference between the plates is V_0. If a conductor is slid between plates, how would charge need to be adjusted to keep same potential difference?



A) $Q_1 < Q_o$ B) $Q_1 = Q_o$ C) $Q_1 > Q_o$

"delta V= E*d, and d is smaller for the second plates as there is an uncharged conducting plate where E=0 inside. As a result, E has to be greater for the second plates, and so Q1 is greater than Q0."

Main Point 3: Capacitors Store Energy in E

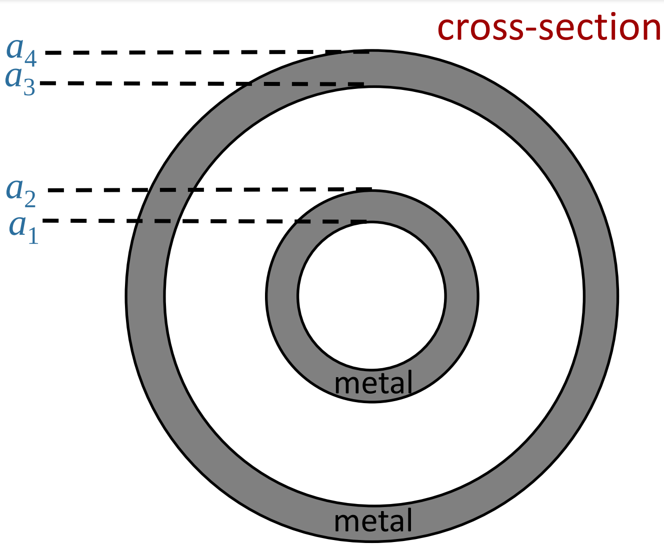


$$u = \frac{1}{2}\varepsilon_o E^2$$ Energy Density

Energy Stored in Capacitors

$$U = \frac{1}{2}QV$$ or $$U = \frac{1}{2}\frac{Q^2}{C}$$ or $$U = \frac{1}{2}CV^2$$

Calculation



A capacitor is constructed from two conducting cylindrical tubes of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

➢ Conceptual Analysis:

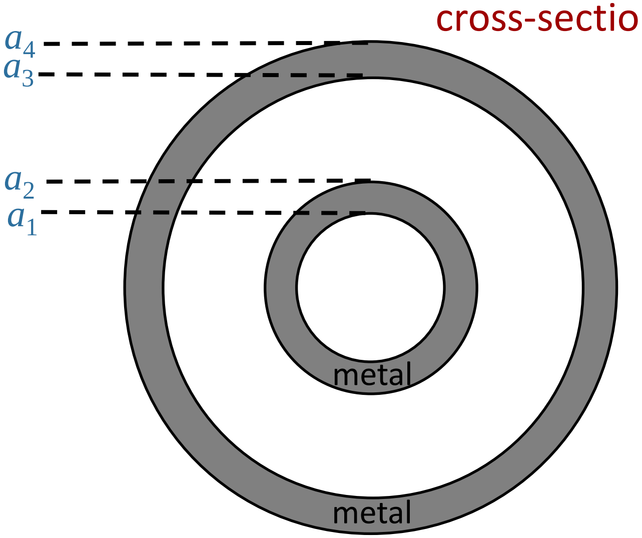
$$C \equiv \frac{Q}{V}$$

But what is Q and what is V ? They are not given?

➢Important Point: C is a property of the object! (concentric cylinders here)

- Assume some Q (i.e., $+Q$ on one conductor and $-Q$ on the other)
- These charges create E field in region between conductors
- This E field determines a potential difference V between the conductors
- V should be proportional to Q; the ratio Q/V is the capacitance.

Calculation



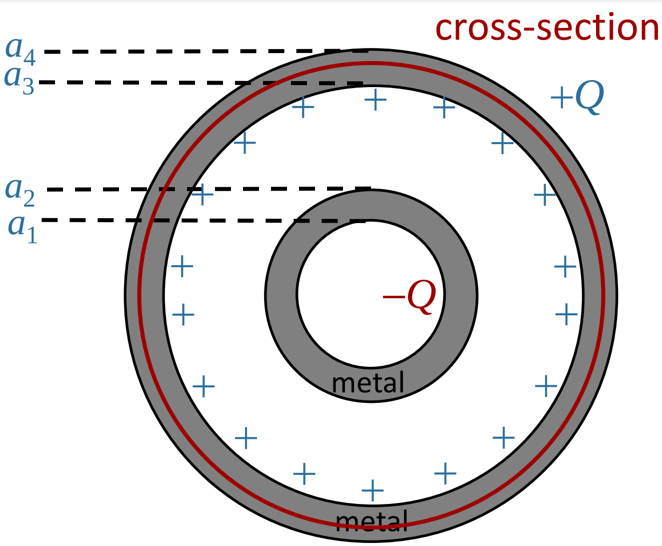
A capacitor is constructed from two conducting cylindrical tubes of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

- Strategic Analysis:
  - Put $+Q$ on outer shell and $-Q$ on inner shell
  - Cylindrical symmetry: Use Gauss' Law to calculate E everywhere
  - Integrate E to get V
  - Take ratio Q/V: should get expression only using geometric parameters (a_i, L)

Calculation



A capacitor is constructed from two conducting cylindrical tubes of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $+Q$ on outer conductor located?

A) at $r = a_4$ B) at $r = a_3$ C) both surfaces D) throughout tube

Why?

Gauss' law: $$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

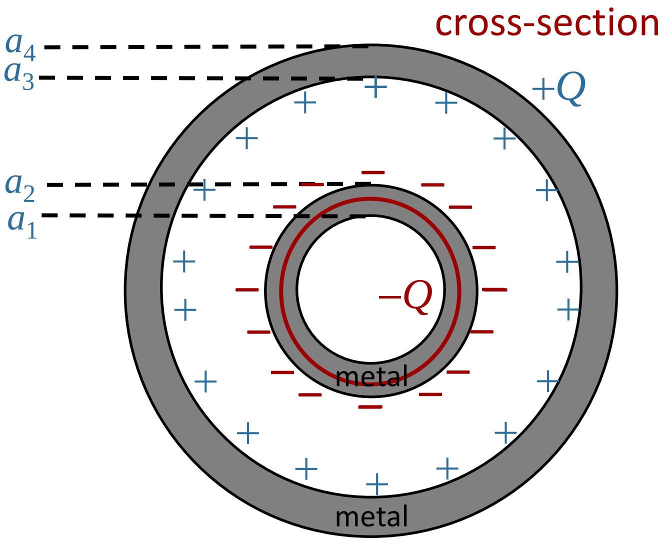
We know that $E = 0$ in conductor (between a_3 and a_4)

$$Q_{enclosed} = 0$$

$$Q_{enclosed} = 0$$

$+Q$ must be on inside surface (a_3), so that $Q_{enclosed} = +Q - Q = 0$

Calculation



A capacitor is constructed from two conducting cylindrical tubes of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $-Q$ on inner conductor located?

A) at $r = a_2$ B) at $r = a_1$ C) both surfaces D) throughout tube

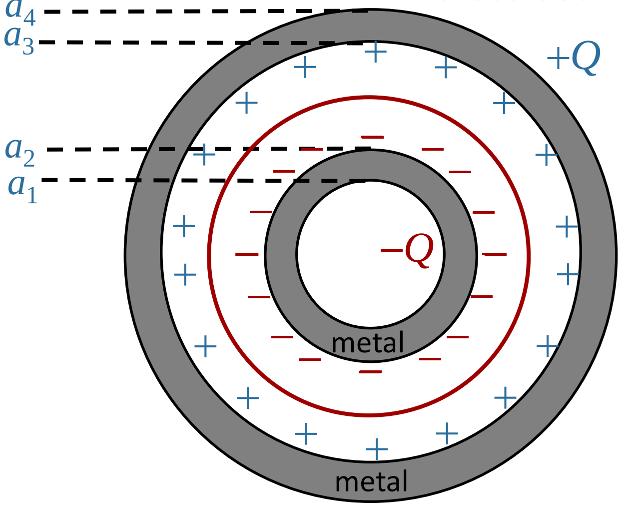
Why?

Gauss' law: $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$

We know that $E = 0$ in conductor (between a_1 and a_2) $\longrightarrow$ $Q_{enclosed} = 0$

$Q_{enclosed} = 0$ $\longrightarrow$ $+Q$ must be on outer surface (a_2), so that $Q_{enclosed} = 0$

Calculation



A capacitor is constructed from two conducting cylindrical tubes of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$: What is $|E(r)|$?

A) 0

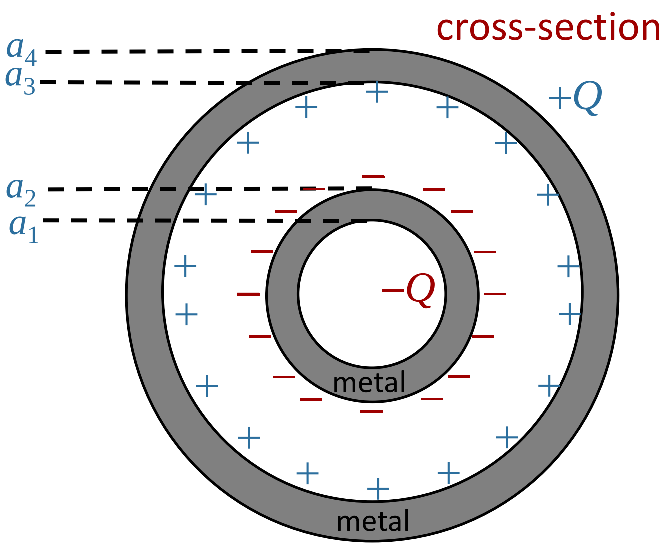
B) $\dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r^2}$

C) $\dfrac{1}{2\pi\varepsilon_o}\dfrac{Q}{Lr}$

D) $\dfrac{1}{2\pi\varepsilon_o}\dfrac{2Q}{Lr}$

E) $\dfrac{1}{4\pi\varepsilon_o}\dfrac{2Q}{r^2}$

Calculation



A capacitor is constructed from two conducting cylindrical tubes of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V} \qquad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

What is $V \equiv V_{outer} - V_{inner}$?

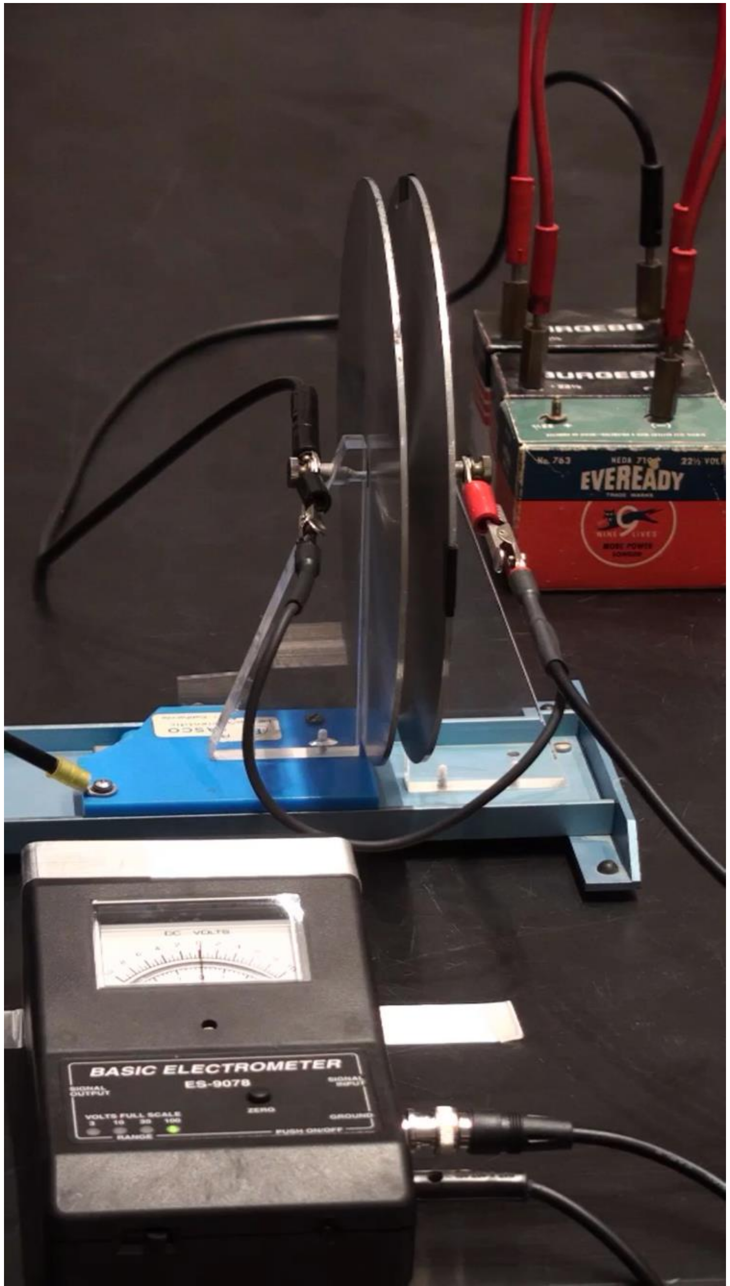
(A) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_1}{a_4}$

(B) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_4}{a_1}$

(C) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_3}{a_2}$

(D) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_2}{a_3}$

Voltage across a parallel plate capacitor



Physics 212
Lecture 8

Today's Concept:

Capacitors

(Capacitors in a circuits, Dielectrics, Energy in capacitors)

So what exactly does capacitance mean? What does a capacitor do in real life?

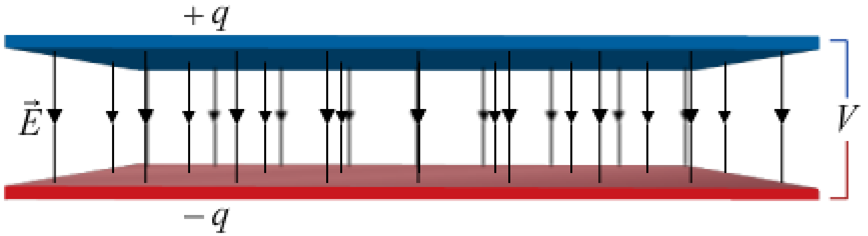
Energy in Capacitors (from lect 7)

Energy Stored in Capacitors

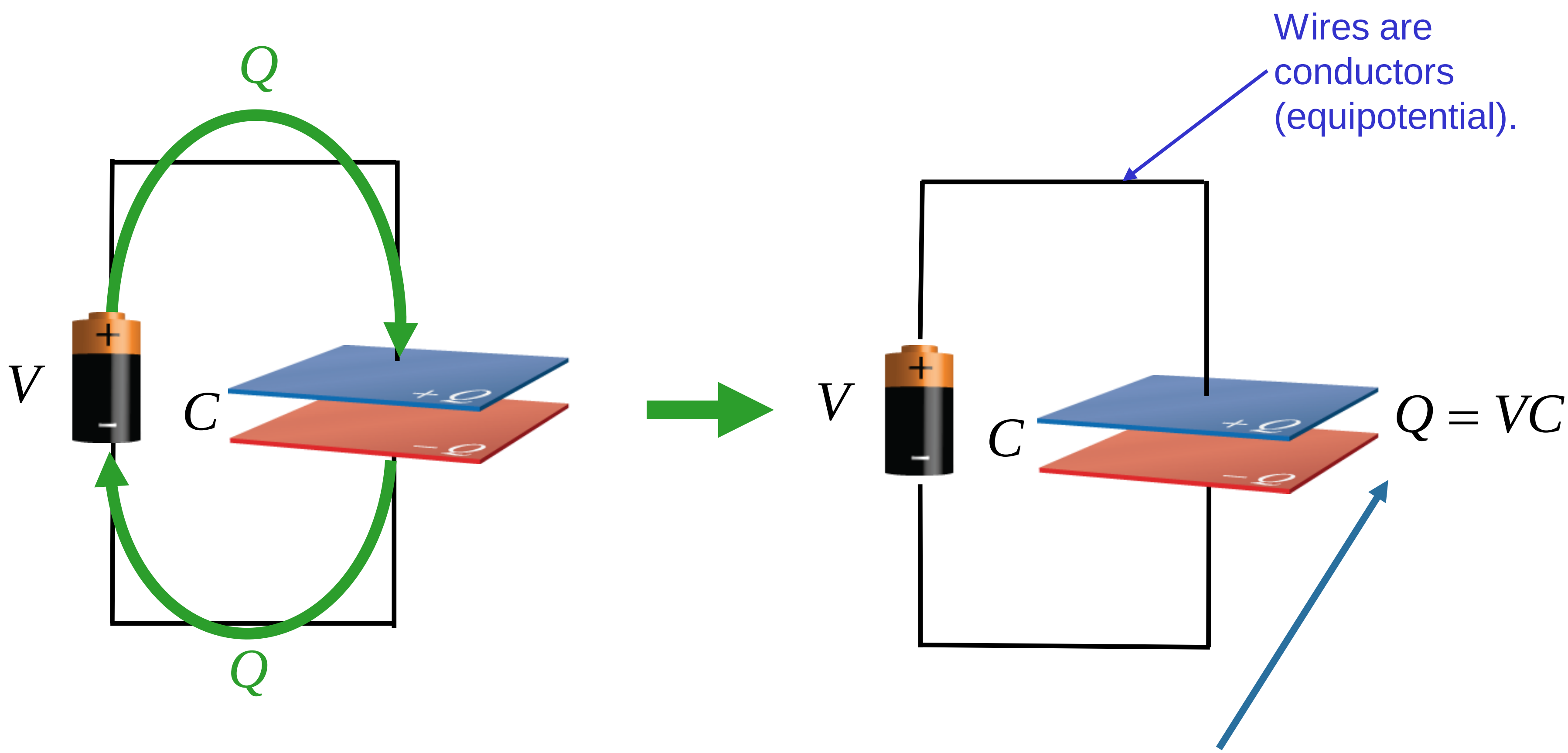
$$\boxed{U = \frac{1}{2} QV}$$ or $$\boxed{U = \frac{1}{2}\frac{Q^2}{C}}$$ or $$\boxed{U = \frac{1}{2} CV^2}$$

Energy Density

$$\boxed{u = \frac{1}{2}\varepsilon_o E^2}$$

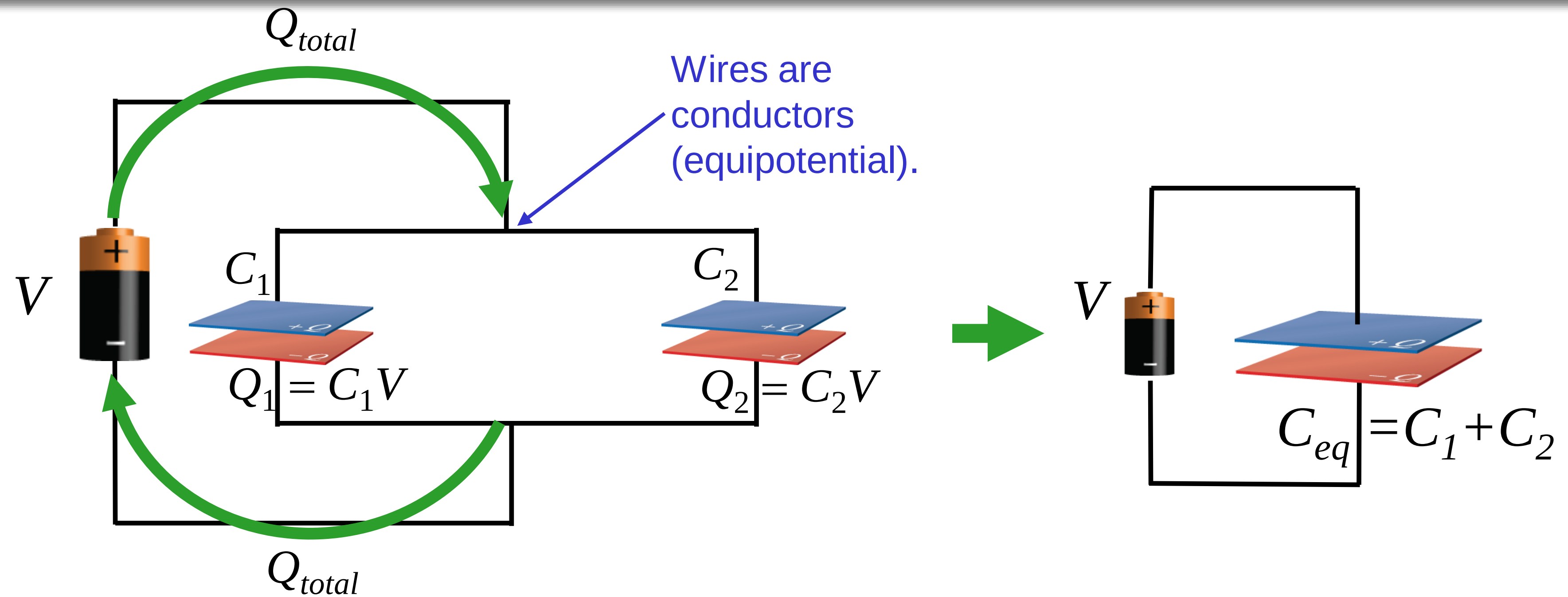


Simple Capacitor Circuit



This "Q" really means that the battery has moved charge Q from one plate to the other, so that one plate holds $+Q$ and the other $-Q$.

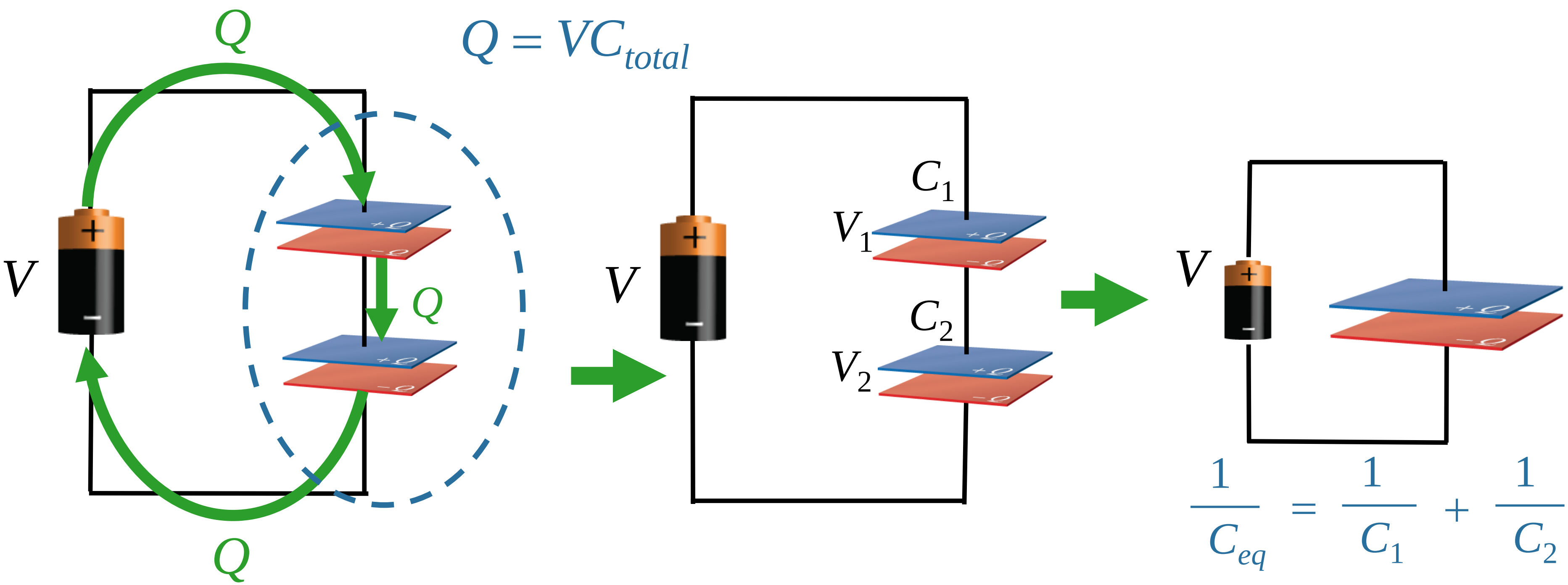
Parallel Capacitor Circuit



Key point: V is the same for both capacitors

Key Point: $Q_{total} = Q_1 + Q_2 = VC_1 + VC_2 = V\underbrace{(C_1 + C_2)}_{C_{total} = C_1 + C_2}$

Series Capacitor Circuit



Key point: Q is the same for both capacitors

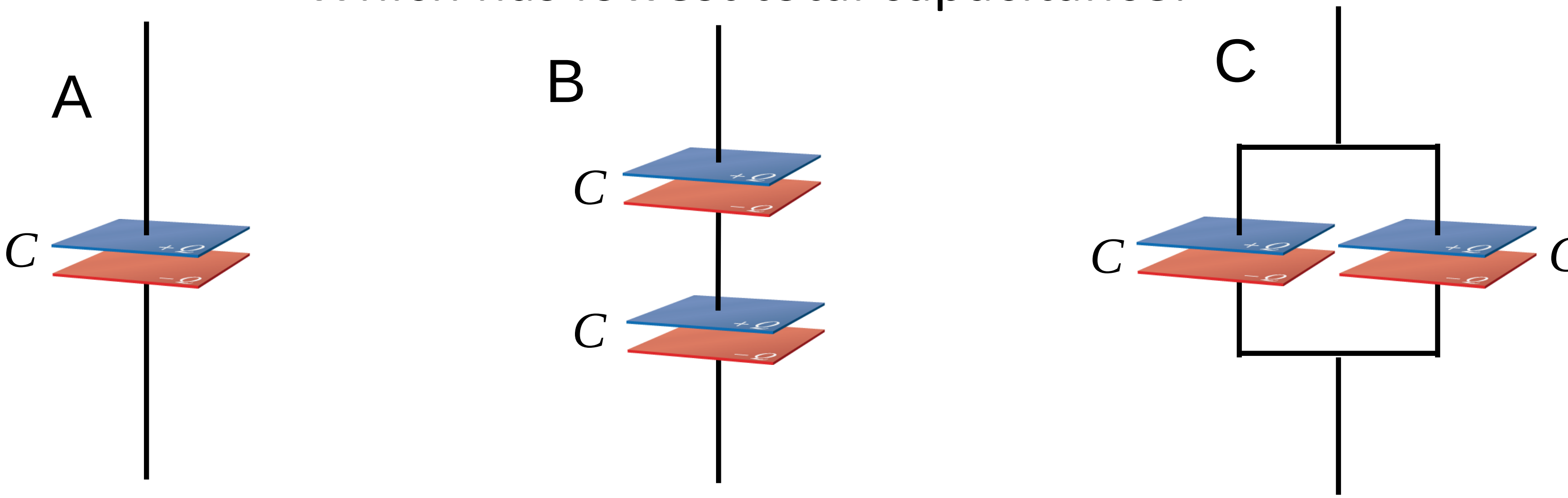
Key point: $Q = VC_{total} = V_1C_1 = V_2C_2$

Also: $V = V_1 + V_2$ $\longrightarrow$ $Q/C_{total} = Q/C_1 + Q/C_2$

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Check Point 1

Which has lowest total capacitance:

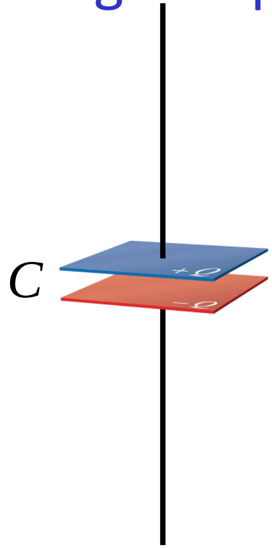


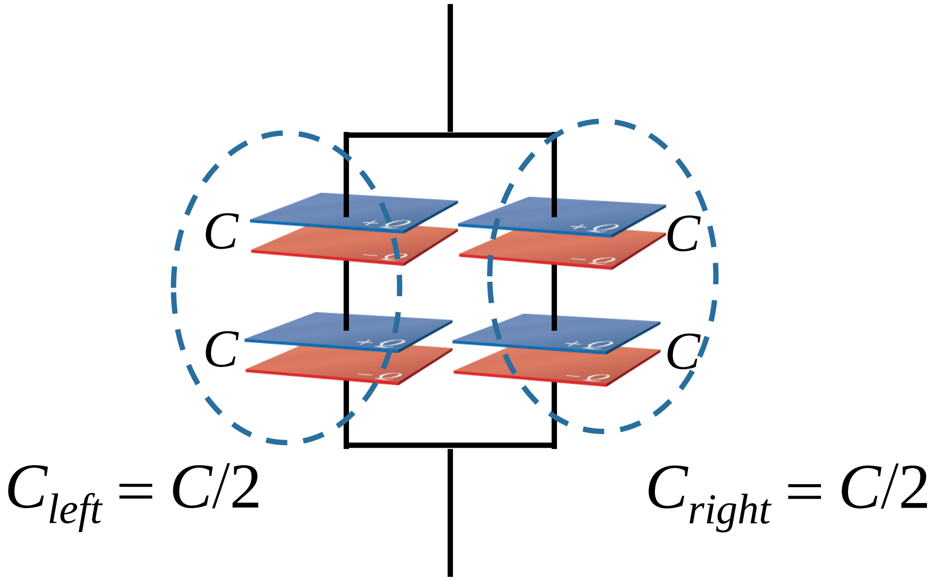
Check Point 2

Which has lowest total capacitance?

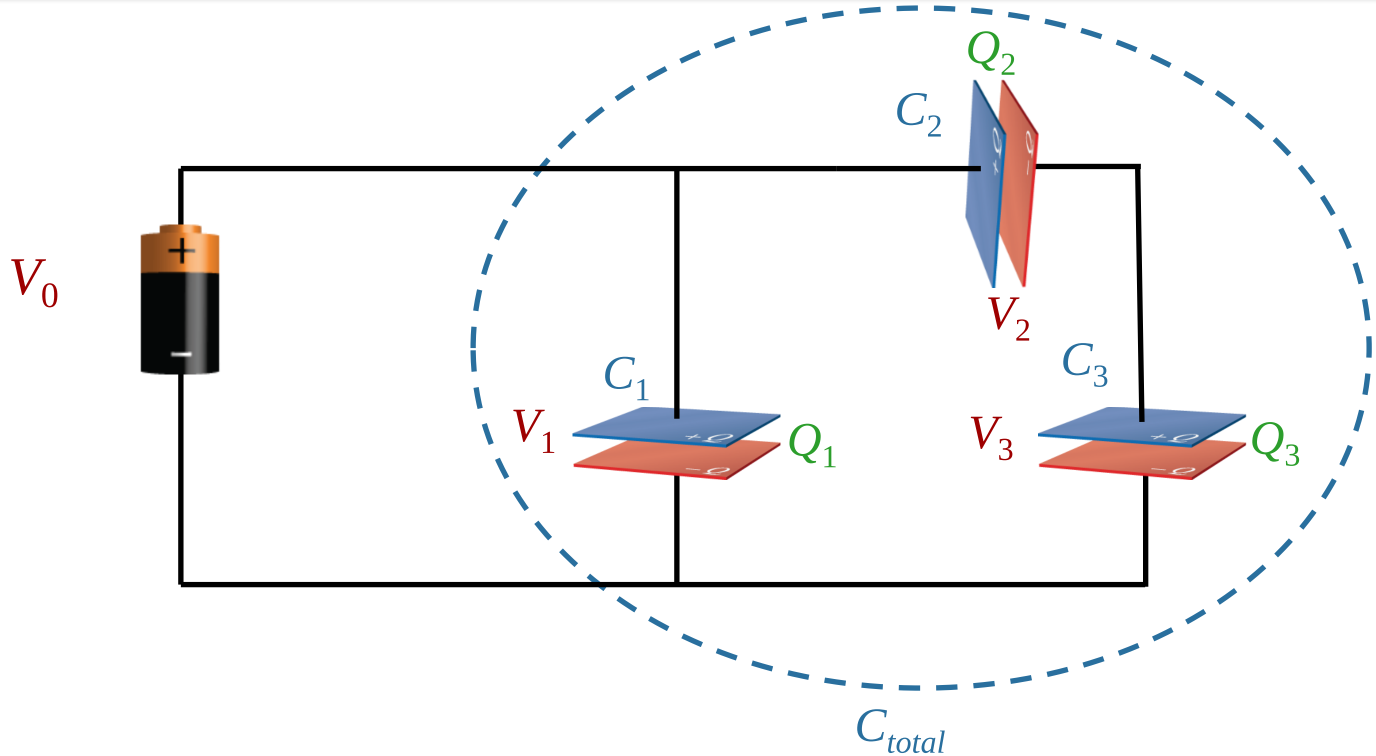
A) Single Capacitor B) 4 Capacitors C) Same

:





Similar to CheckPoint 3

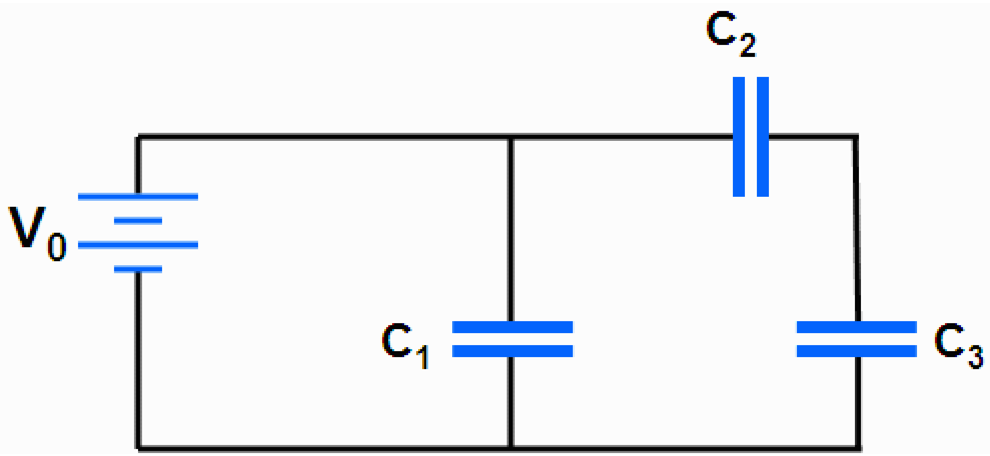


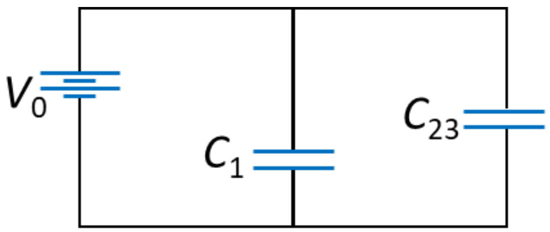
Which of the following is NOT necessarily true:

A) $V_0 = V_1$
B) $C_{total} > C_1$
C) $V_2 = V_3$
D) $Q_2 = Q_3$
E) $V_1 = V_2 + V_3$

Check Point 3

A circuit consists of three unequal capacitors C_1, C_2, and C_3 which are connected to a battery of voltage V_0. The capacitance of C_2 is twice that of C_1. The capacitance of C_3 is three times that of C_1. The capacitors obtain charges Q_1, Q_2, and Q_3.





~~A.~~ $Q_1 > Q_3 > Q_2$ ~~B.~~ $Q_1 > Q_2 > Q_3$ **C.** $Q_1 > Q_2 = Q_3$ **D.** $Q_1 = Q_2 = Q_3$ **E.** $Q_1 < Q_2 = Q_3$

1. : $Q_2 = Q_3$ (capacitors in series)

2. How about Q_1 vs. Q_2 and Q_3? Calculate C_{23} first.

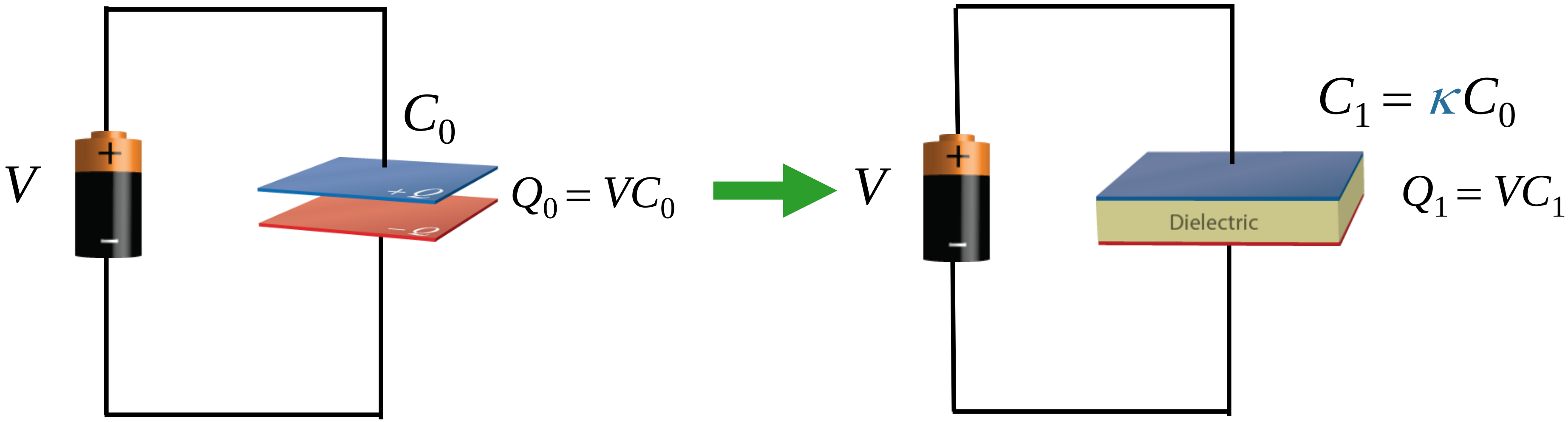
$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{2C_1} + \frac{1}{3C_1} = \frac{5}{6C_1}$$

$$\boxed{C_{23} = \frac{6}{5}C_1}$$

$$Q_1 = C_1 V_0$$

$$Q_{23} = Q_2 = Q_3 = C_{23}V_0 = \frac{6}{5}C_1 V_0$$

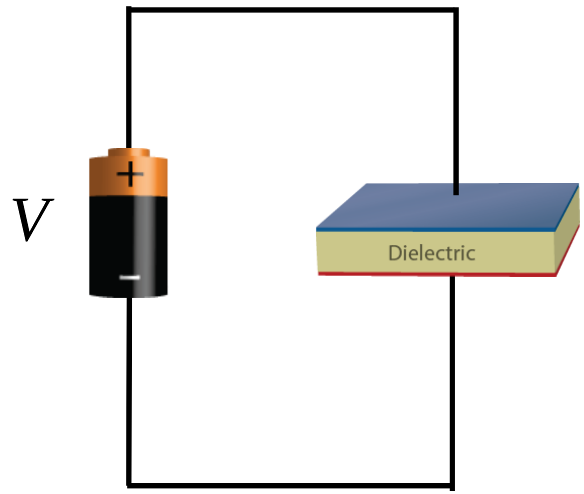
Dielectrics



By adding a dielectric, you are just making a new capacitor with larger capacitance (factor of κ)

Messing with Capacitors

If connected to a battery V stays constant



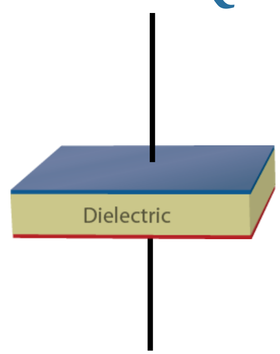
$$V_1 = V$$

$$C_1 = \kappa C$$

$$Q_1 = C_1 V_1$$

$$= \kappa C V = \kappa Q$$

If isolated, then total Q stays constant



$$Q_1 = Q$$

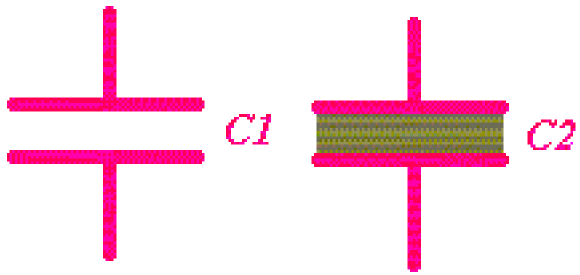
$$C_1 = \kappa C$$

$$V_1 = Q_1 / C_1$$

$$= Q / \kappa C = V / \kappa$$

Check Point 4a

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. Then, a dielectric is placed between the plates of C_2



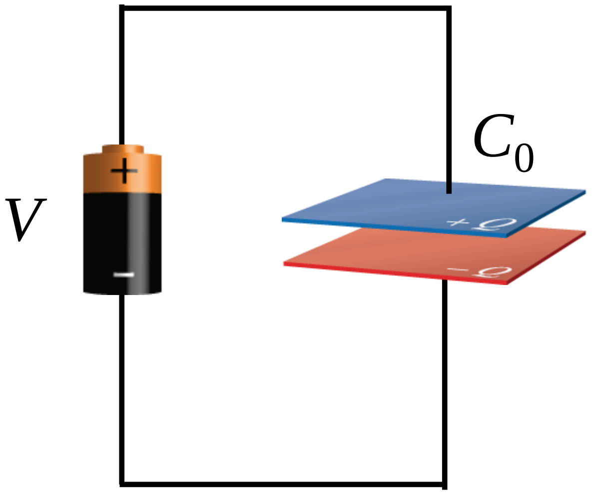
Compare the voltages of the two capacitors.

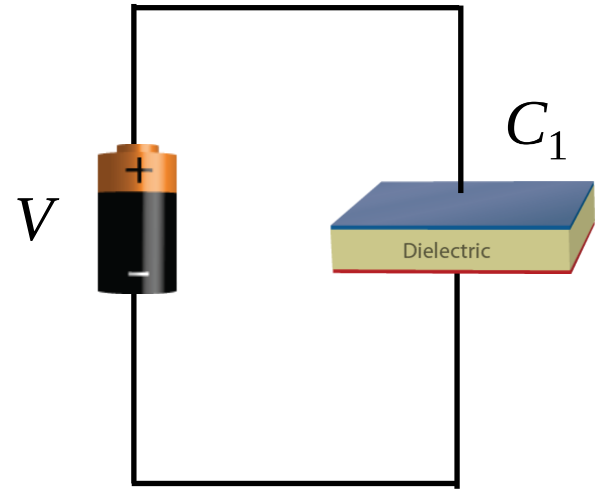
A $V_1 > V_2$ B $V_1 = V_2$ C $V_1 < V_2$

"Q is constant, C increases, C = Q/V."

Messing with Capacitors Clicker Question

Two identical parallel plate capacitors are connected to identical batteries. Then a dielectric is inserted between the plates of capacitor C_1. Compare the energy stored in the two capacitors.





A) $U_1 < U_0$ B) $U_0 = U_1$ C) $U_1 > U_0$

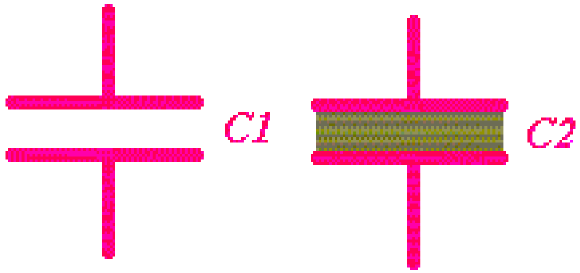
Compare using $U = {}^1\!/_2 CV^2$

$$U_1/U_0 = \kappa$$

→ Potential Energy goes UP

CheckPoint 4b

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. Then, a dielectric is placed between the plates of C_2

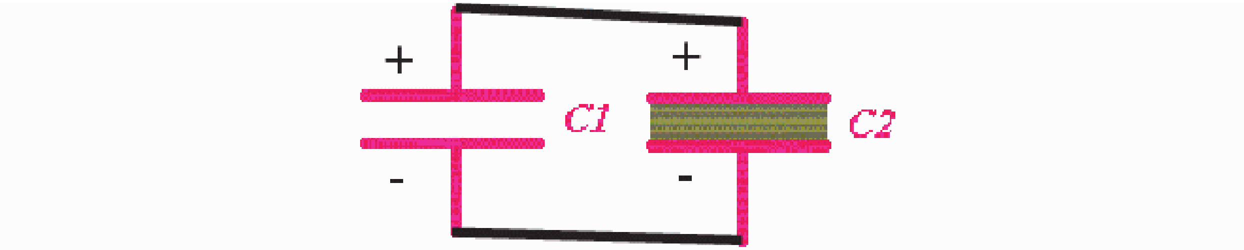


Compare the potential energy stored by the two capacitors.

A) $U_1 > U_2$ B) $U_1 = U_2$ C) $U_1 < U_2$

CheckPoint 4c

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C_2 has been charged and disconnected, it is filled with a dielectric. **The two capacitors are now connected to each other by wires as shown. How will the charge redistribute itself, if at all?**



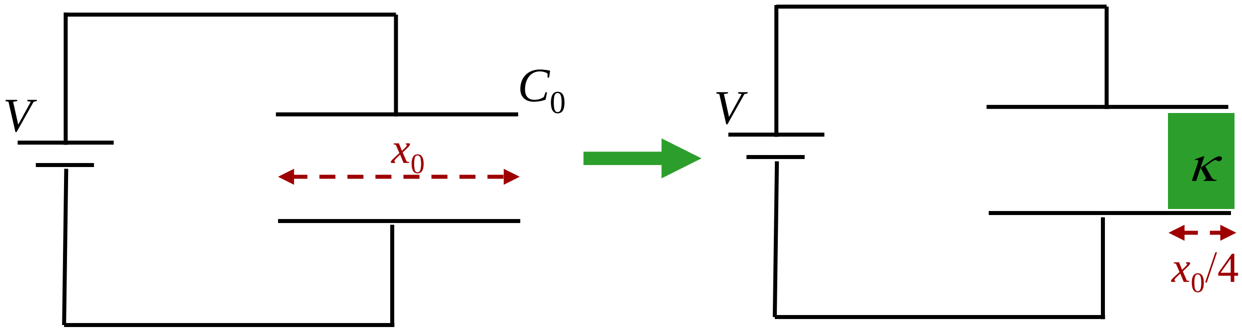
A. The charges will flow so that the charge on C_1 will become equal to the charge on C_2.
B. The charges will flow so that the energy stored in C_1 will become equal to the energy stored in C_2.
C. The charges will flow so that the potential difference across C_1 will become the same as the potential difference across C_2.
D. No charges will flow. The charge on the capacitors will remain what it was before they were connected.

V must be the same !!

Q: $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ $\Rightarrow$ $\boxed{Q_1 = \frac{C_1}{C_2} Q_2}$

U: $U_1 = \frac{1}{2} C_1 V^2$, $U_2 = \frac{1}{2} C_2 V^2$ $\Rightarrow$ $\boxed{U_1 = \frac{C_1}{C_2} U_2}$

Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

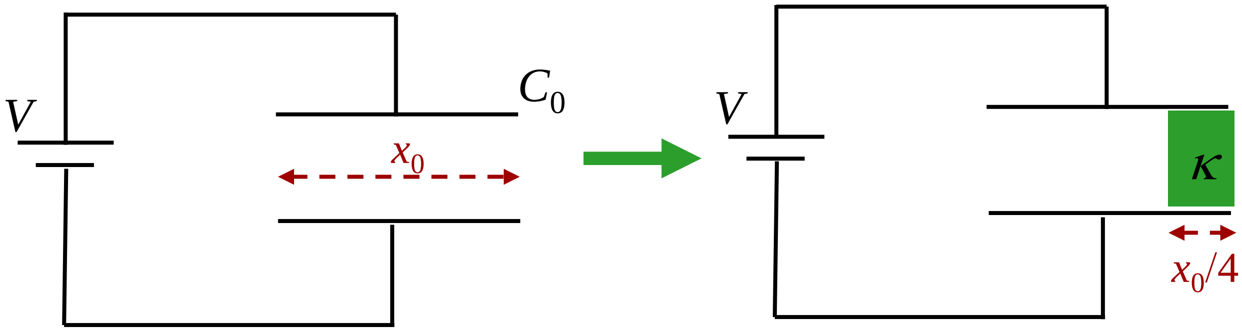
What changes when the dielectric added?

A) Only C B) only Q C) only V D) C and Q E) C and V

Adding dielectric changes the physical capacitor → C changes

V does not change and C changes → Q changes

Calculation



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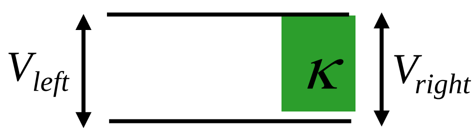
A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Strategic Analysis:

- Calculate new capacitance C
- Apply definition of capacitance to determine Q

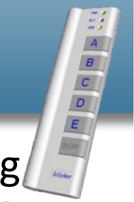
To calculate C, let's first look at:

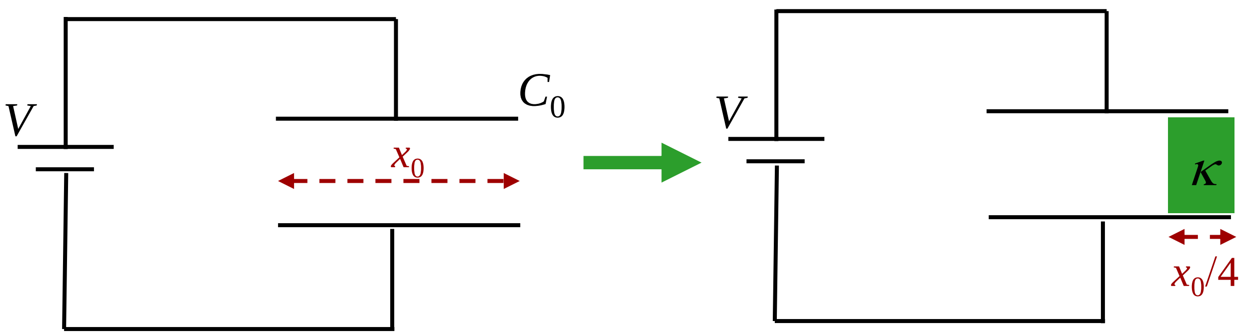


A) $V_{left} < V_{right}$ B) $V_{left} = V_{right}$ C) $V_{left} > V_{right}$

The conducting plate is an equipotential !

Calculation





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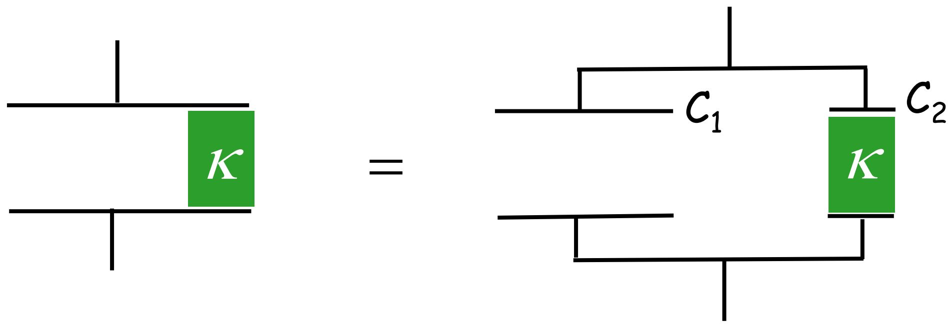
A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

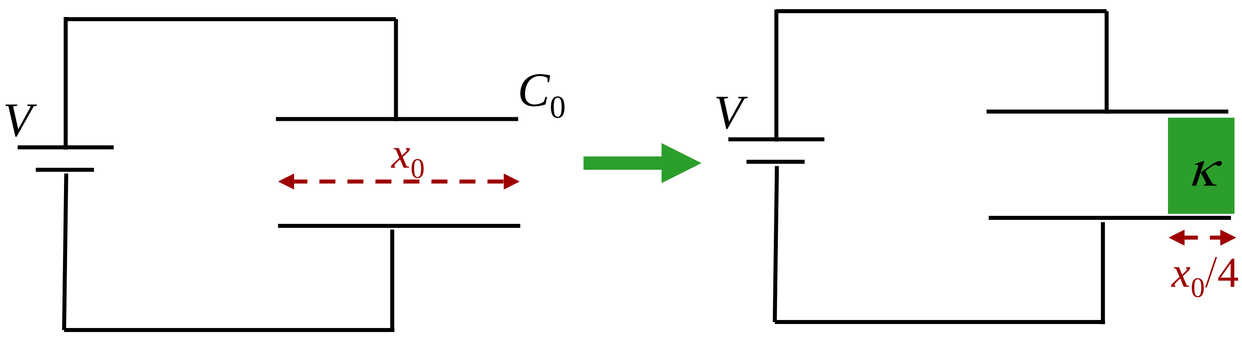
Can consider capacitor to be two capacitances, C_1 and C_2, in parallel



What is C_1 ?

A) $C_1 = C_0$ B) $C_1 = {}^3/_4 C_0$ C) $C_1 = {}^4/_3 C_0$ D) $C_1 = {}^9/_{16} C_0$

Calculation



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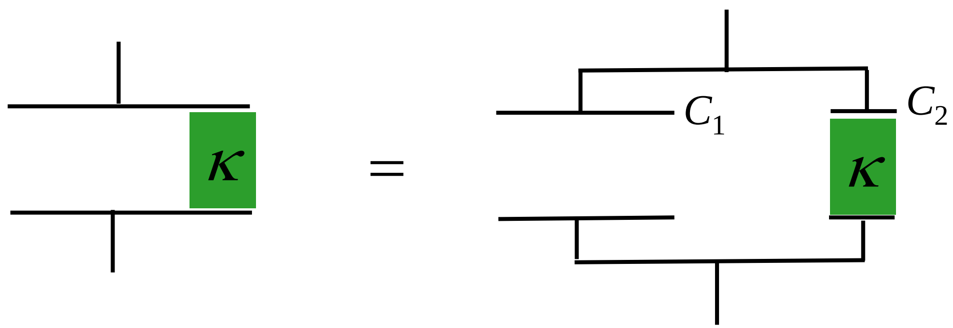
What is Q_f, the final charge on the capacitor?



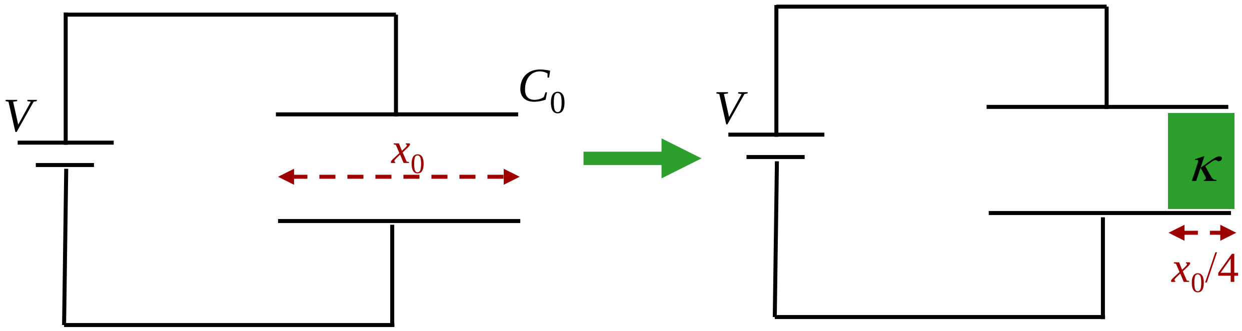
$C_1 = {}^3/_4 C_0$

What is C_2 ?

A) $C_2 = \kappa C_0$ B) $C_2 = {}^3/_4\, \kappa C_0$ C) $C_2 = {}^4/_3\, \kappa C_0$ D) $C_2 = {}^1/_4\, \kappa C_0$

In general. For parallel plate capacitor filled with dielectric: $C = \kappa \varepsilon_0 A/d$

Calculation



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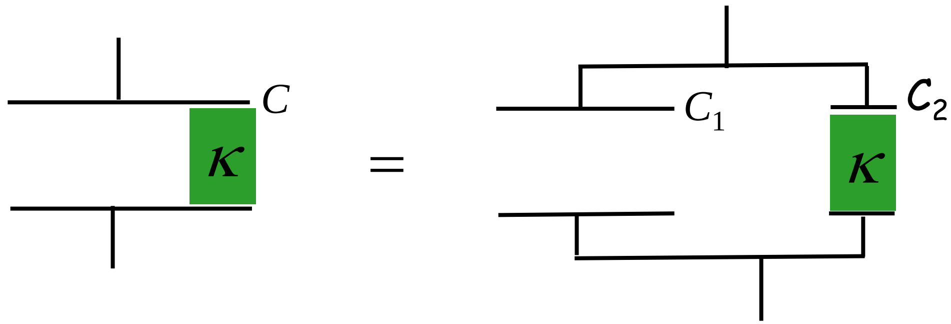
What is Q_f, the final charge on the capacitor?



$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa C_0$

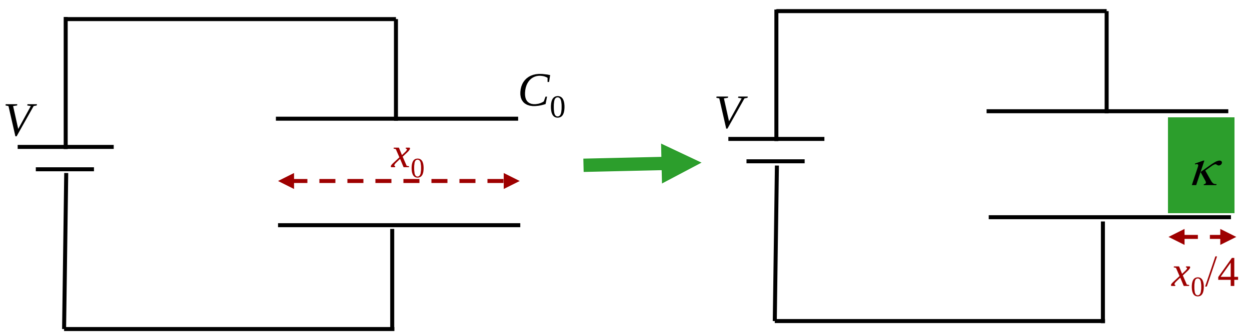
What is C?

A) $C = C_1 + C_2$ $\qquad$ B) $\qquad$ C) $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)^{-1}$

C = parallel combination of C_1 and C_2: $\quad C = C_1 + C_2$

$$C = C_0\,({}^3/_4 + {}^1/_4\,\kappa)$$

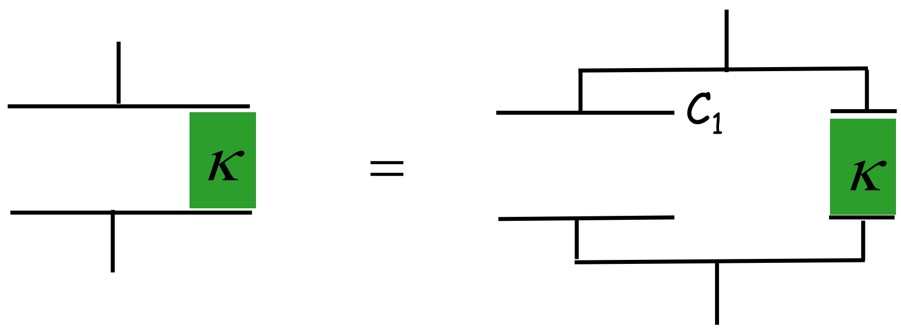
Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?



$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa C_0$

$\rightarrow$ $C = C_0\,({}^3/_4 + {}^1/_4\, \kappa)$

What is Q?

$$C \equiv \frac{Q}{V} \;\rightarrow\; Q = VC$$

$$Q_f = VC_0\left(\frac{3}{4} + \frac{1}{4}\kappa\right)$$

Electric Current

Physics 212
Lecture 9

Today's Concept:

Ohm's Law, Resistors in circuits

Current and Resistance

Key Concepts:

1) How resistance depends on A, L, σ, r

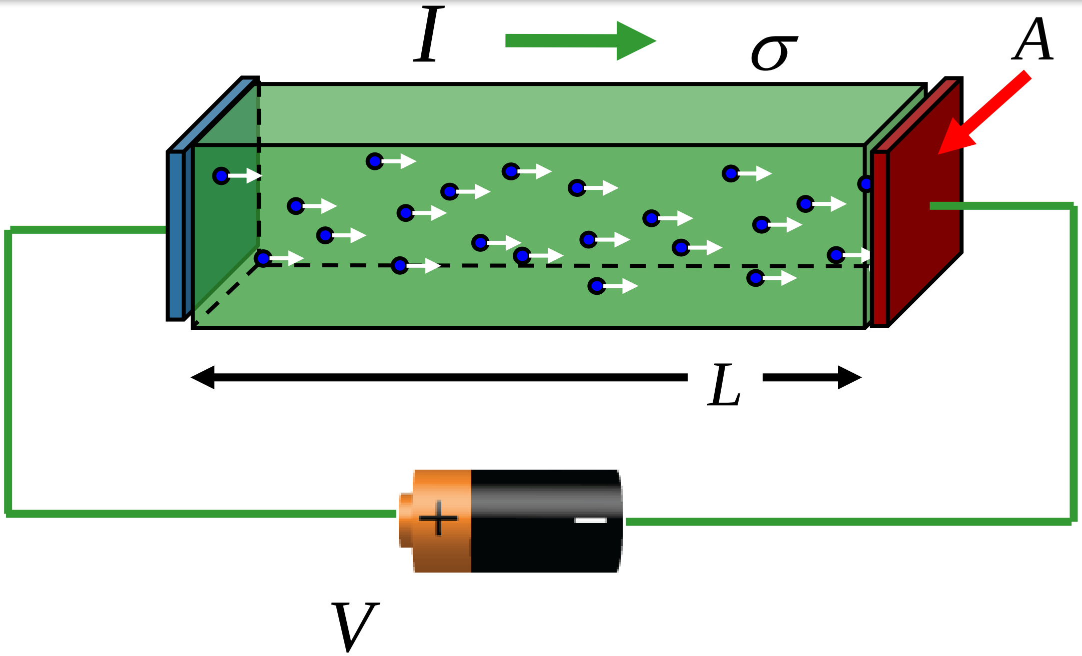
2) How to combine resistors in series and parallel

3) Understanding resistors in circuits

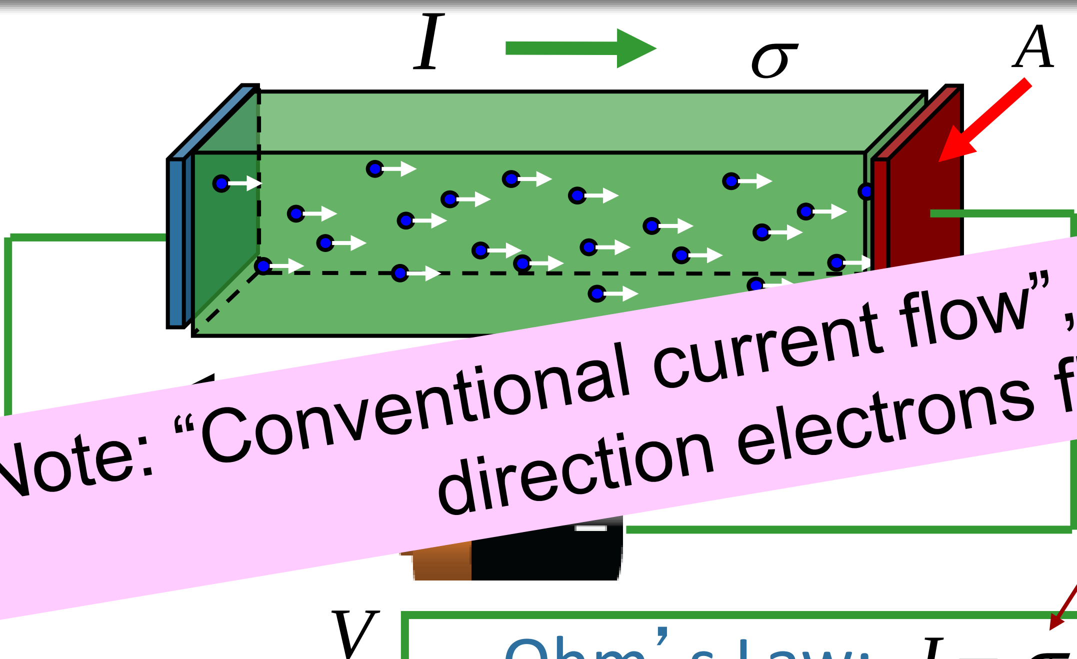
Today's Plan:

1) Review of resistance & prelectures

2) Work out a circuit problem in detail



I
σ
A
L
+
V



Note: "Conventional current flow", I, is opposite to direction electrons flow

Conductivity – high for good conductors.

$$\boxed{\text{Ohm's Law: } J = \sigma E}$$

Observables:

$$V = EL$$
$$I = JA$$

$$I/A = \sigma V/L$$

$$I = V/(L/\sigma A)$$

$$R = \frac{L}{\sigma A}$$

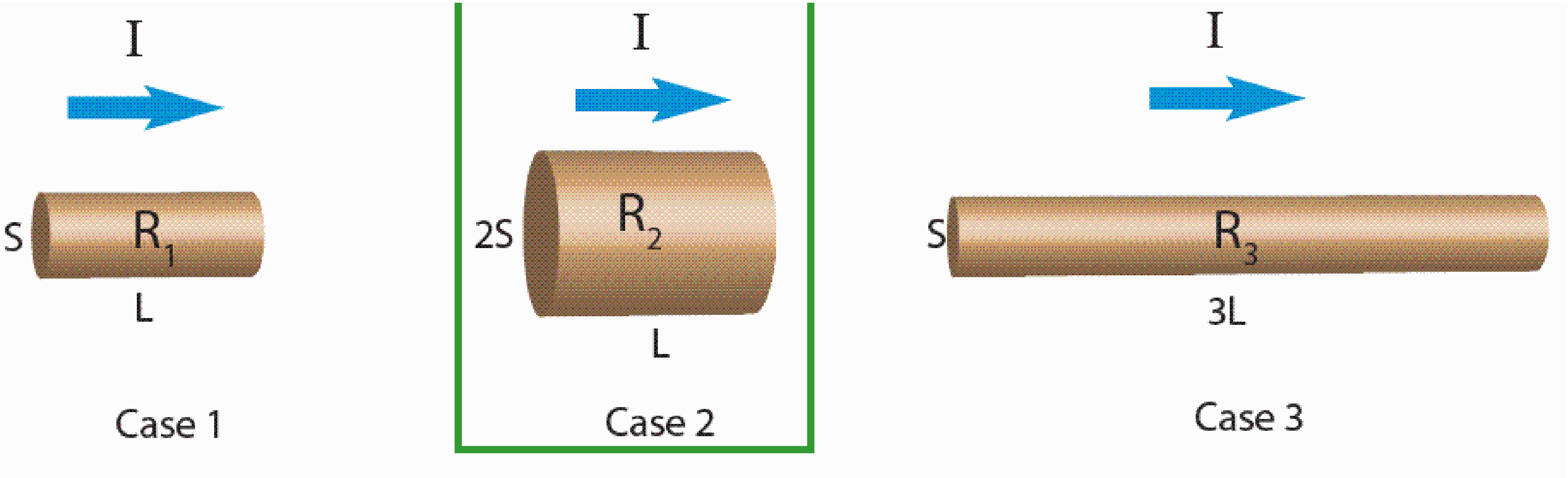
$$\boxed{I = V/R}$$

R = Resistance

$$\rho = 1/\sigma$$

Check Point 1

The SAME amount of current I passes through three different resistors. R_2 has twice the cross-sectional area and the same length as R_1, and R_3 is three times as long as R_1 but has the same cross-sectional area as R_1.



In which case is the CURRENT DENSITY through the resistor the smallest?

A. Case 1 ***B.*** Case 2 ***C.*** Case 3

$$J \equiv \frac{I}{A} \longrightarrow J_1 = J_3 = 2J_2$$

Same Current $\longrightarrow$ $J \propto \dfrac{1}{A}$

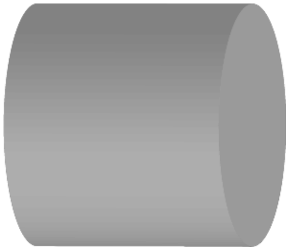
This is just like Plumbing!

I is like flow rate of water (gallons/hour)
V is like pressure
R is how hard it is for water to flow in a pipe

To make R big, make L long or A small

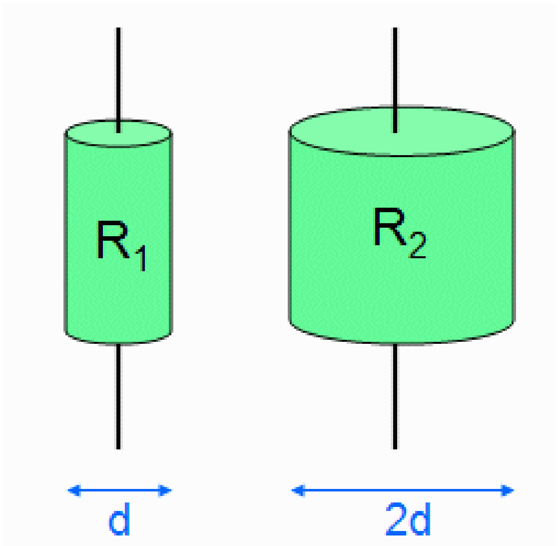
$$R = \frac{L}{\sigma A}$$

To make R small, make L short or A big



Check Point 2a

Check Point 2b



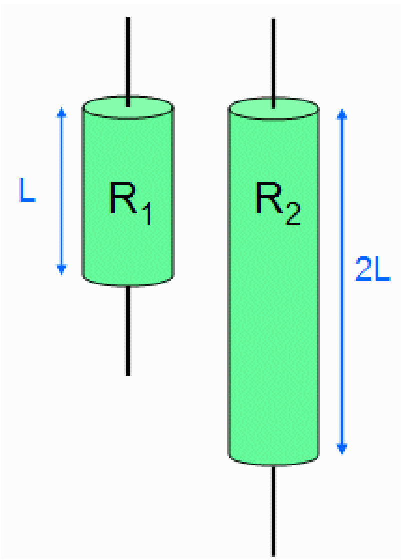
$V_1 > V_2$ A $V_1 = V_2$ B $V_1 < V_2$ C

Same current through both resistors

Compare voltages across resistors

$$R \propto \frac{L}{A}$$

$$V = IR \propto \frac{L}{A}$$

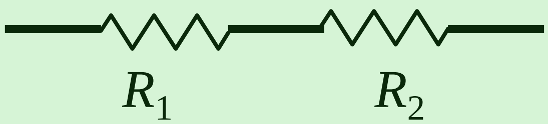


$V_1 > V_2$ A $V_1 = V_2$ B $V_1 < V_2$ C

Resistor Summary

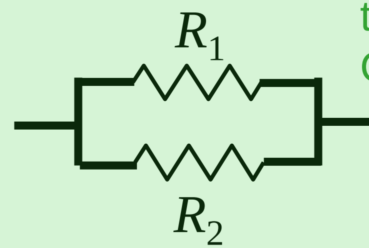
Series

Every loop with R_1 also has R_2



Parallel

There is a loop that contains ONLY R_1 and R_2

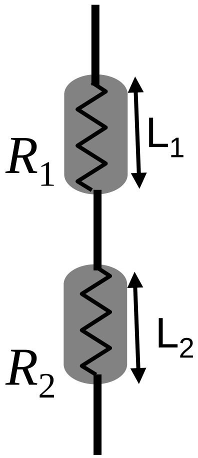


	Series	Parallel
Wiring	Each resistor on the <u>same</u> wire.	Each resistor on a <u>different</u> wire.
Voltage	<u>Different</u> for each resistor. $V_{total} = V_1 + V_2$	<u>Same</u> for each resistor. $V_{total} = V_1 = V_2$
Current	<u>Same</u> for each resistor $I_{total} = I_1 = I_2$	<u>Different</u> for each resistor $I_{total} = I_1 + I_2$
Resistance	<u>Increases</u> $R_{eq} = R_1 + R_2$	<u>Decreases</u> $1/R_{eq} = 1/R_1 + 1/R_2$

Resistors and Capacitors

Can we go over why Capacitors and Resistors are inverses in series and parallel? Like more of a physical reason not just "the math works that way"

Series, you are adding lengths

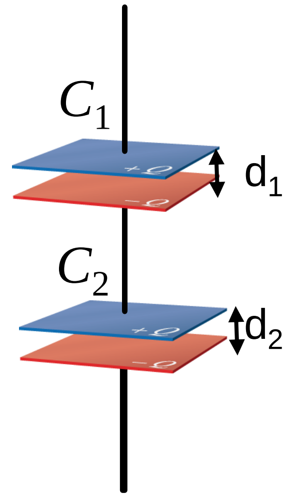


$$R = \rho \frac{L}{A}$$

$$R_{tot} = \rho \frac{L_1 + L_2}{A}$$

$$= \rho \frac{L_1}{A} + \rho \frac{L_2}{A} = R_1 + R_2$$

$$= R_1 + R_2$$



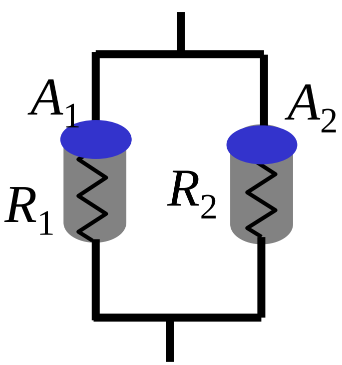
$$C = \varepsilon \frac{A}{d} \qquad \frac{1}{C} = \frac{d}{\varepsilon A}$$

$$\frac{1}{C_{tot}} = \frac{d_1 + d_2}{\varepsilon A}$$

$$= \frac{d_1}{\varepsilon A} + \frac{d_2}{\varepsilon A}$$

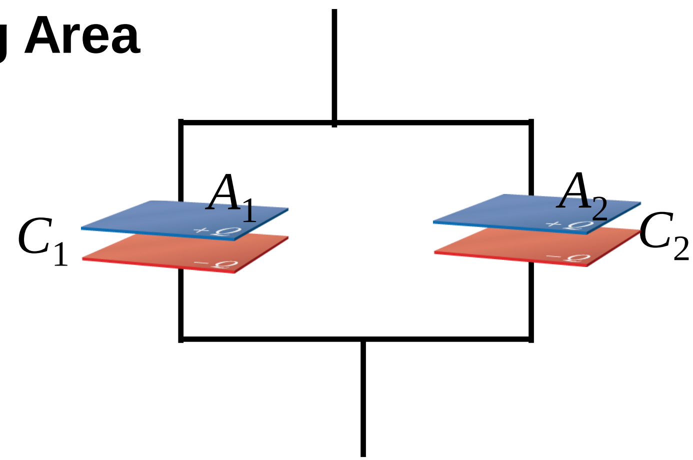
$$= \frac{1}{C_1} + \frac{1}{C_2}$$

Parallel, you are adding Area



$$\frac{1}{R} = \frac{A}{\rho L}$$

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

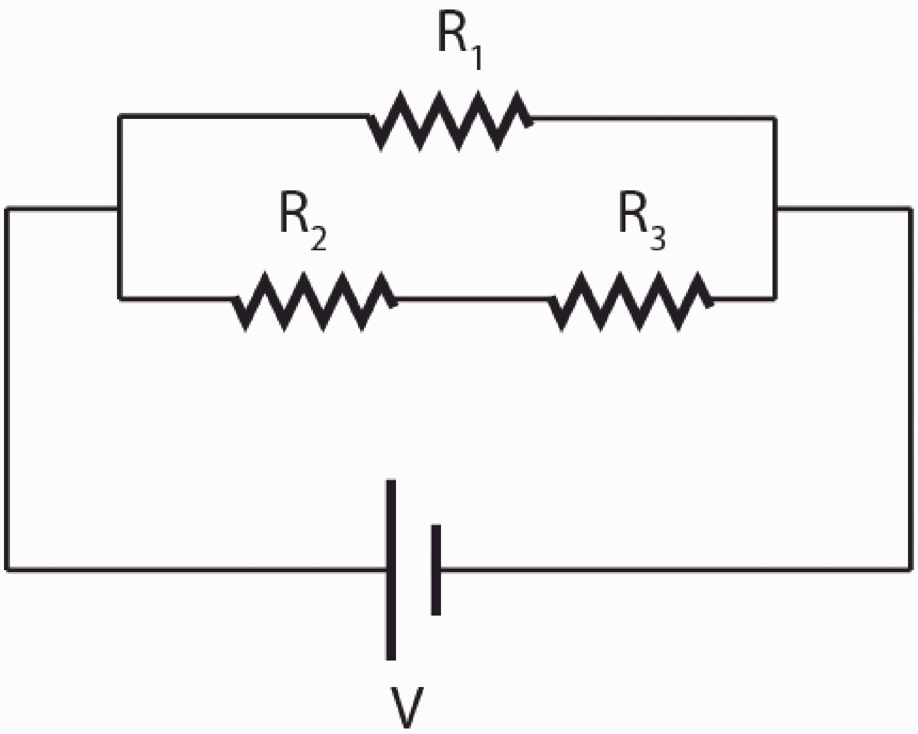


$$C = \varepsilon \frac{A}{d}$$

$$C_{tot} = C_1 + C_2$$

Check Point 3a

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



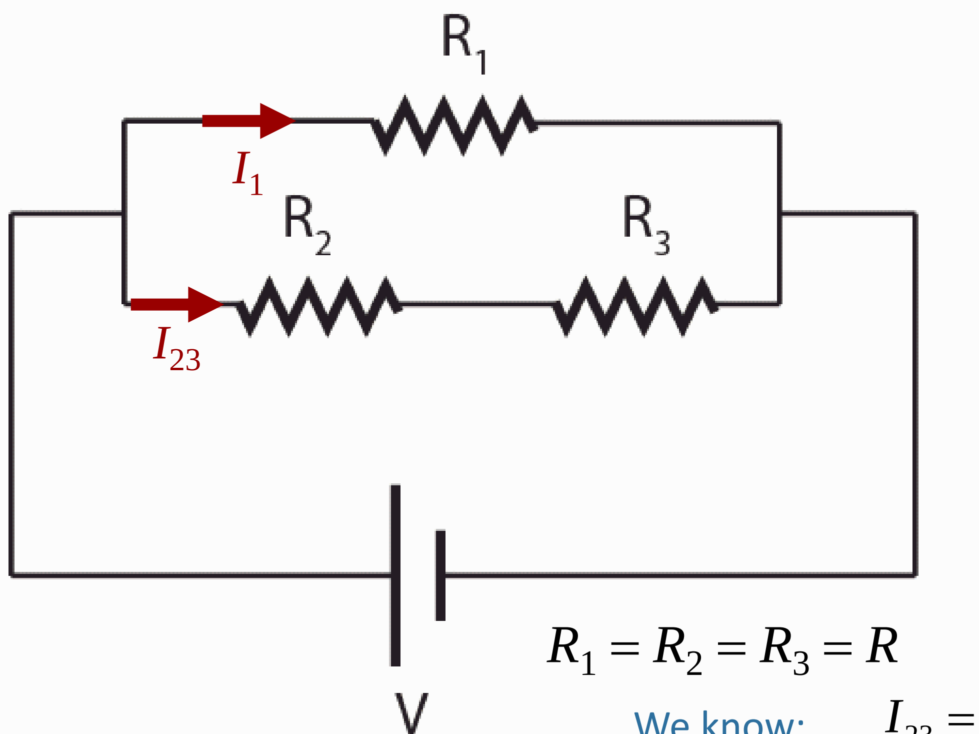
Compare the current through R_2 with the current through R_3:

A. $I_2 > I_3$ ***B.*** $I_2 = I_3$ ***C.*** $I_2 < I_3$

R_2 in series with R_3 → Current through R_2 and R_3 is the same

$$I_{23} = \frac{V}{R_2 + R_3}$$

Check Point 3b



$$R_1 = R_2 = R_3 = R$$

We know: $I_{23} = \dfrac{V}{R_2 + R_3}$

Similarly: $I_1 = \dfrac{V}{R_1}$

$$I_1 = I_{23}\frac{R_2 + R_3}{R_1}$$

Compare the current through R_1 with the current through R_2

A $I_1/I_2 = 1/2$

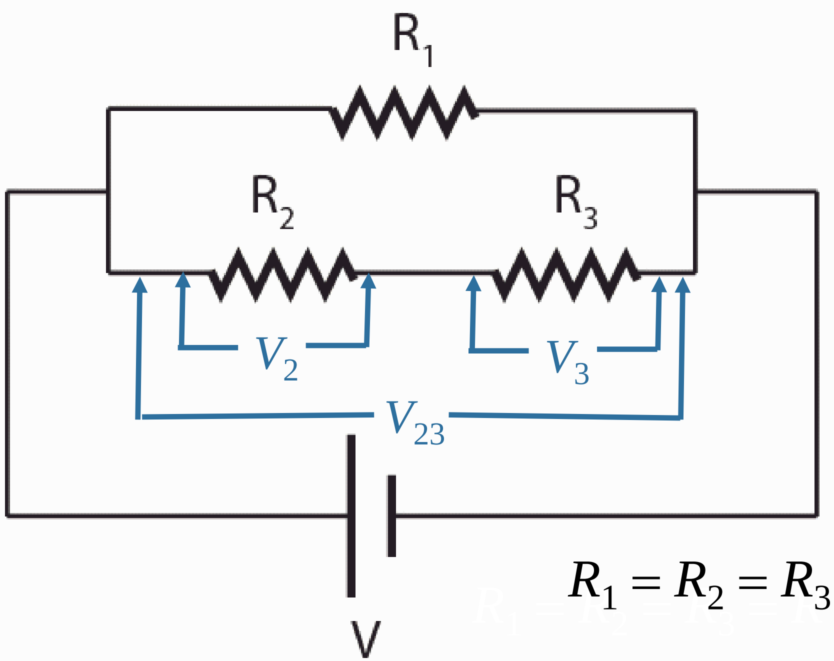
B $I_1/I_2 = 1$

C $I_1/I_2 = 2$

D $I_1/I_2 = 3$

E $I_1/I_2 = 4$

Check Point 3C



$$R_1 = R_2 = R_3 = R$$

Compare the voltage across R_2 with the voltage across R_3

A $V_2 > V_3$

B $V_2 = V_3 = V$

C $V_2 = V_3 < V$

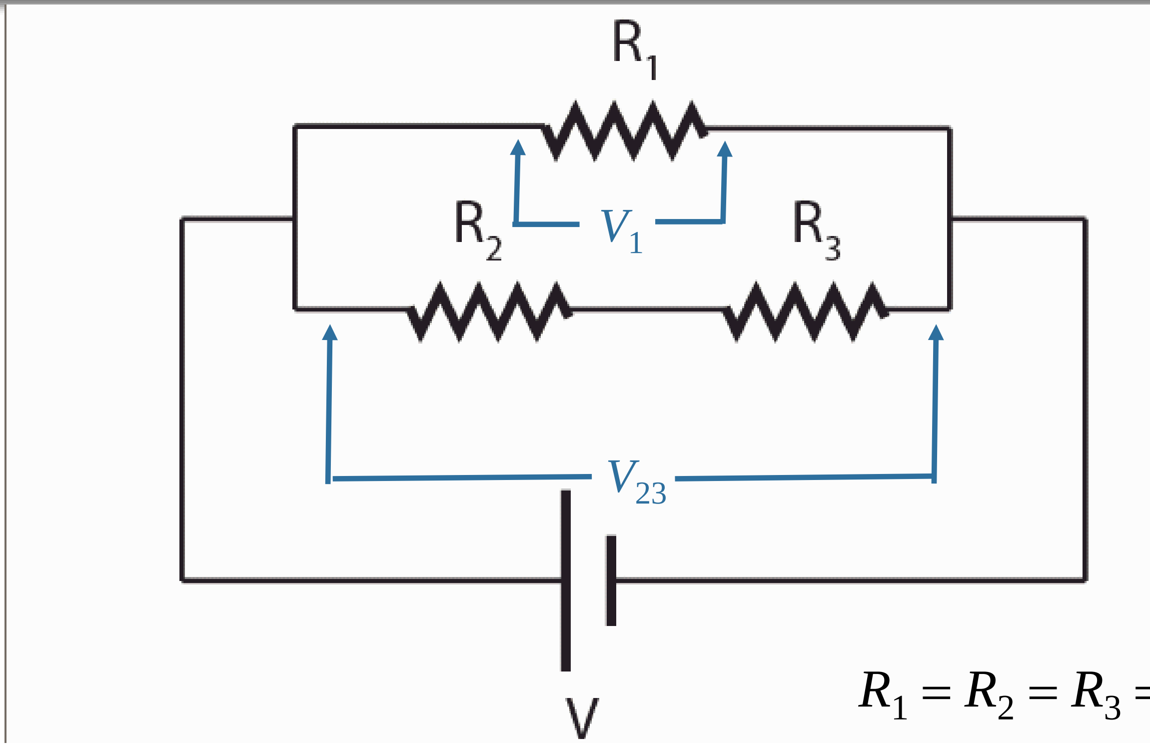
D $V_2 < V_3$

$$V_2 = I_2 R_2$$

$$V_3 = I_3 R_3$$

$I_2 = I_3$ (Series)
$R_2 = R_3$ (Problem statement)

Check Point 3D



R

$$R_1 = R_2 = R_3 = R$$

Compare the voltage across R_1 with the voltage across R_2

R_1 in parallel with series combination of R_2 and R_3

A $\quad V_1 = V_2 = V$

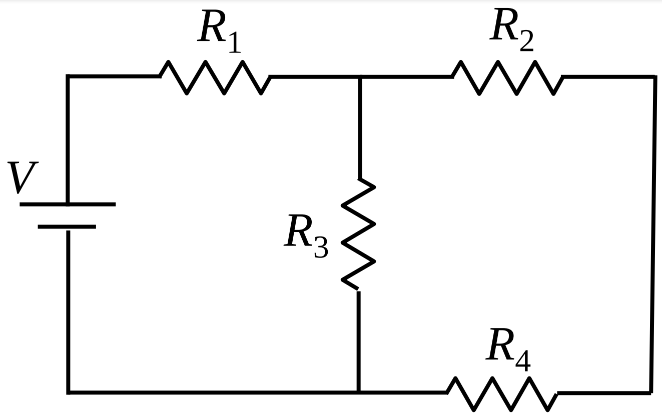
B $\quad V_1 = ½\ V_2 = V$

C $\quad V_1 = ½\ V_2 = 1/5\ V$

D $\quad V_1 = 2V_2 = V$

E $\quad V_1 = ½\ V_2 = ½\ V$

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Conceptual Analysis:

Ohm's Law: when current *I* flows through resistance R, the potential drop V is given by: $V = IR$.

Resistances are combined in series and parallel combinations

$R_{series} = R_a + R_b$

$(1/R_{parallel}) = (1/R_a) + (1/R_b)$

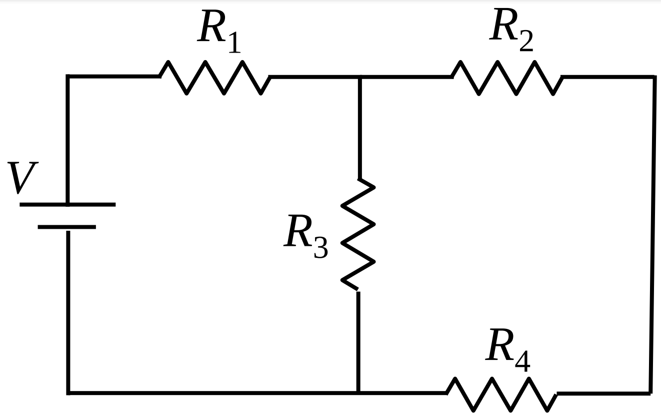
Strategic Analysis:

Combine resistances to form equivalent resistances

Evaluate voltages or currents from Ohm's Law

Expand circuit back using knowledge of voltages and currents

Calculation

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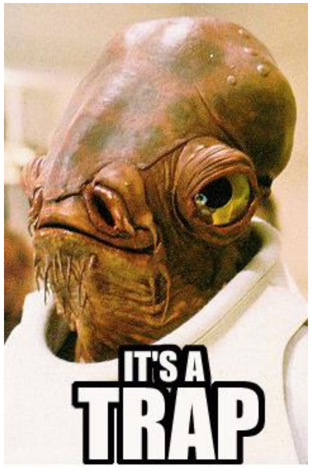
What is V_2, the voltage across R_2?

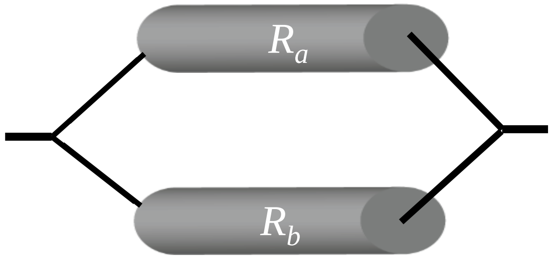


Combine Resistances:

R_1 and R_2 are connected:

A) in series B) in parallel C) neither in series nor in parallel

Parallel Combination

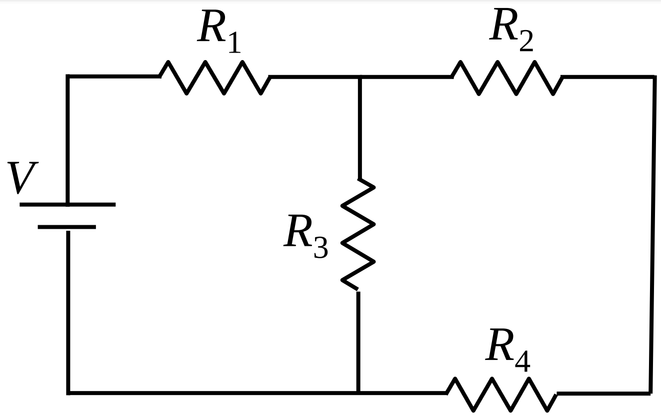


Parallel: Can make a loop that contains only those two resistors

Series Combination

Series : Every loop with resistor 1 also has resistor 2.

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

We first will combine resistances R_2, R_4:

Which of the following is true?

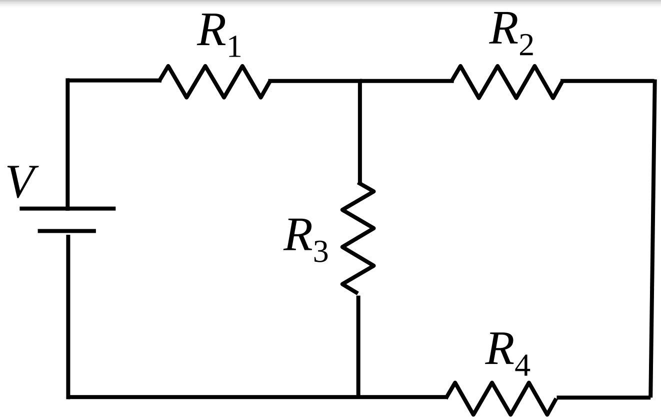
A) R_2 and R_4 are connected in series

B) R_2 and R_4 are connected in parallel

C) R_2 and R_4 are neither in series nor in parallel

D) R_2 and R_4 are both in series and in parallel

Calculation



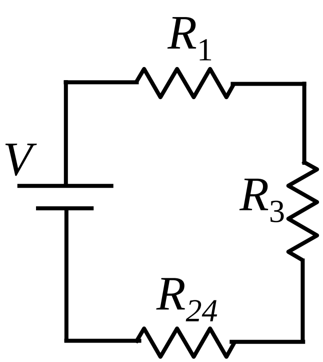
In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

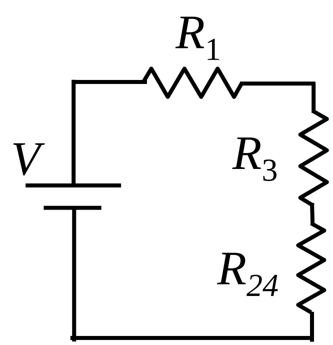
What is V_2, the voltage across R_2?

R_2 and R_4 are connected in series (R_{24})

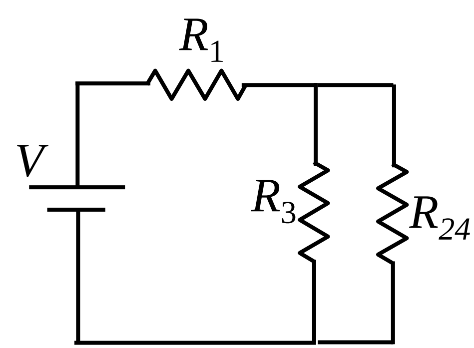
Redraw the circuit using the equivalent resistor $R_{24} =$ series combination of R_2 and R_4.



(A)

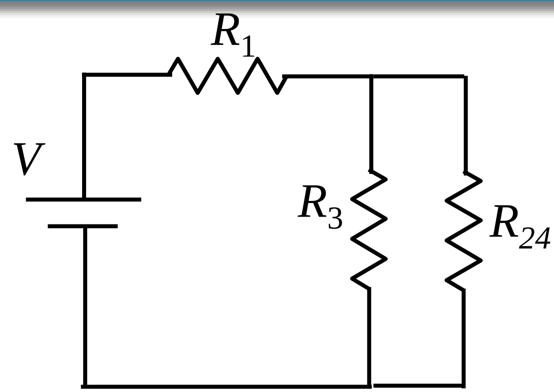


(B)



(C)

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances:

R_2 and R_4 are connected in series $= R_{24}$
R_3 and R_{24} are connected in parallel $= R_{234}$

What is the value of R_{234}?

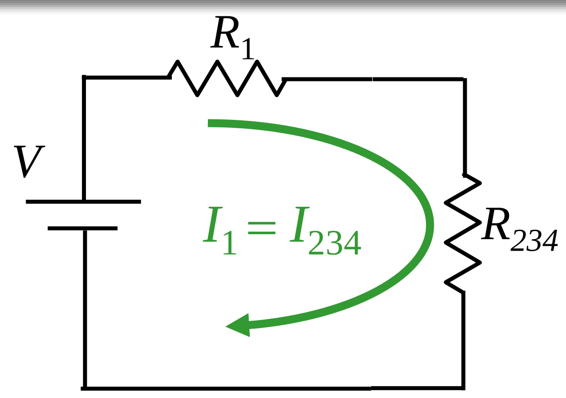
A) $R_{234} = 1\ \Omega$ B) $R_{234} = 2\ \Omega$ C) $R_{234} = 4\ \Omega$ D) $R_{234} = 6\ \Omega$

R_2 and R_4 in series $\rightarrow$ $R_{24} = R_2 + R_4 = 2\Omega + 4\Omega = 6\Omega$

R_3 and R_{24} are connected in parallel

$(1/R_{parallel}) = (1/R_a) + (1/R_b)$

Calculation



In the circuit shown: $V = 18V$,

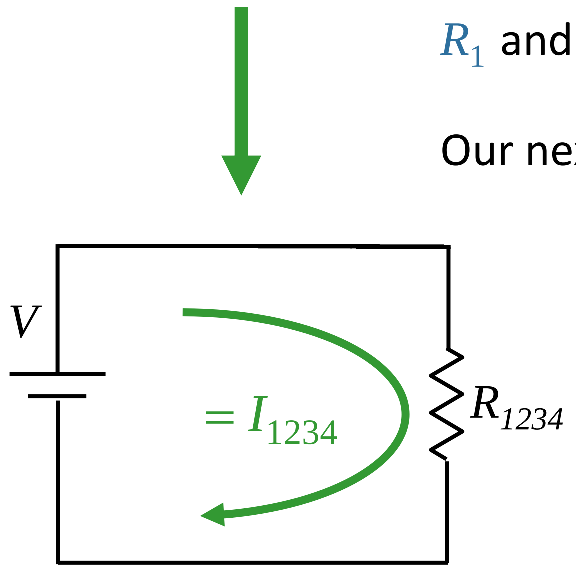
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

$R_{24} = 6\Omega$ $\quad R_{234} = 2\Omega$

What is V_2, the voltage across R_2?

R_1 and R_{234} are in series. $R_{1234} = 1 + 2 = 3\ \Omega$

Our next task is to calculate the total current in the circuit

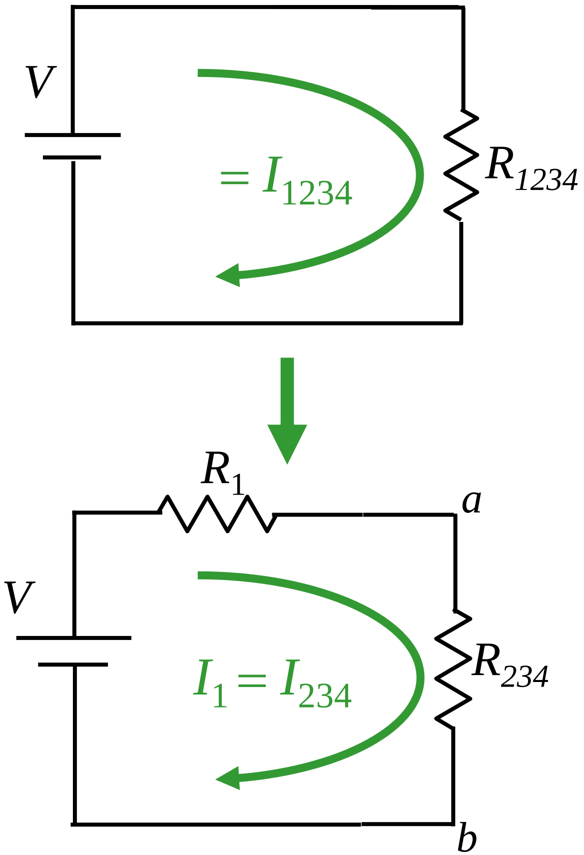


Ohm's Law tells us: $I_{1234} = V/R_{1234}$

$= 18 / 3$

$= 6$ Amps

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

$R_{24} = 6\Omega$ $R_{234} = 2\Omega$ $I_{1234} = 6\ A$

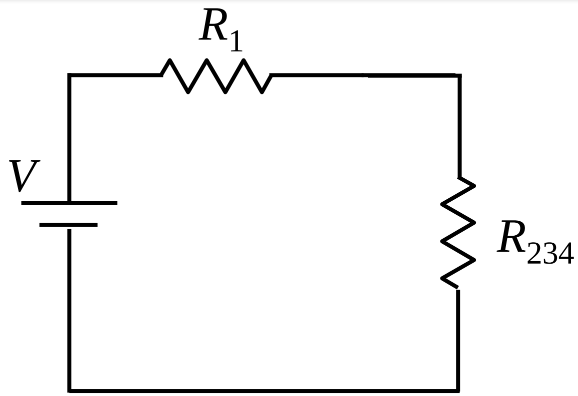
What is V_2, the voltage across R_2?

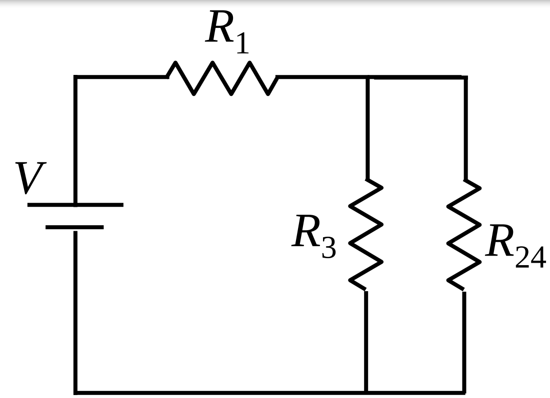
$I_{234} = I_{1234}$ Since R_1 in series with R_{234}

What is V_{ab}, the voltage across R_{234} ?

A) $V_{ab} = 1\ V$ B) $V_{ab} = 2\ V$ C) $V_{ab} = 9\ V$ D) $V_{ab} = 12\ V$ E) $V_{ab} = 16\ V$

Calculation





$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$I_{1234} = 6$ Amps
$I_{234} = 6$ Amps
$V_{234} = 12V$
What is V_2?

Which of the following are true?

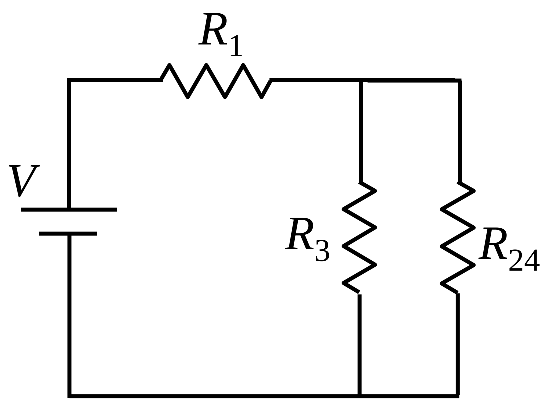
A) $V_{234} = V_{24}$ B) $I_{234} = I_{24}$ C) Both A+B D) None

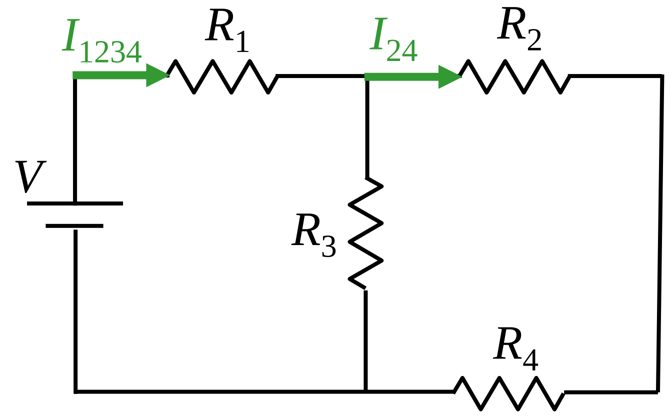
R_3 and R_{24} were combined in parallel to get R_{234} → Voltages are same!

Ohm's Law

$$I_{24} = V_{24} / R_{24}$$
$$= 12 / 6$$
$$= 2 \text{ Amps}$$

Calculation





$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega.$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$I_{1234} = 6$ Amps
$I_{234} = 6$ Amps
$V_{234} = 12V$
$V_{24} = 12V$
$I_{24} = 2$ Amps
What is V_2?

Which of the following are true?

A) $V_{24} = V_2$ B) $I_{24} = I_2$ C) Both A+B D) None

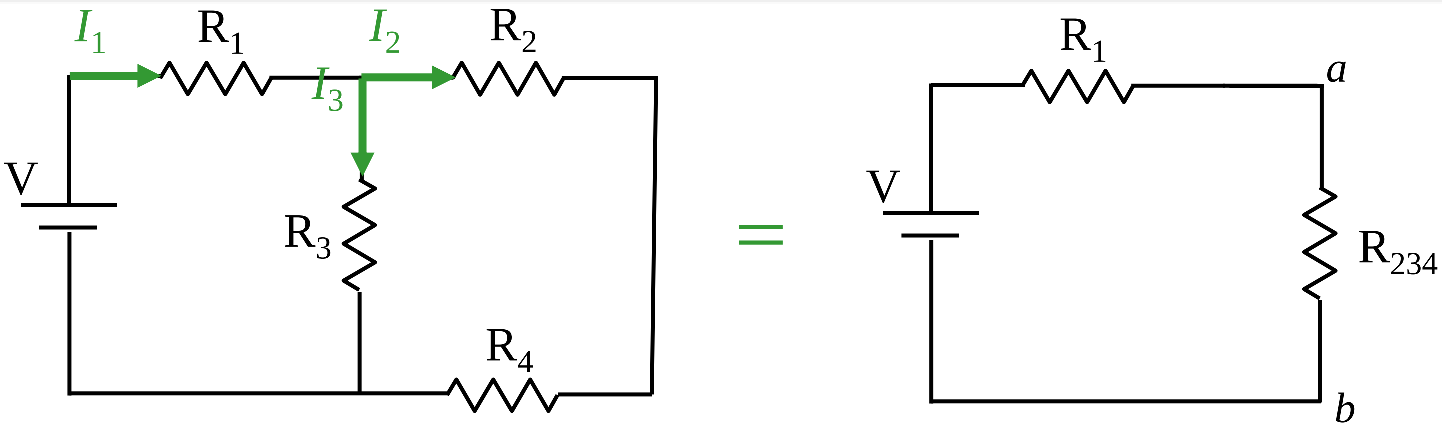
R_2 and R_4 where combined in series to get R_{24} → Currents are same!

Ohm's Law

The Problem Can Now Be Solved!

$$V_2 = I_2 \ R_2$$
$$= 2 \text{ x } 2$$
$$= 4 \text{ Volts!}$$

Quick Follow-Ups



$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$V_{234} = 12V$
$V_2 = 4V$
$I_{1234} = 6$ Amps

What is I_3 ?

A) $I_3 = 2\ A$ B) $I_3 = 3\ A$ C) $I_3 = 4\ A$

$V_3 = V_{234} = 12V$

What is I_1 ?

We know $I_1 = I_{1234} = 6\ A$

NOTE: $I_2 = V_2/R_2 = 4/2 = 2\ A$ → $I_1 = I_2 + I_3$ Make Sense?

Current and Resistance

Key Concepts:

1) How resistance depends on A, L, σ, r

2) How to combine resistors in series and parallel

3) Understanding resistors in circuits

4) Solve a network circuit

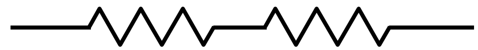
Physics 212
Lecture 10

Today's Concept:

Kirchhoff's Rules

Last Time

Resistors in series:

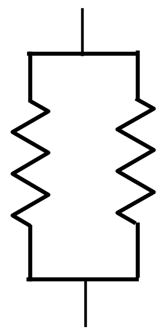


Current through is same.

Voltage drop across each resistor i is IR_i

$$R_{effective} = R_1 + R_2$$

Resistors in parallel:

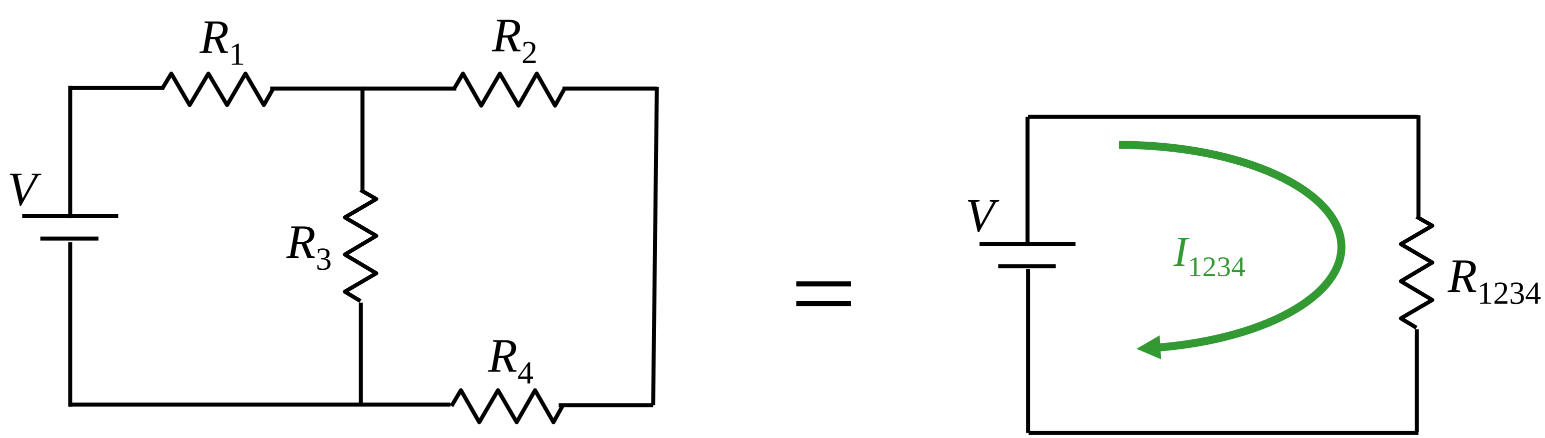


Voltage drop across is same.

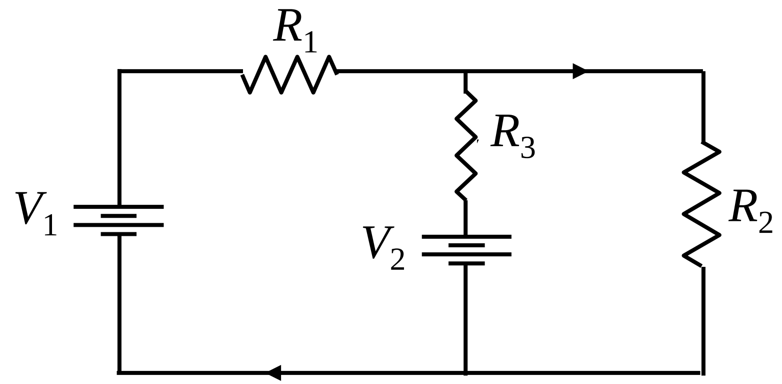
Current through is V/R_i

$$\frac{1}{R_{effective}} = \frac{1}{R_1} + \frac{1}{R_2}$$

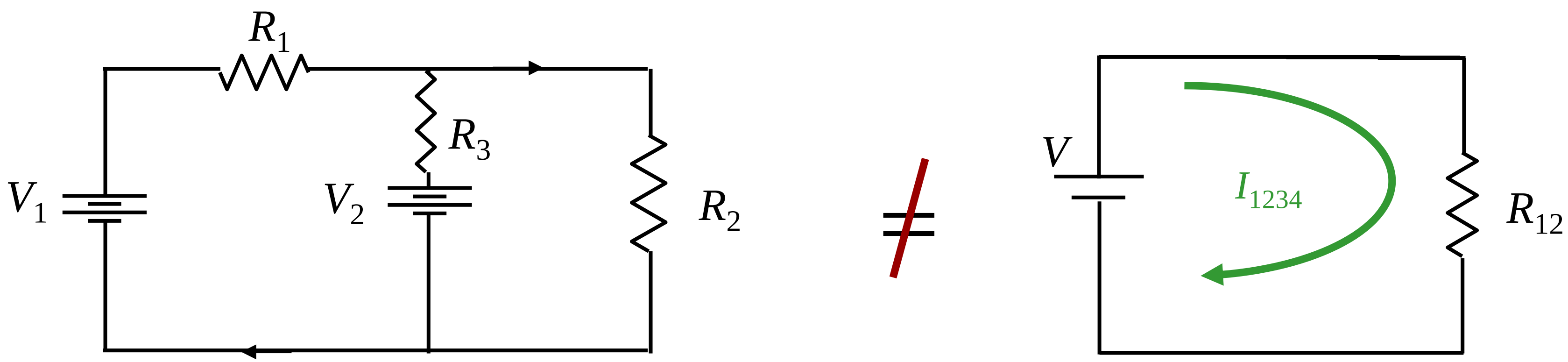
Solved Circuits



New Circuit



How Can We Solve This One?



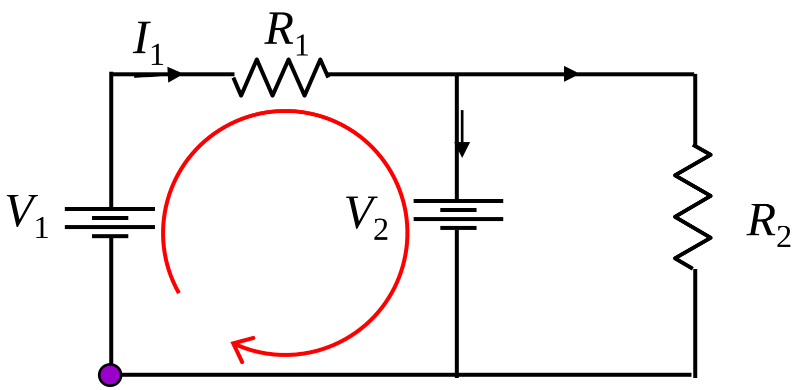
THE ANSWER: Kirchhoff' s Rules

Kirchhoff's Voltage Rule

$$\sum \Delta V_i = 0$$

Kirchhoff's Voltage Rule states that the sum of the voltage changes caused by any elements (like wires, batteries, and resistors) around a circuit must be zero.

$$\sum \mathrm{D} V_i = -V_1 + IR_1 + V_2 = 0$$



WHY?

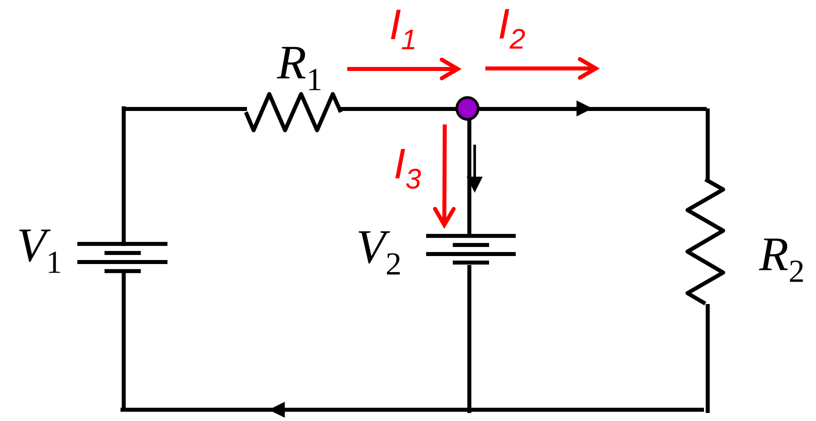
The potential difference between a point and itself is zero!

Kirchhoff's Current Rule

$$\sum I_{in} = \sum I_{out}$$

Kirchhoff's Current Rule states that the sum of all currents entering any given point in a circuit must equal the sum of all currents leaving the same point.

$I_1 = I_2 + I_3$



WHY?

Electric Charge is Conserved

Applying Kirchhoff's Laws in 5 easy steps

1) Label all currents

   Choose any direction

2) Label +/– for all elements

   Current goes + ⇒ – (for resistors)

   Long side is + for battery

3) Choose loop and direction

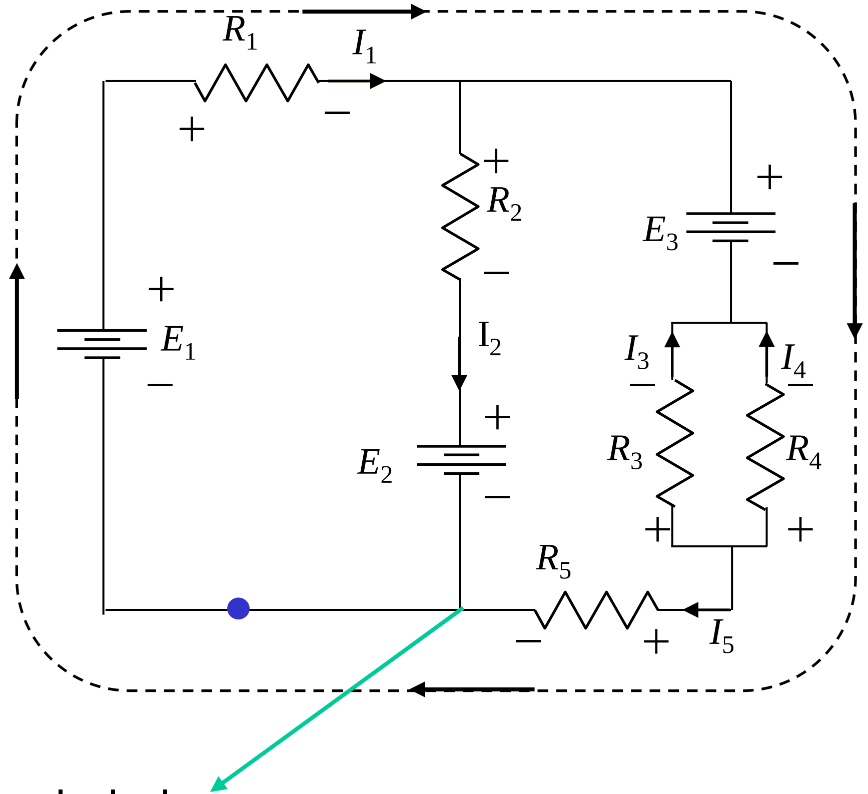
   Must start on wire, not element.

4) Write down voltage drops

   First sign you hit is sign to use.

5) Write down node equation $I_{in} = I_{out}$

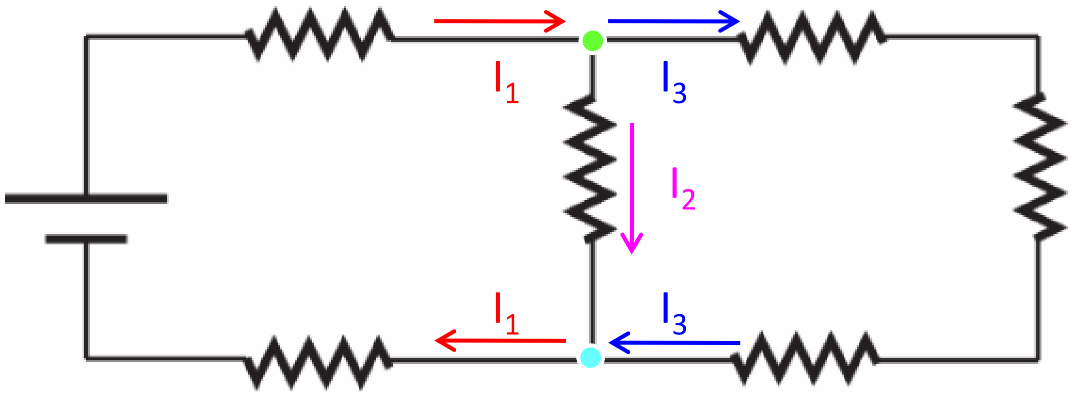
$-E_1 + I_1R_1 + E_3 - I_4R_4 + I_5R_5 = 0$



$I_2 + I_5 = I_1$

Check Point 1

How many potentially different currents are there in the circuit shown?



A. 3 ***B.*** 4 ***C.*** 5 ***D.*** 6 ***E.*** 7

Look at the nodes!

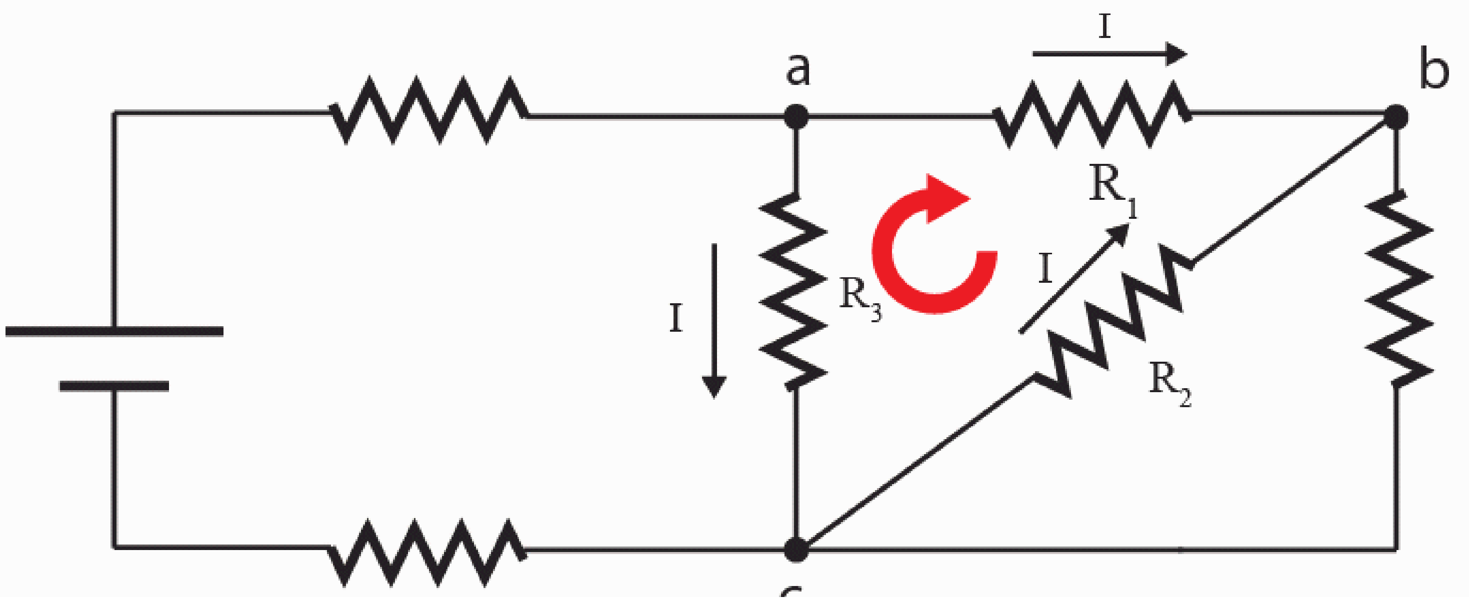
Top node: I_1 flows in, I_2 and I_3 flow out

Bottom node: I_2 and I_3 flow in, I_1 flows out

That's all of them!

Check Point 2

In the following circuit, consider the loop abc. The direction of the current through each resistor is indicated by black arrows.



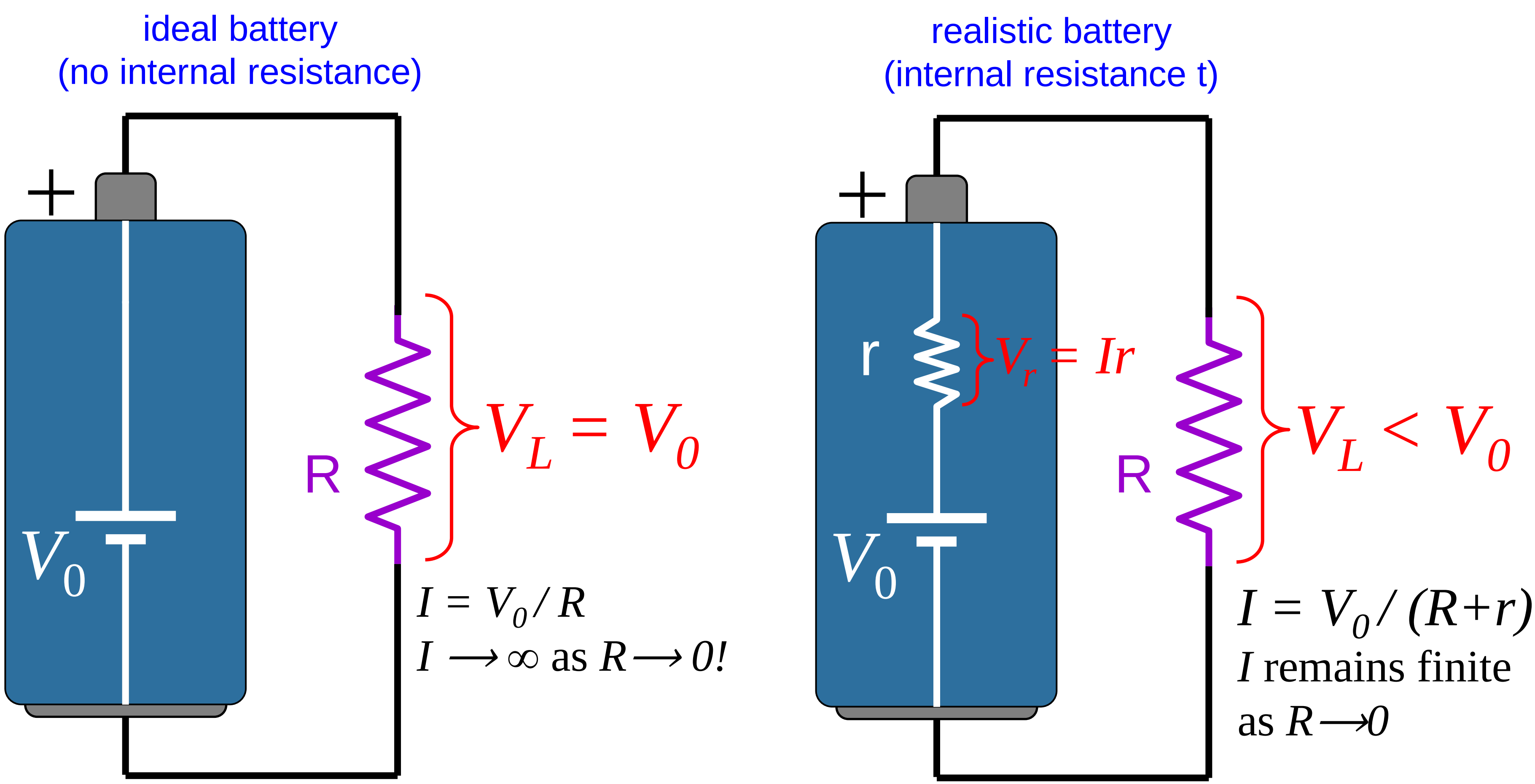
If we are to write Kirchoff's voltage equation for this loop in the clockwise direction starting from point a, what is the correct order of voltage gains/drops that we will encounter for resistors R1, R2 and R3?

A. drop, drop, drop
B. gain, gain, gain
C. drop, gain, gain
D. gain, drop, drop
E. drop, drop, gain

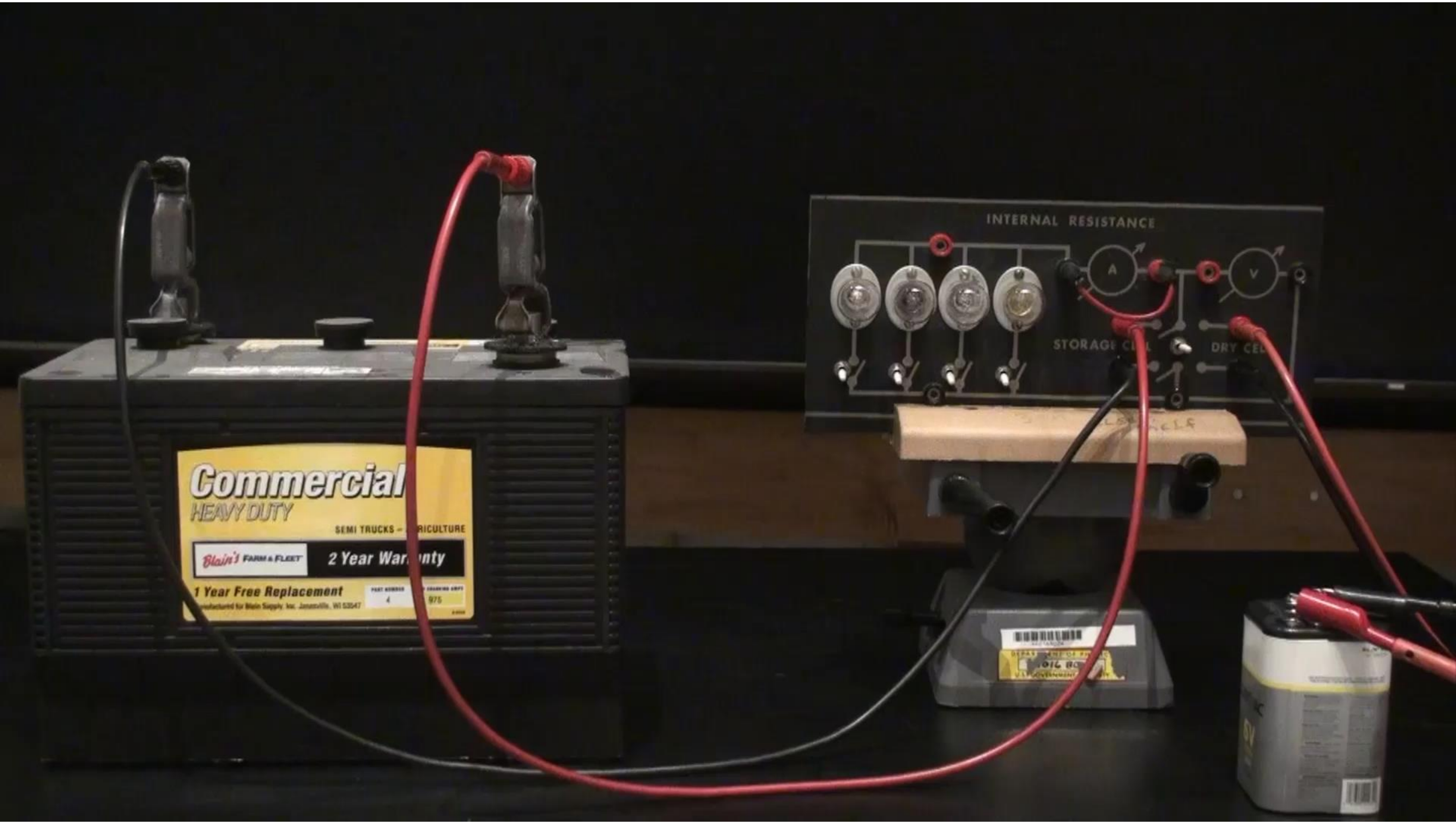
With the current → VOLTAGE DROP

Against the current → VOLTAGE GAIN

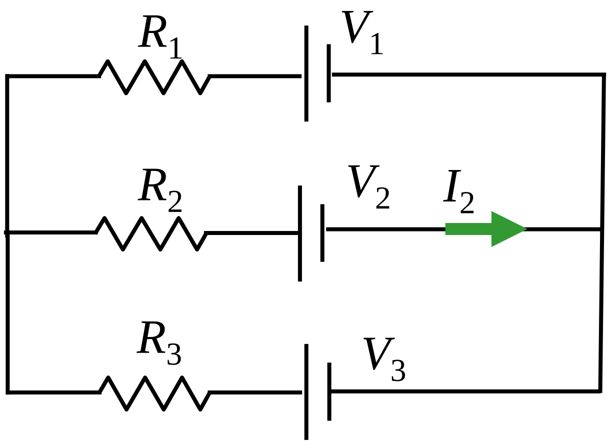
Model for Real Battery: Internal Resistance



Usually, can't supply too much current to the load without voltage "sagging"


Commercial
HEAVY DUTY
2 Year War nty
1 Year Free Replacement
INTERNAL RESISTANCE
STORAGE CE
DRY CE

Calculation



In this circuit, we are given the resistances and battery voltages and are asked to calculate the current through resistor 2.

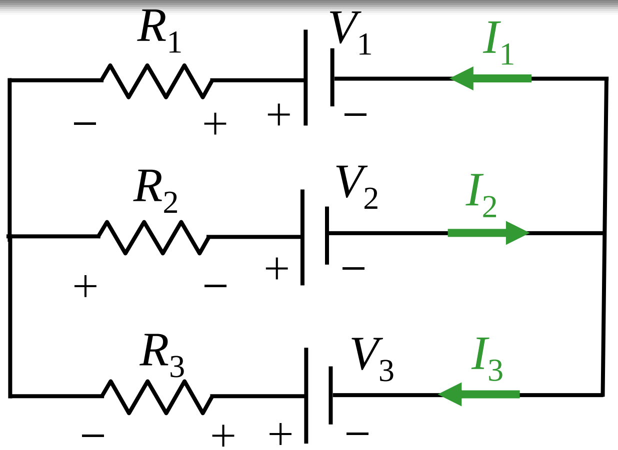
Conceptual Analysis:

- Circuit behavior described by Kirchhoff's Rules:
  - KVR: $\Sigma V_{drops} = 0$
  - KCR: $\Sigma I_{in} = \Sigma I_{out}$

Strategic Analysis

- Write down Loop Equations (KVR)
- Write down Node Equations (KCR)
- Solve

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

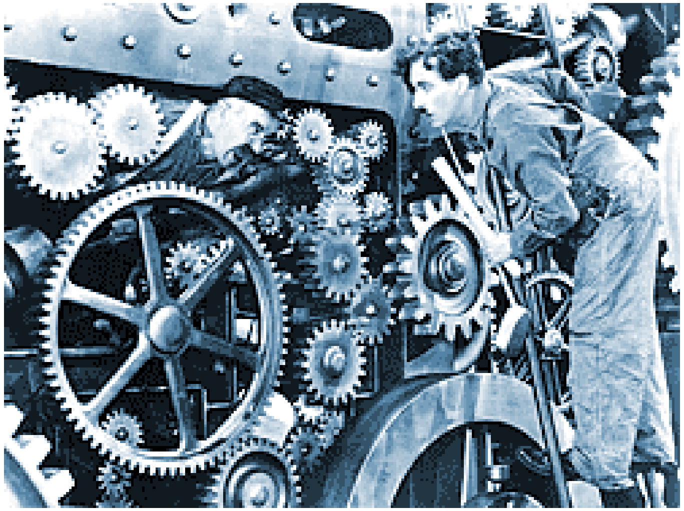
1) Label and pick directions for each current

2) Label the + and – side of each element

This is easy for batteries Long side is +

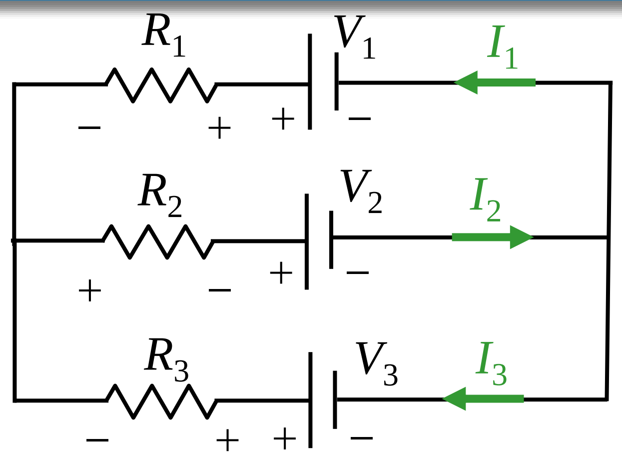
For resistors, the "upstream" side is +

Now write down loop and node equations



Just turn the crank.

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

How many equations do we need to write down in order to solve for I_2?

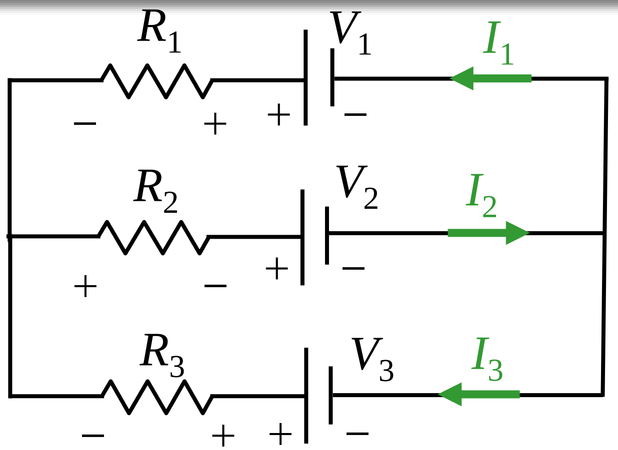
A) 1 B) 2 C) 3 D) 4 E) 5

Why?

- We have 3 unknowns: I_1, I_2, and I_3
- We need 3 independent equations to solve for these unknowns

3) Choose Loops and Directions

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

Which of the following equations is NOT correct?

A) $I_2 = I_1 + I_3$

B) $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$

C) $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$

D) $-V_2 - I_2R_2 + I_1R_1 + V_1 = 0$

4) Write down voltage drops

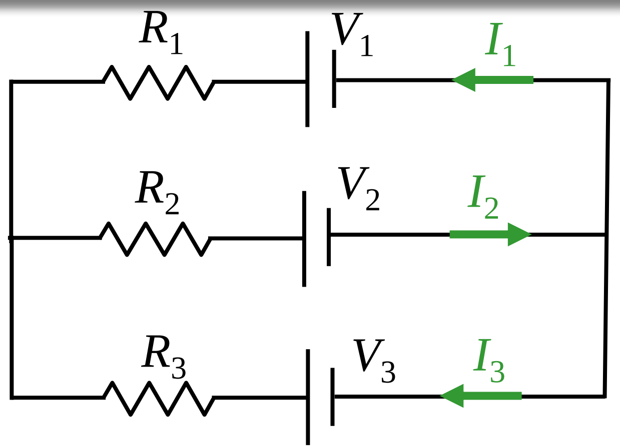
5) Write down node equation

Why?

- (D) is an attempt to write down KVR for the top loop
- Start at negative terminal of V_2 and go clockwise

$V_{gain}\ (-V_2)$ then $V_{gain}\ (-I_2R_2)$ then $V_{gain}\ (-I_1R_1)$ then $V_{drop}\ (+V_1)$

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

We have the following 4 equations:

1. $I_2 = I_1 + I_3$
2. $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$
3. $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$
4. $-V_2 - I_2R_2 - I_1R_1 + V_1 = 0$

We need 3 equations:
Which 3 should we use?

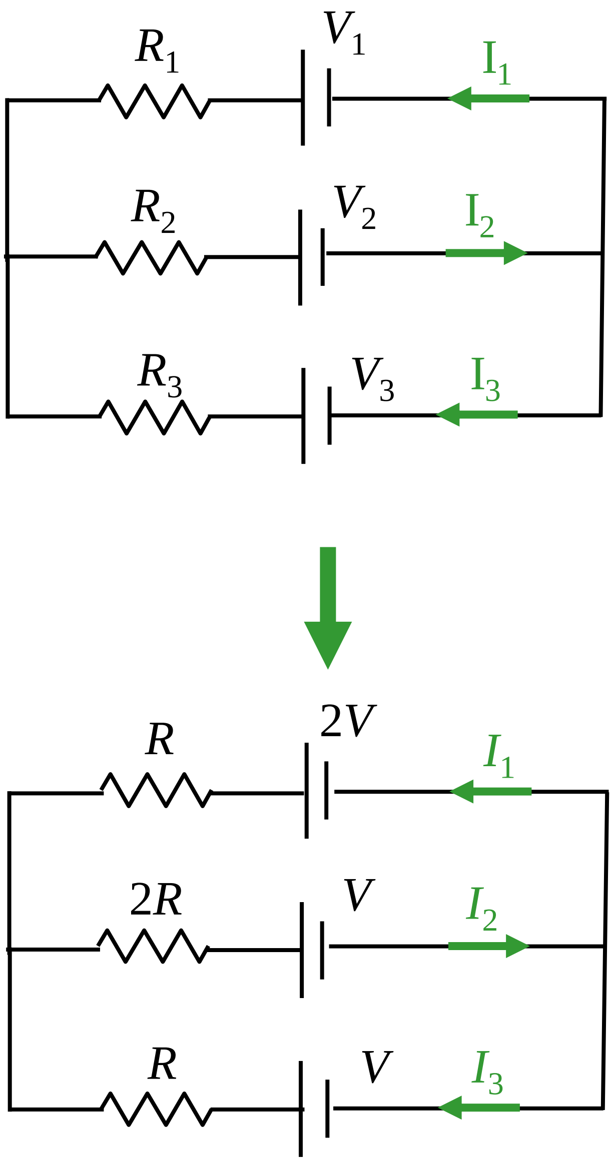
A) Any 3 will do
B) 1, 2, and 4
C) 2, 3, and 4

Why?

- We need 3 INDEPENDENT equations
- Equations 2, 3, and 4 are NOT INDEPENDENT

  $\text{Eqn } 2 + \text{Eqn } 3 = -\text{Eqn } 4$
- We must choose Equation 1 and any two of the remaining (2, 3, and 4)

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$
$$V_1 + I_1R_1 - I_3R_3 + V_3 = 0$$
$$V_2 - I_2R_2 - I_1R_1 + V_1 = 0$$

Now just need to solve ☺

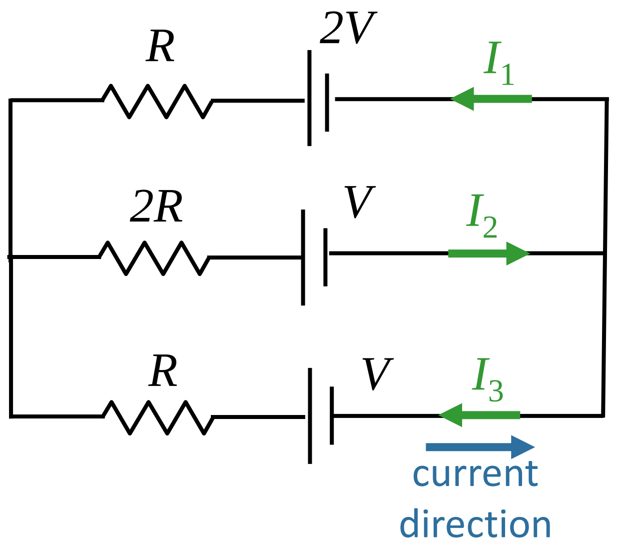
The solution will get very messy!

Simplify: assume $V_2 = V_3 = V$

$$V_1 = 2V$$
$$R_1 = R_3 = R$$
$$R_2 = 2R$$

Calculation: Simplify

In this circuit, assume V and R are known. What is I_2 ?



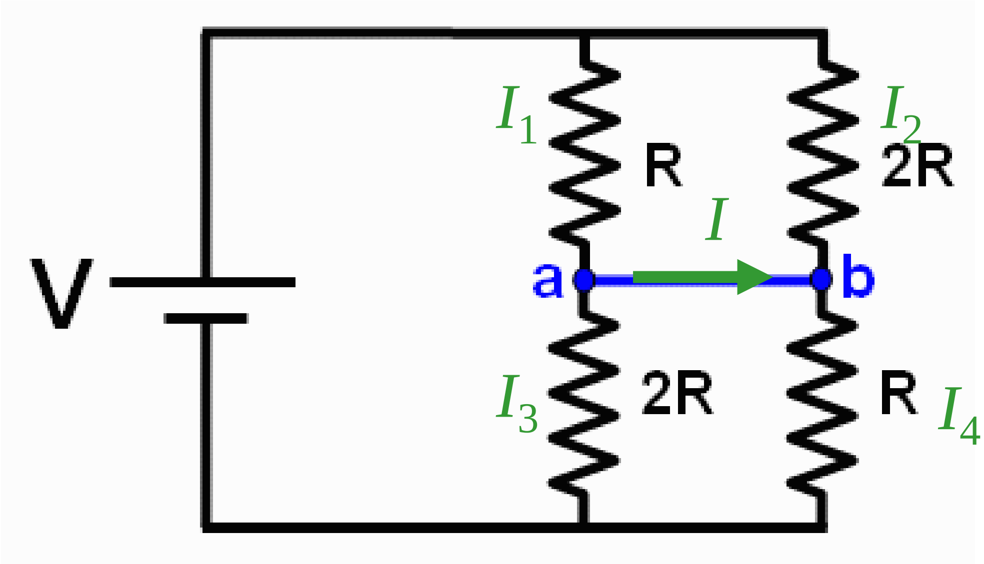
We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$
$$-2V + I_1 R - I_3 R + V = 0 \quad \text{(outside)}$$
$$-V - I_2(2R) - I_1 R + 2V = 0 \quad \text{(top)}$$

With this simplification, you can verify:

$$I_2 = (\ 1/5)\ V/R$$
$$I_1 = (\ 3/5)\ V/R$$
$$I_3 = (-2/5)\ V/R$$

Check Point 3a



Which of the following best describes the current flowing in the blue wire connecting points **a** and **b**?

A. Positive current flows from *a* to *b*

B. Positive current flows from *b* to *a*

C. No current flows between *a* and *b*

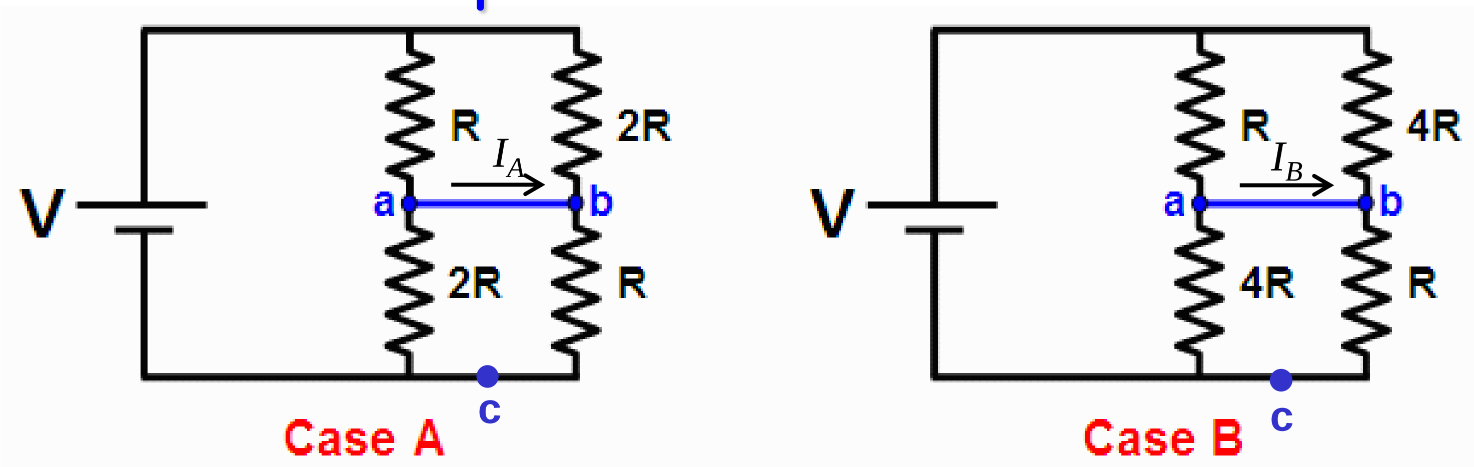
$I_1 R - I_2\,(2R) = 0 \quad \rightarrow \quad I_2 = ½\ I_1$

$I_4 R - I_3\,(2R) = 0 \quad \rightarrow \quad I_4 = 2\ I_3$

$I = I_1 - I_3$

$I + I_2 = I_4 \quad \rightarrow \quad I_1 - I_3 + ½\ I_1 = 2I_3 \quad \rightarrow \quad I_1 = 2I_3 \quad \rightarrow \quad I = +I_3$

Check point 3b



which case is the current flowing in the blue wire connecting points **a** and **b** the largest?

A. Case A ***B.*** Case B ***C.*** They are both the same

Current will flow from left to right in both cases.

In both cases, $V_{ac} = V/2$

$$I_{2R} = 2I_{4R}$$

$$I_A = I_R - I_{2R} = I_R - 2I_{4R}$$

$$I_B = I_R - I_{4R}$$

Summary

1) Label all currents

Choose any direction

2) Label +/– for all elements

Current goes + ⇒ – (for resistors)

Long side is + for battery

3) Choose loop and direction

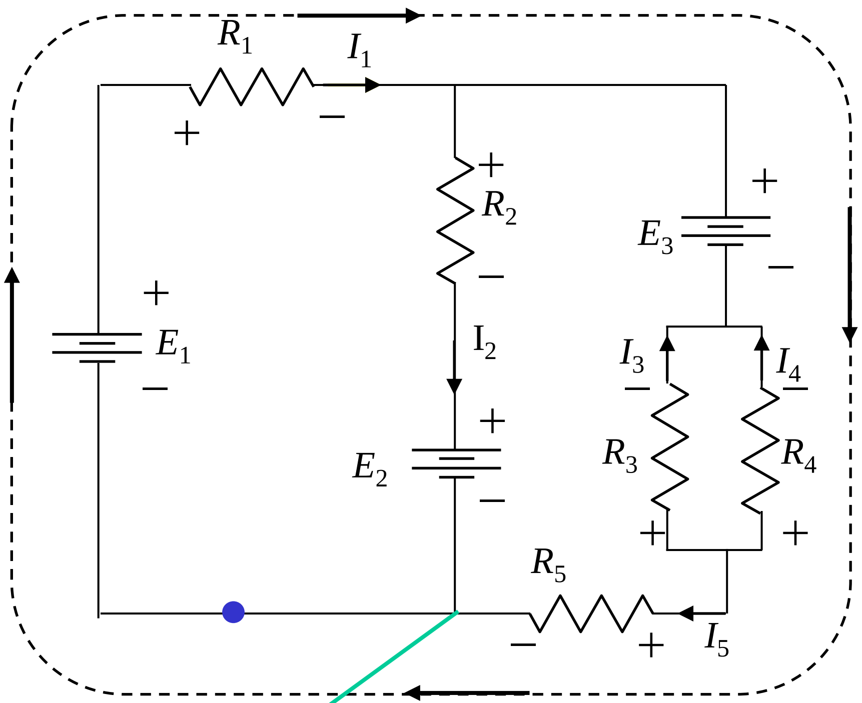
Must start on wire, not element.

4) Write down voltage drops

First sign you hit is sign to use.

5) Write down node equation $I_{\text{in}} = I_{\text{out}}$

$-E_1 + I_1R_1 + E_3 - I_4R_4 + I_5R_5 = 0$



$I_2 + I_5 = I_1$

Physics 212
Lecture 11

Today's Concept:

RC Circuits

The 212 Differential Equations

We describe the world (electrical circuits, problems in heat transfer, control systems, financial markets, etc.) using differential equations

You only need to know the solutions of two basic differential equations

$$\frac{dq(t)}{dt} + \frac{1}{t} q(t) = 0 \quad \rightarrow \quad q(t) = q_0 e^{-t/t}$$

$$\frac{d^2 q(t)}{dt^2} + w^2 q(t) = 0 \quad \rightarrow \quad q(t) = q_0 \sin(wt + f)$$

Capacitors in RC Circuits

Solve by applying Kirchhoff's Rules to circuit.
Need to understand some key phrases.

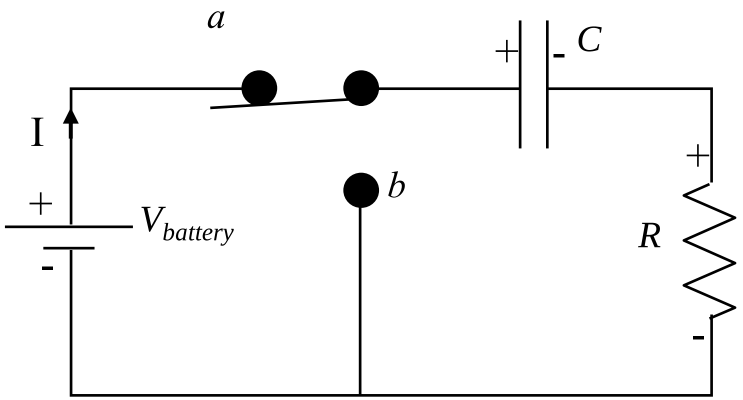
IMMEDIATELY After === Charge on capacitor is same as immediately before

After a LONG TIME === Current through capacitor = 0

After xx seconds === *Exponential solutions*

RC Circuit (Charging)

Capacitor uncharged, Switch is moved to position "a"



Kirchoff's Voltage Rule

$$-V_{battery} + \frac{q}{C} + IR = 0$$

Short Term $(q = q_0 = 0)$

$$-V_{battery} + 0 + I_0 R = 0$$

$$I_0 = \frac{V_{battery}}{R}$$

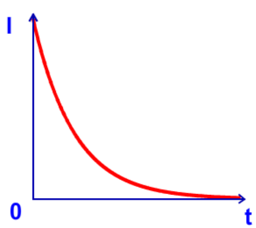
Long Term $(I_c = 0)$

$$-V_{battery} + \frac{q_\infty}{C} + 0 \cdot R = 0$$

$$q_\infty = CV_{battery}$$

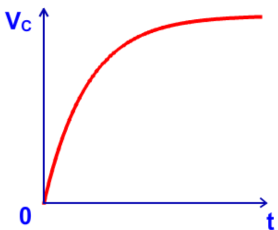
Intermediate

$$-V_{battery} + \frac{q}{C} + \frac{dq}{dt} R = 0$$



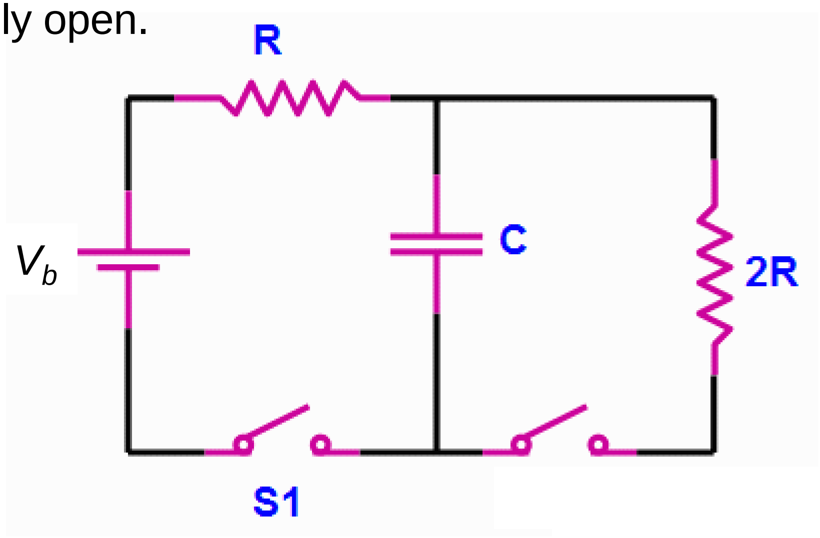
$$q(t) = q_\infty \left(1 - e^{-t/RC}\right)$$

$$I(t) = \frac{dq}{dt} = I_0 e^{-t/RC}$$



CheckPoint 1

A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.



Close $S1$,
$V_1 =$ voltage across C immediately after
$V_2 =$ voltage across C a long time after

A) $V_1 = V_b \quad V_2 = V_b$

B) $V_1 = 0 \quad V_2 = V_b$

C) $V_1 = 0 \quad V_2 = 0$

D) $V_1 = V_b \quad V_2 = 0$

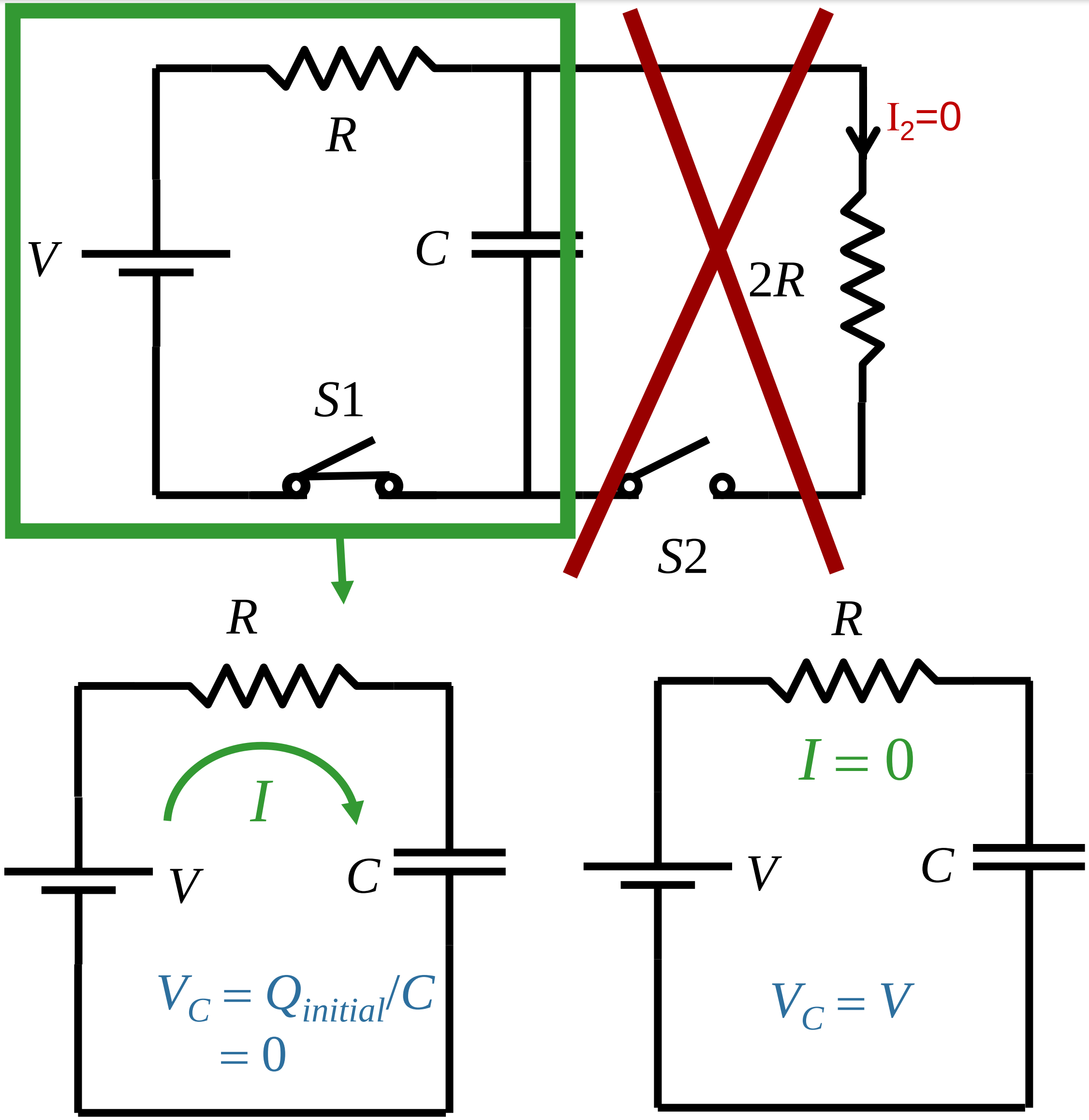
Immediately after the switch S_1 is closed:

Q *is same as immediately before*

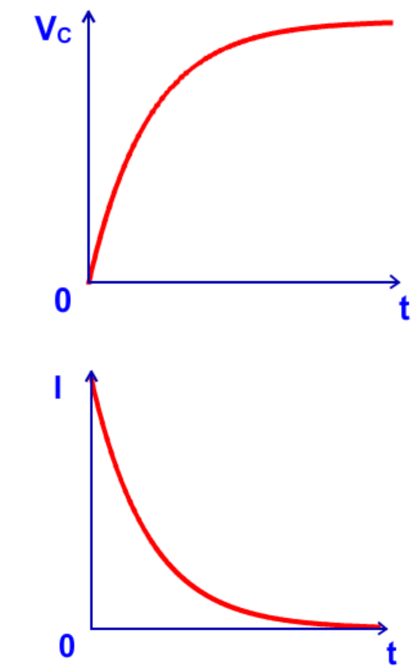
After the switch S_1 has been closed for a long time

$I_C = 0$

Close S_1 at $t = 0$
(leave S_2 open)



At $t = 0$ At $t = \text{big}$



CheckPoint 1

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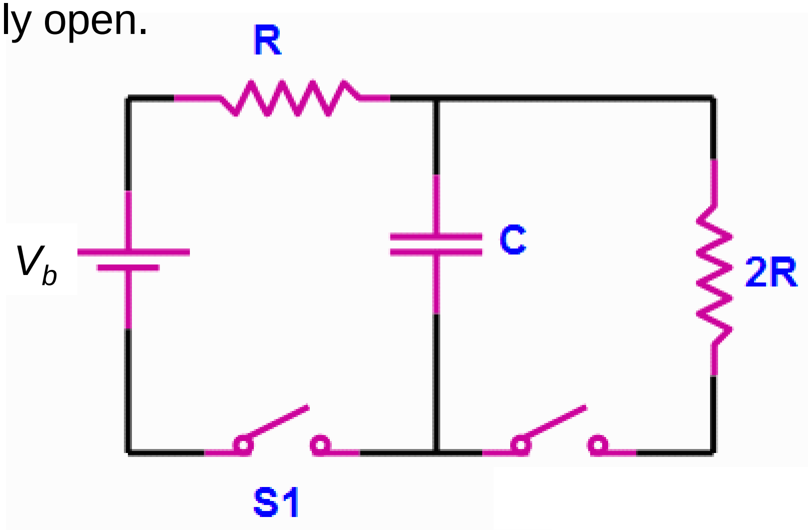


Close $S1$,

$V_1 =$ voltage across C immediately after

$V_2 =$ voltage across C a long time after

A) $V_1 = V_b \qquad V_2 = V_b$

B) $V_1 = 0 \qquad V_2 = V_b$

C) $V_1 = 0 \qquad V_2 = 0$

D) $V_1 = V_b \qquad V_2 = 0$

Immediately after the switch S_1 is closed:

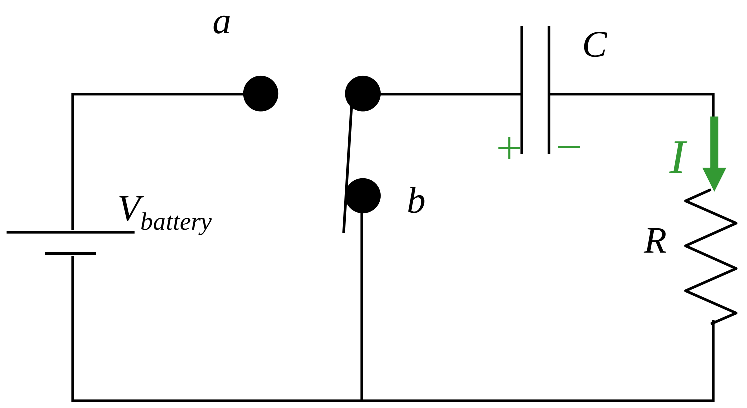
Q *is same as immediately before*

After the switch S_1 has been closed for a long time

$I_C = 0$

RC Circuit (Discharging)

Capacitor has $q_0 = CV_{battery}$, Switch is moved to position "b"



Kirchoff's Voltage Rule

$$+\frac{q}{C} + IR = 0$$

Short Term $(q = q_0)$

$$V_{battery} + IR = 0$$

$$I_0 = \frac{-V_{battery}}{R}$$

Long Term $(I_c = 0)$

$$\frac{q_\infty}{C} + 0 \cdot R = 0$$

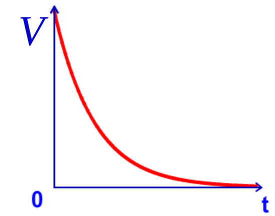
$$q_\infty = 0$$

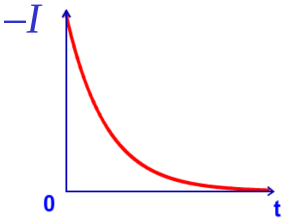
Intermediate

$$+\frac{q}{C} + \frac{dq}{dt} R = 0$$

$$q(t) = q_0 e^{-t/RC}$$

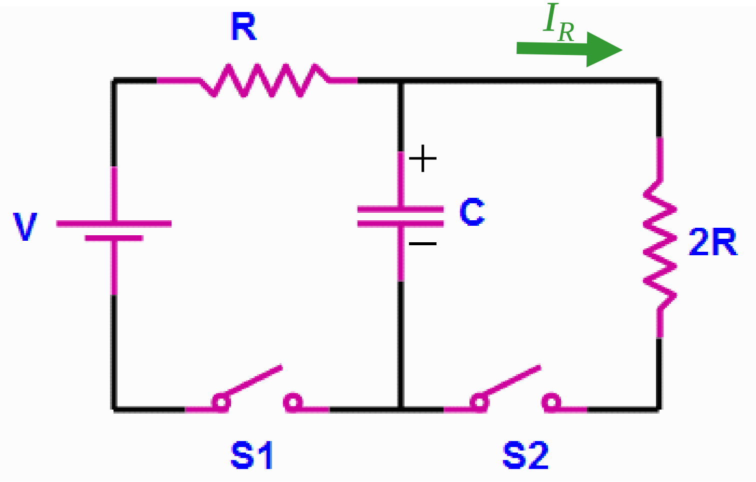
$$I(t) = I_0 e^{-t/RC}$$





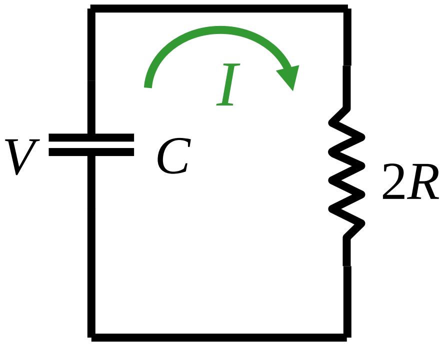
CheckPoint 1c

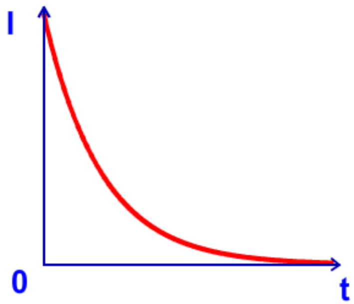
A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.

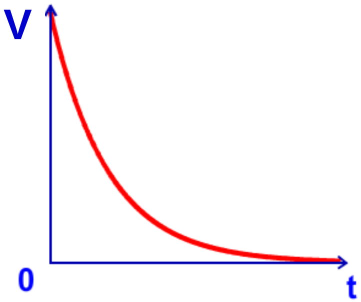


Switch 1 is closed for a long time. Then, switch 1 is opened and switch 2 is closed. What is the current through the right resistor immediately after switch 2 is closed?

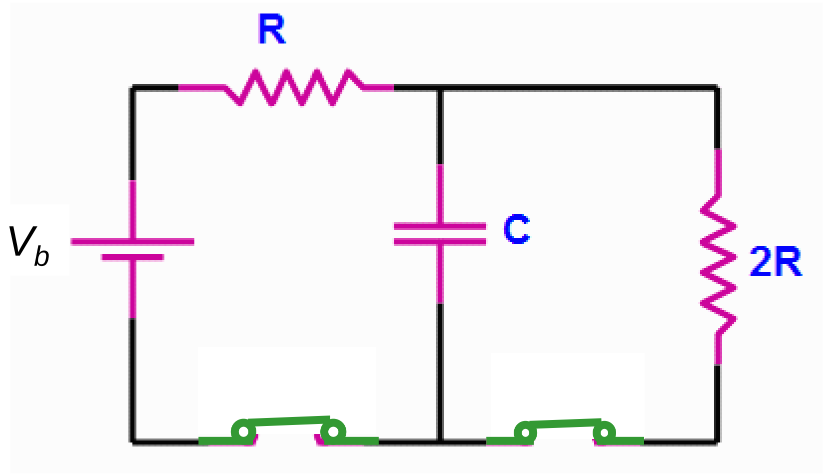
A. $I_R = 0$ ***B.*** $I_R = V/3R$ ***C.*** $I_R = V/2R$ ***D.*** $I_R = V/R$





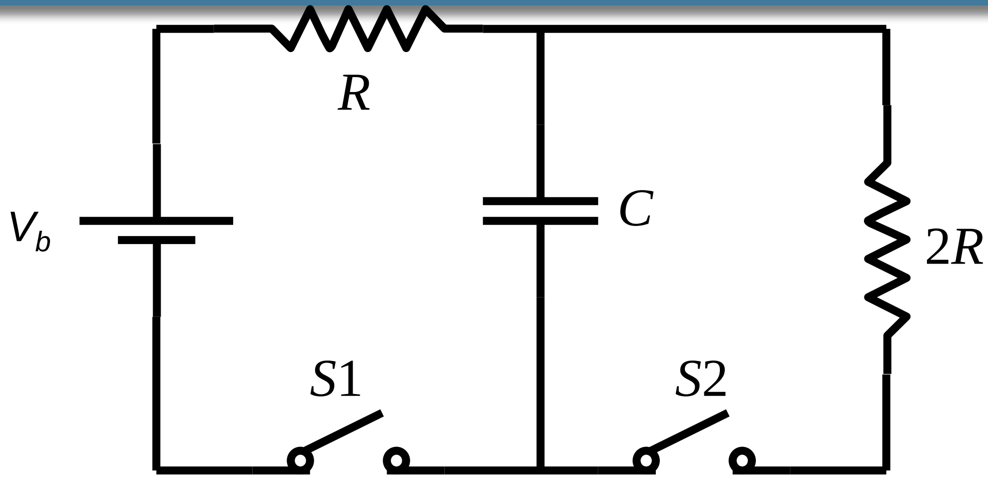


CheckPoint 1 d

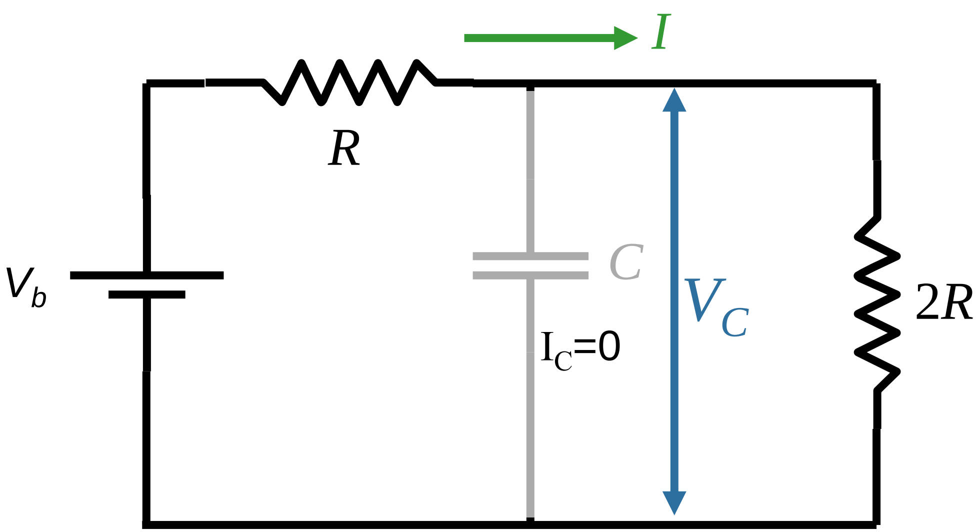


Now suppose both switches are closed. What is the voltage across the capacitor after a very long time?

A. $V_C = 0$ ***B.*** $V_C = V_b$ ***C.*** $V_C = 2V_b/3$



Close both $S1$ and $S2$ and wait a long time…



No current flows through the capacitor after a long time. This will always be the case in any static circuit!!

Outer Loop

$$IR + 2IR - V_b = 0$$

$$I = V_b/(3R)$$

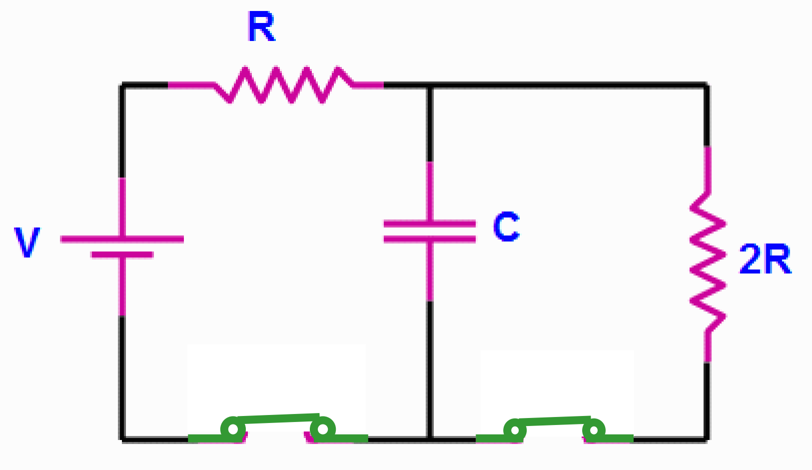
Right Loop

$$+V_C - 2IR = 0$$

$$V_C = 2IR$$

$$\rightarrow \quad V_C = (2/3)V_b$$

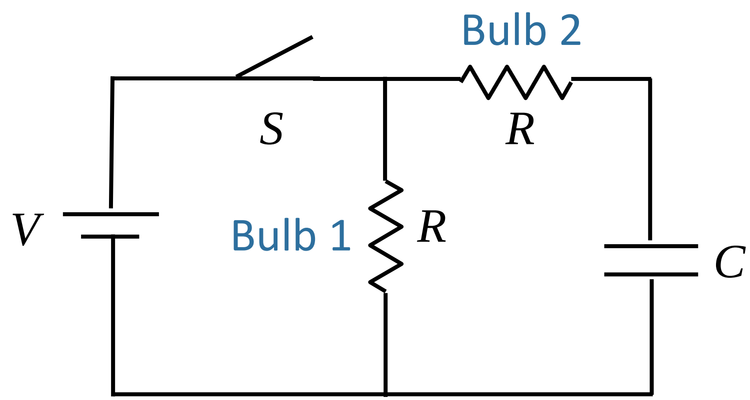
CheckPoint 1 d



Now suppose both switches are closed. What is the voltage across the capacitor after a very long time?

A. $V_C = 0$ ***B.*** $V_C = V_b$ ***C.*** $V_C = 2V_b/3$

DEMO – Clicker Question 1

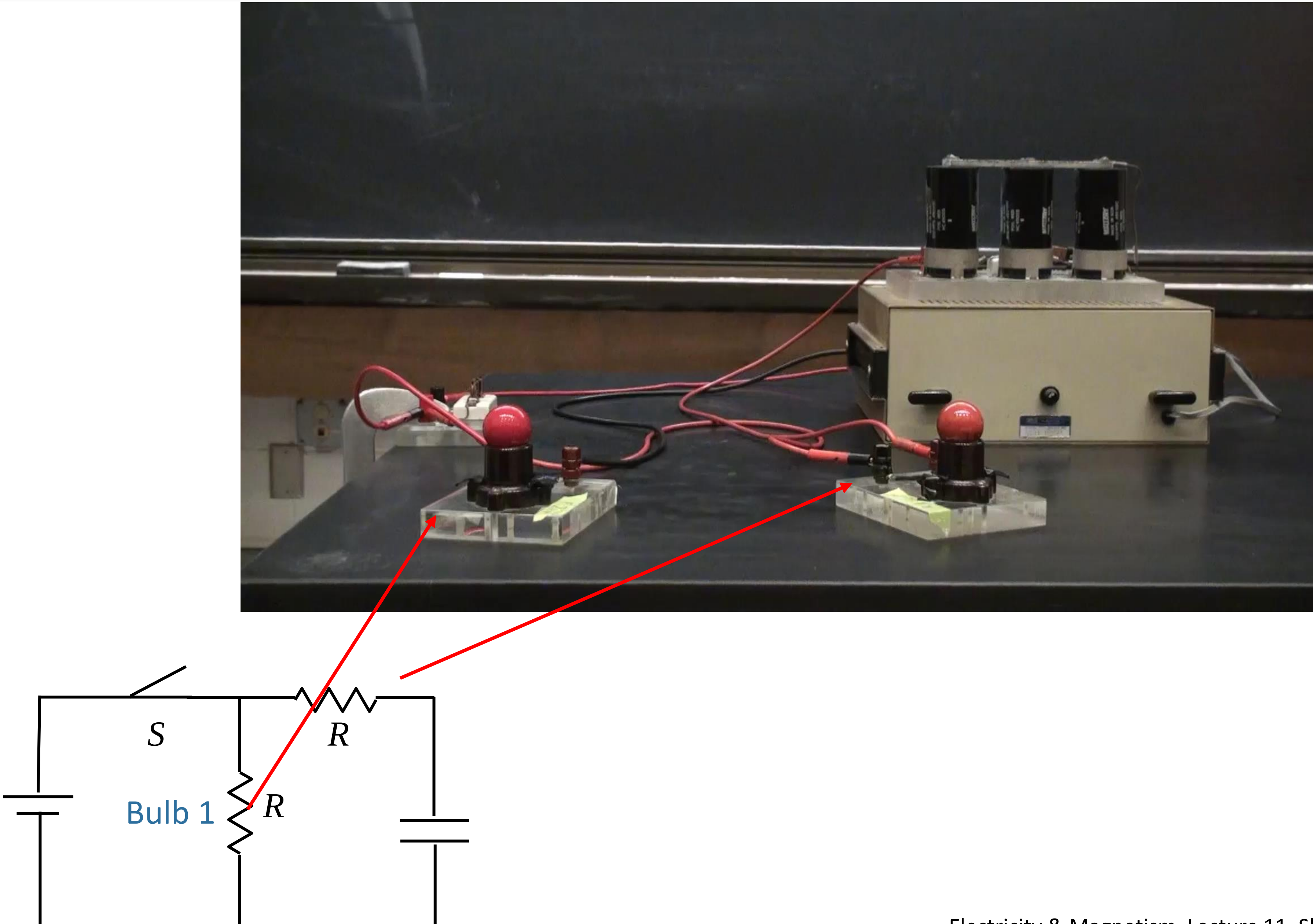


What will happen after I close the switch?

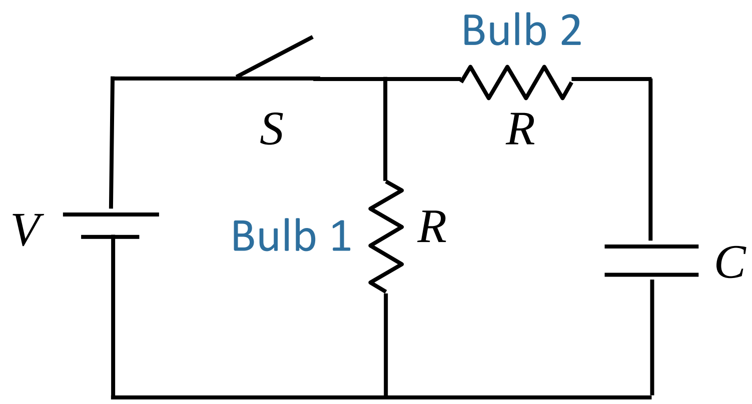
A) Both bulbs come on and stay on.
B) Both bulbs come on but then bulb 2 fades out.
C) Both bulbs come on but then bulb 1 fades out.
D) Both bulbs come on and then both fade out.

No initial charge on capacitor → $V(\text{bulb 1}) = V(\text{bulb 2}) = V$ → Both bulbs light

No final current through capacitor → $V(\text{bulb 2}) = 0$


S
R
Bulb 1
R

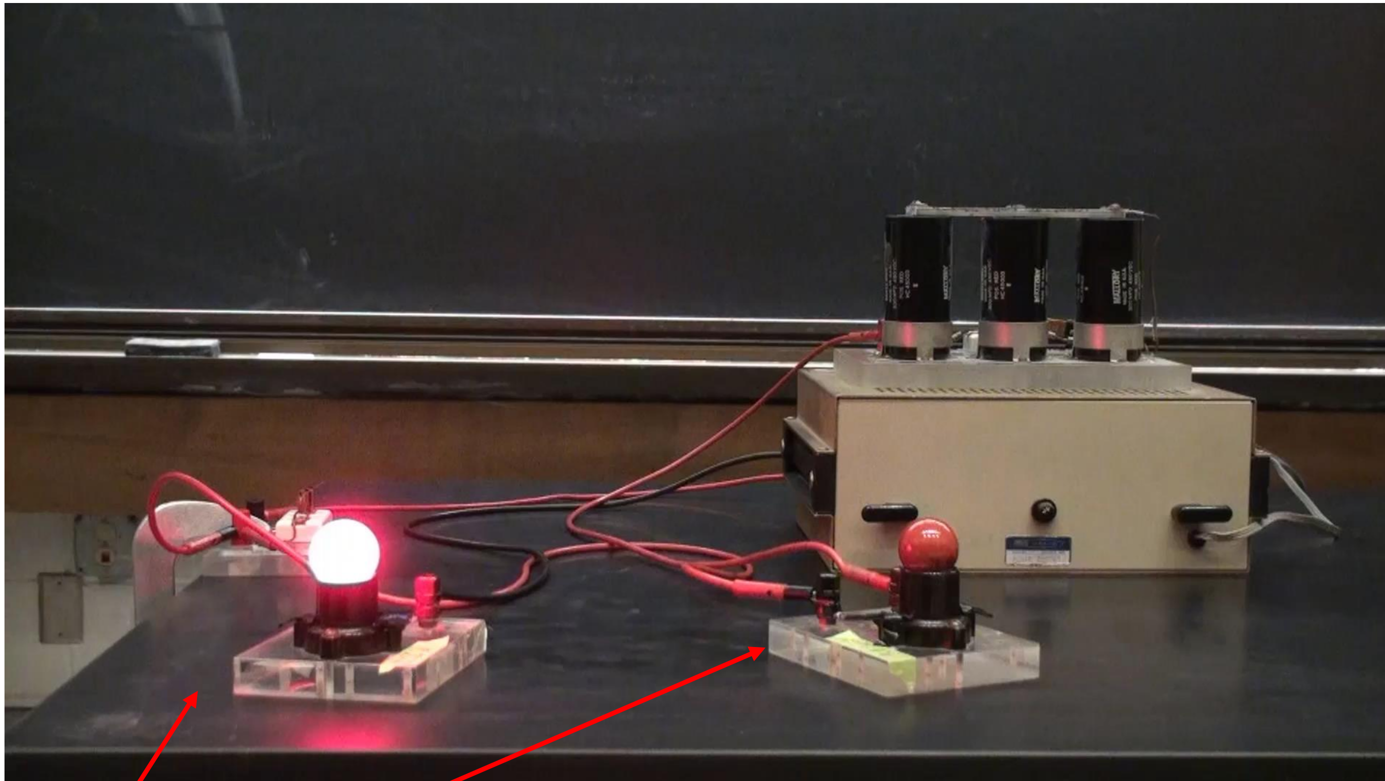
DEMO Clicker Question 2

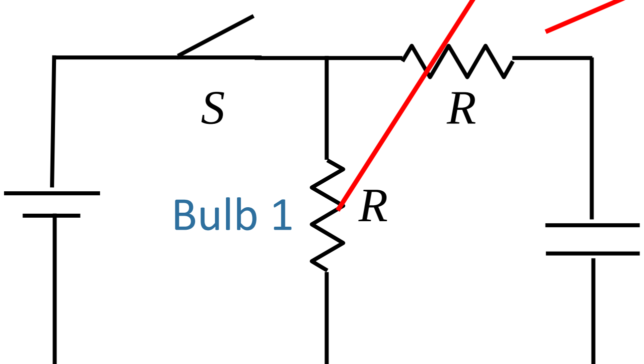


Suppose the switch has been closed a long time.
Now what will happen after opening the switch?

A) Both bulbs come on and stay on.
B) Both bulbs come on but then bulb 2 fades out.
C) Both bulbs come on but then bulb 1 fades out.
D) Both bulbs come on and then both fade out.

Capacitor has charge (=CV) → Capacitor discharges through both resistors

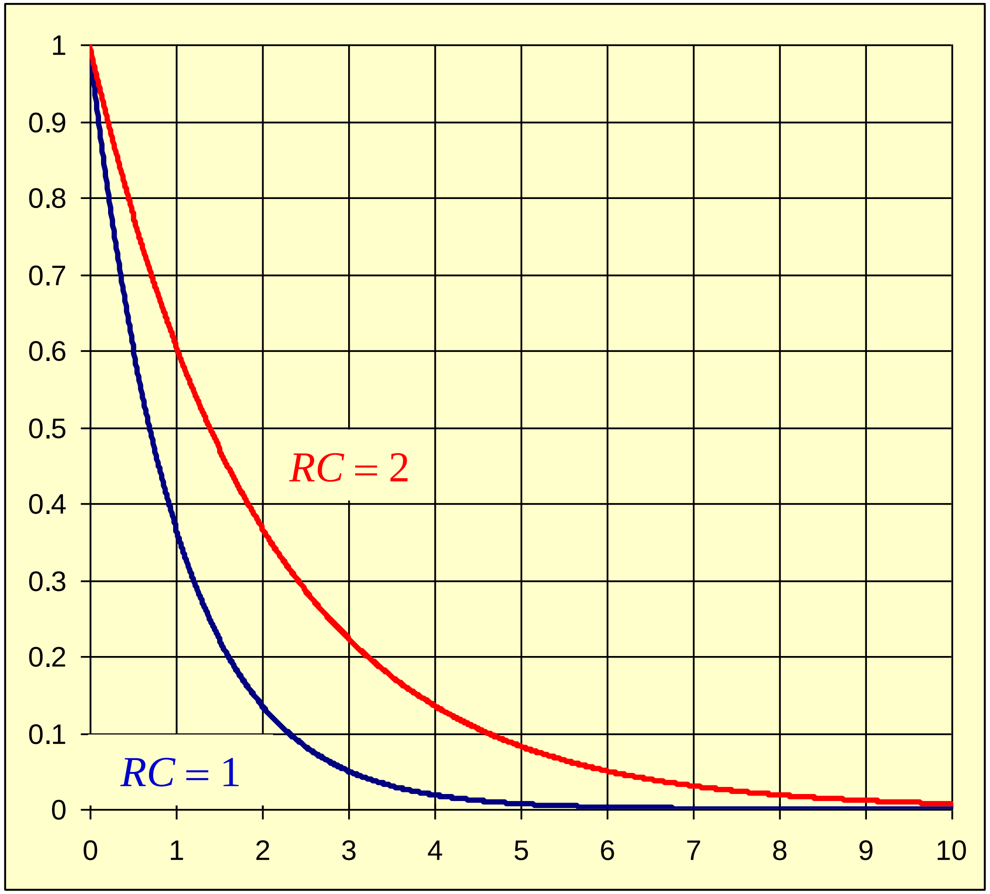



S
R
Bulb 1
R

How do Exponentials Work?

$$Q(t) = Q_0 e^{-t/RC}$$

$$\frac{Q(t)}{Q_0}$$



t

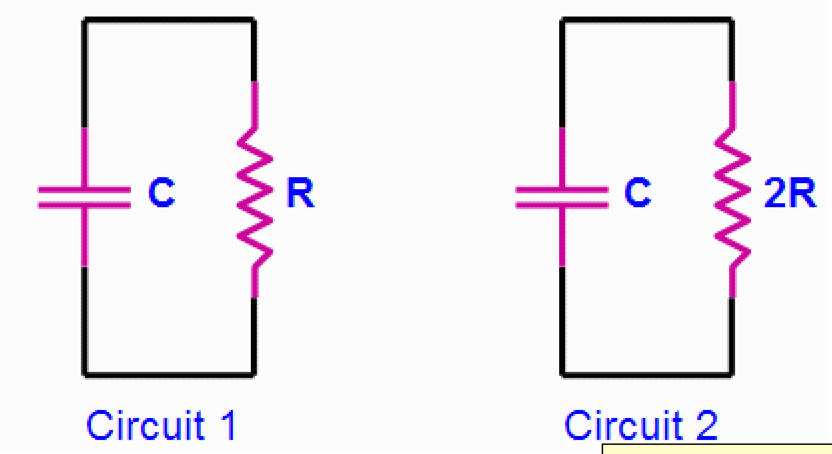
Time constant:

$\tau = RC$

The bigger τ is, the longer it takes to get the same change…

CheckPoint 2

The two circuits shown below contain identical capacitors that hold the same charge at t = 0. Circuit 2 has twice as much resistance as circuit 1.



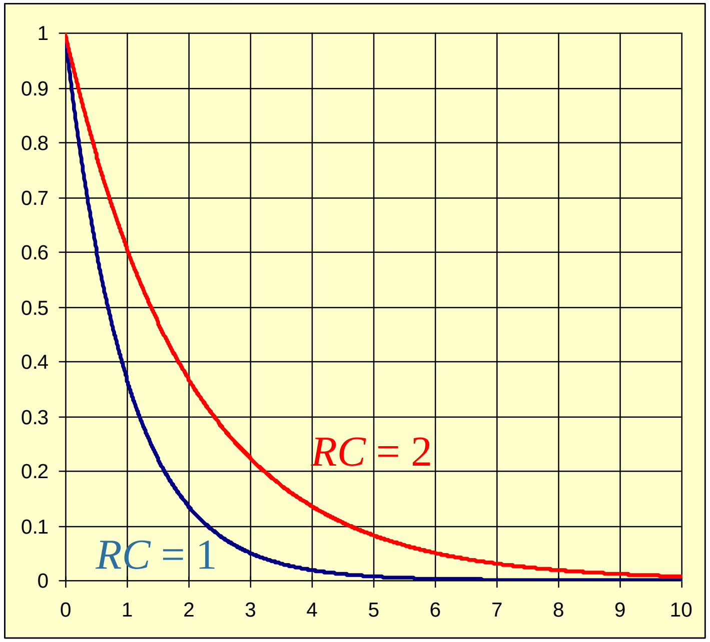
Which circuit has the largest time constant?

A) Circuit 1

B) Circuit 2

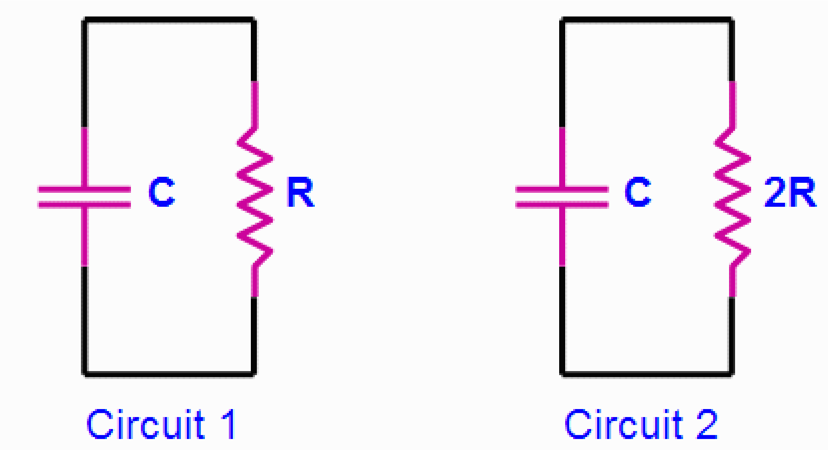
C) Same

$$\tau = R_{equiv}\,C$$



CheckPoint 2

The two circuits shown below contain identical capacitors that hold the same charge at t = 0. Circuit 2 has twice as much resistance as circuit 1.



Which of the following statements best describes the charge remaining on each of the the two capacitors for for any time after t = 0?

A $Q_1 < Q_2$

B $Q_1 > Q_2$

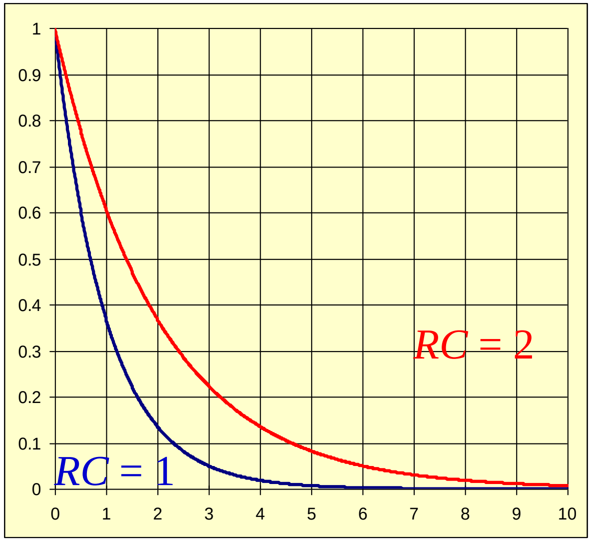
C $Q_1 = Q_2$

D $Q_1 < Q_2$ at first and then $Q_1 > Q_2$ after a long time

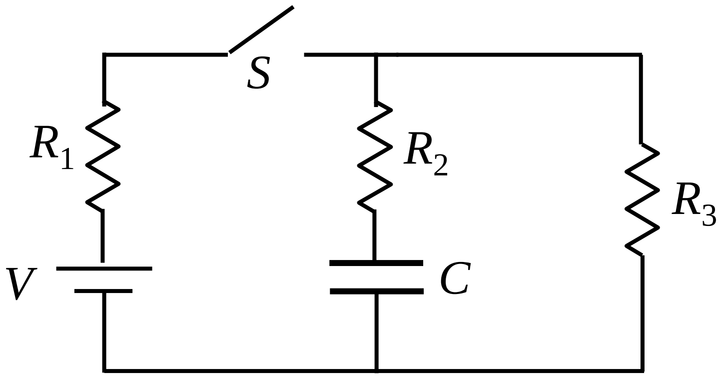
E $Q_1 > Q_2$ at first and then $Q_1 < Q_2$ after a long time

$$\boxed{Q = Q_0 e^{-t/RC}}$$

Look at plot!



Calculation



In this circuit, assume V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Conceptual Analysis:

Circuit behavior described by Kirchhoff's Rules:

$$\Sigma V_{drops} = 0$$

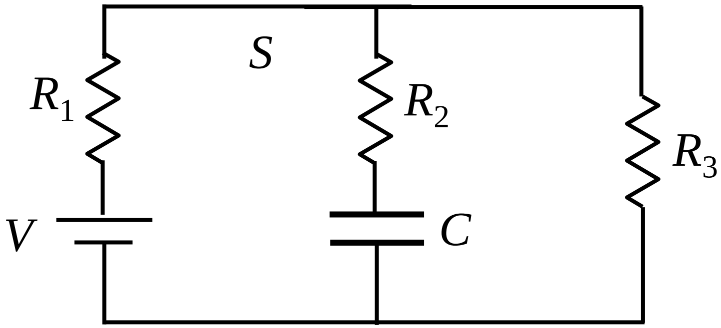
$$\Sigma I_{in} = \Sigma I_{out}$$

S closed and C charges to some voltage with some time constant

Strategic Analysis

Determine currents and voltages in circuit a long time after S closed

Calculation



In this circuit, assume V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Immediately after S is closed:

what is I_2, the current through C

what is V_C, the voltage across C?

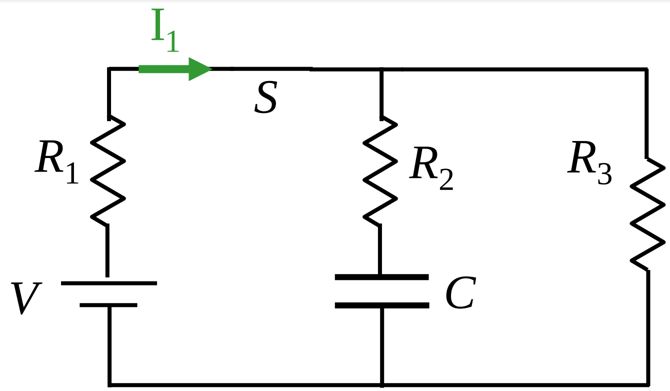
A) Only $I_2 = 0$ B) Only $V_C = 0$ C) Both I_2 and $V_C = 0$ D) Neither I_2 nor $V_C = 0$

Why?

We are told that C is initially uncharged $(V_C = Q/C)$

I_2 cannot be zero because $V_{R2}+V_C = V_{R3}$ (Right Loop)

Calculation



In this circuit, V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Immediately after S is closed, what is I_1, the current through R_1 ?

A: $\dfrac{V}{R_1}$

B: $\dfrac{V}{R_1 + R_3}$

C: $\dfrac{V}{R_1 + R_2 + R_3}$

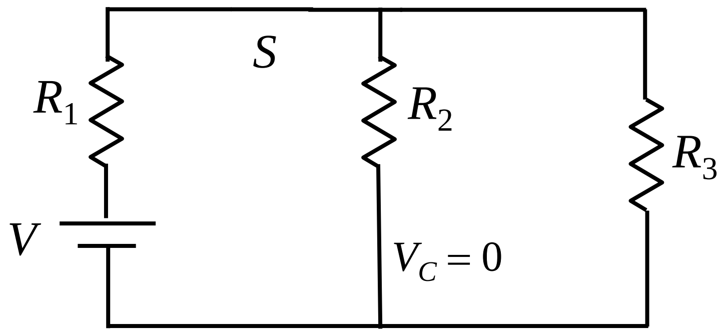
D: $\dfrac{V}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}}$

E: $V\dfrac{R_1 + R_2 + R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$

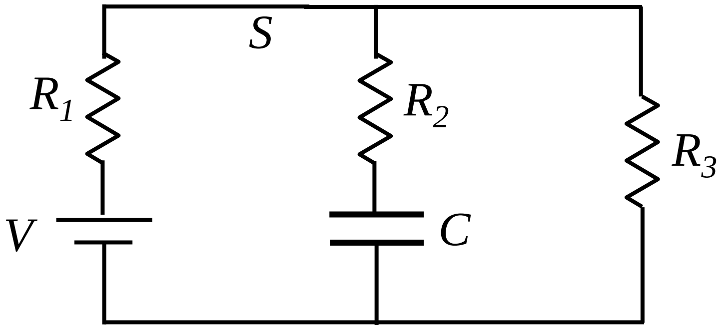
Why?

Draw circuit just after S closed (knowing $V_C = 0$)

R_1 is in series with the parallel combination of R_2 and R_3



Calculation



In this circuit, assume V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

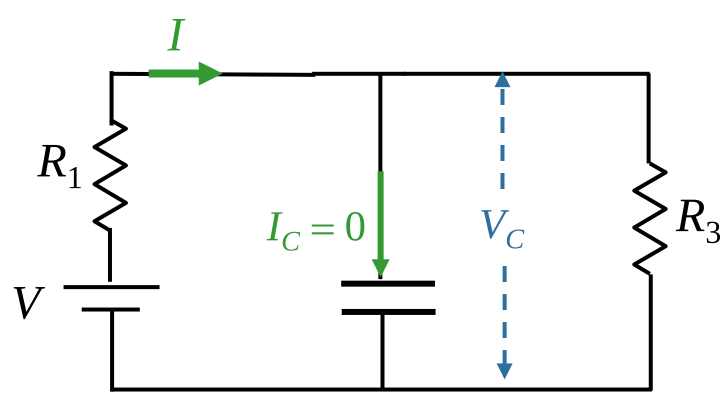
After S has been closed "for a long time", what is I_2, the current through R_2 ?

$\dfrac{V}{R_2}$	$\dfrac{V}{R_1}$	0
A	B	C

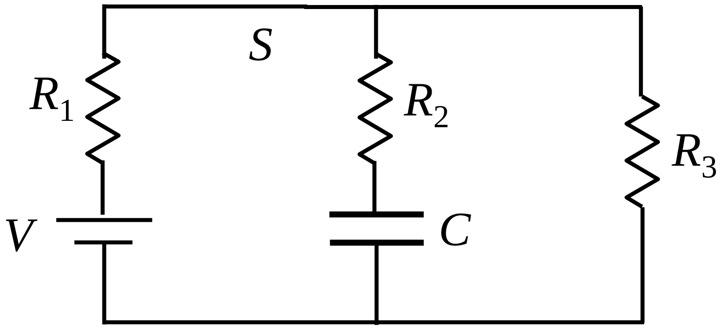
Why?

After a long time in a static circuit, the current through any capacitor approaches 0 !

This means we Redraw circuit with open circuit in middle leg



Calculation



In this circuit, assume V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

After S has been closed "for a long time", what is V_C, the voltage across C ?

A: $V\dfrac{R_3}{R_1+R_3}$

B: $V\dfrac{R_2}{R_1+R_2}$

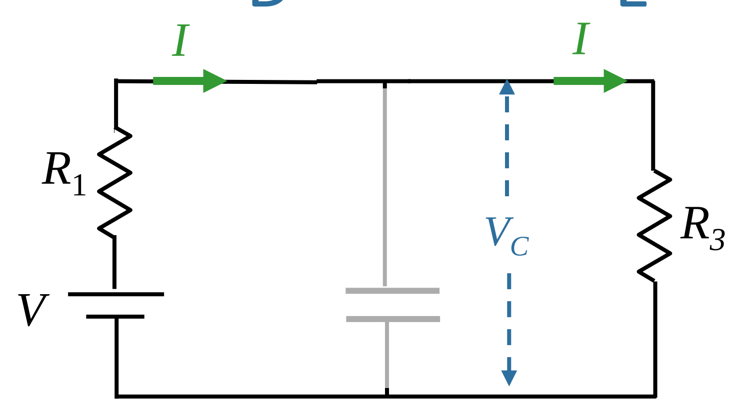
C: V

D: $V\dfrac{R_2}{R_1+\dfrac{R_2R_3}{R_2+R_3}}$

E: 0

Why?

– $V_C = V_3 = IR_3 = (V/(R_1+R_3))R_3$



Next up --> Magnetism!

Physics 212
Lecture 12

Today's Concept:

Magnetic Force on Moving Charges

Key Concepts:

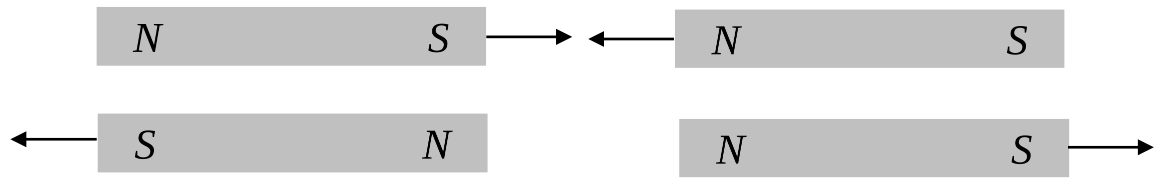
1) The force on moving charges due to a magnetic field.
2) The cross product.

Today's Plan:

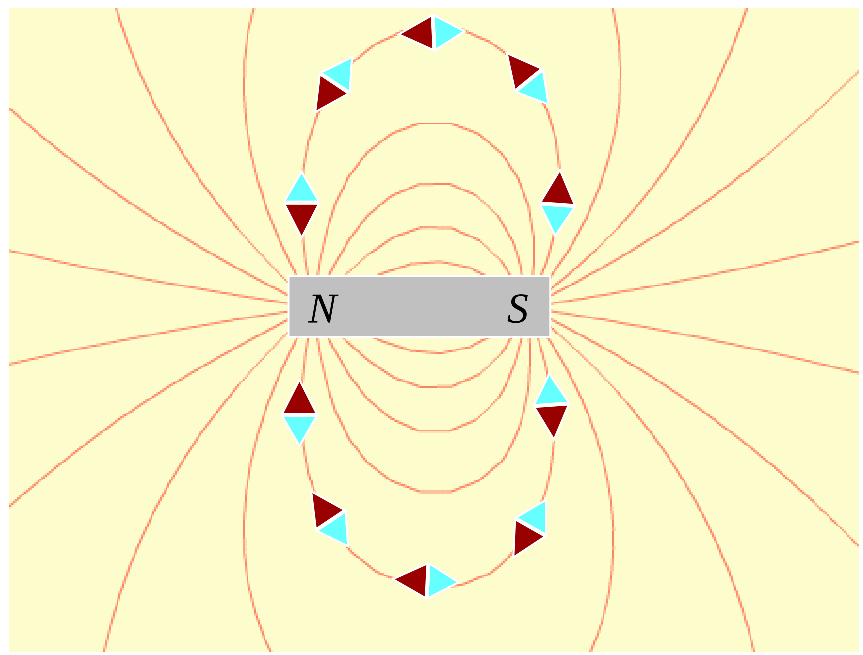
1) Review of magnetism
2) Review of cross product
3) Example problem

Magnetic Observations

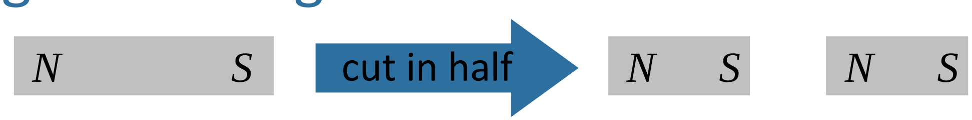
Bar Magnets

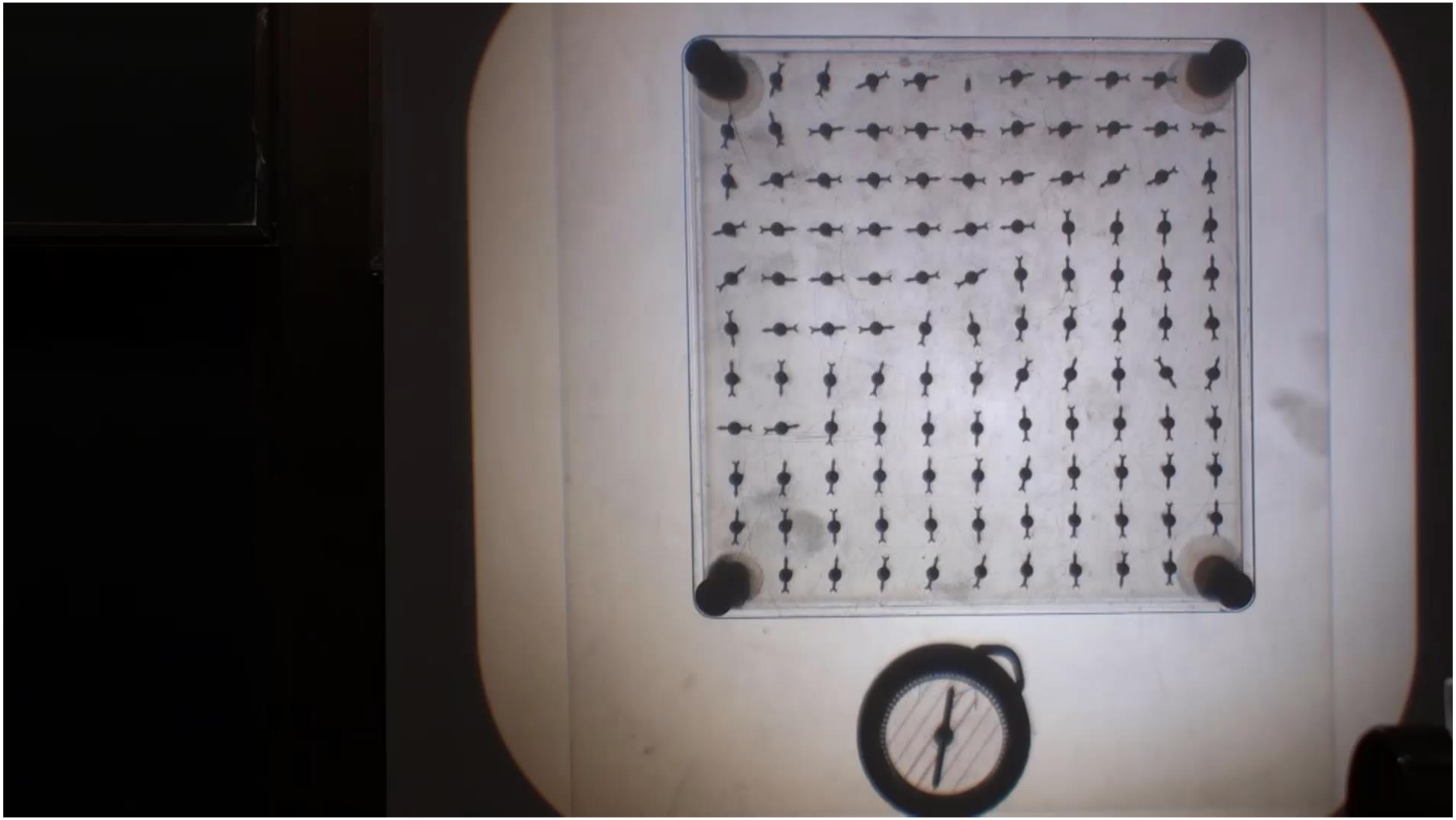


Compass Needles



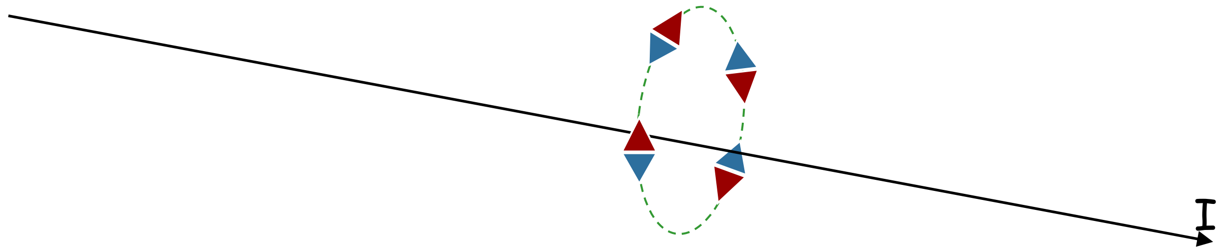
Magnetic Charge?





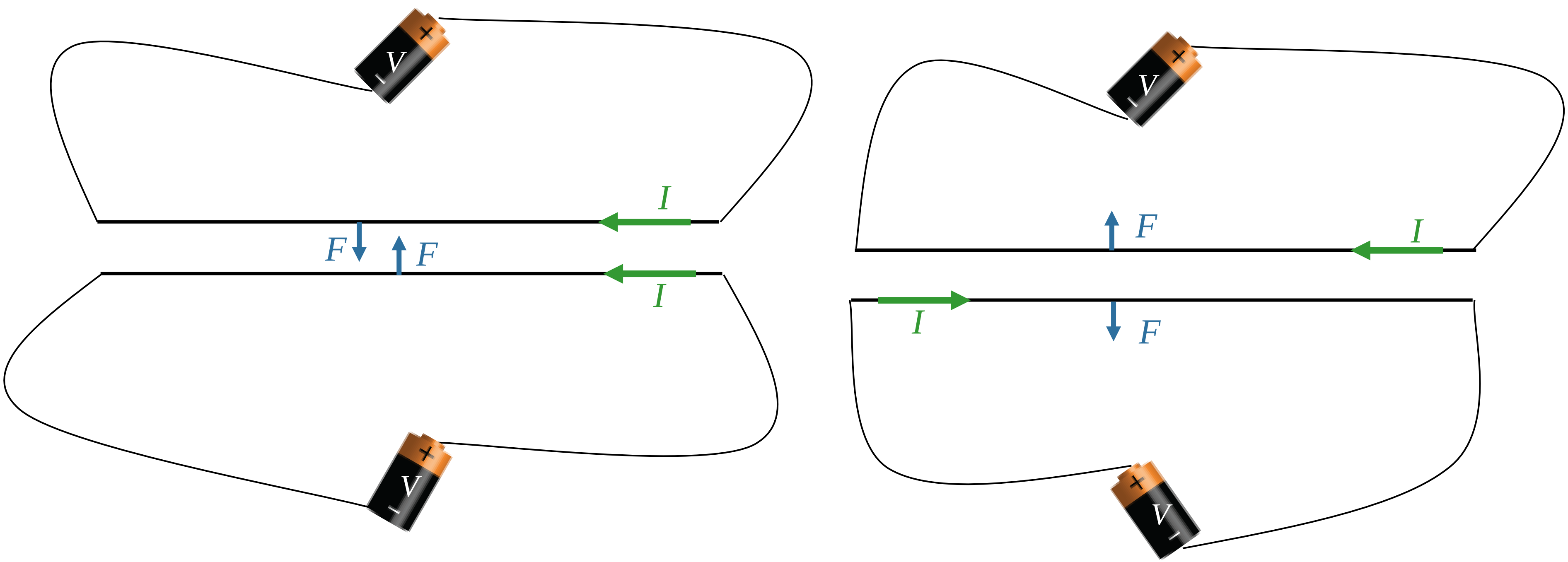
Magnetic Observations

Compass needle deflected by electric current



Magnetic fields created by electric currents

Magnetic fields exert forces on electric currents (charges in motion)



Magnetism & Moving Charges

All observations are explained by two equations:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Today

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

Next Week

Cross Product Review

Cross Product different from Dot Product

$A \bullet B$ is a scalar; $A \times B$ is a vector

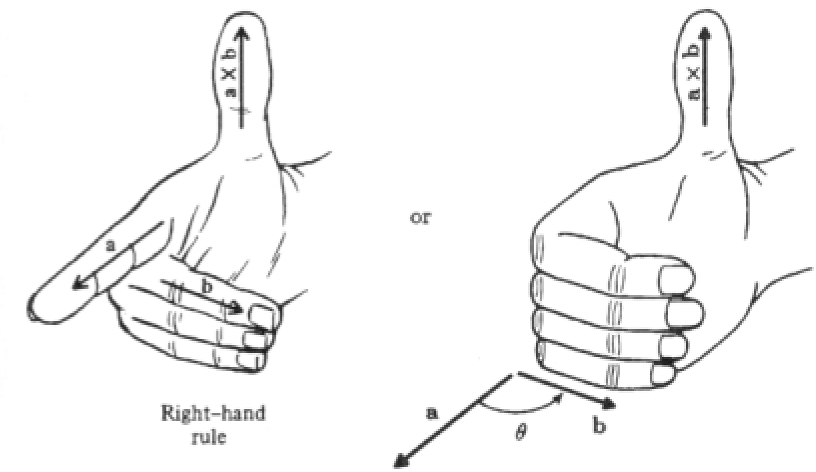
$A \bullet B$ proportional to the component of B parallel to A

$A \times B$ proportional to the component of B perpendicular to A

Definition of $A \times B$

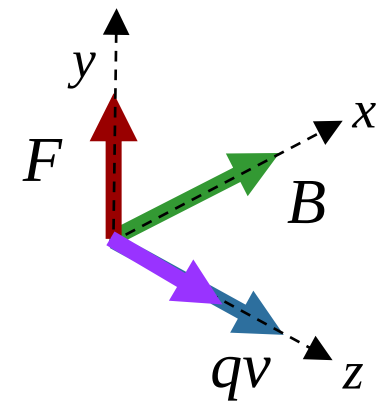
Magnitude: $AB\sin\theta$

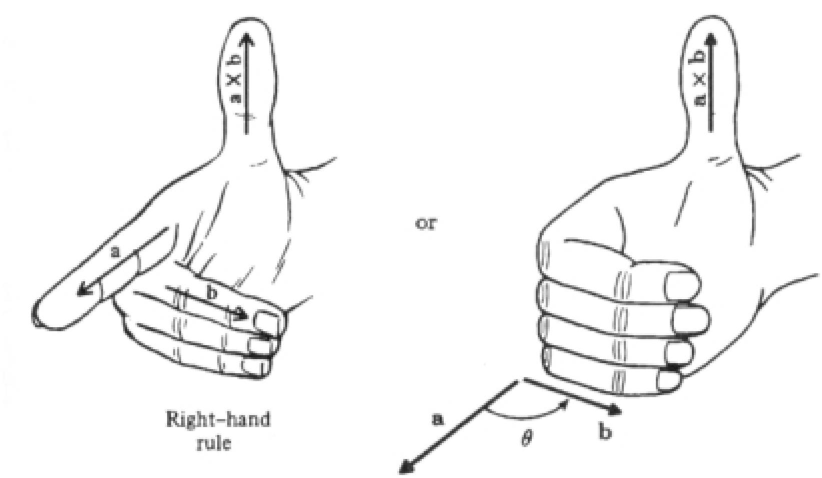
Direction: perpendicular to plane defined by A and B with sense given by right-hand-rule

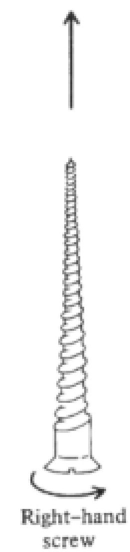


Remembering Directions: The Right Hand Rule

$$\vec{F} = q\vec{v} \times \vec{B}$$

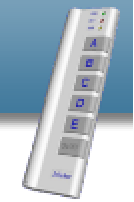




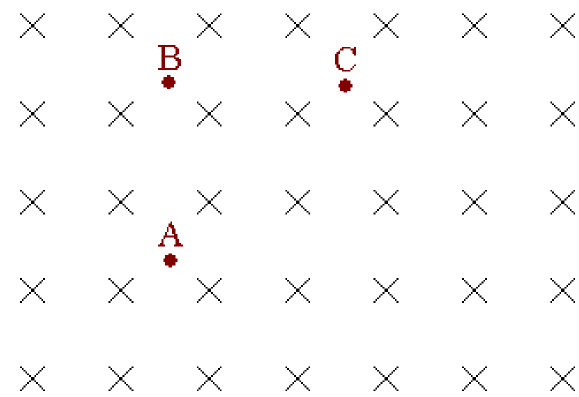


Lots of videos for extra explanation: https://www.youtube.com/watch?v=zGyfiOqiR4s

Check Point 1a



Three points are arranged in a uniform magnetic field. The B field points into the screen.



A positively charged particle is located at point A and is stationary. The direction of the magnetic force on the particle is

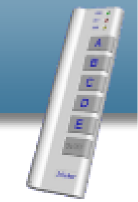
A. right ***B.*** left ***C.*** into the screen ***D.*** out of the screen

E. zero

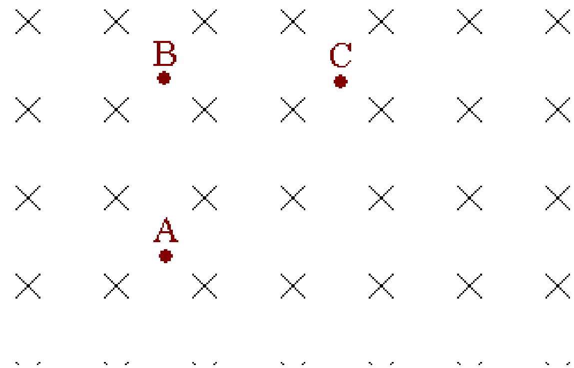
$$F = q\vec{v} \times B$$

Magnetic force only acts on moving charges.

Check Point 1b



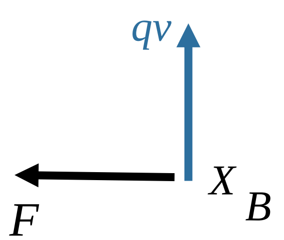
Three points are arranged in a uniform magnetic field. The B field points into the screen.



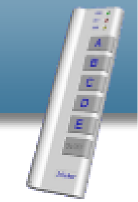
The positive charge moves from A toward B. The direction of the magnetic force on the particle is

A. right ***B.*** left ***C.*** into the screen ***D.*** out of the screen
E. zero

$$\vec{F} = q\vec{v} \times \vec{B}$$



Cross Product Practice



Protons (positive charge) coming out of screen

Magnetic field pointing down

What is direction of force on POSITIVE charge?

$$\vec{F} = q\vec{v} \times \vec{B}$$

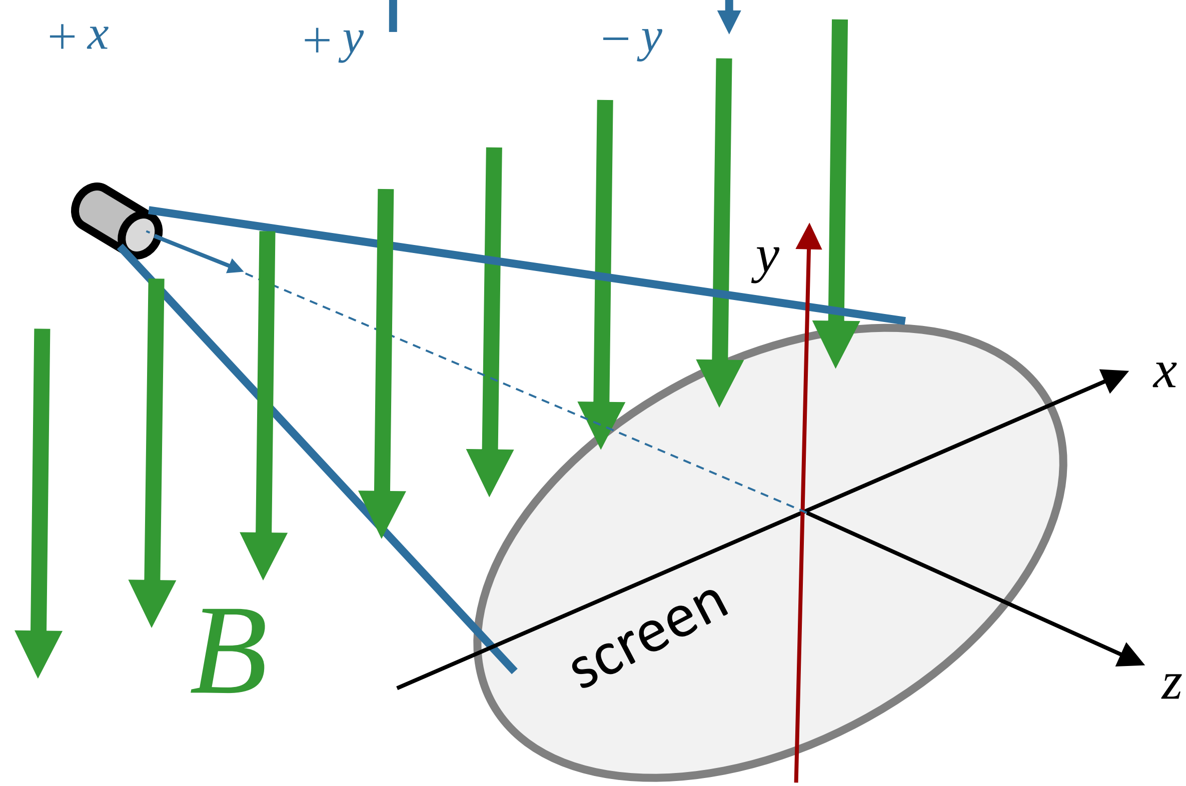
A) Left $-x$

B) Right $+x$

C) UP $+y$

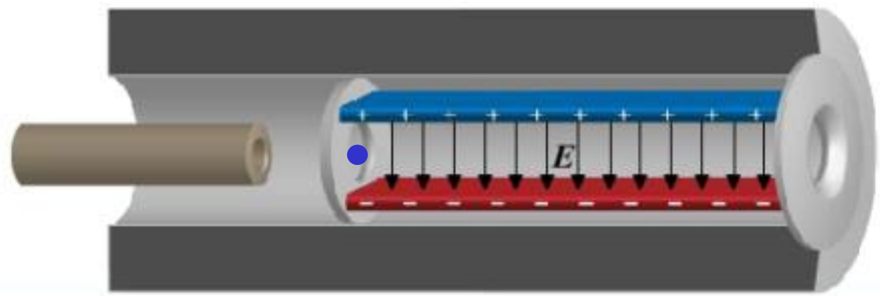
D) Down $-y$

E) Zero



Velocity Selector

Electric force on positive charge will be DOWN.
$F_E = Eq$



Need magnetic force UP

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

Magnetic field should be
A) Up
B) Down
C) Into page
D) Out of page

Fingers to right (v) thumb up (F) curl fingers (B) into page

$F_E + F_B = 0$

$Eq = qvB$ Since v is to right and B is into page

$v = E/B$

Motion of Charge q in Uniform B Field

Force is perpendicular to v

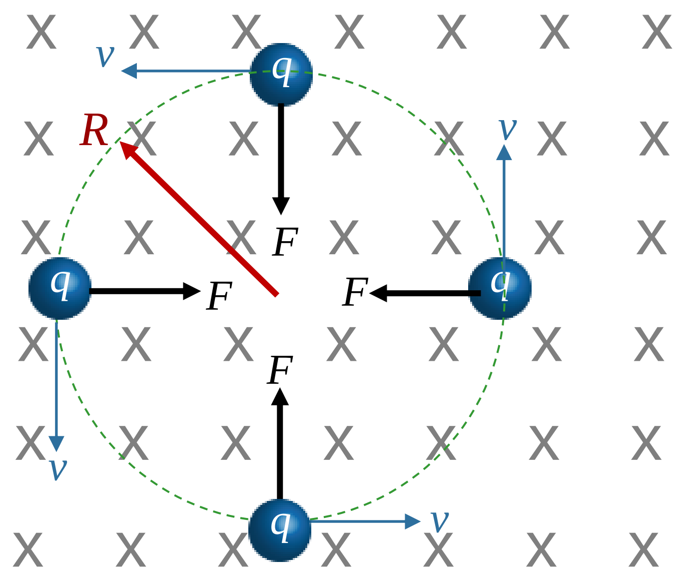
Speed does not change

Uniform Circular Motion

Solve for R:

$$\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB$$

$$a = \frac{v^2}{R}$$

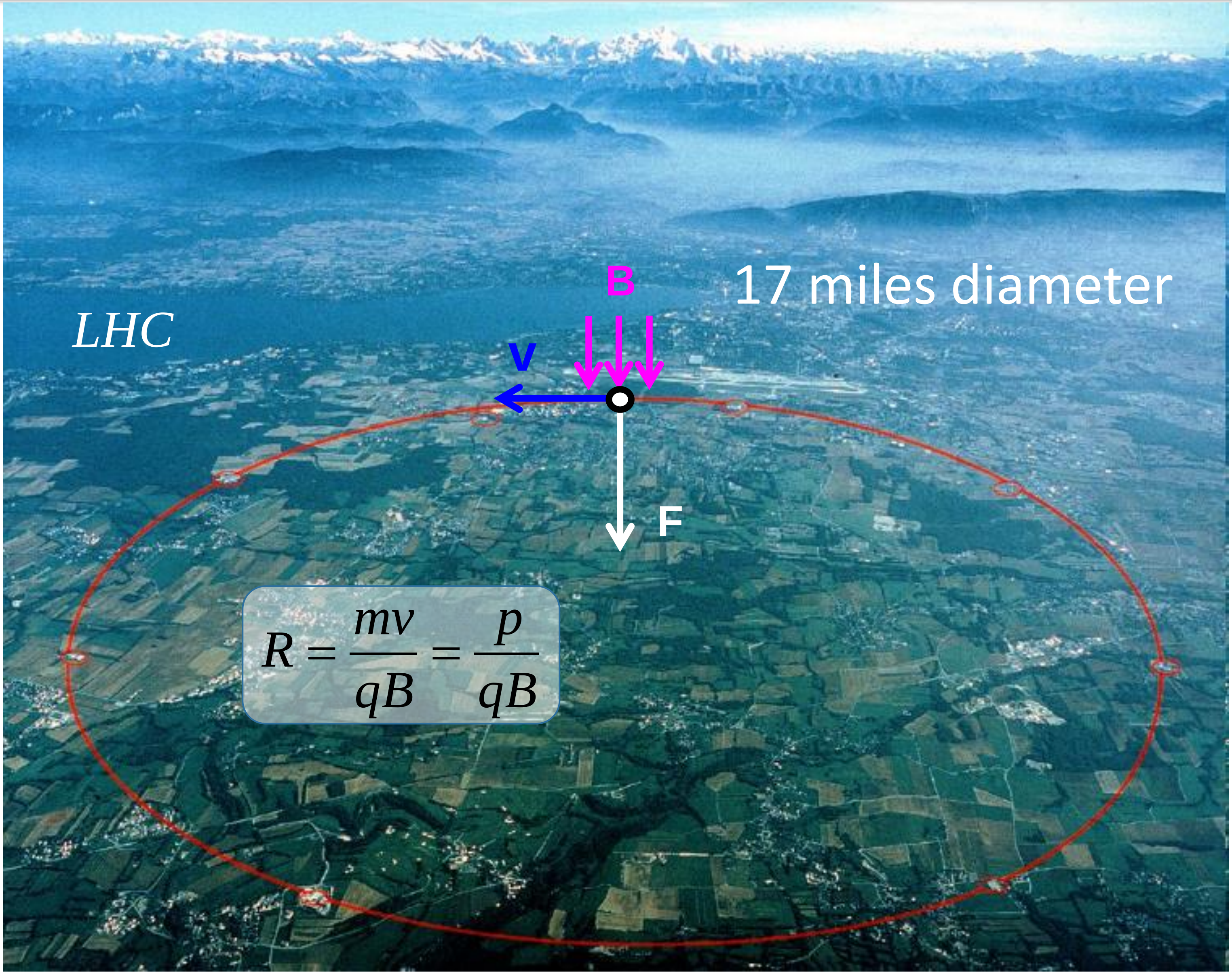


Uniform B into page

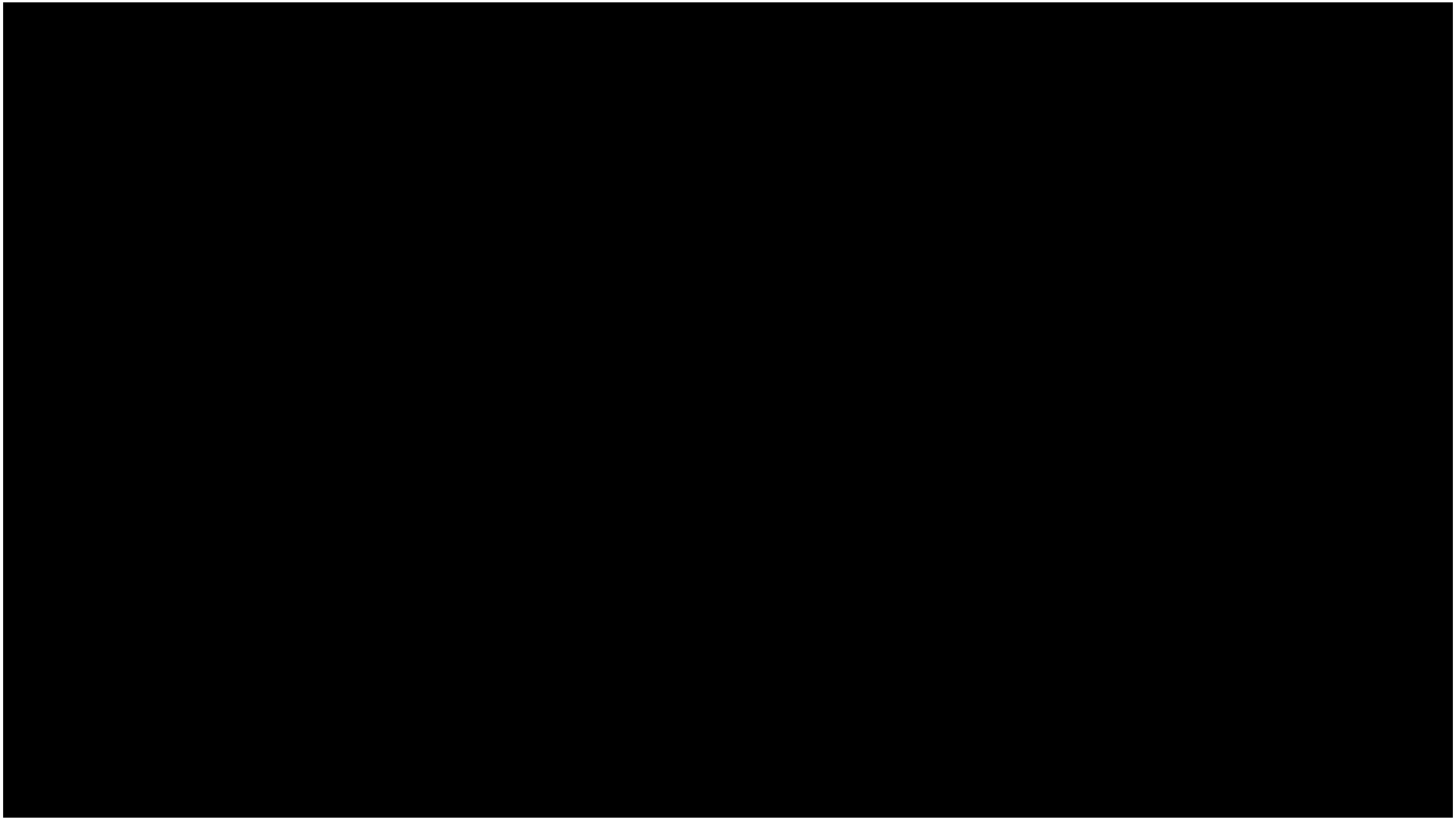
$$qvB = m\frac{v^2}{R} \quad \rightarrow \quad R = \frac{mv}{qB} \quad \rightarrow \quad p = qBR$$

$$(p = mv)$$

Can you take us to the LHC to see some of the big magnets in the ring and the detectors there?

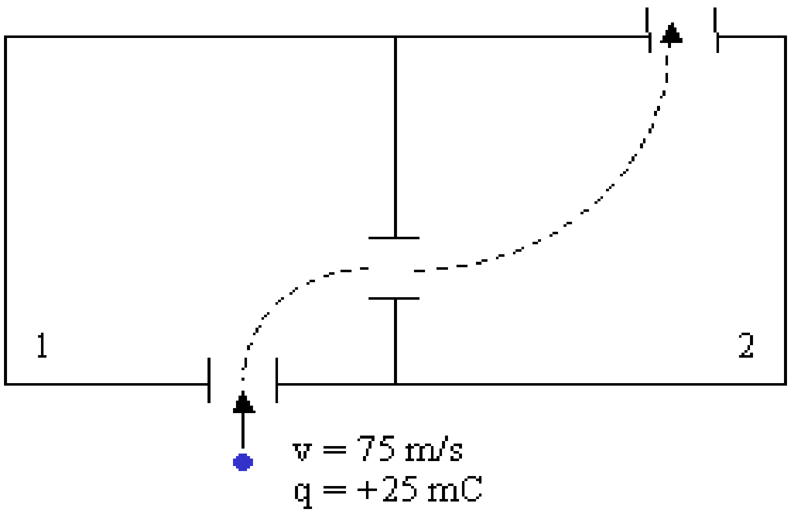


proton-proton Collision Event at the LHC



Check Point 2a

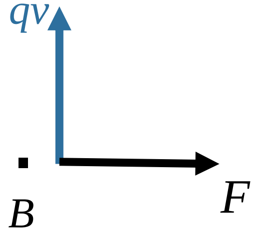
The drawing below shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.



What is the direction of the magnetic field in chamber 1?

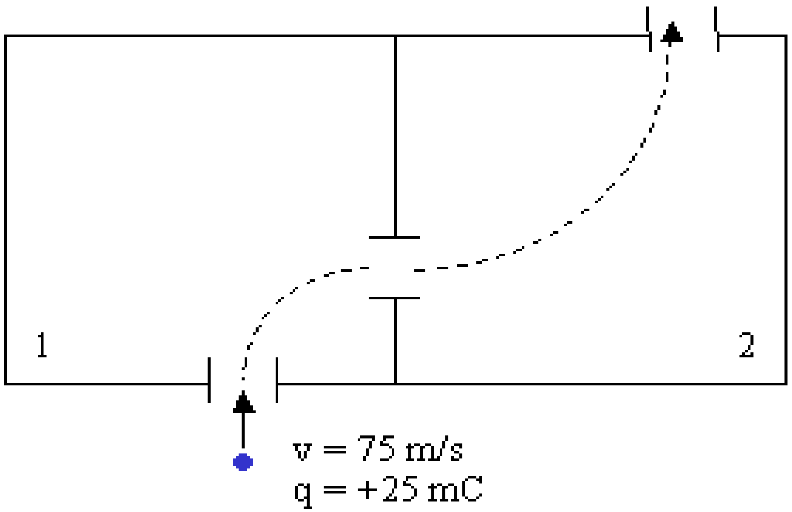
A. up **B.** down **C.** into the page **D.** out of the page

$$\vec{F} = q\vec{v} \times \vec{B}$$



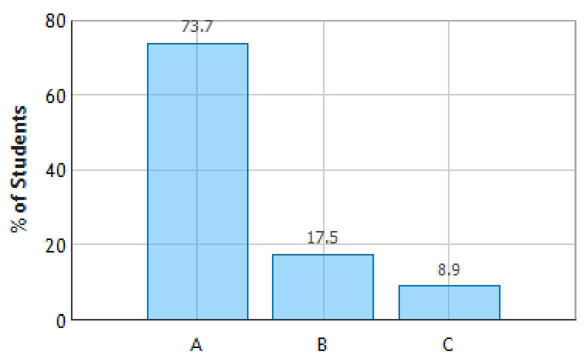
Check Point 2c

The drawing below shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.



Compare the magnitude of the magnetic field in chamber 1 to the magnitude of the magnetic field in chamber 2

A. $|B_1| > |B_2|$ ***B.*** $|B_1| = |B_2|$ ***C.*** $|B_1| < |B_2|$

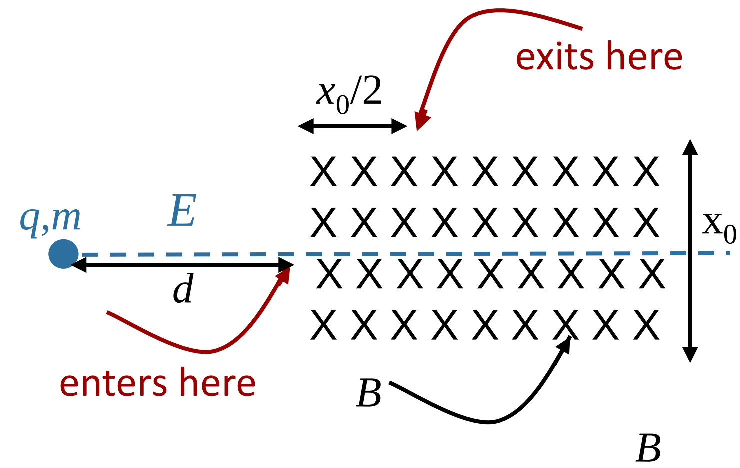


Observation: $R_2 > R_1$

$$R = \frac{mv}{qB} \rightarrow |B_1| > |B_2|$$

Calculation

A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.



What is B?

Strategic Analysis

Calculate v, the velocity of the particle as it enters the magnetic field

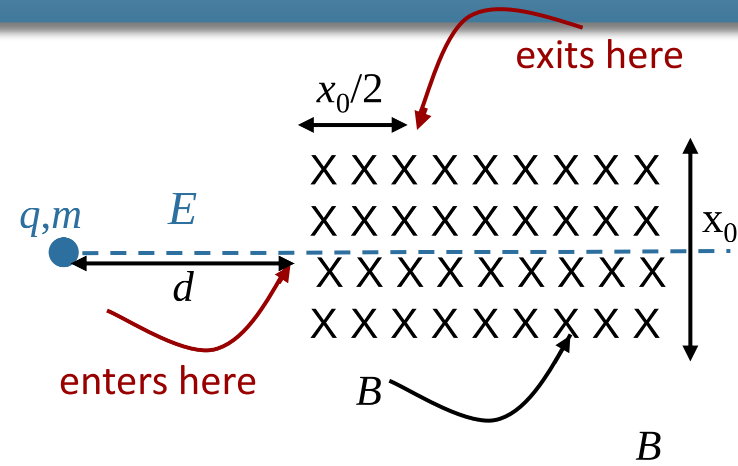
Use Lorentz Force equation to determine the path in the field as a function of B

Apply the entrance-exit information to determine B

Let's Do It !

Calculation

A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.



What is B?

- What is the change in the particle's potential energy after travelling distance d?

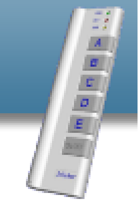
$\Delta U = -qEd$ (A)

$\Delta U = -Ed$ (B)

$\Delta U = 0$ (C)

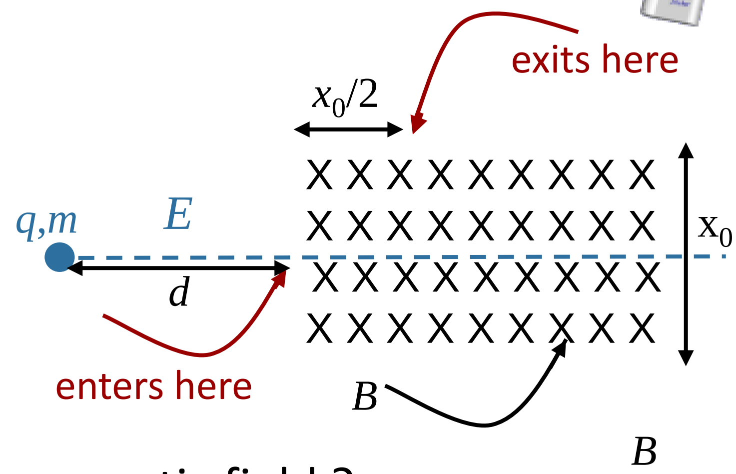
- Why??
  - How do you calculate change in the electric potential given an electric field? $\Delta V = -\int \vec{E} \cdot d\vec{\ell} = -Ed$
  - What is the relation between the electric potential and the potential energy? $\Delta U = q\Delta V$

Calculation



A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.

What is B?



What is v_0, the speed of the particle as it enters the magnetic field ?

$$v_o = \sqrt{\frac{2E}{m}}$$
A

$$v_o = \sqrt{\frac{2qEd}{m}}$$
B

$$v_0 = \sqrt{2gd}$$
C

$$v_o = \sqrt{\frac{2qE}{md}}$$
D

$$v_o = \sqrt{\frac{qEd}{m}}$$
E

Why?

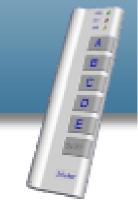
Conservation of Energy

Initial: $Energy = U + 0 = qV = qEd$

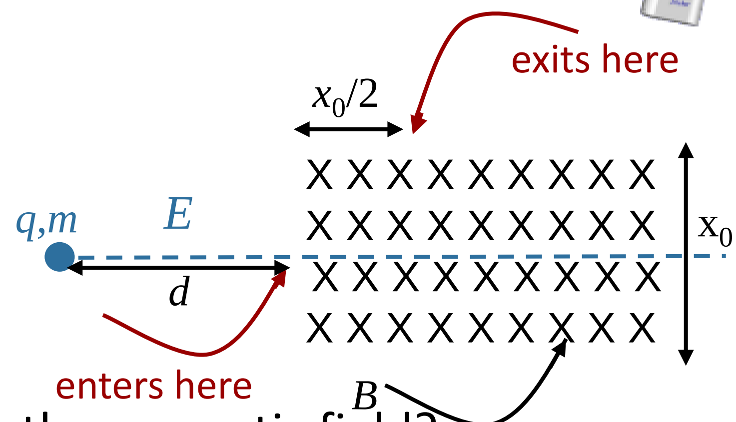
Final: $Energy = 0 + KE = ½\, mv_0^2$

$$\rightarrow \frac{1}{2}mv_o^2 = qEd \rightarrow v_o = \sqrt{\frac{2qEd}{m}}$$

Calculation



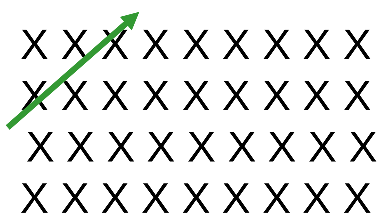
A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.

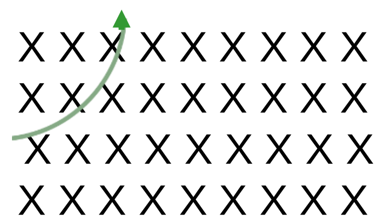


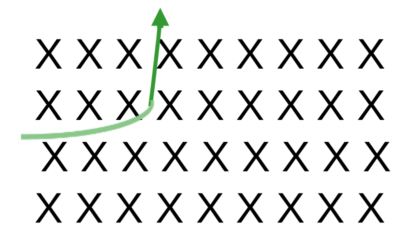
What is B? $v_o = \sqrt{\dfrac{2qEd}{m}}$

What is the path of the particle as it moves through the magnetic field?

B

A 

B 

C 

Why?

Path is circle!

Force is perpendicular to the velocity

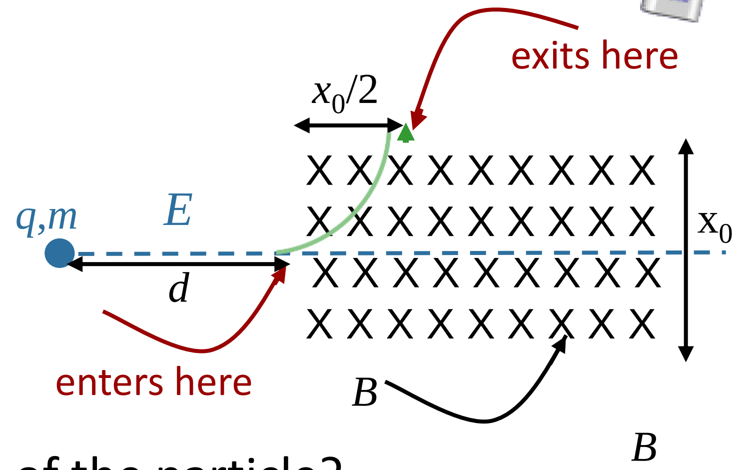
Force produces centripetal acceleration

Particle moves with uniform circular motion

Calculation

A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.

What is B? $v_o = \sqrt{\dfrac{2qEd}{m}}$



What can we use to calculate the radius of the path of the particle?

$R = x_o$ A

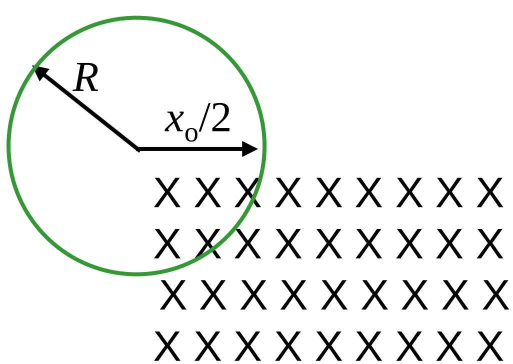
$R = 2x_o$ B

$R = \frac{1}{2}x_o$ C

$R = d + x_0/2$ D

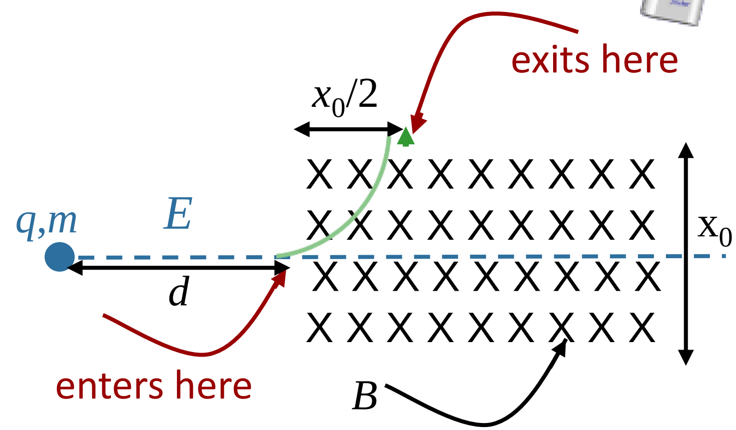
$R = d$ E

Why?



Calculation

A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.



$\boxed{\text{What is } B\text{?}}$

$$v_o = \sqrt{\frac{2qEd}{m}} \qquad R = \tfrac{1}{2} x_0$$

A: $B = \dfrac{2}{x_o}\sqrt{\dfrac{2mEd}{q}}$

B: $B = \dfrac{E}{v}$

C: $B = E\sqrt{\dfrac{m}{2qEd}}$

D: $B = \dfrac{1}{x_o}\sqrt{\dfrac{2mEd}{q}}$

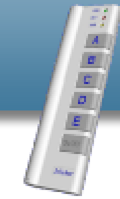
E: $B = \dfrac{mv_o}{qx_o}$

Why?

$$\vec{F} = m\vec{a} \;\rightarrow\; qv_oB = m\frac{v_o^2}{R} \;\rightarrow\; B = \frac{m}{q}\frac{v_o}{R} \;\rightarrow\; B = \frac{m}{q}\frac{2}{x_o}\sqrt{\frac{2qEd}{m}}$$

$$\downarrow$$

$$B = \frac{2}{x_o}\sqrt{\frac{2mEd}{q}}$$



See you next week!

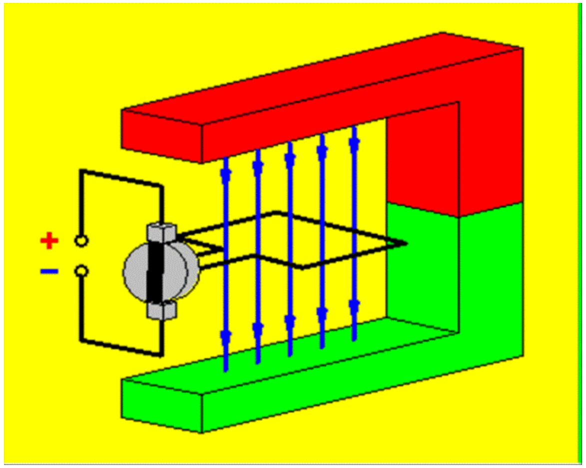
It’s a honeypot.
Don’t answer!

[redacted]

Physics 212
Lecture 13

Today's Concept:

Forces and Torques on current



$$\vec{F} = I\vec{L} \times \vec{B}$$

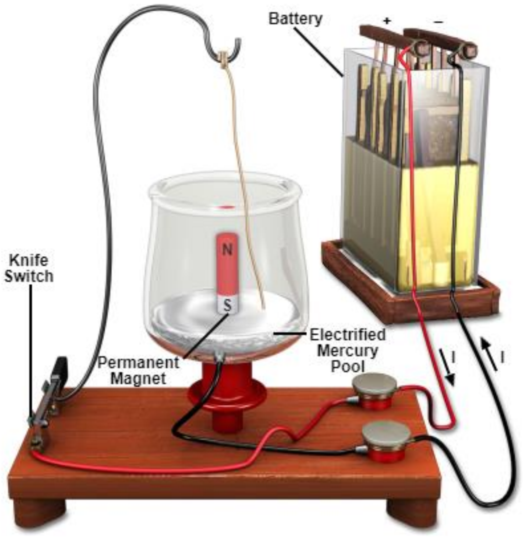
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The torque always wants to line μ up with B!

An early example of a magnetic torque

1820 --> Orsted discovers compass needles get deflected by current

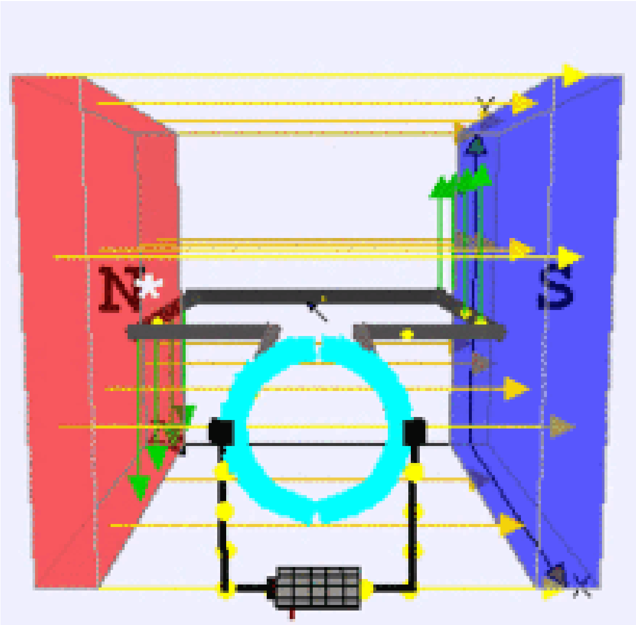
1821 --> Faraday lets use that to convert electricity into mechanical rotation
--> the first electric motor!



Faraday's motor

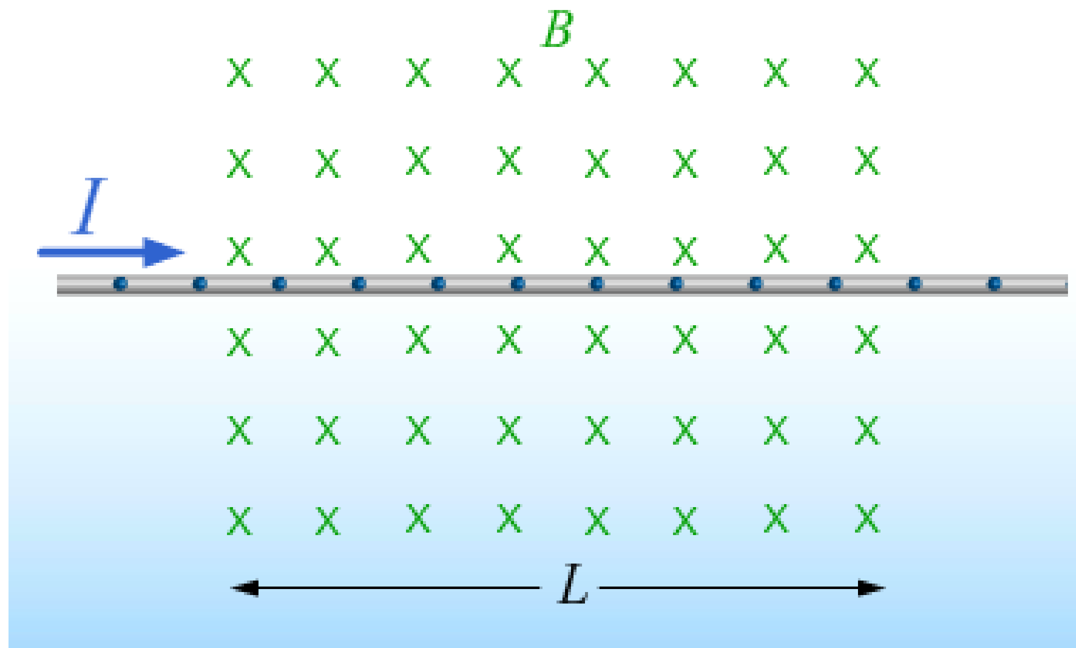
https://nationalmaglab.org/education/magnet-academy/watch-play/interactive/faraday-motor

Modern DC motor



Last Time:

$$\vec{F} = q\vec{v} \times \vec{B}$$

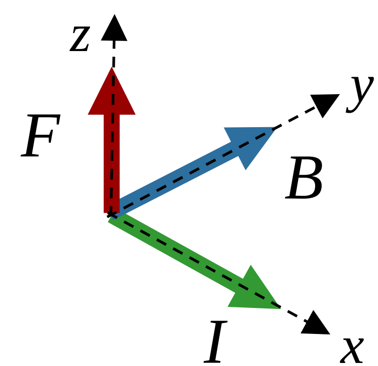


This Time:

$$\vec{F} = q\sum_i \vec{v}_i \times \vec{B}$$

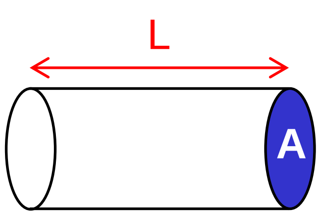
$$\vec{F} = qN\vec{v}_{avg} \times \vec{B}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$



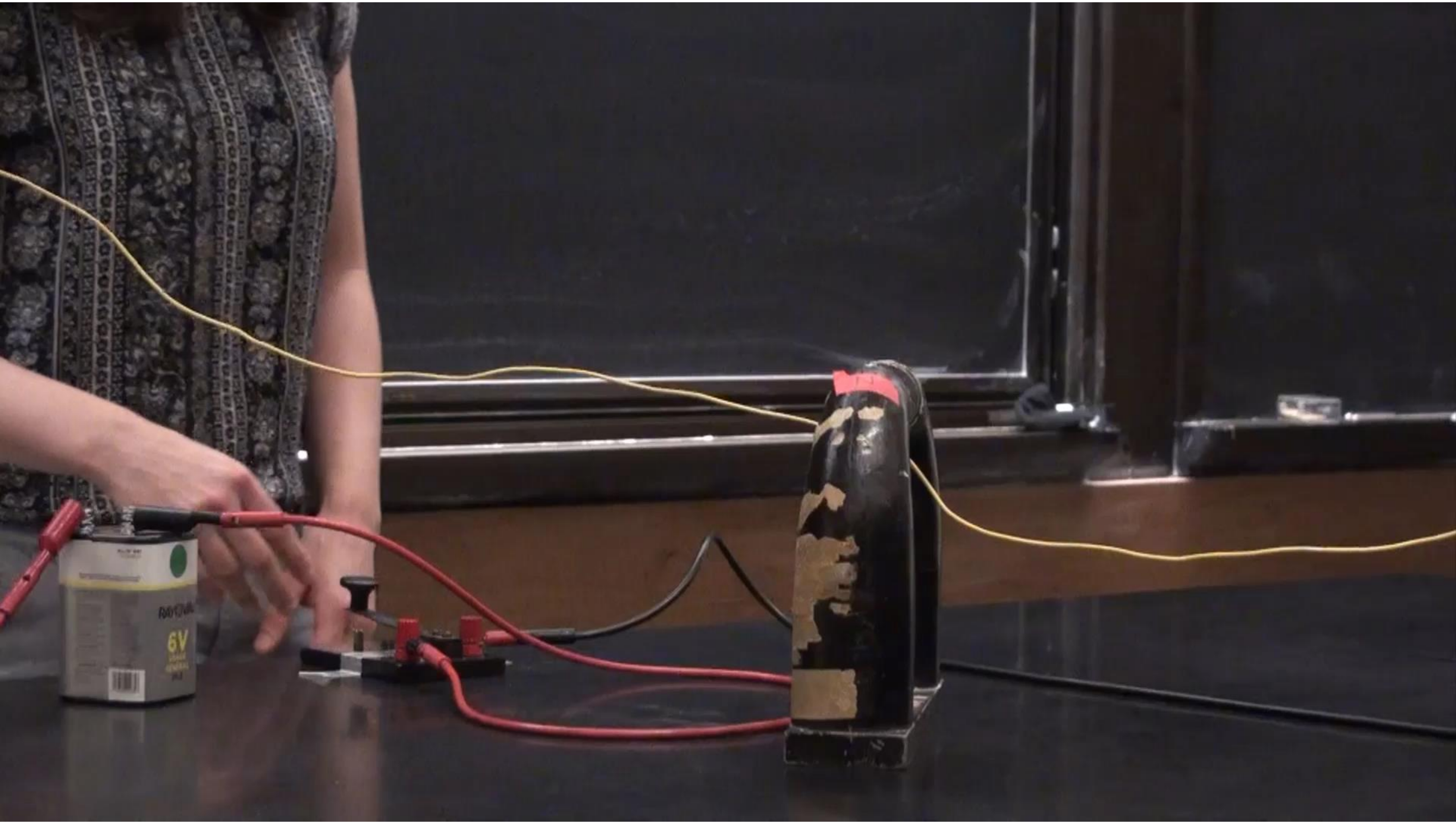
$$N = nAL$$

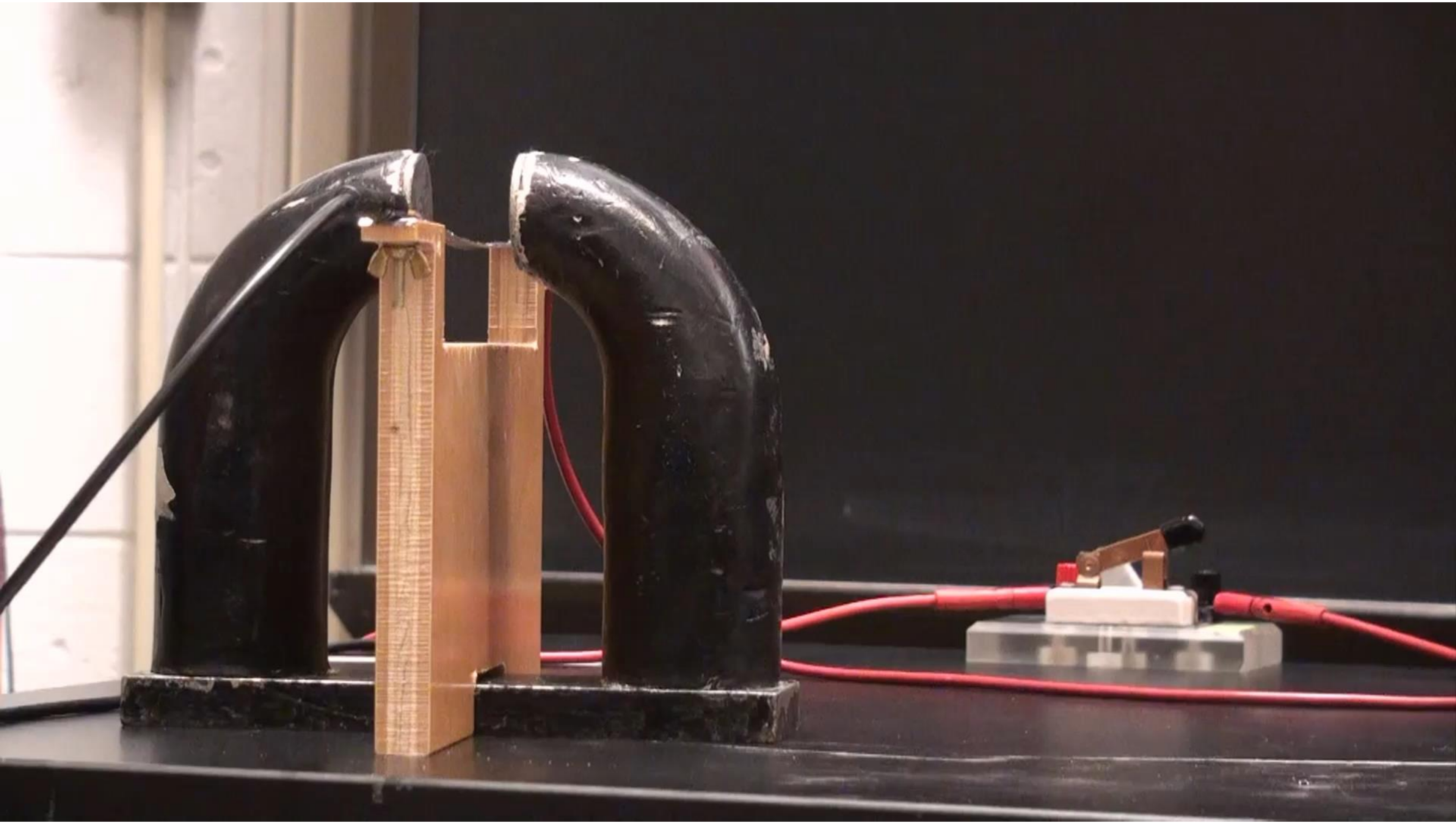
$$I = qnAv_{avg}$$



N: total charge carriers
n = N / volume: charge carriers per unit volume

Force on a current in a magnetic field

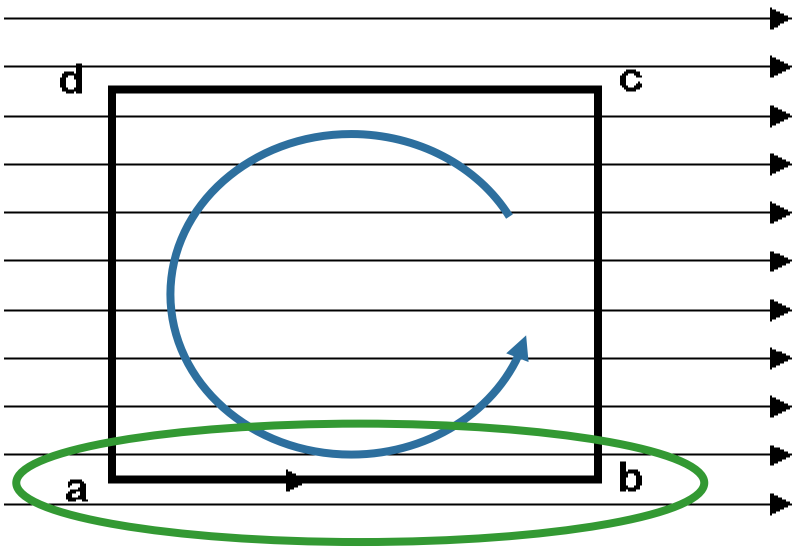




Clicker Question

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$



What is the force on section a-b of the loop?

A. zero

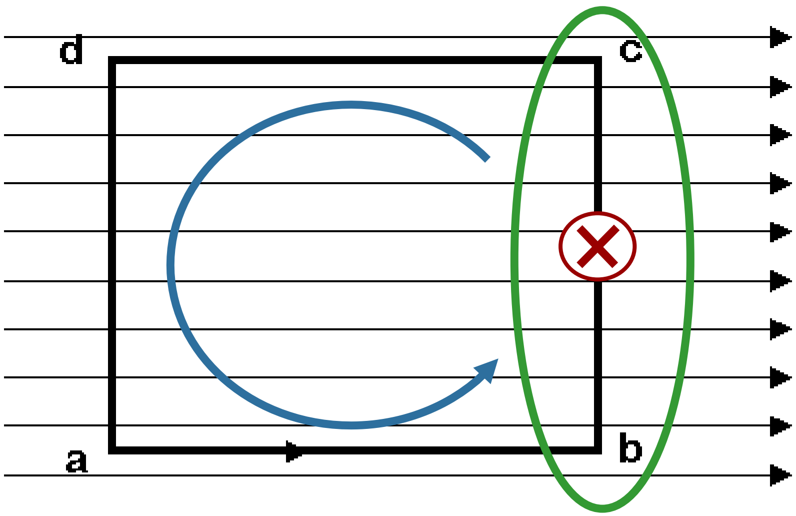
B. out of the page

C. into the page

Clicker Question

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$



What is the force on section b-c of the loop?

A. zero

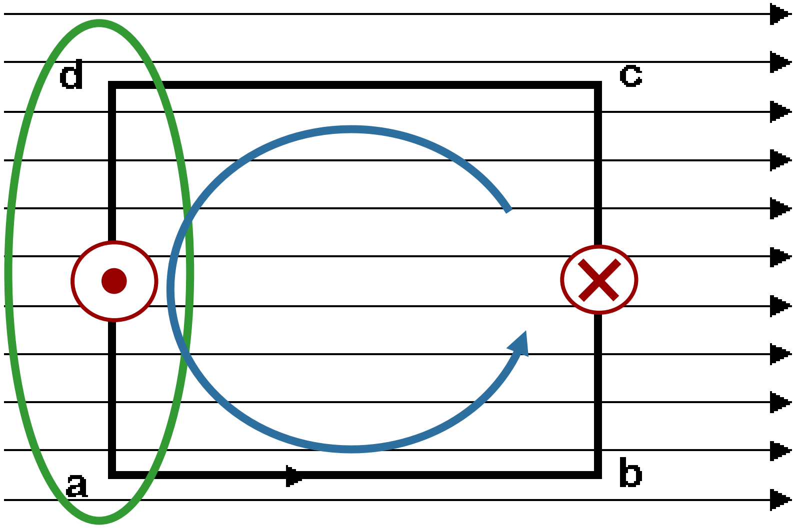
B. out of the page

C. into the page

Clicker Question

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$



What is the force on section d-a of the loop?

A) Zero

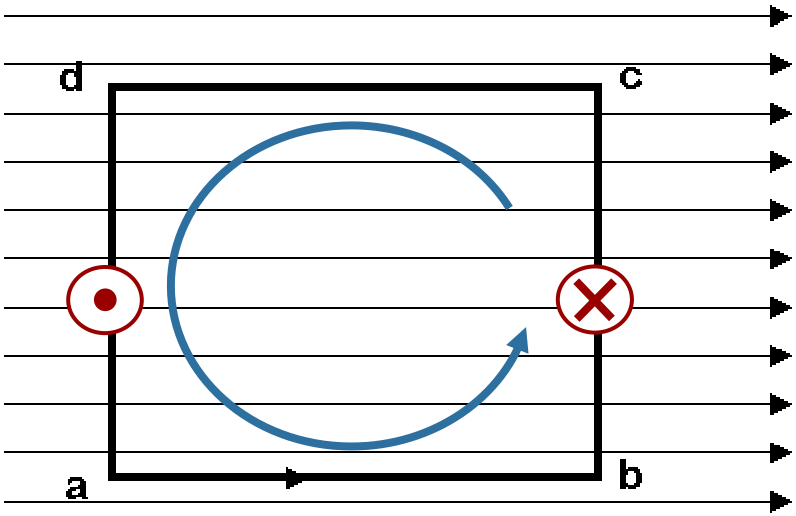
B) Out of the page

C) Into the page

Check Point 1a

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$

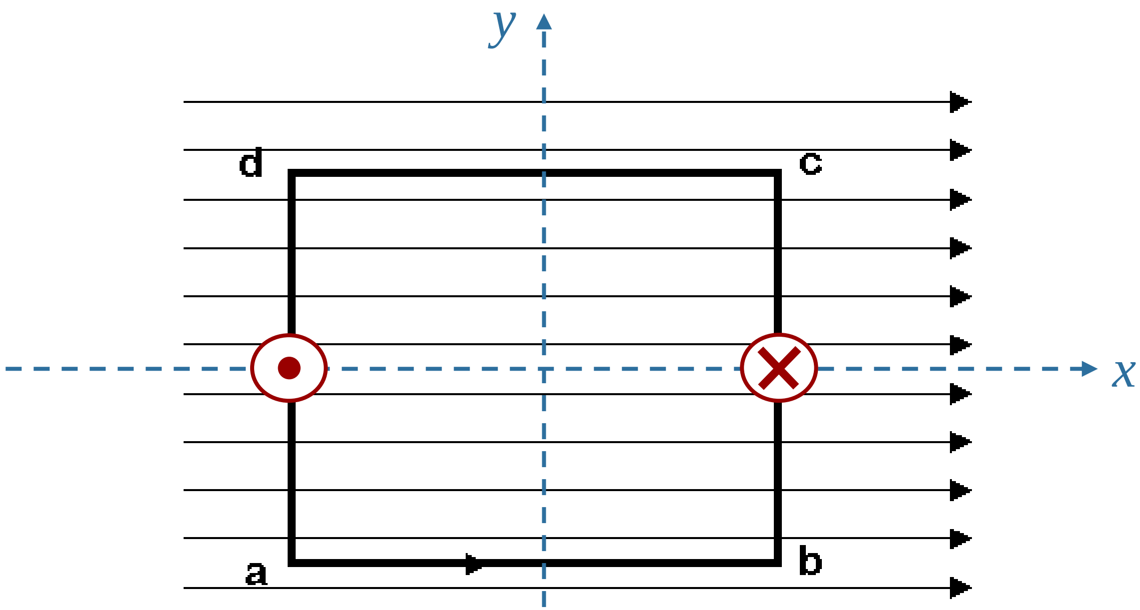


What is the direction of the net force on the loop?

A. Out of the page ***B.*** Into of the page
C. The net force on the loop is zero

Check Point 1b

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.

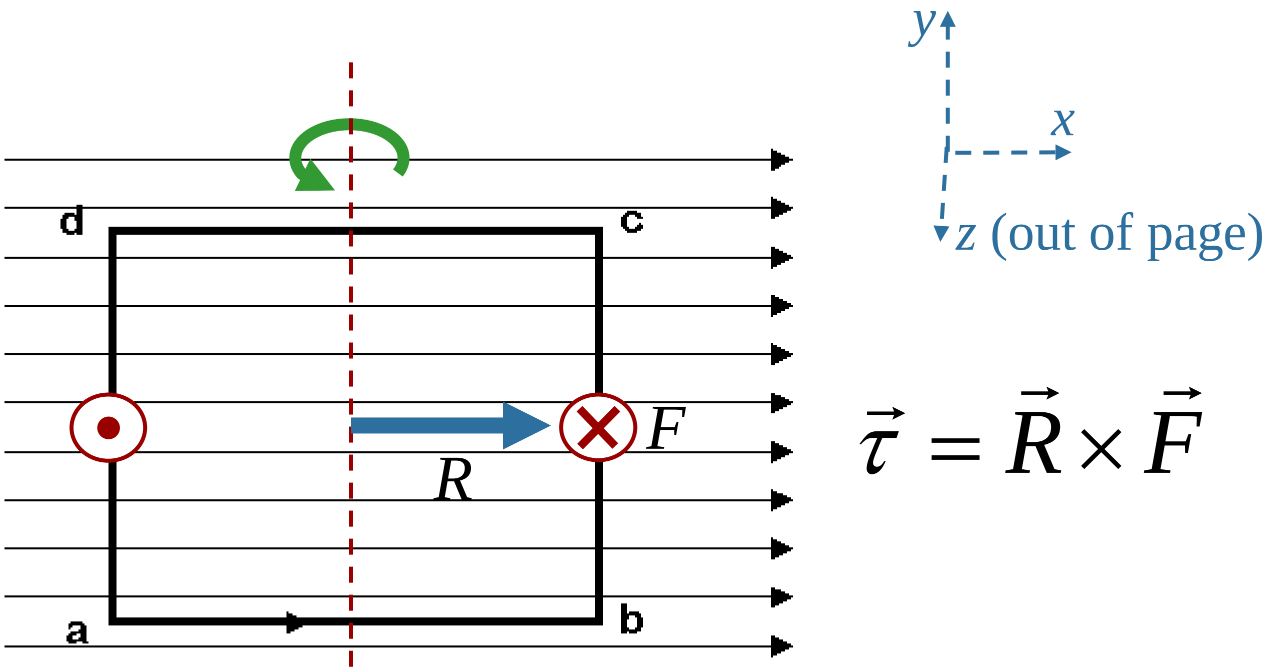


In which direction will the loop rotate?
(the z axis is out of the page)

A) Around the x axis
B) Around the y axis
C) Around the z axis
D) It will not rotate

Check Point 1c

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.



$$\vec{\tau} = \vec{R} \times \vec{F}$$

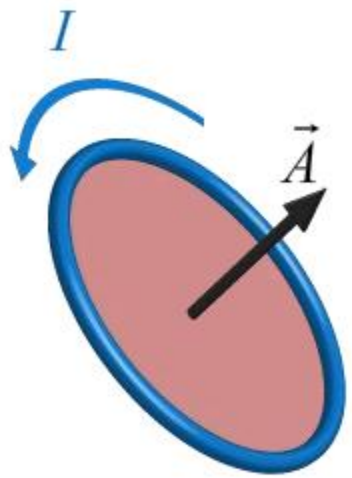
What is the direction of the net torque on the loop?

A. Up (+y)
B. Down (-y)
C. Out of the page (+z)
D. Into the page (-z)
E. The net torque is zero (0)

r x f = torque

Magnetic Dipole Moment

What the heck is a magnetic dipole moment?

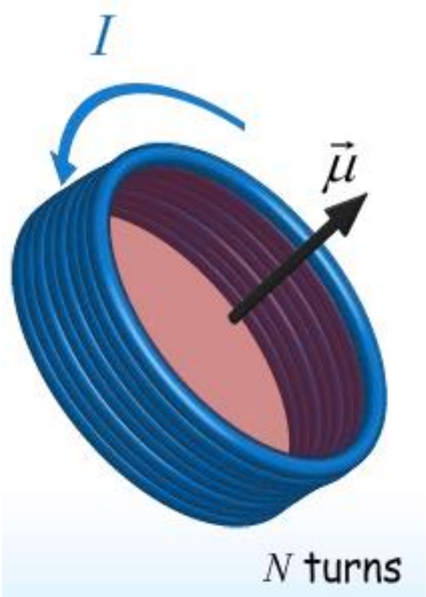


Area vector

Magnitude = Area

Direction uses R.H.R.

$$\vec{\mu} \equiv I\vec{A}$$



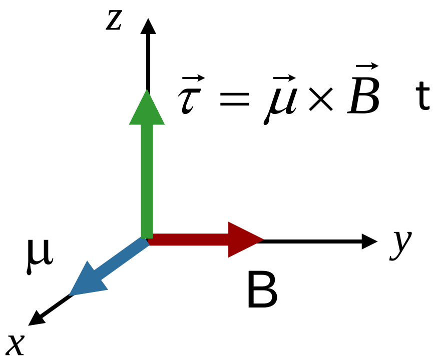
Magnetic Dipole moment

$$\vec{\mu} \equiv NI\vec{A}$$

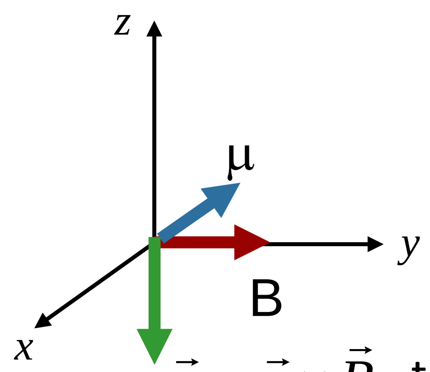
μ Makes Torque Easy!

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The torque always wants to line μ up with B!



$\vec{\tau} = \vec{\mu} \times \vec{B}$ turns μ toward B

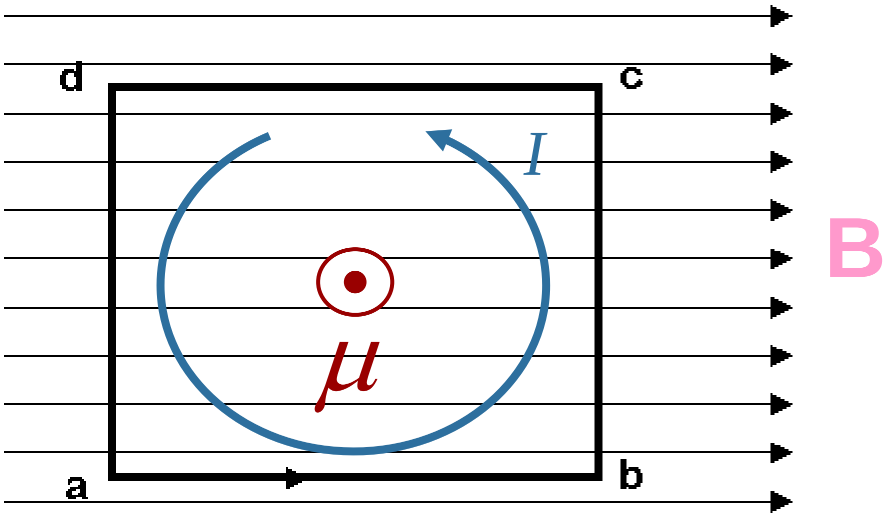


$\vec{\tau} = \vec{\mu} \times \vec{B}$ turns μ toward B

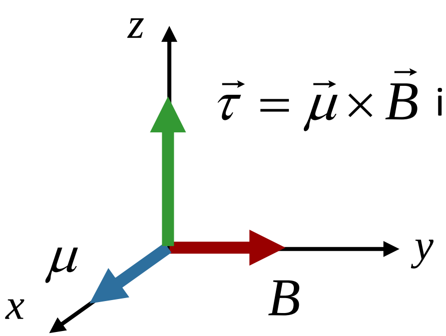
Torque on a current loop

Practice with μ and τ

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



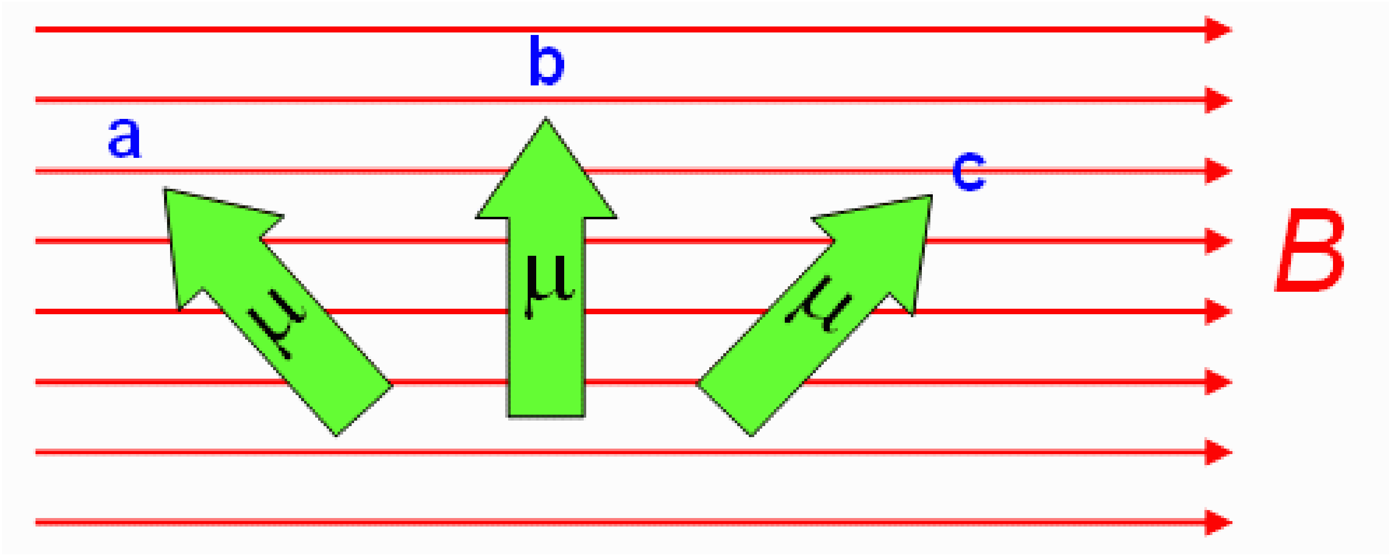
In this case μ is out of the page (using right hand rule)



$\vec{\tau} = \vec{\mu} \times \vec{B}$ is up (turns μ toward B)

Check Point 2a

Three different orientations of a magnetic dipole moment in a constant magnetic field are shown below. Which orientation results in the largest magnetic torque on the dipole?



$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Biggest when $\vec{\mu} \perp \vec{B}$

Magnetic Field can do Work on $\vec{\mu}$

From Physics 211: $$W = \int \vec{\tau} \cdot d\vec{\theta}$$

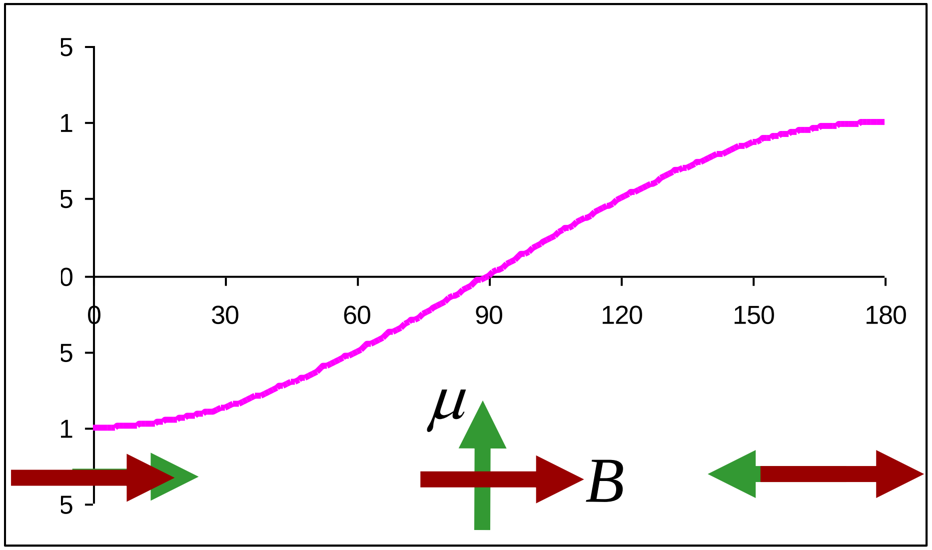
From Physics 212: $$\vec{\tau} = \vec{\mu} \times \vec{B} = \mu B \sin(\theta)$$

$$W = \int_{90}^{\theta} -\mu B \sin(\theta) d\theta = \mu B \cos(\theta) = \vec{\mu} \cdot \vec{B}$$

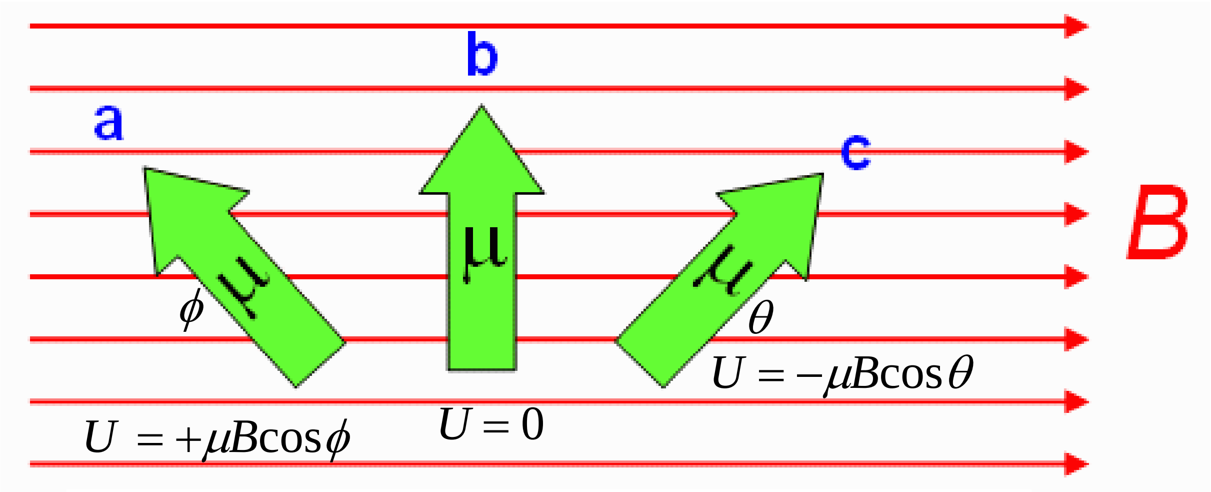
Take max torque as θ initial

$$\Delta U = -W$$

$$U \equiv -\vec{\mu} \cdot \vec{B}$$



Check Point 2b

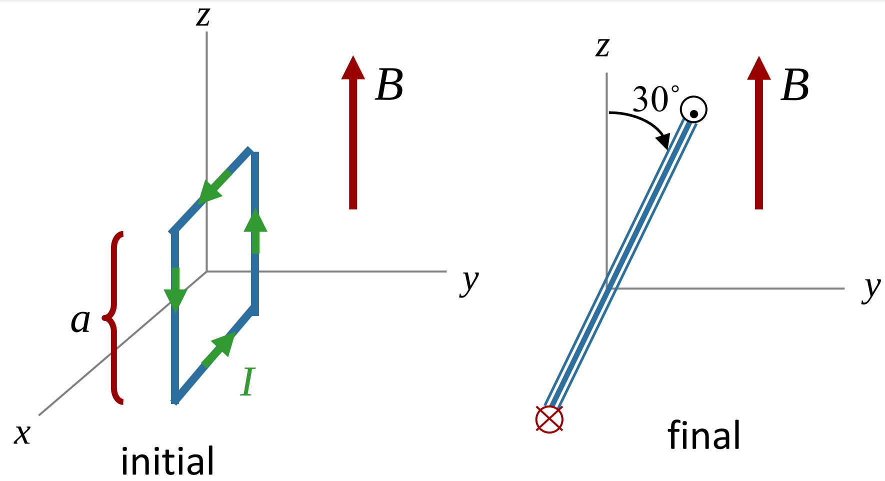


Which orientation has the most potential energy?

$$U = -\vec{\mu} \cdot \vec{B}$$

Calculation

A square loop of side a lies in the x–z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the $+z$ axis. Assume a, I, and B are known.



How much does the potential energy of the system change as the coil moves from its initial position to its final position.

Conceptual Analysis

A current loop may experience a torque in a constant magnetic field

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

We can associate a potential energy with the orientation of loop

$$U = -\vec{\mu} \cdot \vec{B}$$

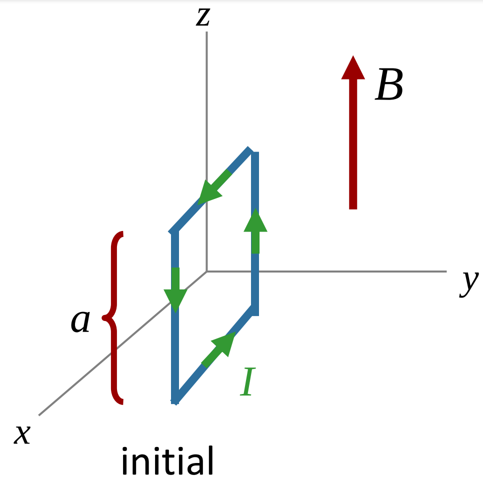
Strategic Analysis

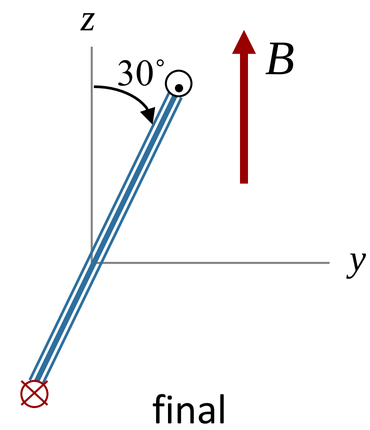
Find μ

Calculate the change in potential energy from initial to final

Calculation

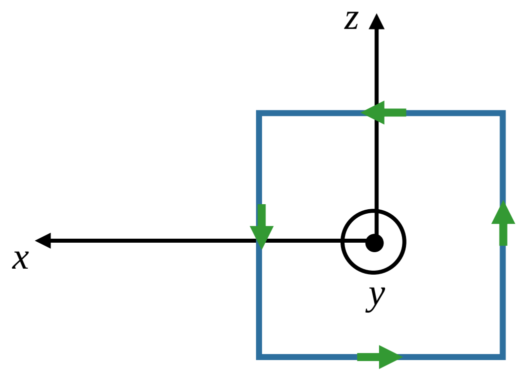
A square loop of side a lies in the x–z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the $+z$ axis. Assume a, I, and B are known.





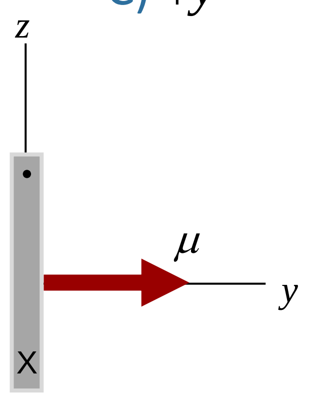
What is the direction of the magnetic moment of this current loop in its initial position?

A) $+x$ B) $-x$ C) $+y$ D) $-y$



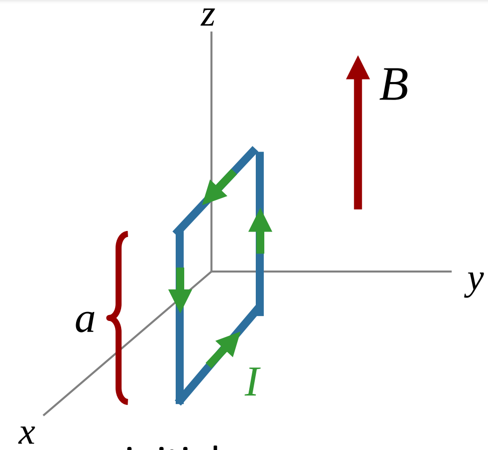
$$\vec{\mu} = I\vec{A}$$

Right Hand Rule

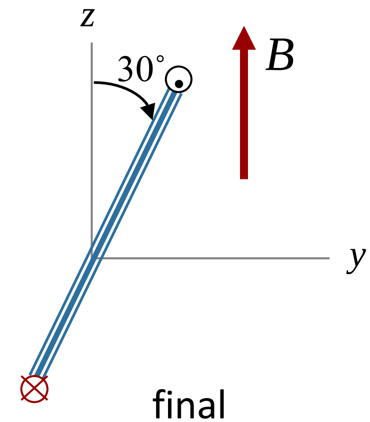


Calculation

A square loop of side a lies in the x–z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the $+z$ axis. Assume a, I, and B are known.



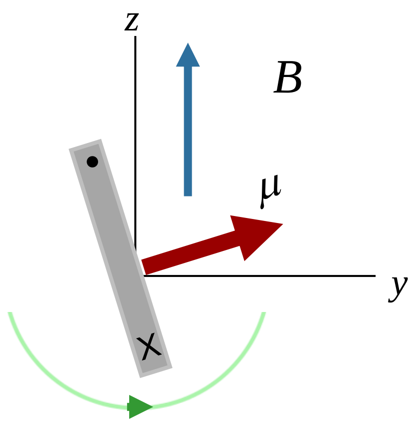
initial



final

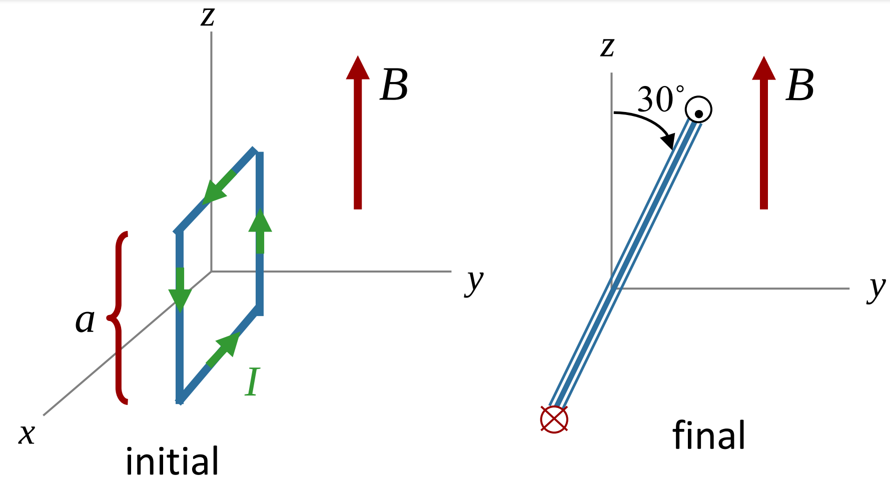
What is the direction of the torque on this current loop in the initial position?

A) $+x$ B) $-x$ C) $+y$ D) $-y$



Calculation

A square loop of side a lies in the x–z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the $+z$ axis. Assume a, I, and B are known.



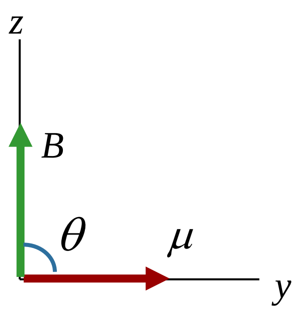
$$U = -\vec{\mu} \cdot \vec{B}$$

What is the potential energy of the initial state?

A) $U_{initial} < 0$

B) $U_{initial} = 0$

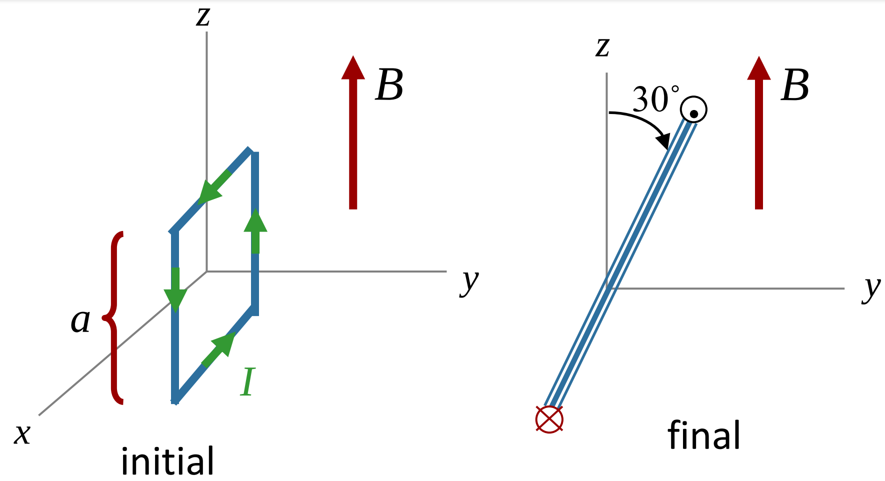
C) $U_{initial} > 0$



$\theta = 90^0 \longrightarrow \vec{\mu} \bullet \vec{B} = 0$

Calculation

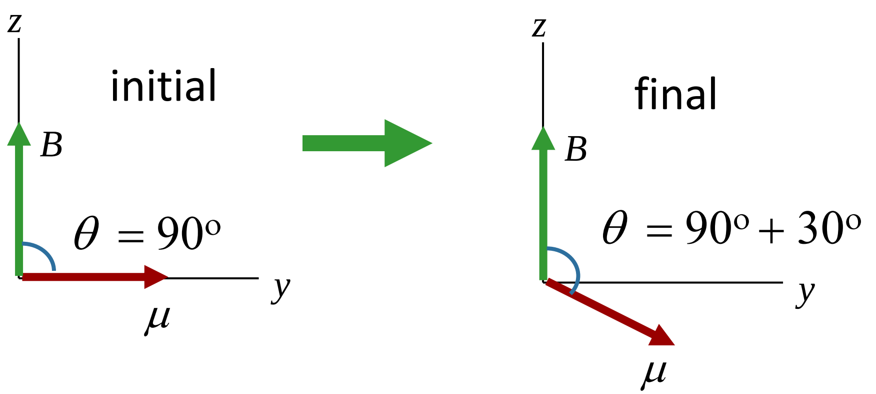
A square loop of side a lies in the x–z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the $+z$ axis. Assume a, I, and B are known.



$$U = -\vec{\mu}\cdot\vec{B}$$

What is the potential energy of the final state?

A) $U_{final} < 0$ B) $U_{final} = 0$ C) $U_{final} > 0$



$\theta = 120^0$

$\vec{\mu}\bullet\vec{B} < 0$

$U = -\vec{\mu}\cdot\vec{B} > 0$

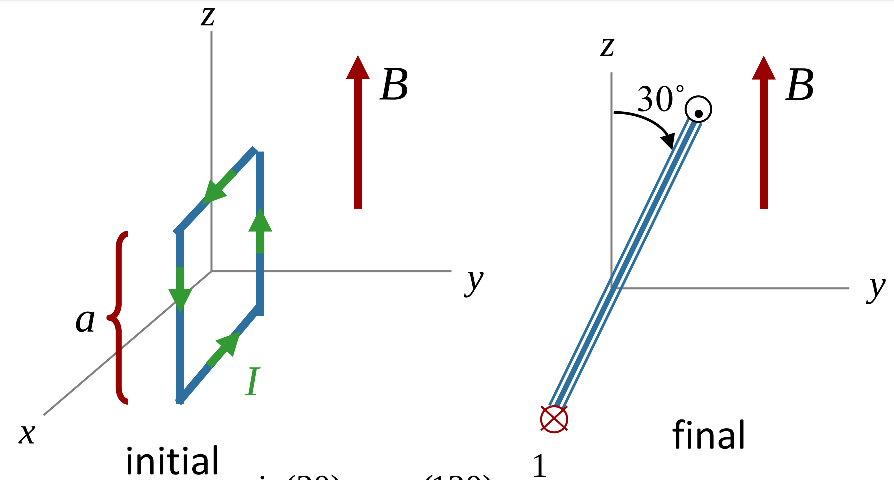
Check: μ moves away from B

Energy must increase !

Calculation

A square loop of side a lies in the x–z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the $+z$ axis. Assume a, I, and B are known.

$$\boxed{U = -\vec{\mu}\cdot\vec{B}}$$



$$\sin(30) = \cos(120) = \frac{1}{2}$$

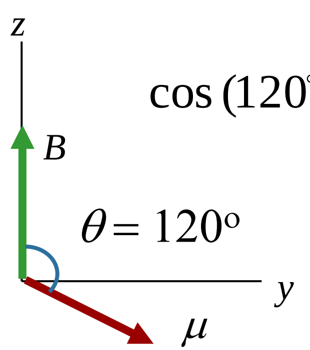
$$\sin(120) = \cos(30) = \frac{\sqrt{3}}{2}$$

What is the potential energy of the final state?

A) $U = Ia^2B$

B) $U = \dfrac{\sqrt{3}}{2}Ia^2B$

C) $U = \dfrac{1}{2}Ia^2B$



$$\cos(120^{\circ}) = -\frac{1}{2}$$

$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos(120^{o}) = \frac{1}{2}\mu B$$

$$\mu = Ia^2$$

$$U = \frac{1}{2}Ia^2B$$

Physics 212
Lecture 13

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

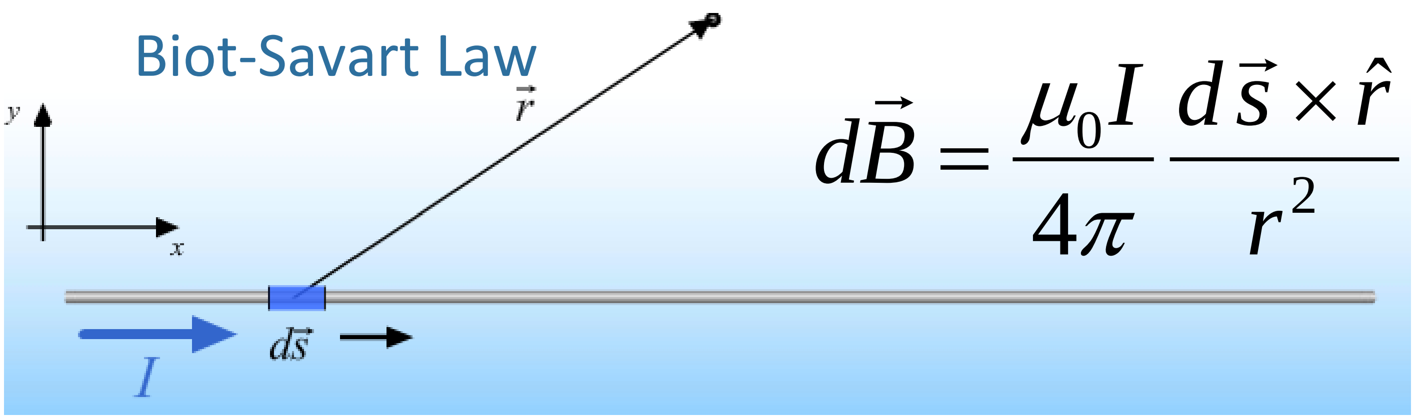
The torque always wants to line μ up with B!

$$U = -\vec{\mu} \cdot \vec{B}$$

Physics 212
Lecture 14

Today's Concept:

Biot-Savart Law



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

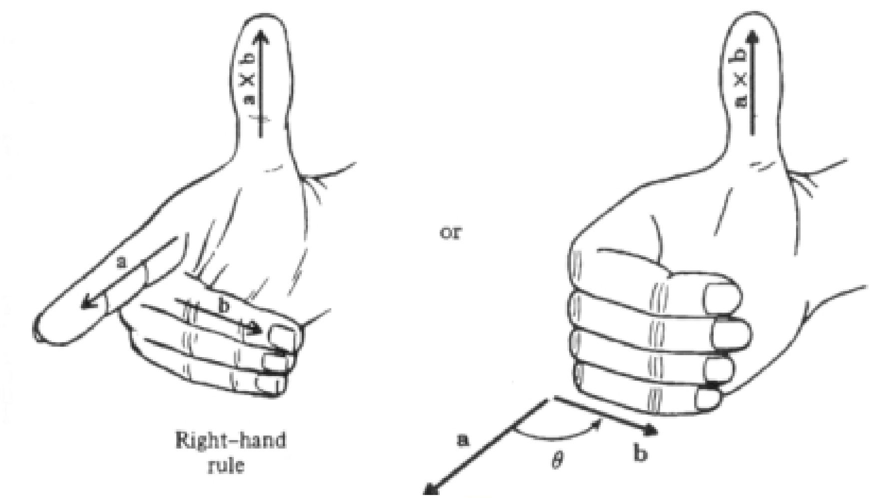
Right Hand Rule Review

1. ANY CROSS PRODUCT

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = I\vec{L} \times \vec{B}$$

$$\tau = \vec{r} \times \vec{F} \qquad \tau = \vec{\mu} \times \vec{B}$$

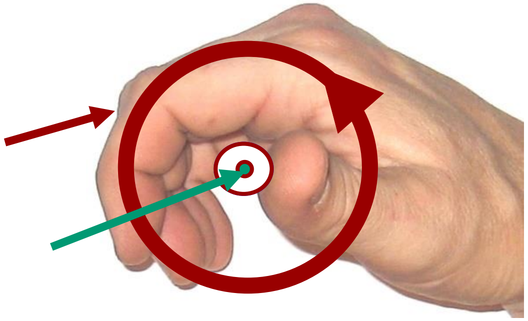
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$



2. Direction of Magnetic Moment

Fingers: Current in Loop

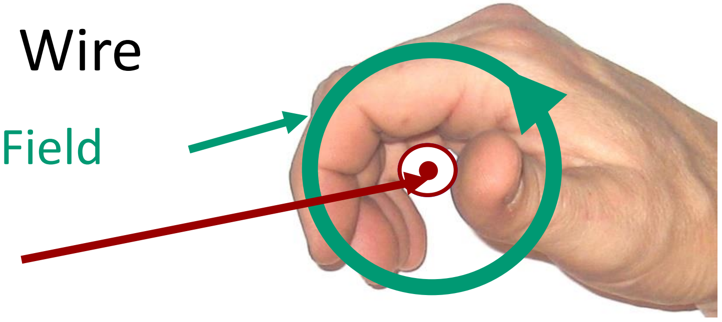
Thumb: Magnetic Moment



3. Direction of Magnetic Field from Wire

Fingers: Magnetic Field

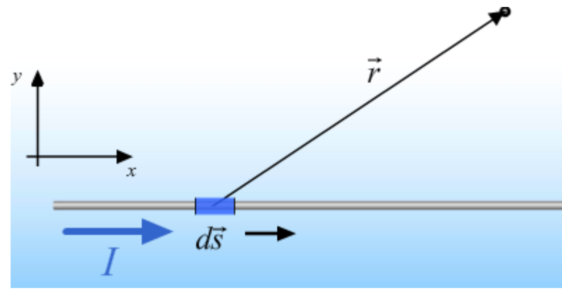
Thumb: Current



Biot-Savart Law:

What is it?

Fundamental law for determining the direction and magnitude of the magnetic field due to an element of current

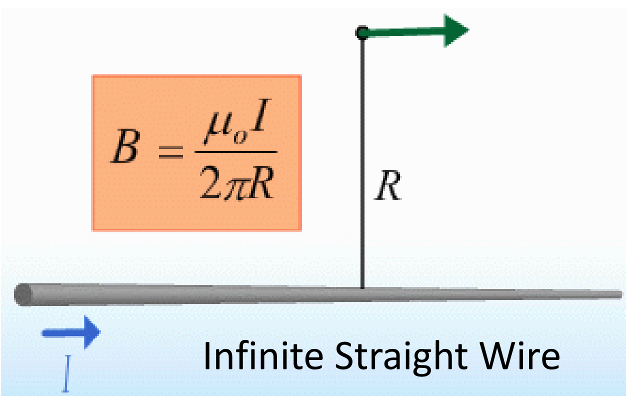


$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

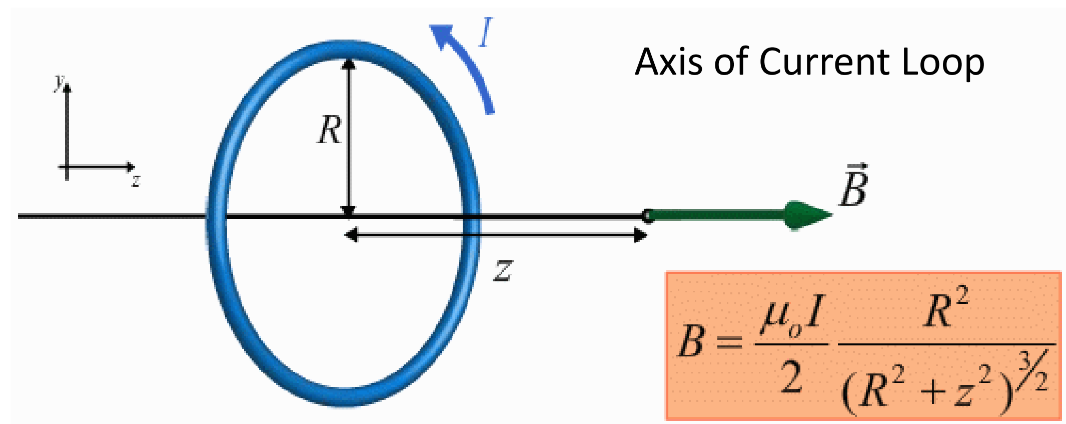
We can use this law to calculate the magnetic field produced by ANY current distribution

BUT

Easy analytic calculations are possible only for a few distributions:



$$B = \frac{\mu_o I}{2\pi R}$$



$$B = \frac{\mu_o I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$$

Plan for Today: Mainly use the results of these calculations!

GOOD NEWS: Remember Gauss' Law? Allowed us to calculate E for symmetrical charge distributions

NEXT TIME: Introduce Ampere's Law Allows us to calculate B for symmetrical current distributions

B from Infinite Line of Current

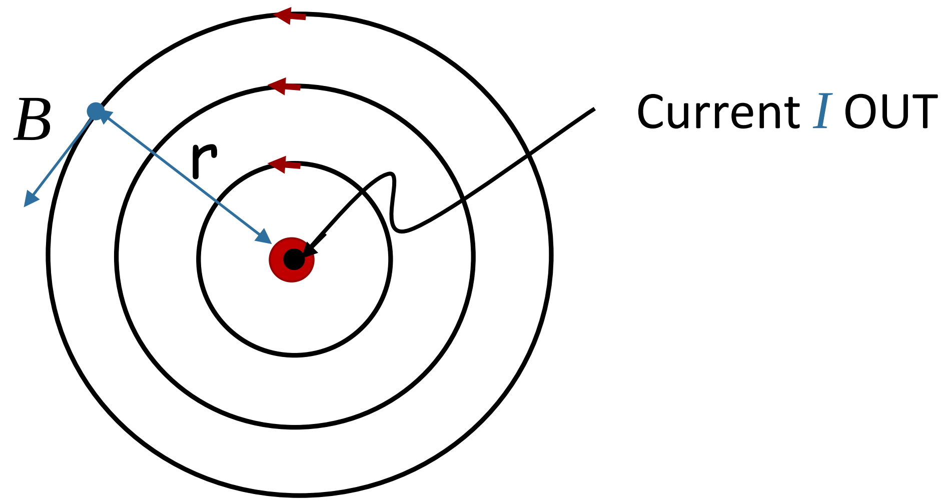
Integrating $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$ gives result

Magnitude:

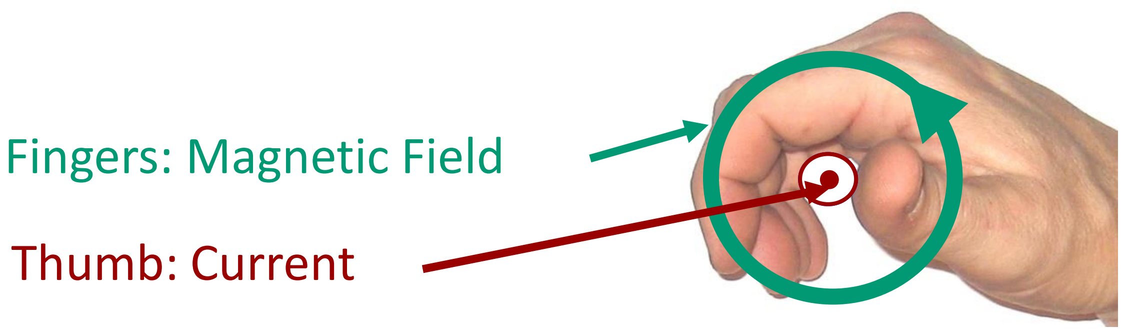
$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mu_0 = 4\pi \times 10^{-7}\, Tm/A$$

$r =$ distance from wire

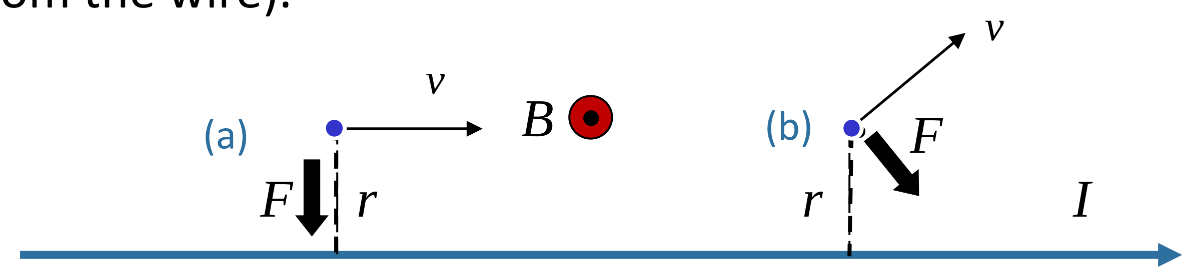


Direction:



Currents + Charges

A long straight wire is carrying current from left to right. Two identical charges are moving with equal speed. Compare the magnitude of the force on charge a moving directly to the right, to the magnitude of the force on charge b moving up and to the right at the instant shown (i.e. same distance from the wire).



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\left|\vec{F}\right| = qvB\sin\theta$$

Same q, $|v|$, B and $\theta\,(=90)$

A) $|F_a| > |F_b|$

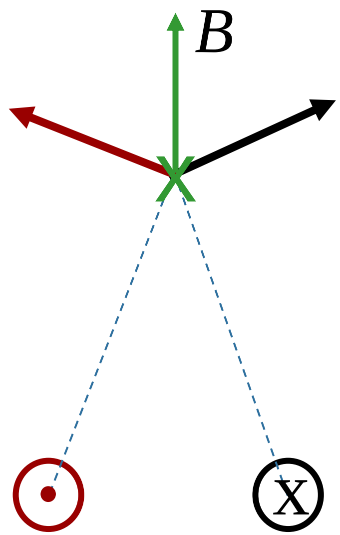
B) $|F_a| = |F_b|$

C) $|F_a| < |F_b|$

Forces are in different directions

Adding Magnetic Fields

Two long wires carry opposite current

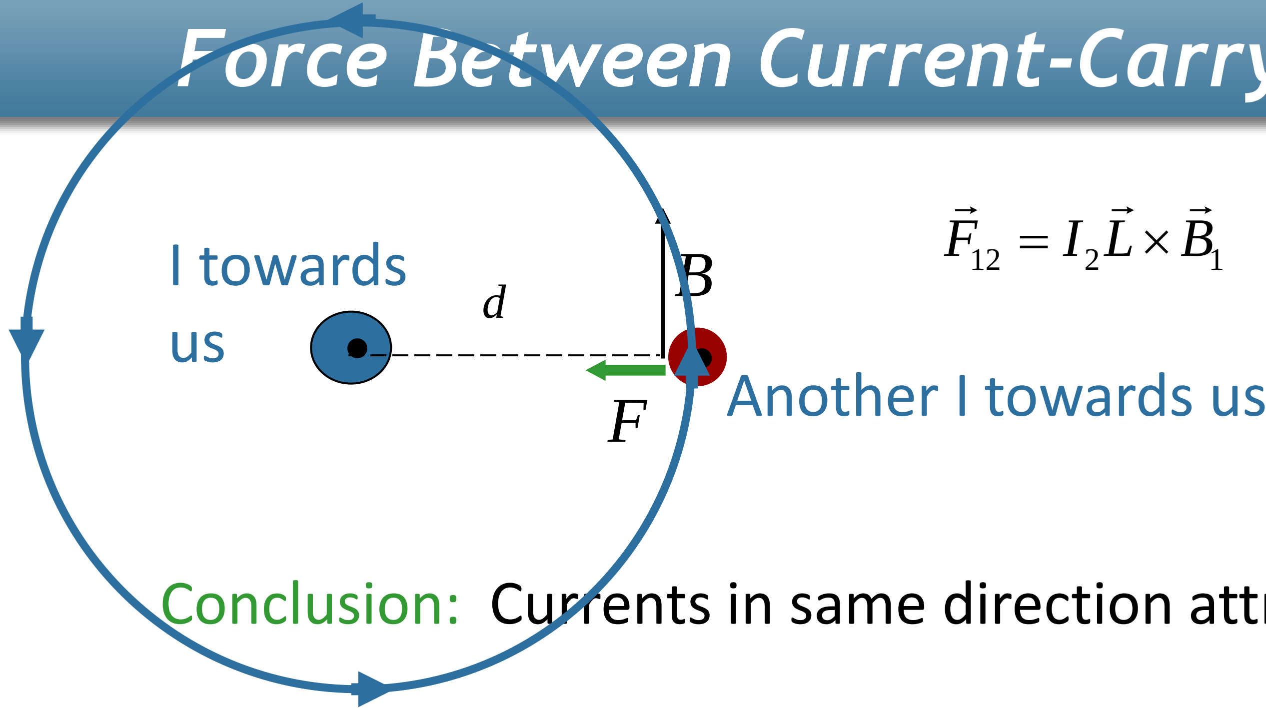


What is the direction of the magnetic field above, and midway between the two wires carrying current – at the point marked "X"?

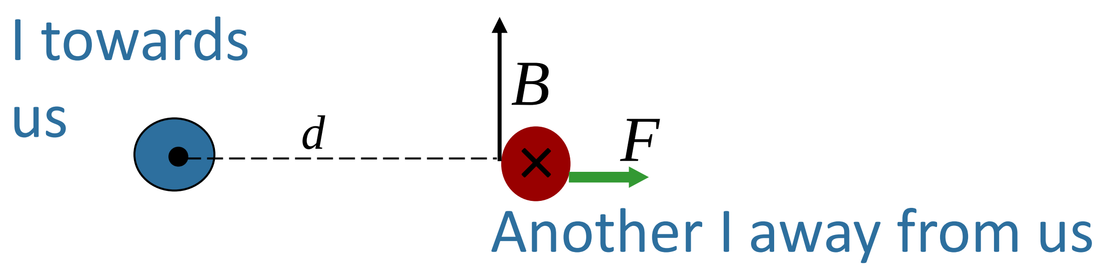
A) Left B) Right C) Up D) Down E) Zero

Force Between Current-Carrying Wires

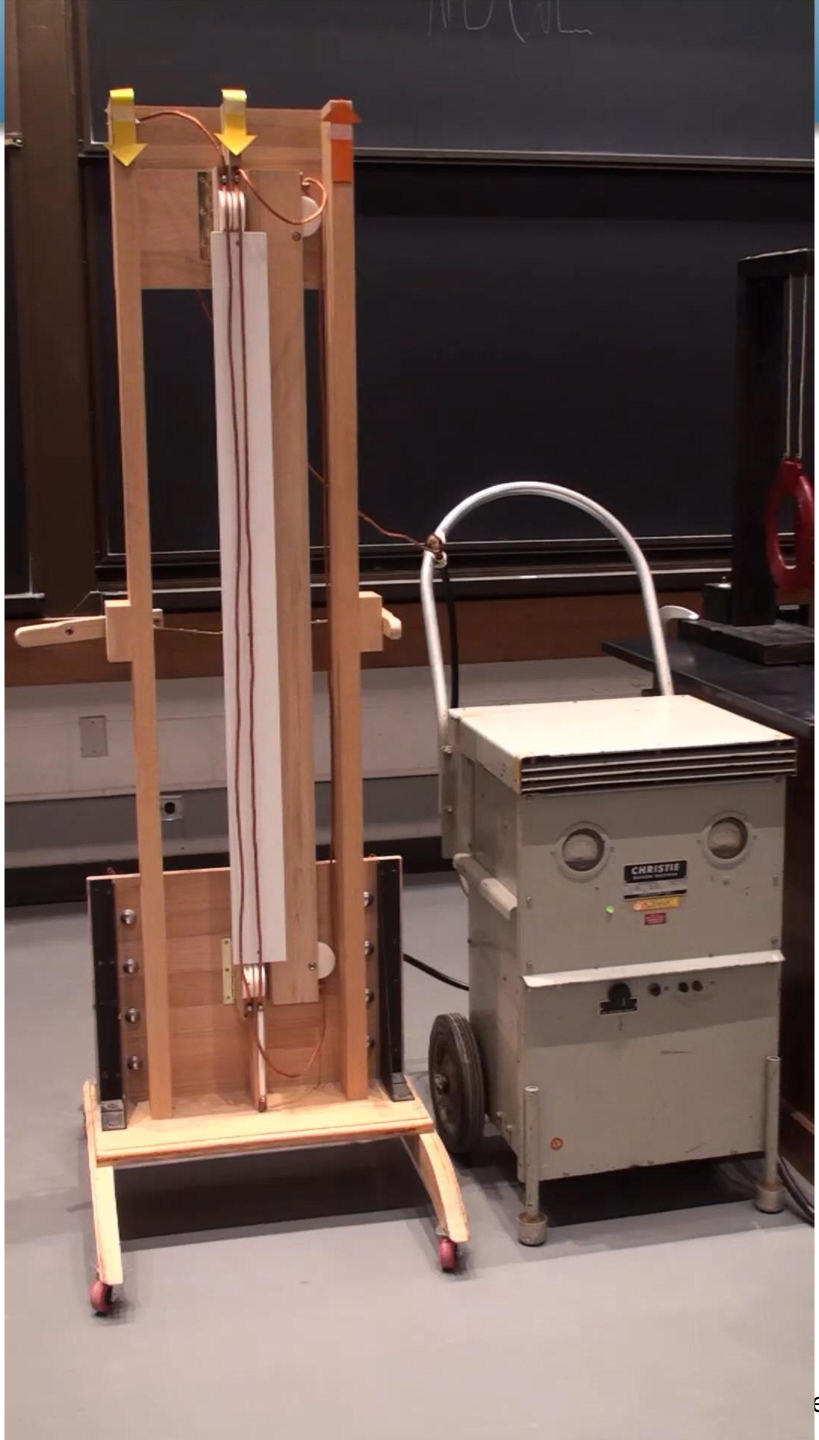
$$\vec{F}_{12} = I_2 \vec{L} \times \vec{B}_1 \quad \Rightarrow \quad F_{12} = I_2 L \cdot \frac{\mu_o}{2\pi d} I_1$$



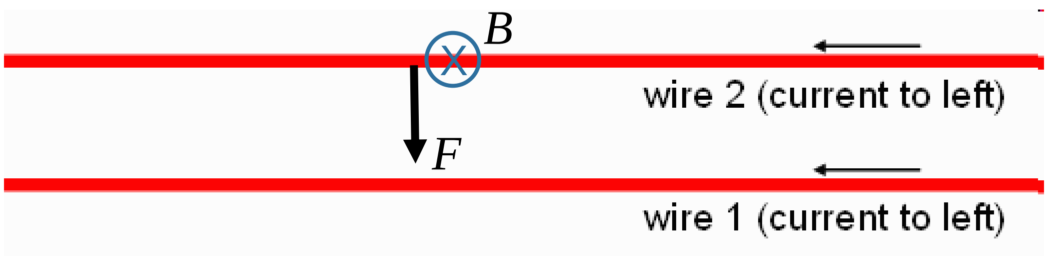
Conclusion: Currents in same direction attract!



Conclusion: Currents in opposite direction repel!


CHRISTIE

Check Point 1



$$\vec{F}_{12} = I_2 \vec{L} \times \vec{B}_1$$

What is the direction of the force on wire 2 due to wire 1?

A) Up B) Down C) Into Screen D) Out of screen E) Zero

2 wires with same-direction currents are attracted

What is the direction of the torque on wire 2 due to wire 1?

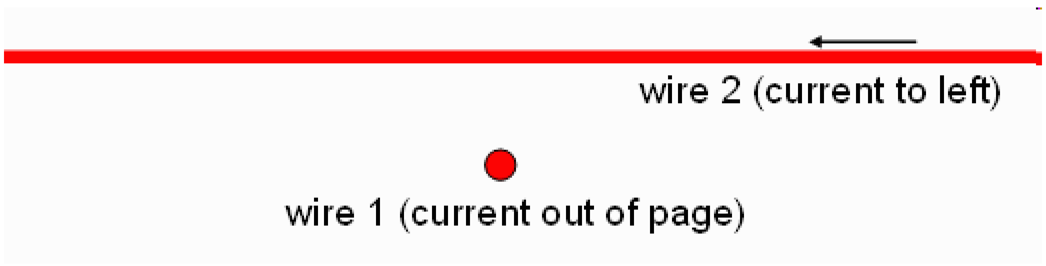
A) Up B) Down C) Into Screen D) Out of screen E) Zero

Uniform force at every segment of wire

$\downarrow$

No torque about any axis

Check Point 2

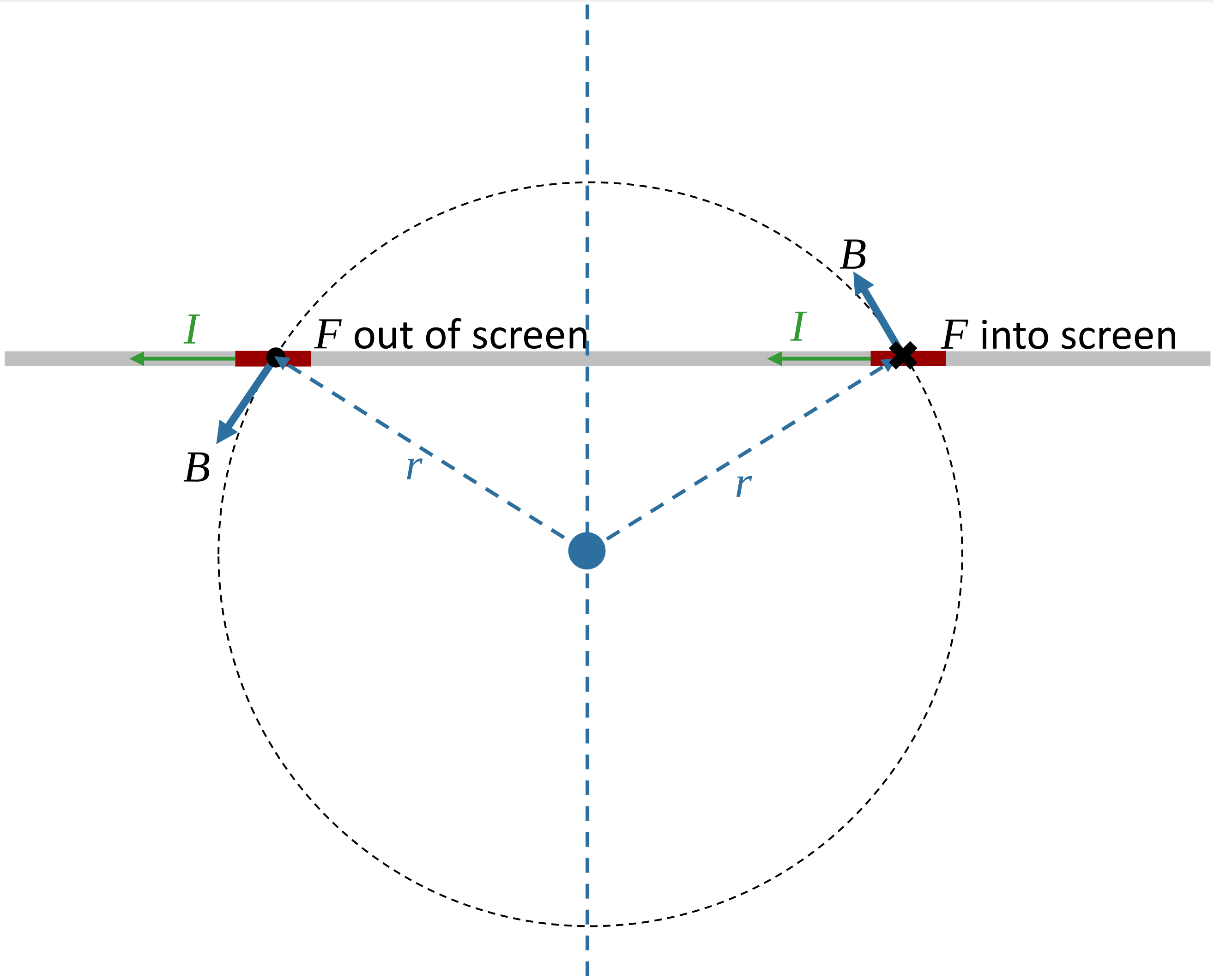


What is the direction of the force on wire 2 due to wire 1?

A) Up   B) Down   C) Into Screen   D) Out of screen   E) Zero

WHY?
DRAW PICTURE!

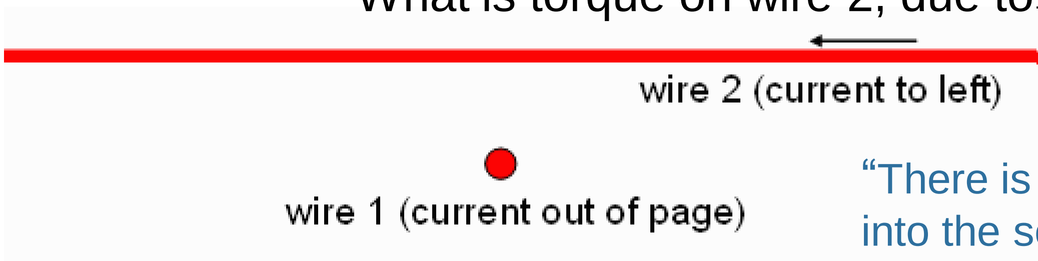
Consider Force on Symmetric Segments



Net Force is Zero!

Check Point 3

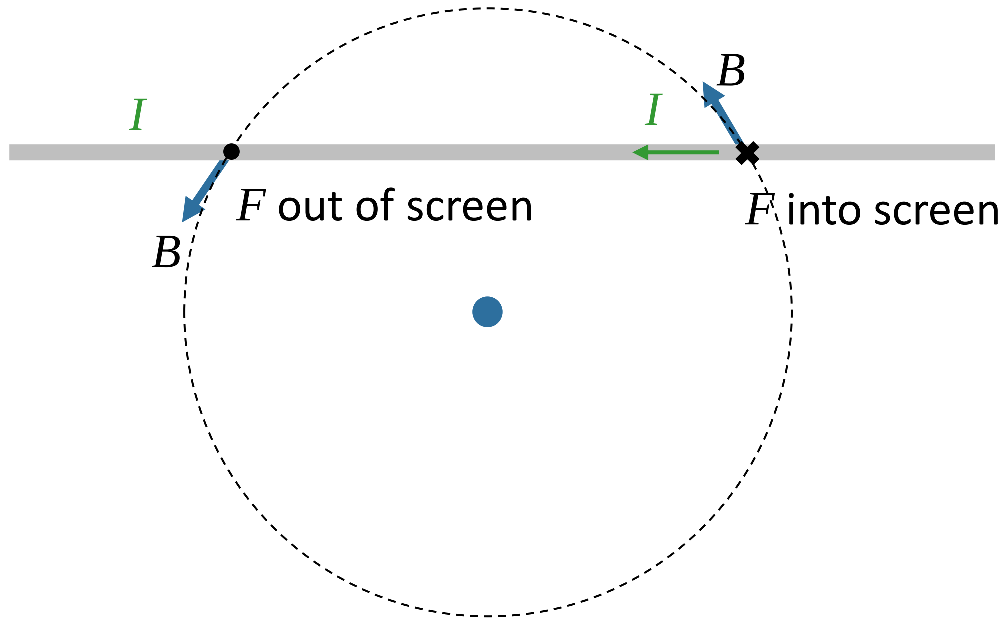
What is torque on wire 2, due to wire 1?



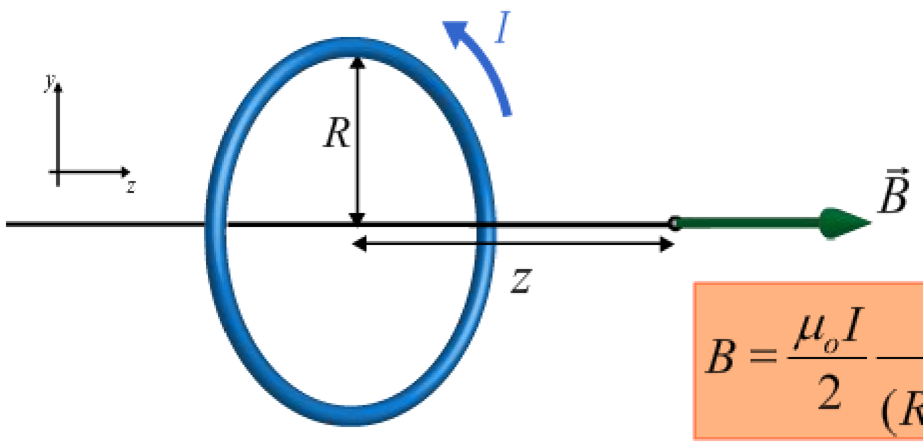
"There is a net force on the right side pointing into the screen and a net force on the left side pointing out of the screen. Using the right hand rule, this means that the torque is pointing up."

The wire will try to align with wire 1.

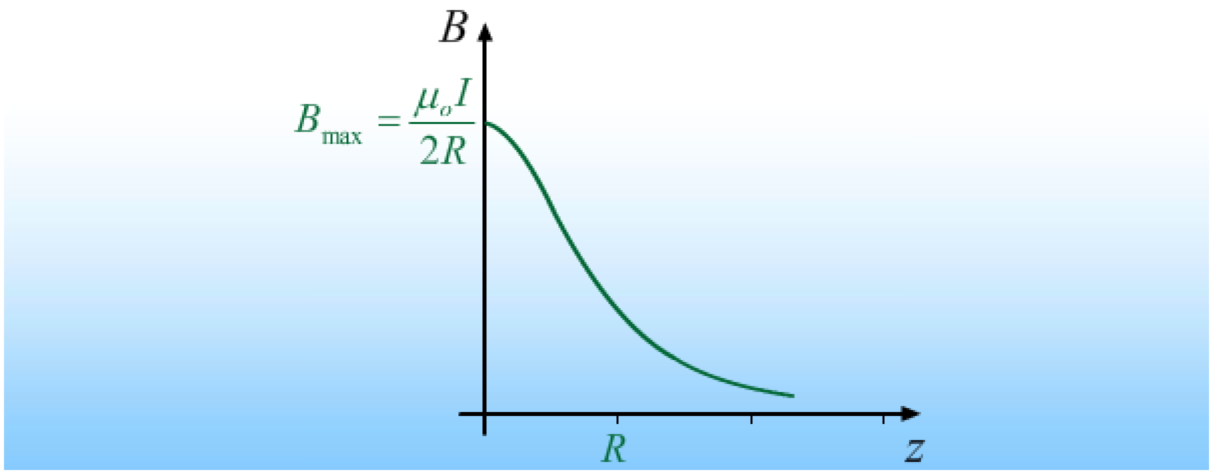
A) Up B) Down C) Into Screen D) Out of screen E) Zero

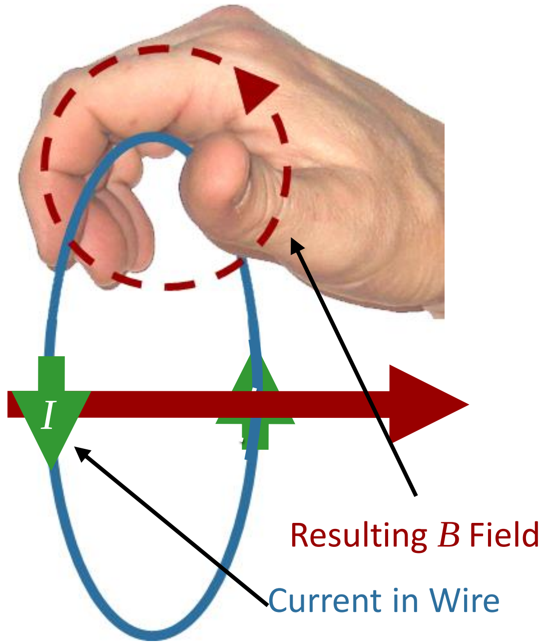


B on axis from Current Loop

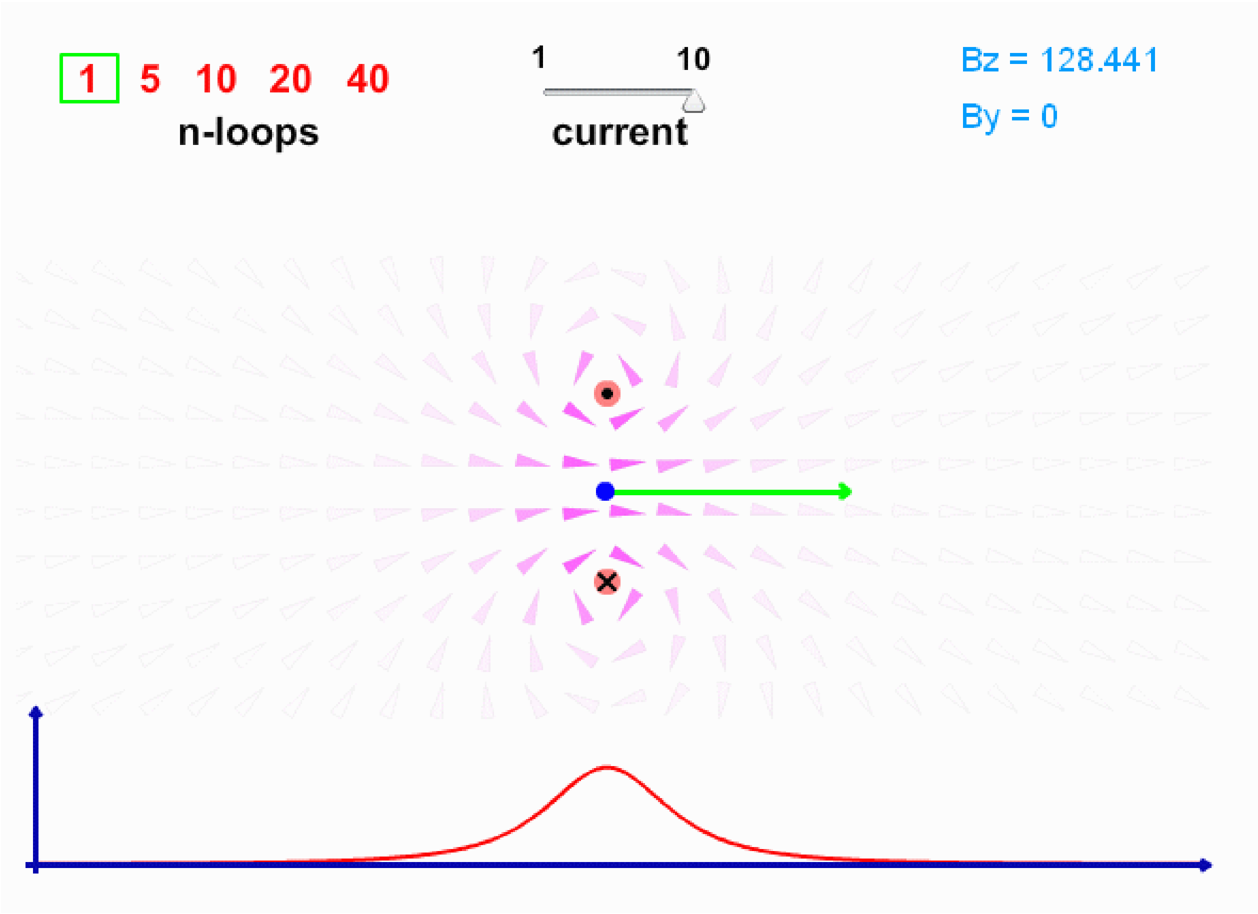


$$B = \frac{\mu_o I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$$



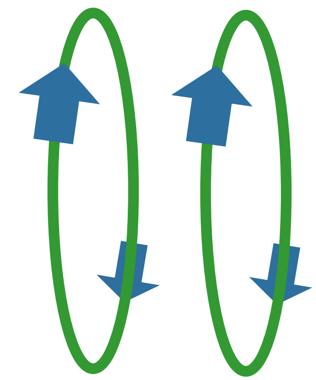


What about Off-Axis ?



Two Current Loops

Two identical loops are hung next to each other. Current flows in the same direction in both.



The loops will:

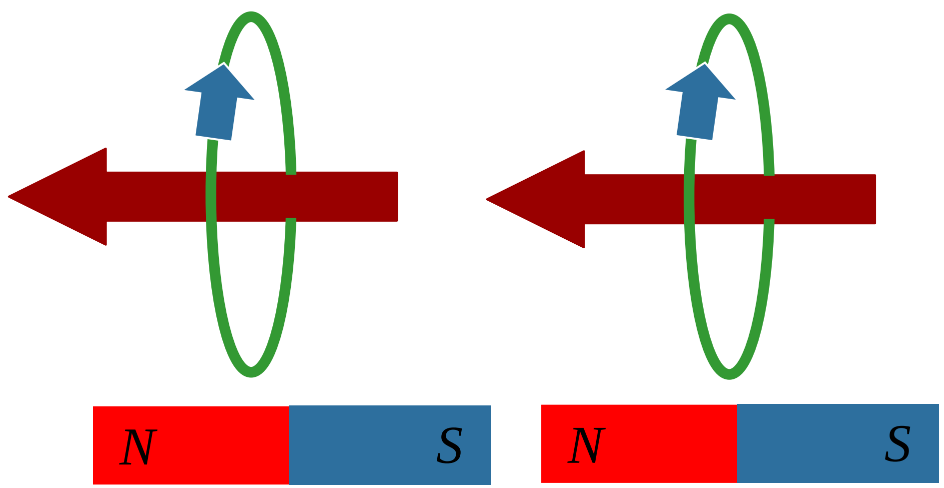
A) Attract each other

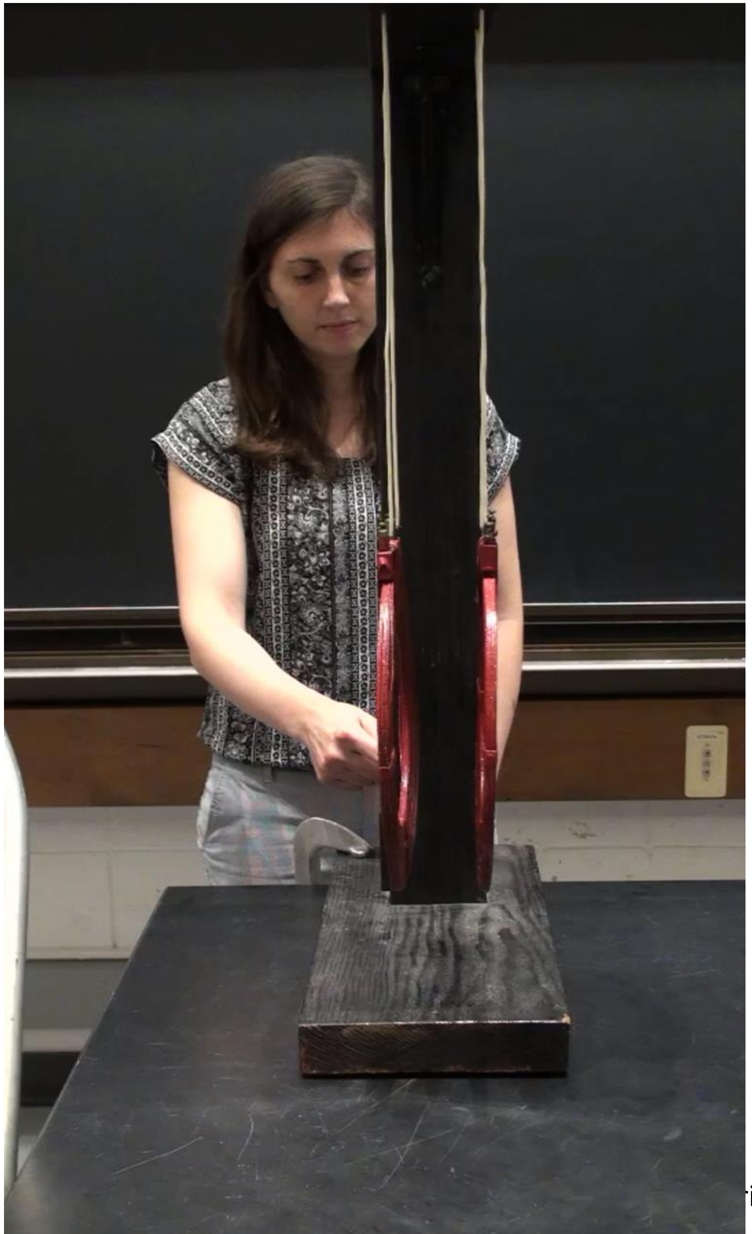
B) Repel each other

Two ways to see this:

1) Like currents attract

2) Look like bar magnets

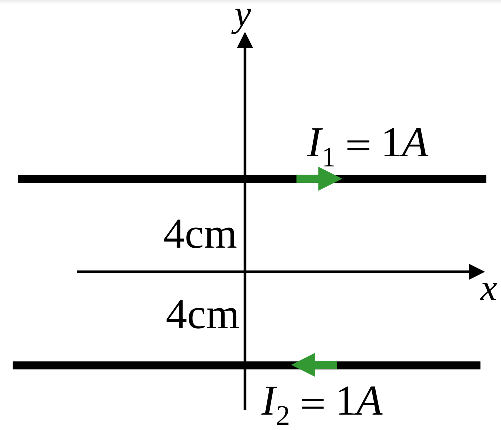




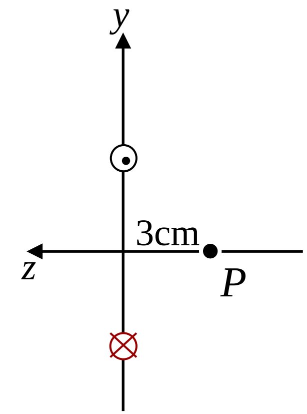
Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?



Front view



Side view

Conceptual Analysis

Each wire creates a magnetic field at P

B from infinite wire: $B = \mu_0 I / 2\pi r$

Total magnetic field at P obtained from superposition

Strategic Analysis

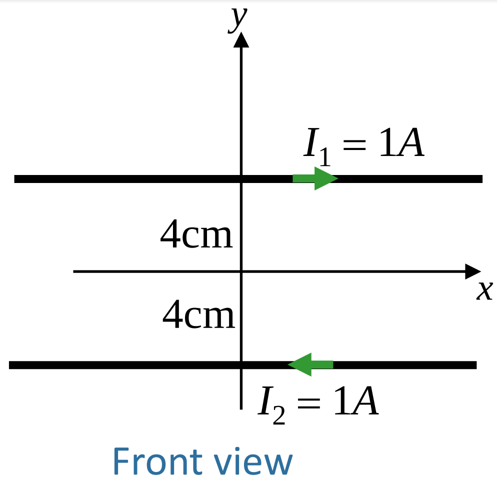
Calculate B at P from each wire separately

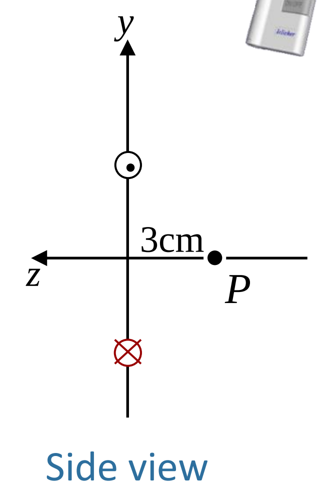
Total B = vector sum of individual B fields

Calculation

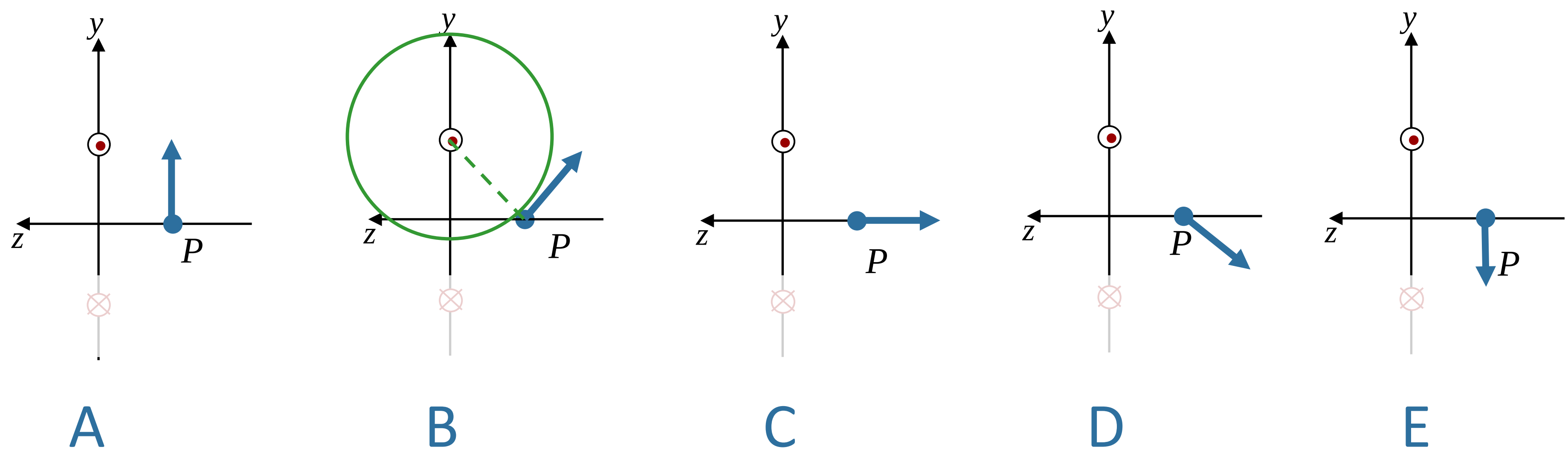
Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?





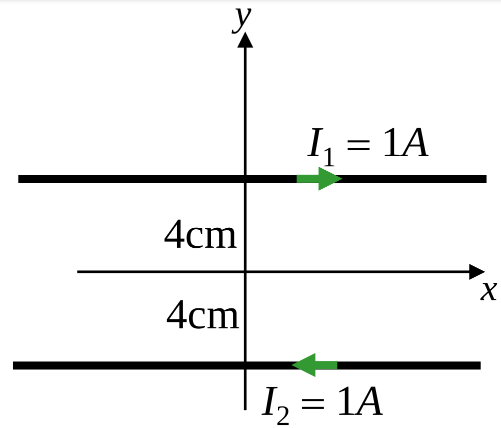
What is the direction of *Magnetic Field* at P produced by the top current I_1?



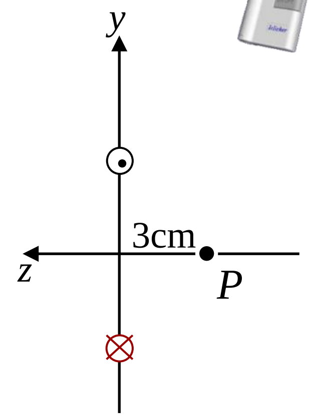
Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?

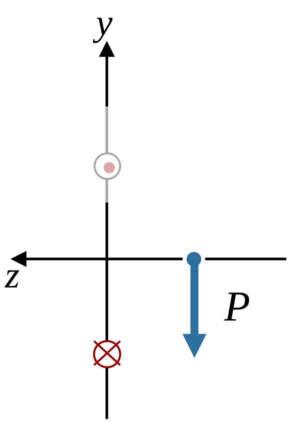


Front view

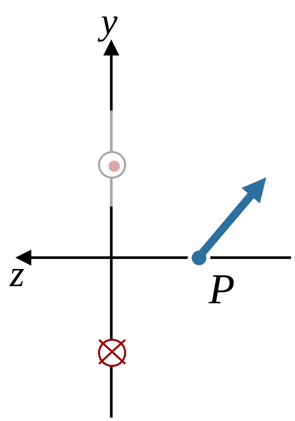


Side view

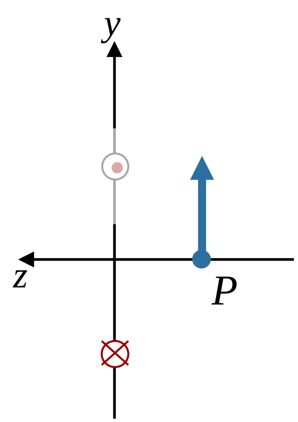
What is the direction of *Magnetic Field* at P produced by the bottom current I_2?



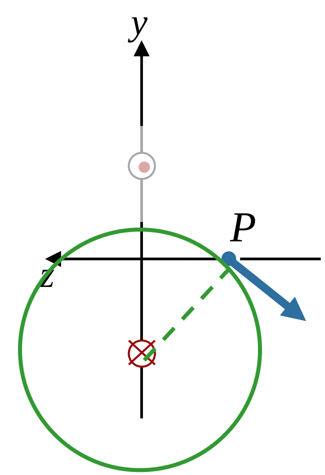
A



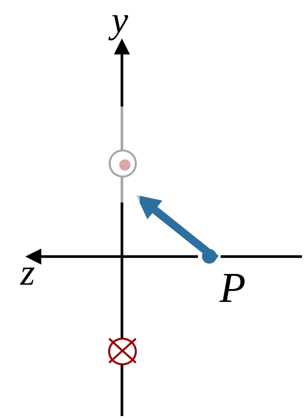
B



C



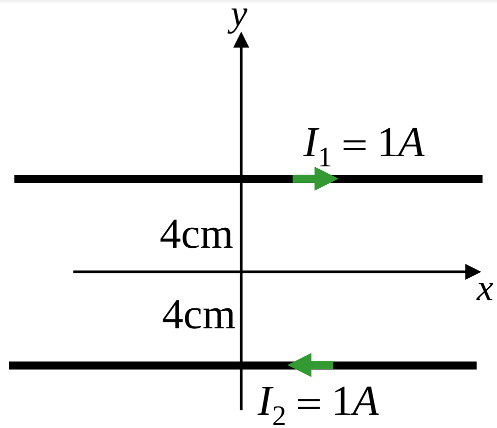
D



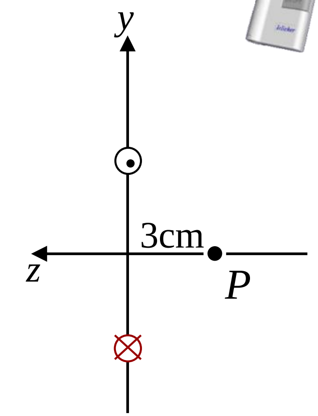
E

Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.



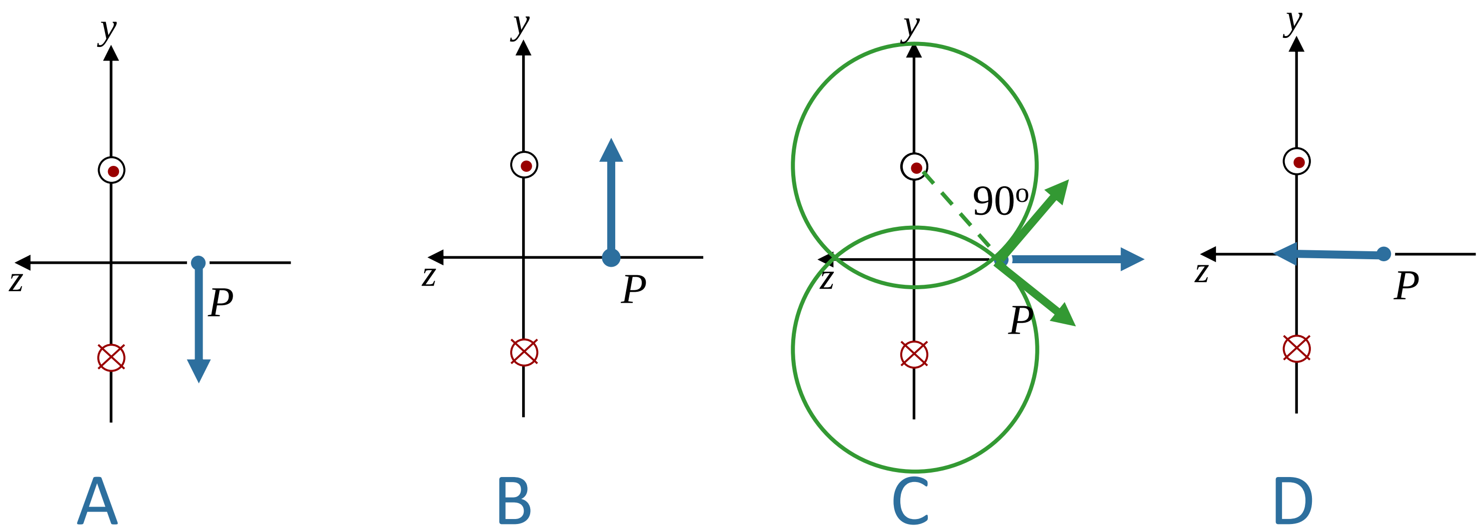
Front view



Side view

What is the B field at point P?

What is the direction of *Magnetic Field* at P?

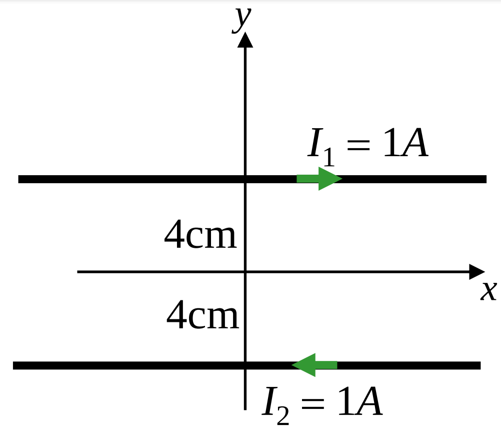


Calculation

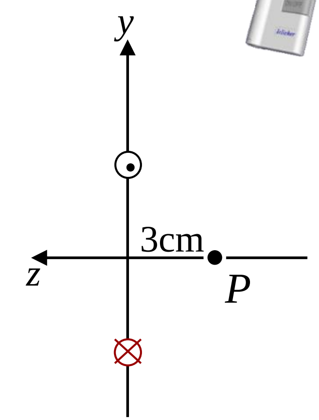
Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?

$$B = \frac{\mu_0 I}{2\pi r}$$



Front view



Side view

What is the magnitude of B at P produced by the top current I_1?

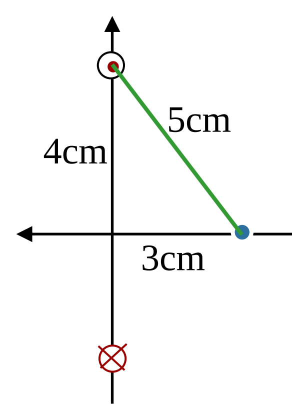
$(\mu_0 = 4\pi \times 10^{-7}\ T\text{–}m/A)$

A) $4.0 \times 10^{-6}\ T$ B) $5.0 \times 10^{-6}\ T$ C) $6.7 \times 10^{-6}\ T$

What is r?

$r =$ distance from wire axis to P

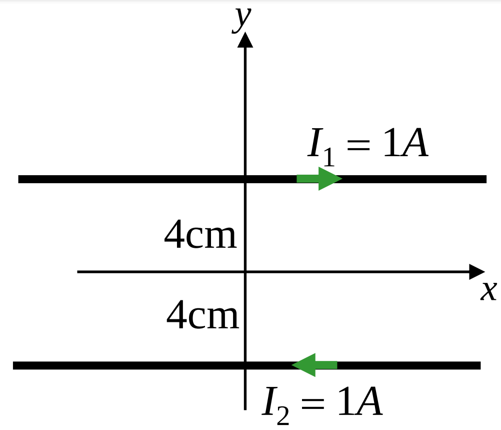
$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times 1}{2\pi r} = 40 \times 10^{-7}$$



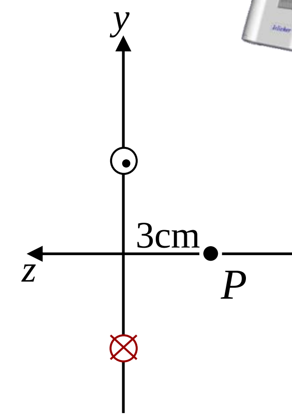
$$r = \sqrt{3^2 + 4^2} = 5cm$$

Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.



Front view

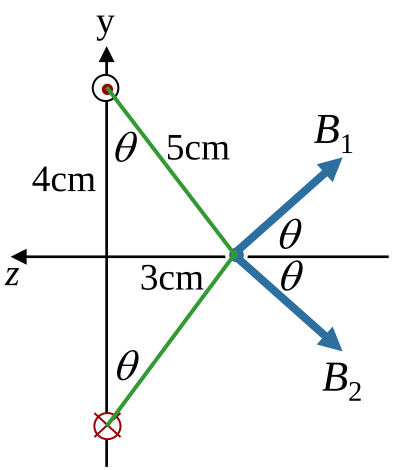


What is the B field at point P?

$$\boxed{B_{top} = 4 \times 10^{-6}\ T}$$

What is the magnitude of B at P? $(\mu_0 = 4\pi \times 10^{-7}\ T - m/A)$

A) $3.2 \times 10^{-6}\ T$ B) $4.8 \times 10^{-6}\ T$ C) $6.4 \times 10^{-6}\ T$ D) $8.0 \times 10^{-6}\ T$



$$B_{1x} = B_1 \cos\theta$$

$$B_{2x} = B_2 \cos\theta$$

$$B_x = 2B_1\cos\theta = 2\times 4\times 10^{-6} \times \left(\frac{4}{5}\right) = 6.4\times 10^{-6}$$

Have a great weekend!

Physics 212
Lecture 15

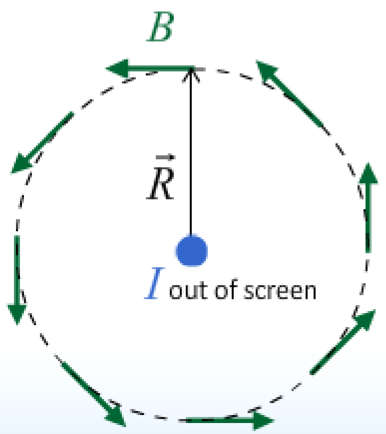
Today's Concept:

Ampere's Law

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enclosed}$$

Ampere's Law

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_o I}$$



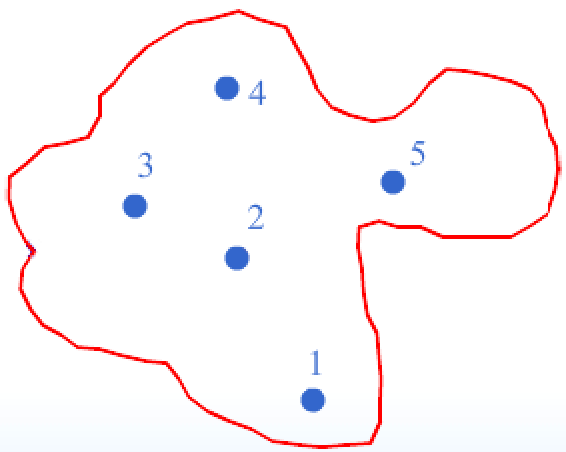
Infinite current-carrying wire

LHS: $\oint \vec{B} \bullet d\vec{\ell} = \oint B d\ell = B\oint d\ell = B \cdot 2\pi R$

RHS: $I_{enclosed} = I$

$$\longrightarrow \quad B = \frac{\mu_o I}{2\pi R}$$

General Case



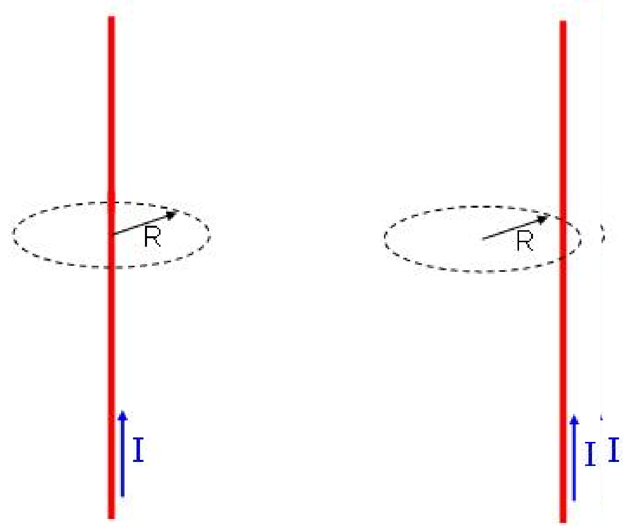
Ampere's Law

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_o I_{enclosed}}$$

Practice on Enclosed Currents

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I_{enclosed}$$

Check Point 1a

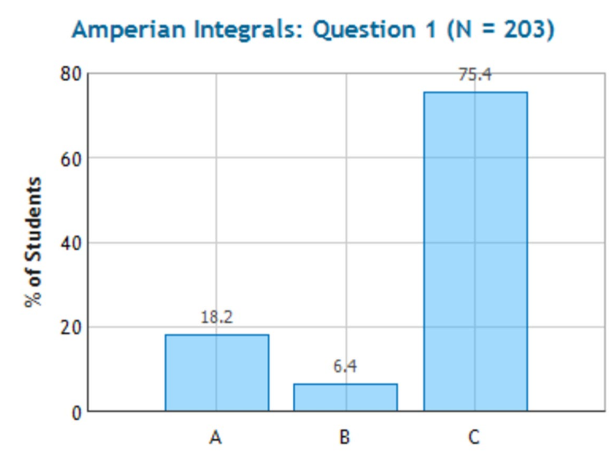


Case 1 $I_{enclosed} = I$

Case 2 $I_{enclosed} = I$

For which loop is ∫**B**·d**l** the greatest?

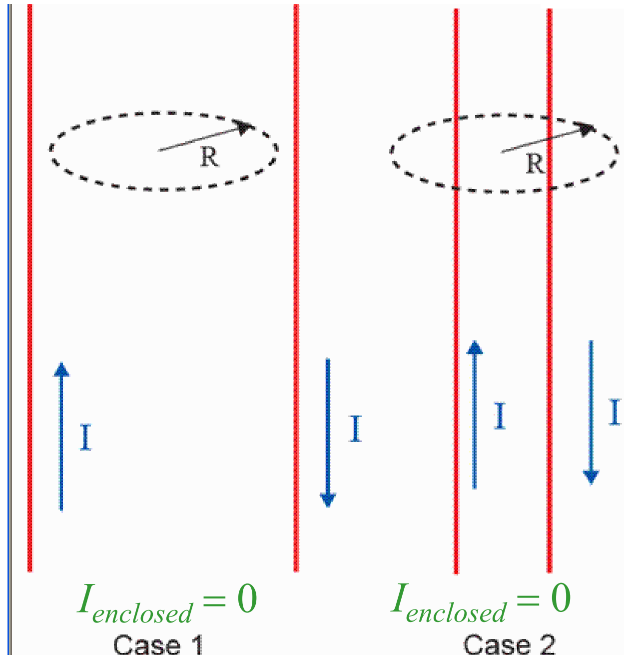
A. Case 1 ***B.*** Case 2 $\boxed{\textit{\textbf{C.}}\text{ Same}}$



Practice on Enclosed Currents

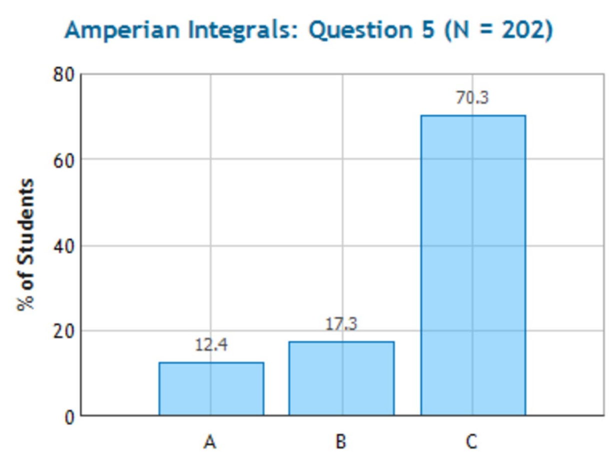
$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$

Check Point 1b



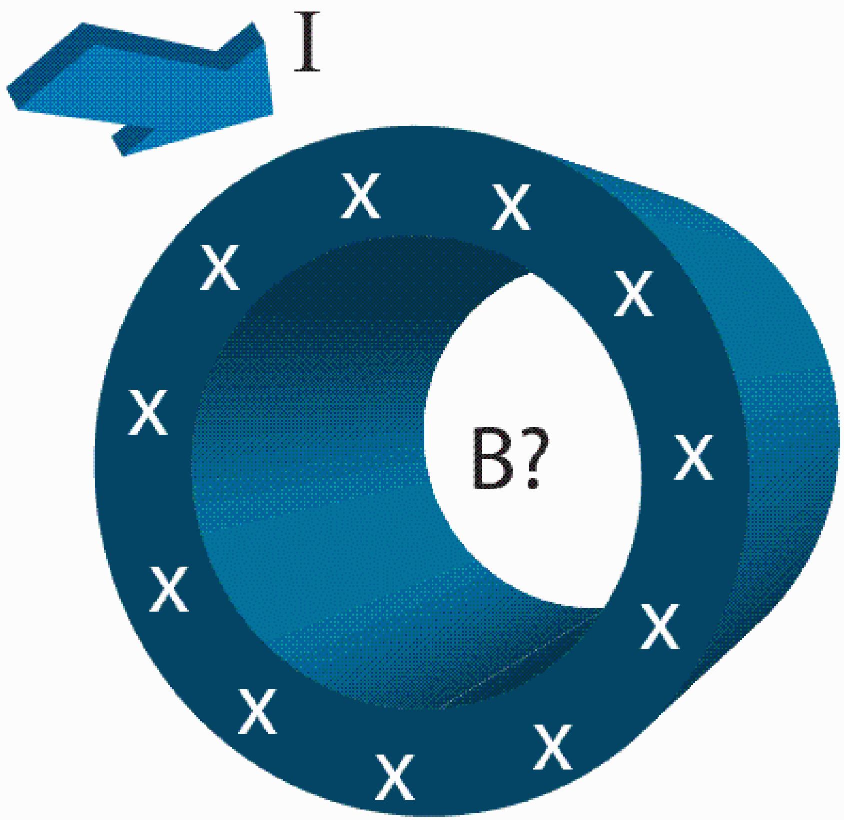
For which loop is ∫**B**·d**l** the greatest?

A. Case 1 ***B.*** Case 2 $\boxed{\textbf{\textit{C.}}\text{ Same}}$



Check Point 2a

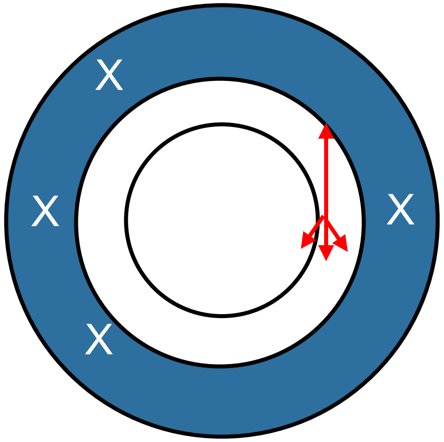
An infinitely long hollow conducting tube carries current I in the direction shown.



What is the direction of the magnetic field inside the tube?

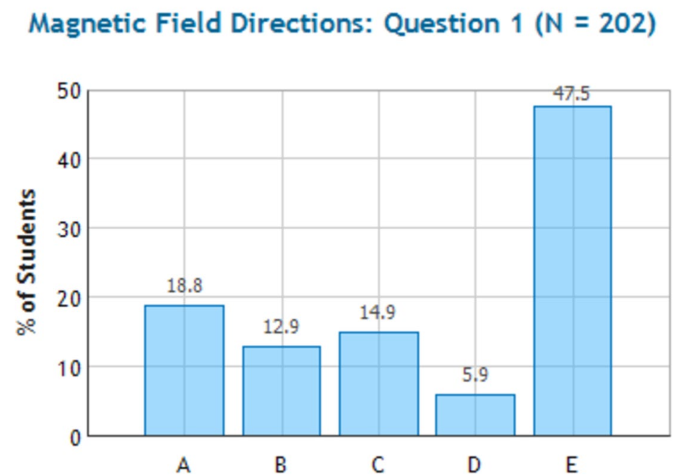
A. clockwise
B. counterclockwise
C. radially inward to the center
D. radially outward from the center
E. the magnetic field is zero

Cylindrical Symmetry



Enclosed Current = 0

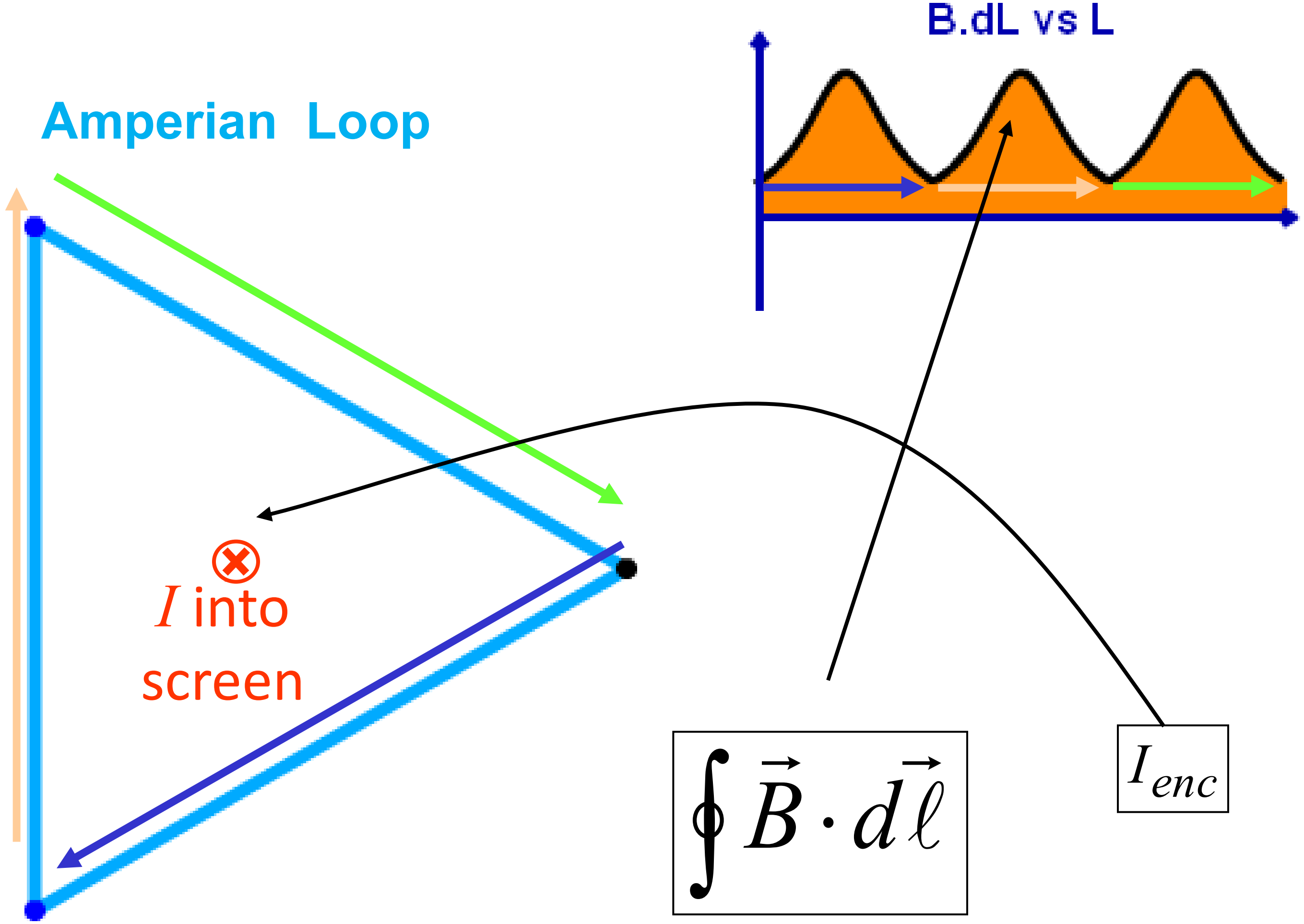
Check cancellations



Ampere's Law

"I was a bit confused with the meaning of the integral of B dot dl and its use."

+integrals + magnetic field directions

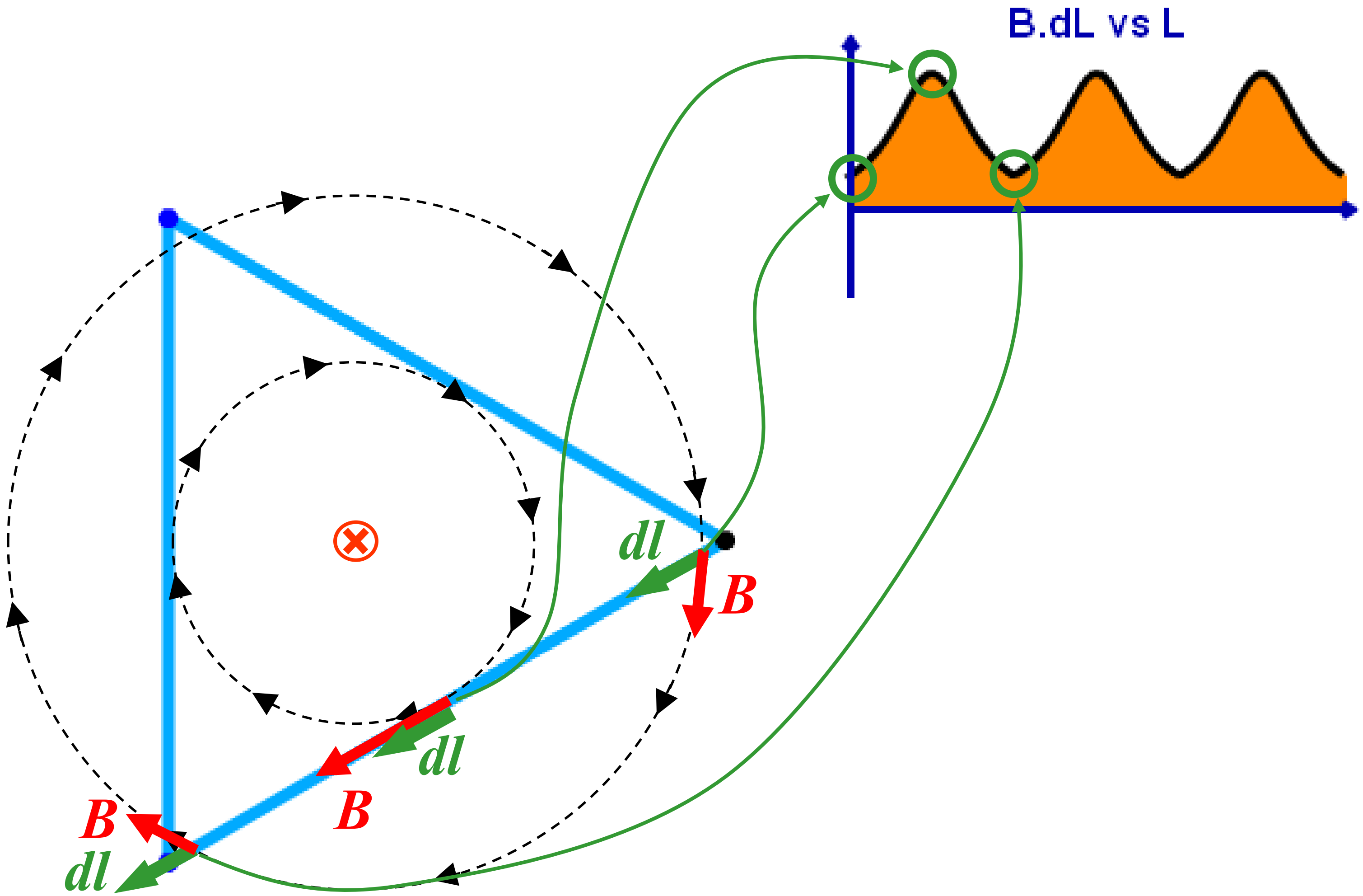


$$\oint \vec{B} \cdot d\vec{\ell}$$

$$I_{enc}$$

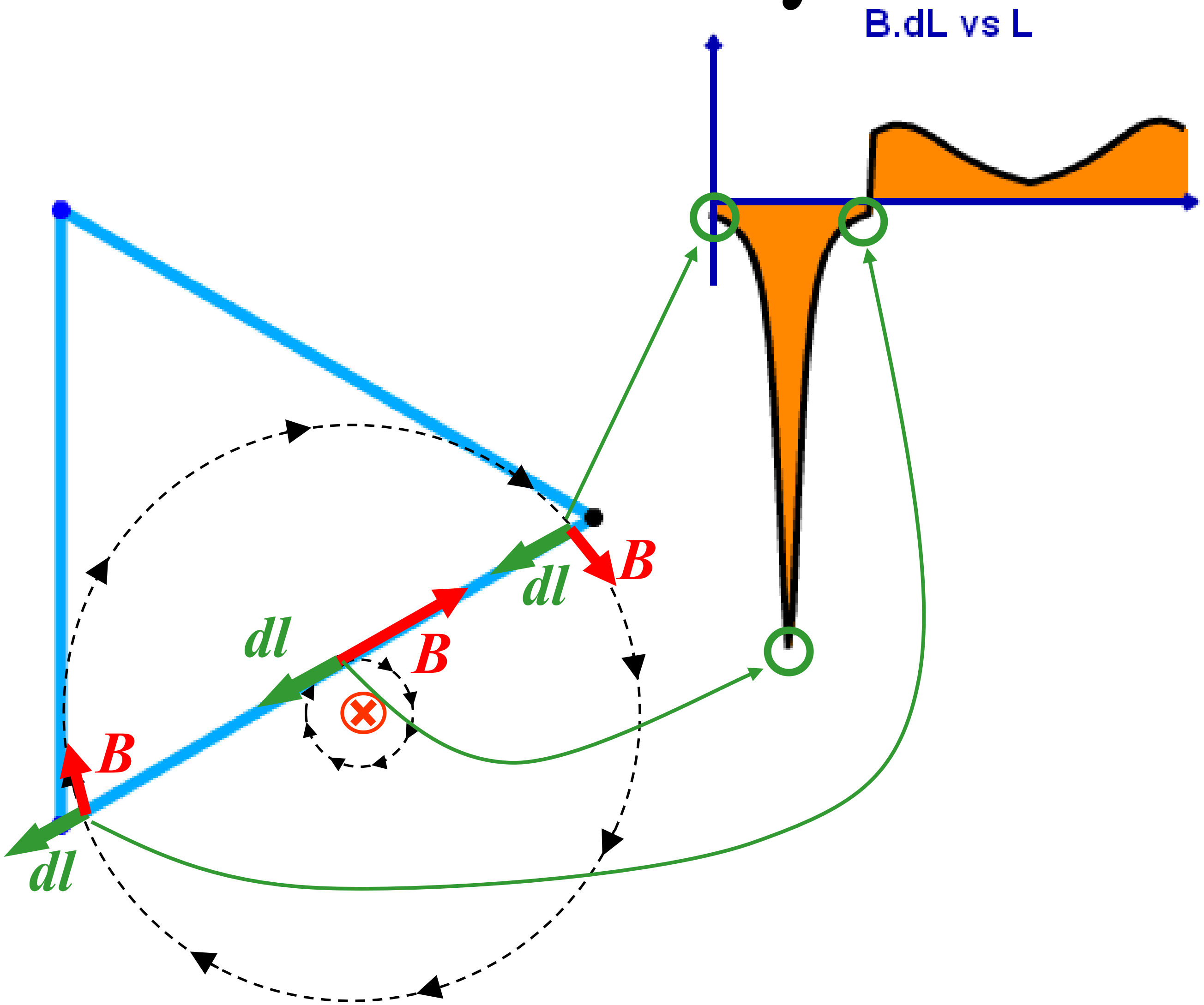
Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$



Ampere's Law

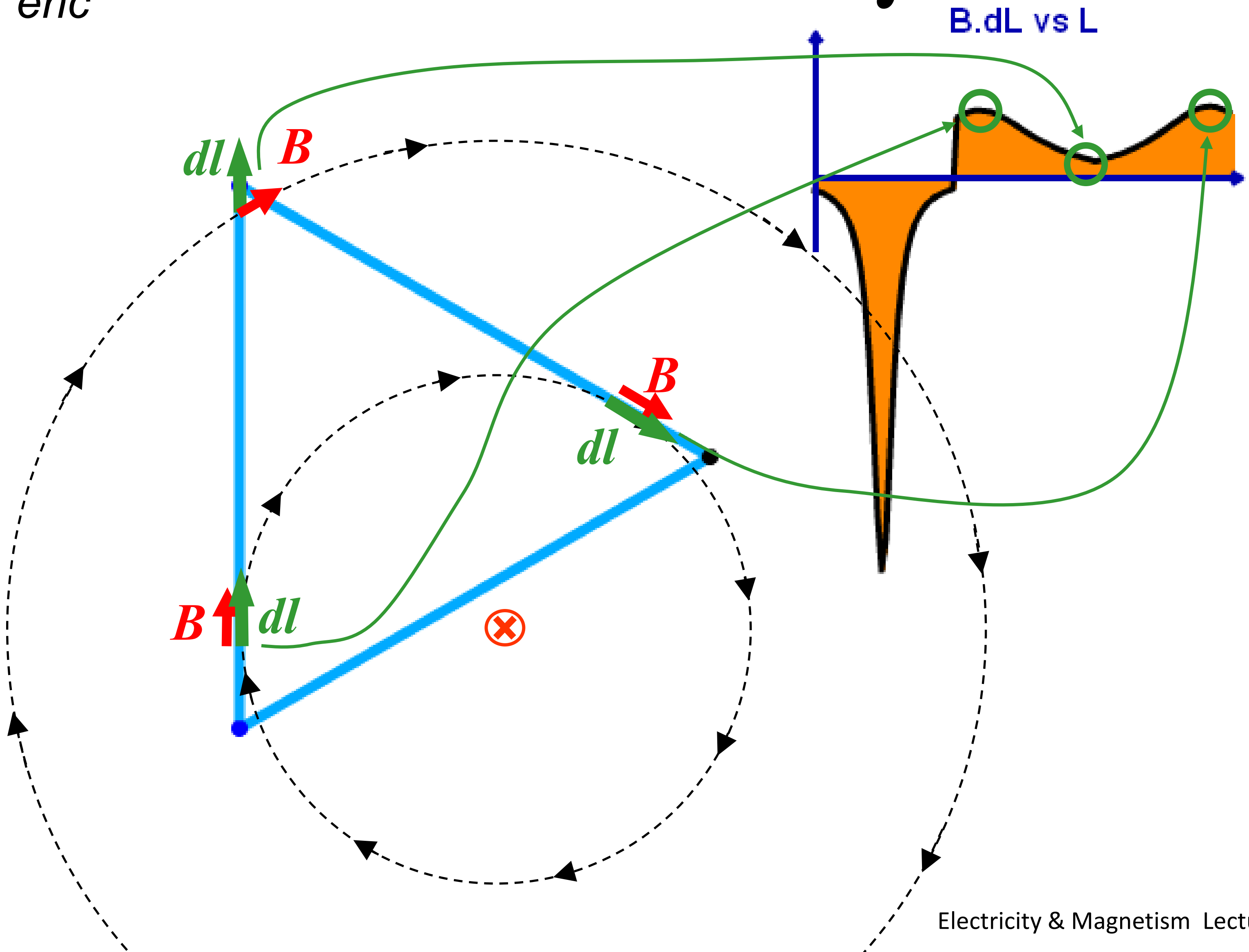
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$



Ampere's Law

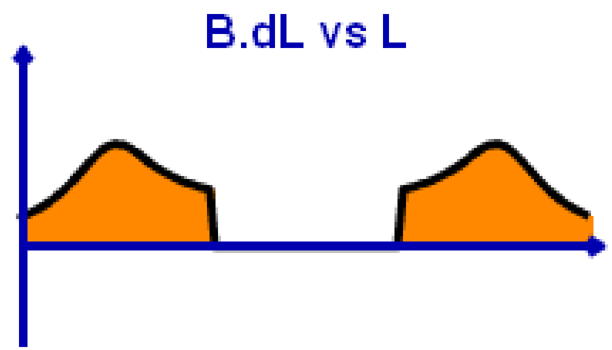
$$I_{enc} = 0!$$

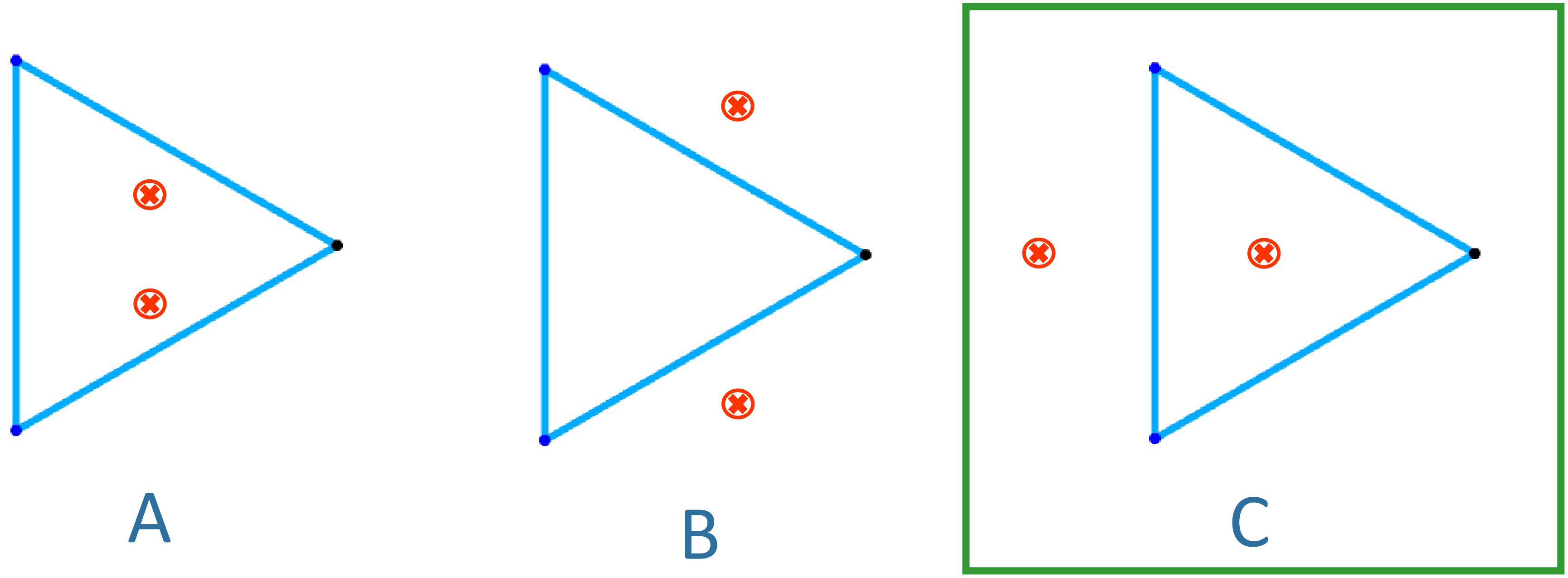
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$



Ampere's Law Clicker Question

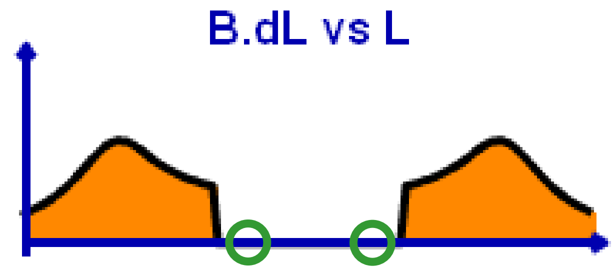
Which of the following current distributions would give rise to the $B \cdot dL$ distribution at the right?

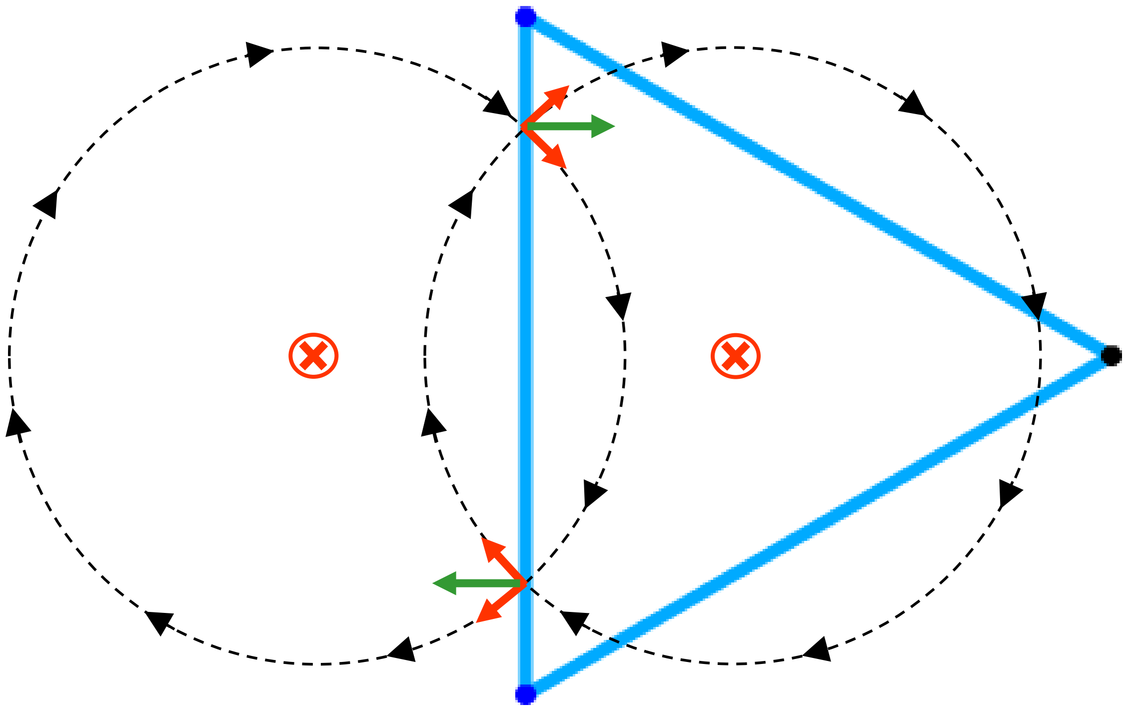




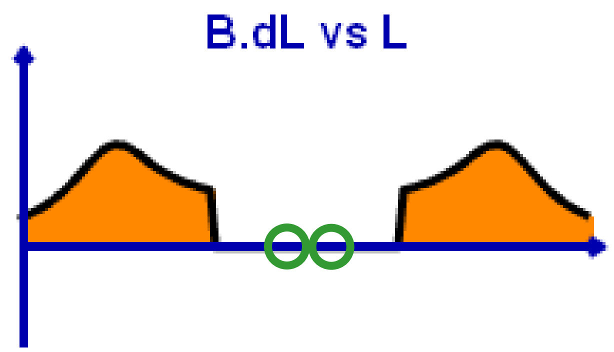
1 point bonus

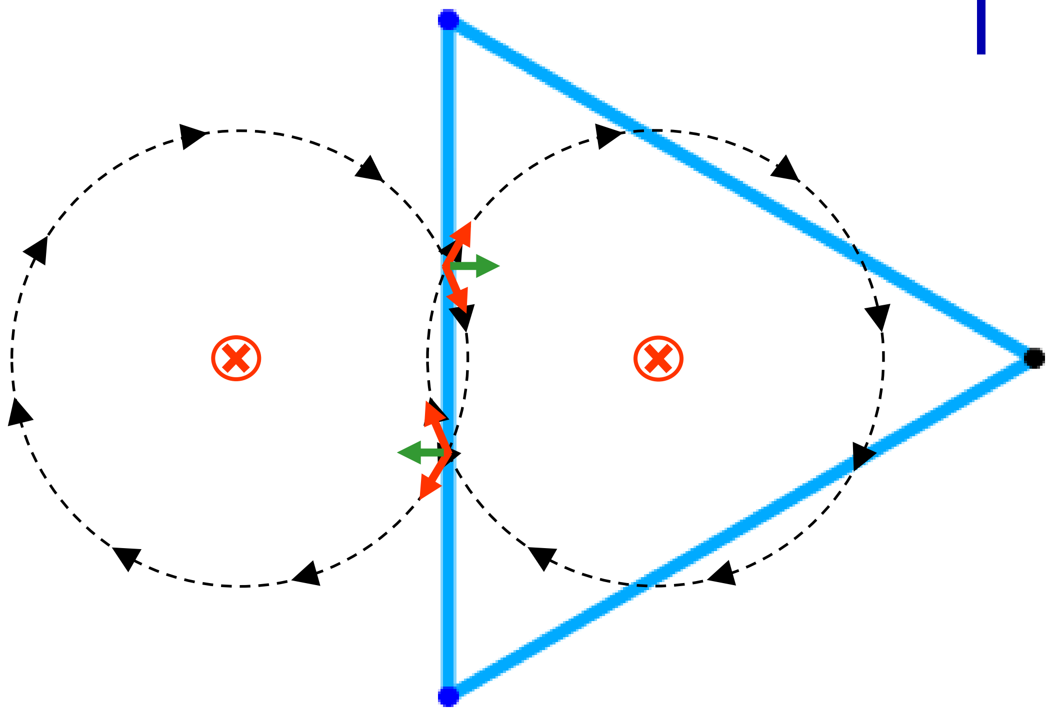
Ampere's Law Clicker Question



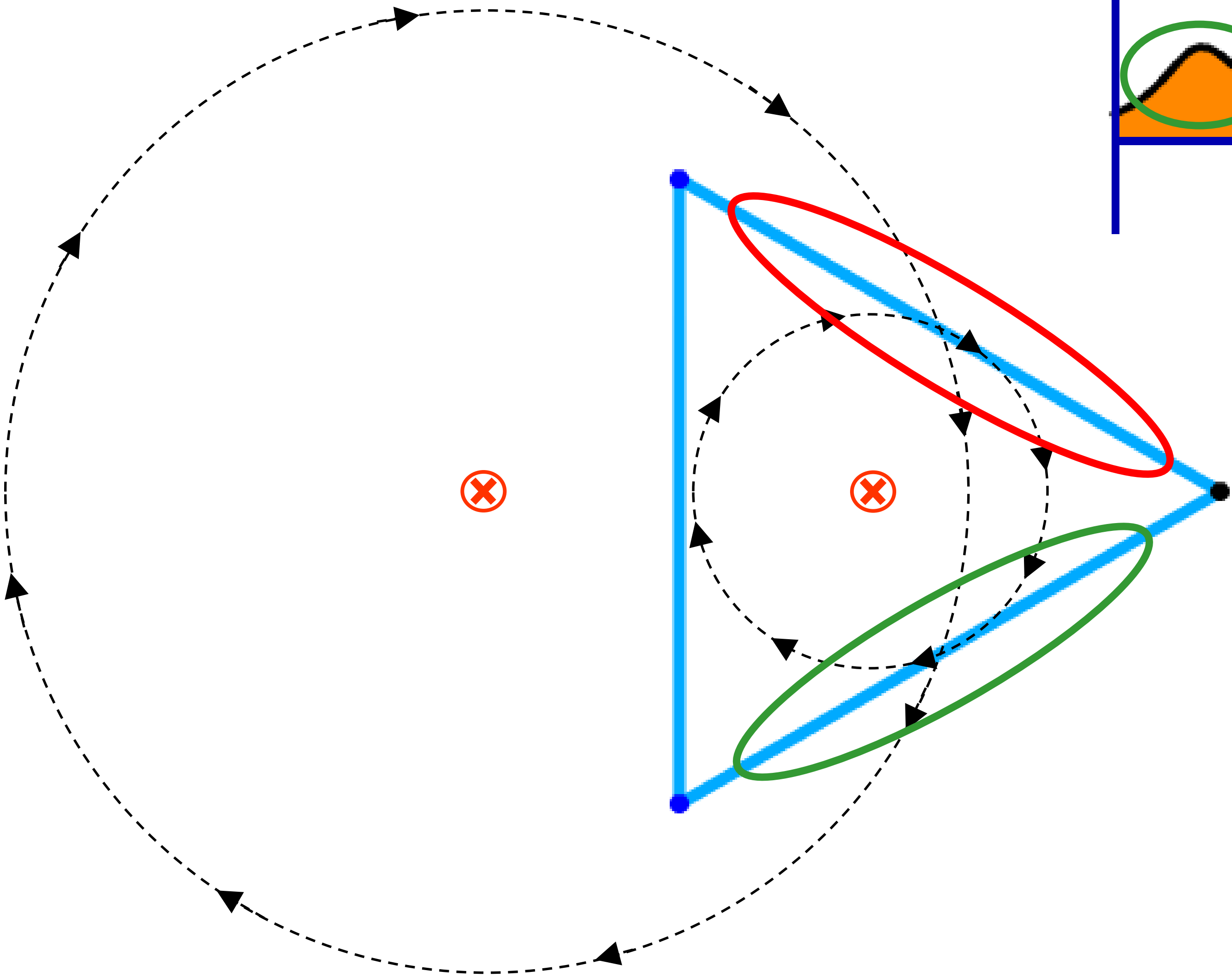


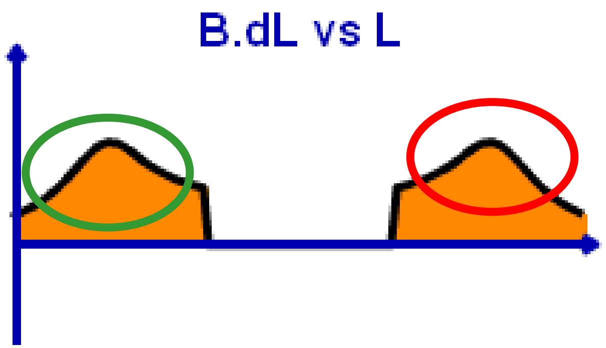
Ampere's Law Clicker Question



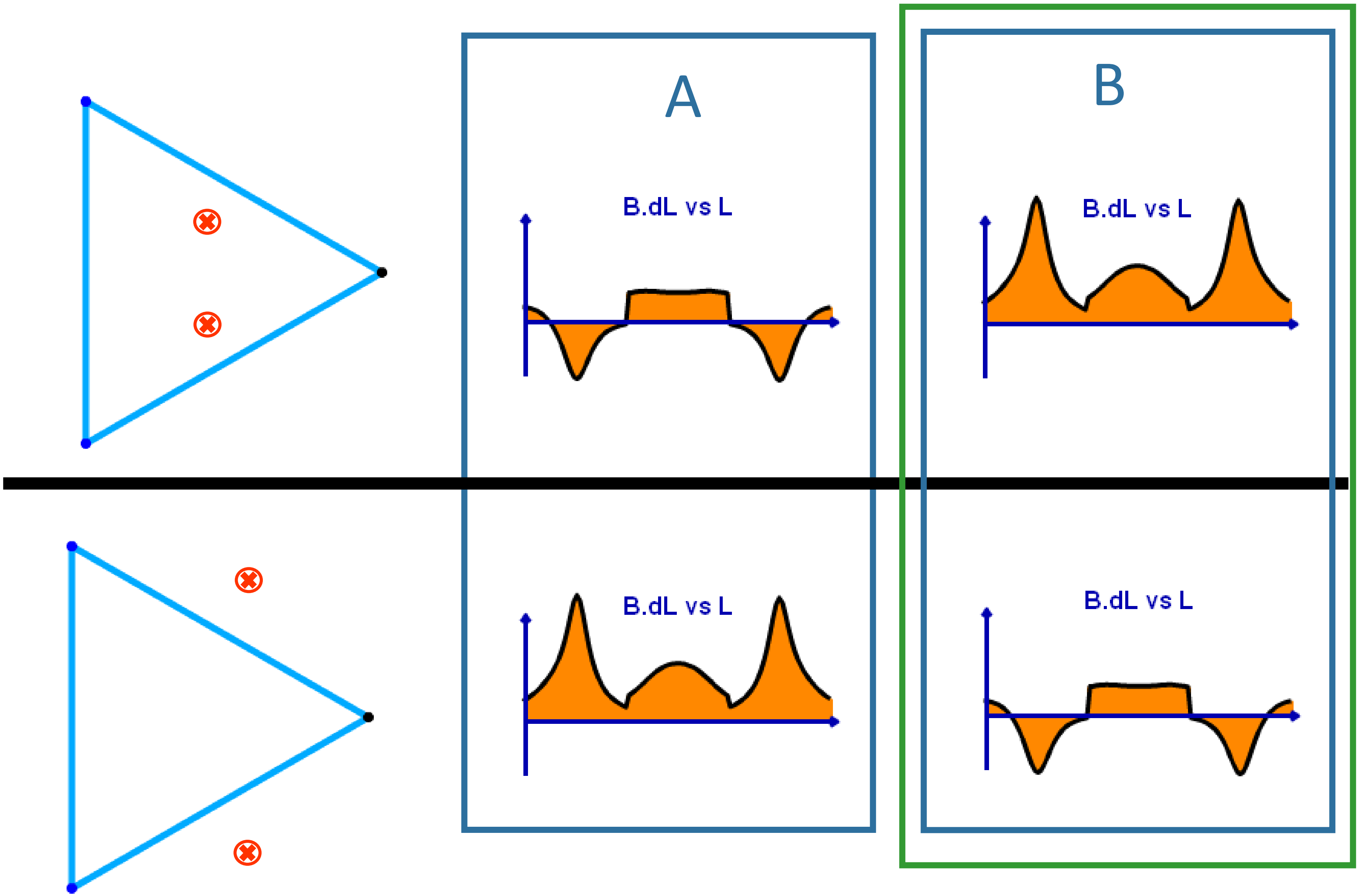


Ampere's Law Clicker Question





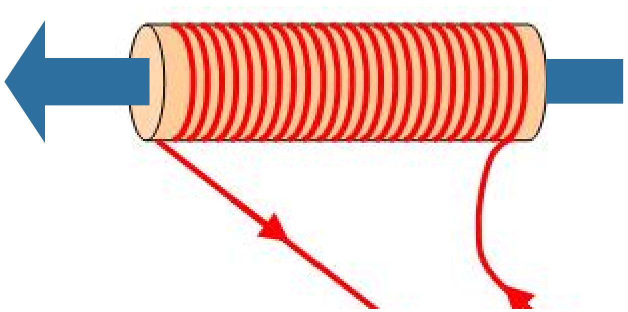
Match the other two:

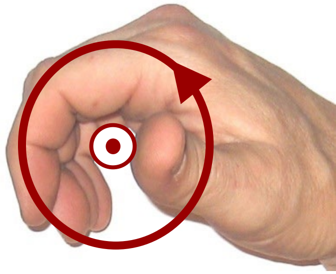


1 point bonus

Check Point 2b

A current carrying wire is wrapped around cardboard tube as shown below.

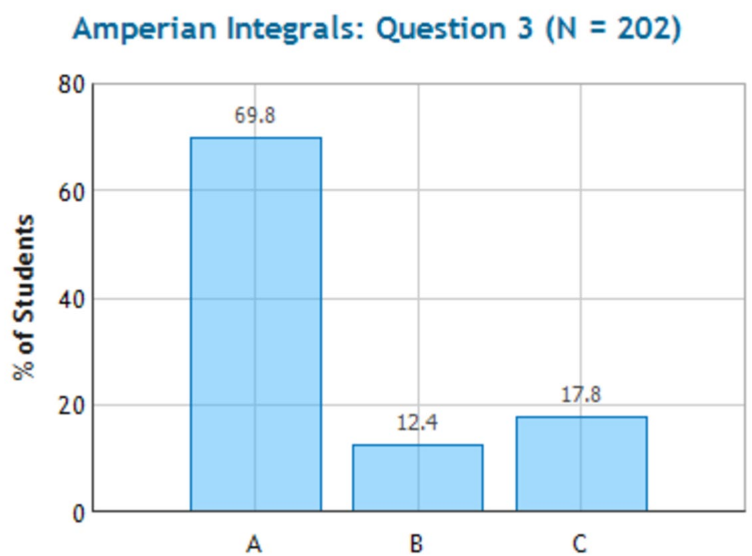




In which direction does the magnetic field point inside the tube?

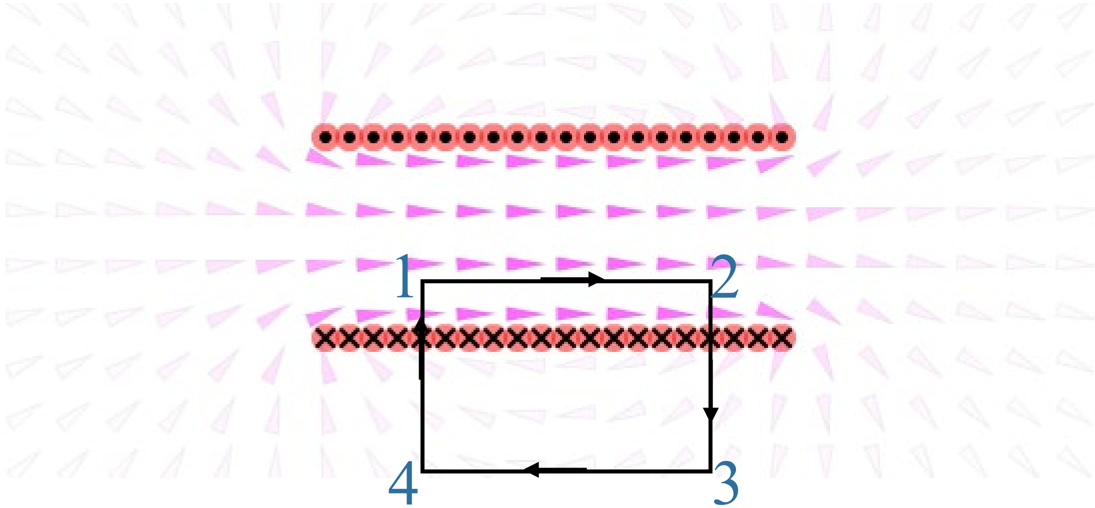
A. Left **B.** Right **C.** Up **D.** Down **E.** Out of screen

Use the right hand rule and curl your fingers along the direction of the current.



Solenoid

Several loops packed tightly together form a uniform magnetic field inside, and nearly zero magnetic field outside.



From this case we can assume a constant field inside the solenoid and zero field outside the solenoid, and apply Ampere's law to find the magnitude of the constant field inside the solenoid!

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc} \longrightarrow \int_1^2 \vec{B} \bullet d\vec{\ell} + \int_2^3 \vec{B} \bullet d\vec{\ell} + \int_3^4 \vec{B} \bullet d\vec{\ell} + \int_4^1 \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

$$BL + 0 + 0 + 0 = \mu_o I_{enc}$$

n = # turns/length

$$\longrightarrow BL = \mu_o n L I \longrightarrow B = \mu_o n I$$

Solenoid in real life

World's strongest magnet! At the National Mag Lab in Florida

B = 45 T !

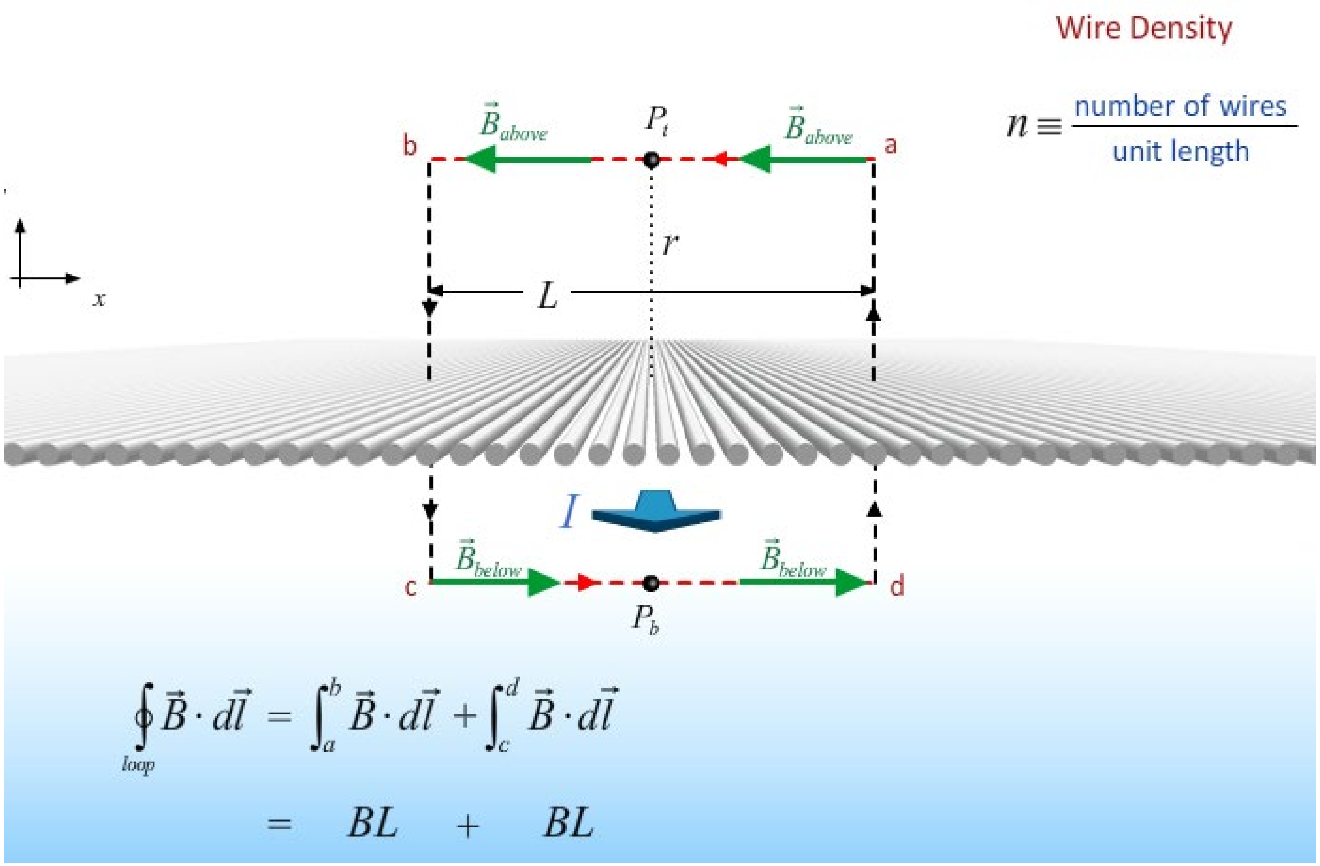
MRI = 0.5 T

Bar magnet = 0.01 T

Earth's magnetic field = 10^{-5} T

https://nationalmaglab.org/about/around-the-lab/meet-the-magnets/meet-the-45-tesla-hybrid-magnet

Similar to the Current Sheet

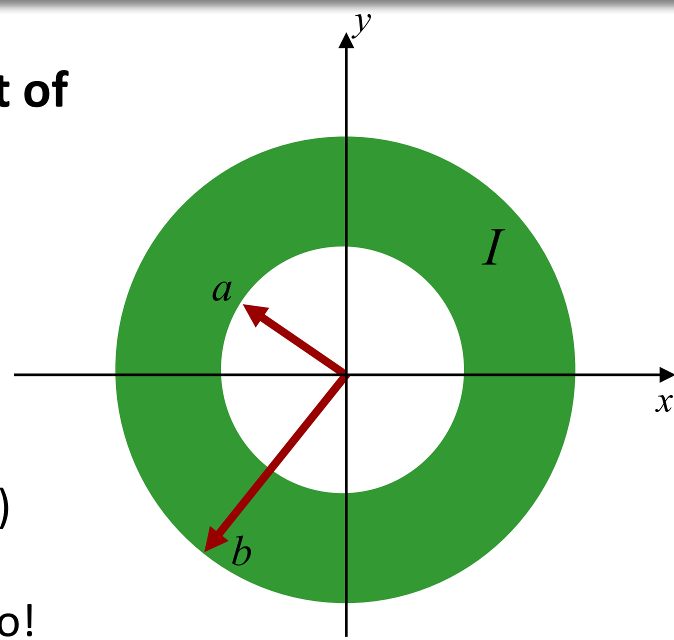


Total integral around the loop

$$2BL = \mu_0 I_{enclosed}$$

$$\therefore B = \frac{\mu_0 N I}{2L} = \frac{\mu_0 n I}{2}$$

Example Problem



An infinitely long cylindrical shell with inner radius a and outer radius b carries a uniformly distributed current I **out of the screen.**

Sketch $|B|$ as a function of r.

Conceptual Analysis

Complete cylindrical symmetry (can only depend on r)

$\Rightarrow$ can use Ampere's law to calculate B

B field can only be clockwise, counterclockwise or zero!

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

$$B \oint d\ell = \mu_o I_{enc}$$ For circular path concentric with shell.

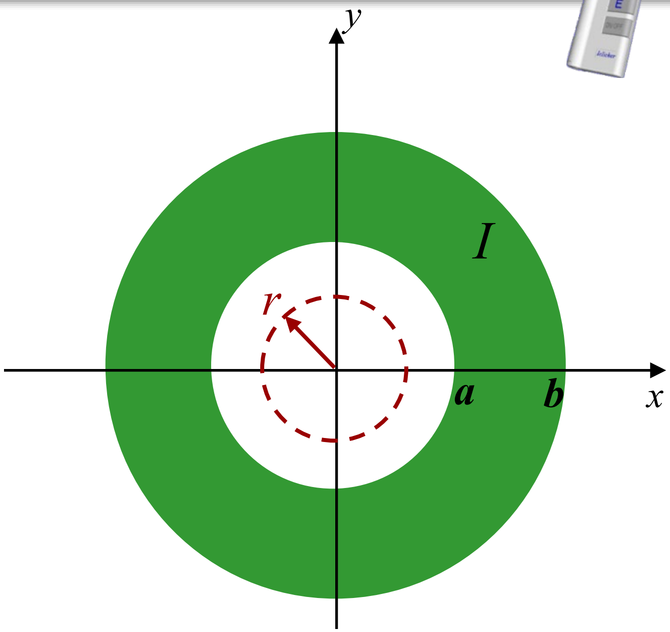
Strategic Analysis

Calculate B for the three regions separately:

1) $r < a$
2) $a < r < b$
3) $r > b$

Example Problem

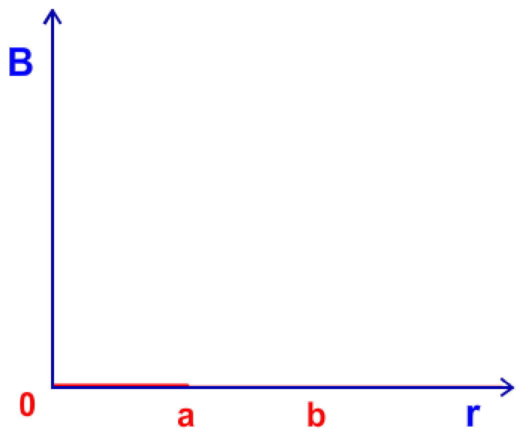
What does $|B|$ look like for $r < a$?



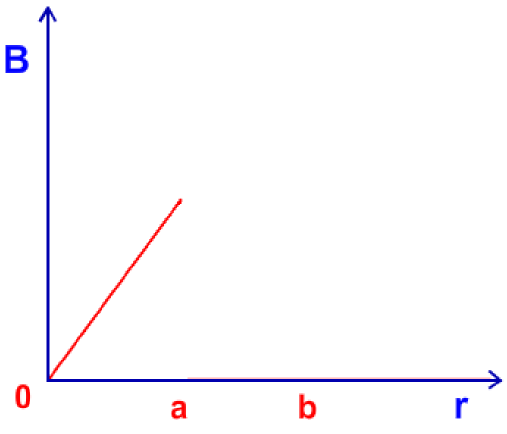
$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

0

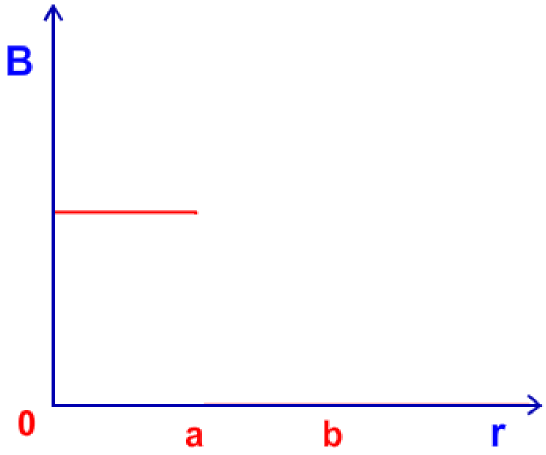
so $\vec{B} = 0$



A



B



C

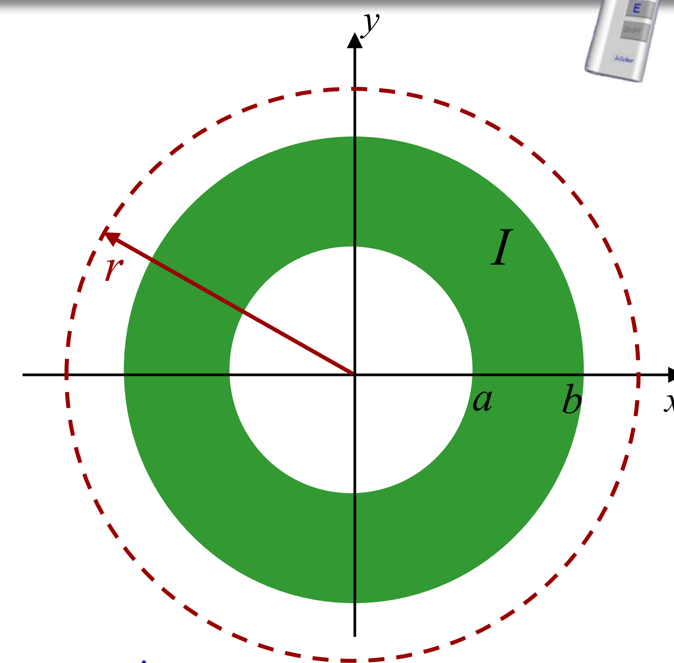
1 point bonus

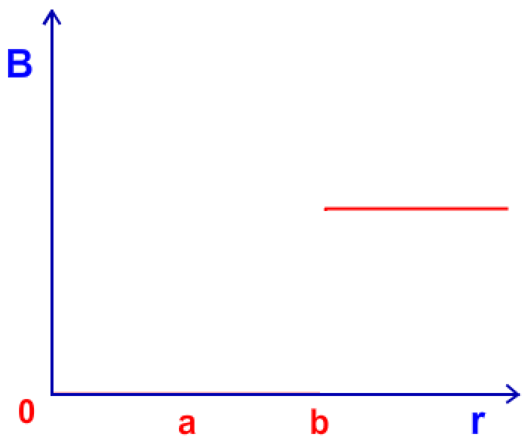
Example Problem

What does $|B|$ look like for $r > b$?

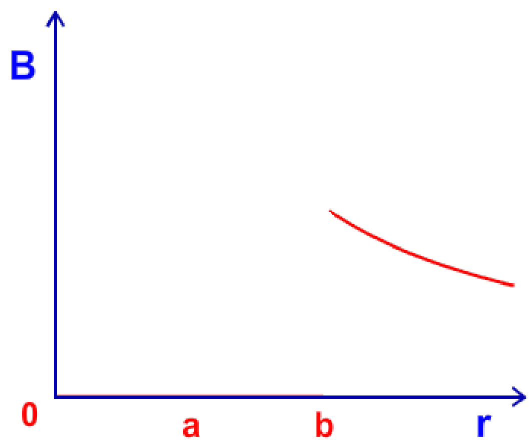
$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{inc}$$

I

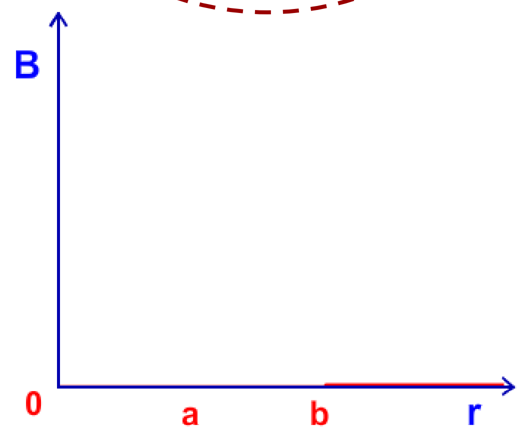




A



B



C

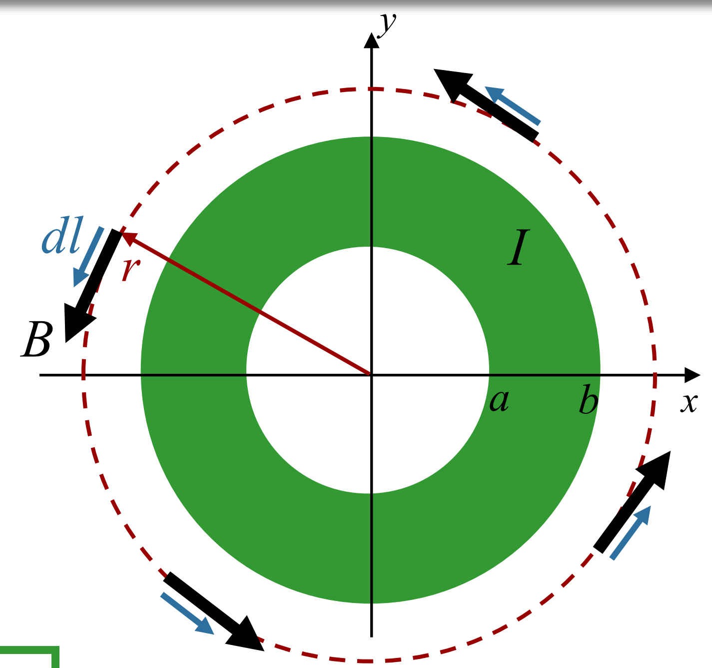
Example Problem

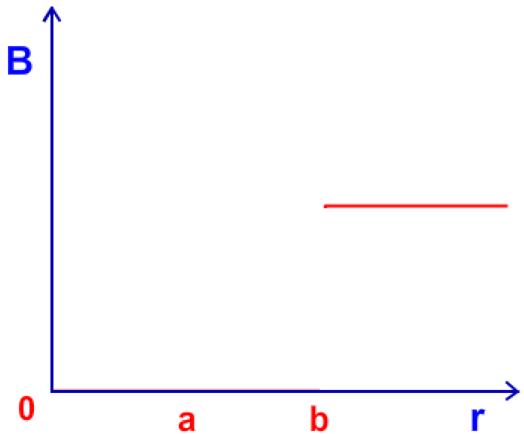
What does $|B|$ look like for $r > b$?

LHS: $\oint \vec{B} \bullet d\vec{\ell} = \oint B d\ell = B \oint d\ell = B \cdot 2\pi r$

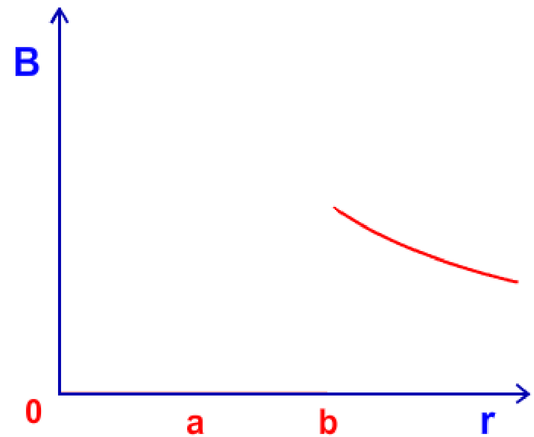
RHS: $I_{enclosed} = I$

$$B = \frac{\mu_o I}{2\pi r}$$

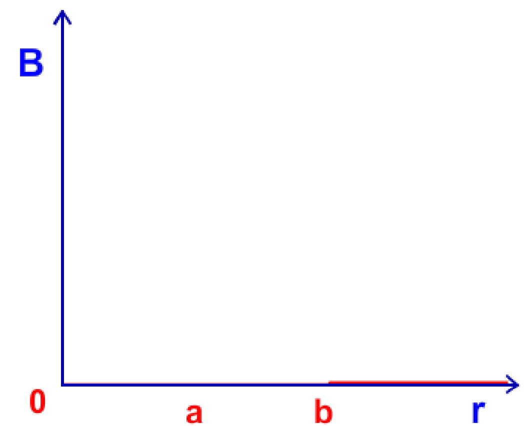




A



B

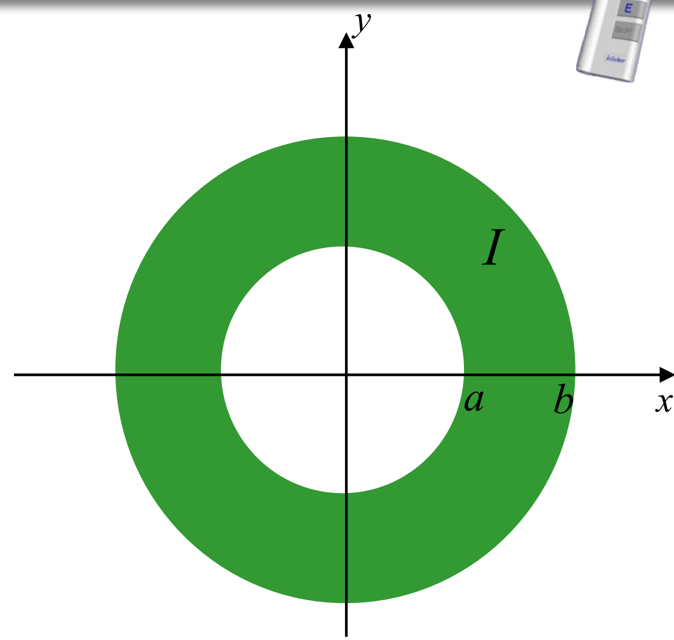


C

1 point bonus

Example Problem

What is the current density j (Amp/m^2) in the conductor?



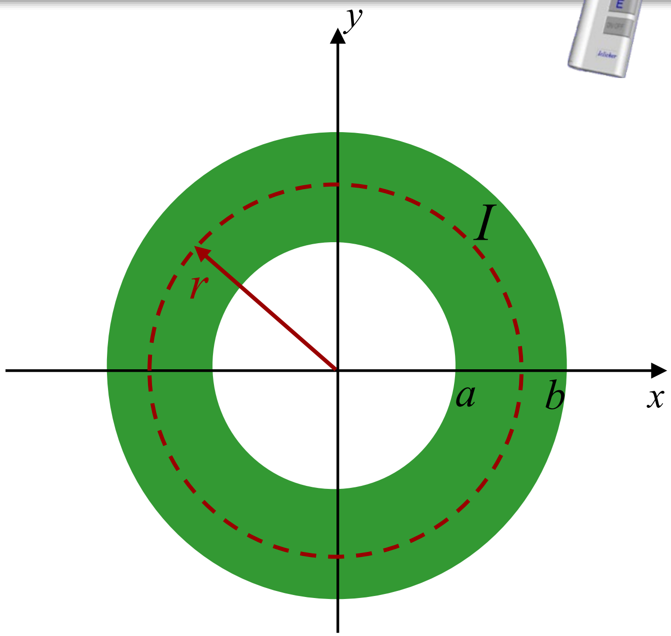
A) $j = \dfrac{I}{\pi b^2}$ B) $j = \dfrac{I}{\pi b^2 + \pi a^2}$ C) $\boxed{j = \dfrac{I}{\pi b^2 - \pi a^2}}$

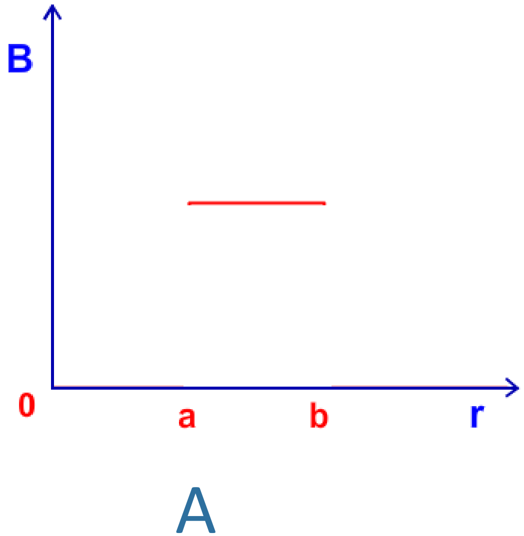
$$\underbrace{j = I/area \qquad area = \pi b^2 - \pi a^2}$$

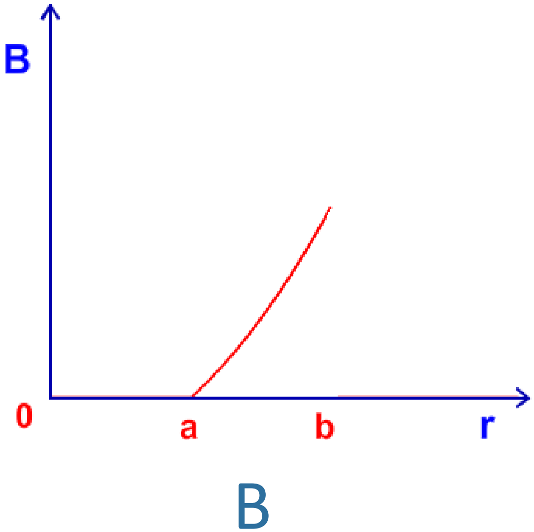
$$j = \frac{I}{\pi b^2 - \pi a^2}$$

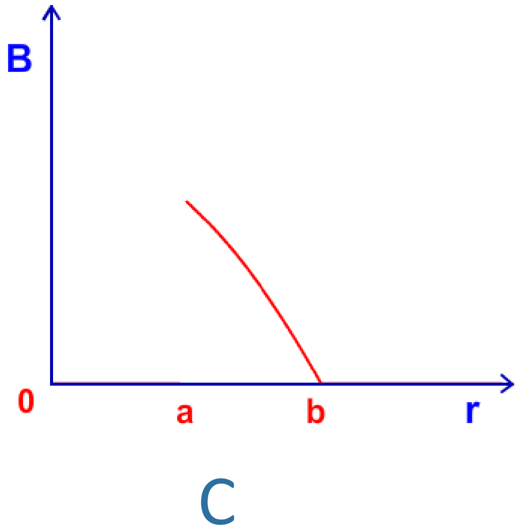
Example Problem

What does $|B|$ look like for $a < r < b$?





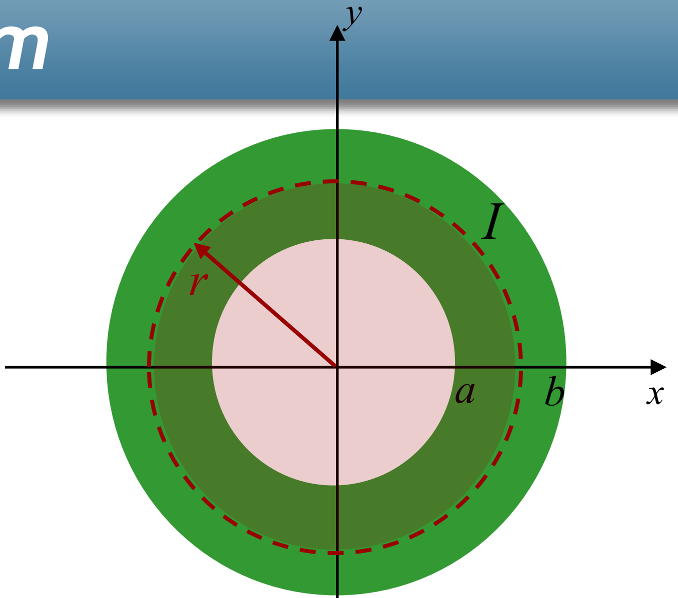




Example Problem

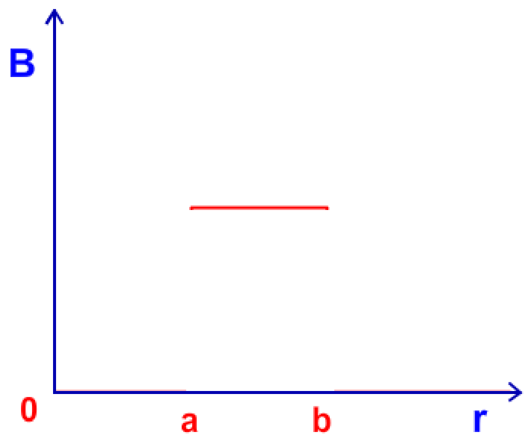
What does $|B|$ look like for $a < r < b$?

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc} \rightarrow B \cdot 2\pi r = \mu_o \cdot j A_{enc}$$

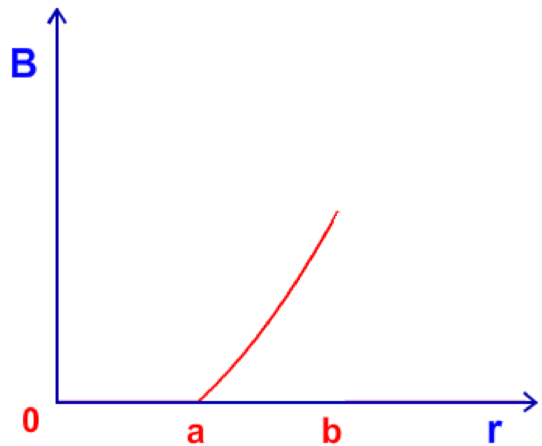


$$B \cdot 2\pi r = \mu_o \cdot \frac{I}{\pi(b^2 - a^2)} \cdot \pi(r^2 - a^2) \rightarrow B = \frac{\mu_o I}{2\pi r} \cdot \frac{(r^2 - a^2)}{(b^2 - a^2)}$$

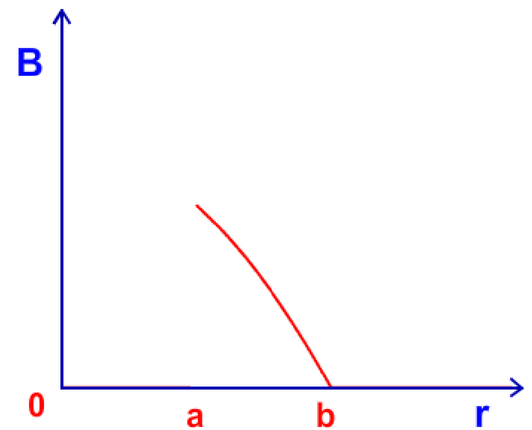
Starts at 0 and increases almost linearly



A



B

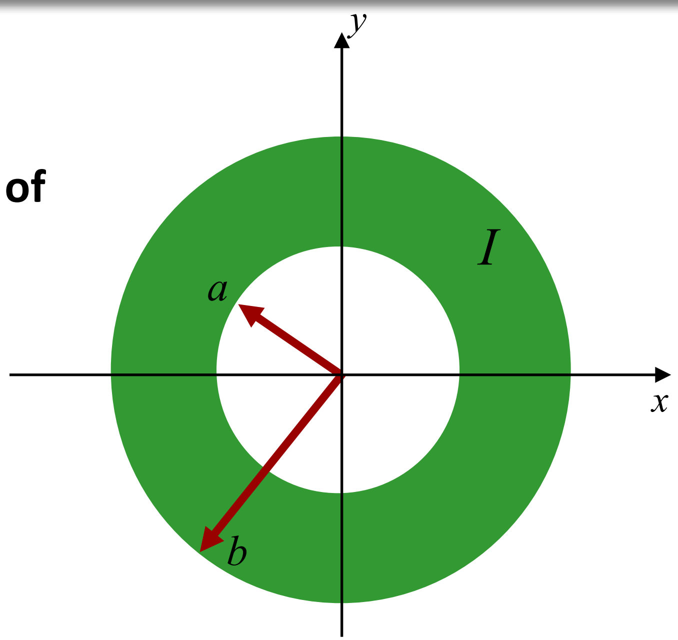


C

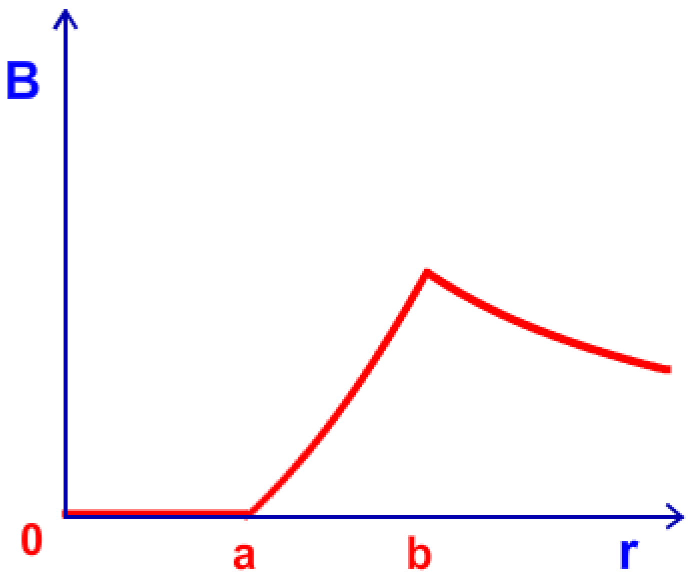
1 point bonus

Example Problem

An infinitely long cylindrical shell with inner radius a and outer radius b carries a uniformly distributed current **I out of the screen.**



Sketch $|B|$ as a function of r.

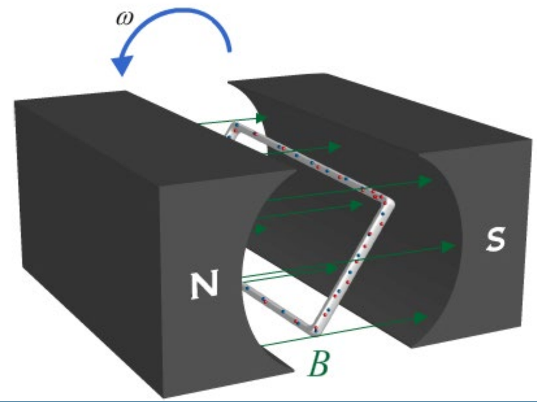


Exam Logistics

1) Exam 2 – units 9 to 16
    - 11/11/2024 – 6 : 00 pm

2) EXAM 2 PREPARATION
    - Demo exam at SmartPhysics (Solutions)

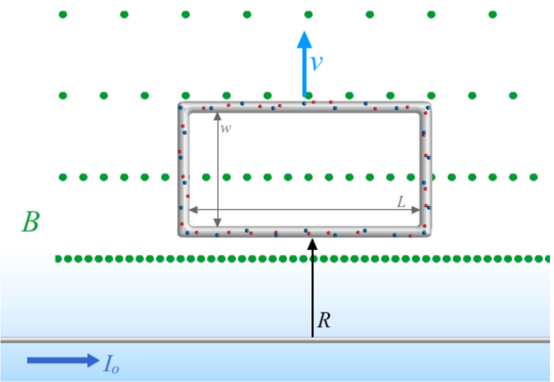
3) Office Hours
    - 9:00 AM – 9:50 AM, Monday （1C BUILDING, B308）

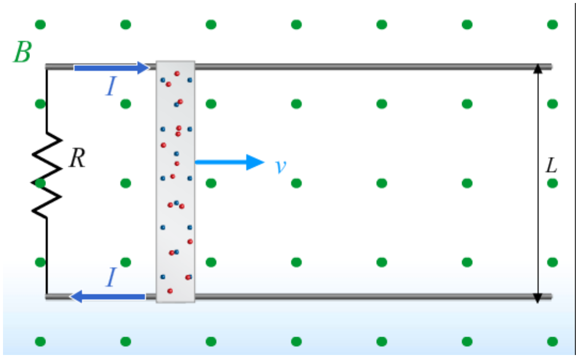


Physics 212
Lecture 16

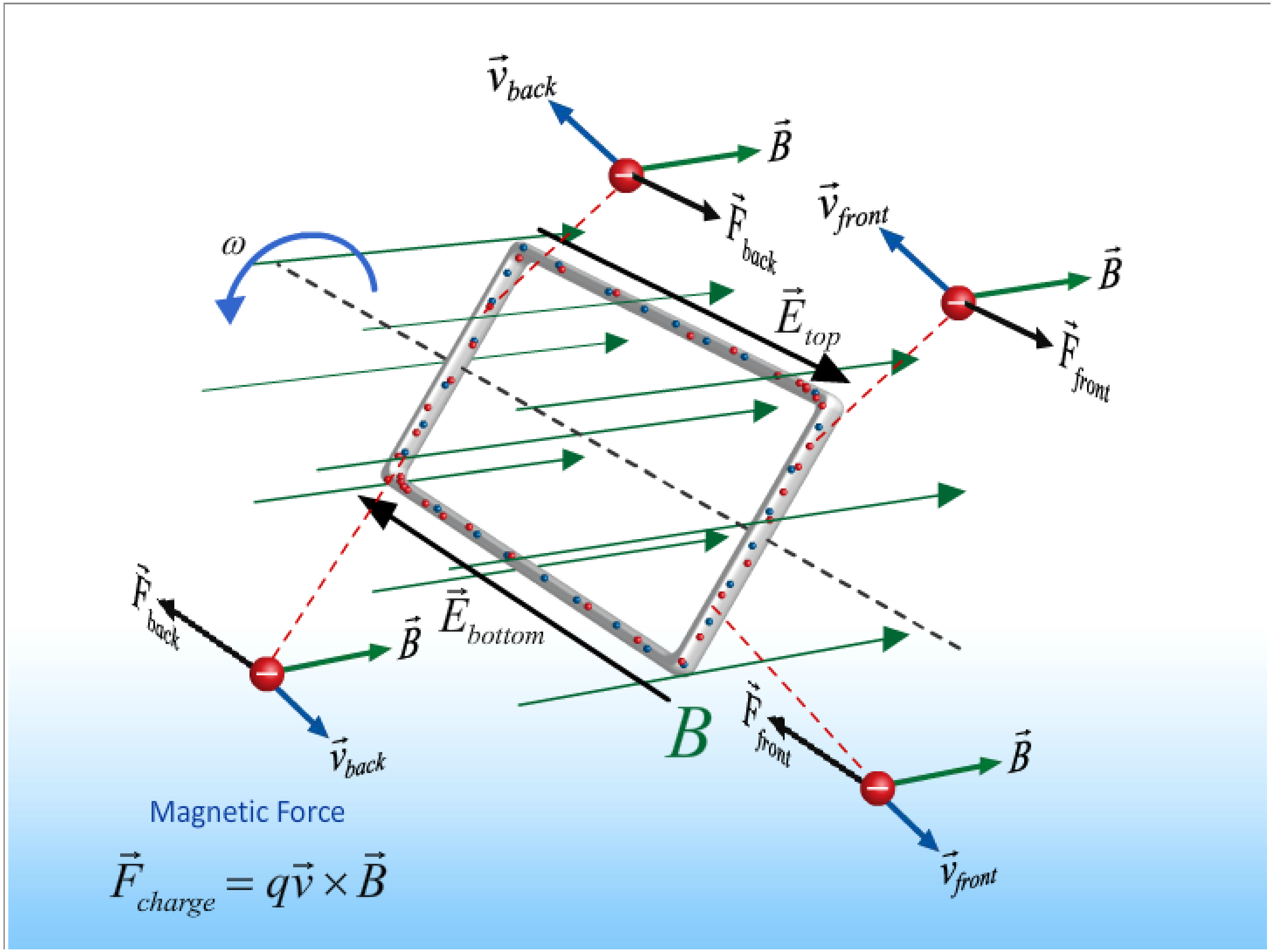
Today's Concept:

Motional EMF





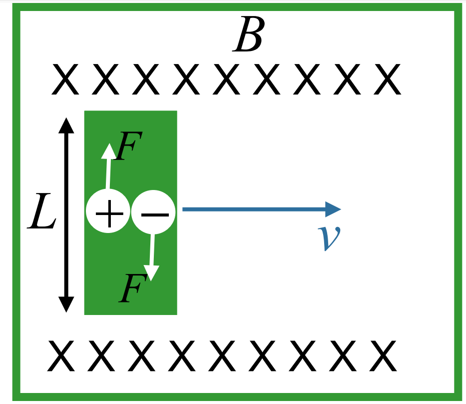
What part of



don’t you understand?

The Big Idea

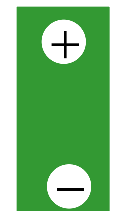
When a conductor moves through a region containing a magnetic field:



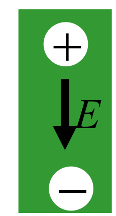
Magnetic forces may be exerted on the charge carriers in the conductor

$$\vec{F} = q\vec{v} \times \vec{B}$$

These forces produce a charge separation in the conductor



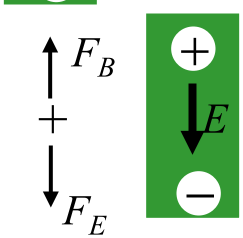
This charge distribution creates an electric field in the conductor



The equilibrium distribution is reached when the forces from the electric and magnetic fields cancel

$$qvB = qE$$

$$E = vB$$

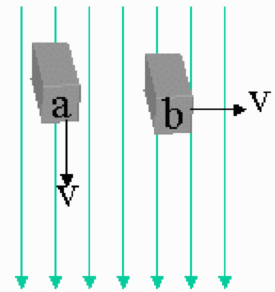


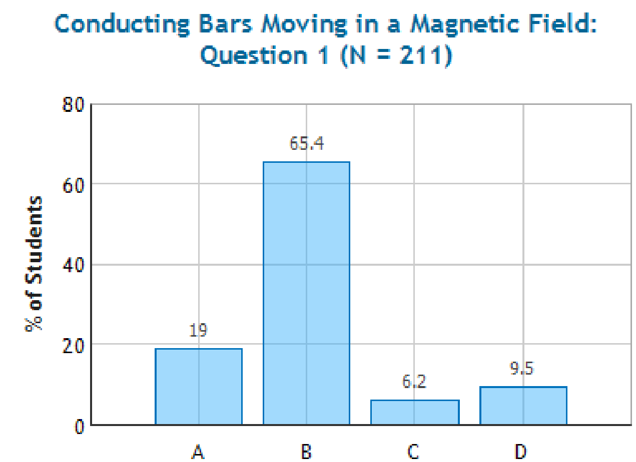
The equilibrium electric field produces a potential difference (emf) in the conductor

$$V = EL \rightarrow \boxed{V = vBL}$$

Check Point 1

Two identical conducting bars (shown in end view) are moving through a vertical magnetic field. Bar (a) is moving vertically and bar (b) is moving horizontally.





Which of the following is true?

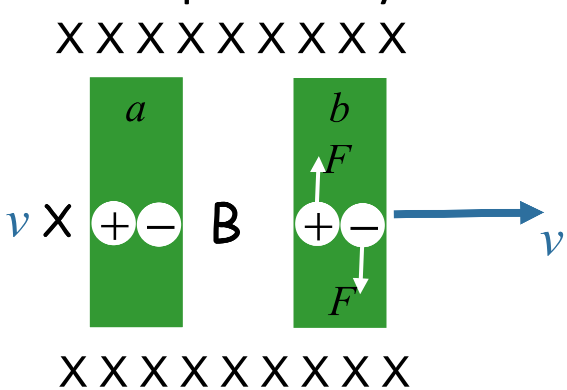
A. A motional emf exists in the bar for case (a), but not (b)

$\boxed{\textbf{B.}\ \text{A motional emf exists in the bar for case (b), but not (a)}}$

C. A motional emf exists in the bar for both cases (a) and (b)

D. A motional emf exists in the bar for neither case (a) nor case (b)

Rotate picture by 90^o



$F_a = 0$ $\quad$ $F_b = qvB$

Bar a

No force on charges

No charge separation

No E field

No emf

Bar b

Opposite forces on charges

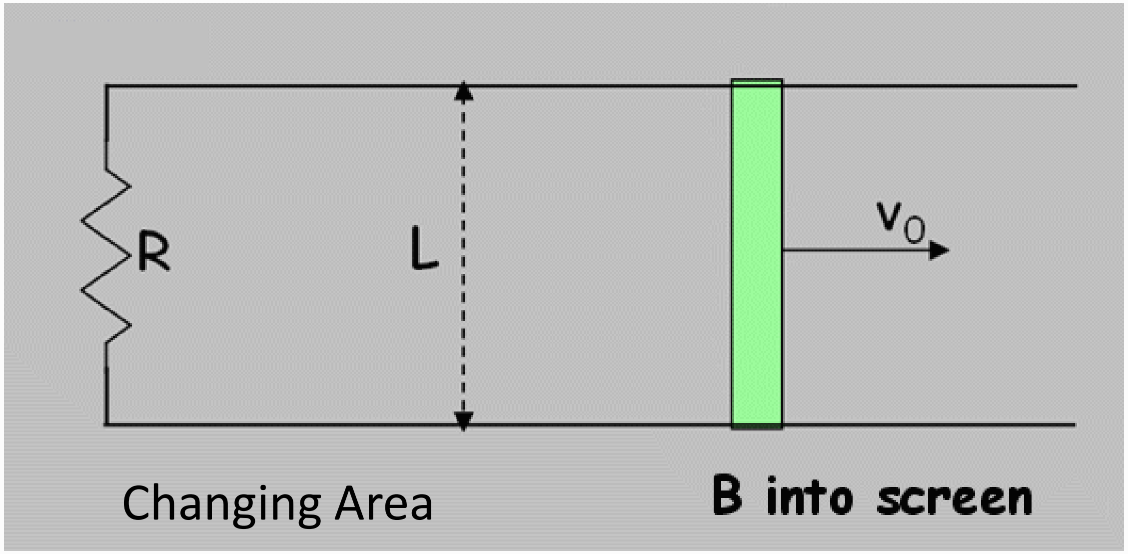
Charge separation

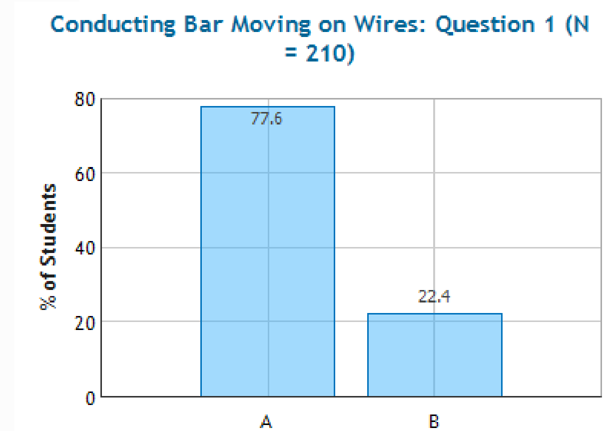
$E = vB$

$emf = EL = vBL$

Check Point 2a

A conducting bar (green) rests on two frictionless wires connected by a resistor as shown.



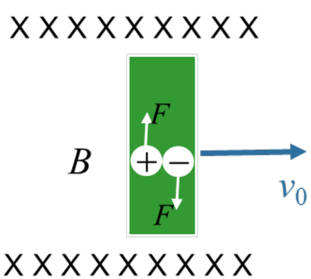


The entire apparatus is placed in a uniform magnetic field pointing into the screen, and the bar is given an initial velocity to the right.

The motion of the green bar creates a current through the bar

A. going up

B. going down



$F_b = qv_0B$

Bar

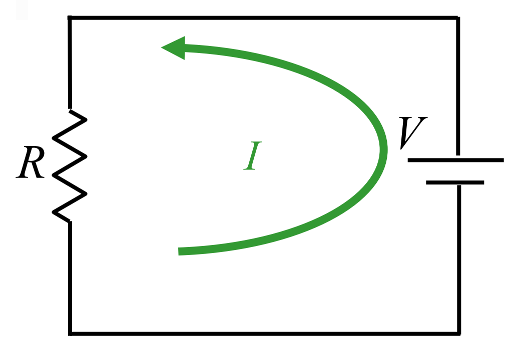
Opposite forces on charges

Charge separation

$E = v_0B$

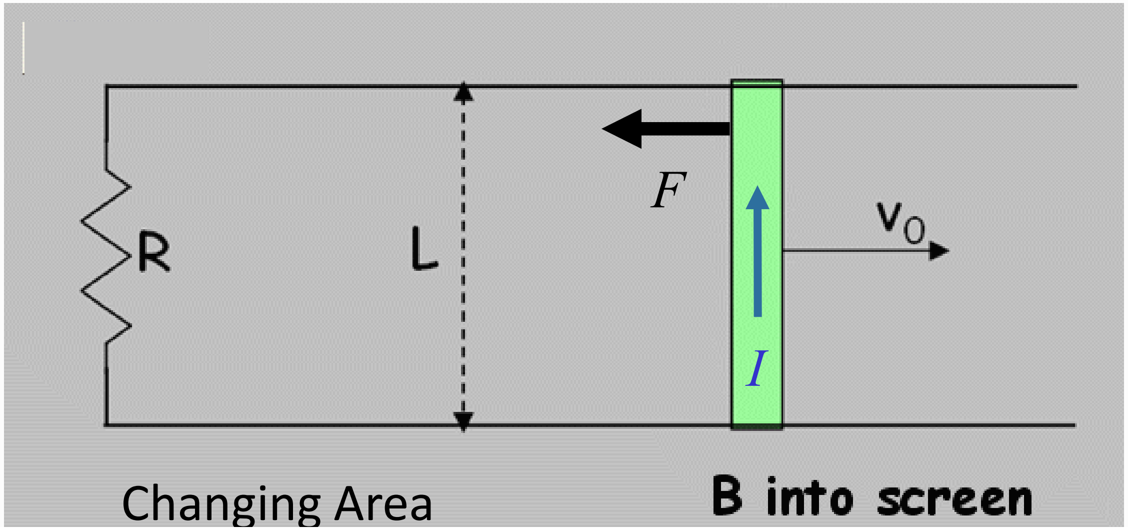
$emf = EL = v_0BL$

Equivalent circuit



Check Point 2b

A conducting bar (green) rests on two frictionless wires connected by a resistor as shown.



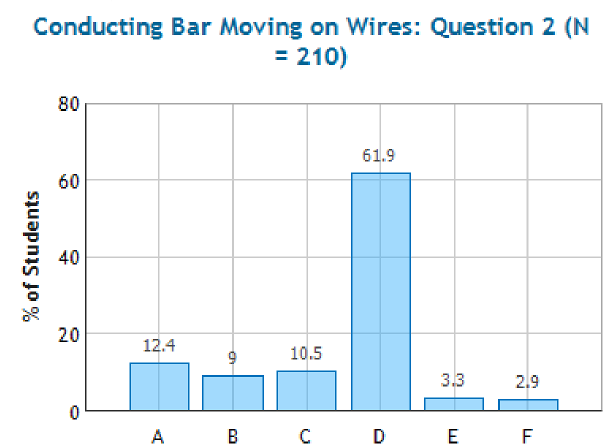
The entire apparatus is placed in a uniform magnetic field pointing into the screen, and the bar is given an initial velocity to the right.

The current through this bar results in a force on the bar

A. down
B. up
C. right
D. left
E. into the screen

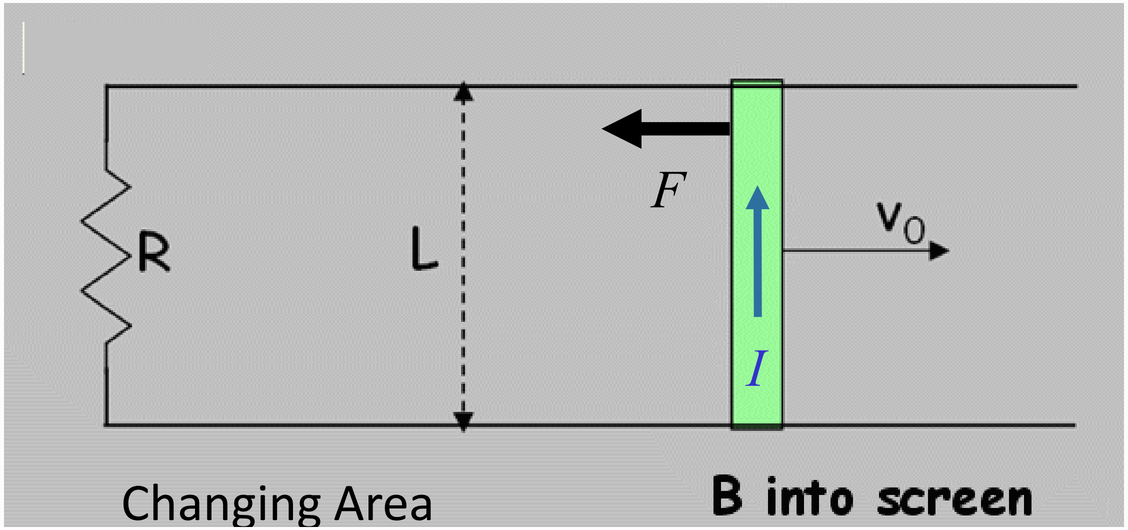
Current up through bar

$$\vec{F} = I\vec{L} \times \vec{B} \longrightarrow F \text{ points to left}$$



CheckPoint 2b

A conducting bar (green) rests on two frictionless wires connected by a resistor as shown.



Energy

External agent must exert force F to the right to maintain constant v

This energy is dissipated in the resistor!

The entire apparatus is placed in a uniform magnetic field pointing into the screen, and the bar is given an initial velocity to the right.

The current through this bar results in a force on the bar

A. down
B. up
C. right
$\boxed{\textbf{D. } \text{left}}$
E. into the screen

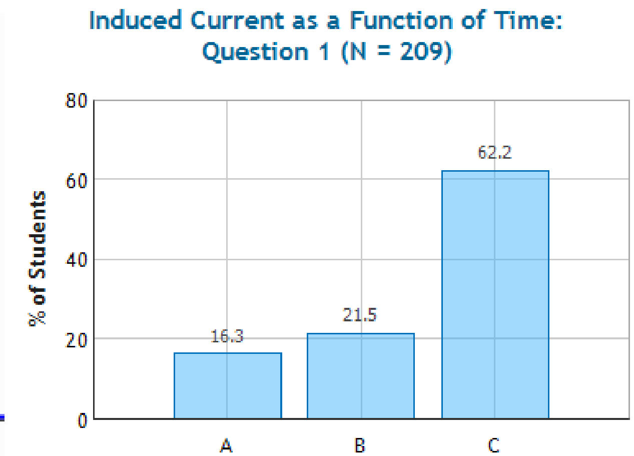
Current up through bar

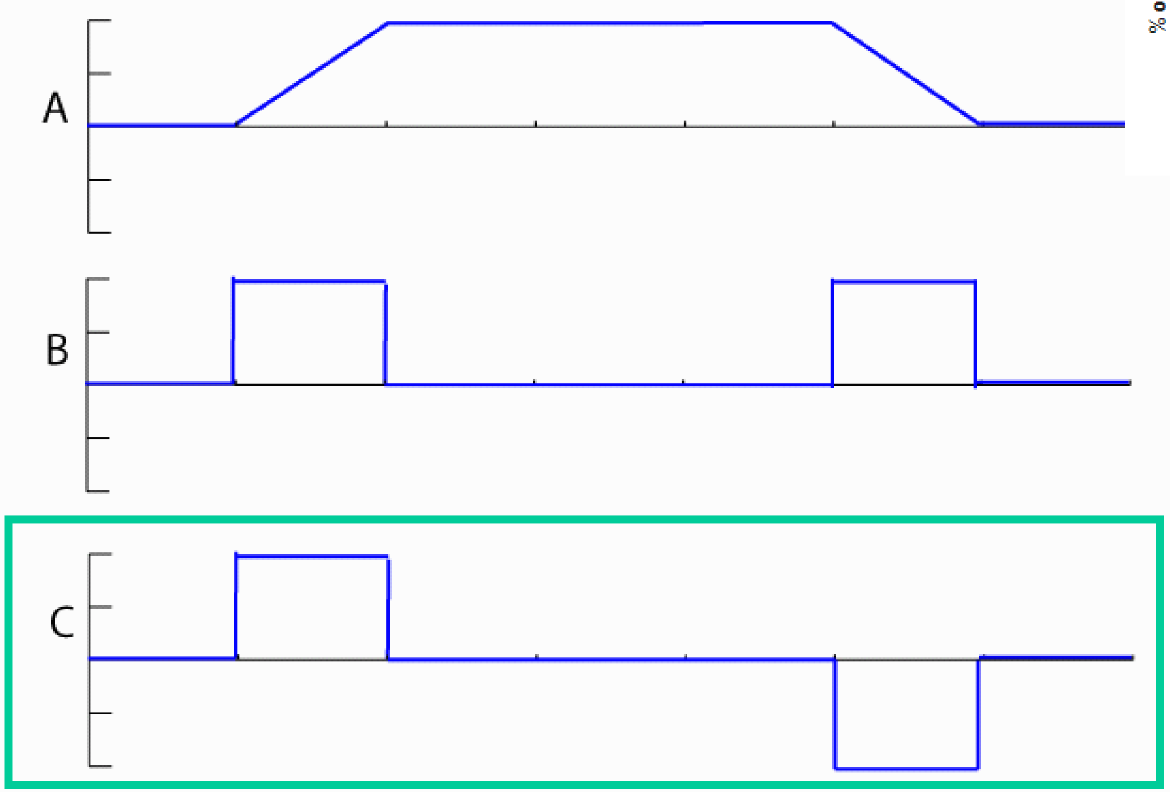
$$\vec{F} = I\vec{L} \times \vec{B} \longrightarrow F \text{ points to left}$$

$$F = \left(\frac{vBL}{R}\right)LB \longrightarrow P = Fv = \left(\frac{vBL}{R}\right)LBv = I^2R$$

Check Point 3

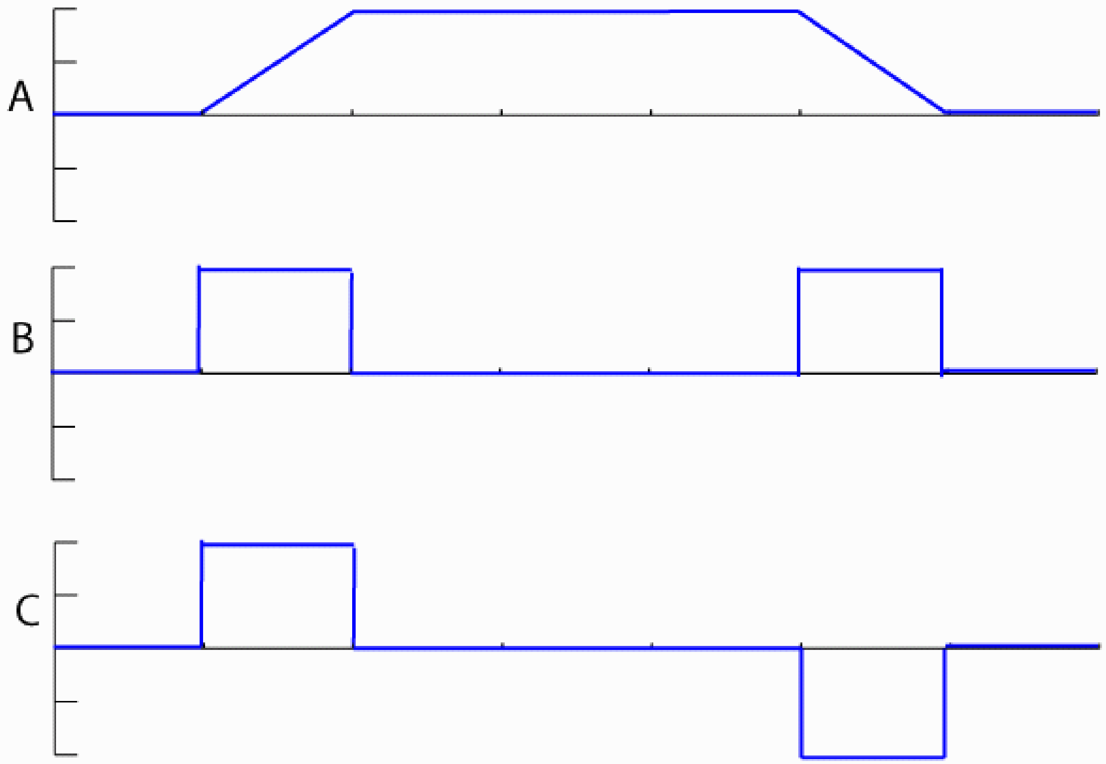
A wire loop travels to the right at a constant velocity. Which plot best represents the induced current in the loop as it travels from left of the region of magnetic field, through the magnetic field, and then entirely out on the right side?



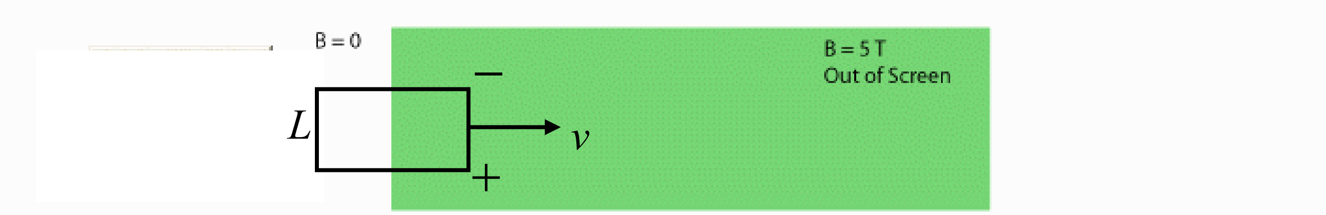


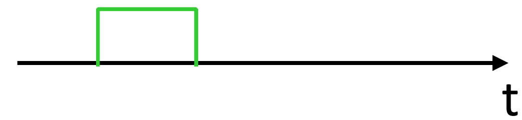
Before Check Point 3

A wire loop travels to the right at a constant velocity. Which plot best represents the induced current in the loop as it travels from left of the region of magnetic field, through the magnetic field, and then entirely out on the right side?



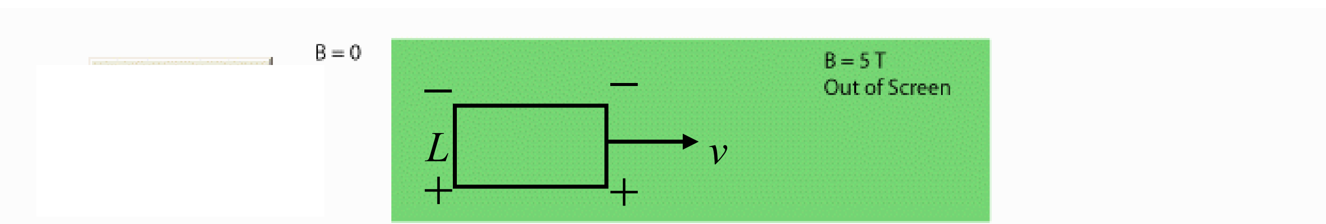
Let's step through this one





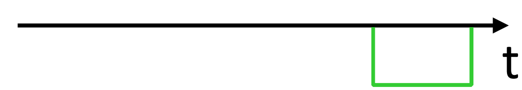
Only leading side has charge separation

$emf = BLv$ (cw current)



Leading and trailing sides have charge separation

$emf = BLv - BLv = 0$ (no current)

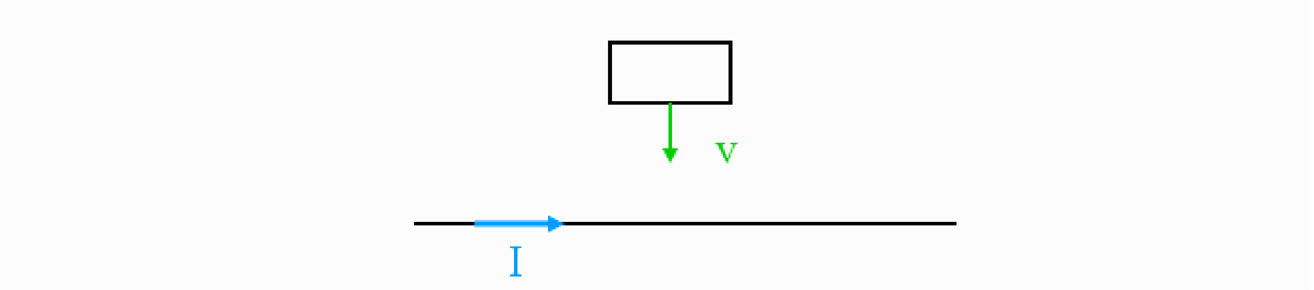


Only trailing side has charge separation

$emf = BLv$ (ccw current)

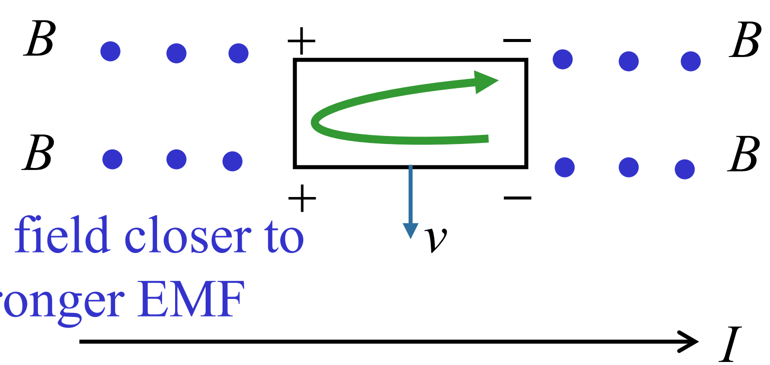
Changing B Field

A conducting rectangular loop moves with velocity *v* toward an infinite straight wire carrying current as shown.



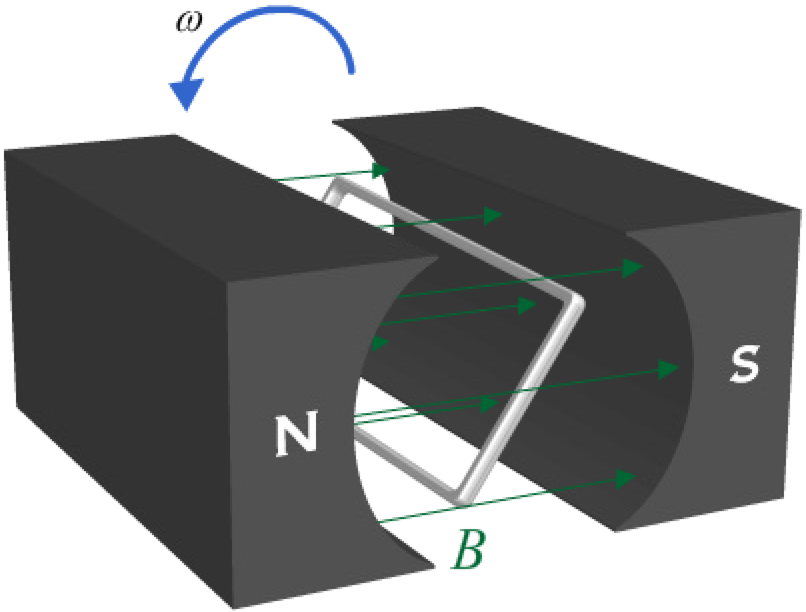
What is the direction of the induced current in the loop?

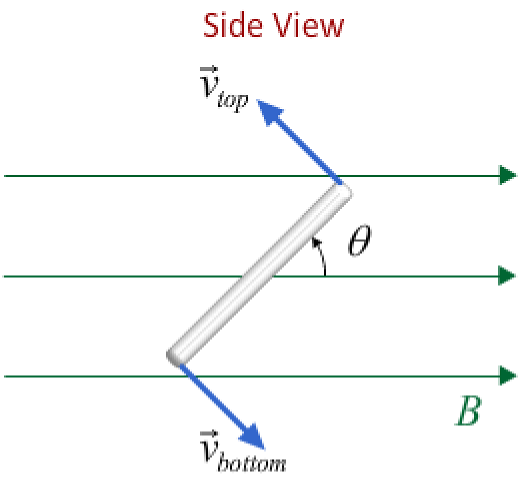
A. clockwise
B. counter-clockwise
C. there is no induced current in the loop



Stronger B field closer to wire, so stronger EMF

Generator: Changing Orientation





On which legs of the loop is charge separated?

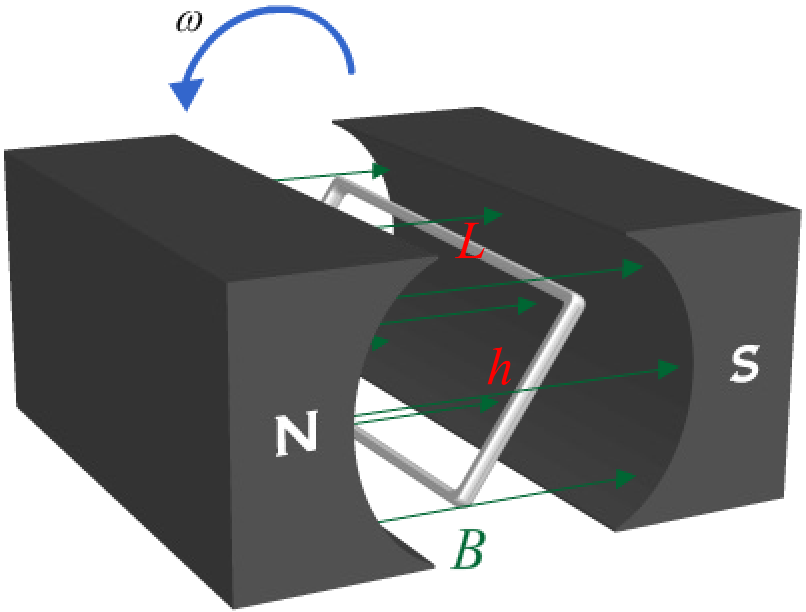
A) Top and Bottom legs only
B) Front and Back legs only
C) All legs
D) None of the legs

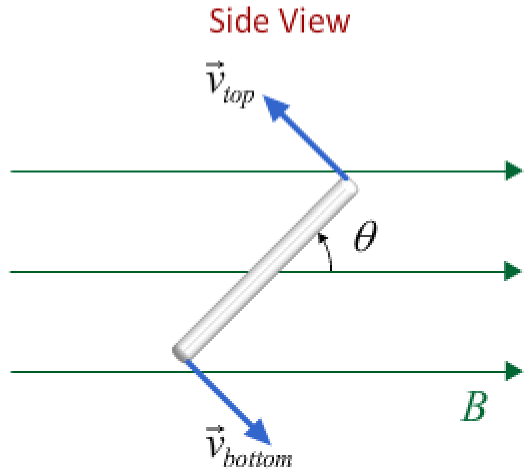
$$\vec{v} \times \vec{B}$$

Parallel to top and bottom legs

Perpendicular to front and back legs

Generator: Changing Orientation





At what angle θ is emf the largest?

A) $\boxed{\theta = 0}$
B) $\theta = 45^o$
C) $\theta = 90^o$
D) emf is same at all angles

$$\vec{v} \times \vec{B}$$

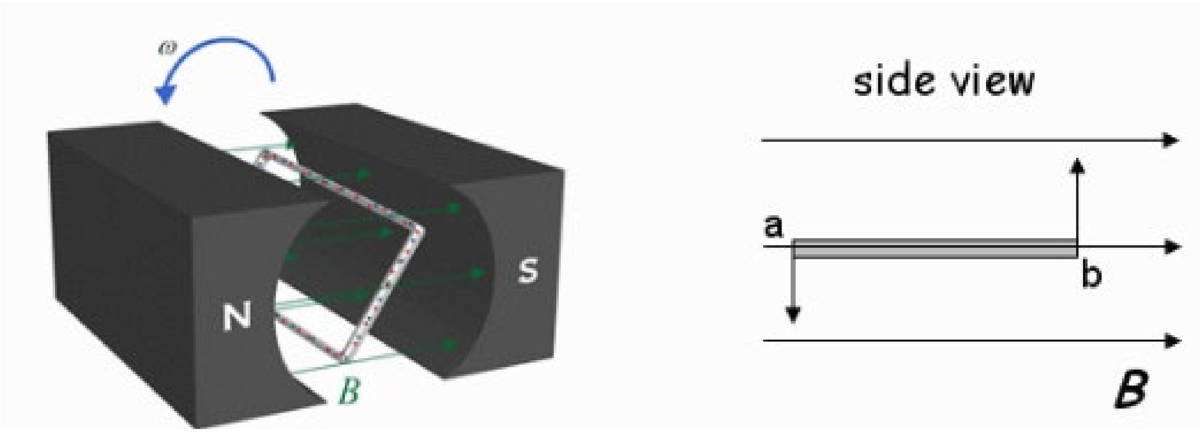
Largest for $\theta = 0$ (v perp to B)

$$\varepsilon = 2EL$$

Bonus Point 1

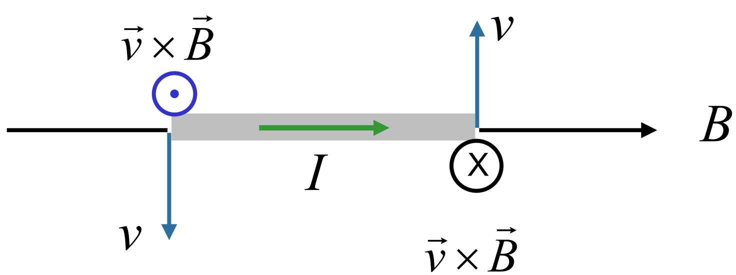
Changing Orientation

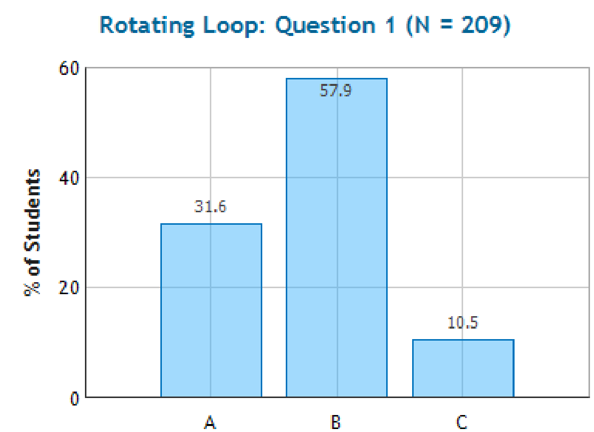
A rectangular loop rotates in a region containing a constant magnetic field as shown.



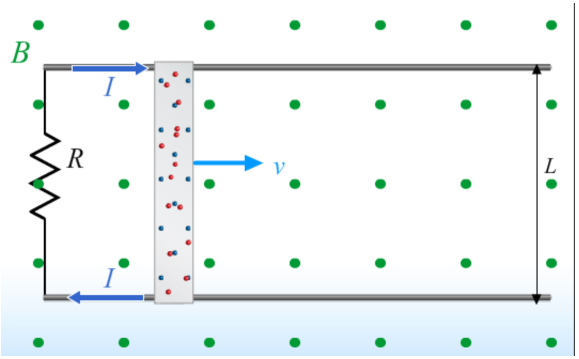
The side view of the loop is shown at a particular time during the rotation. At this time, what is the direction of the induced (positive) current in segment *ab*?

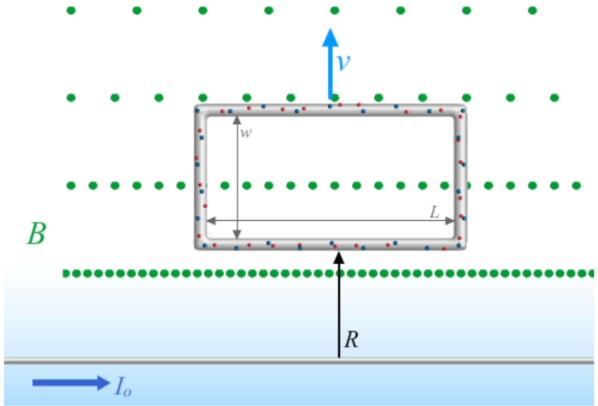
A. from *b* to *a*
B. from *a* to *b*
C. there is no induced current in the loop at this time

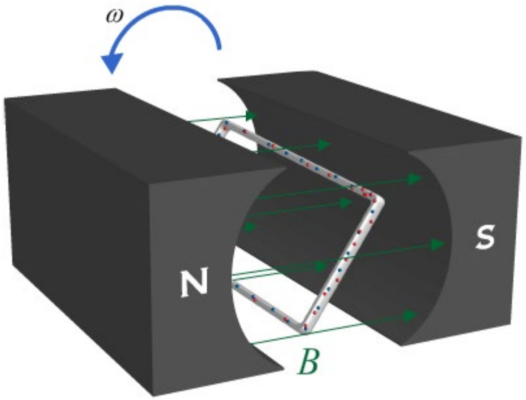




Putting it Together







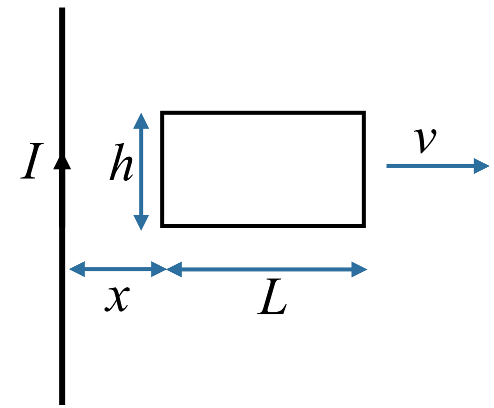
Faraday's Law

$$\Phi \equiv \int \vec{B} \cdot d\vec{A} \qquad \varepsilon = -\frac{d\Phi}{dt}$$

We will study this law in detail next time !

Example Problem

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps. What is the induced current in the loop when it is a distance $x = 0.7\ m$ from the wire?



Conceptual Analysis:

Long straight current creates magnetic field in region of the loop.

Vertical sides develop *emf* due to motion through B field

Net *emf* produces current

Strategic Analysis:

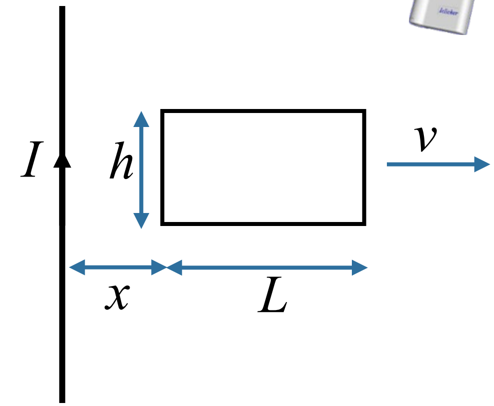
Calculate B field due to wire.

Calculate motional *emf* for each segment

Use net *emf* and Ohm’s law to get current

Example Problem

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps. What is the induced current in the loop when it is a distance $x = 0.7\ m$ from the wire?



What is the direction of the B field produced by the wire in the region of the loop?

A) Into the page

B) Out of the page

C) Left

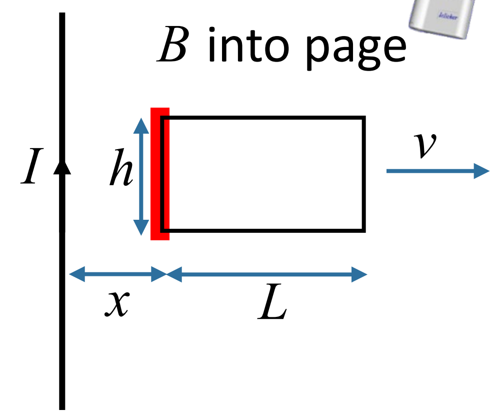
D) Right

E) Up

Bonus Point 2

Example Problem

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps. What is the induced current in the loop when it is a distance $x = 0.7\ m$ from the wire?



What is the *emf* induced on the left segment?

A) Top is positive

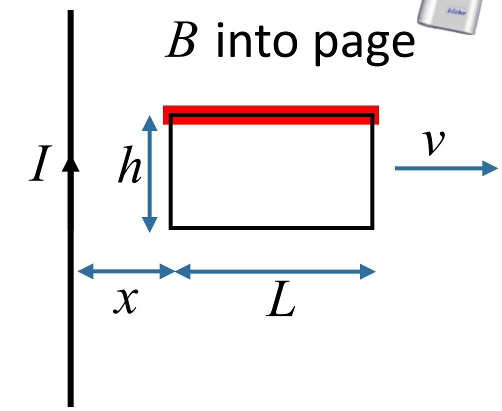
B) Top is negative

C) Zero

$$\vec{v} \times \vec{B}$$

Example Problem

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps. What is the induced current in the loop when it is a distance $x = 0.7\ m$ from the wire?



What is the *emf* induced on the top segment?

A) left is positive

B) left is negative

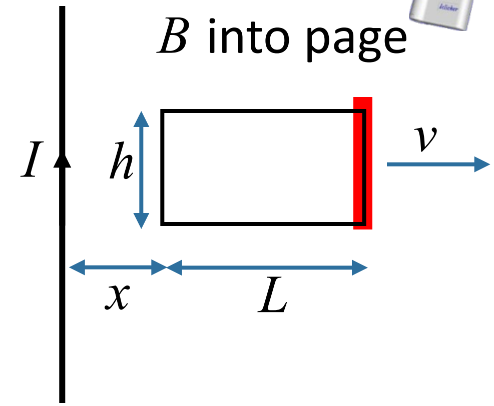
C) Zero

$$\vec{v} \times \vec{B}$$

perpendicular to wire

Example Problem

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps. What is the induced current in the loop when it is a distance $x = 0.7\ m$ from the wire?



What is the emf induced on the right segment?

A) top is positive

B) top is negative

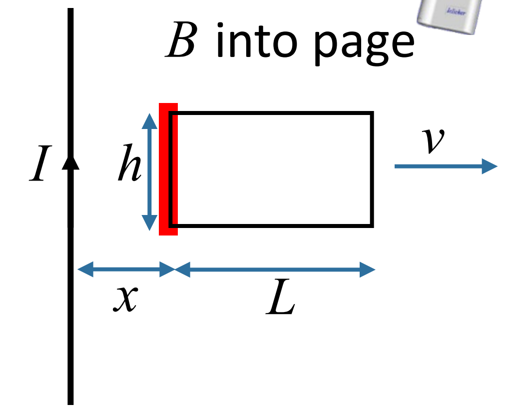
C) Zero

$$\vec{v} \times \vec{B}$$

Bonus Point 3

Example Problem

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps. What is the induced current in the loop when it is a distance $x = 0.7\ m$ from the wire?



Which expression represents the *emf* induced in the left wire?

A) $\varepsilon_{left} = \dfrac{\mu_o I}{2\pi x} L v$

B) $\varepsilon_{left} = \dfrac{\mu_o I}{2\pi x} h v$

C) $\varepsilon_{left} = \dfrac{\mu_o I}{2\pi (L + x)} L v$

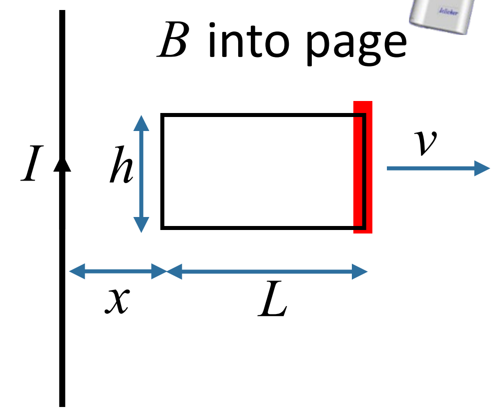
$$qvB = qE \rightarrow E = vB \rightarrow \varepsilon = Eh = vBh$$

$$B = \frac{\mu_o I}{2\pi x} \rightarrow \varepsilon = \frac{\mu_o I}{2\pi x} h v$$

Bonus Point 4

Example Problem

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps. What is the induced current in the loop when it is a distance $x = 0.7\ m$ from the wire?



Which expression represents the *emf* induced in the right wire?

A) $\varepsilon_{right} = \dfrac{\mu_o I}{2\pi(L+x)} hv$

B) $\varepsilon_{right} = \dfrac{\mu_o I}{2\pi x} hv$

C) $\varepsilon_{right} = \dfrac{\mu_o I}{2\pi(h+x)} Lv$

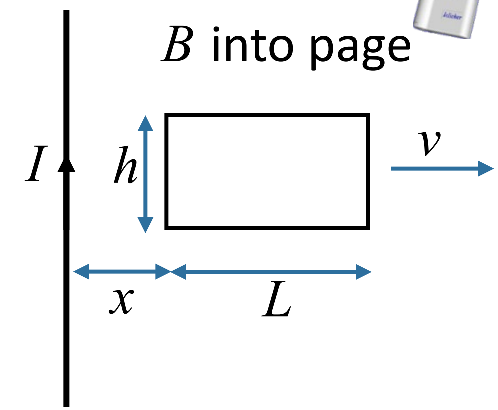
$$qvB = qE \rightarrow E = vB \rightarrow \varepsilon = Eh = vBh$$

$$B = \frac{\mu_o I}{2\pi(L+x)} \rightarrow \varepsilon = \frac{\mu_o I}{2\pi(L+x)} hv$$

Bonus Point 5

Example Problem

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps. What is the induced current in the loop when it is a distance $x = 0.7\ m$ from the wire?



Which expression represents the total *emf* in the loop?

A) $\varepsilon_{loop} = \dfrac{\mu_o I}{2\pi x} hv + \dfrac{\mu_o I}{2\pi (L+x)} hv$

B) $\varepsilon_{loop} = \dfrac{\mu_o I}{2\pi x} hv - \dfrac{\mu_o I}{2\pi (L+x)} hv$

C) $\varepsilon_{loop} = 0$

$$I_{loop} = \frac{\varepsilon_{loop}}{R}$$

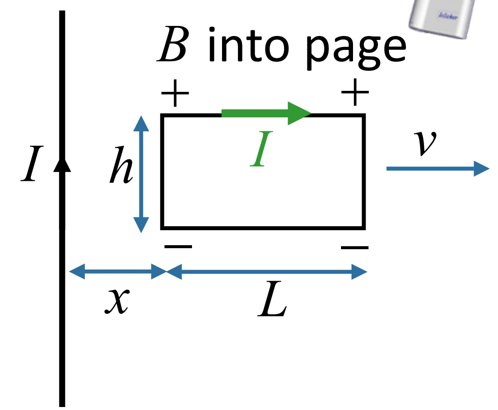
$$I_{loop} = \frac{\mu_o I}{2\pi R} hv \left(\frac{1}{x} - \frac{1}{L+x} \right)$$

Bonus Point 6

Have a great weekend!

Follow-Up

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps.



What is the direction of the induced current?

A) Clockwise

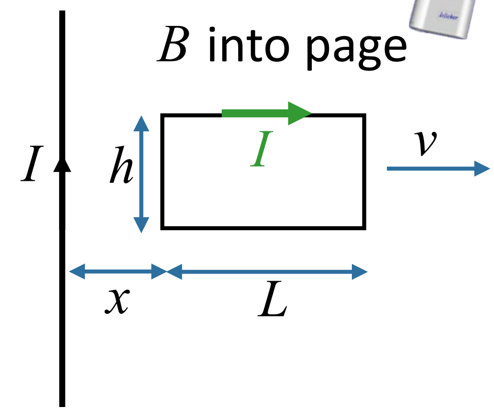
B) Counterclockwise

$$\varepsilon_{left} > \varepsilon_{right}$$

Clockwise current

Follow-Up

A rectangular loop $(h = 0.3m\ L = 1.2\ m)$ with total resistance of 5Ω is moving away from a long straight wire carrying total current 8 amps.



What is the direction of the force exerted by the magnetic field on the loop?

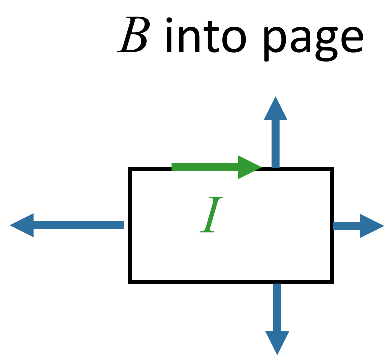
A) UP

B) DOWN

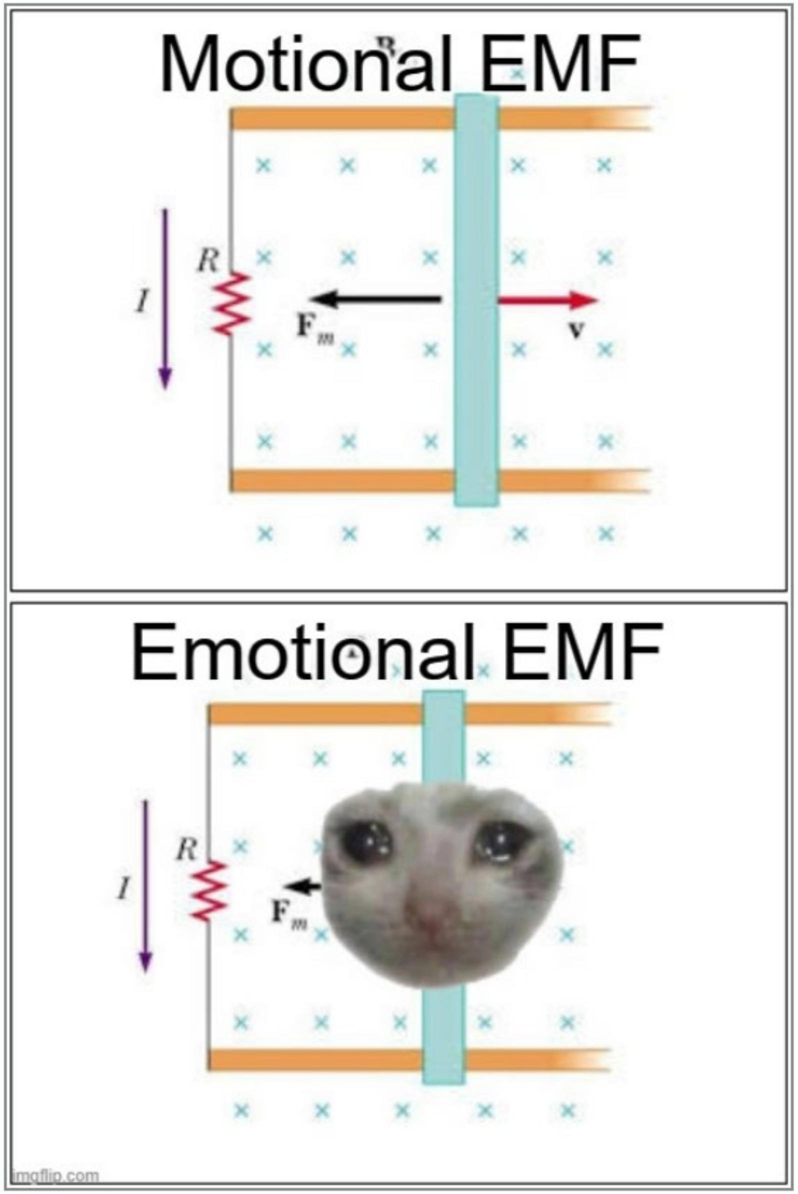
C) LEFT

D) RIGHT

E) $F = 0$



Total force from B
Points to the left !



Gabriel Grais

Exam Details

1) Exam 2 – units 9 to 16 (today's lecture **not** on exam)

   - 11/11/2024 (today) – 6pm - 7:30 pm
   - location can be found in BB

2) EXAM 2 PREPARATION
   - Old Exams are a good way to assess what you need to know
   - Old Exams and solutions in SmartPhysics

Physics 212
Lecture 17

Today's Concept:

Faraday's Law

$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

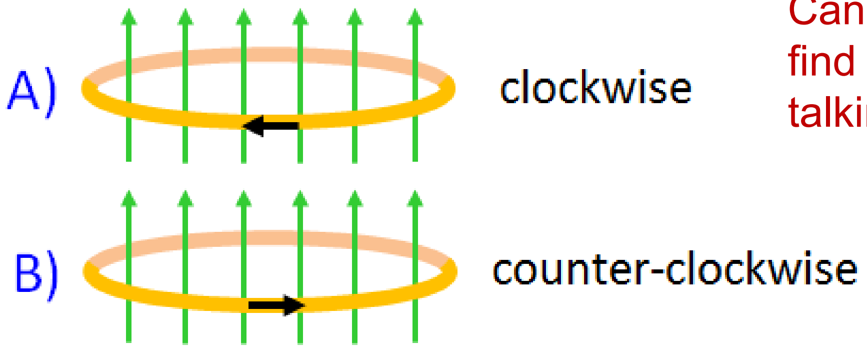
Looks scary but it's not – its amazing and beautiful!

A changing magnetic flux produces an electric field.

Electricity and magnetism are deeply connected.

Check Point 1

Suppose a current flows in a horizontal conducting loop in such a way that the magnetic flux produced by this current points upward.



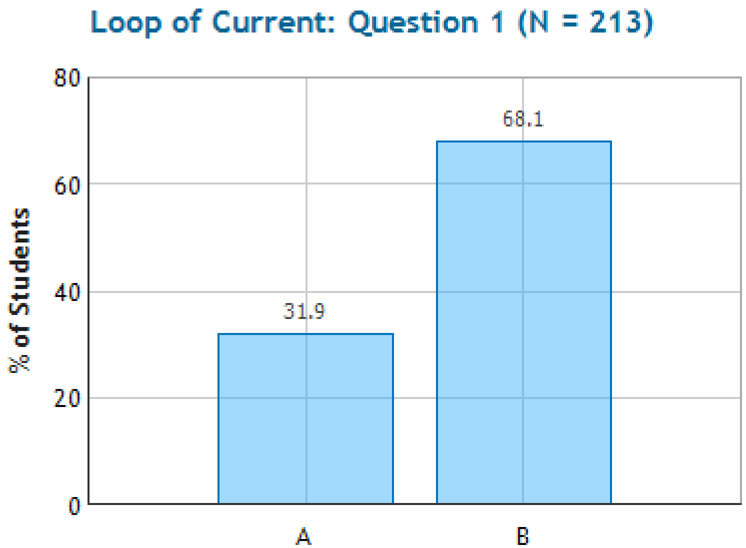
Can we go over again how to find the current direction when talking about a loop?

As viewed from above, in which direction is this current flowing?

A. clockwise

B. counterclockwise

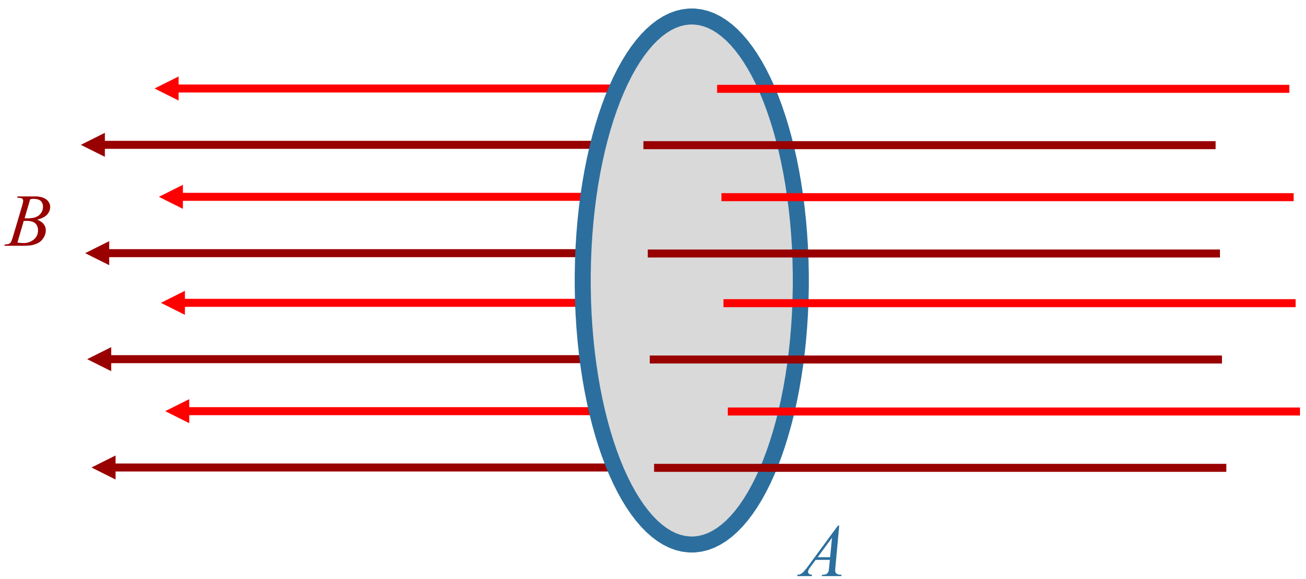
wrap your fingers in the direction of the current and your thumb points up..



Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

In Practical Words:

1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.



Flux

Think of Φ_B as the number of field lines passing through the surface

There are many ways to change this…

Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

In Practical Words:

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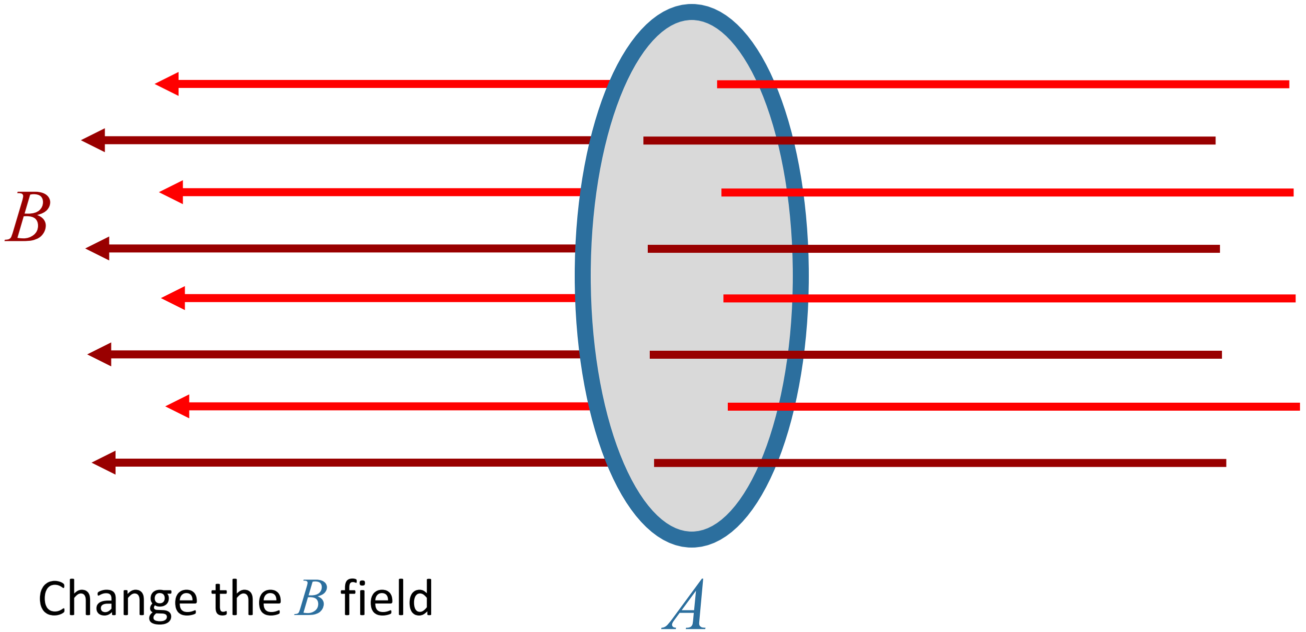
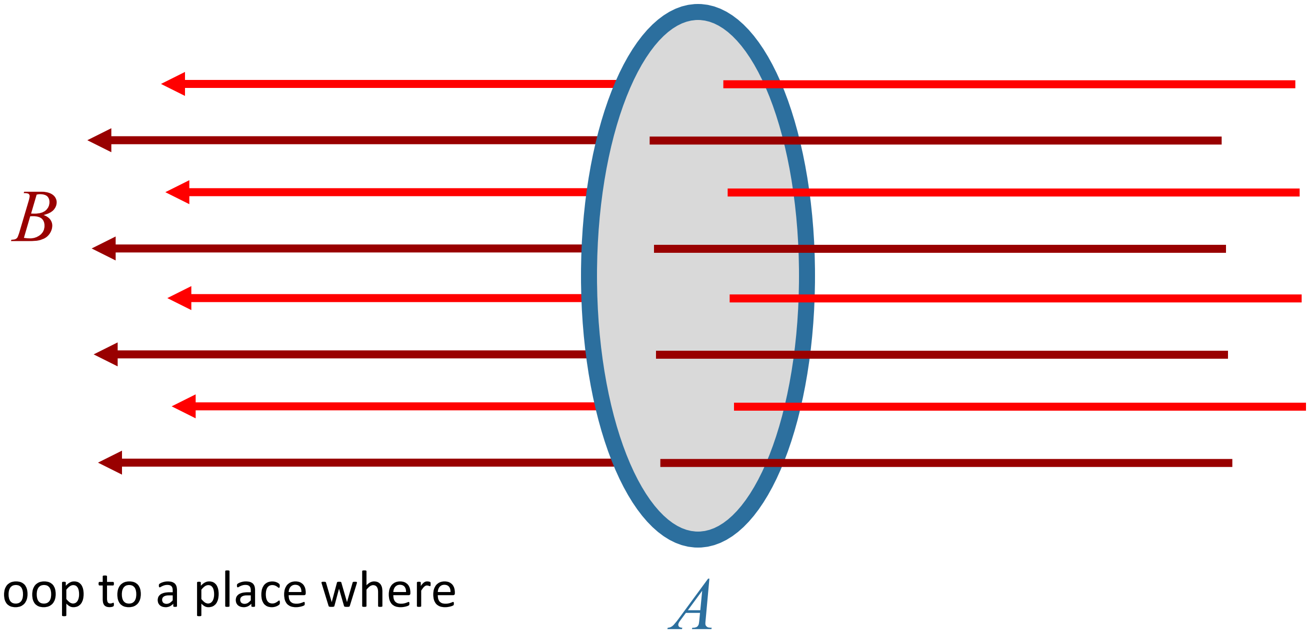


Change the B field

Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

In Practical Words:

1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.

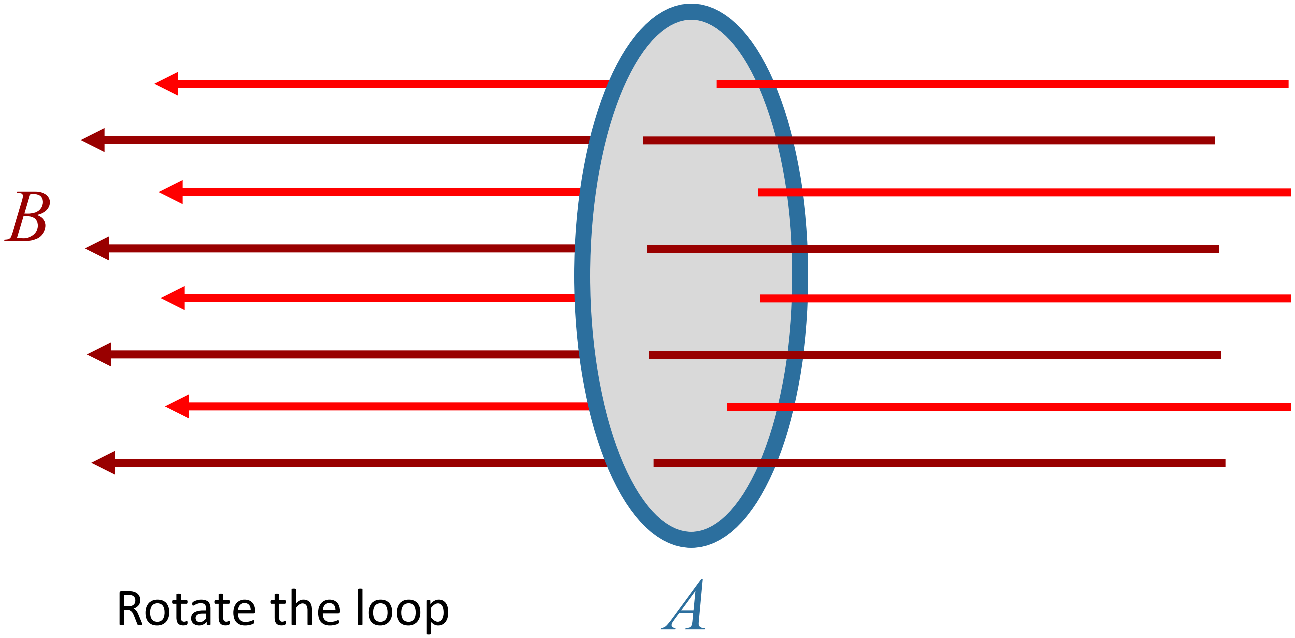


Move loop to a place where the B field is different

Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

In Practical Words:

1) When the flux Φ_B through a loop changes, an emf is induced in the loop.

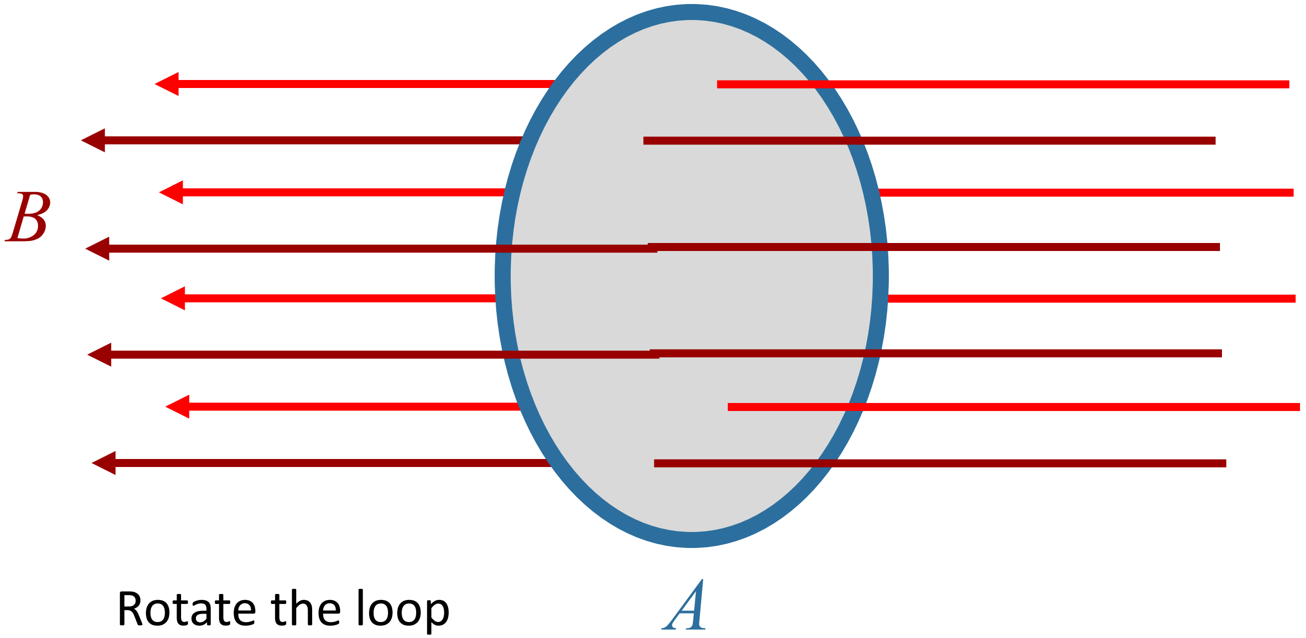


Rotate the loop

Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

In Practical Words:

1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.

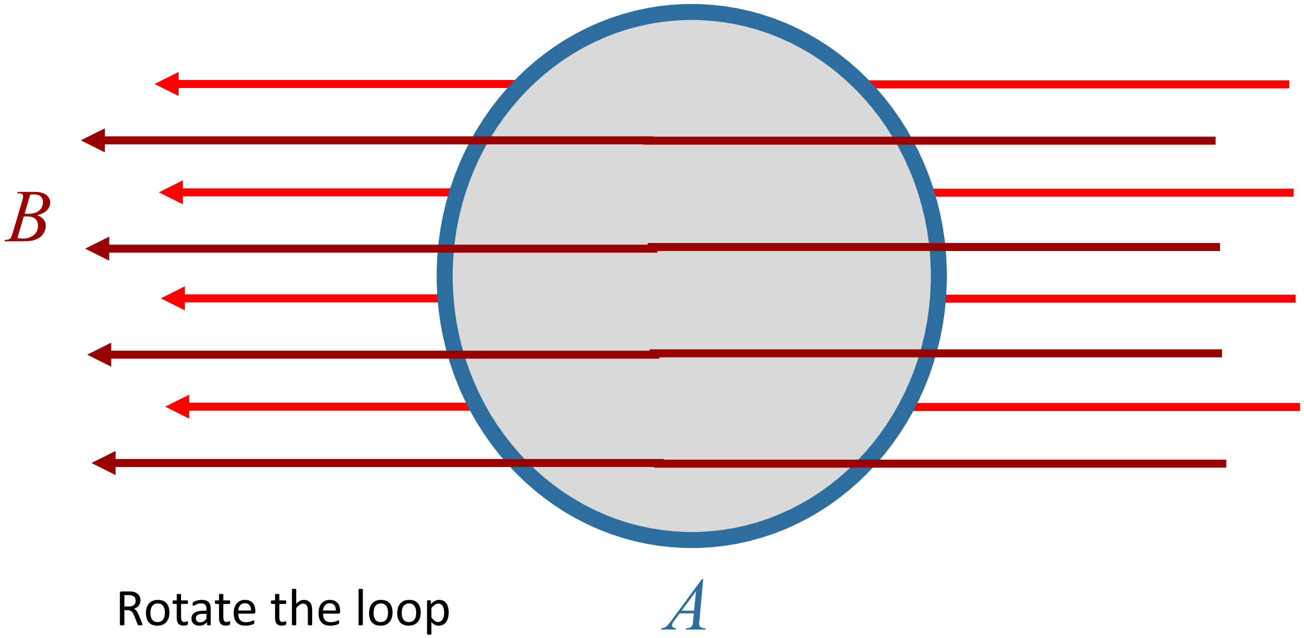


Rotate the loop

Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\dfrac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

In Practical Words:

1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.



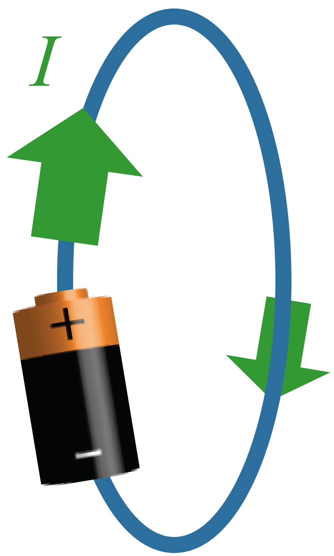
Rotate the loop

Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

In Practical Words:

1) When the flux Φ_B through a loop changes, an emf is induced in the loop.

2) The emf will make a current flow if it can (like a battery).



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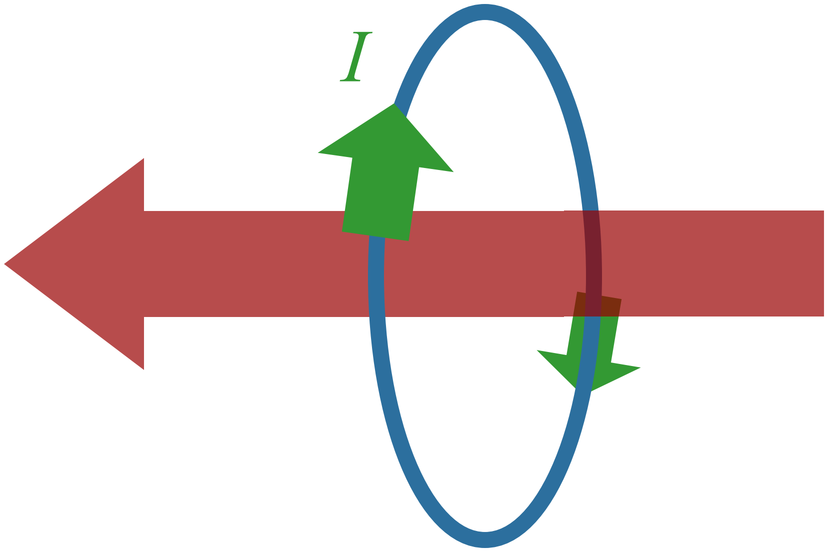
In Practical Words:

1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.
2) The *emf* will make a current flow if it can (like a battery).
3) The current that flows induces a new magnetic field.



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1) When the flux Φ_B through a loop changes, an emf is induced in the loop.

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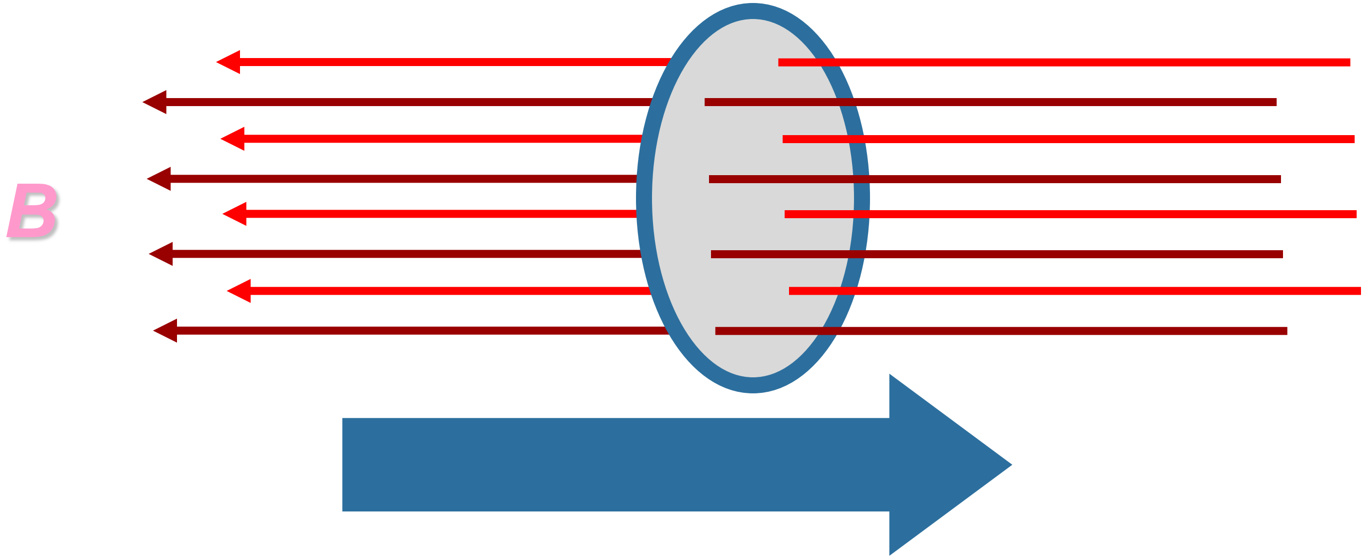
4) The new magnetic field opposes the change in the original magnetic field that created it. (Lenz' Law)



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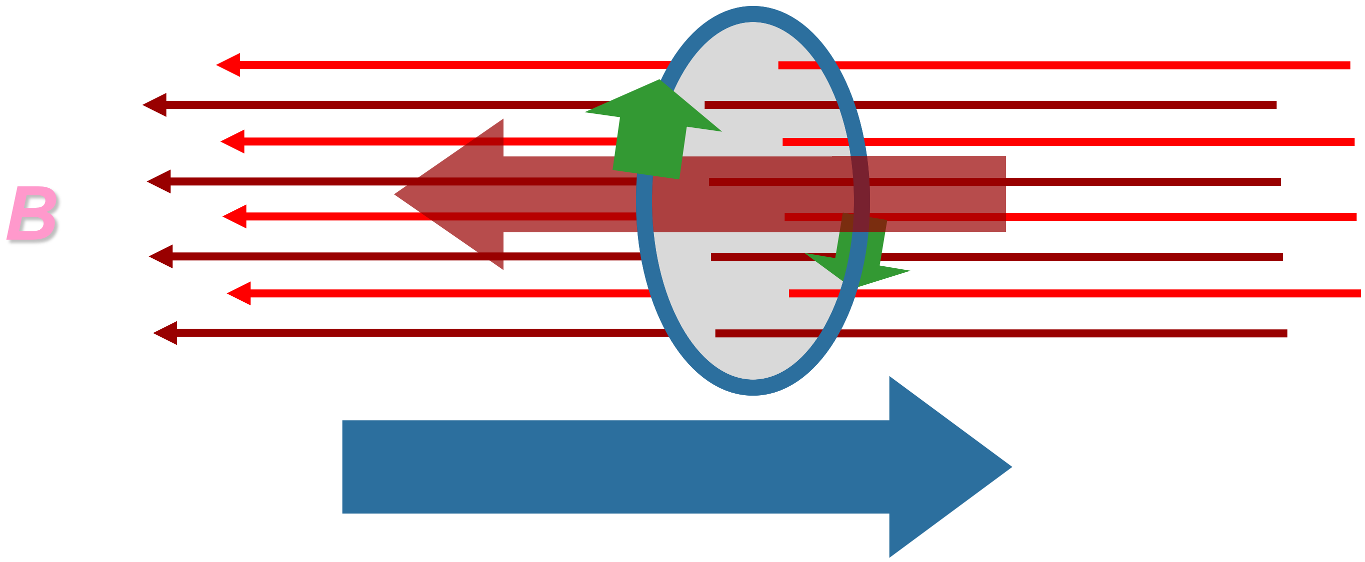


Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

Summary:

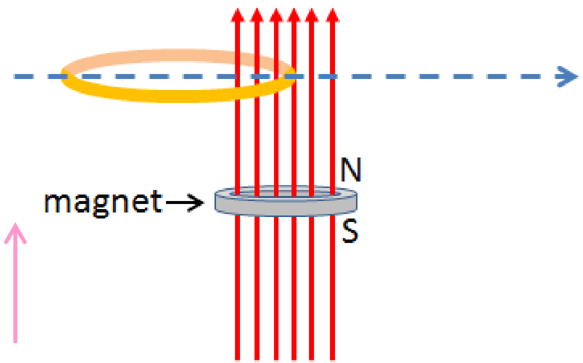
$emf \longrightarrow$ current $\longrightarrow$ field a) induced only when flux is changing

b) opposes the change

"Nature abhors a change in magnetic flux"

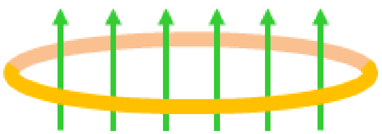
Check Point 2

A magnet makes the vertical magnetic field shown by the red arrows. A horizontal conducting loop is entering the field as shown.

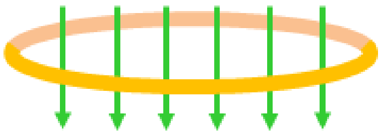


Direction of positive flux

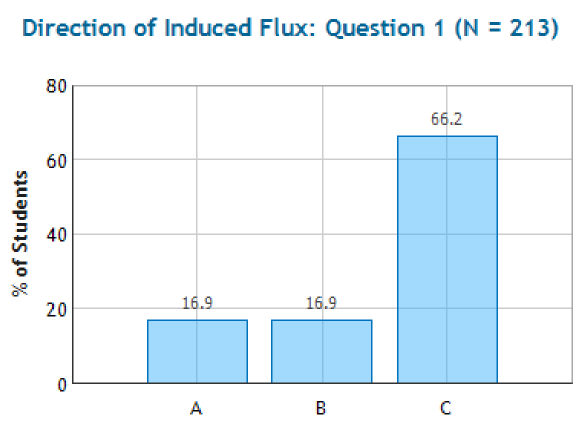
At the instant shown above, what is the sign of the additional flux produced by the current induced in the loop?

A) 
Positive

B) 
Zero

C) 
Negative

“Oppose the direction of increasing B field from magnet.”



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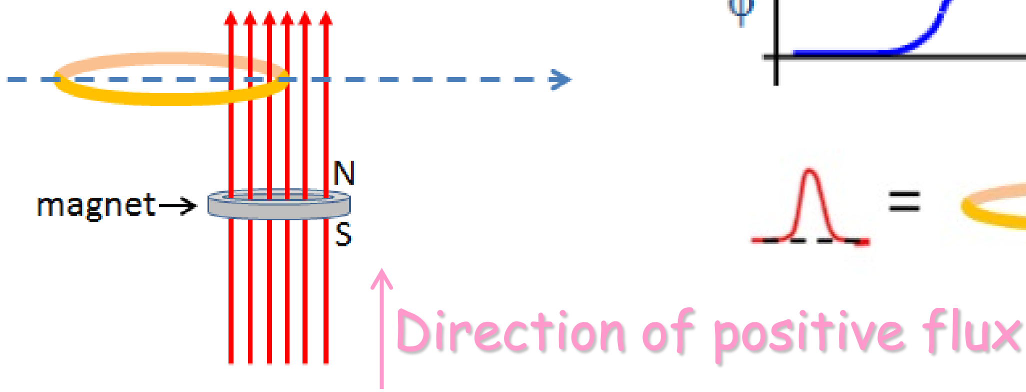
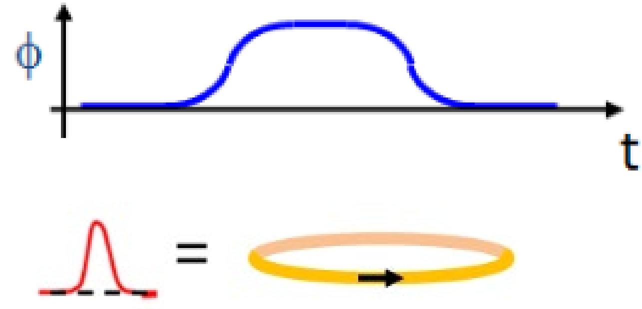
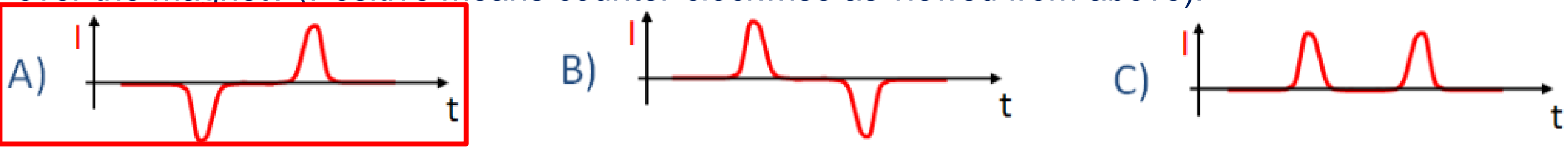




The upward flux through the loop as a function of time is shown by the blue trace. Which of the red traces below it best represents the current induced in the loop as a function of time as it passes over the magnet? (Positive means counter-clockwise as viewed from above):

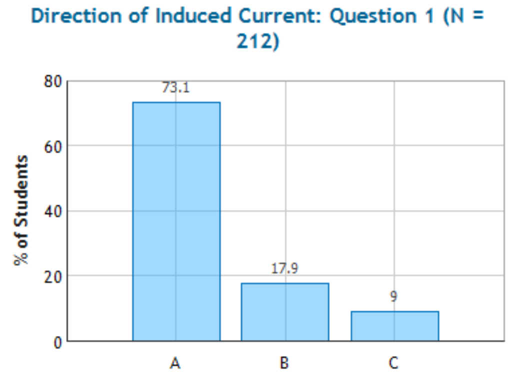


Flux is changing!

Induced flux is initially negative (opposing increasing positive flux – last checkpoint)

THEREFORE, initial *induced current* must be CW as viewed from above

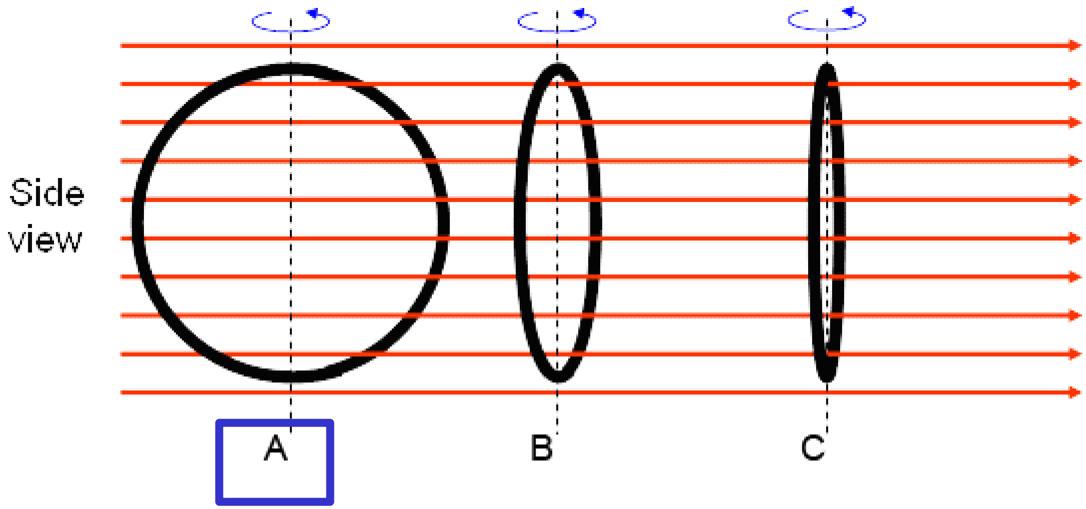
- Current direction from right-hand rule ☺



Prelecture: Faraday's Law

A circular wire loop is placed in a uniform magnetic field pointing to the right. The loop is rotated with *constant angular velocity* around a vertical axis (dashed line).

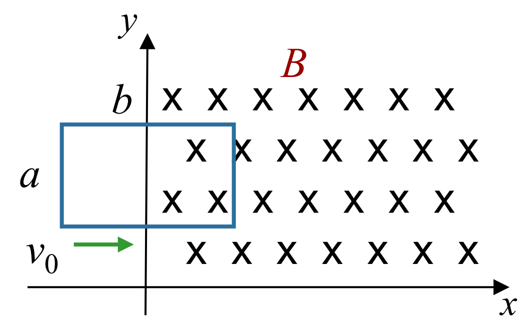
At which of the three times shown is the induced *emf* greatest?



emf = $-d\Phi/dt$ → largest where slope of Φ vs t largest

Calculation

A rectangular loop (height $= a$, length $= b$, resistance $= R$, mass $= m$) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the direction and the magnitude of the force on the loop when half of it is in the field?

Conceptual Analysis

Once loop enters B field region, flux will be changing in time

Faraday's Law then says emf will be induced

Strategic Analysis

Find the emf

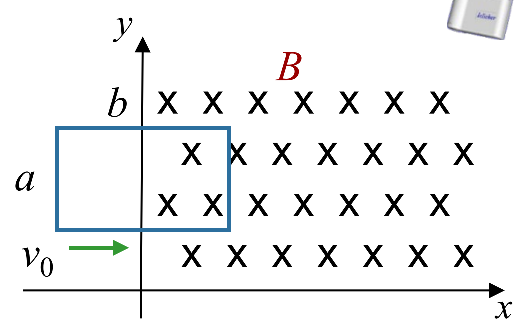
Find the current in the loop

Find the force on the current

Bonus Point 1

Calculation

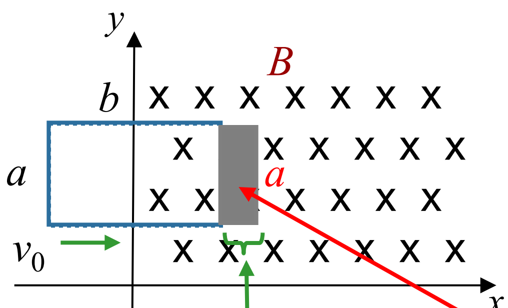
A rectangular loop (height $= a$, length $= b$, resistance $= R$, mass $= m$) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the magnitude of the emf induced in the loop just after it enters the field?

$$emf = -\frac{d\Phi_B}{dt}$$

A) $\varepsilon = Babv_0^2$ B) $\varepsilon = ½\, Bav_0$ C) $\varepsilon = ½\, Bbv_0$ $\boxed{\text{D) } \varepsilon = Bav_0}$ E) $\varepsilon = Bbv_0$



In a time dt
it moves by v_0dt

The area in field
changes by $dA = v_0dt\ a$

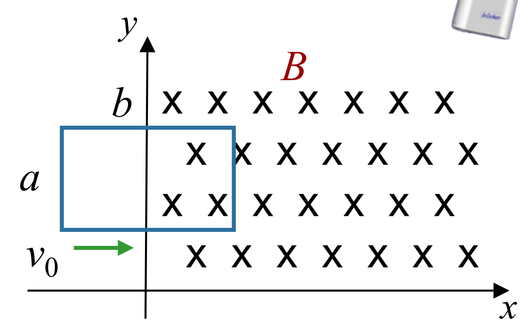
Change in Flux $= d\Phi_B = BdA = Bav_0dt$

$$\frac{d\Phi_B}{dt} = Bav_o$$

Bonus Point 2

Calculation

A rectangular loop (height $= a$, length $= b$, resistance $= R$, mass $= m$) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the direction of the current induced in the loop just after it enters the field?

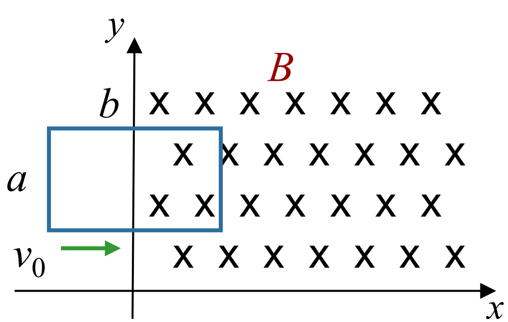
$$emf = -\frac{d\Phi_B}{dt}$$

A) clockwise

B) counterclockwise

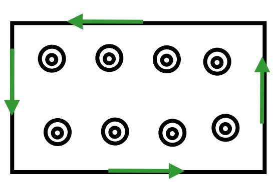
C) no current is induced

emf is induced in direction to oppose the change in flux that produced it



Flux is increasing into the screen

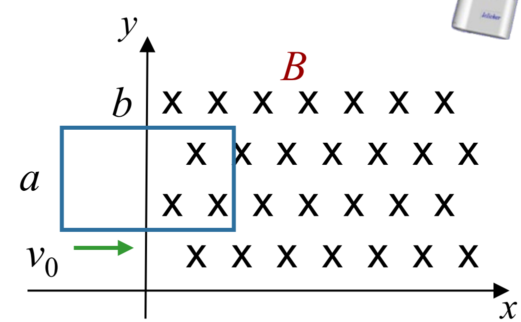
Induced emf produces flux out of screen



Bonus Point 3

Calculation

A rectangular loop (height $= a$, length $= b$, resistance $= R$, mass $= m$) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.

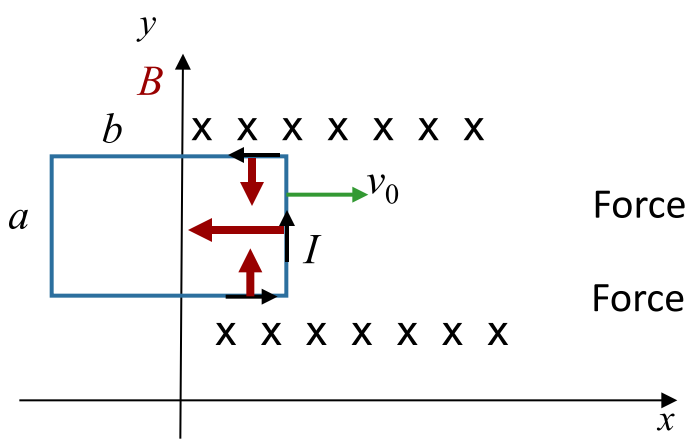


What is the direction of the net force on the loop just after it enters the field?

$$emf = -\frac{d\Phi_B}{dt}$$

A) $+y$ B) $-y$ C) $+x$ $\boxed{\text{D) } -x}$

Force on a current in a magnetic field: $\vec{F} = I\vec{L} \times \vec{B}$



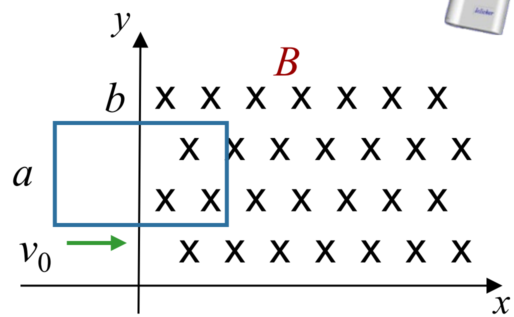
Force on top and bottom segments cancel (red arrows)

Force on right segment is directed in $-x$ direction.

Bonus Point 4

Calculation

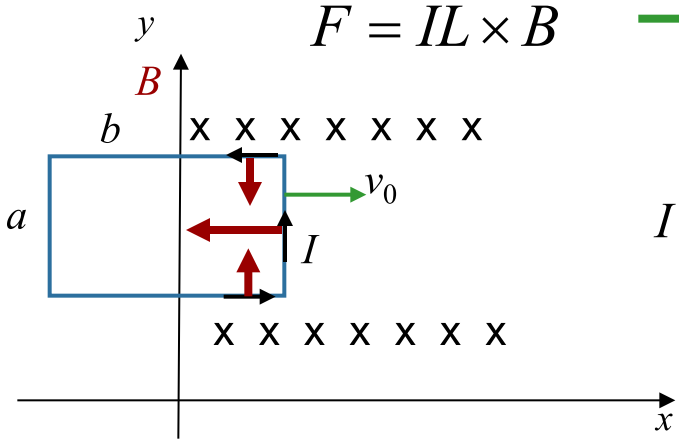
A rectangular loop (height $= a$, length $= b$, resistance $= R$, mass $= m$) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the magnitude of the net force on the loop just after it enters the field?

$$\vec{F} = I\vec{L} \times \vec{B} \qquad \varepsilon = Bav_0 \qquad emf = -\frac{d\Phi_B}{dt}$$

A) $F = 4aBv_oR$ B) $F = a^2Bv_oR$ C) $F = a^2B^2v_o^2 / R$ D) $\boxed{F = a^2B^2v_o / R}$



$$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB \text{ since } \vec{L} \perp \vec{B}$$

$$I = \frac{\varepsilon}{R} = \frac{Bav_o}{R} \rightarrow F = \left(\frac{Bav_o}{R}\right)aB = \frac{B^2a^2v_o}{R}$$

ILB

Bonus Point 5

Have a great break !

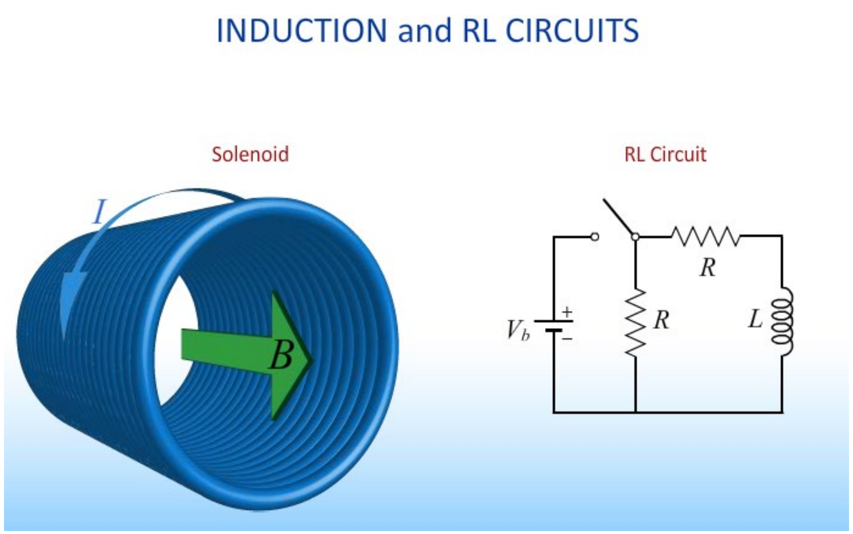
Good luck on Exam 2 !

Physics 212
Lecture 18

Today's Concepts:

A) Induction

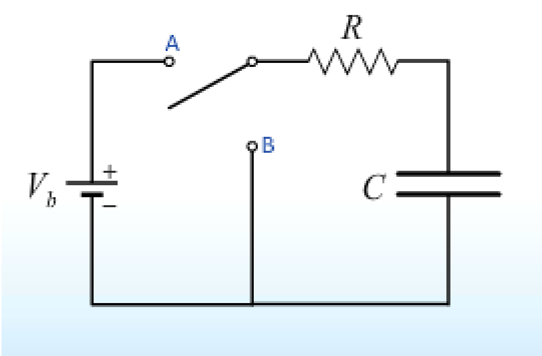
B) RL Circuits



RC Circuit Review

For an RC circuit is ALWAYS true "immediately after…"?

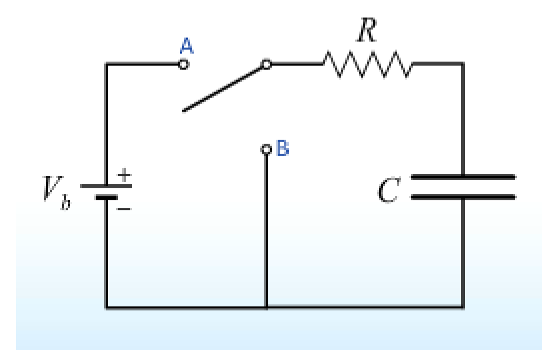
a) Charge on capacitor is same as immediately before
b) Current though capacitor is same as immediately before
c) Current through capacitor is zero
d) Both a and b
e) Both a and c



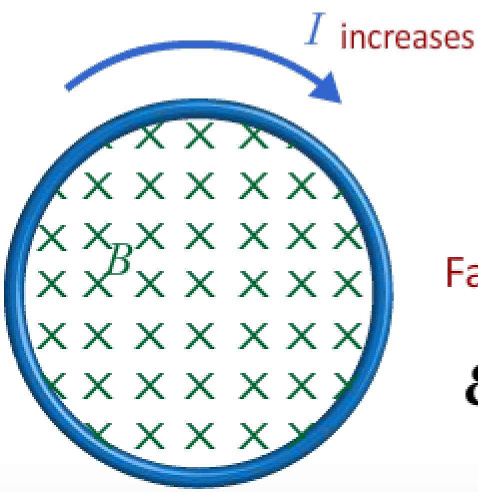
RC Circuit Review 2

For an RC charging circuit what is ALWAYS true "after a long time…"?

a)Voltage across capacitor is same as battery voltage
b)Voltage across capacitor is zero
c)Current through capacitor is zero
d)Both a and b
e)Both a and c



From the Prelecture: Self Inductance



Self-Inductance

$$\boxed{L \equiv \frac{\Phi_B}{I}}$$

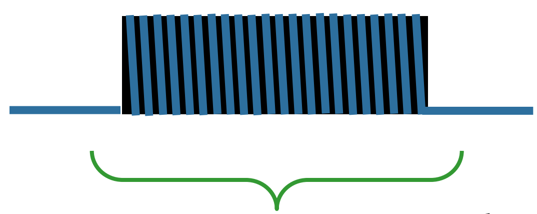
SI Unit

H = T-m²/A

Faraday's Law

$$\mathcal{E} = -\frac{d\boxed{\Phi_B}}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

Wrap a wire into a coil to make an "inductor"…



$$\mathcal{E} = -L\frac{dI}{dt}$$

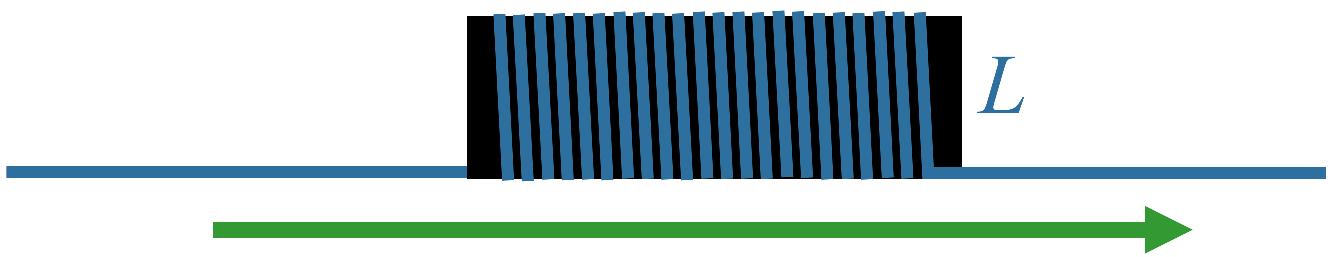
What this really means:

emf induced across L tries to keep I constant.

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

Short Term $I_{\text{before}} = I_{\text{after}}$

Long Term $V_L = 0$



current I

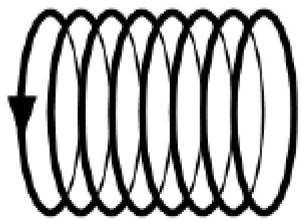
Inductors prevent discontinuous current changes!

It's like inertia!

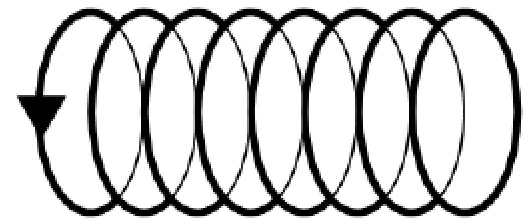
Check Point 1

Two solenoids are made with the same cross sectional area and total number of turns. Inductor B is twice as long as inductor A

$$L_B = \mu_0 \underbrace{n^2}_{(1/2)^2} \pi r^2 \underbrace{z}_{2}$$



A

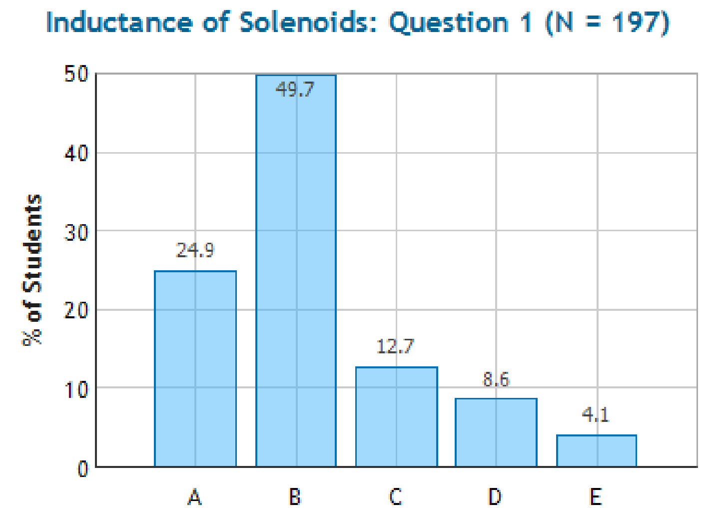


B

$$\longrightarrow \quad L_B = \tfrac{1}{2} L_A$$

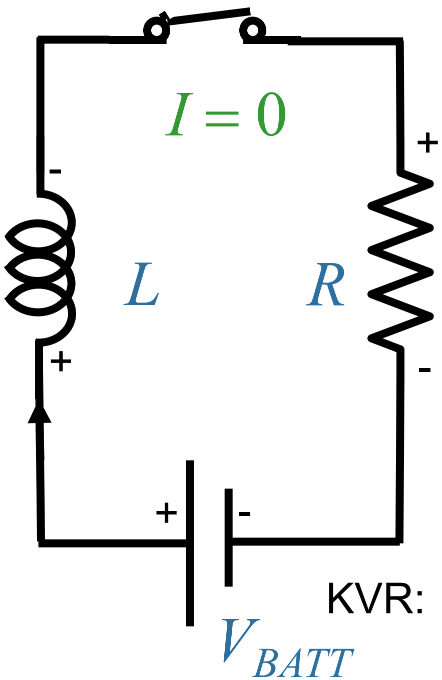
Compare the inductance of the two solenoids

A) $L_A = 4\, L_B$
B) $\boxed{L_A = 2\, L_B}$
C) $L_A = L_B$
D) $L_A = (1/2)\, L_B$
E) $L_A = (1/4)\, L_B$

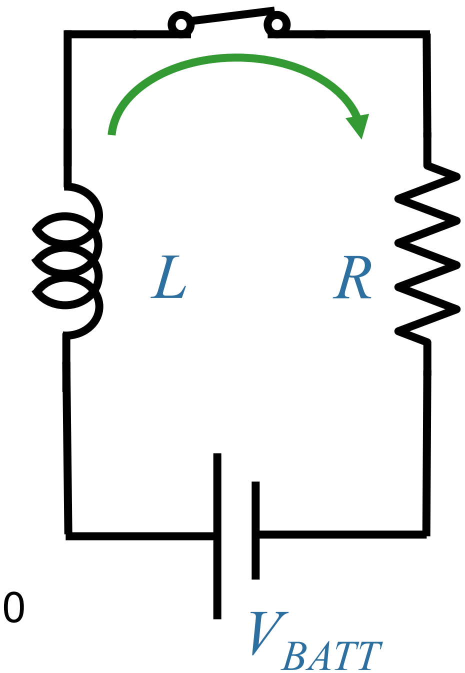


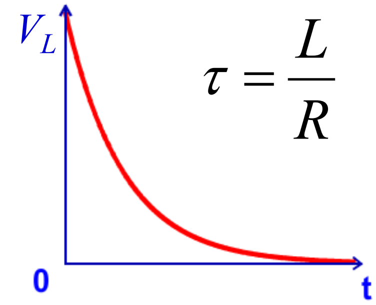
How to think about RL circuits Episode 1:

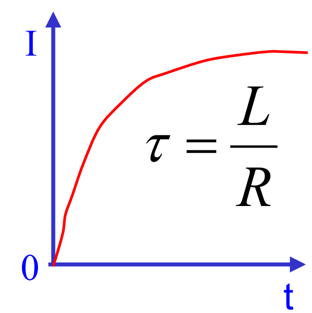
When no current is flowing initially:



KVR: $-V_{BATT} + V_L + IR = 0$







At $t = 0$: I_L unchanged

$I_L = 0$

Ohm's Law $V_R = IR = 0$

Using KVR $V_L = V_{BATT}$

At $t >> L/R$: $V_L = 0$

$V_L = 0$

Using KVR $V_R = V_{BATT}$

Ohm's Law $I = V_{BATT}/R$

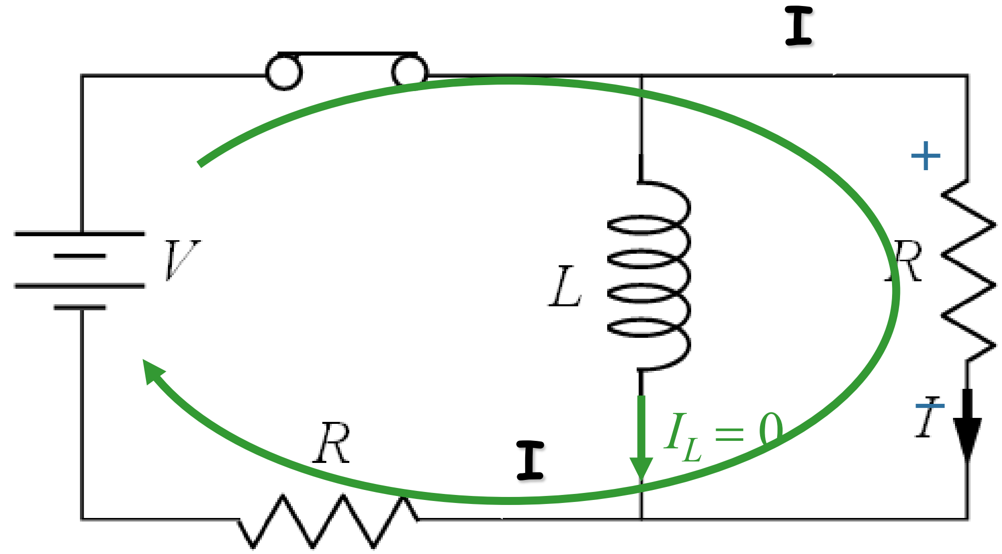
Check Point 2a

In the circuit, the switch has been open for a long time, and the current is zero everywhere.

At time $t = 0$ the switch is closed.

What is the current I through the vertical resistor immediately after the switch is closed?

(+ is in the direction of the arrow)



$$KVR:\ \ -V + IR + IR = 0$$

A) $I = V/R$

B) $\boxed{I = V/2R}$

C) $I = 0$

D) $I = -V/2R$

E) $I = -V/R$

Before: $I_L = 0$

After: $I_L = 0$

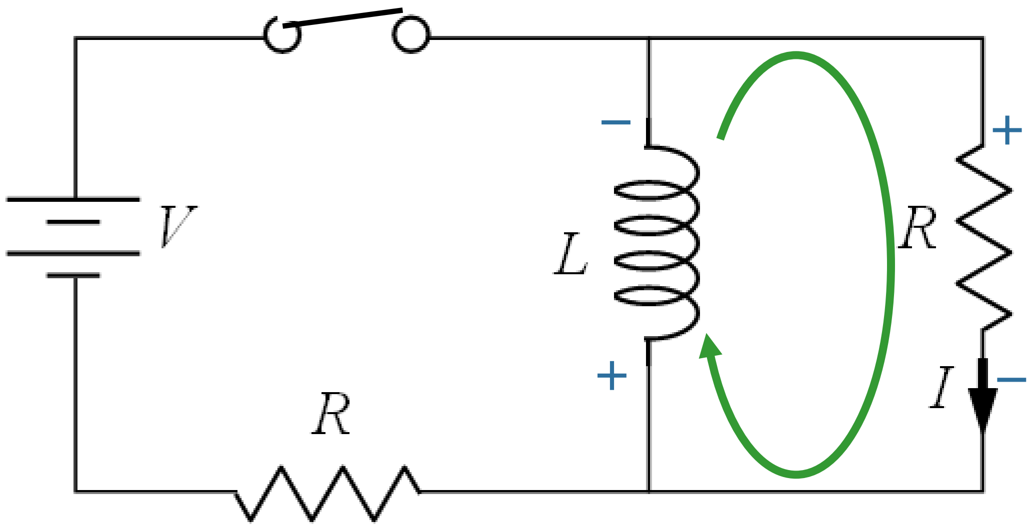
$\rightarrow$ $I = +\,V/2R$

RL Circuit (Long Time)

What is the current I through the vertical resistor after the switch has been closed for a long time?

(+ is in the direction of the arrow)

A) $I = V/R$
B) $I = V/2R$
C) $\boxed{I = 0}$
D) $I = -V/2R$
E) $I = -V/R$

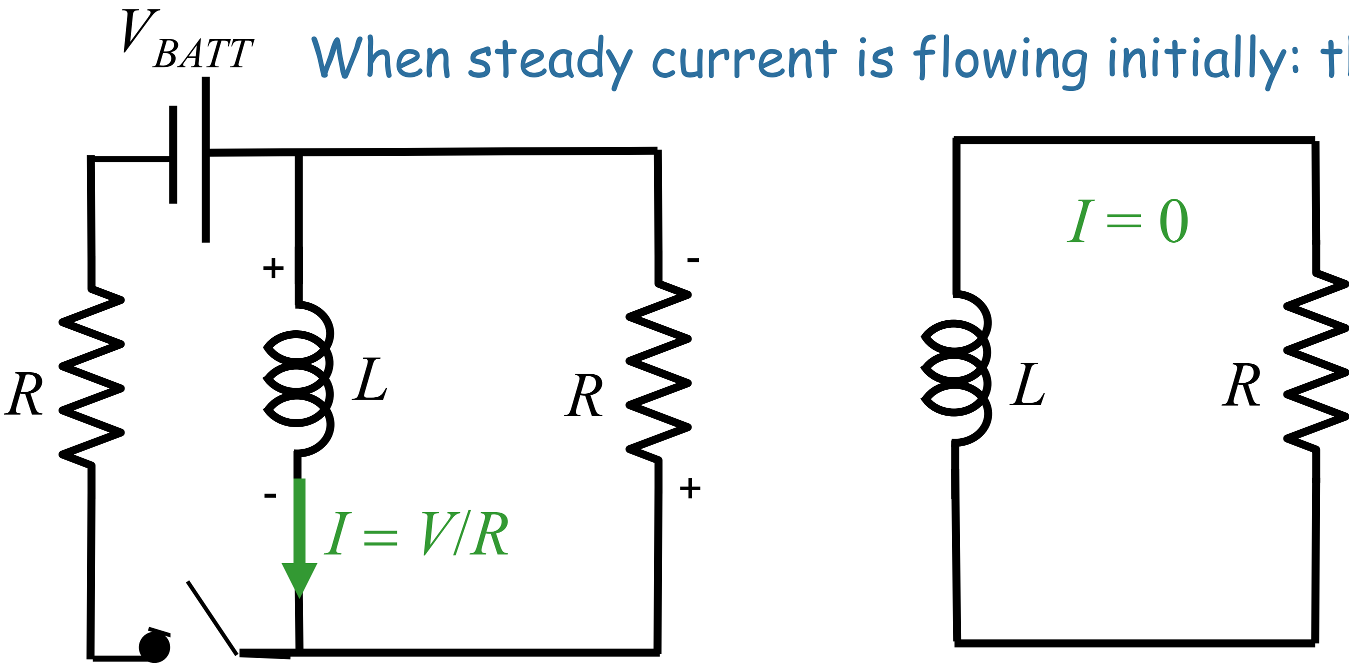


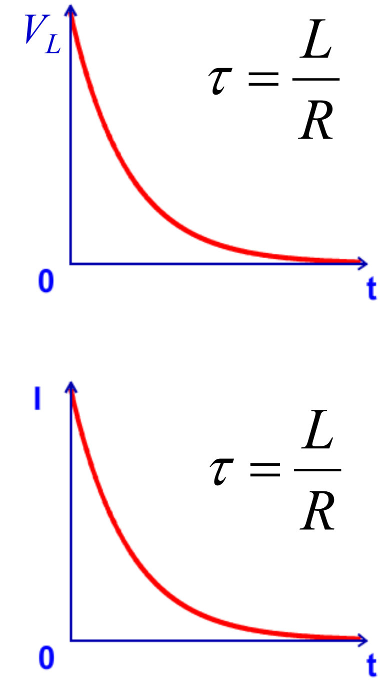
After a long time in any static circuit: $V_L = 0$

KVR:
$$V_L + IR = 0$$

How to Think about RL Circuits Episode 2:

When steady current is flowing initially: then switch is opened





KVR: $+V_L + V_R = 0$

$L\,dI/dt + IR = 0$

At $t = 0$: I_L unchanged

$I = V_{BATT}/R$

Ohm's Law $V_R = IR$

Using KVR $V_L = V_R$

At $t >> L/R$: $V_L = 0$

$V_L = 0$

Using KVR $V_R = 0$

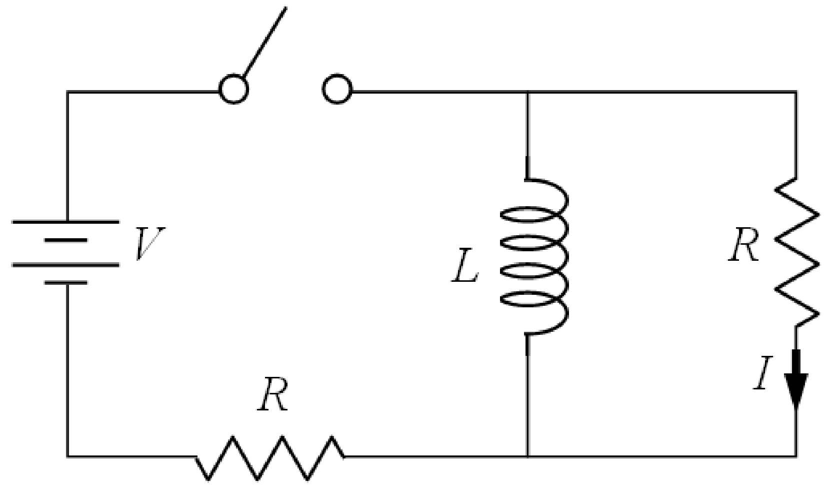
Ohm's Law $I = 0$

Check Point 2b

After a long time, the switch is opened, abruptly disconnecting the battery from the circuit.

What is the current I through the vertical resistor immediately after the switch is opened?

(+ is in the direction of the arrow)



A) $I = V/R$

B) $I = V/2R$

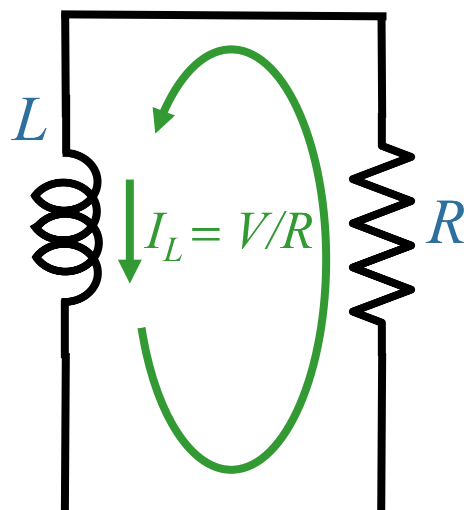
C) $I = 0$

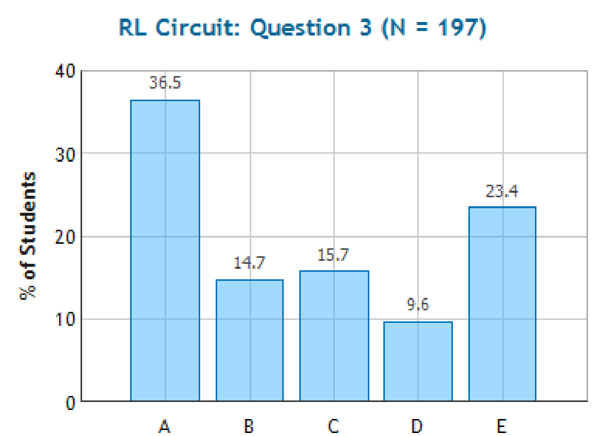
D) $I = -V/2R$

E) $\boxed{I = -V/R}$

Current through inductor cannot change DISCONTINUOUSLY

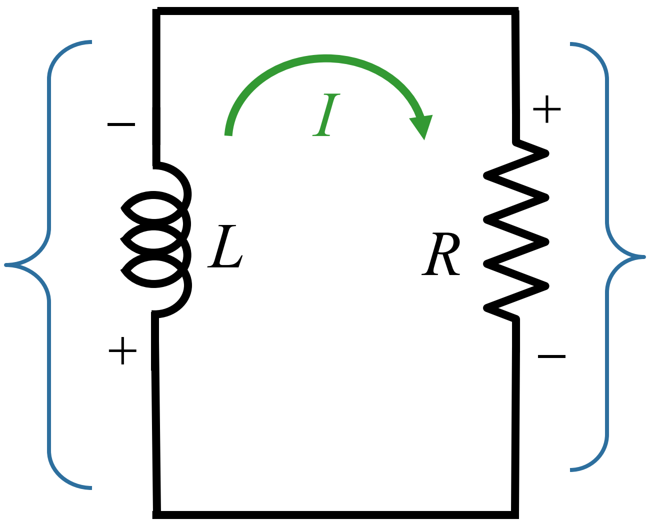
circuit when switch opened



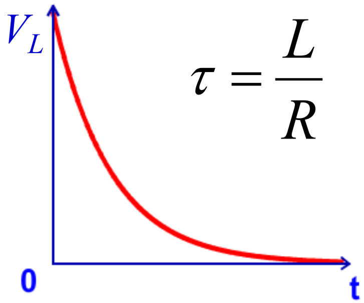


Why is there Exponential Behavior?

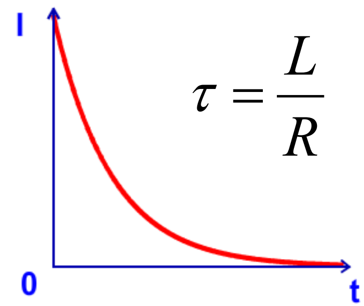
$$V = L\frac{dI}{dt}$$



$$V = IR$$



$$\tau = \frac{L}{R}$$



$$\tau = \frac{L}{R}$$

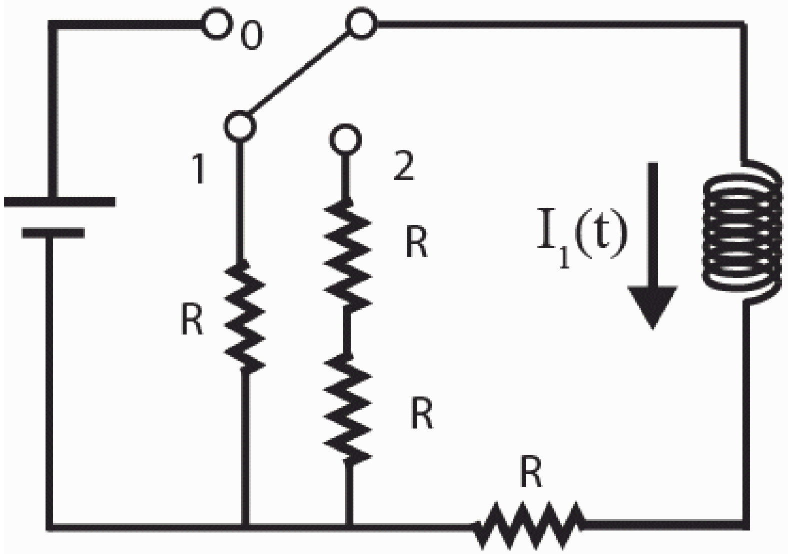
$$L\frac{dI}{dt} + IR = 0$$

$$I(t) = I_0 e^{-tR/L} = I_0 e^{-t/\tau}$$

where $\tau = \frac{L}{R}$

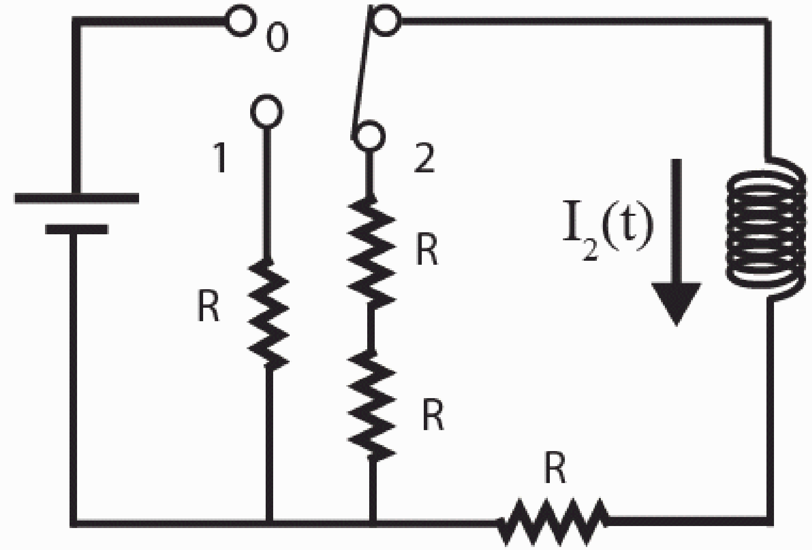
Check Point 3a

After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



Case 2

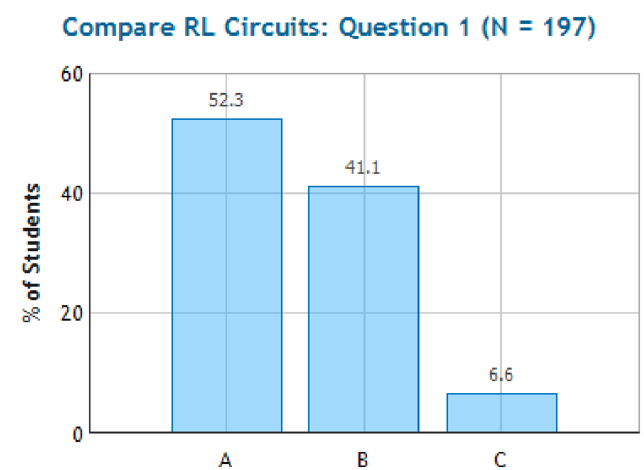
After switch moved, which case has larger time constant?

A) Case 1

B) Case 2

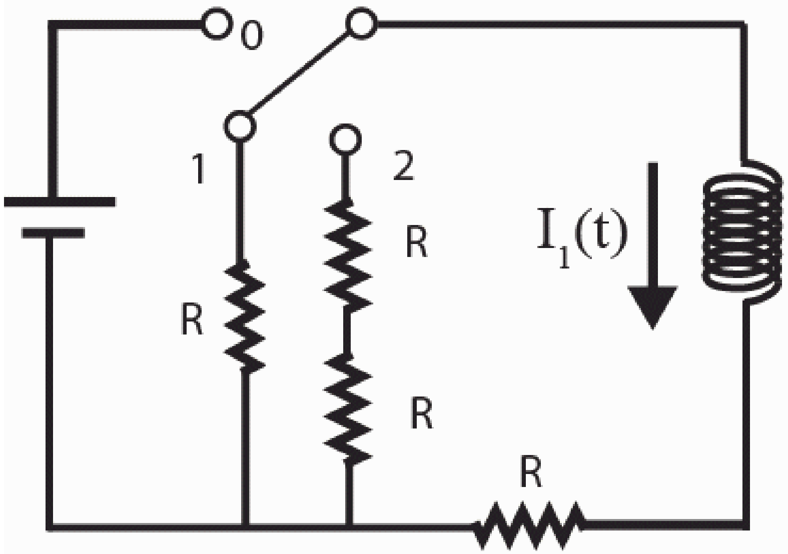
C) The same

$$\tau_1 = \frac{L}{2R} \qquad \tau_2 = \frac{L}{3R}$$



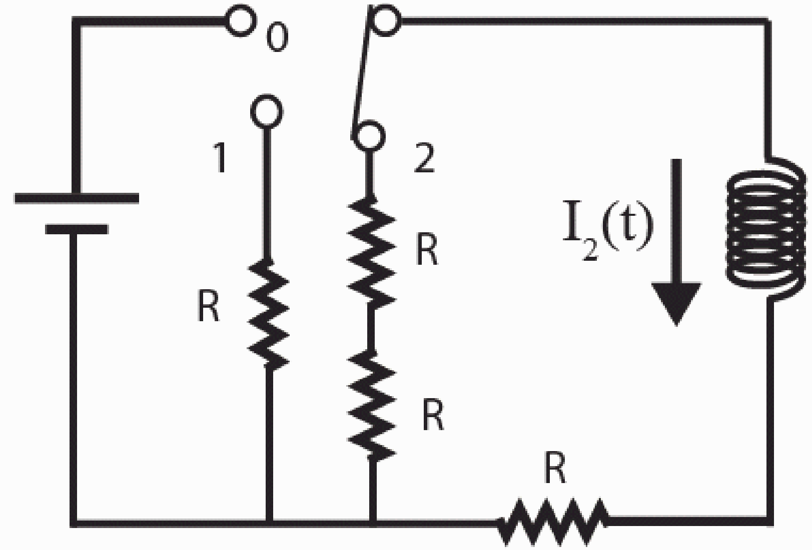
Check Point 3b

After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



Case 2

Immediately after switch moved, in which case is the voltage across the inductor larger?

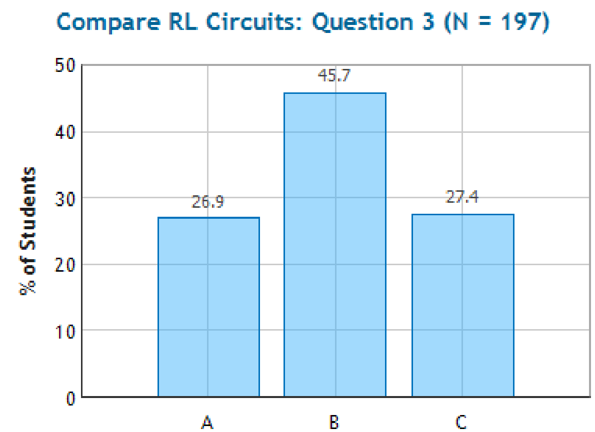
A) Case 1
B) $\boxed{\text{Case 2}}$
C) The same

After switch moved:

$$V_{L1} = \frac{V}{R}2R$$

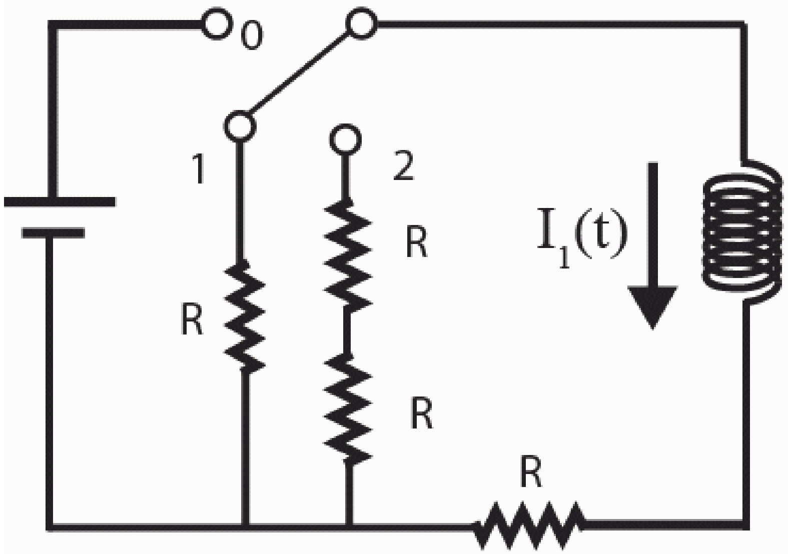
Before switch moved: $I = \dfrac{V}{R}$

$$V_{L2} = \frac{V}{R}3R$$



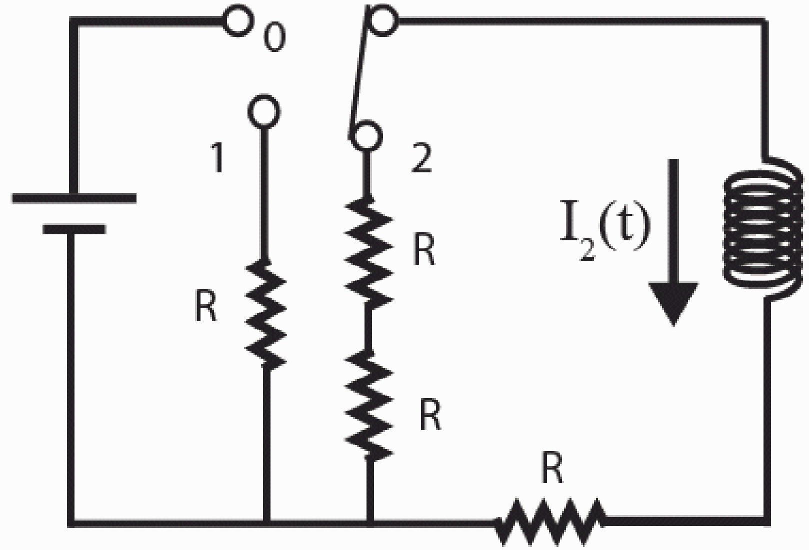
Check Point 3c

After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



Case 2

After switch moved for finite time, in which case is the current through the inductor larger?

- A) Case 1
- B) Case 2
- C) The same

After awhile

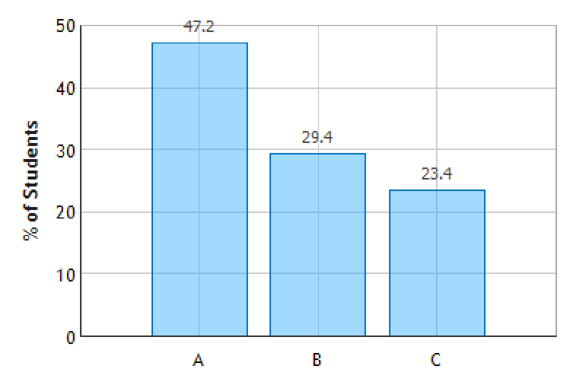
$$I_1 = Ie^{-t/\tau_1}$$

$$I_2 = Ie^{-t/\tau_2}$$

$$\tau_1 > \tau_2$$

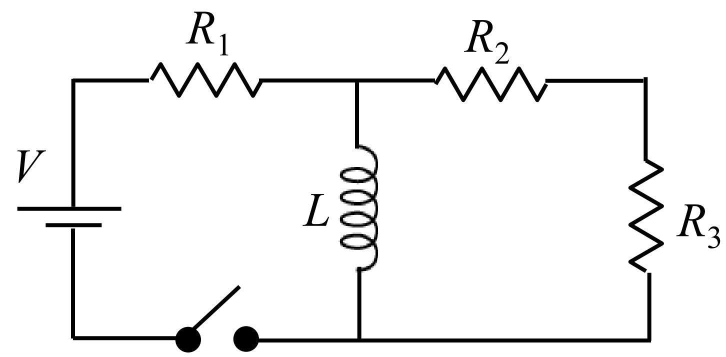
Immediately after: $I_1 = I_2$

Compare RL Circuits: Question 5 (N = 197)



Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is dI_L/dt, the time rate of change of the current through the inductor immediately after switch is closed

Conceptual Analysis

Once switch is closed, currents will flow through this 2-loop circuit.

KVR and KCR can be used to determine currents as a function of time.

Strategic Analysis

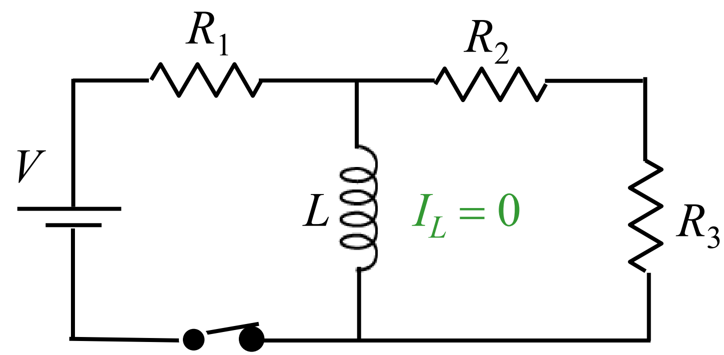
Determine currents immediately after switch is closed.

Determine voltage across inductor immediately after switch is closed.

Determine dI_L/dt immediately after switch is closed.

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is I_L, the current in the inductor, immediately after the switch is closed?

A) $I_L = V/R_1$ up

B) $I_L = V/R_1$ down

C) $\boxed{I_L = 0}$

INDUCTORS: Current cannot change discontinuously !

↓

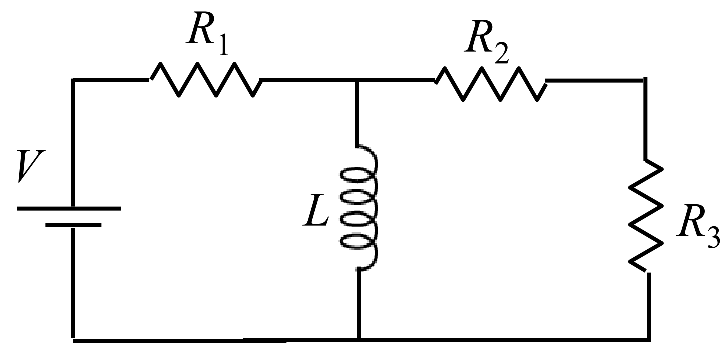
Current through inductor immediately after switch is closed
is the same as
the current through inductor immediately before switch is closed

Immediately before switch is closed: $I_L = 0$ since no battery in loop

Bonus Point 1

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



$$I_L(t = 0+) = 0$$

What is the magnitude of I_2, the current in R_2, immediately after the switch is closed?

A) $I_2 = \dfrac{V}{R_1}$

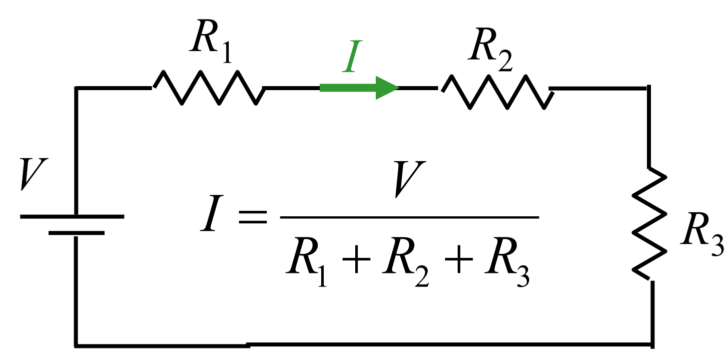
B) $I_2 = \dfrac{V}{R_2 + R_3}$

C) $\boxed{I_2 = \dfrac{V}{R_1 + R_2 + R_3}}$

D) $I_2 = \dfrac{V R_2 R_3}{R_2 + R_3}$

We know $I_L = 0$ immediately after switch is closed

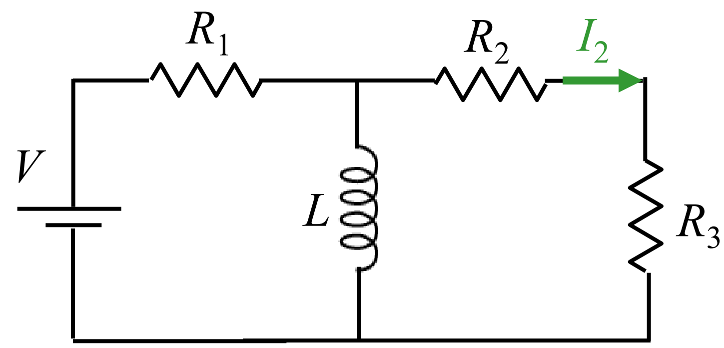
Immediately after switch is closed, circuit looks like:



Bonus Point 2

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



$I_L(t = 0+) = 0 \quad I_2(t = 0+) = V/(R_1 + R_2 + R_3)$

What is the magnitude of V_L, the voltage across the inductor, immediately after the switch is closed?

A) $V_L = V\dfrac{R_2 R_3}{R_1}$ B) $V_L = V$ C) $V_L = 0$ D) $V_L = V\dfrac{R_2 R_3}{R_1(R_2 + R_3)}$ $\boxed{\text{E) } V_L = V\dfrac{R_2 + R_3}{R_1 + R_2 + R_3}}$

Kirchhoff's Voltage Law,

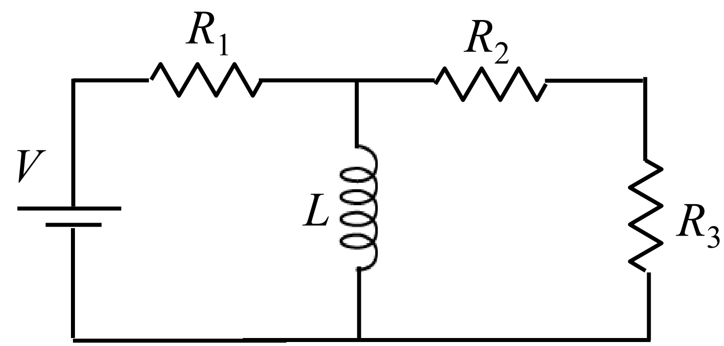
$V_L - I_2 R_2 - I_2 R_3 = 0 \quad V_L = I_2 (R_2 + R_3)$

$$V_L = \frac{V}{R_1 + R_2 + R_3}(R_2 + R_3)$$

Bonus Point 3

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is dI_L/dt, the time rate of change of the current through the inductor immediately after switch is closed

$V_L(t = 0 +) = V(R_2 + R_3)/(R_1 + R_2 + R_3)$

A) $\frac{dI_L}{dt} = \frac{V}{L}\frac{R_2 + R_3}{R_1}$ B) $\frac{dI_L}{dt} = 0$ C) $\boxed{\frac{dI_L}{dt} = \frac{V}{L}\frac{R_2 + R_3}{R_1 + R_2 + R_3}}$ D) $\frac{dI_L}{dt} = \frac{V}{L}$

The time rate of change of current through the inductor $(dI_L/dt) = V_L/L$

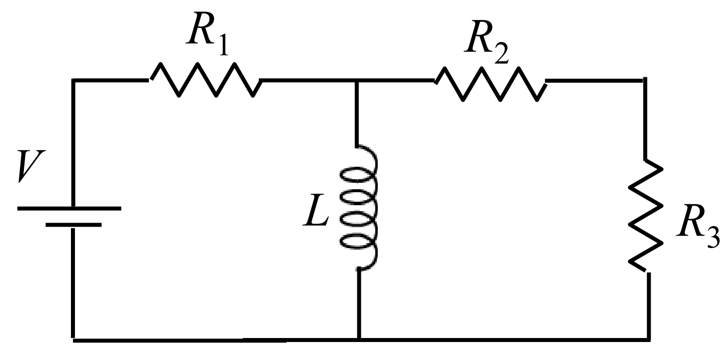
$$\frac{dI_L}{dt} = \frac{V}{L}\frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

Bonus Point 4

Follow Up

The switch in the circuit shown has been closed for a long time.

What is I_2, the current through R_2 ?
(Positive values indicate current flows to the right)



A) $I_2 = +\dfrac{V}{R_2 + R_3}$ B) $I_2 = +\dfrac{V(R_2R_3)}{R_1 + R_2 + R_3}$ C) $\boxed{I_2 = 0}$ D) $I_2 = -\dfrac{V}{R_2 + R_3}$

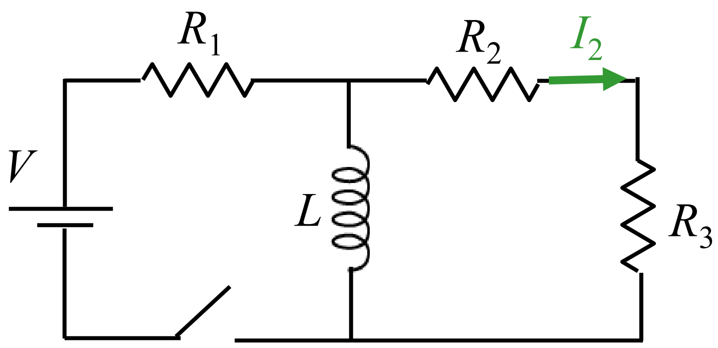
After a long time, $dI/dt = 0$
Therefore, the voltage across $L = 0$
Therefore, the voltage across $R_2 + R_3 = 0$
Therefore, the current through $R_2 + R_3$ must be zero!

Bonus Point 5

Follow Up 2

The switch in the circuit shown has been closed for a long time at which point, the switch is opened.

What is I_2, the current through R_2 immediately after switch is opened ?
(Positive values indicate current flows to the right)



A) $I_2 = +\dfrac{V}{R_1 + R_2 + R_3}$ B) $I_2 = +\dfrac{V}{R_1}$ C) $I_2 = 0$ D) $\boxed{I_2 = -\dfrac{V}{R_1}}$ E) $I_2 = -\dfrac{V}{R_1 + R_2 + R_3}$

Current through inductor immediately after switch is opened
is the same as
the current through inductor immediately before switch is opened

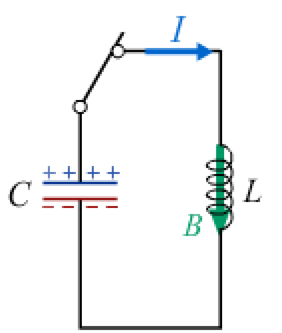
Immediately before switch is opened: $I_L = V/R_1$

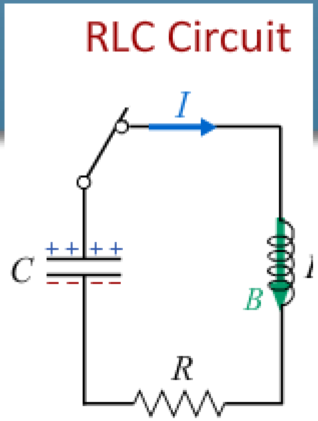
Immediately after switch is opened: I_L flows in right loop
Therefore, $I_L = -V/R_1$

Midterm III

11	**Monday** 11/18/2024	**Lecture 19:** Oscillations: LC Circuits	14.5-14.6	
	Monday 11/18/2024	Discussion 1-8		Quiz 8
	Thursday 11/21/2024	**Lecture 20:** AC Circuits	15.1-15.3	
12	**Monday** 11/25/2024	**Lecture 21:** AC Power & Resonant Circuits	15.4-15.6	
	Monday 11/25/2024	Discussion 1-8		Quiz 9
	Thursday 11/28/2024	**Lecture 22:** Maxwell's Displacement Current and Electromagnetic Waves	16.1-16.2	
13	**Monday** 12/2/2024	**Lecture 23:** Electromagnetic Waves	16.3-16.5	
	Monday 12/2/2024	**Exam 3**		**(Units 17-23)**
	Thursday 12/5/2024	**Lecture 24:** Polarization	Vol.3 1.7	

Physics 212
Lecture 19



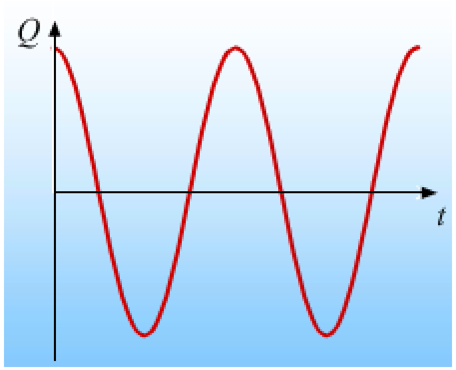


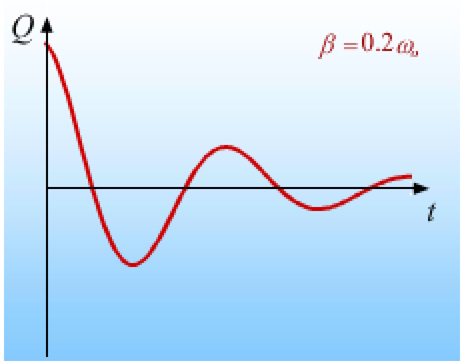
Today's Concepts:

A) Oscillation Frequency

B) Energy

C) Damping





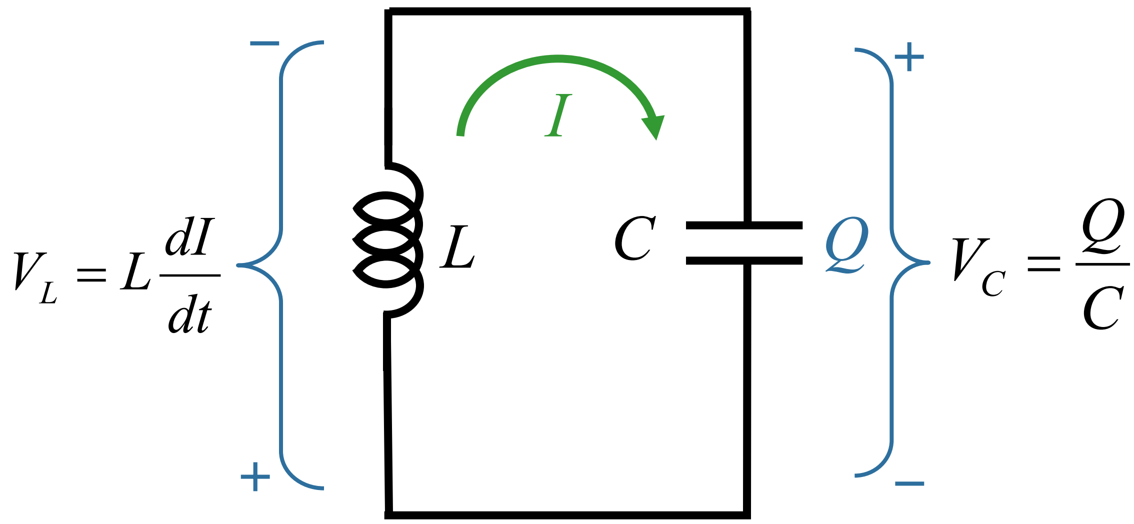
Energy in an inductor

$$V = L \frac{dI}{dt}$$

$$P = IV = IL\frac{dI}{dt}$$

Energy = $\int_0^t P\,dt = \int_0^I IL\,dI = \frac{1}{2}LI^2$

LC Circuit



KVL Circuit Equation: $\frac{Q}{C} + L\frac{dI}{dt} = 0$

$$I = \frac{dQ}{dt} \longrightarrow \frac{d^2Q}{dt^2} = -\frac{Q}{LC} \longrightarrow \frac{d^2Q}{dt^2} = -\omega^2 Q$$

where

$$\omega = \frac{1}{\sqrt{LC}}$$

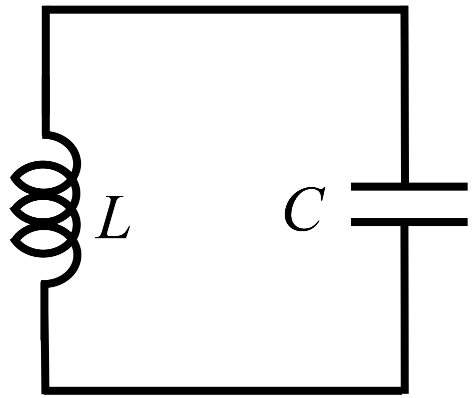
Solution:

$$Q(t) = Q_{max}\cos(\omega t + \phi)$$

$$I(t) = -\omega Q_{max}\sin(\omega t + \phi)$$

Check Point 1a

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.



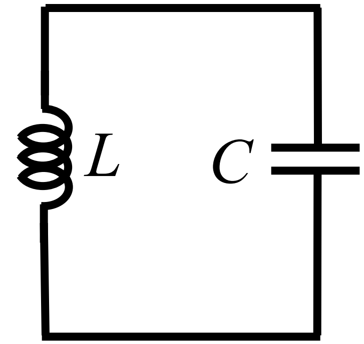
What is the potential difference across the inductor at $t = 0$?

A) $V_L = 0$
B) $\boxed{V_L = Q_{max}/C}$ since $V_L = V_C$
C) $V_L = Q_{max}/2C$

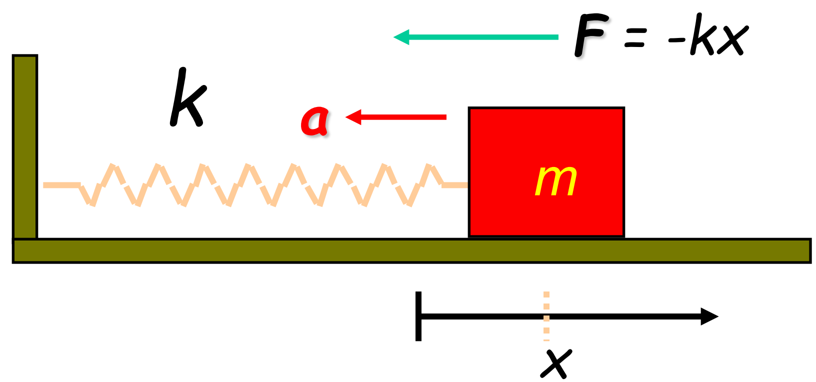
Voltage is the same across the capacitor and the inductor. Voltage across capacitor is Q/C

LC Circuits analogous to mass on spring

$$\frac{d^2Q}{dt^2} = -\omega^2 Q \qquad \omega = \frac{1}{\sqrt{LC}}$$

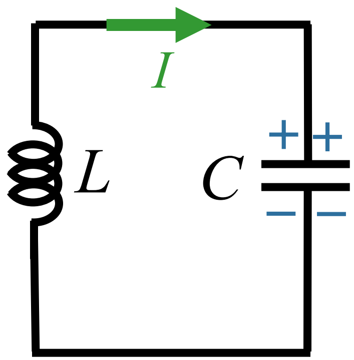


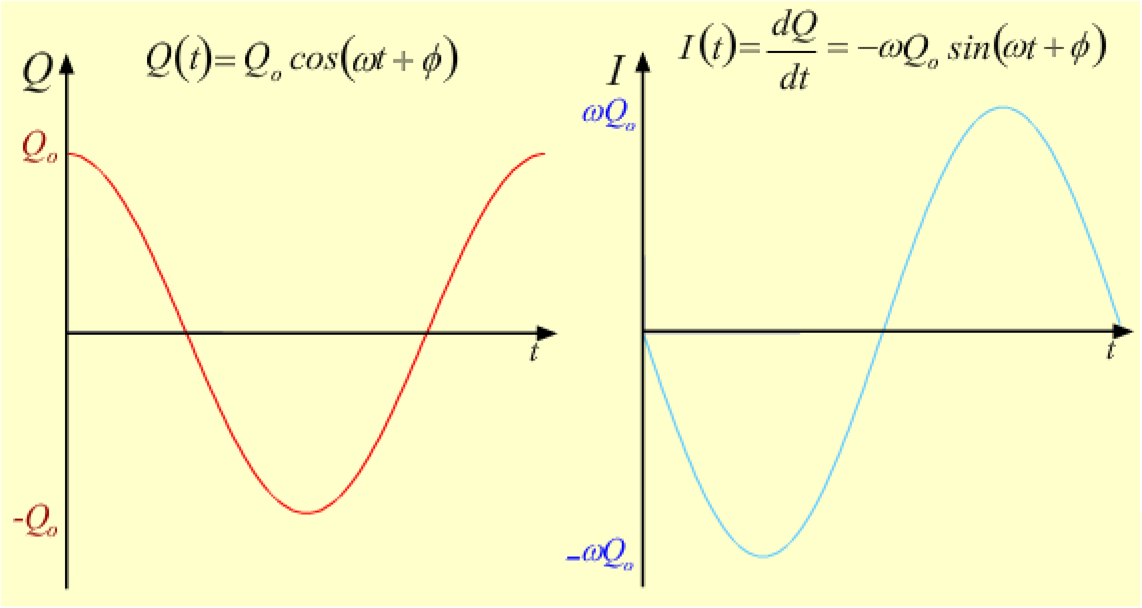
$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad \omega = \sqrt{\frac{k}{m}}$$



Same thing if we notice that $\boxed{k \leftrightarrow \frac{1}{C}}$ and $\boxed{m \leftrightarrow L}$

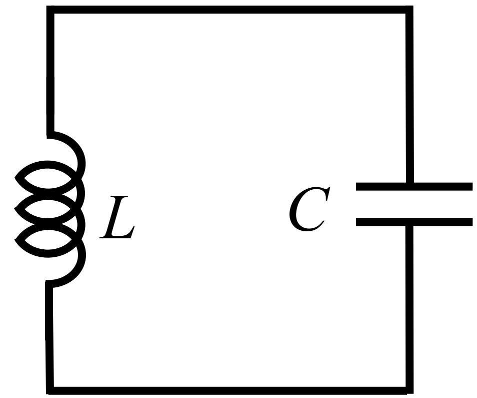
Time Dependence





Check Point 1b

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.



What is the potential difference across the inductor when the current is maximum?

A) $\boxed{V_L = 0}$
B) $V_L = Q_{max}/C$
C) $V_L = Q_{max}/2C$

Solution: $V_L = V_C$

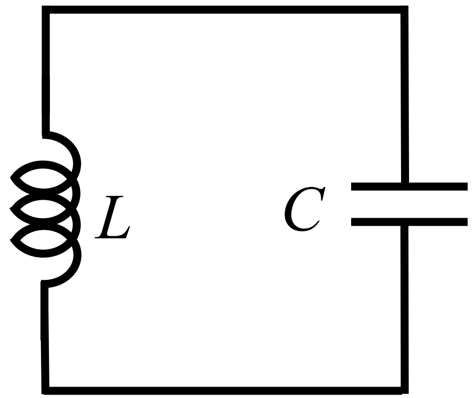
$$Q(t) = Q_{max}\cos(\omega t + \phi)$$

$$I(t) = -\omega Q_{max}\sin(\omega t + \phi)$$

When the current is at a maximum, its rate of change is zero, so the voltage across the inductor will also be zero.

Check Point 1c

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.



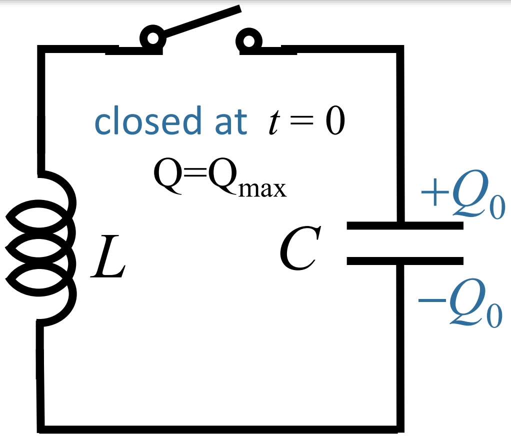
How much energy is stored in the capacitor when the current is a maximum ?

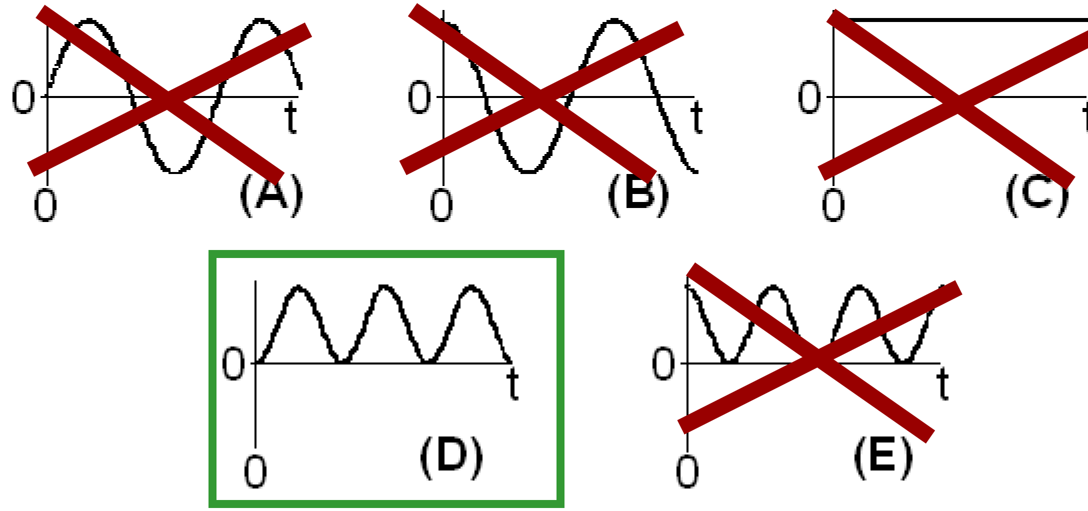
A) $U_C = Q_{max}{}^2/(2C)$
B) $U_C = Q_{max}{}^2/(4C)$
C) $U_C = 0$

Total Energy is constant!

$U_{Lmax} = \frac{1}{2} L I_{max}{}^2$
$U_{Cmax} = Q_{max}{}^2/2C$
$I = max$ when $Q = 0$

Check Point 2b





Which plot best represents the energy in the inductor as a function of time starting just after the switch is closed?

$$U_L = \frac{1}{2} L I^2$$

Energy proportional to $I^2 \rightarrow U_L$ cannot be negative

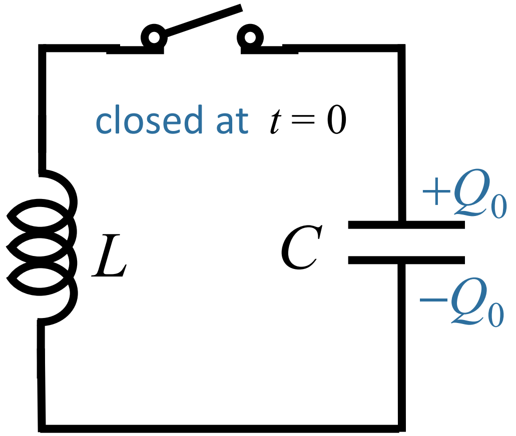
Current is changing $\rightarrow U_L$ is not constant

Initial current is zero

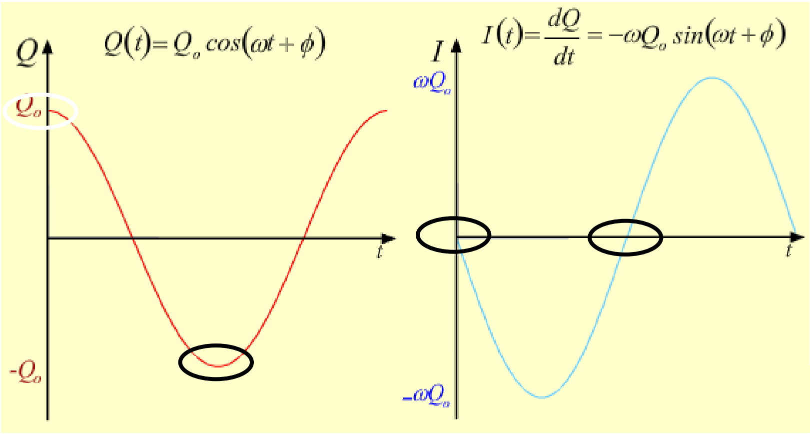
A

Check Point 2c

When the energy stored in the capacitor reaches its maximum again for the first time after $t = 0$, how much charge is stored on the top plate of the capacitor?



A) $+Q_0$
B) $+Q_0 /2$
C) 0
D) $-Q_0/2$
E) $\boxed{-Q_0}$



Q is maximum when current goes to zero

$$I = \frac{dQ}{dt}$$

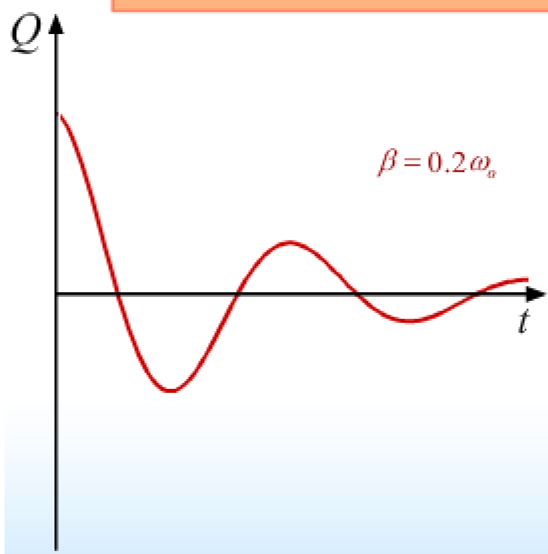
Current goes to zero twice during one cycle

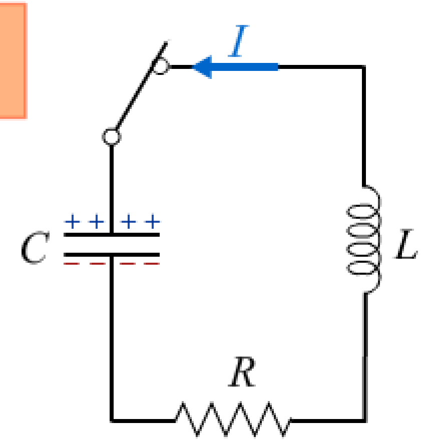
T

Add R: Damping

Just like LC circuit but energy and the oscillations get smaller because of R

$$Q(t) = Q_o e^{-\beta t}\cos(\omega' t + \phi)$$





Damping Factor $\beta = \dfrac{R}{2L}$

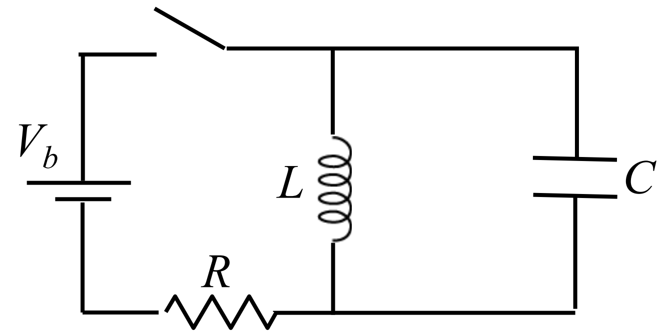
Oscillation Frequency $\omega'^2 = \omega_o^2 - \beta^2$

Natural Frequency $\omega_o = \dfrac{1}{\sqrt{LC}}$

Concept makes sense… …but answer looks kind of complicated

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



What is Q_{MAX}, the maximum charge on the capacitor?

Conceptual Analysis

Once switch is opened, we have an LC circuit

Current will oscillate with natural frequency ω_0

Strategic Analysis

Determine initial current

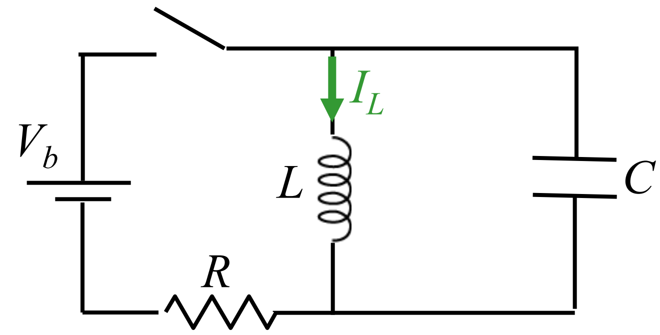
Determine oscillation frequency ω_0

Find maximum charge on capacitor

Calculation

Bonus Point 1

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



What is I_L, the current in the inductor, immediately after the switch is opened? Take positive direction as shown.

A) $I_L < 0$ B) $I_L = 0$ $\boxed{\text{C) } I_L > 0}$

Current through inductor immediately after switch is opened

is the same as

the current through inductor immediately before switch is opened

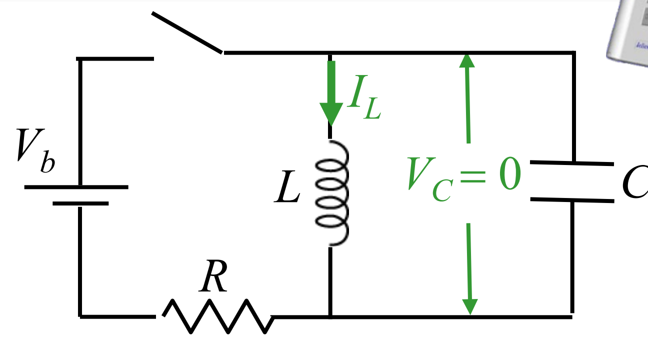
before switch is opened:

all current goes through inductor in direction shown

Calculation

Bonus Point 2

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$

The energy stored in the capacitor immediately after the switch is opened is zero.

A) TRUE B) FALSE

before switch is opened:

$dI_L/dt \sim 0 \Rightarrow V_L = 0$

BUT: $V_L = V_C$

since they are in parallel

$\rightarrow V_C = 0$

after switch is opened:

V_C cannot change abruptly

$\rightarrow V_C = 0$

$\rightarrow U_C = \frac{1}{2} CV_C^2 = 0$!

IMPORTANT: NOTE DIFFERENT CONSTRAINTS AFTER SWITCH OPENED

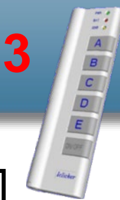
CURRENT through INDUCTOR cannot change abruptly

VOLTAGE across CAPACITOR cannot change abruptly

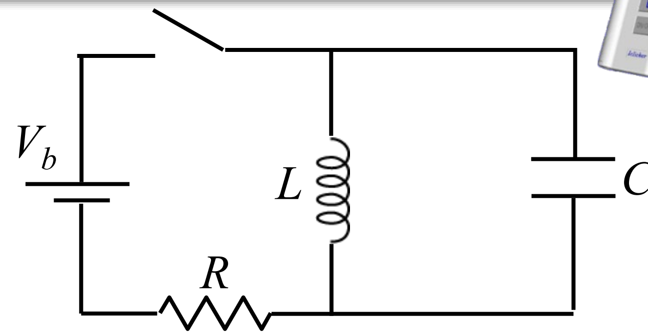
T

Calculation

Bonus Point 3



The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$ $V_C(t = 0+) = 0$

What is the magnitude of the current right after the switch is opened?

A) $I_o = V_b\sqrt{\dfrac{C}{L}}$

B) $I_o = \dfrac{V_b}{R^2}\sqrt{\dfrac{L}{C}}$

C) $\boxed{I_o = \dfrac{V_b}{R}}$

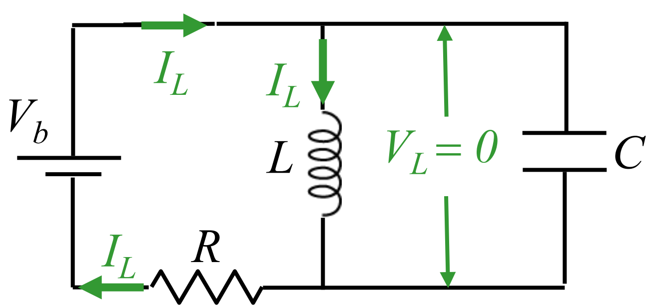
D) $I_o = \dfrac{V_b}{2R}$

Current through inductor immediately after switch is opened

is the same as

the current through inductor immediately before switch is opened

Before switch is opened:



$V_L = 0$

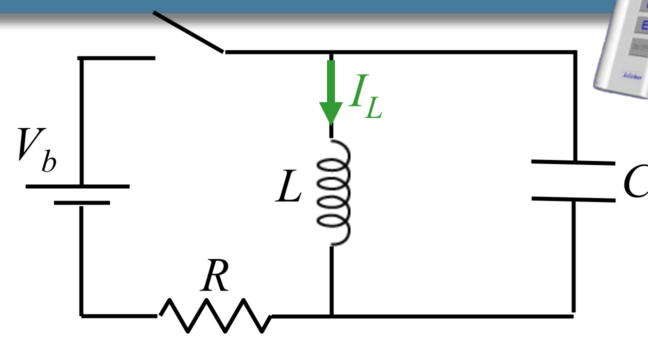
$V = I_L R$

Calculation

Bonus Point 4

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.

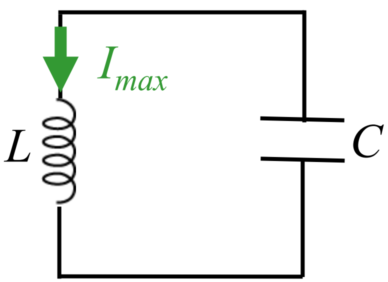
Hint: Energy is conserved



$I_L(t = 0+) = V/R \qquad V_C(t = 0+) = 0$

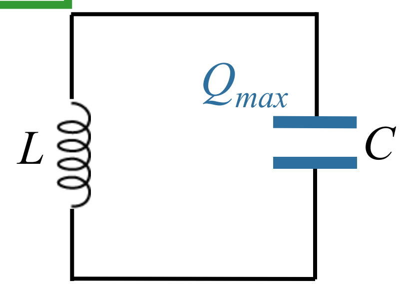
What is Q_{max}, the maximum charge on the capacitor during the oscillations?

A) $\boxed{Q_{\max} = \frac{V_b}{R}\sqrt{LC}}$ B) $Q_{\max} = \frac{1}{2} C V_b$ C) $Q_{\max} = C V_b$ D) $Q_{\max} = \frac{V_b}{R\sqrt{LC}}$



When I is max (and Q is 0)

$$U = \frac{1}{2} L I_{\max}^2$$

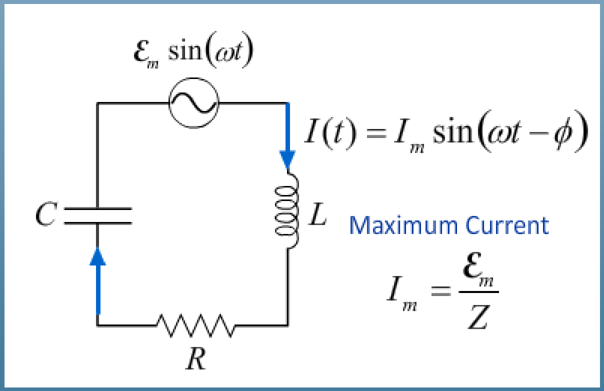


When Q is max (and I is 0)

$$U = \frac{1}{2} \frac{Q_{\max}^2}{C}$$

$$\frac{1}{2} L I_{\max}^2 = \frac{1}{2} \frac{Q_{\max}^2}{C}$$

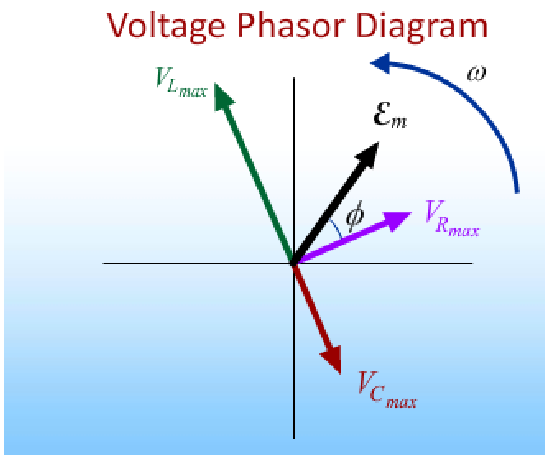
$$Q_{\max} = I_{\max}\sqrt{LC} = \frac{V}{R}\sqrt{LC}$$



Physics 212
Lecture 20

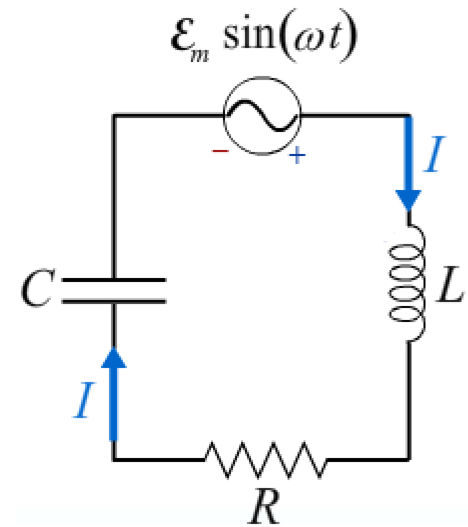
Today's Concept:

AC Circuits
Maximum currents & voltages
Phasors: A Useful Tool



Big Idea

KVR



Maximum Values (nice V=IR)

$$V_{Rmax} = I_{max} R$$

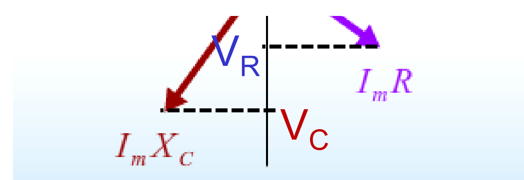
Differential equation solution:

Approximate form

$$Q(t) = c_1 e^{\frac{1}{2} t \left(-\frac{\sqrt{C R^2 - 4 L}}{\sqrt{C}\, L} - \frac{R}{L}\right)} + c_2 e^{\frac{1}{2} t \left(\frac{\sqrt{C R^2 - 4 L}}{\sqrt{C}\, L} - \frac{R}{L}\right)} - \frac{C^2 L V w^2 \sin(t w)}{C^2 w^2 (L^2 w^2 + R^2) - 2 C L w^2 + 1} + \frac{C V \sin(t w)}{C^2 w^2 (L^2 w^2 + R^2) - 2 C L w^2 + 1} - \frac{C^2 R V w \cos(t w)}{C^2 w^2 (L^2 w^2 + R^2) - 2 C L w^2 + 1}$$

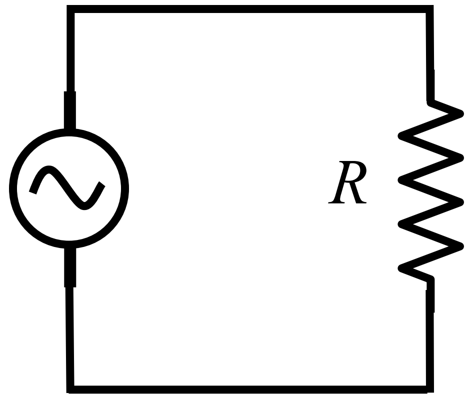
$V_{Inductor}$ Leads current

$V_{Capacitor}$ Lags current



Resistors

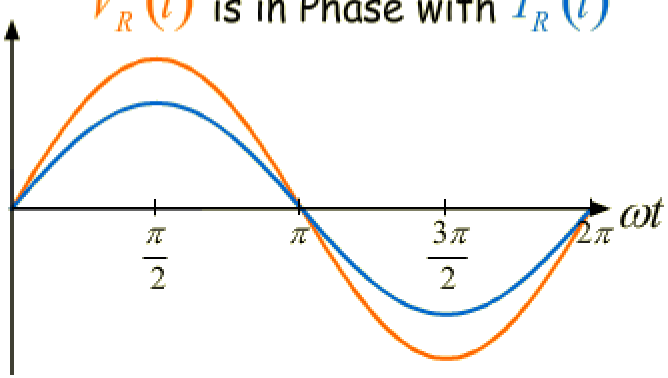
$\mathcal{E} = V_{max}\sin(\omega t)$



$I = V_R/R = \boxed{(V_{max}/R)}\sin(\omega t)$

Amplitude = V_{max}/R

$V_R(t)$ is in Phase with $I_R(t)$



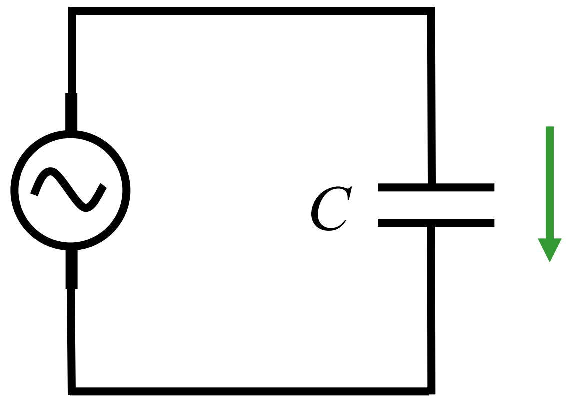
Capacitors

$$KVR: \quad V_{max}\sin(\omega t) - Q/C = 0$$

$$Q = CV_{max}\sin(\omega t)$$

$$I = dQ/dt$$

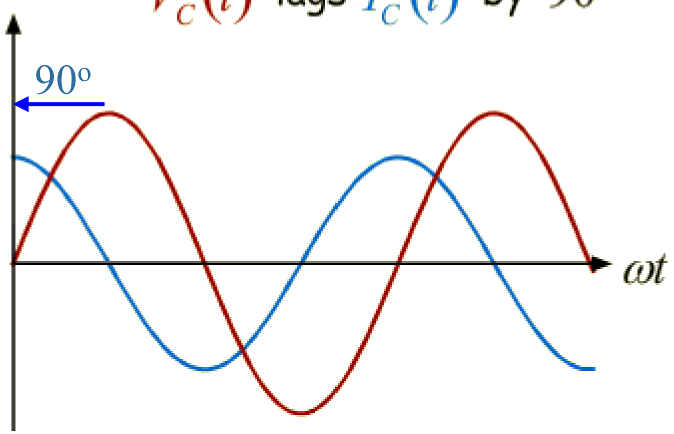
$$\mathcal{E} = V_{max}\sin(\omega t)$$



$$I = \boxed{V_{max}\omega C}\cos(\omega t)$$

$$\boxed{\text{Amplitude} = V_{max}/X_C}$$

$V_C(t)$ lags $I_C(t)$ by 90°

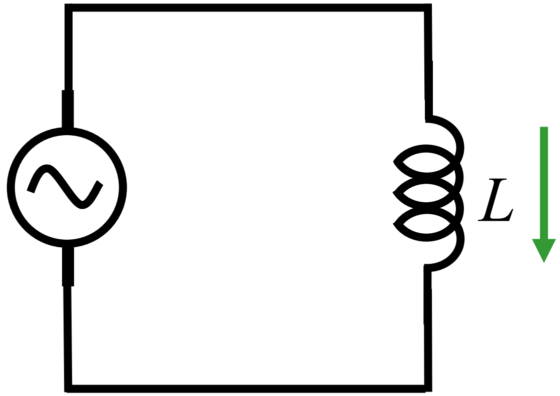


where $X_C = 1/\omega C$
is like the "resistance"
of the capacitor
X_C depends on ω

Inductors

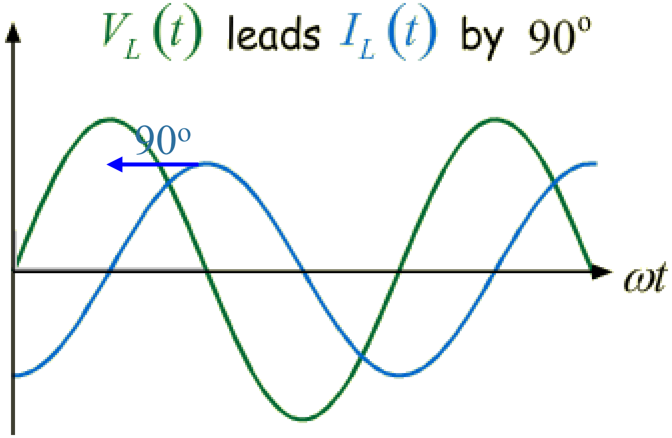
$$L\, dI/dt = V_L = V_{max}\sin(\omega t)$$

$$\mathcal{E} = V_{max}\sin(\omega t)$$



$$I = \boxed{-V_{max}/\omega L}\cos(\omega t)$$

$$\boxed{\text{Amplitude} = V_{max}/X_L}$$

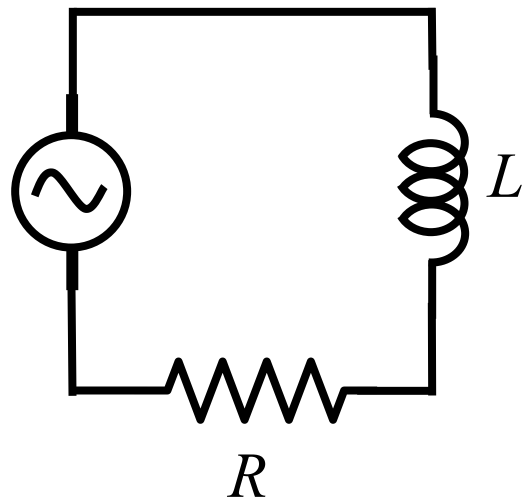


where $X_L = \omega L$
is like the "resistance"
of the inductor
X_L depends on ω

Question

Bonus Point 1

An RL circuit is driven by a 50 volt AC generator as shown in the figure.



$$X_L = \omega L$$

As ω -> 0, so does X_L

As ω -> 0,
current gets bigger

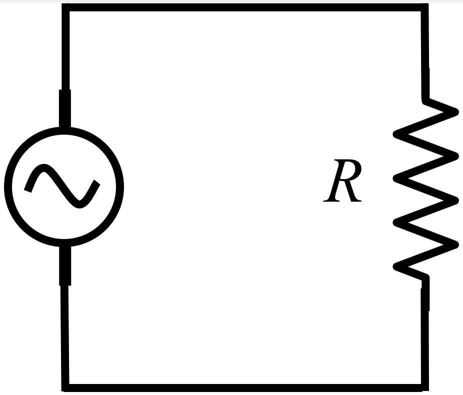
For what driving frequency ω of the generator will the current through the resistor be largest

A) ω large

B) Current through R doesn't depend on ω

C) ω small

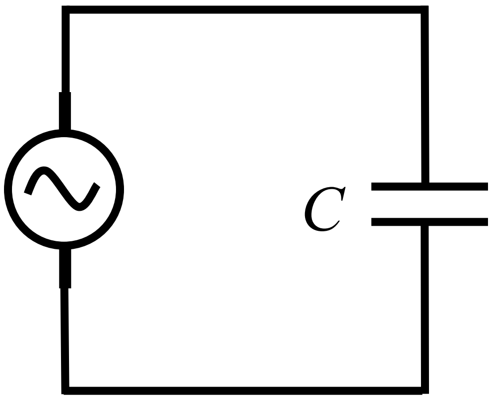
Summary



$I_{max} = V_{max}/R$

V_R in phase with I

Because resistors are simple



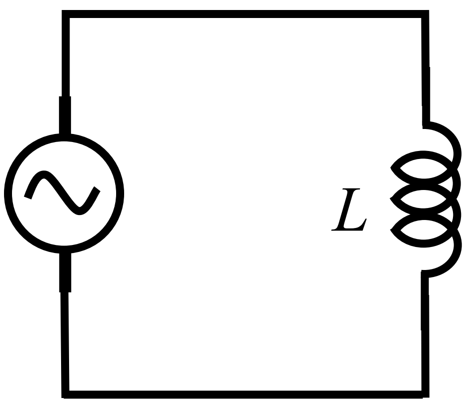
$I_{max} = V_{max}/X_C$

$X_C = 1/\omega C$

V_C 90^o behind I

Current comes first since it charges capacitor

Like a wire at high ω



$I_{max} = V_{max}/X_L$

$X_L = \omega L$

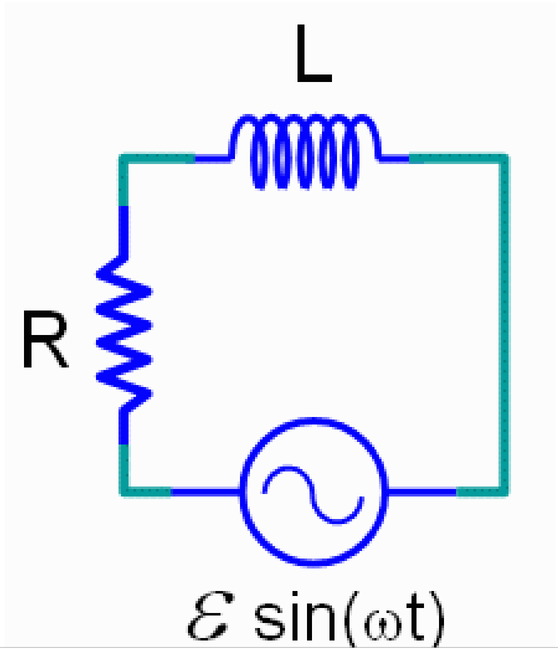
V_L 90^o ahead of I

Opposite of capacitor

Like a wire at low ω

Check Point 1a

A RL circuit is driven by an AC generator as shown in the figure.



The phase difference between the CURRENT through the resistor and the CURRENT through the inductor is

A) Is always zero
B) Is always 90°
C) Depends on the value of L and R
D) Depends on L, R and the generator voltage

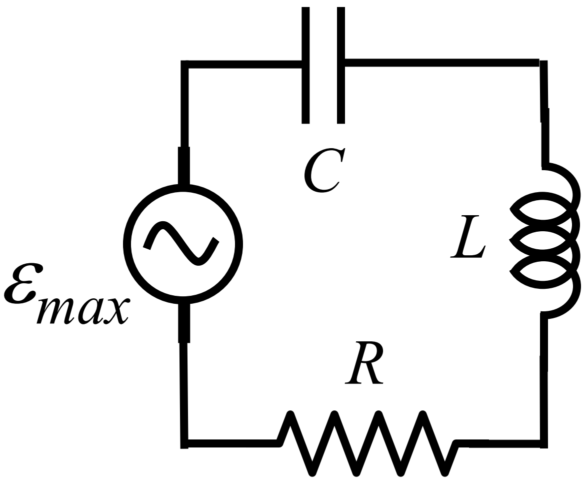
The CURRENT is THE CURRENT
There is only 1 current in this circuit
Same everywhere in circuit

Driven RLC Circuit

Makes sense to write everything in terms of I since this is the same everywhere in a one-loop circuit:

$$V_{max} = I_{max} X_C$$

V_C 90^o behind I



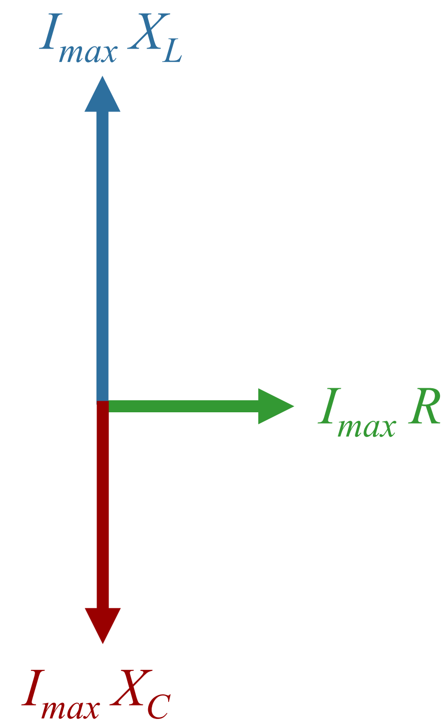
$$V_{max} = I_{max} X_L$$

V_L 90^o ahead of I

$$V_{max} = I_{max} R$$

V_R in phase with I

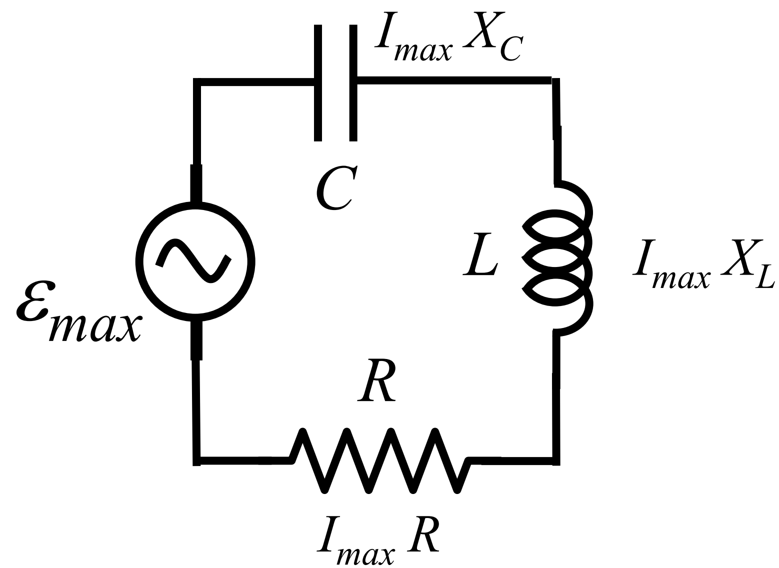
Phasors make this simple to see

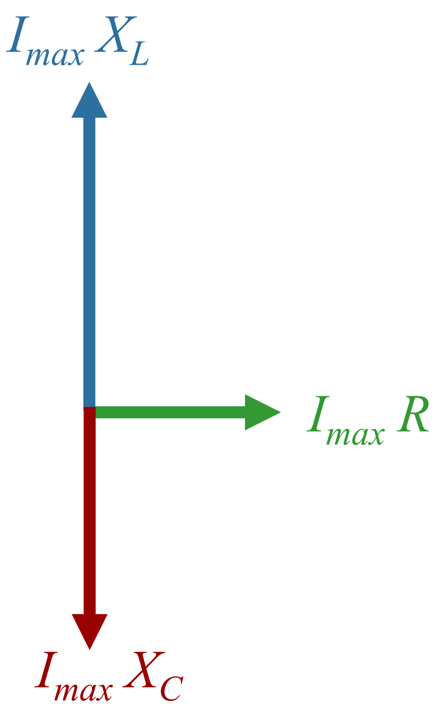


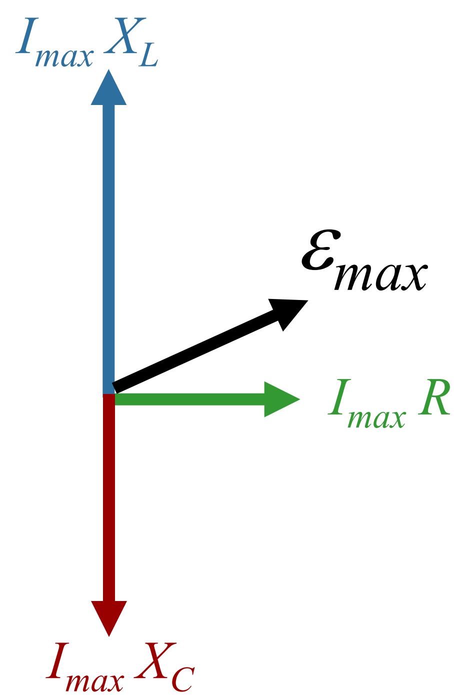
L,R,C always looks the same,
Only the lengths will change

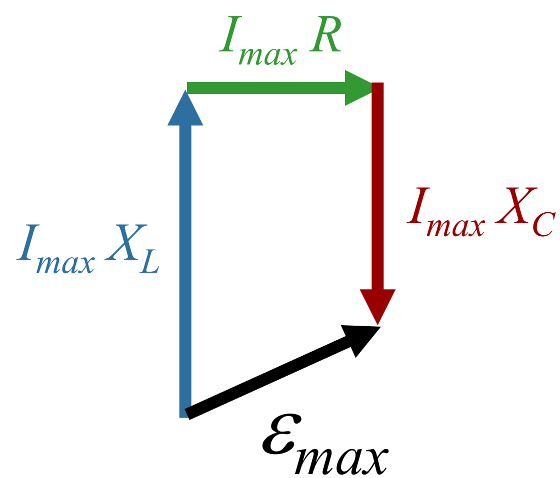
The Voltages still Add Up

But now we are adding vectors:

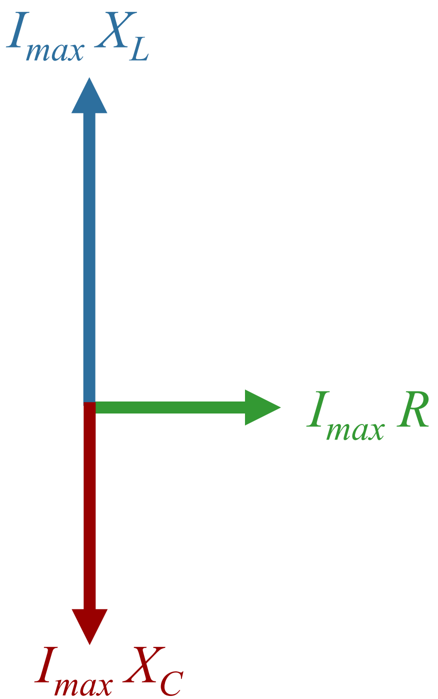


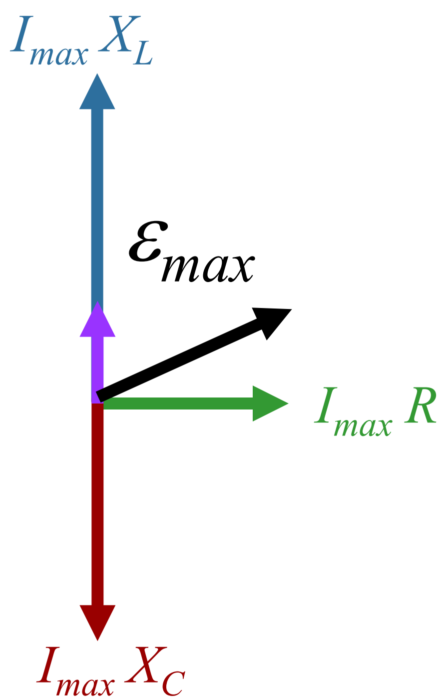


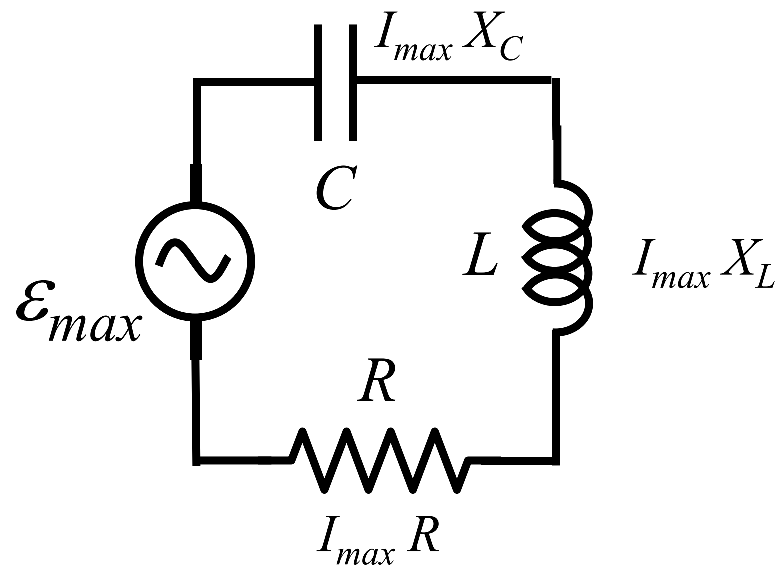




Make this Simpler

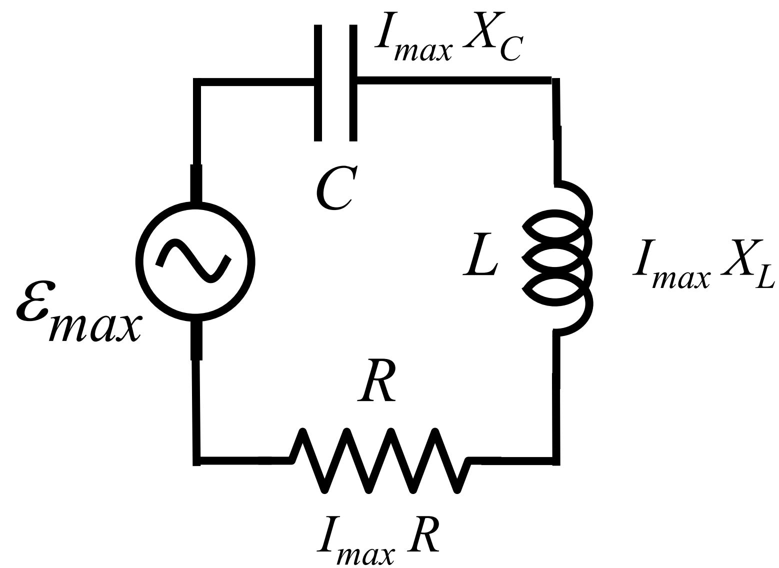


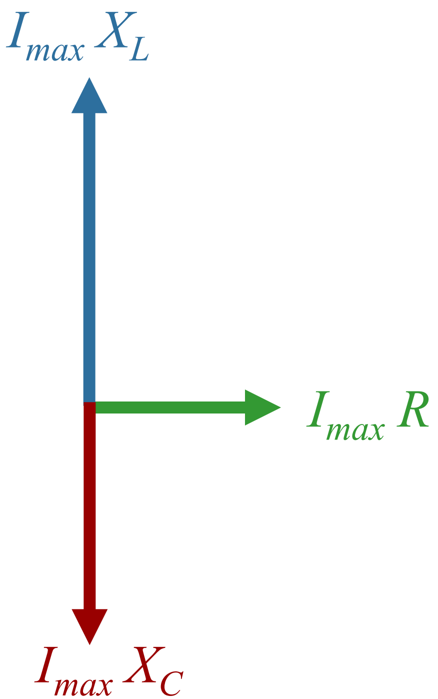




A

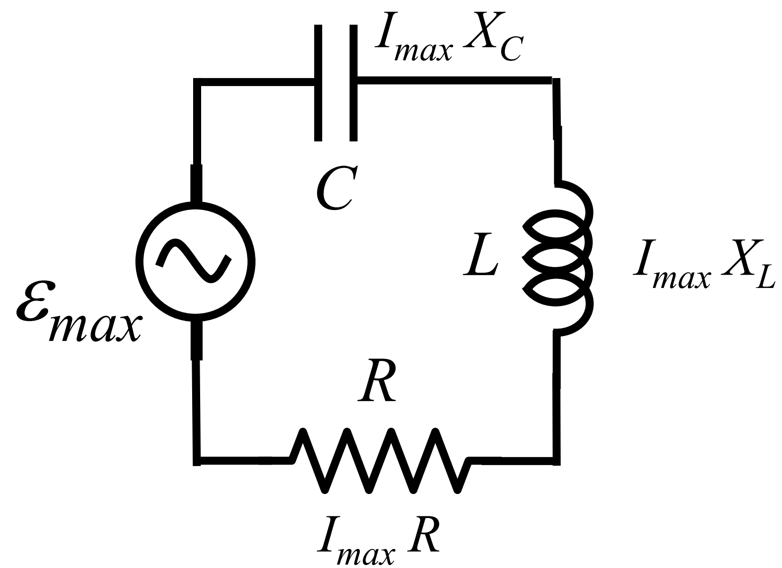
Make this Simpler



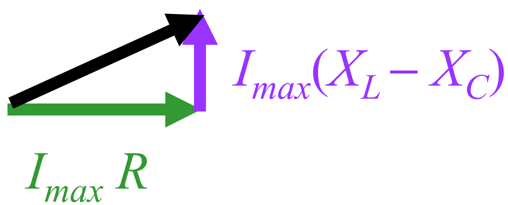


$$\mathcal{E}_{max} = I_{max}Z$$

Make this Simpler



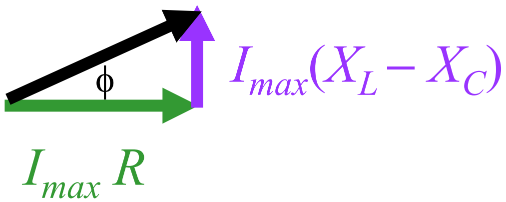
$$\mathcal{E}_{max} = I_{max} Z$$

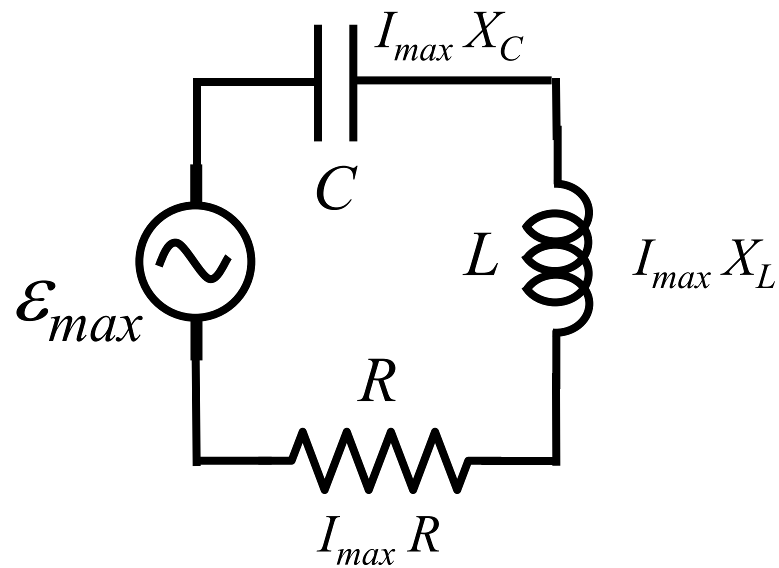


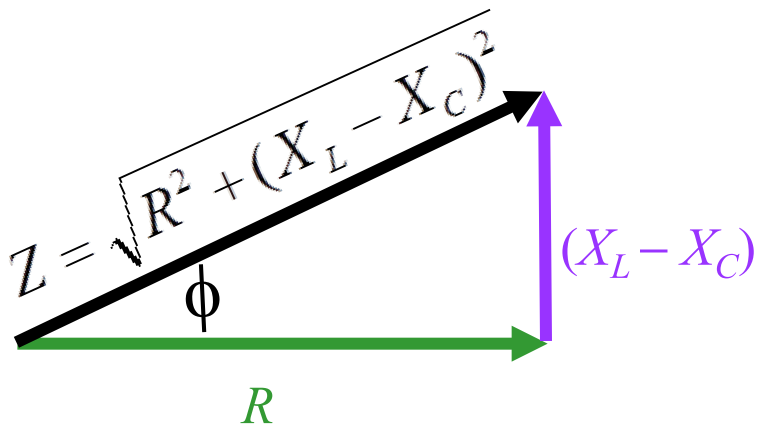
A

Make this Simpler

$$\mathcal{E}_{max} = I_{max} Z$$







Impedance Triangle

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

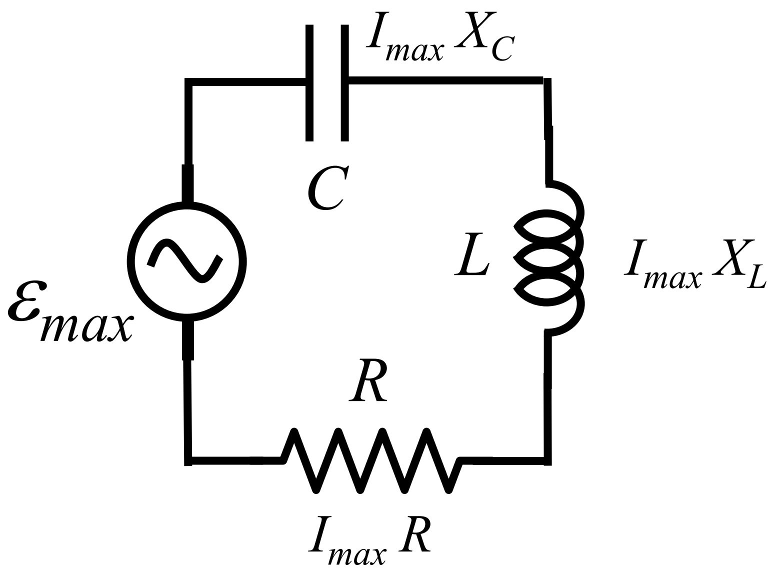
Summary

$$V_{Cmax} = I_{max} X_C$$

$$V_{Lmax} = I_{max} X_L$$

$$V_{Rmax} = I_{max} R$$

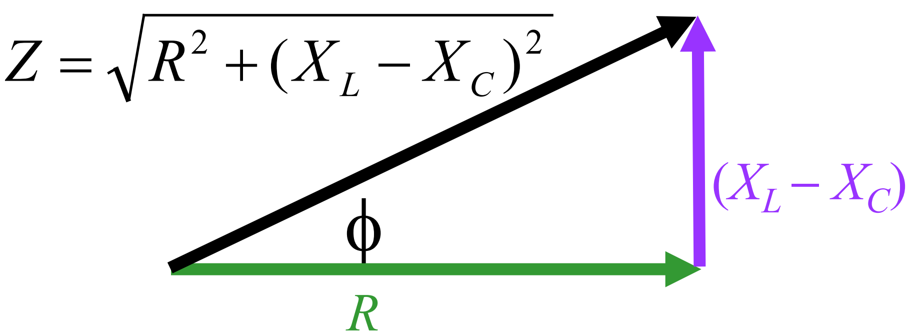
$$\mathcal{E}_{max} = I_{max} Z$$



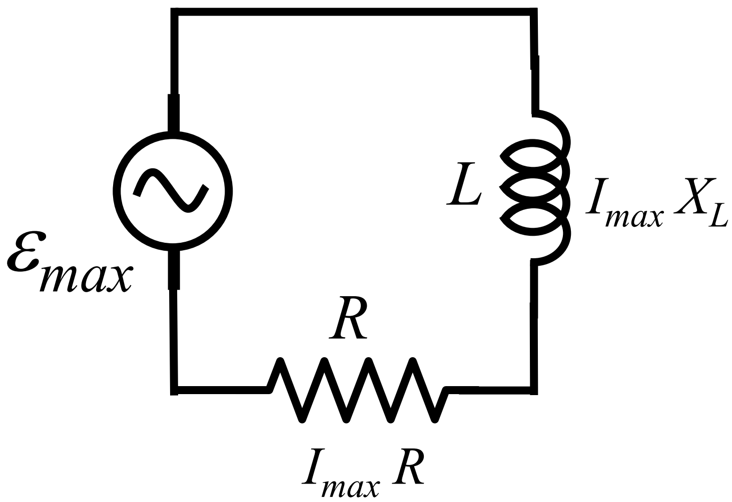
$$I_{max} = \mathcal{E}_{max} / Z$$

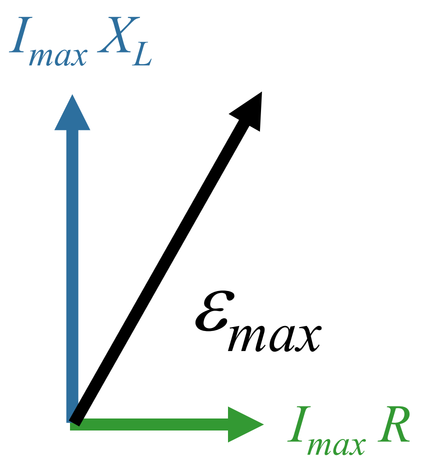
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan(\phi) = \frac{X_L - X_C}{R}$$



Example: RL Circuit $X_c = 0$

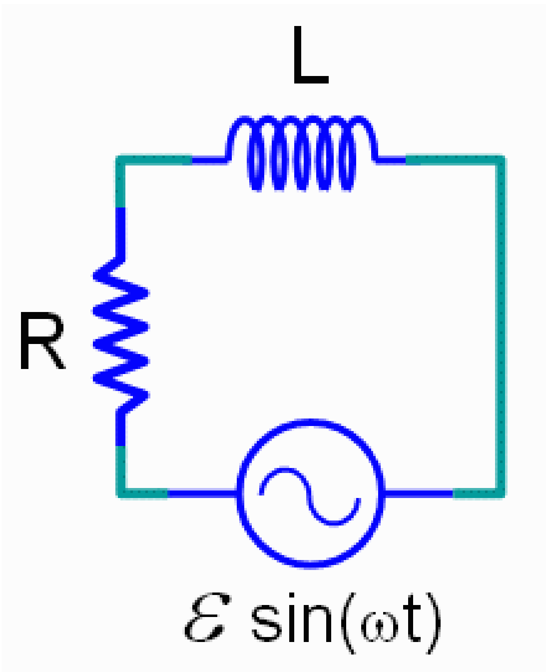




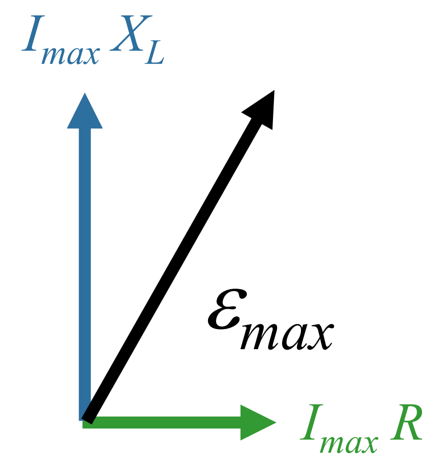
A

Check Point 1b

2) A RL circuit is driven by an AC generator as shown in the figure.



Draw Voltage Phasors



The voltages across the resistor and generator are

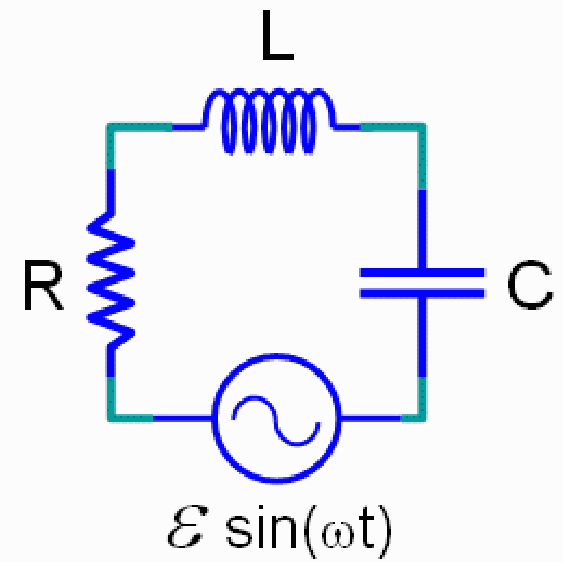
A) Always out of phase ***B)*** Always in phase

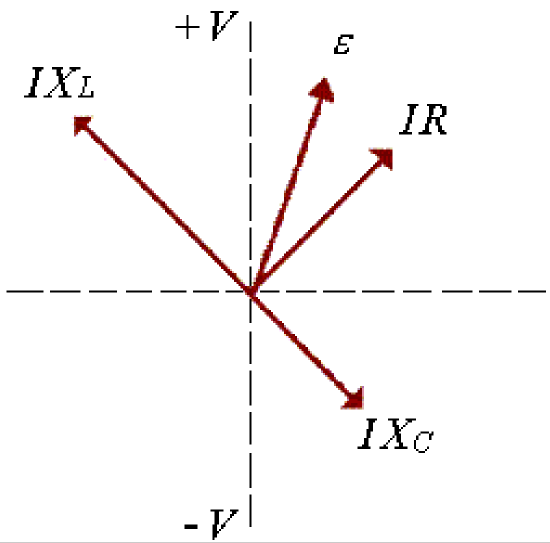
C) Sometimes in and sometimes out of phase

T

Check Point 2a

A driven RLC circuit is represented by the phasor diagram below.

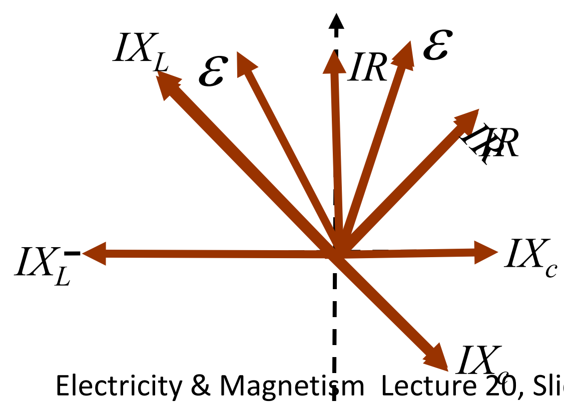




The vertical axis of the phasor diagram represents voltage. When the current through the circuit is maximum, what is the potential difference across the inductor?

A) $V_L = 0$ **B)** $V_L = V_{L,max}/2$ **C)** $V_L = V_{L,max}$

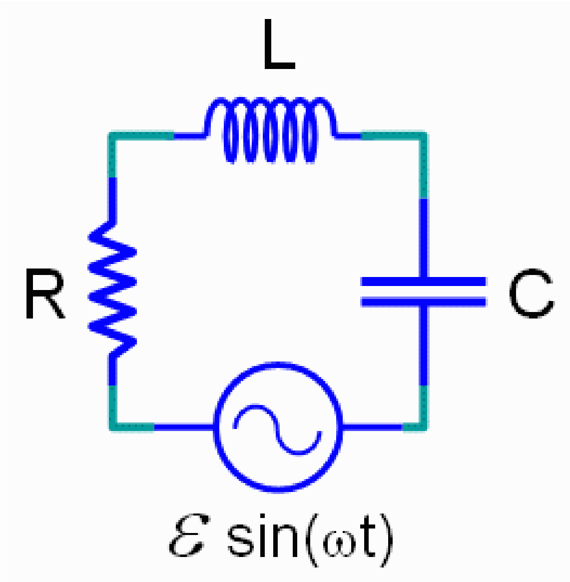
What does the voltage phasor diagram look like when the current is a maximum?

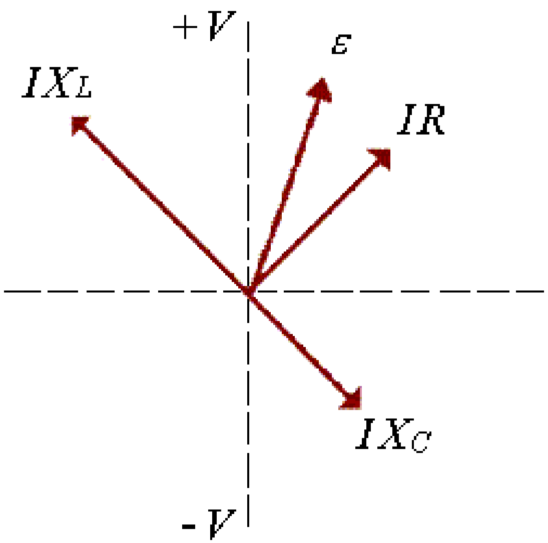


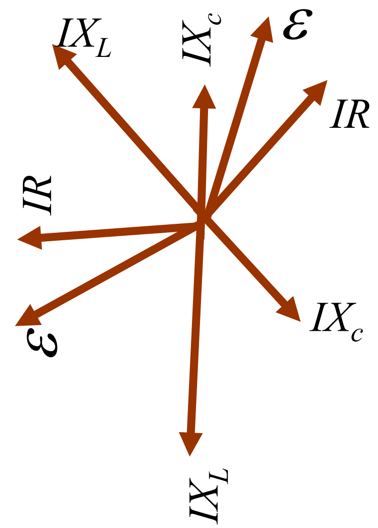
T

Check Point 2b

A driven RLC circuit is represented by the phasor diagram below.







When the capacitor is fully charged, what is the magnitude of the voltage across the inductor?

A) $V_L = 0$

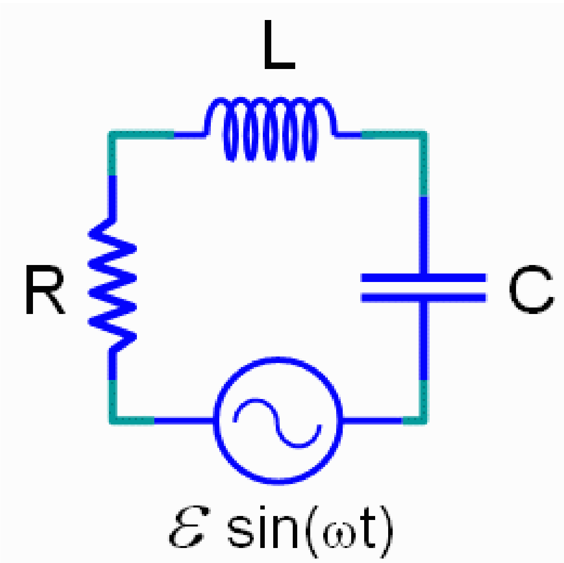
B) $V_L = V_{L,max}/2$

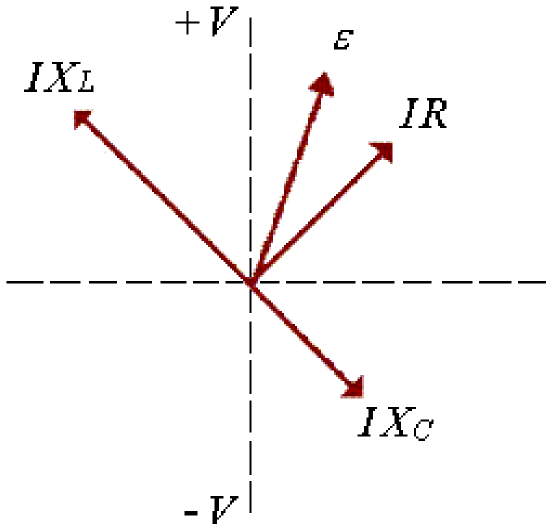
C) $V_L = V_{L,max}$

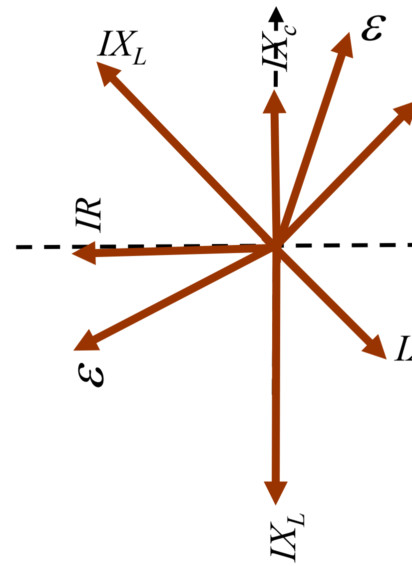
What does the voltage phasor diagram look like when the capacitor is fully charged?

Check Point 2C

A driven RLC circuit is represented by the phasor diagram below.







When the voltage across the capacitor is at its positive maximum, $V_C = +V_{C,max}$, what is the voltage across the inductor ?

A) $V_L = 0$ ***B)*** $V_L = V_{L,max}$ $\boxed{\textbf{\textit{C)}}\ V_L = -V_{L,max}}$

What does the voltage phasor diagram look like when the voltage across capacitor is at its positive maximum?

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

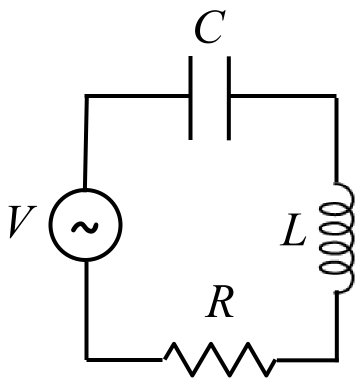
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



Conceptual Analysis

The maximum voltage for each component is related to its reactance and to the maximum current.

The impedance triangle determines the relationship between the maximum voltages for the components

Strategic Analysis

Use V_{max} and I_{max} to determine Z

Use impedance triangle to determine R

Use V_{Cmax} and impedance triangle to determine X_L

Calculation

Bonus Point 2

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

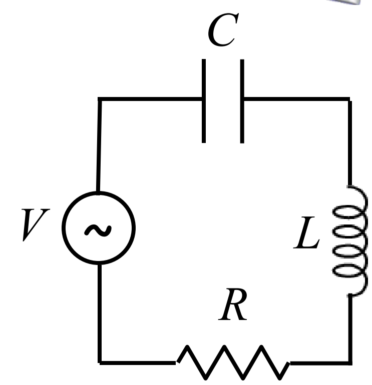
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



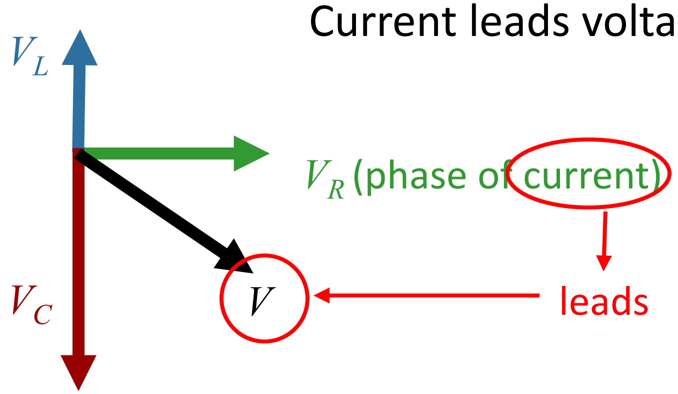
Compare X_L and X_C at this frequency:

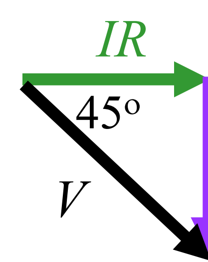
A) $X_L < X_C$ B) $X_L = X_C$ C) $X_L > X_C$ D) Not enough information

This information is determined from the phase

Current leads voltage

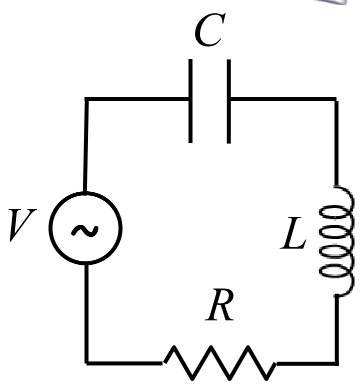
$$V_L = I_{\max} X_L$$
$$V_C = I_{\max} X_C$$





Calculation

Bonus Point 3



Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^{o}

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?

What is Z, the total impedance of the circuit?

A) $70.7\ k\Omega$ B) $50\ k\Omega$ C) $35.4\ k\Omega$ D) $21.1\ k\Omega$

$$Z = \frac{V_{\max}}{I_{\max}} = \frac{100V}{2mA} = 50k\Omega$$

Calculation

Bonus Point 4

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

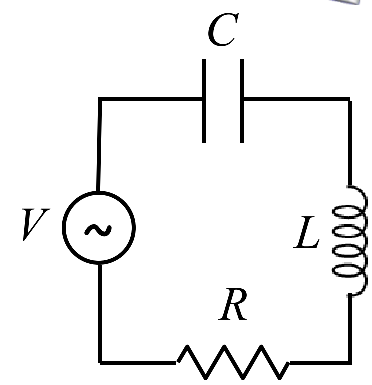
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



$Z = 50k\Omega$

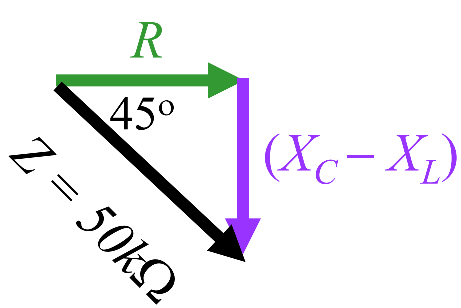
$\sin(45) = .707$

$\cos(45) = .707$

What is R?

A) $70.7\ k\Omega$ B) $50\ k\Omega$ C) $35.4\ k\Omega$ D) $21.1\ k\Omega$

Determined from impedance triangle



$$\cos(45) = \frac{R}{Z} \rightarrow R = Z\cos(45^o)$$

$$= 50\ k\Omega \times 0.707$$

$$= 35.4\ k\Omega$$

T

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

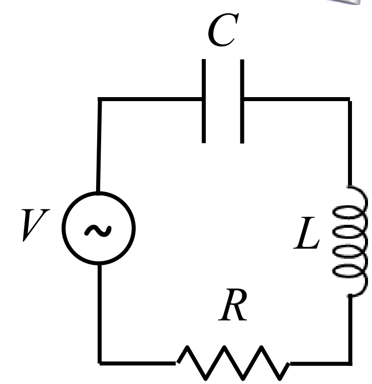
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?

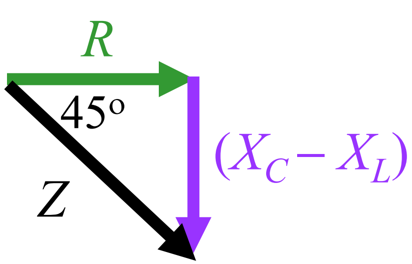


$Z = 50k\Omega$

$R = 35.4k\Omega$

A) $70.7\ k\Omega$ B) $50\ k\Omega$ C) $35.4\ k\Omega$ D) $21.1\ k\Omega$

We start with the impedance triangle:



$$\frac{X_C - X_L}{R} = \tan 45° = 1 \rightarrow X_L = X_C - R$$

What is X_C?

$$V_{Cmax} = I_{max} X_C$$

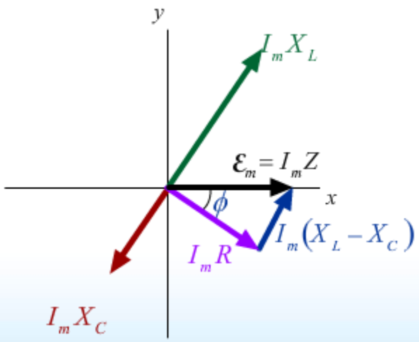
$$X_C = \frac{113}{2} = 56.5k\Omega$$

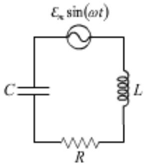
$$X_L = 56.5\ k\Omega - 35.4\ k\Omega$$

A

Physics 212
Lecture 21

Voltage Phasor Diagram





Phase Relation

$$\tan\phi = \frac{X_L - X_C}{R}$$

Impedance

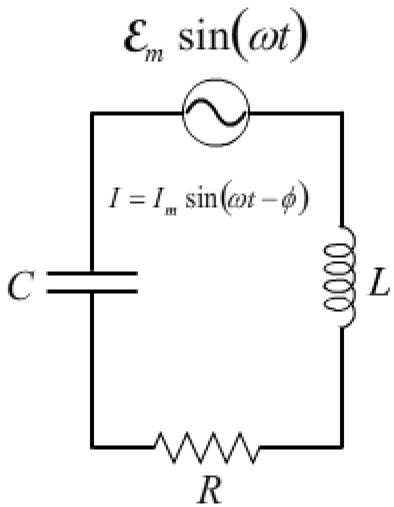
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Maximum Current

$$I_m = \frac{\mathcal{E}_m}{Z}$$

Looks intimidating, but is OK w/ practice

The Driven LCR Circuit



Frequency Dependence of Maximum Current

$$I_m = \frac{\mathcal{E}_m}{R}\frac{1}{\sqrt{1+Q^2\frac{(x^2-1)^2}{x^2}}}$$

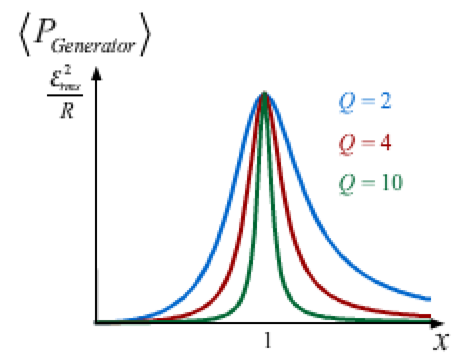
Quality Factor

$$Q \equiv 2\pi\left[\frac{U_{max}}{\Delta U}\right]_{cycle} \xrightarrow{\text{evaluate at}} \omega = \omega_o$$

Average Power per Cycle

$$\langle P_{Generator}\rangle = \frac{\mathcal{E}_{rms}^2}{R}\frac{x^2}{x^2+Q^2(x^2-1)^2}$$

where $x \equiv \dfrac{\omega}{\omega_o}$ & $Q^2 = \dfrac{L}{R^2C}$



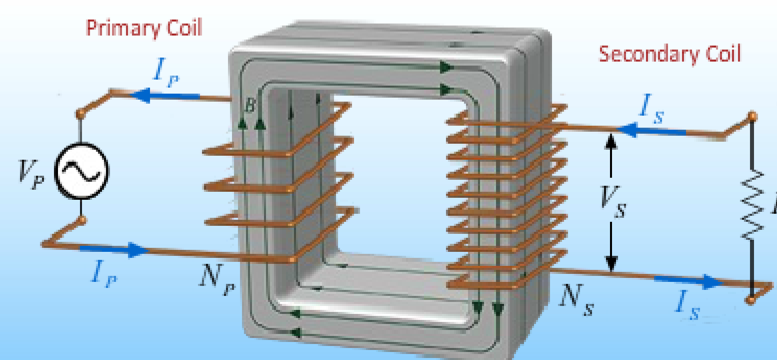
Transformers

Voltage Relation

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Current Relation

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

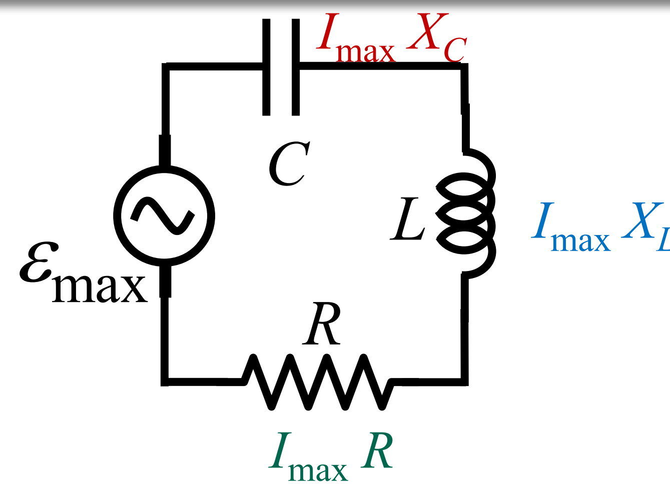


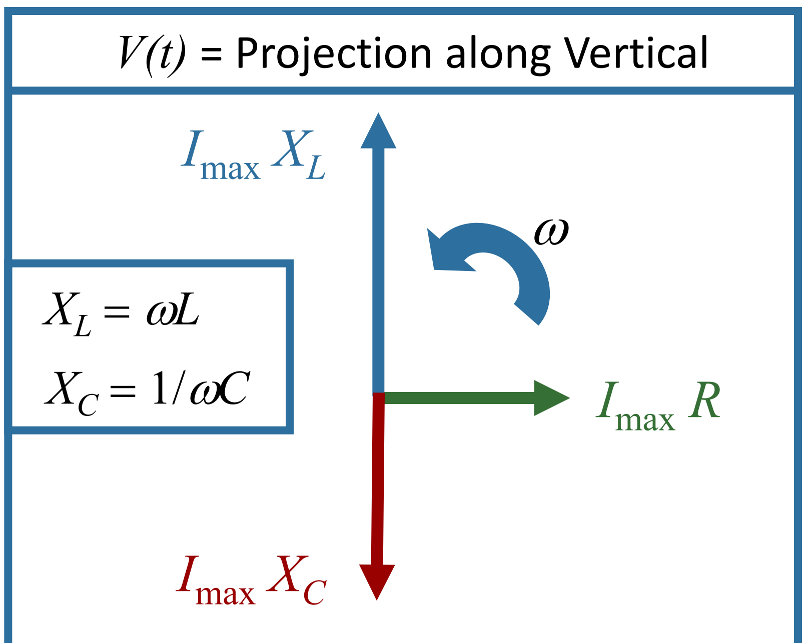
AC Circuits & Phasors

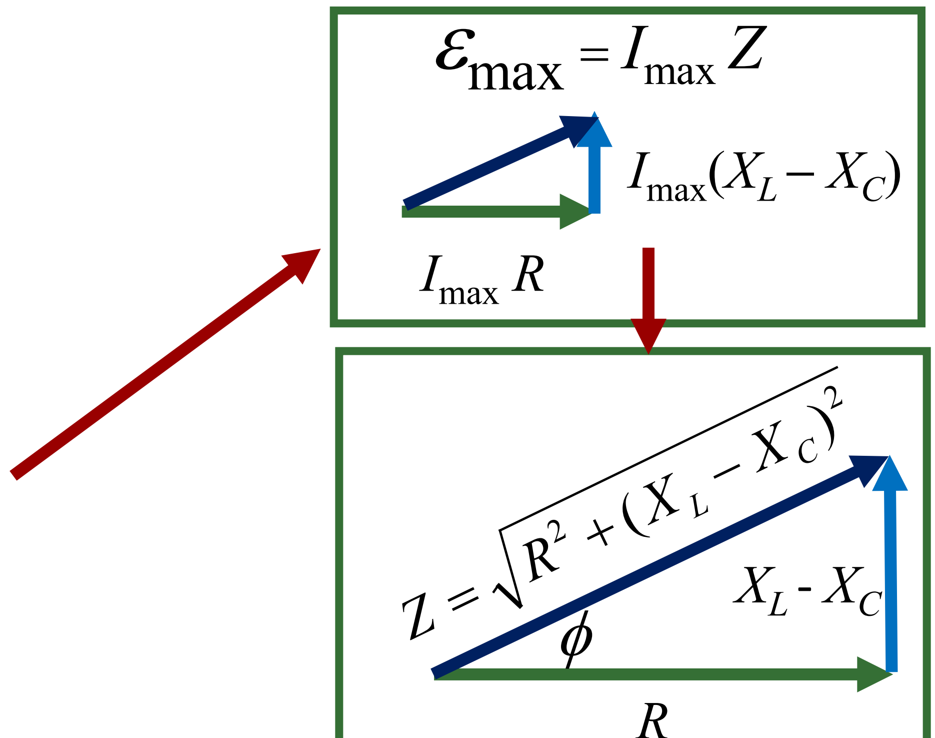
PHASORS ARE THE KEY !
FORMULAS ARE NOT !

START WITH PHASOR DIAGRAM

DEVELOP FORMULAS FROM THE DIAGRAM !!







Peak AC Problems

"Ohms" Law for each element

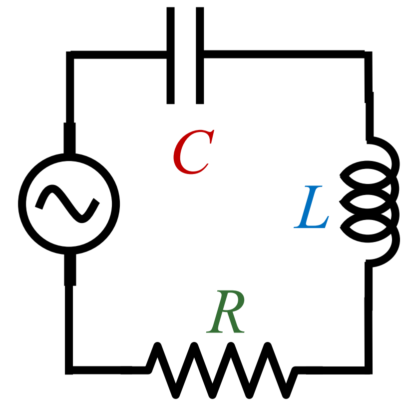
NOTE: Good for MAXIMUM values only!

$$V_{gen} = I_{\max} Z$$
$$V_{Resistor} = I_{\max} R$$
$$V_{inductor} = I_{\max} X_L$$
$$V_{Capacitor} = I_{\max} X_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

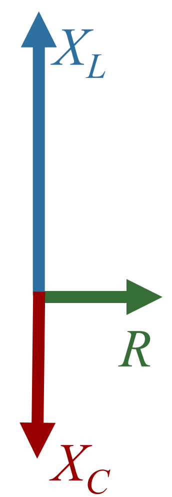
$$X_L = \omega L \quad X_C = \frac{1}{\omega C}$$

$$P_{Average} = \frac{1}{2} I_{\max}^2 R$$



Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit and average power dissipated?



$$X_L = \omega L = 200\,\Omega \qquad Z = \sqrt{R^2 + (X_L - X_C)^2} = 112\,\Omega$$

$$X_C = \frac{1}{\omega C} = 100\,\Omega \qquad I_{\max} = \frac{V_{gen}}{Z} = 0.13\,A$$

$$P_{Average} = \frac{1}{2} I_{\max}^2 R = 0.42 \text{ Watts}$$

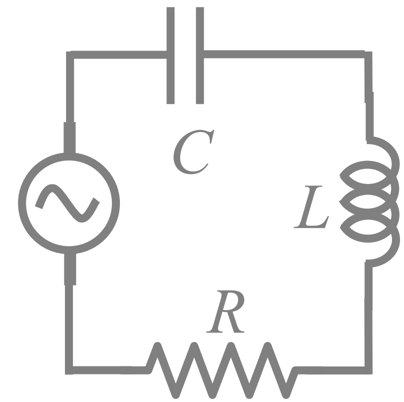
Peak AC Problems

"Ohms" Law for each element

NOTE: Good for PEAK values only)

$$V_{gen} = I_{max} Z$$
$$V_{Resistor} = I_{max} R$$
$$V_{inductor} = I_{max} X_L$$
$$V_{Capacitor} = I_{max} X_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$X_L = \omega L$$
$$X_C = \frac{1}{\omega C}$$



Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

Which element has the largest peak voltage across it?

A) Generator

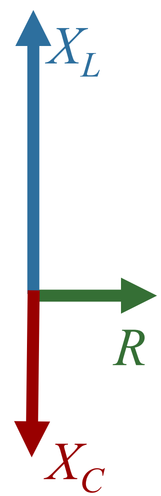
B) Inductor

C) Resistor

D) Capacitor

E) All the same.

$$V_{max} = I_{max} X$$



$$X_L = \omega L = 200\,\Omega$$
$$X_C = \frac{1}{\omega C} = 100\,\Omega$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 122\,\Omega$$
$$I_{max} = \frac{V_{gen}}{Z} = 0.13\,A$$

Peak AC Problems

"Ohms" Law for each element

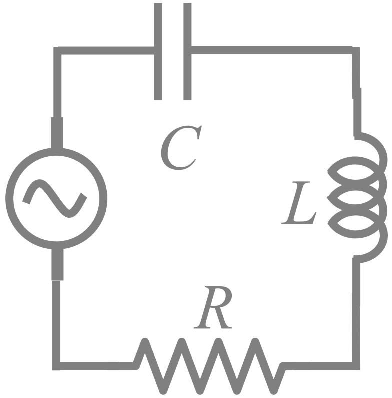
NOTE: Good for PEAK values only)

$$V_{gen} = I_{max} Z$$
$$V_{Resistor} = I_{max} R$$
$$V_{inductor} = I_{max} X_L$$
$$V_{Capacitor} = I_{max} X_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$



Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

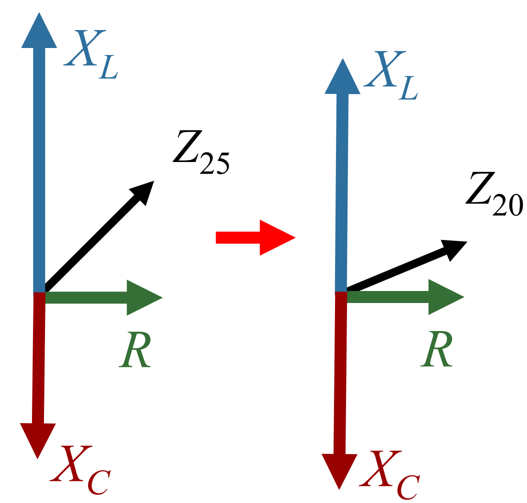
What happens to the impedance if we decrease the angular frequency to 20 rad/sec?

A) Z increases

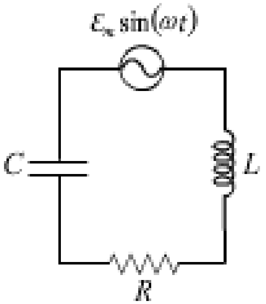
B) Z remains the same

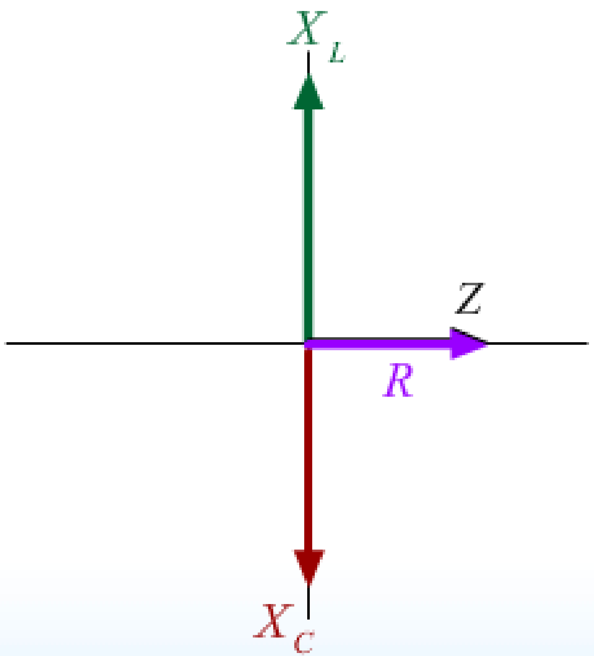
C) Z decreases

$(X_L - X_C)$: (200 – 100) -> (160 – 125)



Resonance





Resonance

I_m is a maximum $\longrightarrow$ $I_m = \dfrac{\mathcal{E}_m}{R}$

$\omega = \omega_o$

Z minimized $\longrightarrow$ $X_L = X_C$

$\phi = 0^o$

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$

Resonance

Frequency at which voltage across inductor and capacitor cancel

R is independent of ω

X_L increases with ω

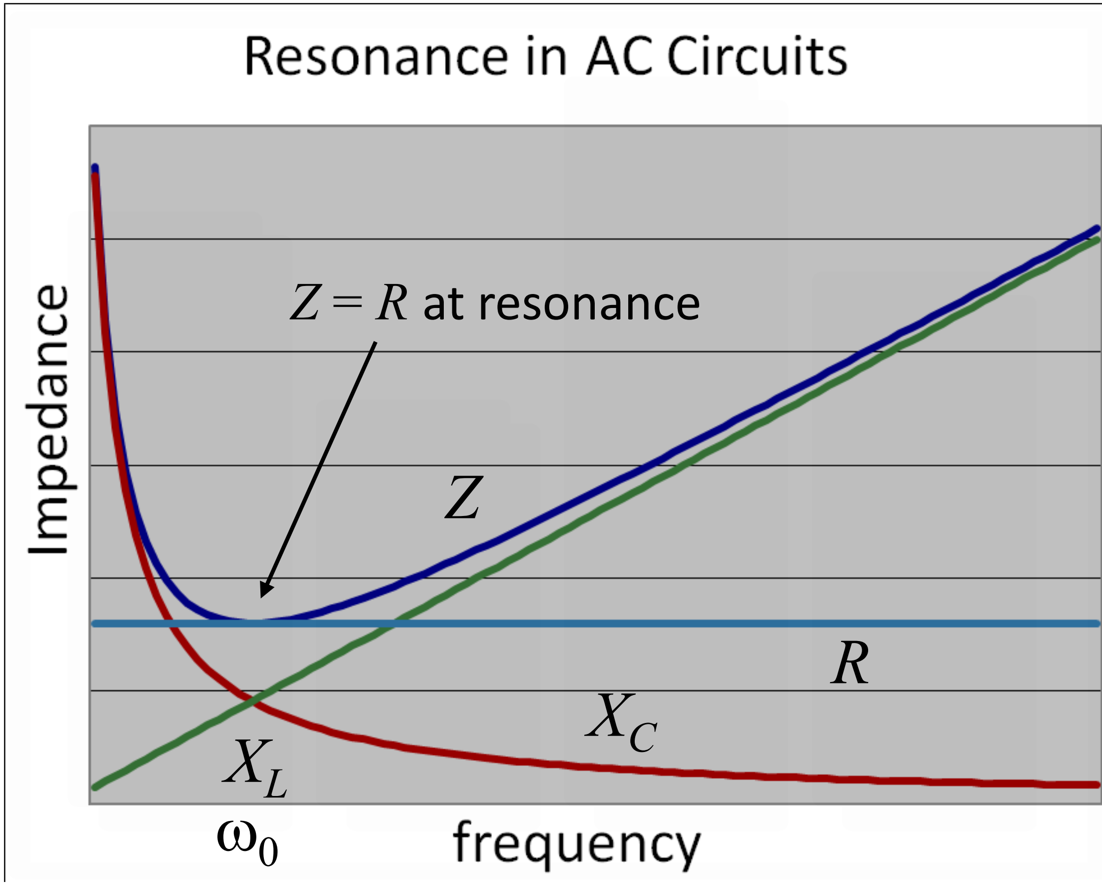
$$X_L = \omega L$$

X_C increases with $1/\omega$

$$X_C = \frac{1}{\omega C}$$

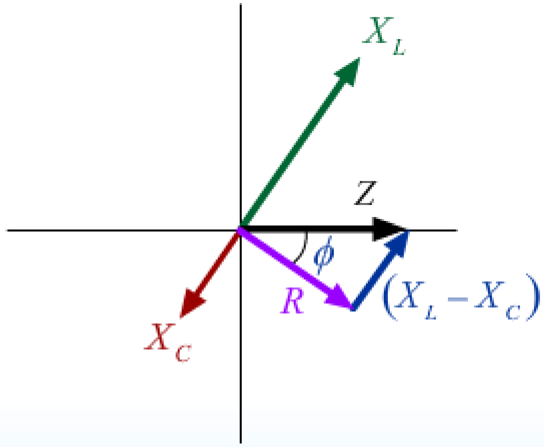
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

is minimum at resonance



Resonance: $X_L = X_C$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

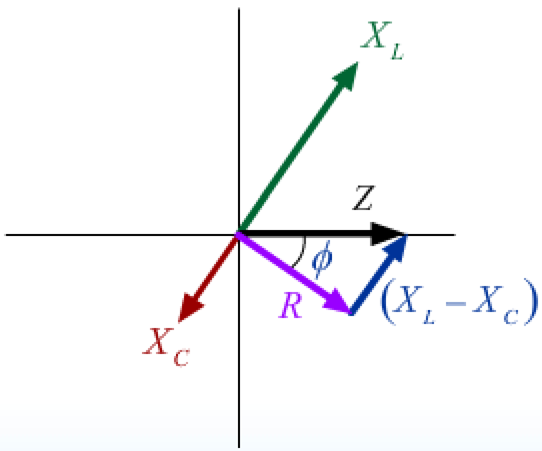
Off Resonance



$$I_m = \frac{\mathcal{E}_m}{Z}$$

$$I_m = \frac{\mathcal{E}_m}{R}\frac{R}{\boxed{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}}$$

$\boldsymbol{Z}$

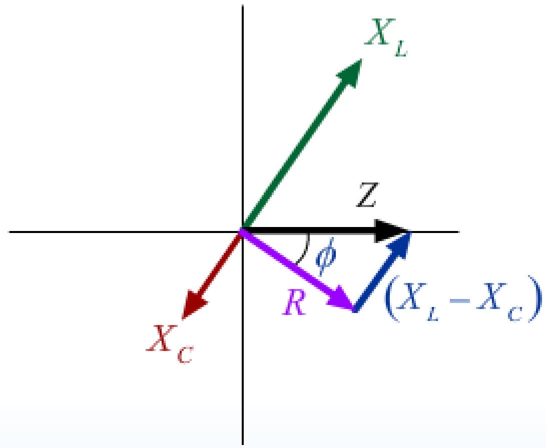


$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C} \longrightarrow Q \equiv 2\pi \frac{U_{\max}}{\Delta U}$$

$$\boxed{I_m = \frac{\mathcal{E}_m}{R}\frac{1}{\sqrt{1 + Q^2 \frac{(x^2-1)^2}{x^2}}}}$$

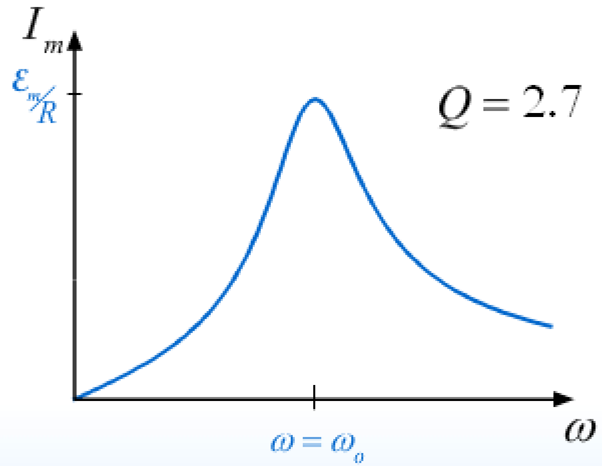
U_{max} = max energy stored
ΔU = energy dissipated
in one cycle at resonance

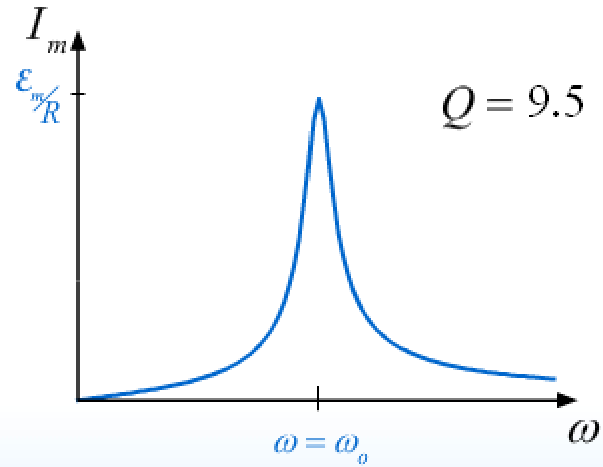
Off Resonance



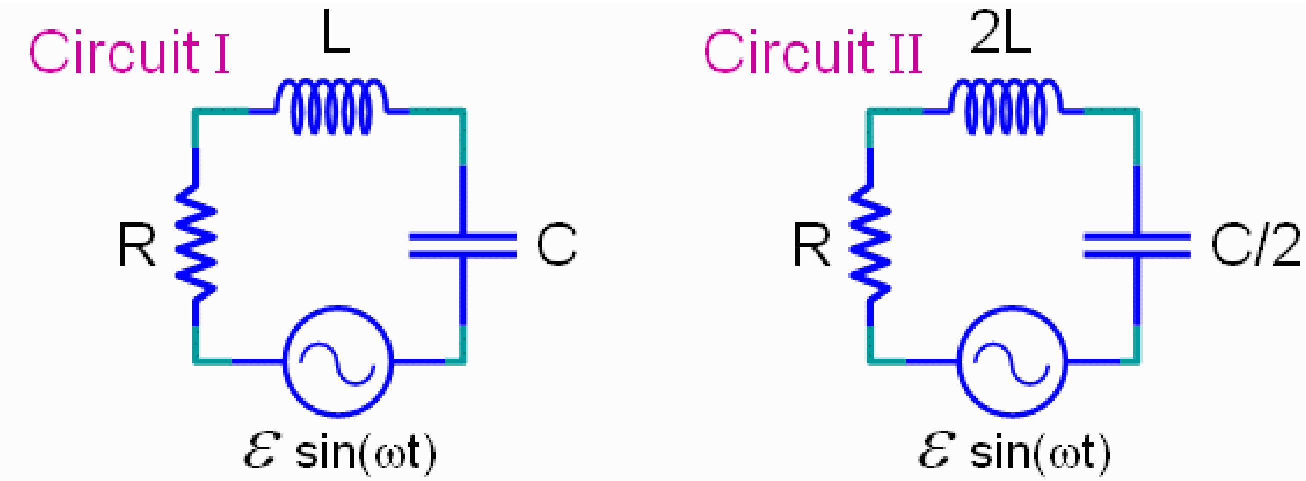
$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C}$$

$$I_m = \frac{\varepsilon_m}{R} \frac{1}{\sqrt{1 + Q^2 \frac{(x^2 - 1)^2}{x^2}}}$$





Check Point 1a



Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

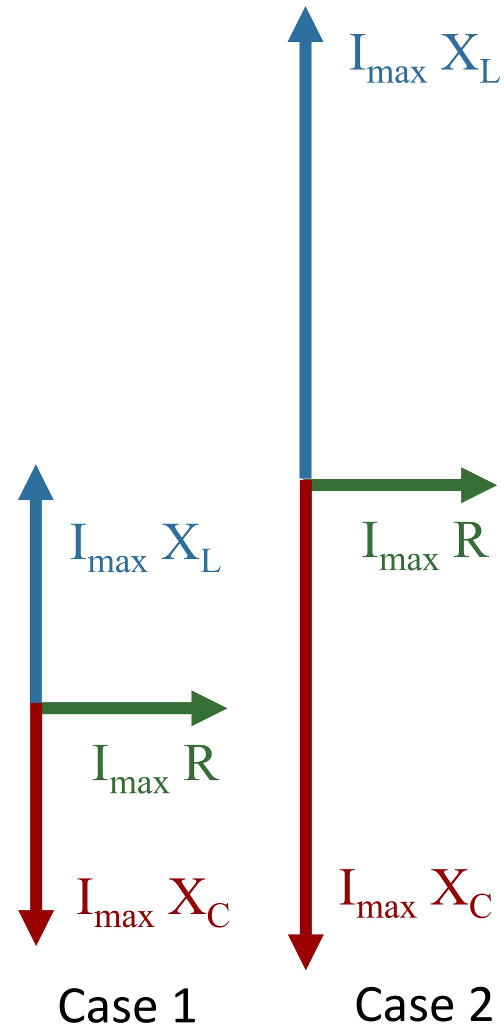
Compare the peak voltage across the resistor in the two circuits

A. $V_I > V_{II}$ ***B.*** $\boxed{V_I = V_{II}}$ ***C.*** $V_I < V_{II}$

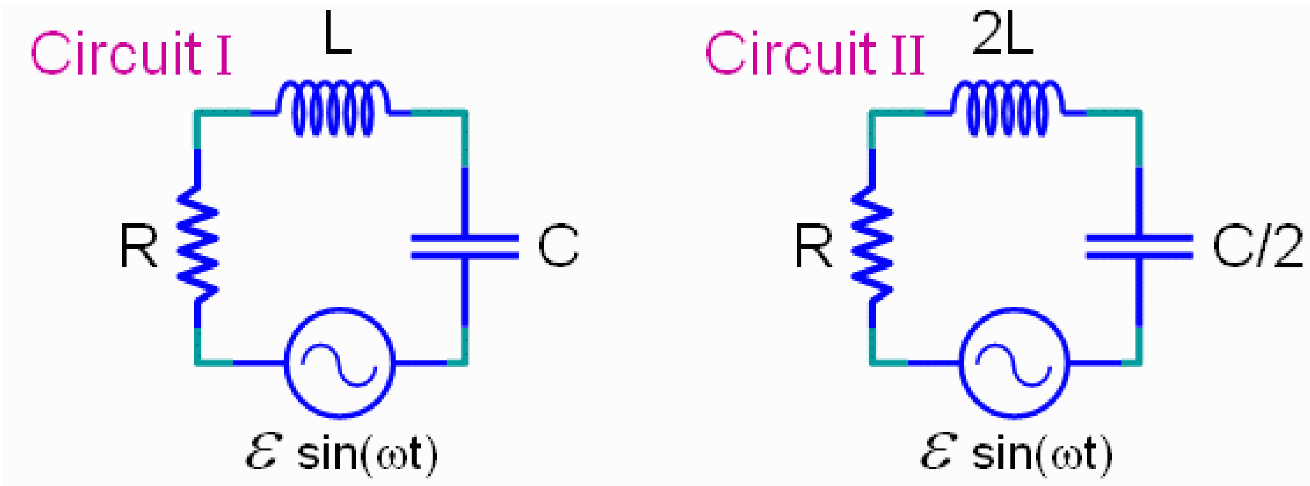
Resonance: $X_L = X_C$

$Z = R$

Same since R doesn't change



Check Point 1b

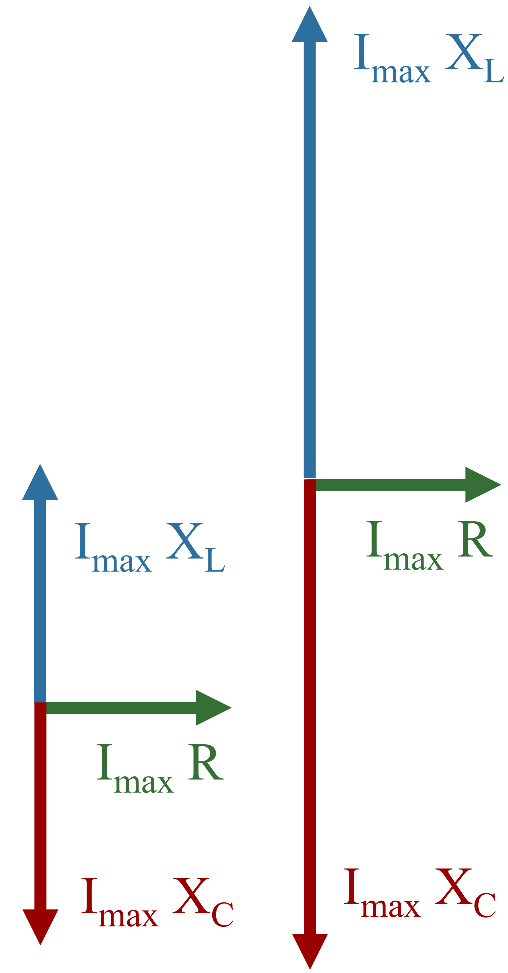


Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

Compare the peak voltage across the inductor in the two circuits

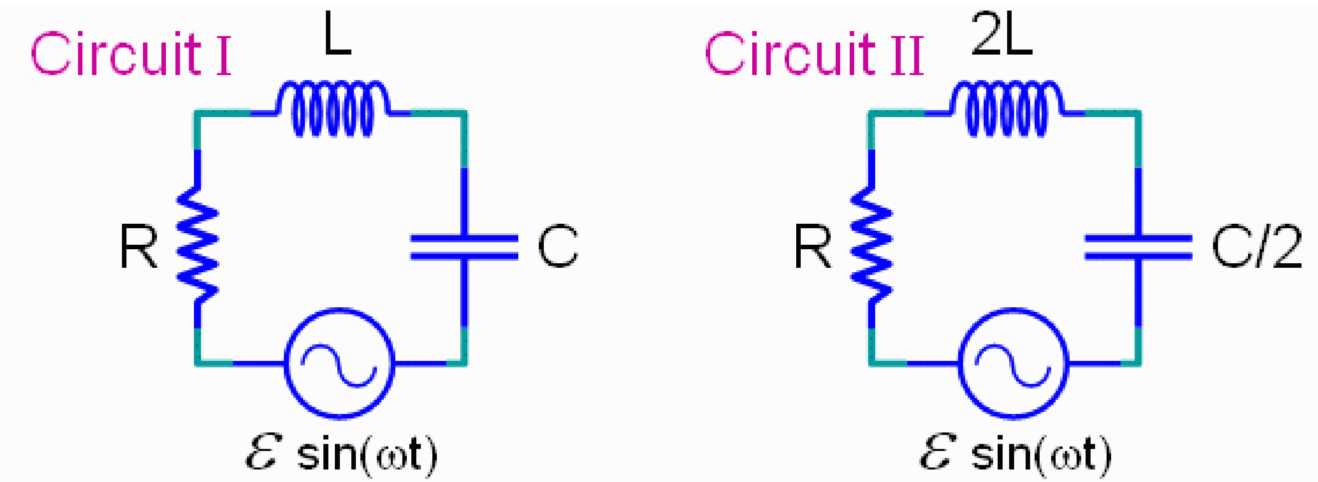
A. $V_I > V_{II}$ ***B.*** $V_I = V_{II}$ ***C.*** $V_I < V_{II}$

circuit 2 has twice the inductance and $V_L = I\,\omega\,L$. I is the same for both



Case 1 Case 2

CheckPoint 1c

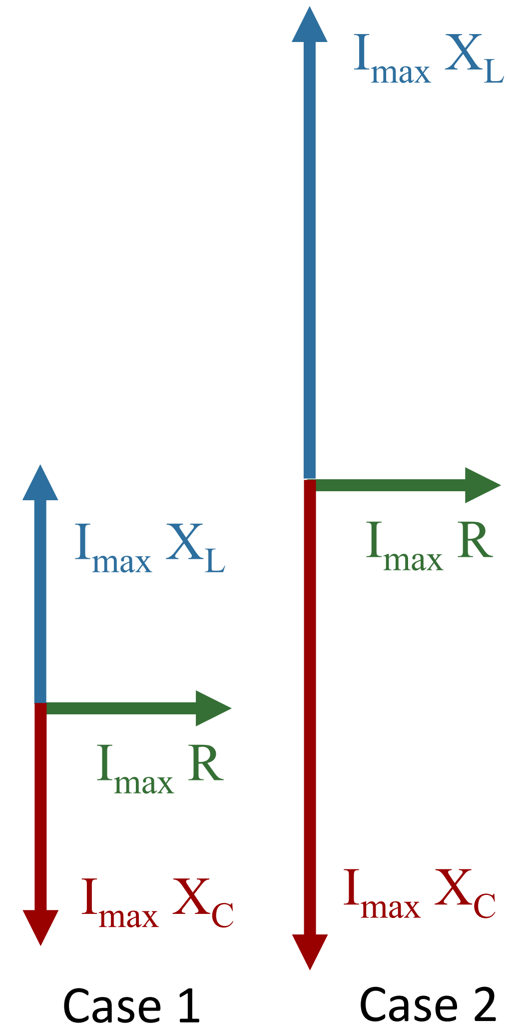


Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

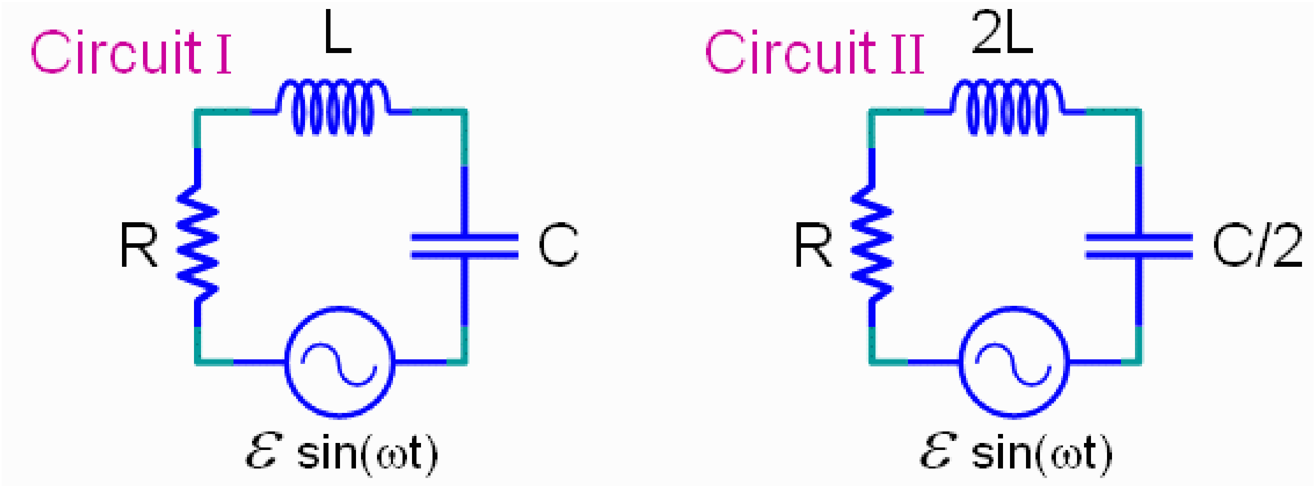
Compare the peak voltage across the capacitor in the two circuits

A. $V_I > V_{II}$ ***B.*** $V_I = V_{II}$ $\boxed{\textbf{\textit{C.}}\ V_I < V_{II}}$

Smaller capacitance, greater reactance therefore greater voltage if current is equal.



Check Point 1d



Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

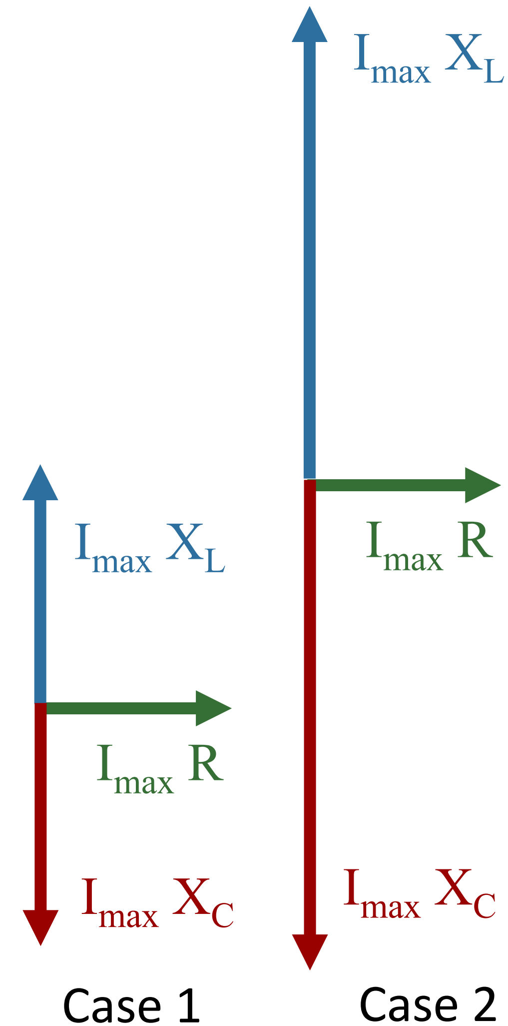
At the resonant frequency, which of the following is true?

A. Current leads voltage across the generator

B. Current lags voltage across the generator

C. Current is in phase with voltage across the generator

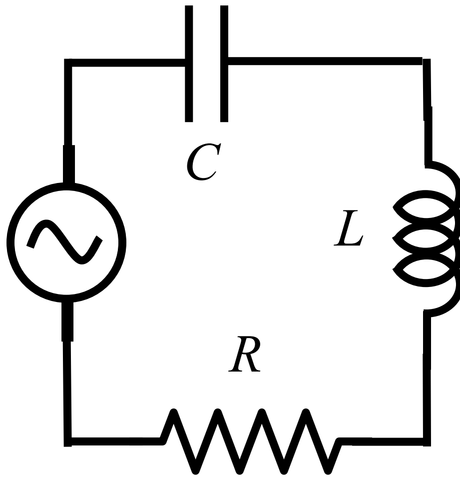
At resonant frequency, the current and the voltage across the generator are in phase because the voltages across the inductor and the capacitor cancel each other out.



Power

$P = IV$ instantaneous always true

- Difficult for Generator, Inductor and Capacitor because of phase
- Resistor I, V are always in phase!



Average power

$$P_{\text{ave}} = \frac{1}{T}\int_0^T p(t)dt$$

where $T = 2\pi/\omega$ is the period of the oscillations. With the substitutions $v(t) = V_0 \sin \omega t$ and $i(t) = I_0 \sin(\omega t - \phi)$, this integral becomes

$$P_{\text{ave}} = \frac{I_0 V_0}{T}\int_0^T \sin(\omega t - \phi)\sin \omega t \, dt$$

Using the trigonometric relation $\sin(A - B) = \sin A \cos B - \sin B \cos A$, we obtain

$$P_{\text{ave}} = \frac{I_0 V_0 \cos\phi}{T}\int_0^T \sin^2 \omega t dt - \frac{I_0 V_0 \sin\phi}{T}\int_0^T \sin \omega t \cos \omega t dt$$

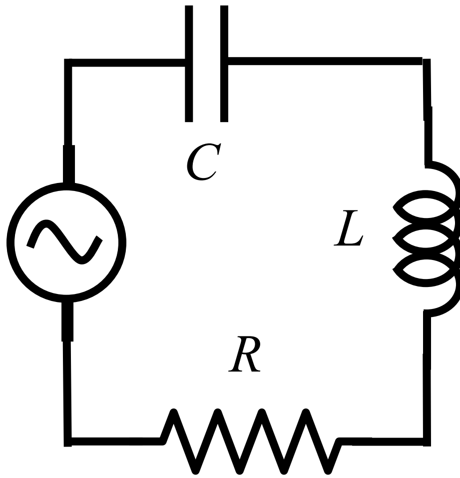
$$\frac{1}{T}\int_0^T \sin^2 \omega t dt = \frac{1}{2} \qquad \frac{1}{T}\int_0^T \sin \omega t \cos \omega t dt = 0$$

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0 \cos\phi$$

Average Power

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0 \cos \phi$$

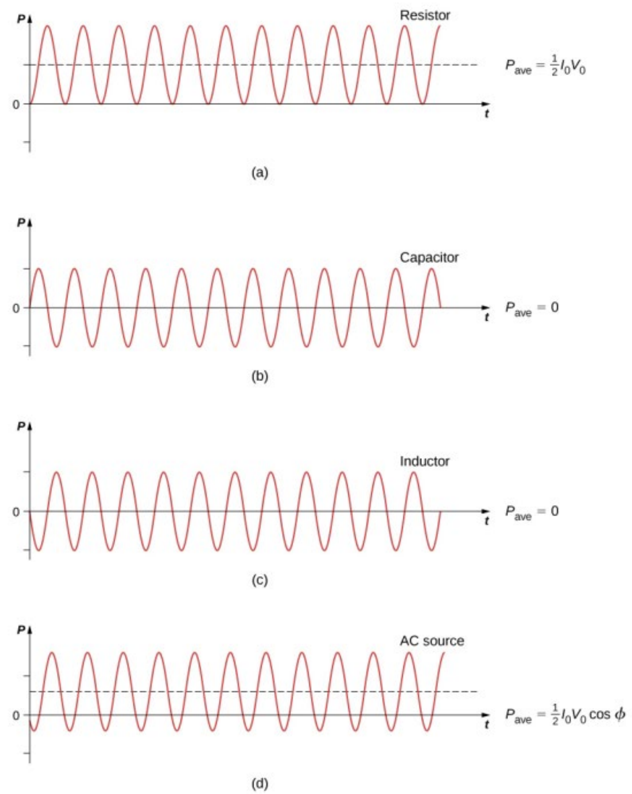
$\cos \phi$ is known as the **power factor**



For a resistor, $\phi = 0$, so the average power dissipated is

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0$$

For capacitor or inductor, $P_{\text{ave}} = 0$!



rms values

To make $P_{ave} = ½\, I_0 V_0$ look like its dc counterpart, we use rms values I_{rms} and V_{rms}

$$I_{rms} = \sqrt{i^2_{ave}} \text{ and } V_{rms} = \sqrt{v^2_{ave}}$$

$$i^2_{ave} = \frac{1}{T}\int_0^T i^2(t)dt \text{ and } v^2_{ave} = \frac{1}{T}\int_0^T v^2(t)dt$$

With $i(t) = I_0 \sin(\omega t - \phi)$ and $v(t) = V_0 \sin \omega t$, we obtain

$$I_{rms} = \frac{1}{\sqrt{2}} I_0 \text{ and } V_{rms} = \frac{1}{\sqrt{2}} V_0$$

P_{ave} dissipated by a resistor:

$$P_{ave} = \frac{1}{2} I_0 V_0 = I_{rms} V_{rms} = I^2_{rms} R$$

The phase angle for an ac generator may have any value. If $\cos\phi > 0$, the generator produces power; if $\cos\phi < 0$, it absorbs power. In terms of rms values, the average power of an ac generator is written as

$$P_{ave} = I_{rms} V_{rms} \cos\phi.$$

Power Line Calculation

Have generator that produces 1,500 Watts at 100 Volts over transmission lines w/ resistance of 5 Ohms. How much power is lost in the lines?

- Current Required: $I = P/V = 15$ Amps
- Loss $= IV$ (on line) $= I^2 R = 15*15 * 5 = 1{,}125$ Watts!

If you deliver 1,500 Watts at 10,000 Volts over the same transmission lines. How much power is lost?

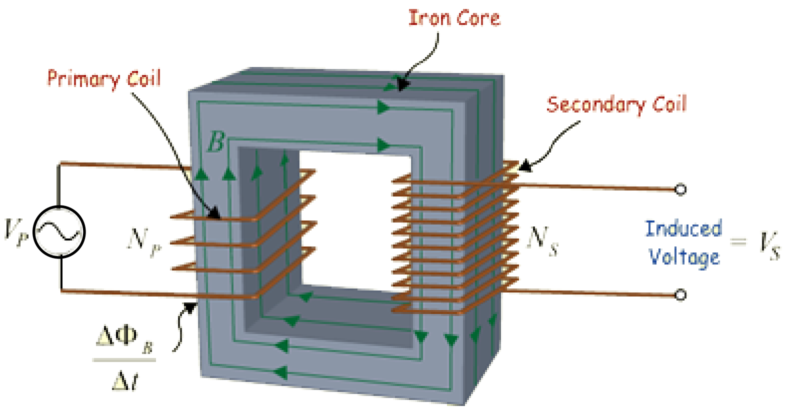
- Current Required: $I = P/V = .15$ Amps
- Loss $= IV$ (on line) $= I^2 R = 0.125$ Watts

Transformers

Application of Faraday's Law

- Changing EMF in Primary creates changing flux
- Changing flux, creates EMF in secondary

$$\frac{V_p}{N_p} = \frac{V_s}{N_s}$$



Efficient method to change voltage for AC.

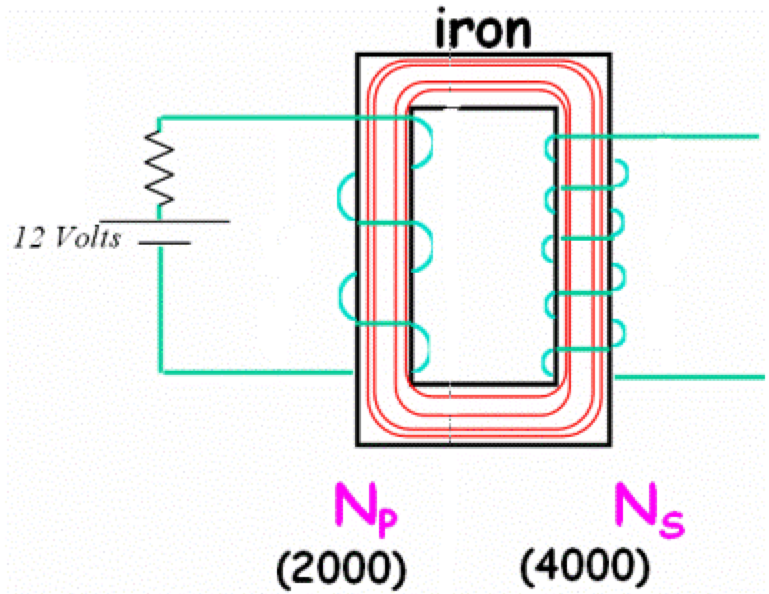
Power Transmission Loss $= I^2R$

Power electronics

Tranformers

For coils connected to battery, what is voltage in secondary?

A) 0 Volts
B) 6 Volts
C) 12 Volts



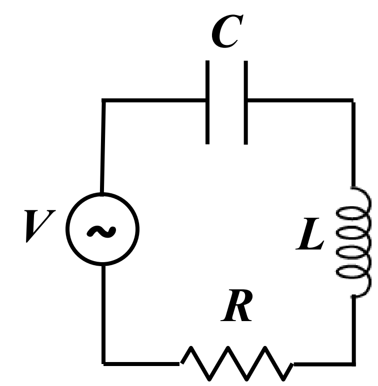
Wrong Answer: 2 ✗

Feedback: Actually if this was connected to an AC source, the secondary would have twice the primary voltage. However, the battery voltage does not change in time, so after the battery has been connected for a while, there will not be a changing current to create a voltage across the secondary.

Calculation from last lecture

Consider the harmonically driven series LCR circuit shown.
$V_{max} = 100\ V$ *(peak voltage across generator)*
$I_{max} = 2\ mA$
$V_{Cmax} = 113\ V$ *(peak voltage across capacitor)*
The current leads generator voltage by 45^o
L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



Conceptual Analysis

The maximum voltage for each component is related to its reactance and to the maximum current.

The impedance triangle determines the relationship between the maximum voltages for the components

Strategic Analysis

Use V_{max} and I_{max} to determine Z

Use impedance triangle to determine R

Use V_{Cmax} and impedance triangle to determine X_L

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$ *(peak voltage across generator)*

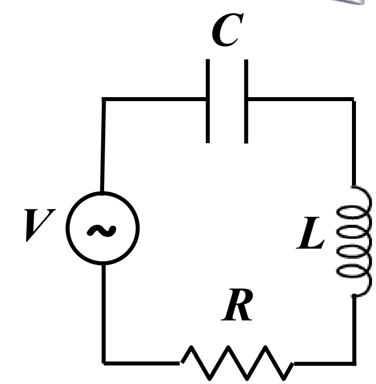
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ *(peak voltage across capacitor)*

The current leads generator voltage by 45^{o}

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



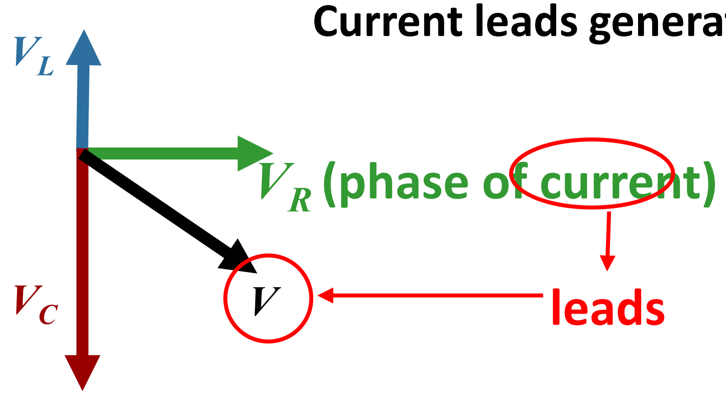
Compare X_L and X_C at this frequency:

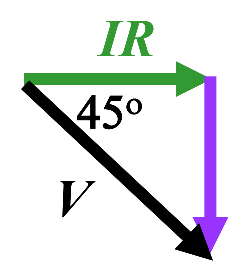
A) $X_L < X_C$ **B)** $X_L = X_C$ **C)** $X_L > X_C$ **D)** Not enough information

This information is determined from the phase

Current leads generator voltage

$$V_L = I_{\max} X_L$$
$$V_C = I_{\max} X_C$$





Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$ *(peak voltage across generator)*

$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ *(peak voltage across capacitor)*

The current leads generator voltage by 45^{o}

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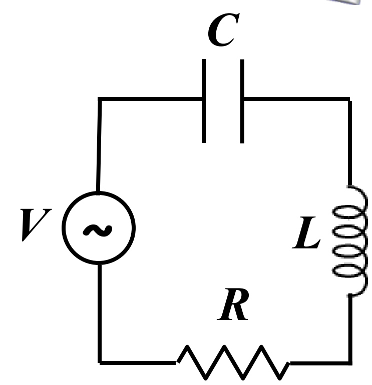
What is X_L, the reactance of the inductor, at this frequency?



What is Z, the total impedance of the circuit?

A) $70.7\ k\Omega$ B) $\boxed{50\ k\Omega}$ C) $35.4\ k\Omega$ D) $21.1\ k\Omega$

$$Z = \frac{V_{\max}}{I_{\max}} = \frac{100V}{2mA} = 50k\Omega$$

Calculation

Consider the harmonically driven series *LCR* circuit shown.

$V_{max} = 100\ V$ *(peak voltage across generator)*

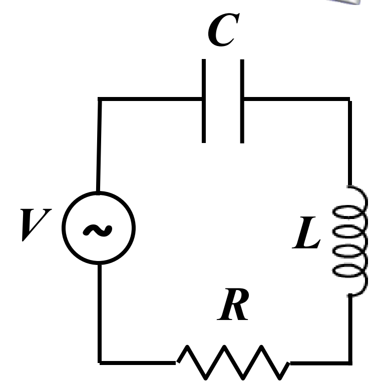
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ *(peak voltage across capacitor)*

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



$Z = 50k\Omega$

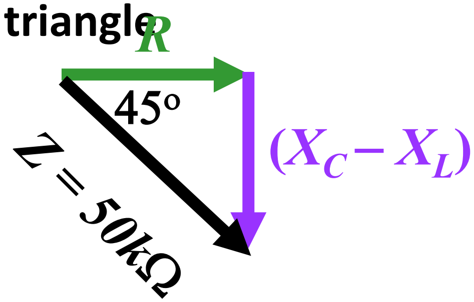
$\sin(45) = .707$

$\cos(45) = .707$

What is R?

A) $70.7\ k\Omega$ B) $50\ k\Omega$ C) $35.4\ k\Omega$ D) $21.1\ k\Omega$

Determined from impedance triangle



$$\cos(45) = \frac{R}{Z} \rightarrow R = Z\cos(45^o)$$

$$= 50\ k\Omega \times 0.707$$

$$= 35.4\ k\Omega$$

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$ *(peak voltage across generator)*

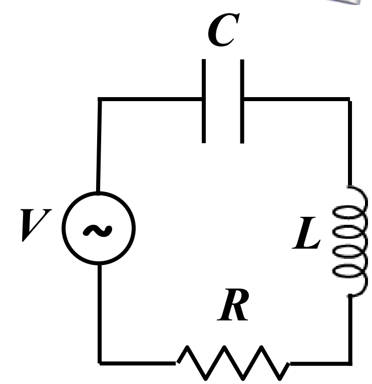
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ *(peak voltage across capacitor)*

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



$Z = 50k\Omega$

A) $70.7\ k\Omega$

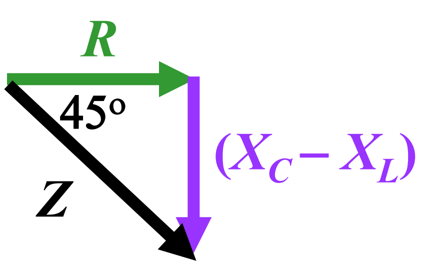
B) $50\ k\Omega$

C) $35.4\ k\Omega$

D) $21.1\ k\Omega$

$R = 35.4k\Omega$

We start with the impedance triangle:



$$\frac{X_C - X_L}{R} = \tan 45^\circ = 1 \rightarrow X_L = X_C - R$$

What is X_C?

$$V_{Cmax} = I_{max} X_C$$

$$X_C = \frac{113}{2} = 56.5k\Omega$$

$$X_L = 56.5\ k\Omega - 35.4\ k\Omega$$

Phasor Diagram

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

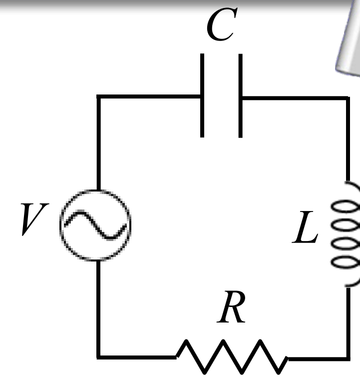
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

The current leads generator voltage by 45^o (cos = sin = 1/sqrt(2))

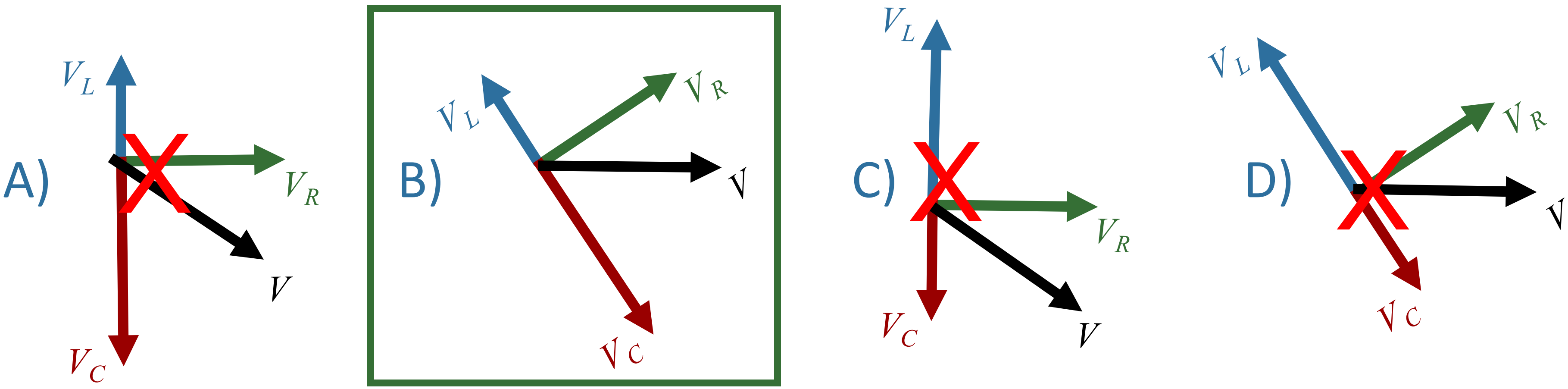
L and R are unknown.

What does the phasor diagram look like at t = 0? (assume $V = V_{max}\sin\omega t$)



$$R = 25\sqrt{2}\ k\Omega$$

$$X_L = 15\sqrt{2}\ k\Omega$$



$V = V_{max}\sin\omega t$ -> V is horizontal at $t = 0$ $(V = 0)$

$\vec{V} = \vec{V}_L + \vec{V}_C + \vec{V}_R$ → $V_L < V_C$ if current leads generator voltage

Bonus point1

Phasor Diagram

Consider the harmonically driven series LCR circuit shown.

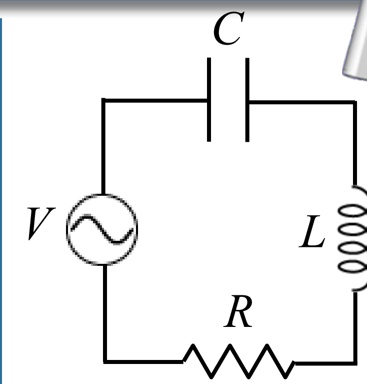
$V_{max} = 100\ V$

$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V\ (= 80\ \mathrm{sqrt}(2))$

The current leads generator voltage by 45^o $(\cos = \sin = 1/\mathrm{sqrt}(2))$

L and R are unknown.



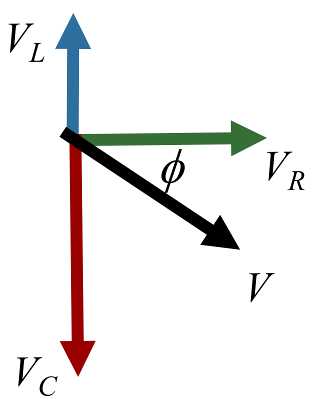
How should we change ω to bring circuit to resonance?

A) decrease ω

B) increase ω

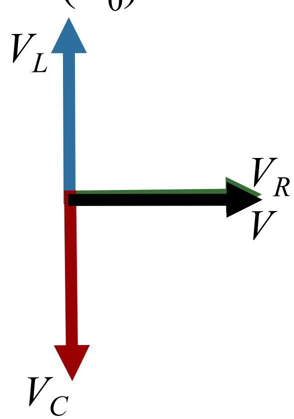
C) Not enough info

Original ω



At resonance

(ω_0)



At resonance

$X_L = X_C$

$\downarrow$

X_L increases

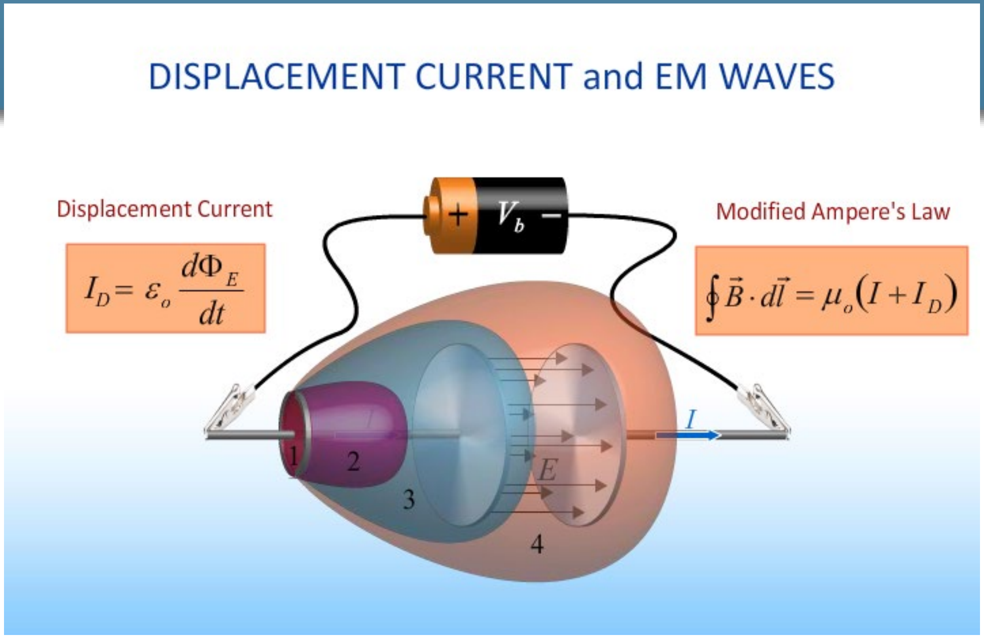
X_C decreases $\rightarrow$ ω increases

Bonus point2

Exam 3

Exam 3: Monday Dec 2nd , 6 - 7:30pm

Physics 212
Lecture 22



Let there be light!

What We Knew Before Prelecture 22

Pre-Maxwell's Equations

Gauss' Law for E Fields

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

Gauss' Law for B Fields

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$

A

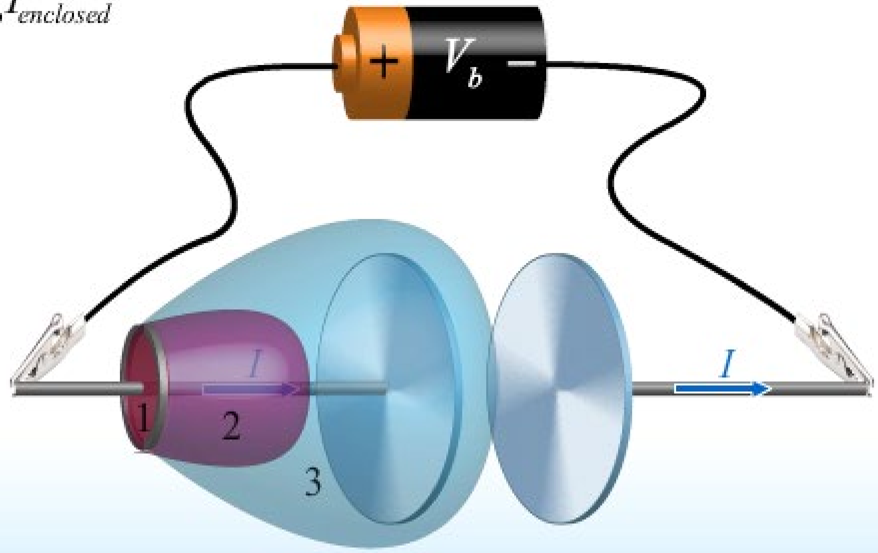
Here's the problem!

$$\oint \vec{B} \cdot d\vec{l}$$ Gives a specific number

$$\mu_0 I_{enclosed}$$ Depends on surface

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$



A

Here's the problem!

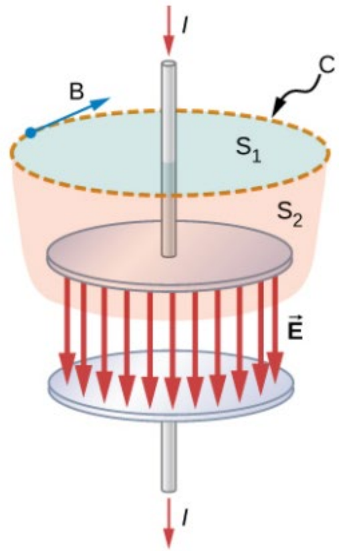
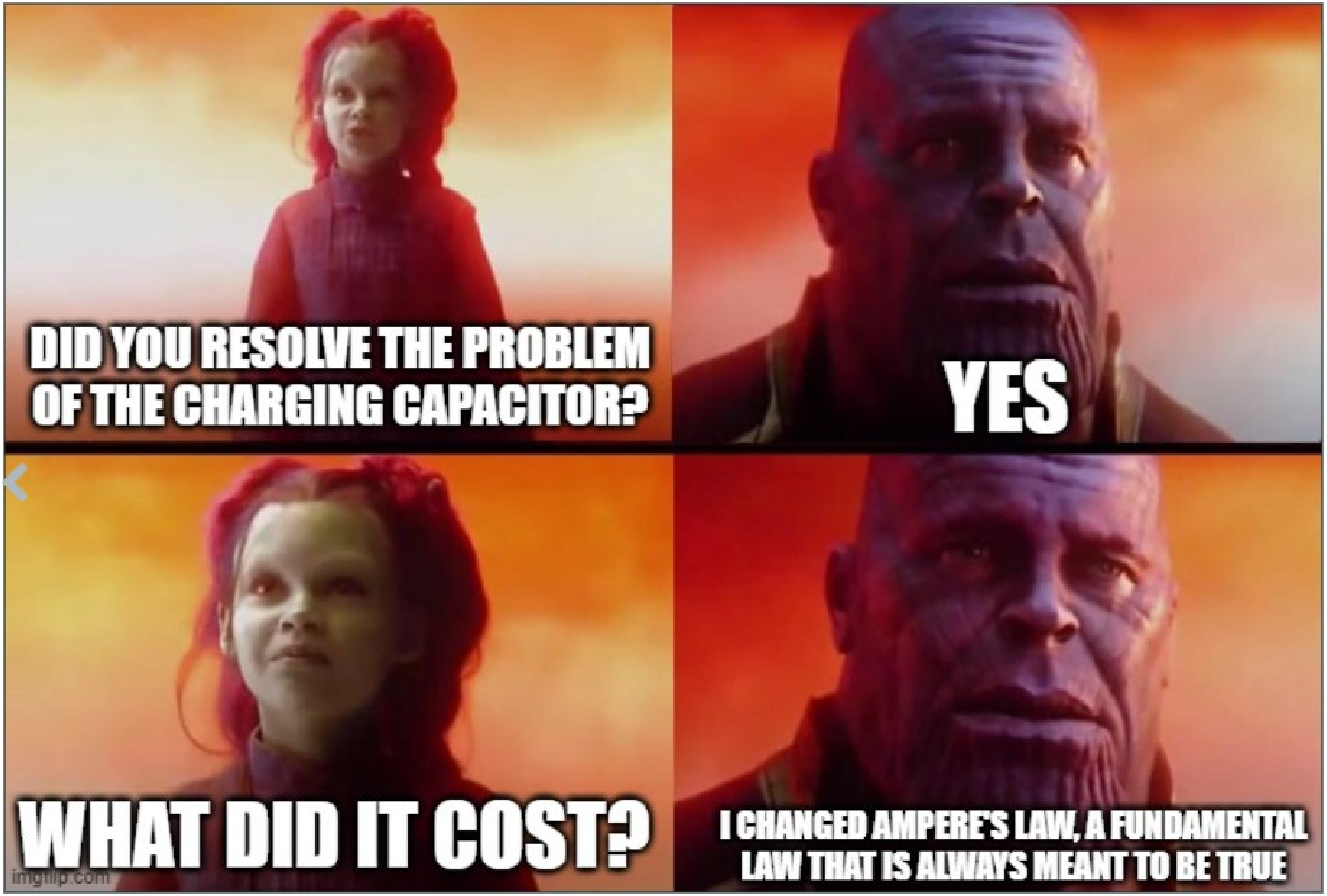


Figure 16.3 The currents through surface S_1 and surface S_2 are unequal, despite having the same boundary loop C.

Consider the set-up in Figure 16.3. A source of emf is abruptly connected across a parallel-plate capacitor so that a time-dependent current I develops in the wire. Suppose we apply Ampère's law to loop C shown at a time before the capacitor is fully charged, so that $I \neq 0$. Surface S_1 gives a nonzero value for the enclosed current I, whereas surface S_2 gives zero for the enclosed current because no current passes through it:

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \begin{cases} \mu_0 I & \text{if surface } S_1 \text{ is used} \\ 0 & \text{if surface } S_2 \text{ is used} \end{cases}.$$

Clearly, Ampère's law in its usual form does not work here. This is an internal contradiction in the theory which requires a modification to the theory, Ampère's law, itself.

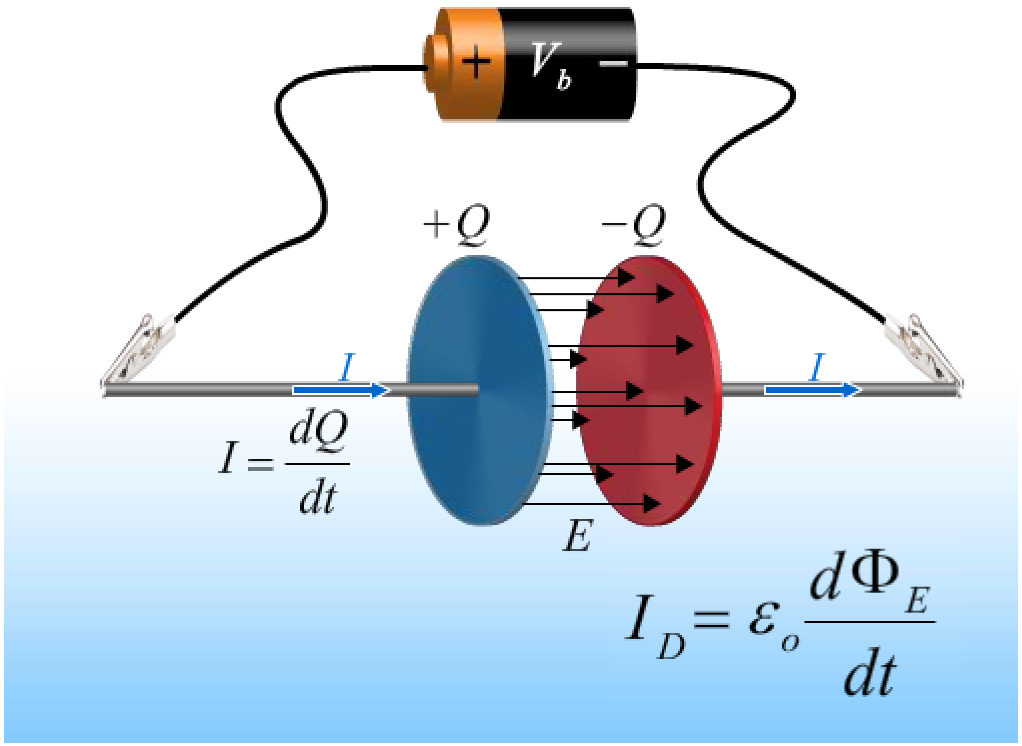


Maxwell suggested including an additional contribution, called the displacement current I_d, to the real current I

After Prelecture 21: Modify Ampere's Law

Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I_{enclosed} = \mu_o\left(I + I_D\right)$$



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$\Phi = EA = \frac{Q}{\varepsilon_0}$$

$$Q = \varepsilon_0 \Phi$$

$$\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi}{dt} \equiv I_D$$

The displacement current is analogous to a real current in Ampère's law, entering into Ampère's law in the same way. It is produced, however, by a changing electric field. It accounts for a changing electric field producing a magnetic field, just as a real current does, but the displacement current can produce a magnetic field even where no real current is present.

A

Displacement Current

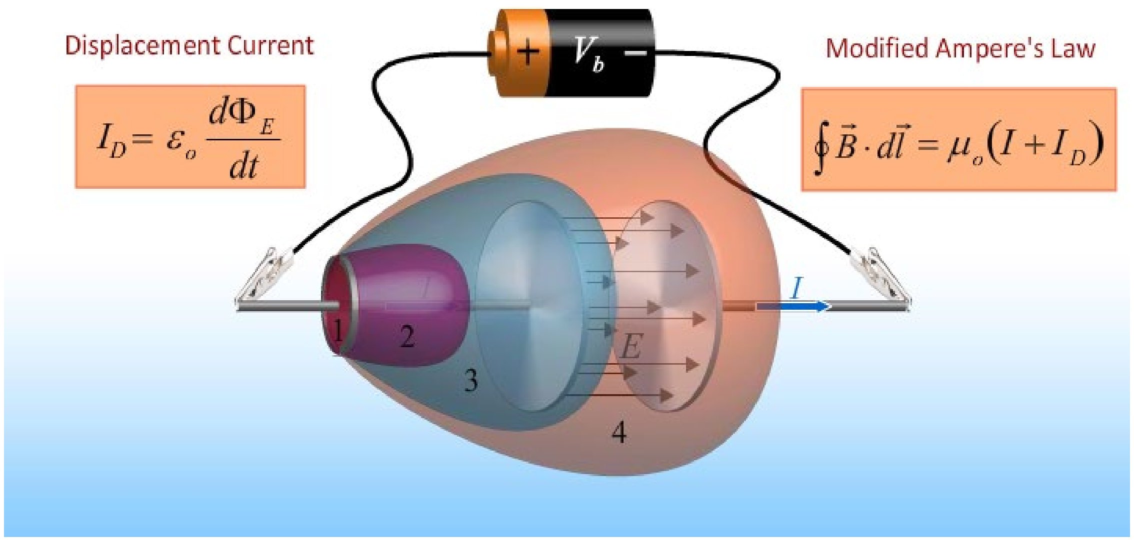
Real Current: Charge Q passes through area A in time t:

$$I = \frac{dQ}{dt}$$

Displacement Current: Electric flux through area A' changes in time

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

DISPLACEMENT CURRENT and EM WAVES



Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Modified Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \varepsilon_o \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

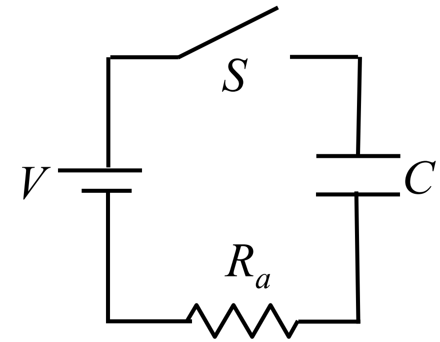
Free space

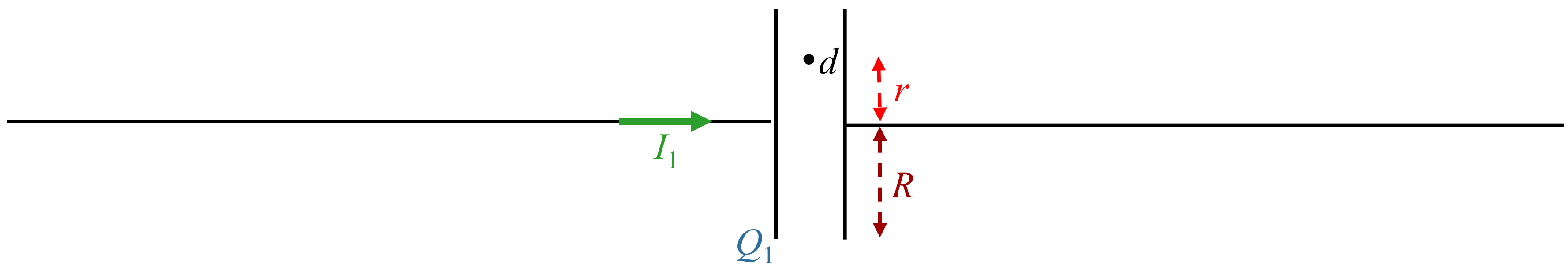
T

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.

At time t_1, what is the magnetic field B_1 at a radius r (point d) in between the plates of the capacitor?





Conceptual and Strategic Analysis

Charge Q_1 creates electric field between the plates of C

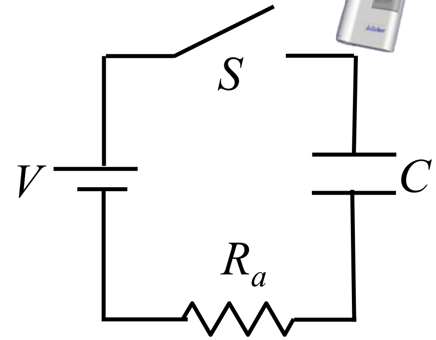
Charge Q_1 changing in time gives rise to a changing electric flux between the plates

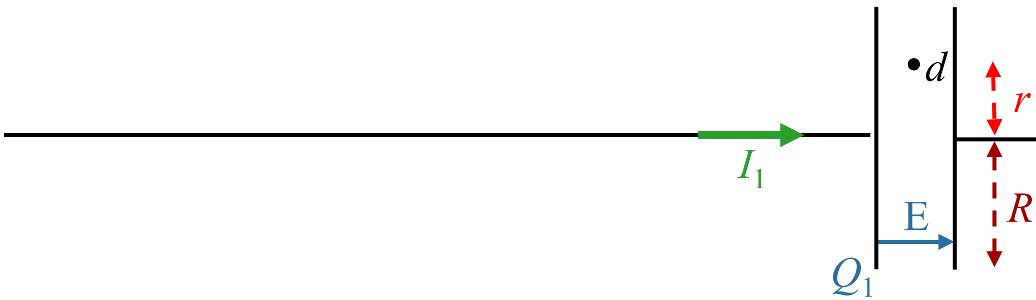
Changing electric flux gives rise to a displacement current I_D in between the plates

Apply (modified) Ampere's law using I_D to determine B

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.





$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

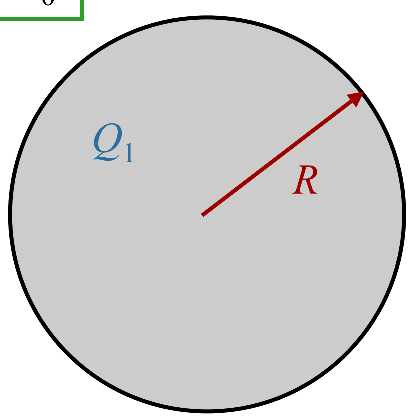
What is the magnitude of the electric field between the plates?

A) $E = \dfrac{Q_1}{\pi R^2 \varepsilon_0}$

B) $E = \dfrac{Q_1 \pi R^2}{\varepsilon_0}$

C) $E = \dfrac{Q_1}{\varepsilon_0}$

D) $E = \dfrac{Q_1}{r}$

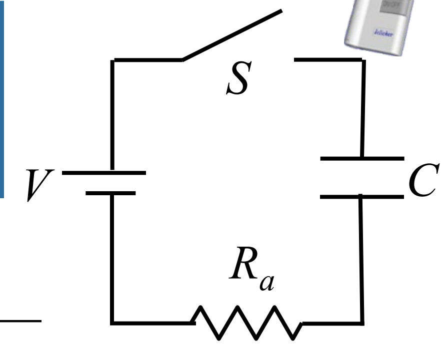


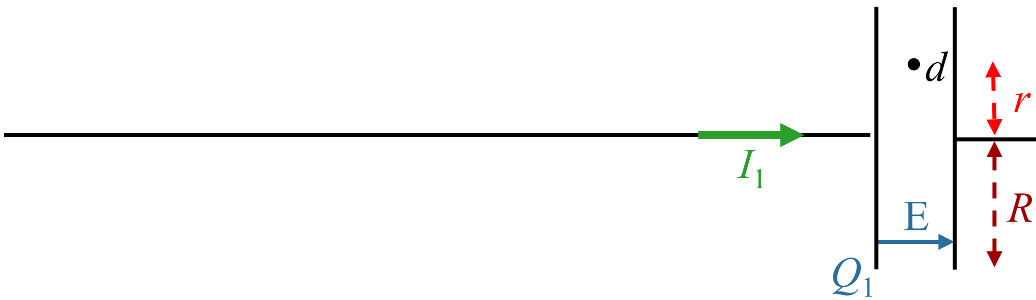
$$E = \frac{\sigma}{\varepsilon_0} \longrightarrow \sigma = \frac{Q_1}{A} = \frac{Q_1}{\pi R^2} \longrightarrow E = \frac{Q_1}{\varepsilon_0 \pi R^2}$$

Bonus point1

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.





$$E = \frac{Q_1}{\pi R^2 \varepsilon_0}$$

$$\Phi_E = \vec{E} \cdot \vec{A}$$

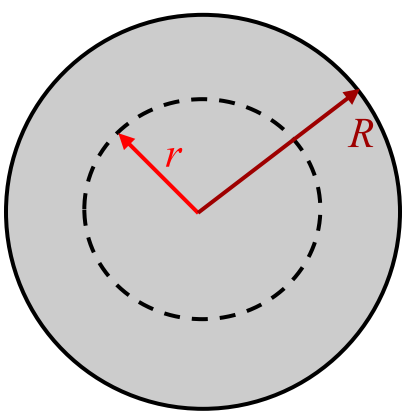
What is the electric flux through a circle of radius r in between the plates?

A) $\Phi_E = \dfrac{Q_1}{\varepsilon_0} \pi r^2$

B) $\Phi_E = \dfrac{Q_1}{\varepsilon_0} \pi R^2$

C) $\boxed{\Phi_E = \dfrac{Q_1 r^2}{\varepsilon_0 R^2}}$

D) $\Phi_E = \dfrac{Q_1 \pi r^2}{\varepsilon_0 R^2}$

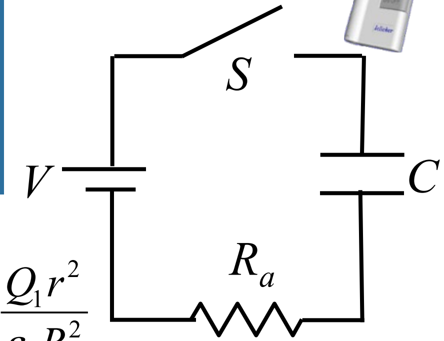


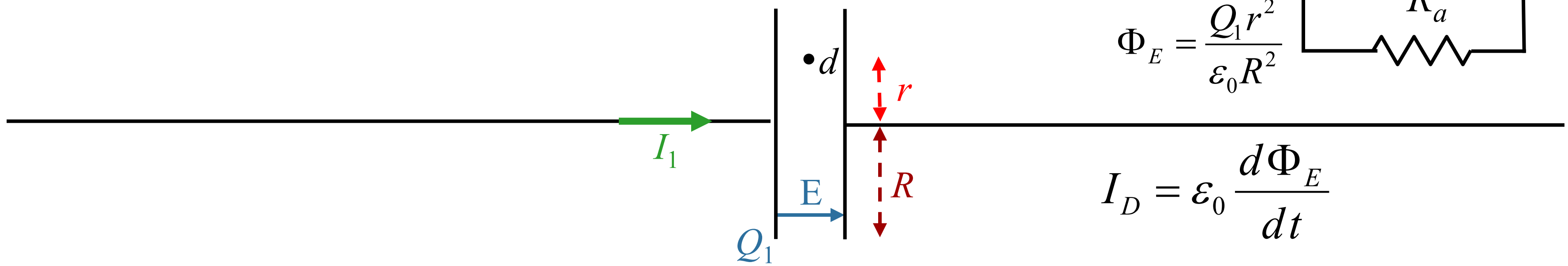
$$\Phi_E = \vec{E} \cdot \vec{A} \rightarrow \Phi_E = \frac{Q_1}{\varepsilon_0 \pi R^2} \pi r^2 \rightarrow \Phi_E = \frac{Q_1}{\varepsilon_0} \frac{r^2}{R^2}$$

Bonus point2

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.



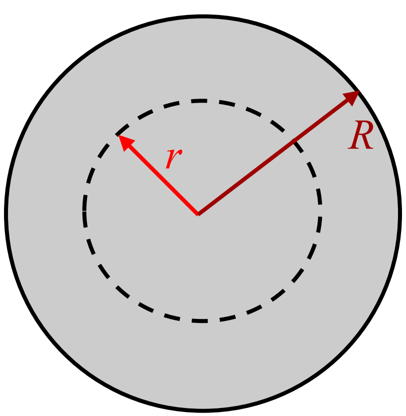


$$\Phi_E = \frac{Q_1 r^2}{\varepsilon_0 R^2}$$

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

What is the displacement current enclosed by circle of radius r ?

A) $I_D = I_1 \dfrac{R^2}{r^2}$ B) $I_D = I_1 \dfrac{r}{R}$ C) $\boxed{I_D = I_1 \dfrac{r^2}{R^2}}$ D) $I_D = I_1 \dfrac{R}{r}$

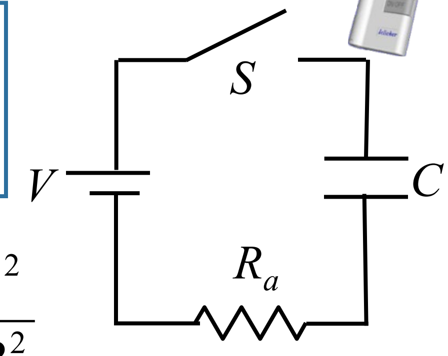


$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

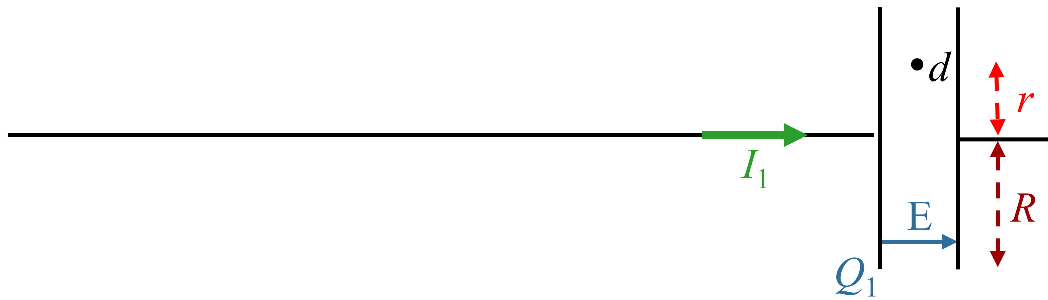
$$\rightarrow \quad I_D = I_1 \frac{r^2}{R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.



$$I_D = I_1 \frac{r^2}{R^2}$$



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o (I + I_D)$$

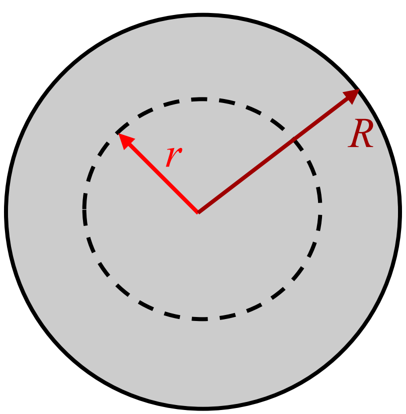
What is the magnitude of the B field at radius r ?

A) $B = \dfrac{\mu_0 I_1}{2\pi R}$

B) $B = \dfrac{\mu_0 I_1}{2\pi r}$

C) $B = \dfrac{\mu_0 I_1}{2\pi} \dfrac{R}{r^2}$

D) $B = \dfrac{\mu_0 I_1}{2\pi} \dfrac{r}{R^2}$



Ampere's Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_o (I + I_D)$

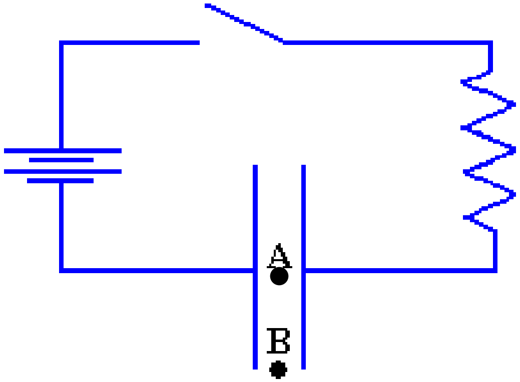
$$\rightarrow B \cdot 2\pi r = \mu_0 \left(0 + I_1 \frac{r^2}{R^2} \right)$$

$$\rightarrow B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

Bonus point3

Check Point 1a

At time t=0 the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; **A is at the center** and B is toward the outer edge.



After the switch is closed, there will be a magnetic field at point A that decreases as the current in the circuit decreases:

A. True

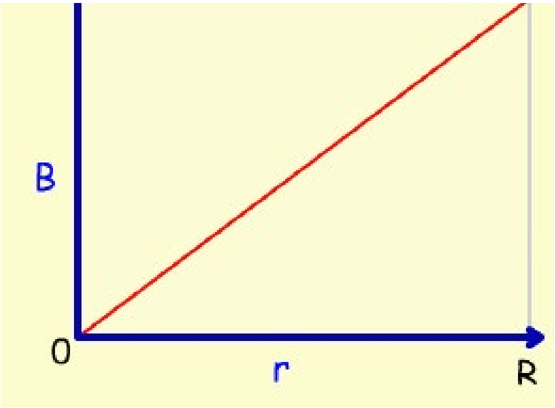
B. False

$$B = \frac{\mu_0 I_1}{2\pi}\frac{r}{R^2}$$

B is proportional to I

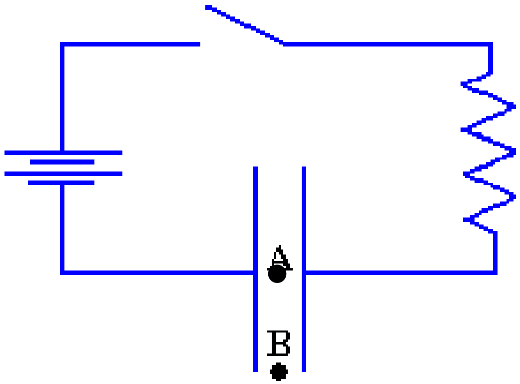
but

At center, $B = 0$!!



Check Point 1B

At time t=0 the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; **A is at the center** and B is toward the outer edge.

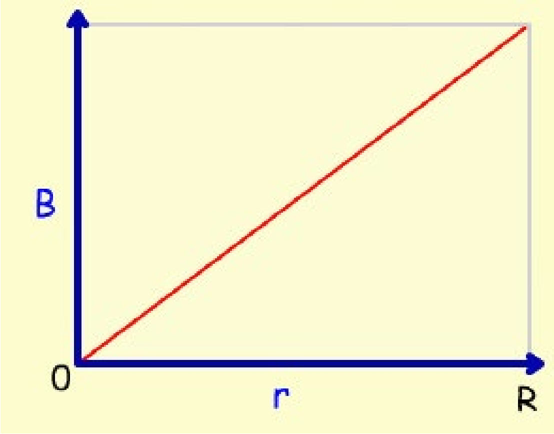


Compare the magnitudes of the magnetic fields at points A and B just after the switch is closed. (note current in circuit not zero just after switch is closed)

A. $B_A < B_B$ ***B.*** $B_A = B_B$ ***C.*** $B_A > B_B$

From the calculation we just did:

$$B = \frac{\mu_0 I_1}{2\pi}\frac{r}{R^2}$$



Let there be light!

Faraday's Law

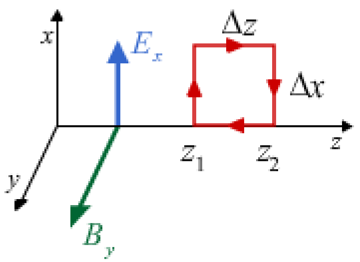
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

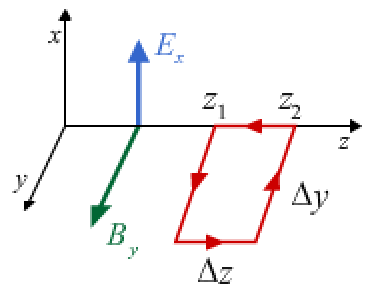
Plane Wave Solution

$$\vec{E} \rightarrow \vec{E}(z,t)$$
$$\vec{B} \rightarrow \vec{B}(z,t)$$

Modified Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_o\varepsilon_o \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$





$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial B_y}{\partial z} = -\mu_o\varepsilon_o\frac{\partial E_x}{\partial t}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z}\frac{\partial B_y}{\partial t}$$

$$\frac{\partial}{\partial t}\frac{\partial B_y}{\partial z} = -\mu_o\varepsilon_o\frac{\partial^2 E_x}{\partial t^2}$$

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_o\varepsilon_o\frac{\partial^2 E_x}{\partial t^2}}$$

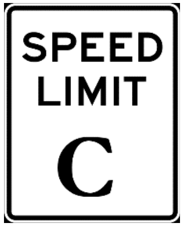
Wave Equation

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_o\varepsilon_o\frac{\partial^2 E_x}{\partial t^2}}$$

Speed of Electromagnetic Wave

$$\boxed{v = \frac{1}{\sqrt{\mu_o\varepsilon_o}}} = c = 3.00\times 10^8 \text{ m/s}$$

Speed of Light !



Special Relativity (1905)

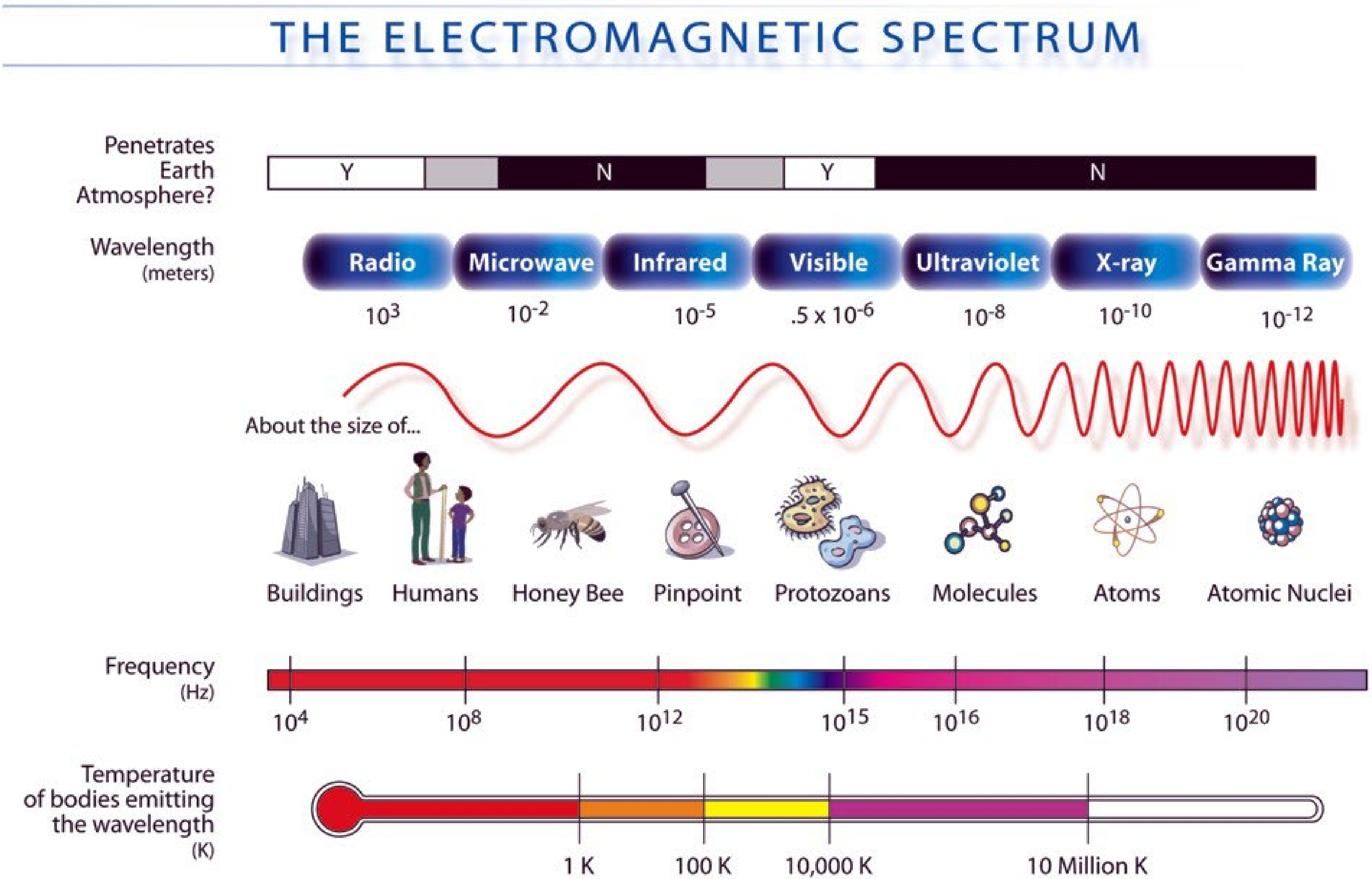
Speed of Light is Constant

Albert Einstein

see PHYS 225

T

Electromagnetic Spectrum



We learned about waves in Physics 211

1-D Wave Equation

$$\frac{d^2h}{dx^2} = \frac{1}{v^2}\frac{d^2h}{dt^2}$$

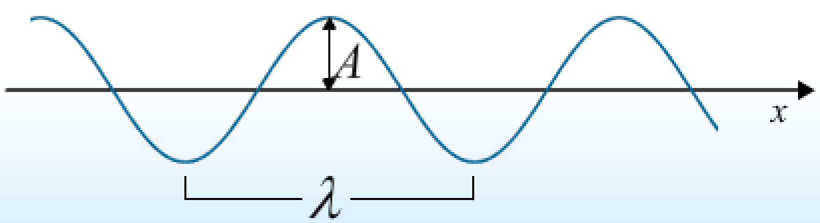
$\longrightarrow$

Solution

$$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

Common Example: Harmonic Plane Wave

$$\boxed{h(x,t) = A\cos(kx - \omega t)}$$



Variable Definitions

Amplitude: A

Wave Number: $k = \frac{2\pi}{\lambda}$

Wavelength: λ

Angular Frequency: $\omega = \frac{2\pi}{T}$

Period: T

Frequency: $f = \frac{1}{T}$

Velocity: $v = \lambda f = \frac{\omega}{k}$

T

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

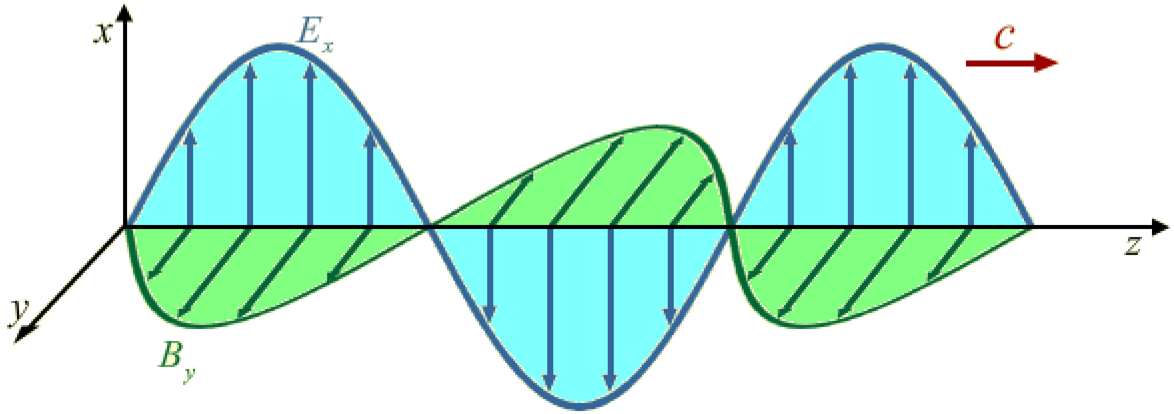
$$\frac{\partial^2 B_y}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 B_y}{\partial t^2}$$

Example: A Harmonic Solution

$$E_x = E_o \cos(kz - \omega t) \xrightarrow{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} B_y = \frac{k}{\omega} E_o \cos(kz - \omega t)$$

Two Important Features

1. B_y is in phase with E_x
2. $B_o = \dfrac{E_o}{c}$

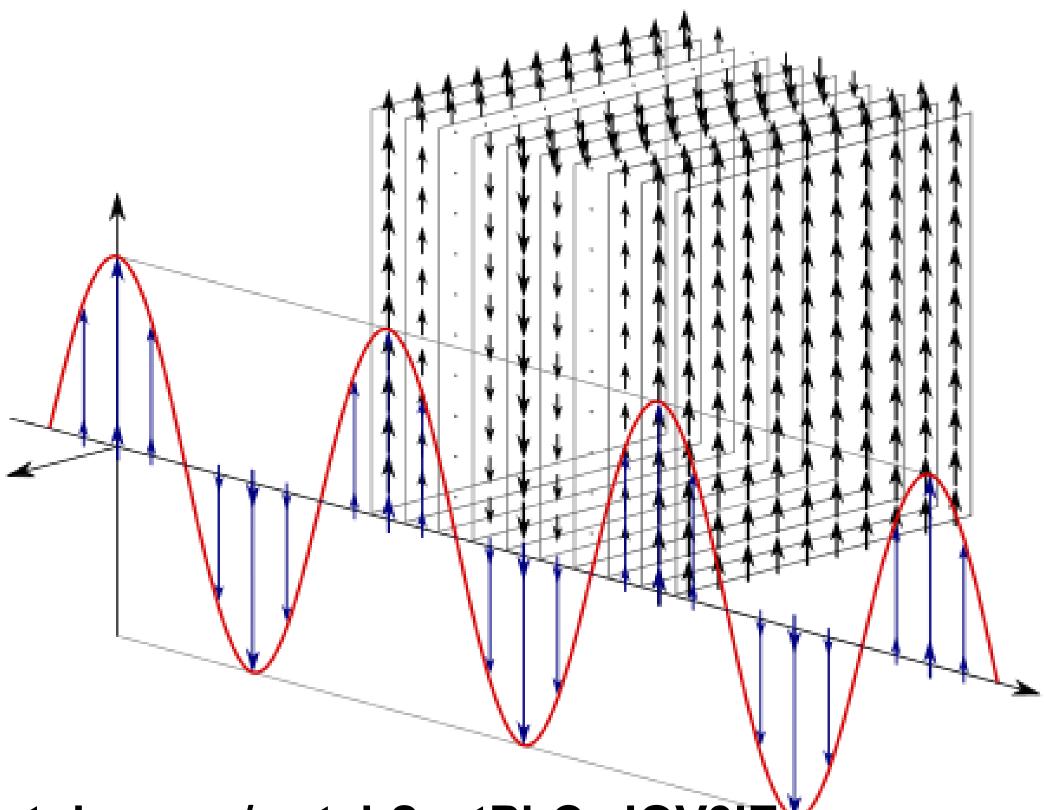


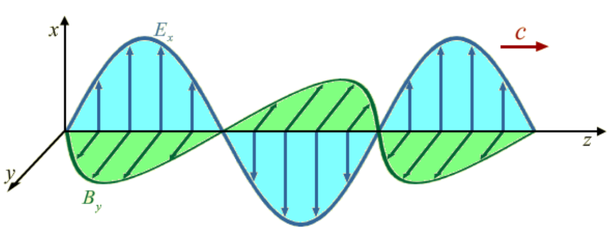
$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 B_y}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 B_y}{\partial t^2}$$

Example: A Harmonic Solution

$$E_x = E_o \cos(kz - \omega t) \xrightarrow{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} B_y = \frac{k}{\omega} E_o \cos(kz - \omega t)$$

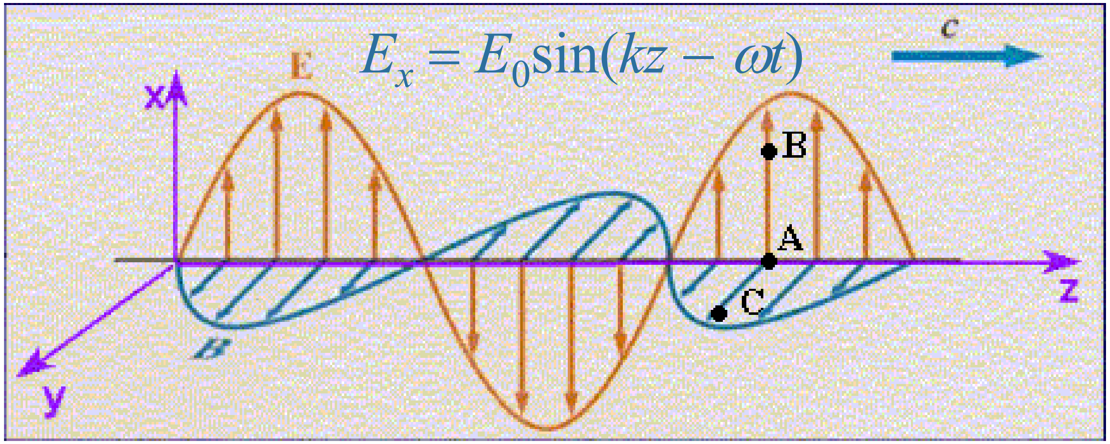




Video: https://www.youtube.com/watch?v=tPbCcJOV3IE

Check Point 2a

An electromagnetic plane-wave is traveling in the +z direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same z coordinate.



Compare the magnitudes of the electric fields at points A and B

A. $E_A < E_B$ **B.** $E_A = E_B$ **C.** $E_A > E_B$

$E = E_0 \sin(kz - \omega t)$:

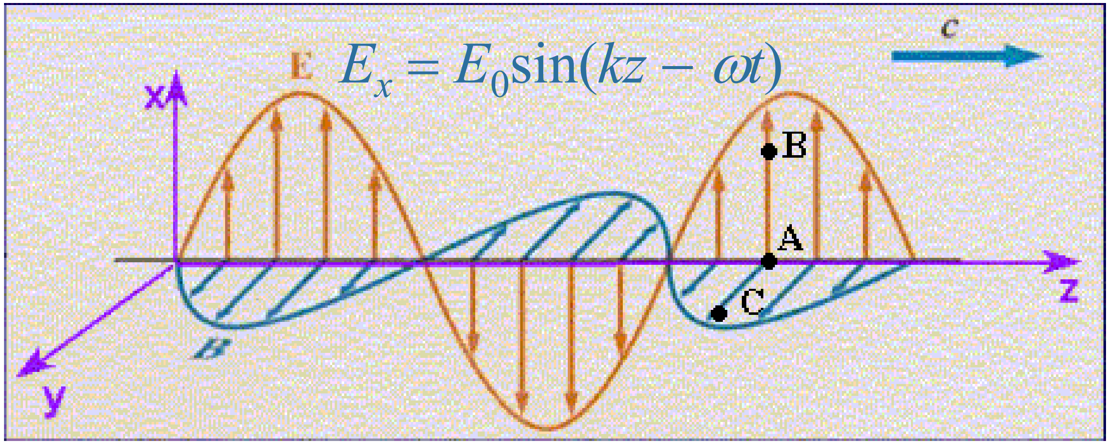
E depends only on z coordinate for constant t.

z coordinate is same for A, B, C.

A

Check Point 2b

An electromagnetic plane-wave is traveling in the +z direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same z coordinate.



Compare the magnitudes of the electric fields at points A and C

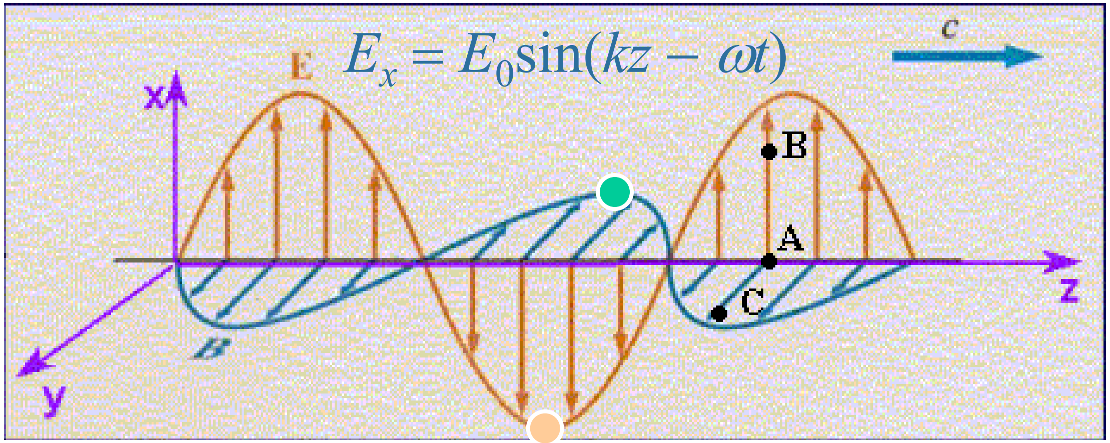
A. $E_A < E_C$ **B.** $E_A = E_C$ **C.** $E_A > E_C$

$E = E_0 \sin(kz - \omega t)$:

E depends only on z coordinate for constant t.

z coordinate is same for A, B, C.

Clicker Question

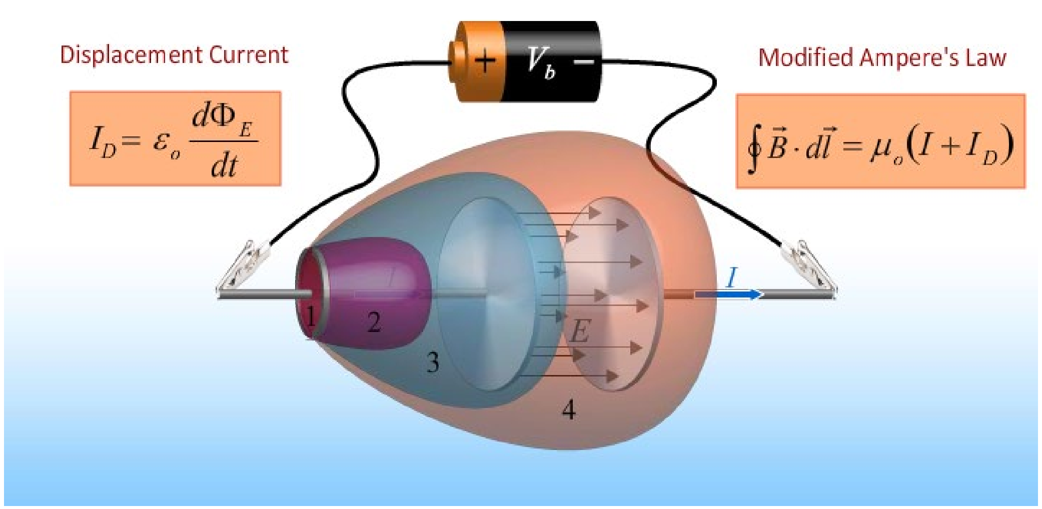


Consider the plot of an electromagnetic plane wave shown above at t=0. At the point (x,y,z) where E_x is negative and has its maximum value what is B_y?

A) B_y is positive and has its maximum value
B) B_y is negative and has its maximum value
C) B_y is zero
D) We do not have enough information

A

DISPLACEMENT CURRENT and EM WAVES



Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Modified Ampere's Law

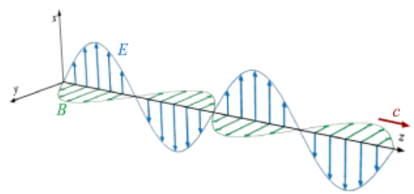
$$\oint \vec{B} \cdot d\vec{l} = \mu_o \varepsilon_o \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Free space

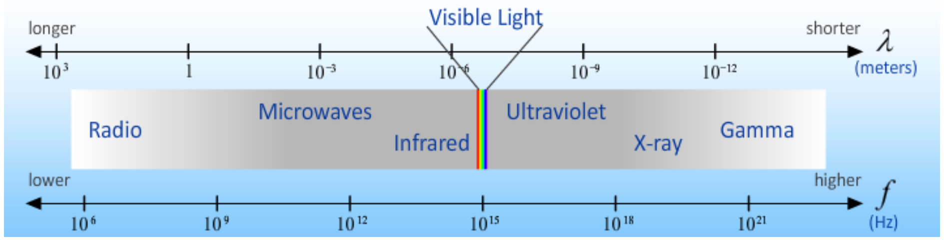
Next time – Properties of EM waves, Doppler shift etc.

Physics 212
Lecture 23

PROPERTIES of ELECTROMAGNETIC WAVES



Electromagnetic Spectrum



Mechanism of EM Wave Propagation

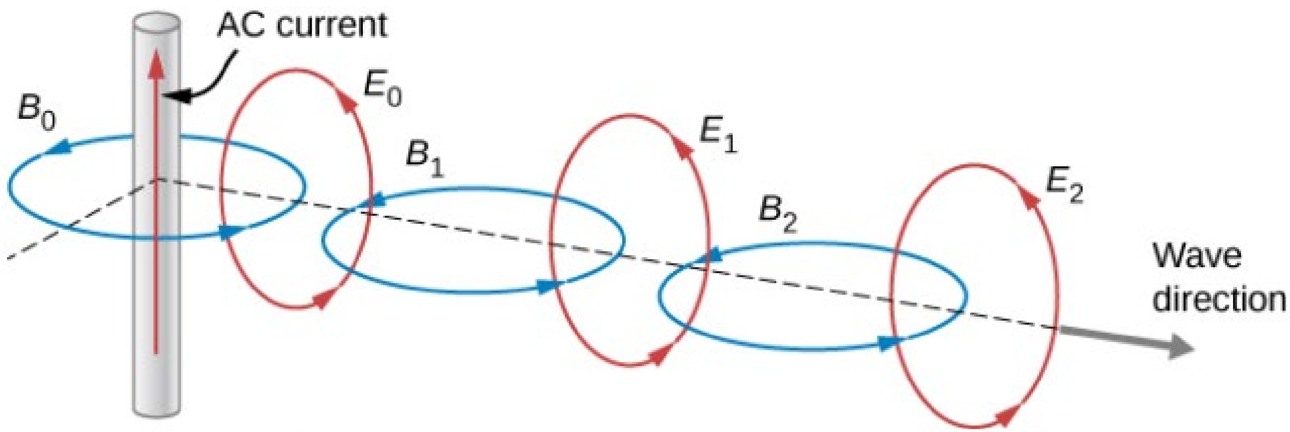


Figure 16.4 How changing $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields propagate through space.

From Faraday's law, the changing magnetic field through a surface induces a time-varying electric field $\vec{\mathbf{E}}_0\,(t)$ at the boundary of that surface. The displacement current source for the electric field, like the Faraday's law source for the magnetic field, produces only closed loops of field lines, because of the mathematical symmetry involved in the equations for the induced electric and induced magnetic fields. A field line representation of $\vec{\mathbf{E}}_0\,(t)$ is shown. In turn, the changing electric field $\vec{\mathbf{E}}_0\,(t)$ creates a magnetic field $\vec{\mathbf{B}}_1\,(t)$ according to the modified Ampère's law. This changing field induces $\vec{\mathbf{E}}_1\,(t)$, which induces $\vec{\mathbf{B}}_2\,(t)$, and so on. We then have a self-continuing process that leads to the creation of time-varying electric and magnetic fields in regions farther and farther away from O. This process may be visualized as the propagation of an electromagnetic wave through space.

Hertz's Observations

- The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory.
- Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves but also verified that they travel at the speed of light.
- High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and helped generate electromagnetic waves.

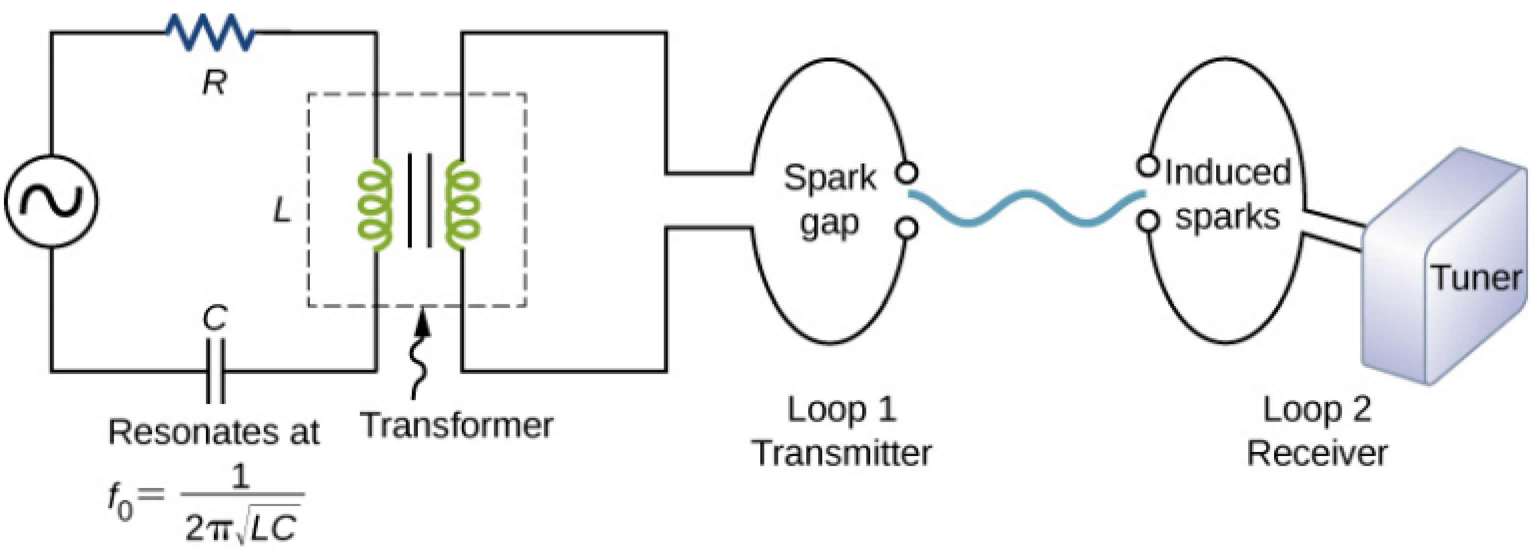
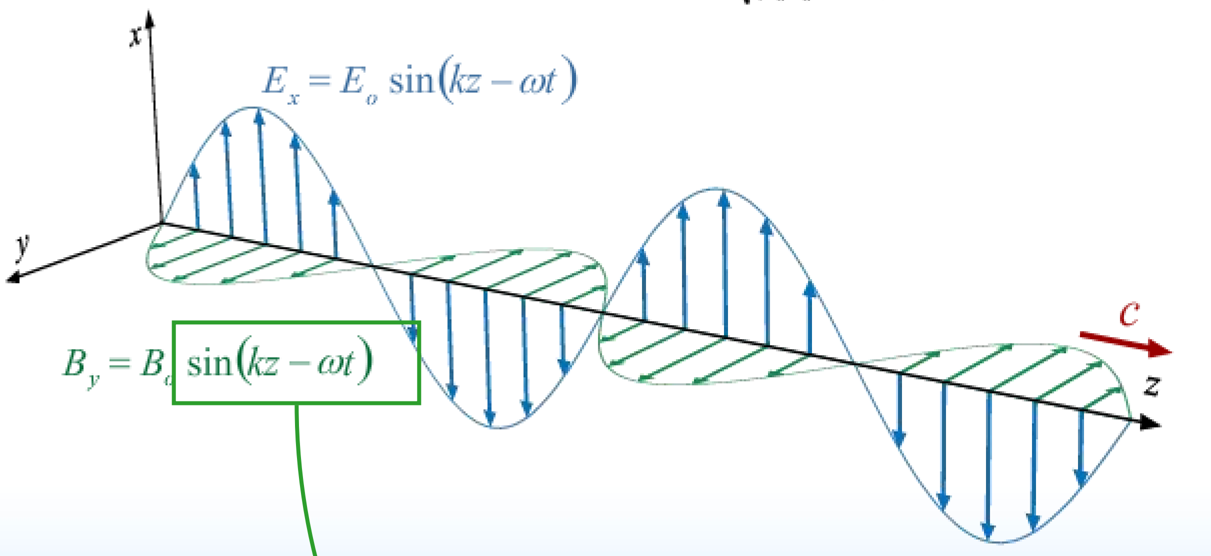


Figure 16.5 The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves.

Plane Waves

Velocity $c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = \frac{E_o}{B_o} = 3\times10^8\ \mathrm{m/s}$



E and B are perpendicular and in phase

Oscillate in time and space

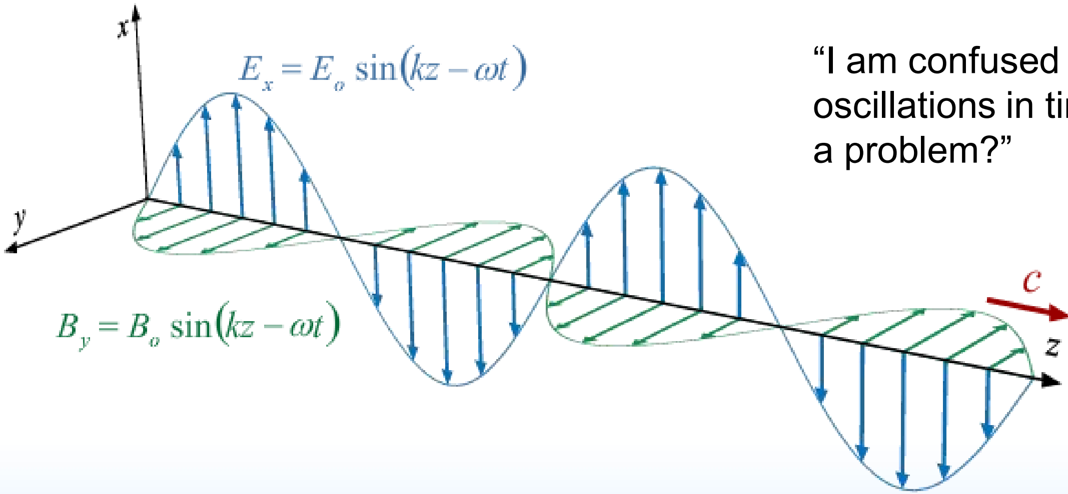
Direction of propagation given by E X B

$E_0 = cB_0$

Argument of sin/cos gives direction of propagation

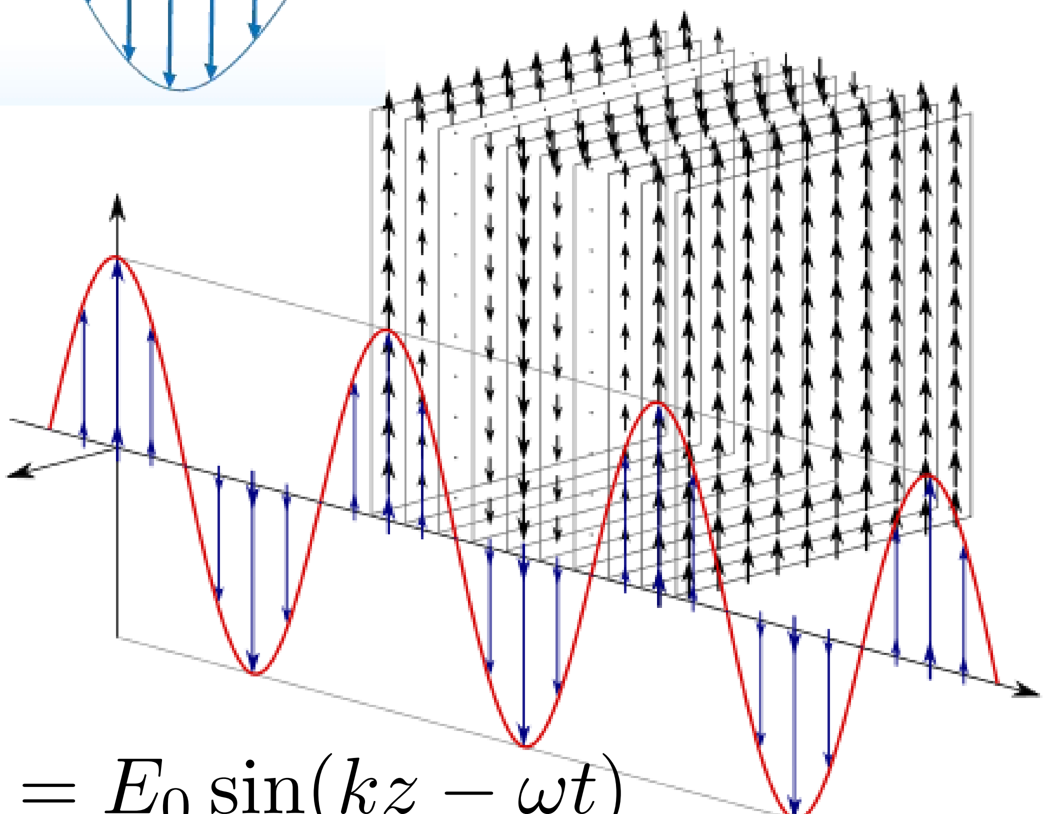
Plane Waves

Velocity $c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = \frac{E_o}{B_o} = 3\times10^8\ \mathrm{m/s}$



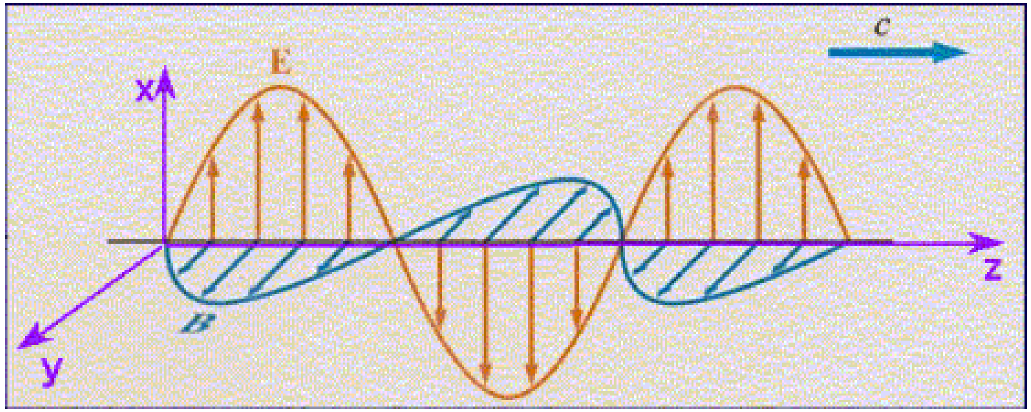
“I am confused about the oscillations in space vs. the oscillations in time. How do we account for both, like in a problem?”

Note: Ex has no x or y dependence



$$E_x = E_0 \sin(kz - \omega t)$$

Check Point 1a



Which equation correctly describes this electromagnetic wave?

- $\mathbf{E}_x = \mathbf{E}_o \sin(kz + \omega t)$ No – moving in the minus *z* direction
- $\mathbf{E}_y = \mathbf{E}_o \sin(kz - \omega t)$ No – has E_y rather than E_x
- $\boxed{\mathbf{B}_y = \mathbf{B}_o \sin(kz - \omega t)}$

Check Point 2a

Your iclicker operates at a frequency of approximately 900 MHz (900×10^6 Hz). What is the approximately wavelength of the EM wave produced by your iclicker?

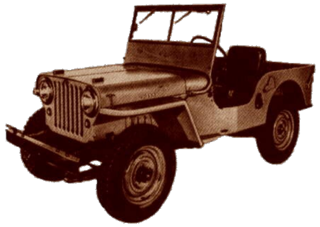
A. 0.03 meters
B. 0.3 meters
C. 3.0 meters
D. 30. meters

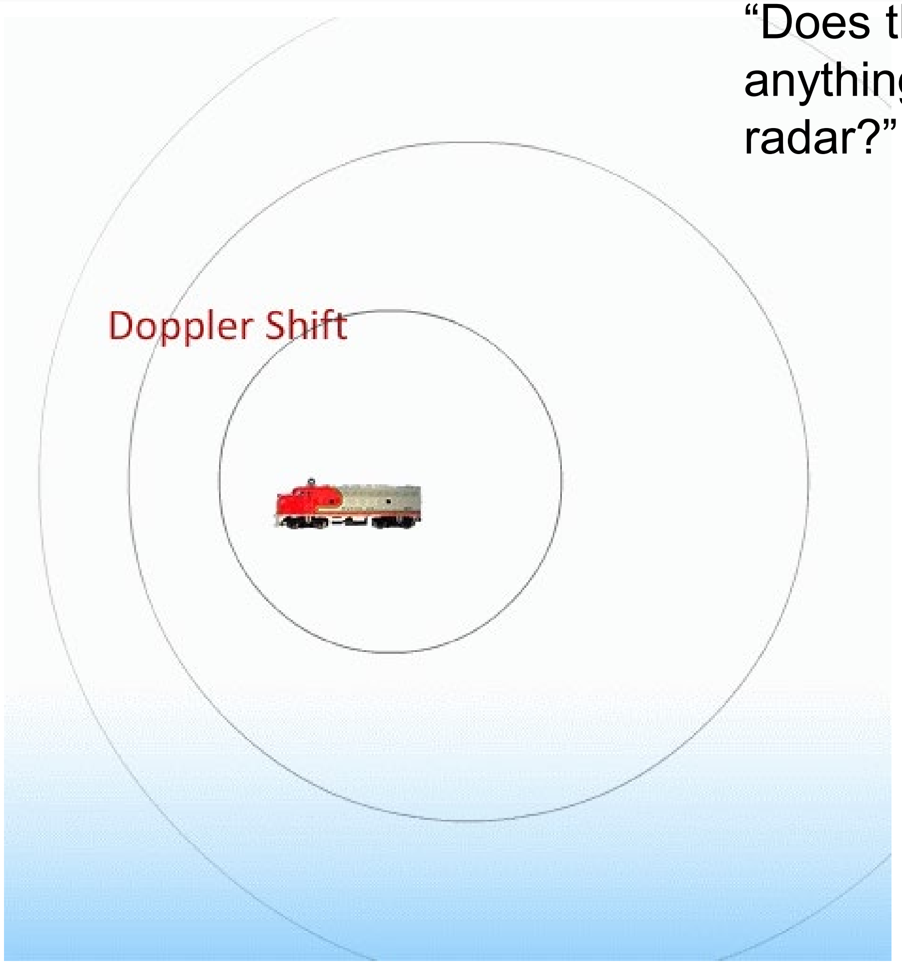
$$c = 3.0 \text{ x } 10^8 \ m/s$$

Wavelength is equal to the speed of light divided by the frequency.

$$\lambda = \frac{c}{f} = \frac{300{,}000{,}000}{900{,}000{,}000} = \frac{1}{3}$$

Doppler Shift





“Does the Doppler effect have anything to do with Doppler radar?”

The Big Idea

As source approaches:
Wavelength decreases
Frequency Increases

Doppler Shift for E-M Waves

What's Different from Sound or Water Waves ?

Sound /Water Waves :

You can calculate (no relativity needed)

BUT

Result is somewhat complicated: is source or observer moving wrt medium?

Electromagnetic Waves :

You need relativity (time dilation) to calculate

BUT

Result is simple: only depends on relative motion of source & observer

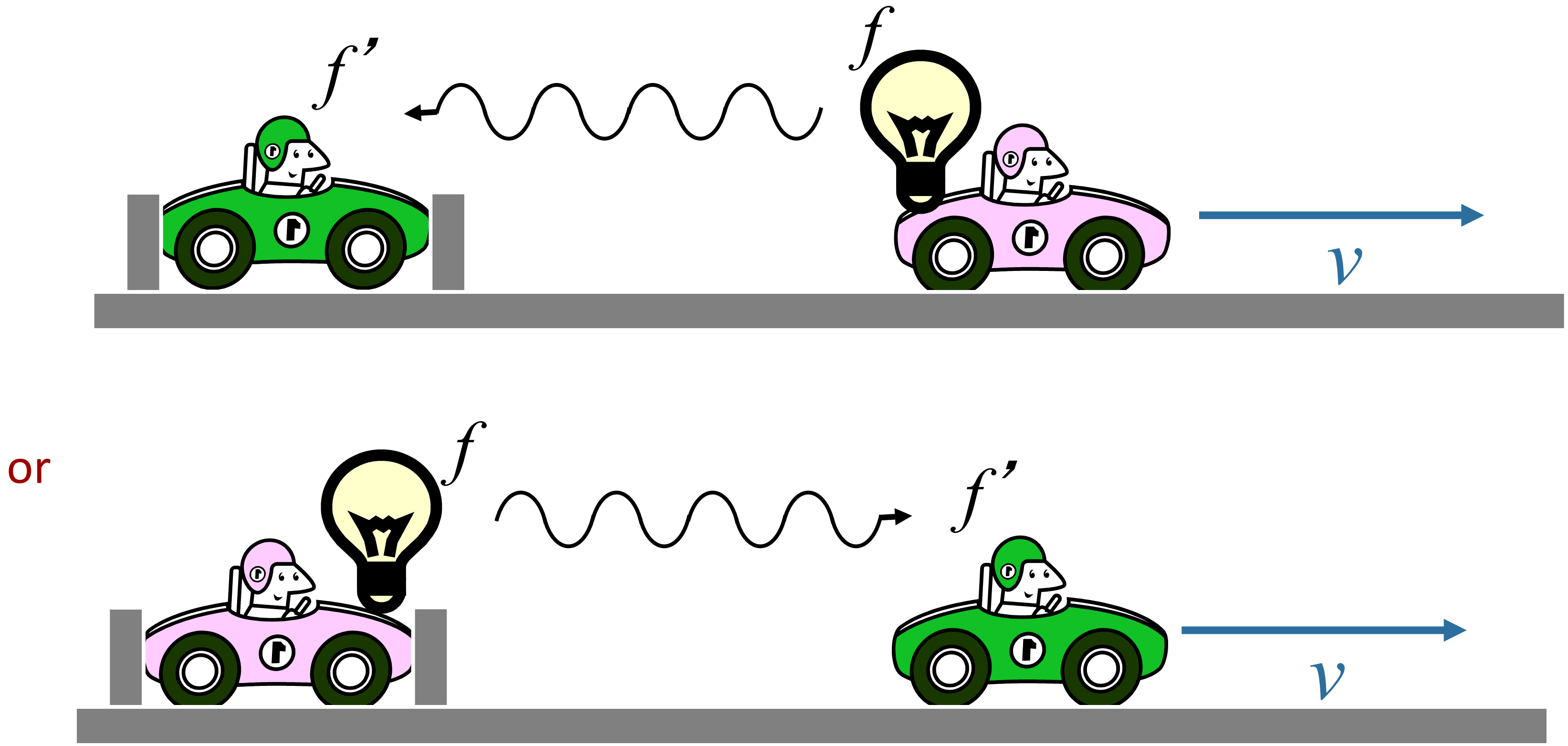
$$f' = f\left(\frac{1+\beta}{1-\beta}\right)^{\frac{1}{2}}$$

$\beta = v/c$

$\beta > 0$ if source & observer are approaching

$\beta < 0$ if source & observer are separating

Doppler Shift for E-M Waves



The Doppler Shift is the SAME for both cases!

f'/f only depends on the relative velocity

$$f' = f\left(\frac{1+\beta}{1-\beta}\right)^{\frac{1}{2}}$$

Doppler Shift for E-M Waves

A Note on Approximations

$$f_o = f_s \left(\frac{1+\beta}{1-\beta} \right)^{\frac{1}{2}} \xrightarrow{\beta \ll 1} f_o \approx f_s \left(1+\beta\right)$$

why?

Taylor Series: Expand $F(\beta) = \left(\frac{1+\beta}{1-\beta} \right)^{\frac{1}{2}}$ around $\beta = 0$

$$F(\beta) = F(0) + \frac{F'(0)}{1!}\beta + \frac{F''(0)}{2!}\beta^2 + \ldots$$

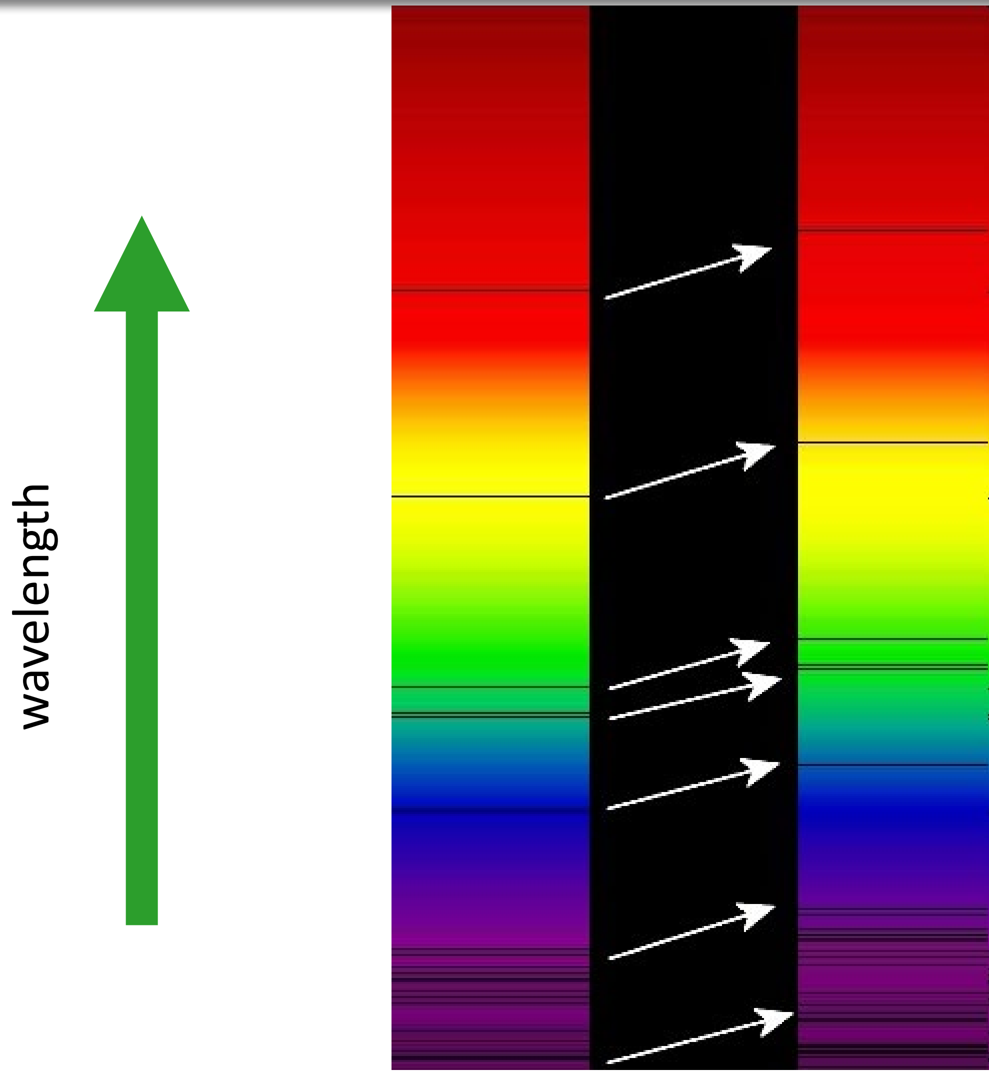
Evaluate:

$F(0) = 1$

$F'(0) = 1$

$$\boxed{F(\beta) \approx 1 + \beta}$$

Red Shift



Wavelengths shifted higher

↓

Frequencies shifted lower

↓

Star separating from us
(Expanding Universe)

Our Sun

Star in a distant galaxy

Check Point 2a

If you wanted to see the EM wave produced by the iclicker with your eyes, which of the following would work? (Note: Your eyes are sensitive to EM waves w/ frequency around 10^{14} Hz)

f_{iclicker} = 900 MHz

A) ○ Run away from the iclicker when it is voting.

B) ○ $\boxed{\text{Run toward the iclicker when it is voting.}}$

C) ○ Neither will work, moving relative to the iclicker won't change the frequency reaching your eyes.

Need to shift frequency UP → Need to approach i>clicker $(\beta > 0)$

How fast would you need to run to see the i>clicker radiation?

$$\frac{f'}{f} = \frac{10^{14}}{10^{9}} = 10^{5} = \left(\frac{1+\beta}{1-\beta}\right)^{1/2}$$

$$10^{10} = \left(\frac{1+\beta}{1-\beta}\right) \quad \rightarrow \quad \beta = \frac{10^{10}-1}{10^{10}+1} = \frac{1-10^{-10}}{1+10^{-10}}$$

Approximation Exercise: $\beta \approx 1-(2\times10^{-10})$

Waves Carry Energy

"Can we go over the vector S? I got confused because in the prelecture they said that S = cE0E^2, but that its magnitude also equals I which is half that value"

Total Energy Density

$$u = \varepsilon_o E^2$$

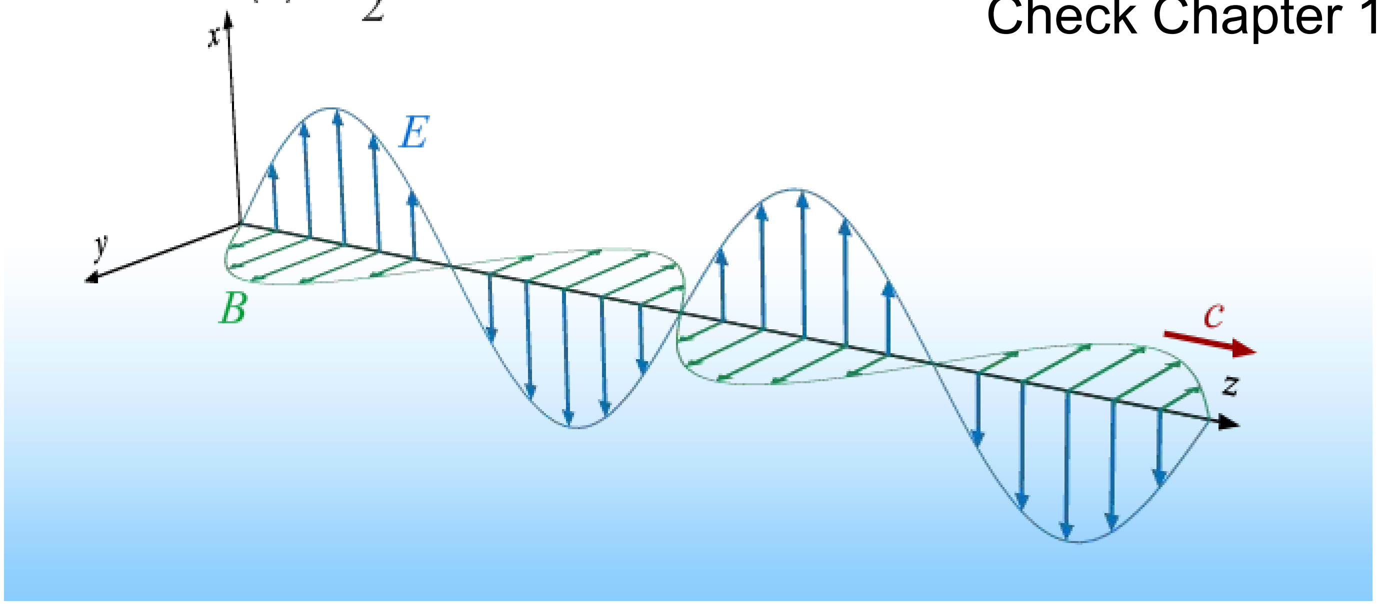
Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o{}^2$$

Intensity

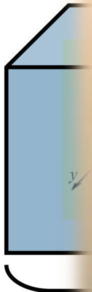
$$\boxed{I = \frac{1}{2} c\,\varepsilon_o E_o{}^2 = c\,\langle u \rangle}$$

Check Chapter 16.3



Intensity

Intensity = Average energy delivered per unit time, per unit area



$$\ldots lume = \langle u \rangle A c\,dt$$

$$\langle dU \rangle / dt = \langle u \rangle A c$$

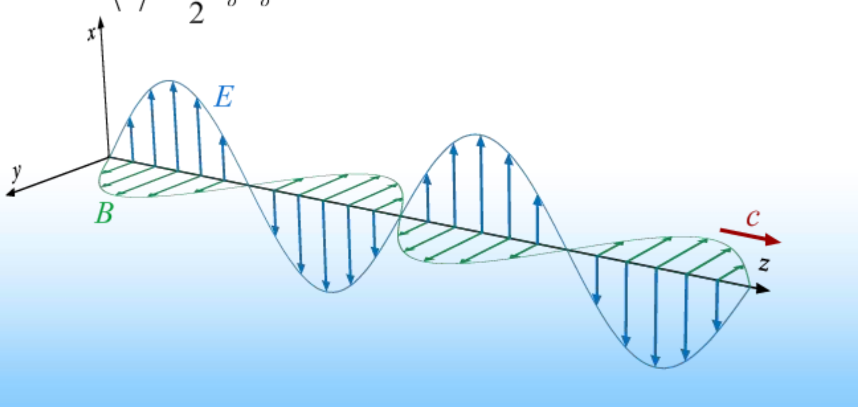
$$= c\langle u \rangle$$

Total En

u

Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\ 2}$$



Sunlight on Earth:

$$I \sim 1000 J/s/m^2$$
$$\sim 1\ kW/m^2$$

Waves Carry Energy

Total Energy Density

$$u = \varepsilon_o E^2$$

Intensity

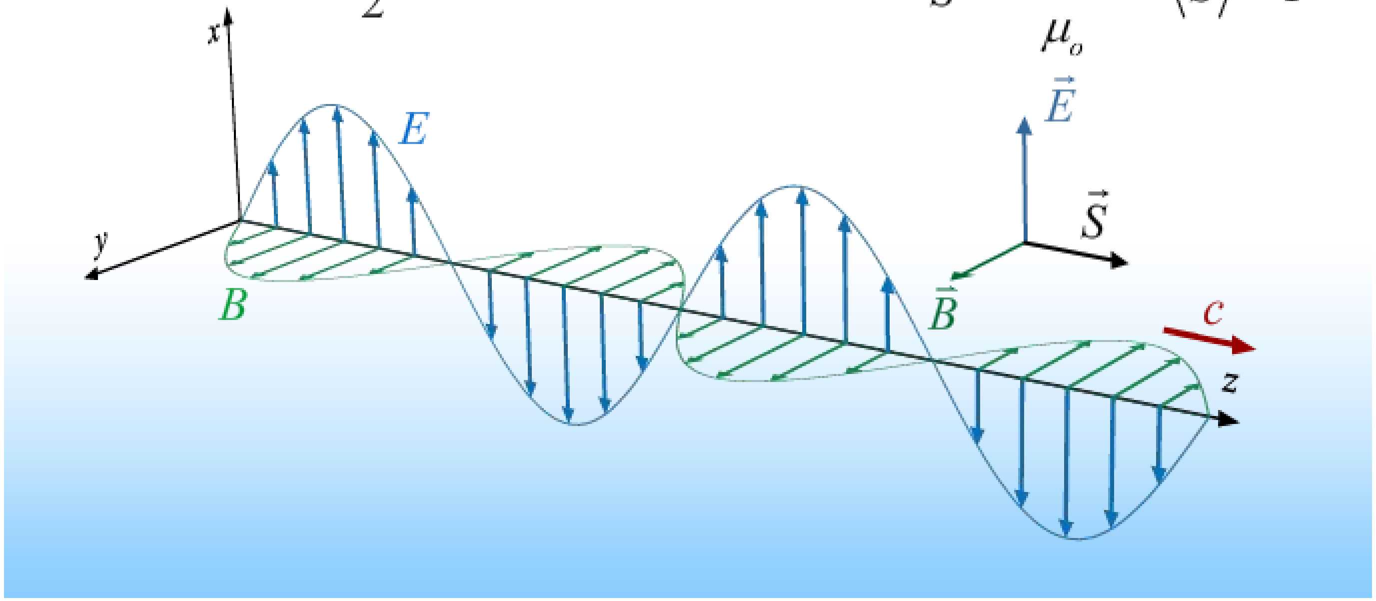
$$I = \frac{1}{2} c \varepsilon_o E_o^{\ 2} = c \langle u \rangle$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \varepsilon_o E_o^{\ 2}$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_o} \qquad \langle S \rangle = I$$



Comment on Poynting Vector

Just another way to keep track of all this:

Its magnitude is equal to I

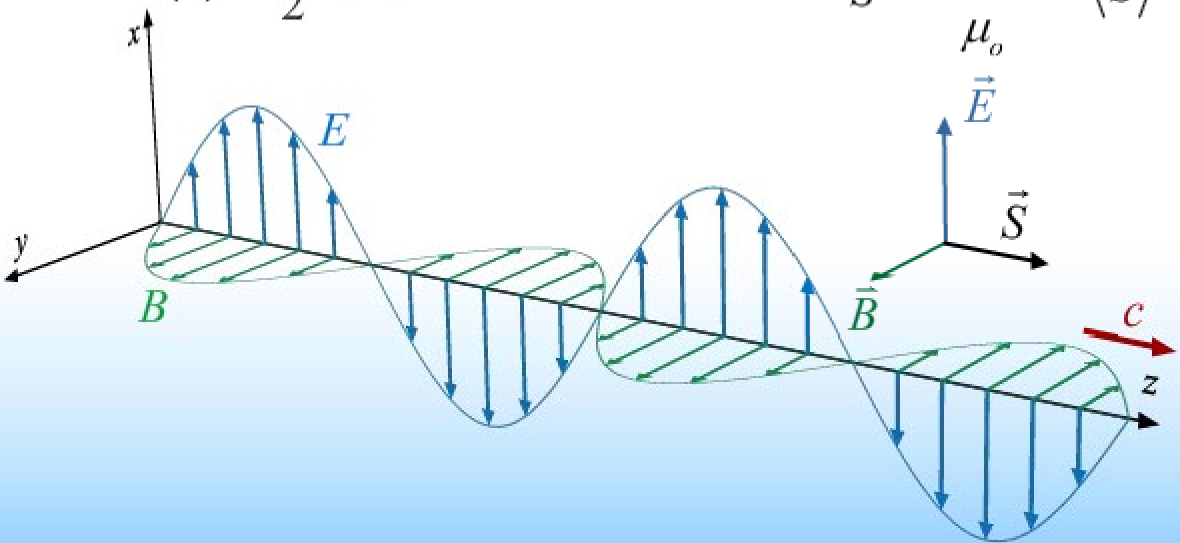
Its direction is the direction of propagation of the wave

Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\;2}$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E}\times\vec{B}}{\mu_o} \qquad \langle S \rangle = I$$



Power in EM Waves: Example

A cell phone tower has a transmitter with a power of 100 W. What is the magnitude of the peak electric field a distance 1500 m (~ 1 mile) from the tower? Assume the transmitter is a point source.

What is the intensity of the wave 1500 m from the tower?

A) 1.5 nW/m^2 ***B)*** 3.5 μW/m^2 ***C)*** 6 mW/m^2

$$I = \frac{P}{4\pi r^2} = \frac{100\ \mathrm{W}}{4\pi\left(1500\ \mathrm{m}\right)^2} = 3.5\,\frac{\mu\mathrm{W}}{m^2}$$

What is the peak value of the electric field?

$$I = \left\langle \left|\vec{S}\right| \right\rangle = \left\langle \frac{\left|\vec{E}\times\vec{B}\right|}{\mu_0} \right\rangle = \left\langle \frac{E}{\mu_0}\frac{E}{c} \right\rangle = \frac{1}{\mu_0 c}\frac{E_0^2}{2} \quad\Rightarrow\quad E_0 = \sqrt{2\mu_0 c I}$$

$$E_0 = \left(2\cdot 4\pi\times 10^{-7}\cdot 3\times 10^{8}\cdot 3.5\times 10^{-6}\right)^{1/2} = 51\,\frac{\mathrm{mV}}{\mathrm{m}}$$

Checkpoint 3a

Which of the following actions will increase the energy carried by an electromagnetic wave?

A. Increase E keeping ω constant

B. Increase ω keeping E constant

C. Both of the above will increase the energy

D. Neither of the above will increase the energy

Intensity

$$I = \frac{1}{2} c \varepsilon_o E_o^{\ 2}$$

Photons

We believe the energy in an e-m wave is carried by photons

Question: What are Photons?

Answer: Photons are Photons.

Photons possess both wave and particle properties

Particle:

Energy and Momentum localized

Wave:

They have definite frequency & wavelength $(f\lambda = c)$

Connections seen in equations:

$$E = hf$$
$$p = h/\lambda$$

Planck's constant

$$h = 6.63e^{-34}\ J-s$$

Question: How can something be both a particle and a wave?

Answer: It can't (when we observe it)

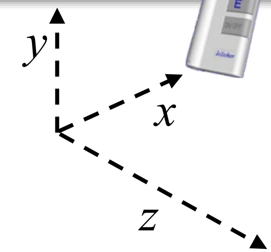
What we see depends on how we choose to measure it!

The mystery of quantum mechanics: More on this in PHYS 214

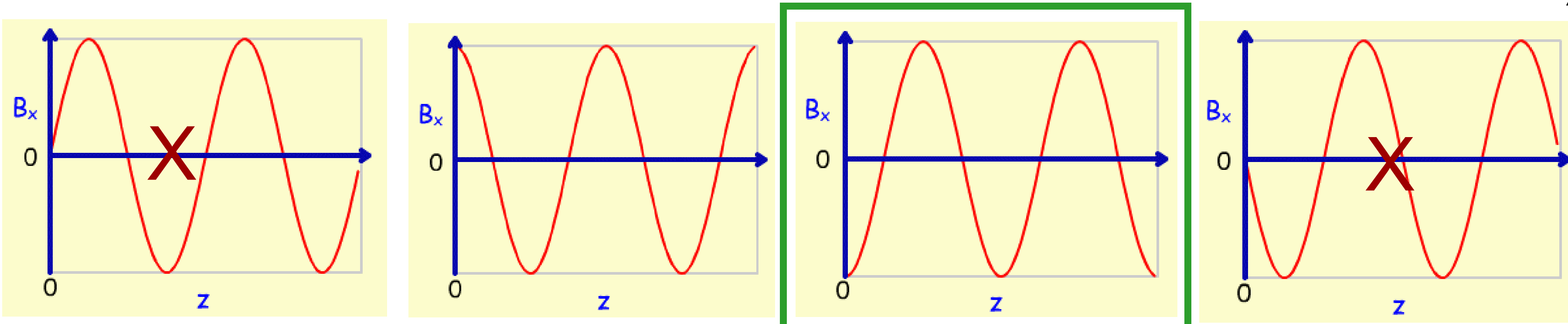
If you're curious: https://en.wikipedia.org/wiki/Photoelectric_effect

Exercise

An electromagnetic wave is described by: $\vec{E} = jE_0 \cos(kz - \omega t)$
where j is the unit vector in the $+y$ direction.



Which of the following graphs represents the z – dependence of B_x at $t = 0$?

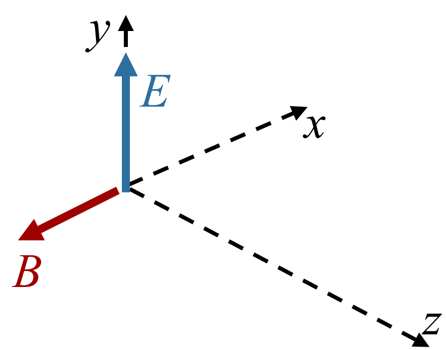


A B C D

E and B are "in phase" (or 180^o out of phase)

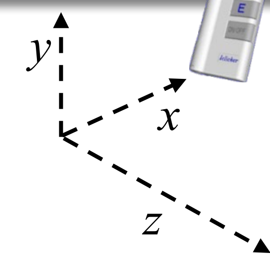
$\vec{E} = jE_0 \cos(kz - \omega t)$ → Wave moves in $+z$ direction

$\vec{E} \times \vec{B}$ Points in direction of propagation



$\vec{B} = -iB_0 \cos(kz - \omega t)$

Exercise



An electromagnetic wave is described by: $\vec{E} = \frac{i + j}{\sqrt{2}} E_0 \cos(kz + \omega t)$

What is the form of B for this wave?

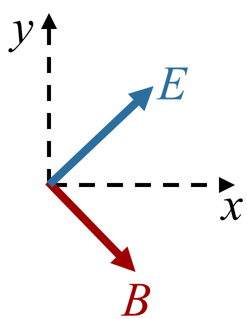
A) $\vec{B} = \frac{i + j}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

B) $\vec{B} = \frac{i - j}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

C) $\vec{B} = \frac{-i + j}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

D) $\vec{B} = \frac{-i - j}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

$\vec{E} = \frac{i + j}{\sqrt{2}} E_0 \cos(kz + \omega t)$ → Wave moves in $-z$ direction



$+z$ points out of screen

$-z$ points into screen

$\vec{E} \times \vec{B}$ Points in direction of propagation

Exercise

A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: “We are both being truthful; you just need to account for the Doppler effect !”

Is it possible that the professor’s argument is correct?

$$(\lambda_{green} = 500\ nm,\ \ \lambda_{red} = 600\ nm)$$

$\boxed{\text{A) YES}}$ B) NO

As professor approaches stoplight, the frequency of its emitted light will be shifted UP

The speed of light does not change

Therefore, the wavelength (c/f) would be shifted DOWN

If he goes fast enough, he could observe a green light !

Exercise

A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: "We are both being truthful; you just need to account for the Doppler effect !"

How fast would the professor have to go to see the light as green?

$$(\lambda_{green} = 500\ nm,\ \ \lambda_{red} = 600\ nm)$$

A) $540\ m/s$ B) $5.4 \times 10^4\ m/s$ $\boxed{\text{C)}\ 5.4 \times 10^7\ m/s}$ D) $5.4 \times 10^8\ m/s$

Relativistic Doppler effect: $f' = f\sqrt{\dfrac{1+\beta}{1-\beta}}$

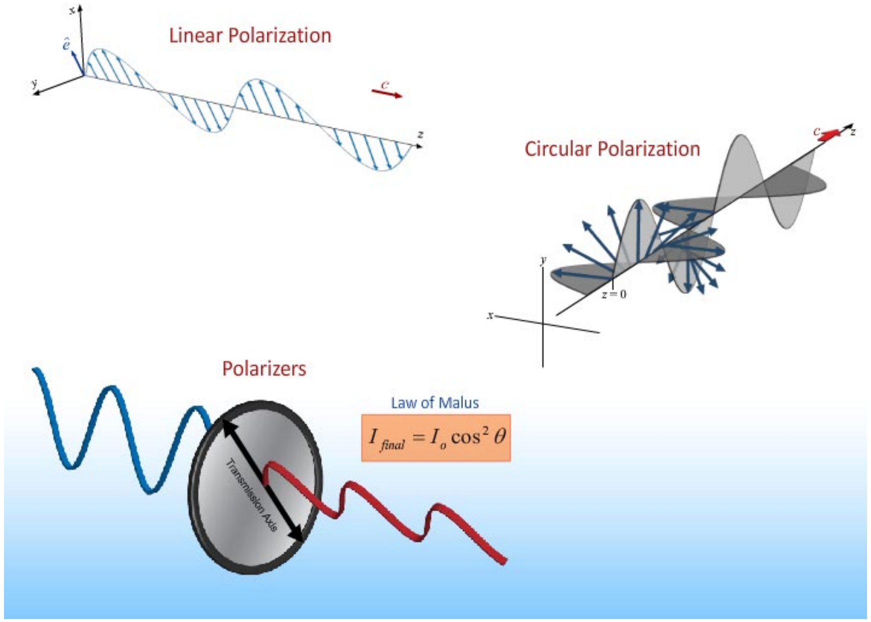
$$\frac{f'}{f} = \frac{600}{500} = \sqrt{\frac{1+\beta}{1-\beta}} \rightarrow 36(1-\beta) = 25(1+\beta) \rightarrow \beta = \frac{11}{61} = 0.18$$

Note approximation for small β is not bad: $f' = f(1+\beta) \rightarrow \beta = \dfrac{1}{5} = 0.2$

$c = 3 \times 10^8\ m/s$ -> $v = 5.4 \times 10^7\ m/s$ $\rightarrow$ Change the charge to speeding!

Only 4 more lectures to go!

Physics 212
Lecture 24
Polarization



Polarization

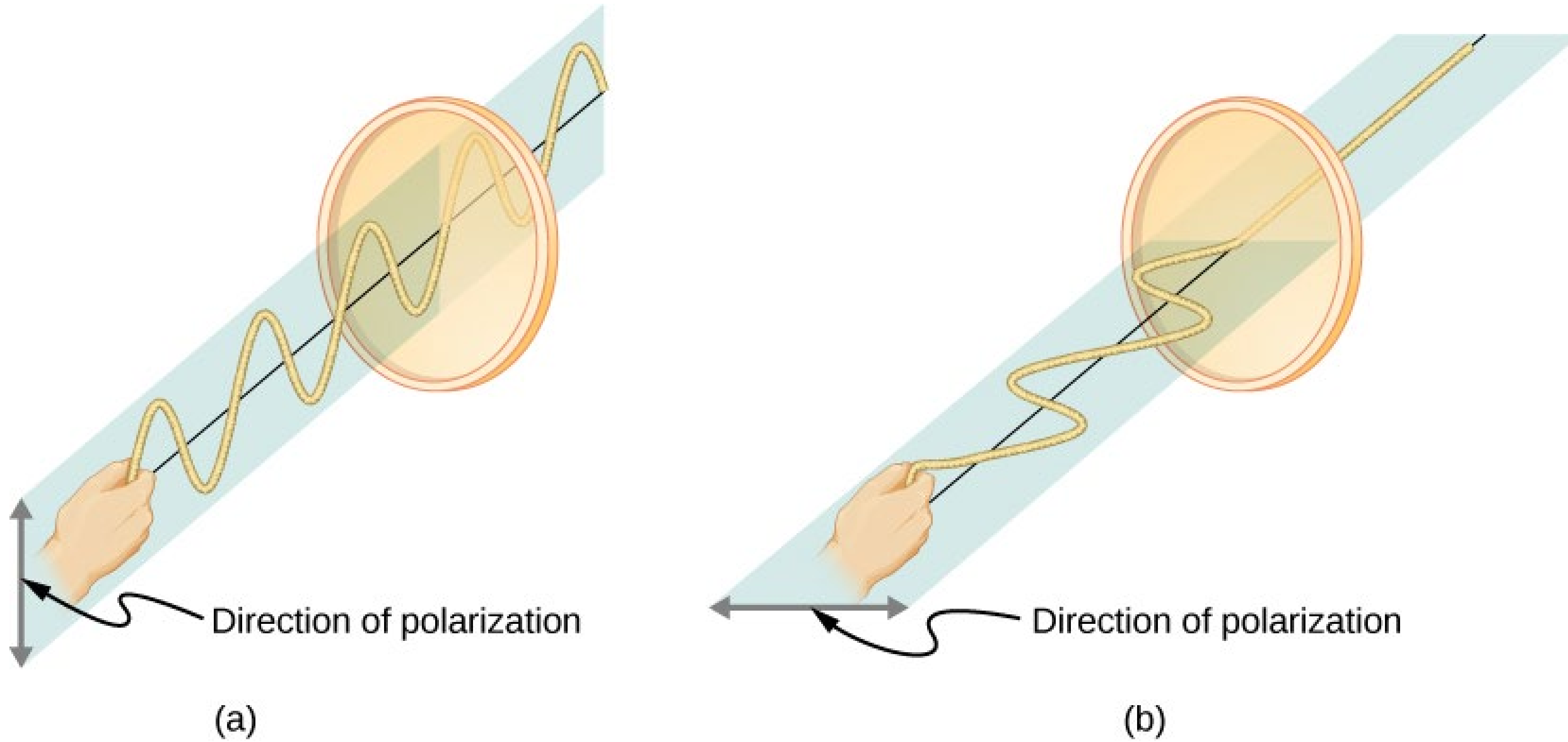
- Light is one type of electromagnetic (EM) wave. EM waves are transverse waves consisting of varying electric and magnetic fields that oscillate perpendicular to the direction of propagation

- In general, there are no specific directions for the oscillations of the electric and magnetic fields; they vibrate in any randomly oriented plane perpendicular to the direction of propagation.

- **Polarization** is the attribute that a wave's oscillations do have a definite direction relative to the direction of propagation of the wave.

- For an EM wave, we define the direction of polarization to be the direction parallel to the electric field.

Polarization



vertically polarized

horizontally polarized

Polarization

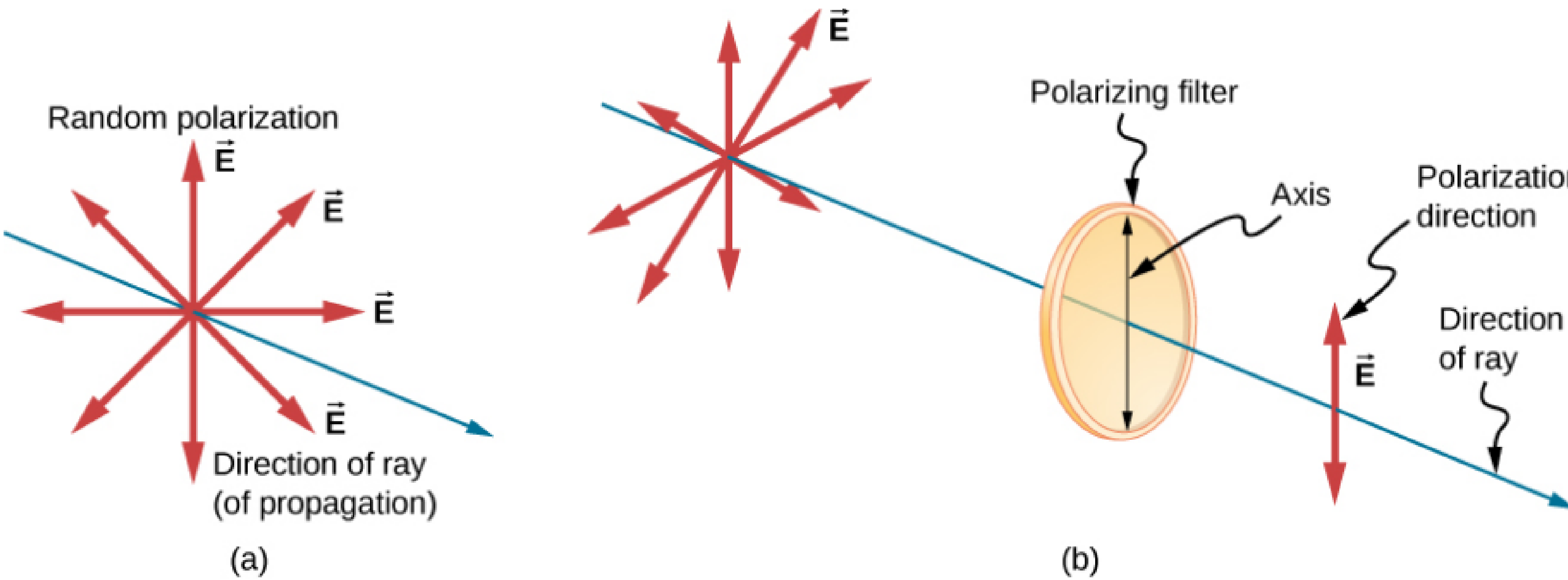
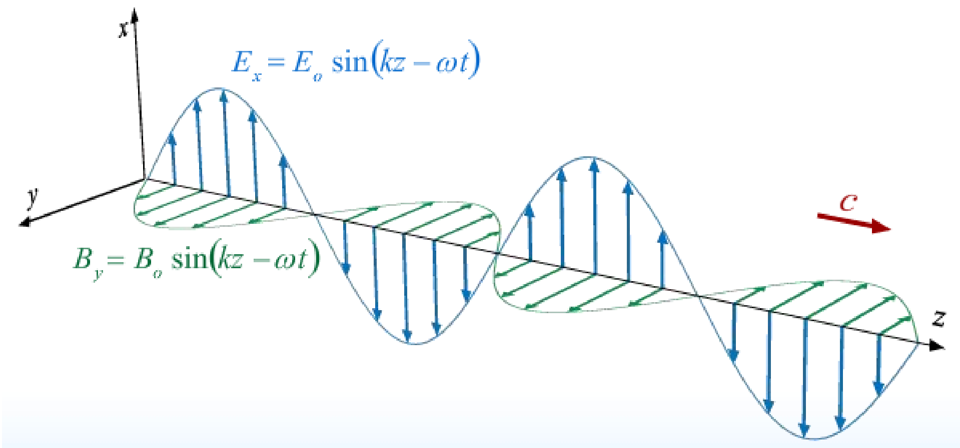


Figure 1.35 The slender arrow represents a ray of unpolarized light. The bold arrows represent the direction of polarization of the individual waves composing the ray. (a) If the light is unpolarized, the arrows point in all directions. (b) A polarizing filter has a polarization axis that acts as a slit passing through electric fields parallel to its direction. The direction of polarization of an EM wave is defined to be the direction of its electric field.

Linearly Polarized Light

So far we have considered plane waves that look like this:



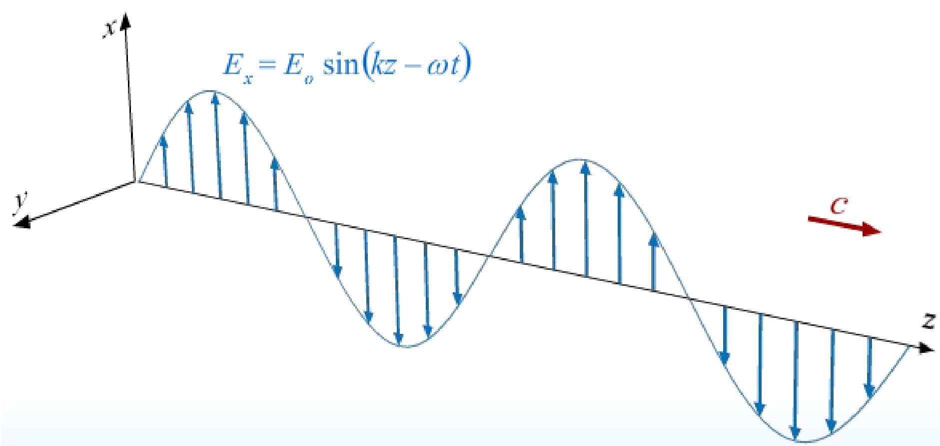
$$\omega = kc$$

$$E_o = cB_o$$

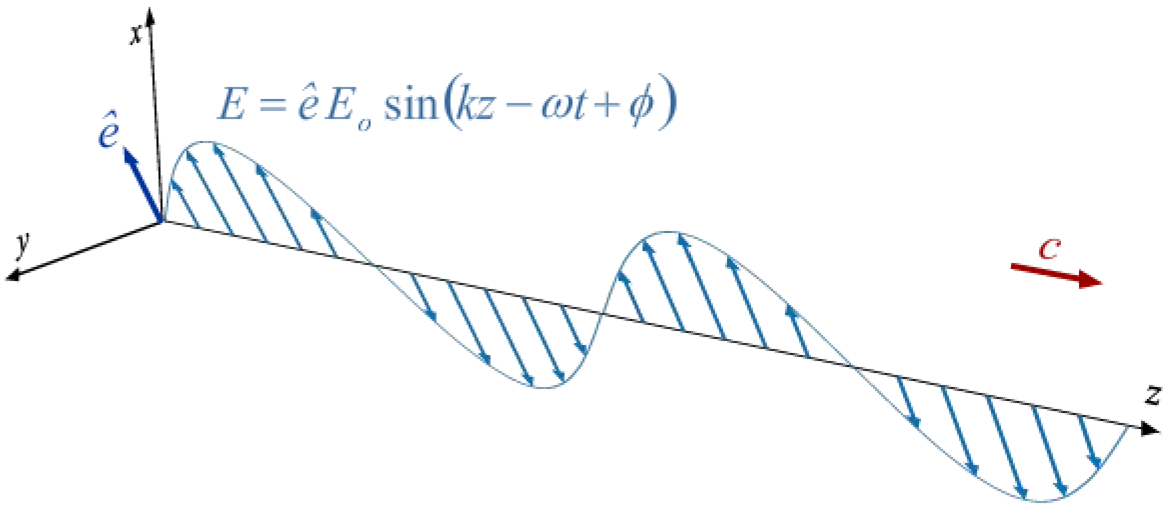
$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

From now on just draw E and remember that B is still there:

$\vec{E}$ Field determines Polarization



Linear Polarization



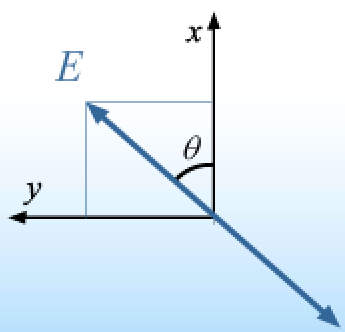
$$\omega = kc$$

$$E_o = cB_o$$

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

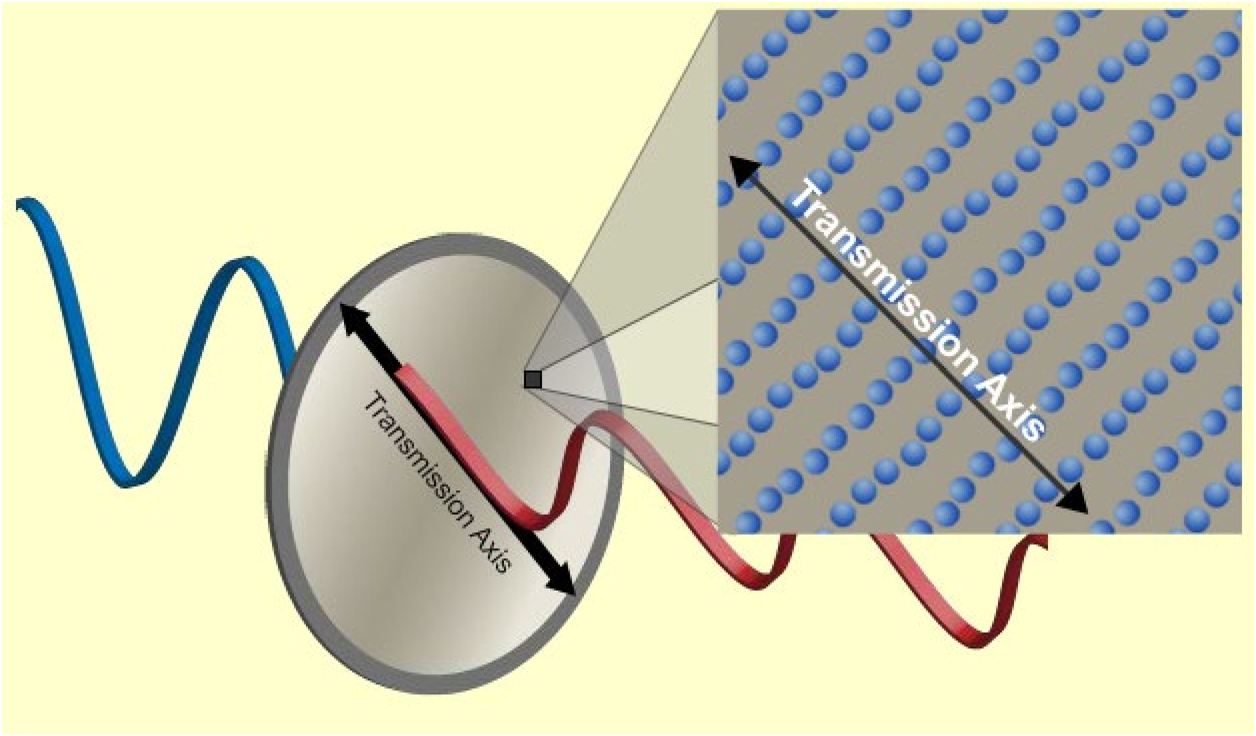
$$E_x = E_o \cos\theta \sin(kz - \omega t + \phi)$$

$$E_y = E_o \sin\theta \sin(kz - \omega t + \phi)$$



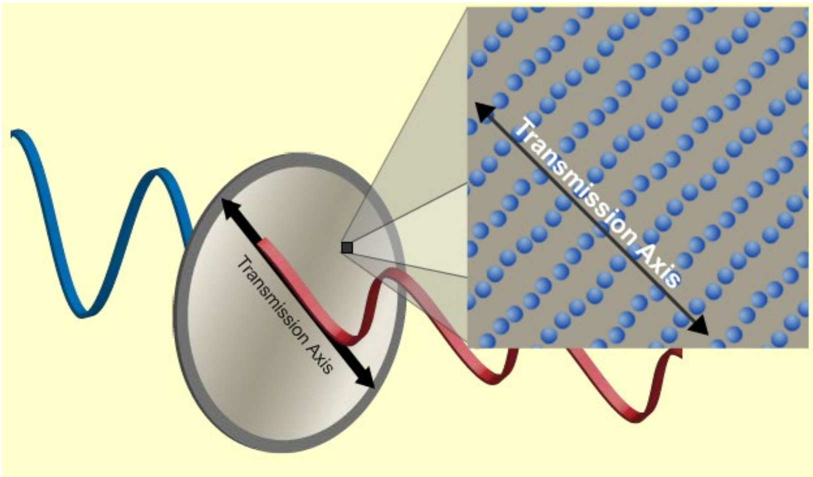
Wave Coming Out of the Screen

Polarizer

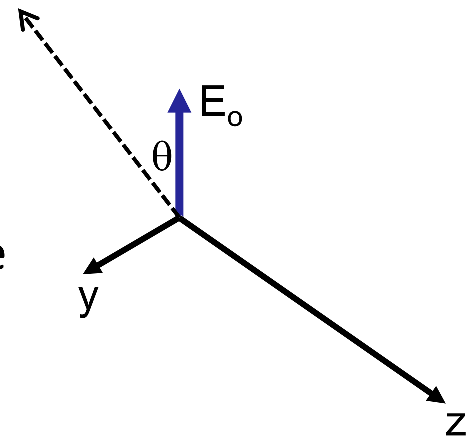


The molecular structure of a polarizer causes the component of the E field perpendicular to the Transmission Axis to be absorbed.

Clicker Question

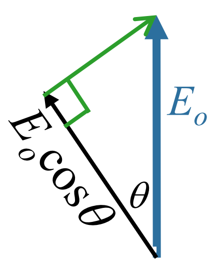


The molecular structure of a polarizer causes the component of the E field perpendicular to the Transmission Axis to be absorbed.



Suppose we have a beam traveling in the $+z-$ direction.
At $t=0$ and $z=0$, the electric field is aligned along the positive $x-$ axis and has a magnitude equal to E_o

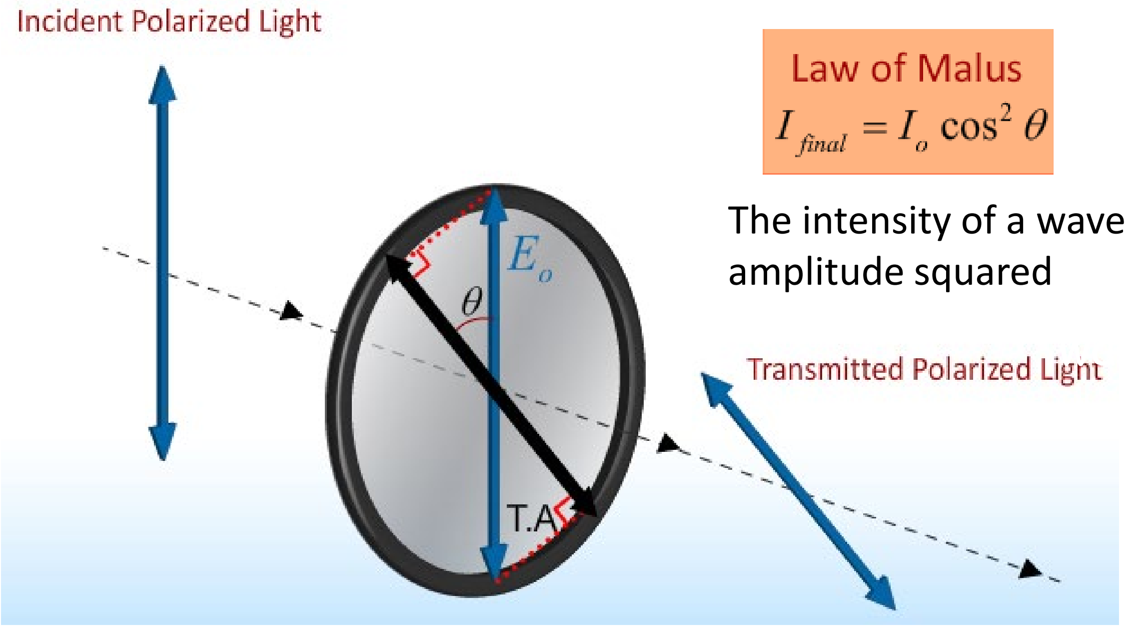
What is the component of E_o along a direction in the $x-y$ plane that makes an angle of θ with respect to the $x-$ axis?



A) $E_o\sin\theta$ $\boxed{\text{B) } E_o\cos\theta}$ C) 0 D) $E_o/\sin\theta$ E) $E_o/\cos\theta$

Bonus point1

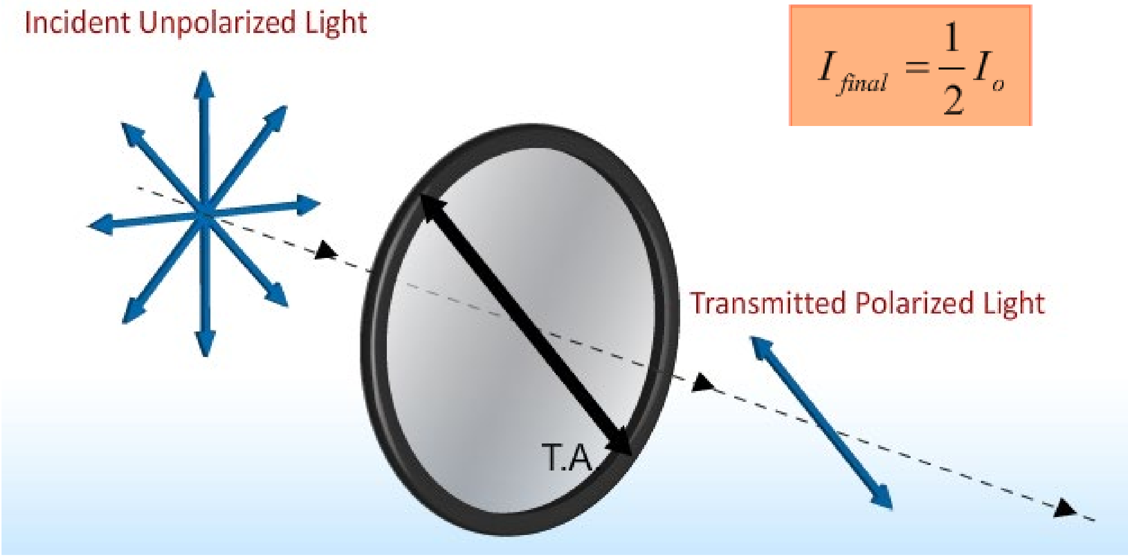
Linear Polarizers



Law of Malus

$$I_{final} = I_o \cos^2 \theta$$

The intensity of a wave is proportional to its amplitude squared

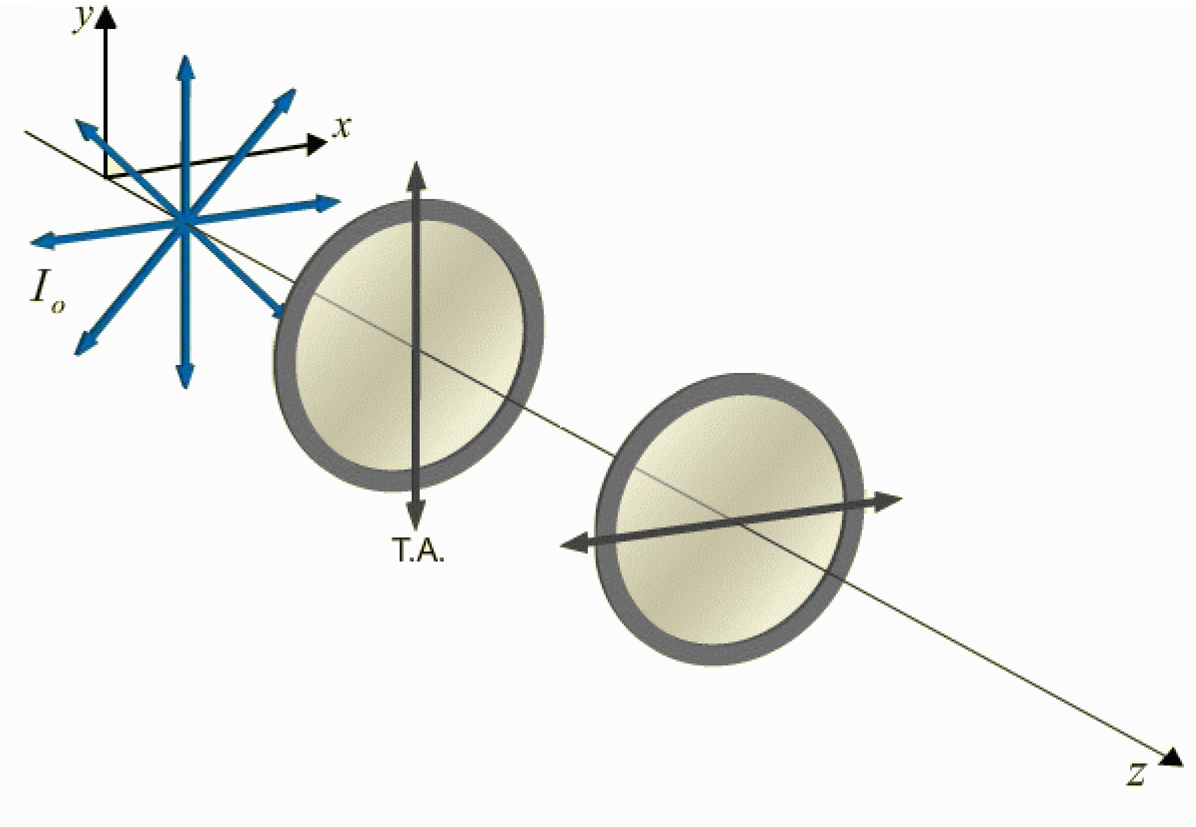


$$I_{final} = \frac{1}{2} I_o$$

Check Point 1a

An unpolarized EM wave is incident on two orthogonal polarizers.

Two Polarizers



What percentage of the intensity gets through both polarizers?

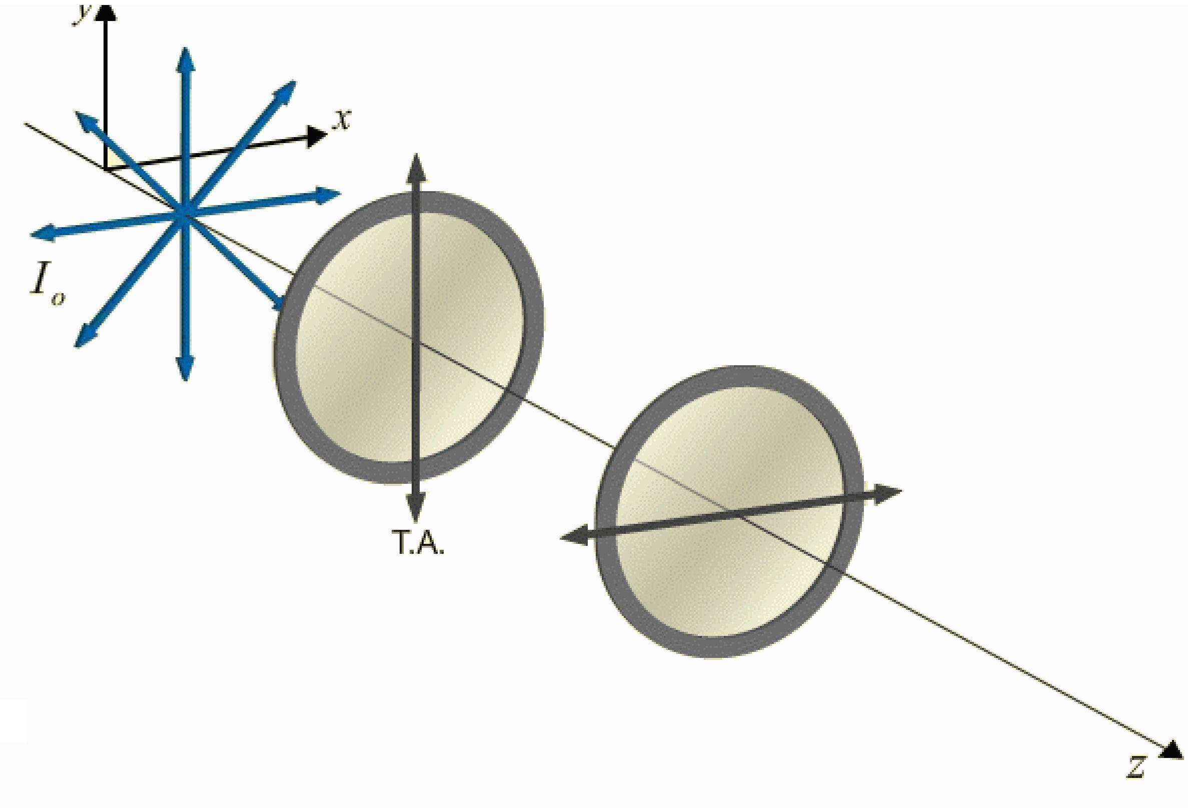
A. 50% ***B.*** 25% ***C.*** 0%

"After the unpolarized light passes through the first polarizer, its intensity was reduced to one half of its original strength. The first and the second polarizer is perpendicular, so the intensity of light passing out will be zero.."

Check Point 1b

An unpolarized EM wave is incident on two orthogonal polarizers.

Two Polarizers



Is it possible to increase this percentage by adding another Polarizer?

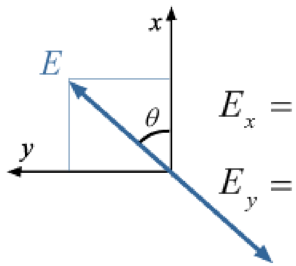
A. Yes **B.** No

Yes, because this would result in some components that are not perpendicular to the last polarizer and will thus pass through

Polarizers in action

Circularly Polarized Light

There is no reason that ϕ has to be the same for E_x and E_y:



$$E_x = E_o \cos\theta \sin(kz - \omega t + \phi_x)$$
$$E_y = E_o \sin\theta \sin(kz - \omega t + \phi_y)$$

Satisfies Wave Equation

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$
$$\frac{\partial^2 E_y}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

Making ϕ_x different from ϕ_y causes circular or elliptical polarization:

Example:

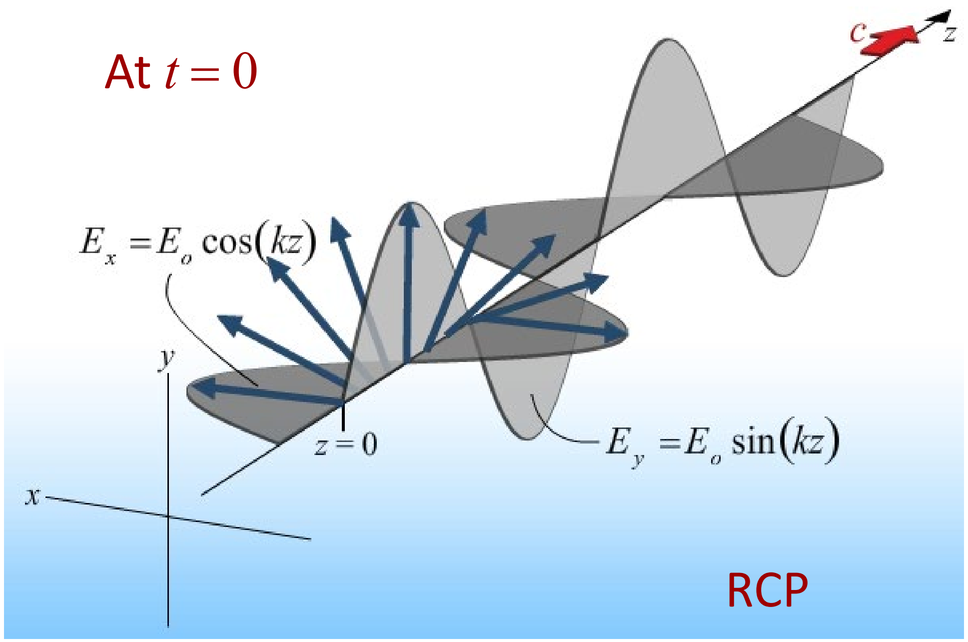
$$\phi_x - \phi_y = 90^\circ = \frac{\pi}{2}$$

$$\theta = 45^\circ = \pi / 4$$

$$E_x = \frac{E_0}{\sqrt{2}} \cos(kz - \omega t)$$

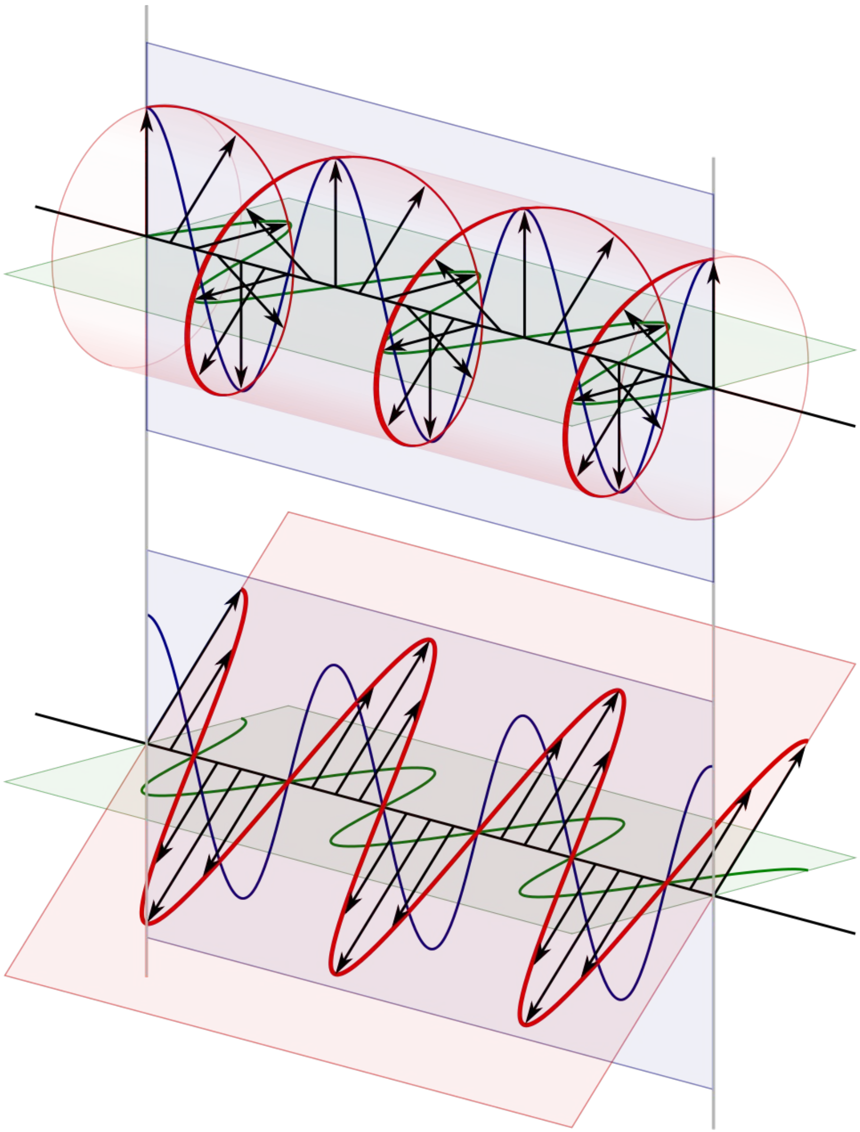
$$E_y = \frac{E_0}{\sqrt{2}} \sin(kz - \omega t)$$

At $t = 0$

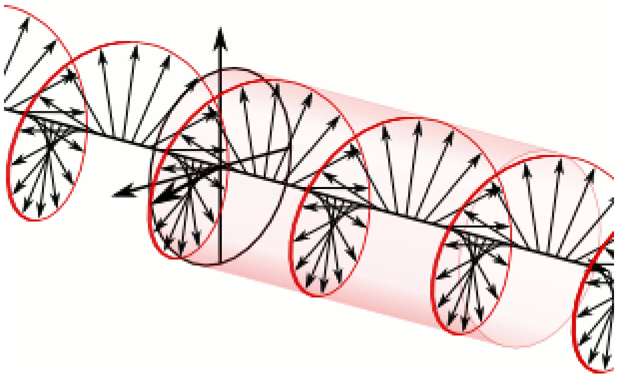


(Right-Circularly Polarized)

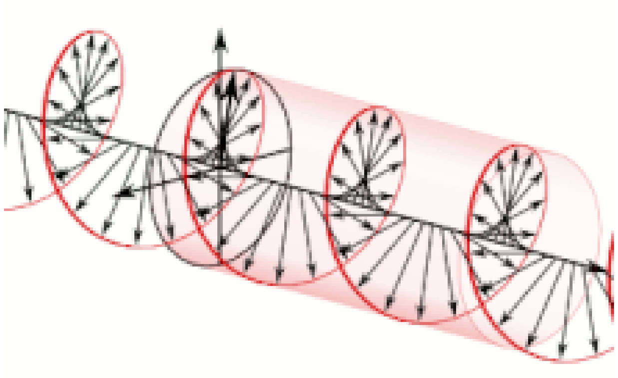
Plane vs. Linear



RCP
(right-circularly polarized)



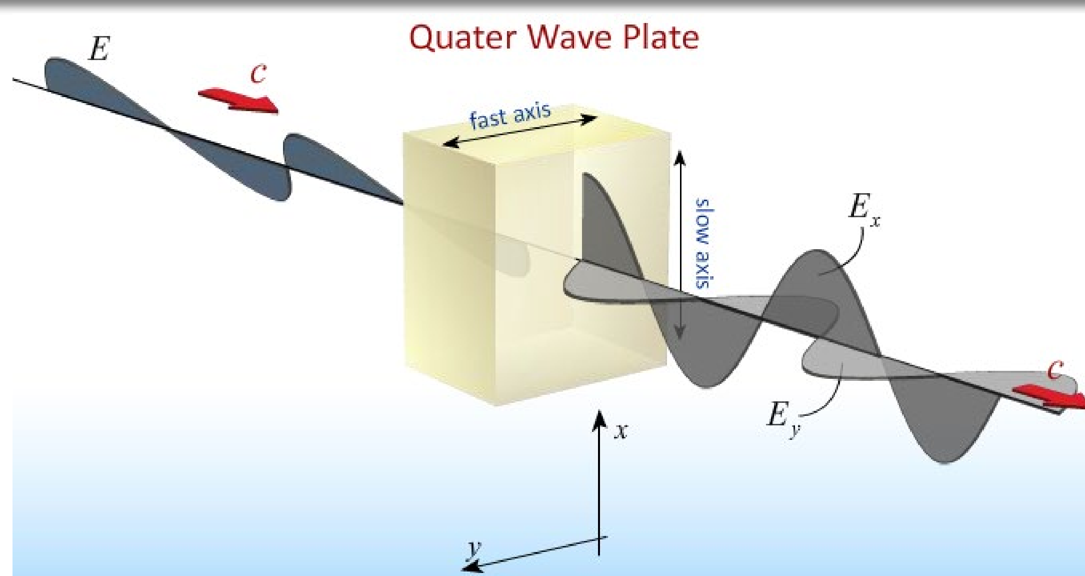
LCP
(left-circularly polarized)

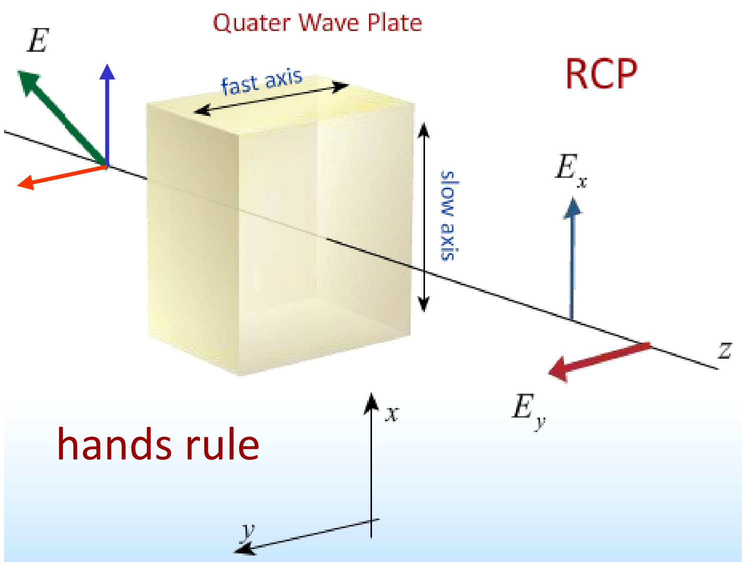


Quarter Waveplates

Q: How do we change the relative phase between E_x and E_y?

A: Birefringence



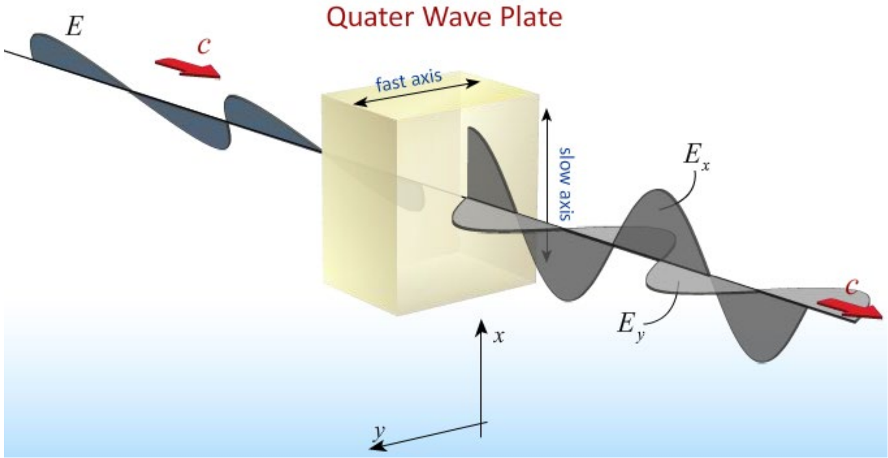


By picking the right thickness we can change the relative phase by exactly 90^o.

This changes linear (if @45°) to circular polarization and is called a *quarter wave plate*

Intensity does not change!

Does the birefringent material absorb any of the light/energy?



NOTE: No Intensity is lost passing through the QWP !

BEFORE QWP:

$$E = E_o \sin(kz - \omega t)\left[\frac{i + j}{\sqrt{2}}\right] \quad \rightarrow \quad I = c\varepsilon_o \langle E^2 \rangle = c\varepsilon_o \langle E_x^2 + E_y^2 \rangle$$

$$= c\varepsilon_o \left(\frac{E_o^2}{2} + \frac{E_o^2}{2}\right)\langle \sin^2(kz - \omega t)\rangle = c\varepsilon_o E_o^2 \frac{1}{2}$$

AFTER QWP:

$$E = \frac{E_o}{\sqrt{2}}\left[i\cos(kz - \omega t) + j\sin(kz - \omega t)\right] \quad \rightarrow \quad I = c\varepsilon_o \langle E^2 \rangle = c\varepsilon_o \langle E_x^2 + E_y^2 \rangle$$

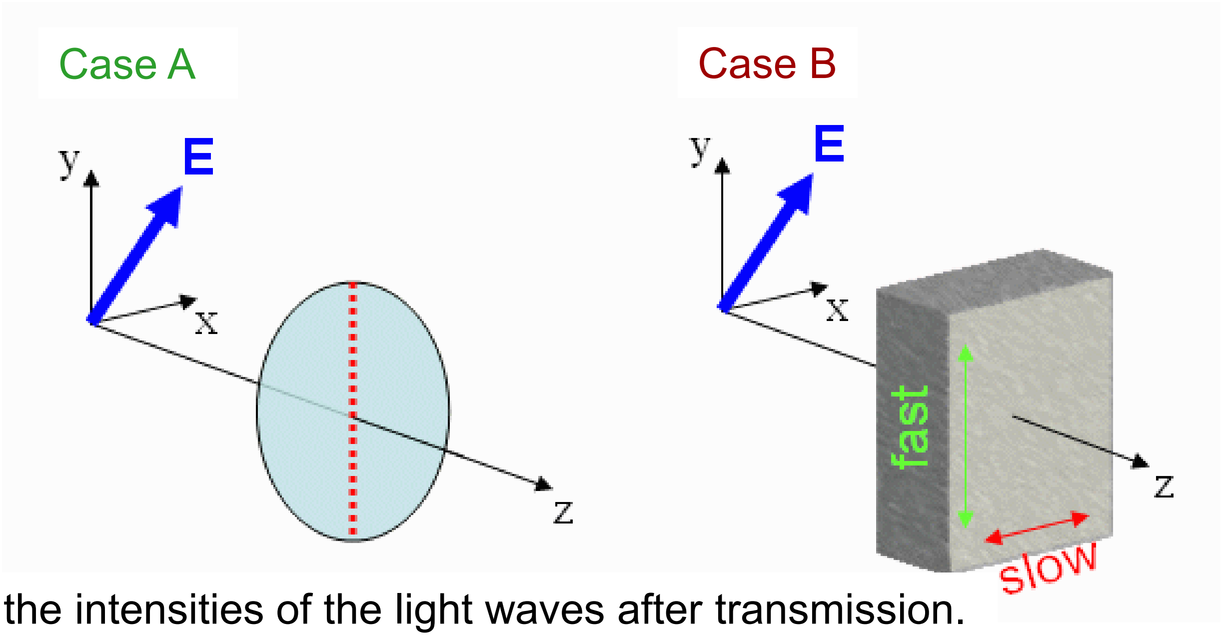
$$= c\varepsilon_o \frac{E_o^2}{2}\langle \cos^2(kz - \omega t) + \sin^2(kz - \omega t)\rangle$$

$$= c\varepsilon_o \frac{E_o^2}{2}\langle 1 \rangle = c\varepsilon_o \frac{E_o^2}{2}$$

THE SAME!

Check Point 2a

Identical linearly polarized light at 45° from the y-axis and propagating along the z axis is incident on two different objects. In Case A the light intercepts a linear polarizer with polarization along the y-axis In Case B, the light intercepts a quarter wave plate with fast axis along the y-axis.



Compare the intensities of the light waves after transmission.

A. $I_A < I_B$ **B.** $I_A = I_B$ **C.** $I_A > I_B$

Case A:

E_x is absorbed

$$I_A = I_0 \cos^2(45^o)$$

$$I_A = \tfrac{1}{2} I_0$$

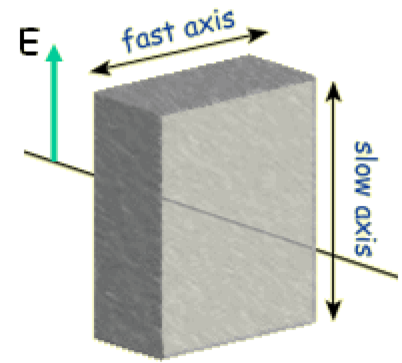
Case B:

(E_x, E_y) phase changed

$$I_B = I_0$$

Quarter Wave Plate (prelecture)

Vertically polarized light is incident on a quarter wave plate which is oriented so that its slow axis is in the vertical direction and its fast axis is in the horizontal direction.

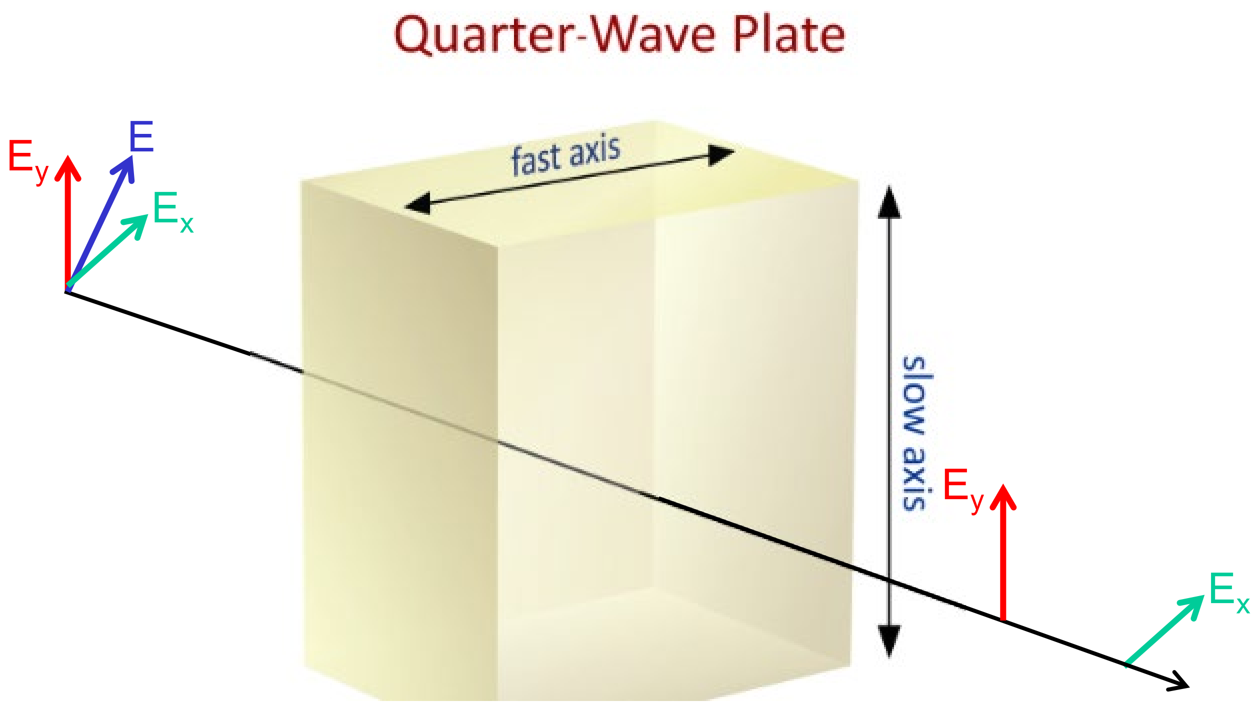


What is the polarization of the light after it exits the quarter wave plate?

A) Vertically polarized
B) Horizontally polarized
C) Left circularly polarized
D) Right circularly polarized

Quarter wave plates only produce circularly polarized light if incoming light is linearly polarized at 45 degrees relative to the fast/slow axis.

Right or Left Circularly Polarized



A linearly polarized EM wave is incident on a quarter-wave plate as shown above. The resulting wave is

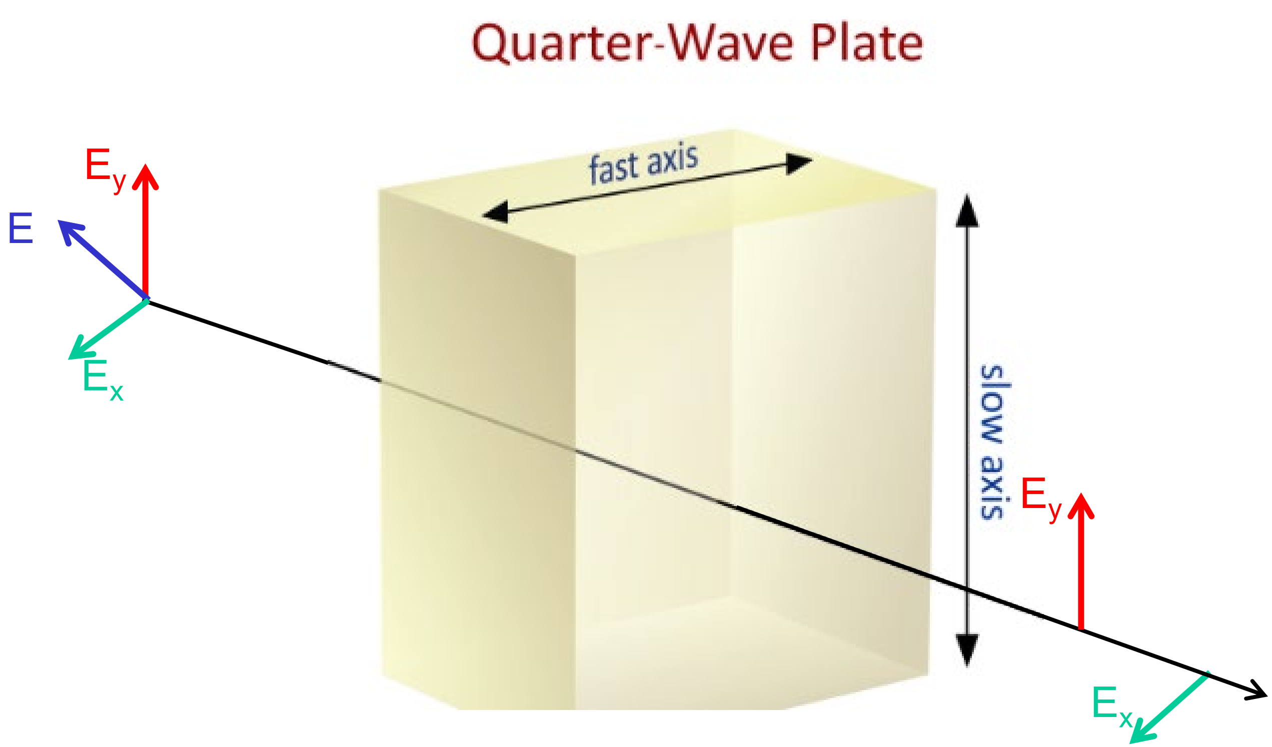
A) Right Circularly Polarized

B) $\boxed{\text{Left Circularly Polarized}}$

C) Linearly Polarized

Curve fingers from slow to fast, thumb must be direction of propagation.

Right or Left Circularly Polarized



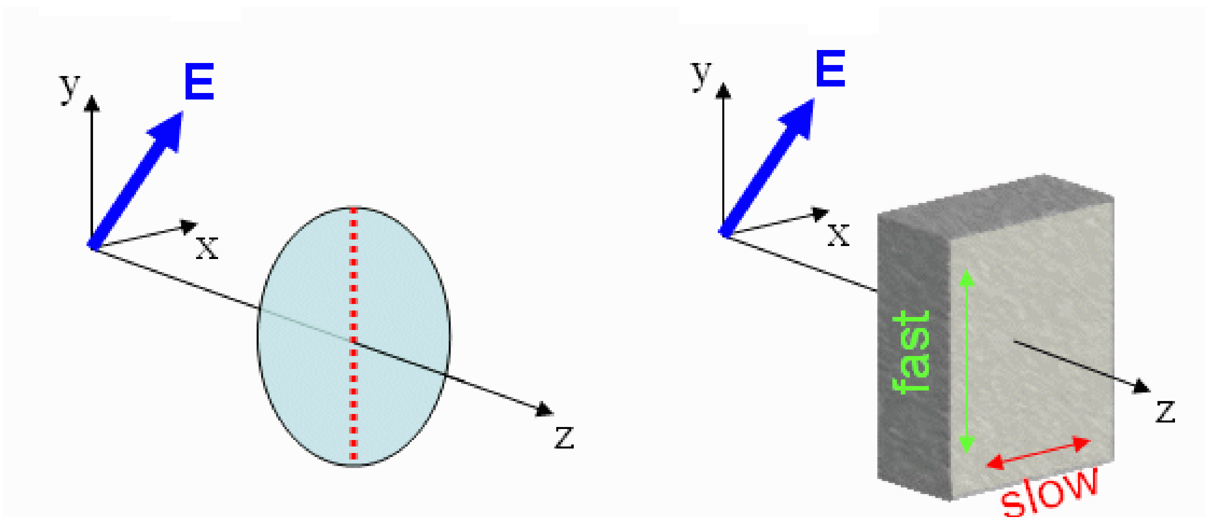
A linearly polarized EM wave is incident on a quarter-wave plate as shown above. The resulting wave is

A) Right Circularly Polarized
B) Left Circularly Polarized
C) Linearly Polarized

Curve fingers from slow to fast, thumb must be direction of propagation.

Check Point 2b

Identical linearly polarized light at 45° from the y-axis and propagating along the z axis is incident on two different objects. In Case A the light intercepts a linear polarizer with polarization along the y-axis In Case B, the light intercepts a quarter wave plate with fast axis along the y-axis.



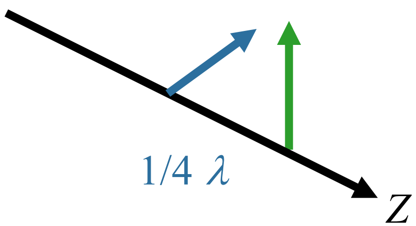
What is the polarization of the light wave in Case B after it passes through the quarter wave plate?.

A. linearly polarized

B. left circularly polarized

C. right circularly polarized

D. undefined



Curl fingers from slow to fast, thumb should point in direction of propagation

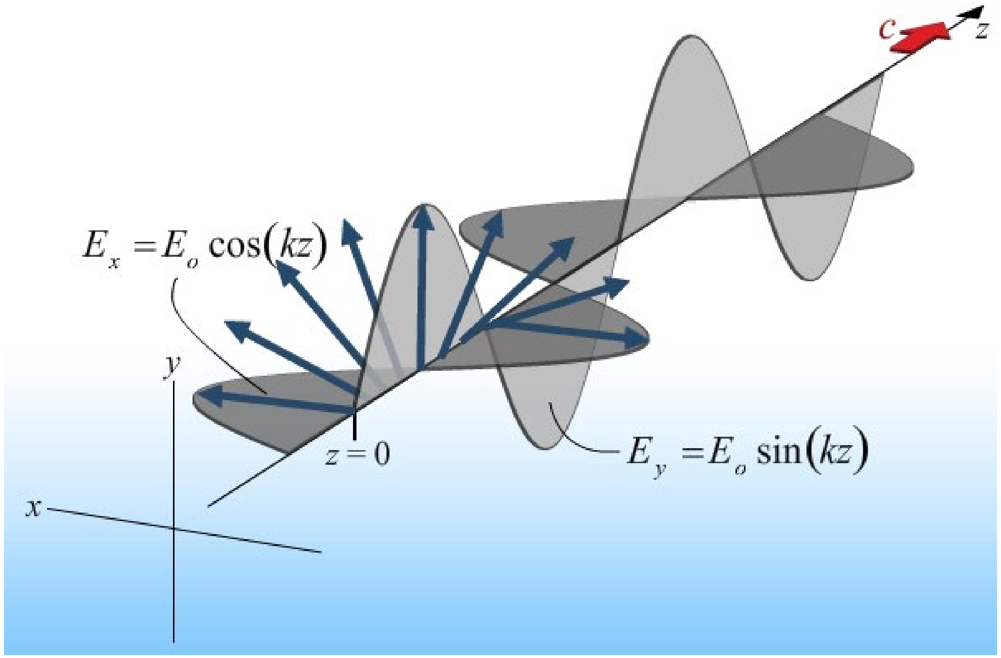
Circular Light on Linear Polarizer

Q: What happens when circularly polarized light is put through a polarizer along the y (or x) axis ?

A) $I = 0$

B) $I = \frac{1}{2} I_0$

C) $I = I_0$



$$I = \varepsilon_0 c \left\langle E^2 \right\rangle$$

$$= \varepsilon_0 c \left\langle E_x^2 + \cancel{E_y^2} \right\rangle$$

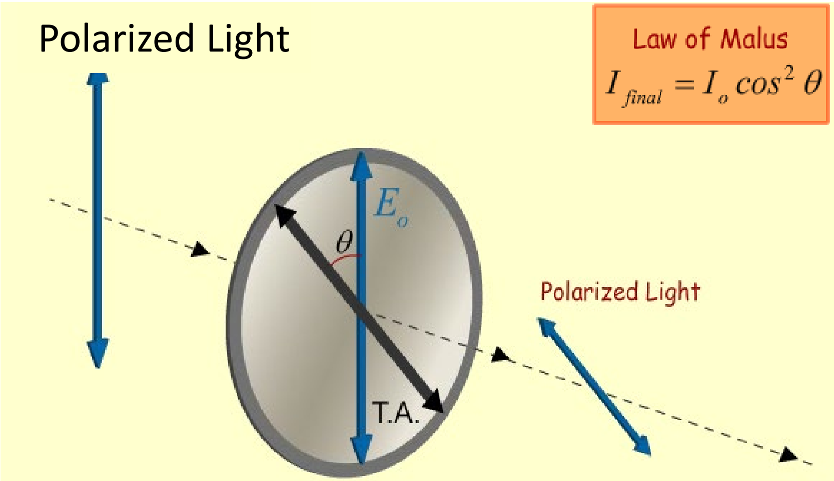
$$= \varepsilon_0 c \frac{E_0^2}{2} \underbrace{\left\langle \cos^2(kz - \omega t) \right\rangle}_{1/2}$$

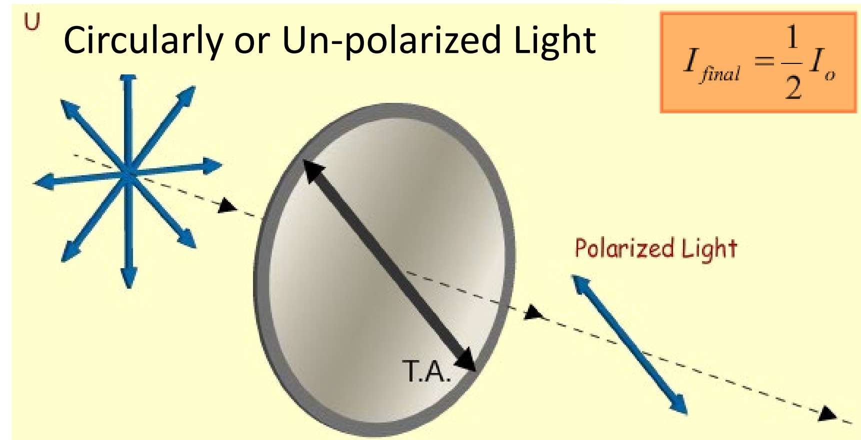
$$= \frac{1}{2} \cdot \frac{1}{2} \varepsilon_0 c E_0^2$$

Half of before

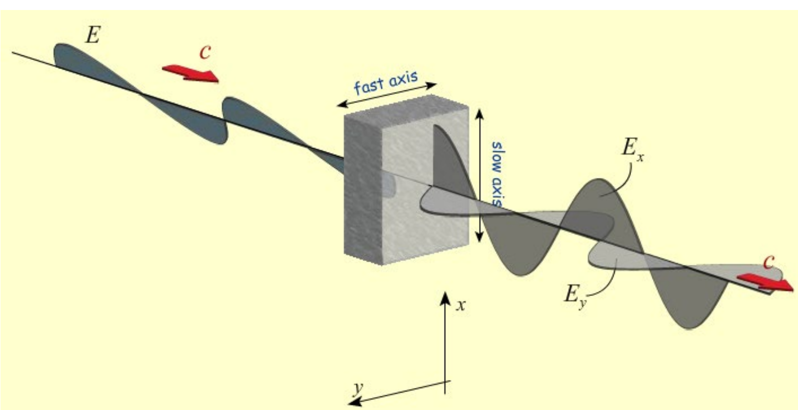
Summary:

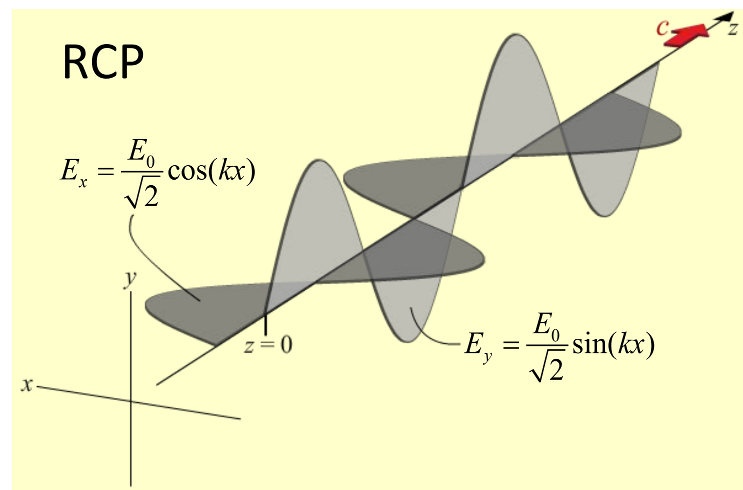
Polarizers & QW Plates:





Birefringence

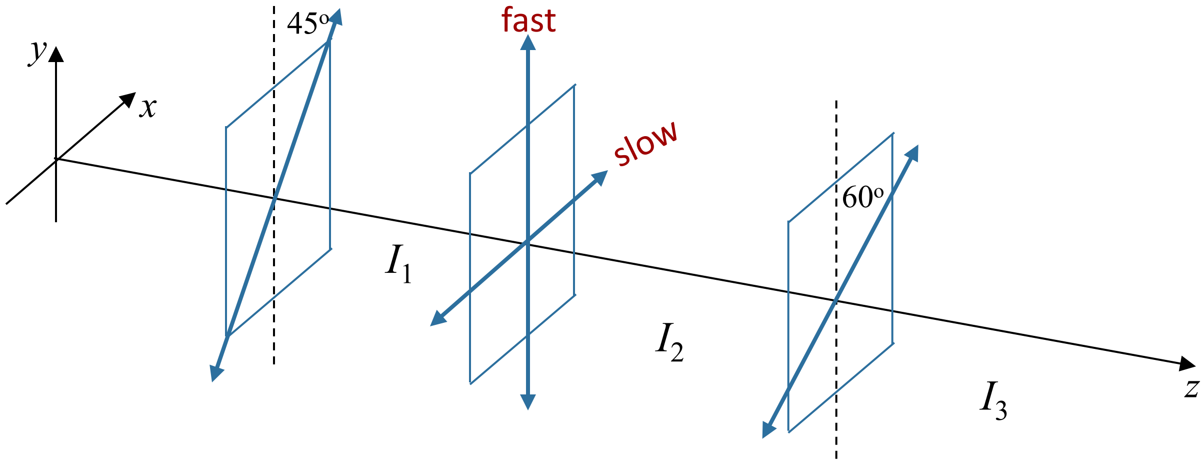




Calculation

Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.

What is the intensity I_3 in terms of I_1?



Conceptual Analysis

Linear Polarizers: absorbs E field component perpendicular to *TA (Transmission Axis)*

Quarter Wave Plates: Shifts phase of E field components in fast-slow directions

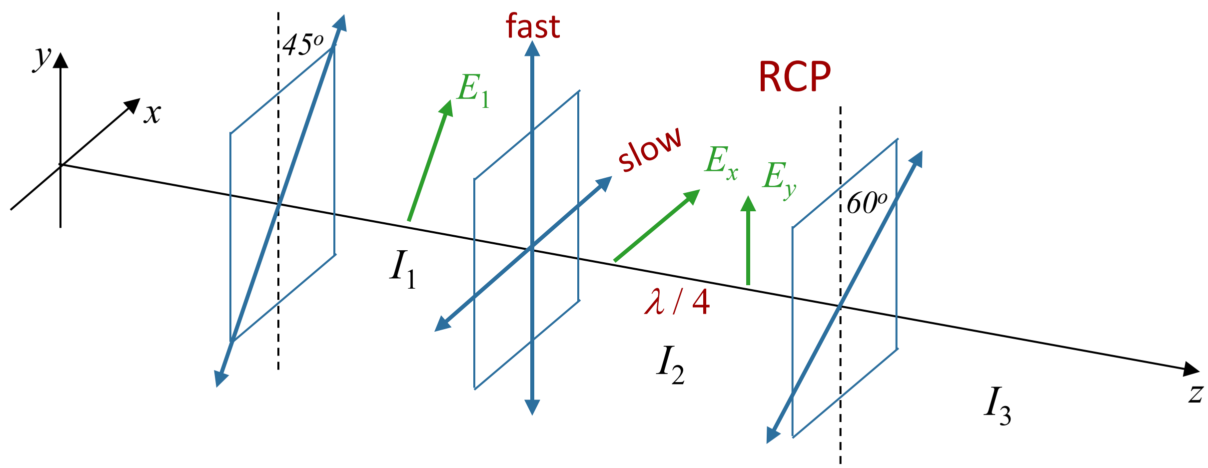
Strategic Analysis

Determine state of polarization and intensity reduction after each object

Multiply individual intensity reductions to get final reduction.

Calculation

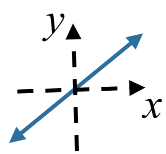
Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.

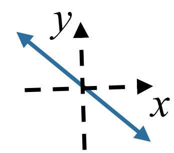


What is the polarization of the light after the QWP?

A) LCP

B) RCP

C) 

D) 

E) un-polarized

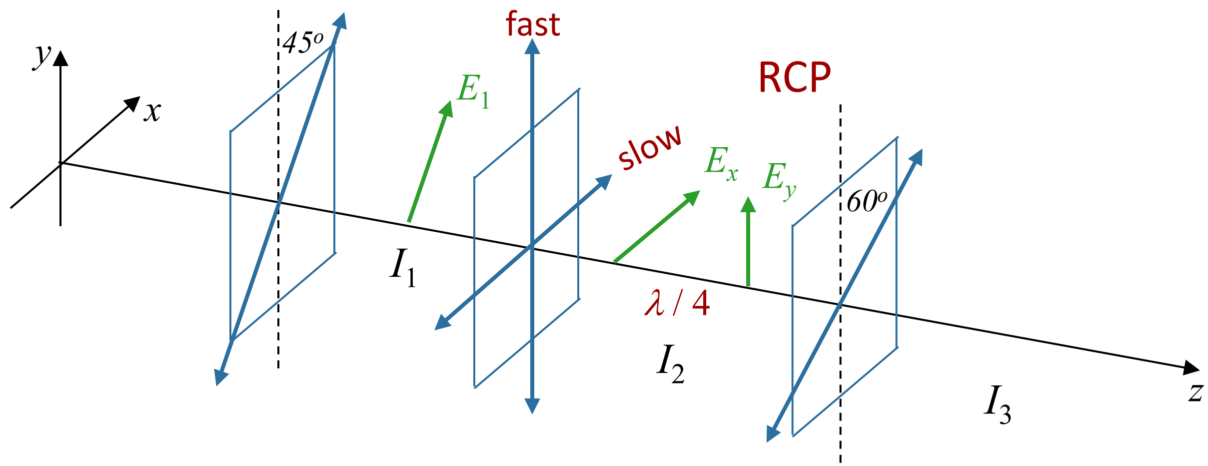
Light incident on QWP is linearly polarized at 45° to fast axis → Light will be circularly polarized after QWP

LCP or RCP? Easiest way: Hands Rule:

Curl fingers of RH back to front
Thumb points in dir of propagation
if right hand polarized. → RCP

Calculation

Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.



What is the intensity I_2 of the light after the QWP?

$\boxed{\text{A) } I_2 = I_1}$ B) $I_2 = ½\, I_1$ C) $I_2 = ¼\, I_1$

Before:

$$E_x = \frac{E_1}{\sqrt{2}} \sin(kz - \omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}} \sin(kz - \omega t)$$

No absorption: Just a phase change!

$$I = \varepsilon_0 c \left[\left\langle E_x^2 \right\rangle + \left\langle E_y^2 \right\rangle \right]$$

Same before & after!

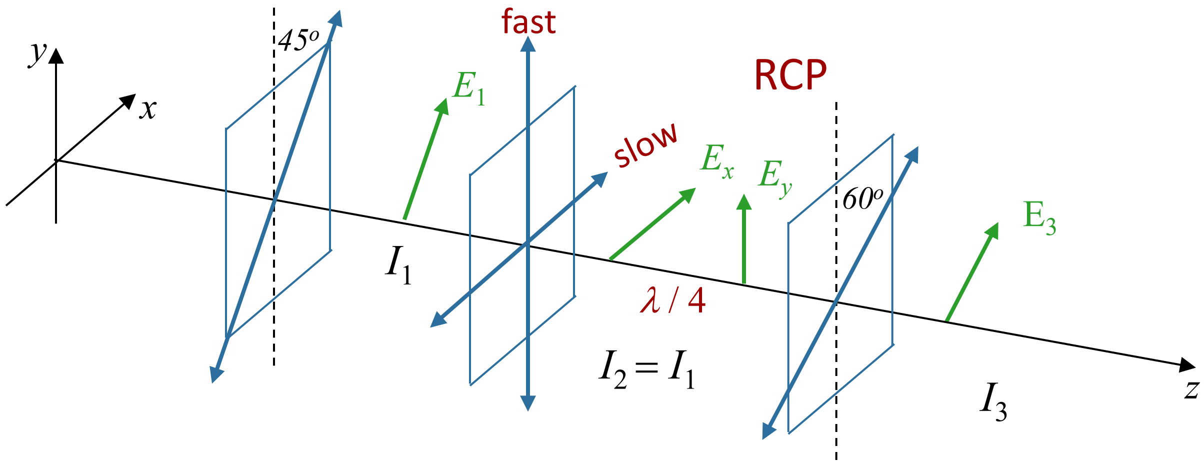
After:

$$E_x = \frac{E_1}{\sqrt{2}} \cos(kz - \omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}} \sin(kz - \omega t)$$

Calculation

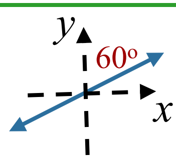
Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.

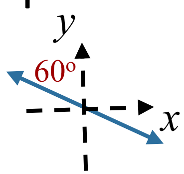


What is the polarization of the light after the 60^o polarizer?

A) LCP

B) RCP

C) 

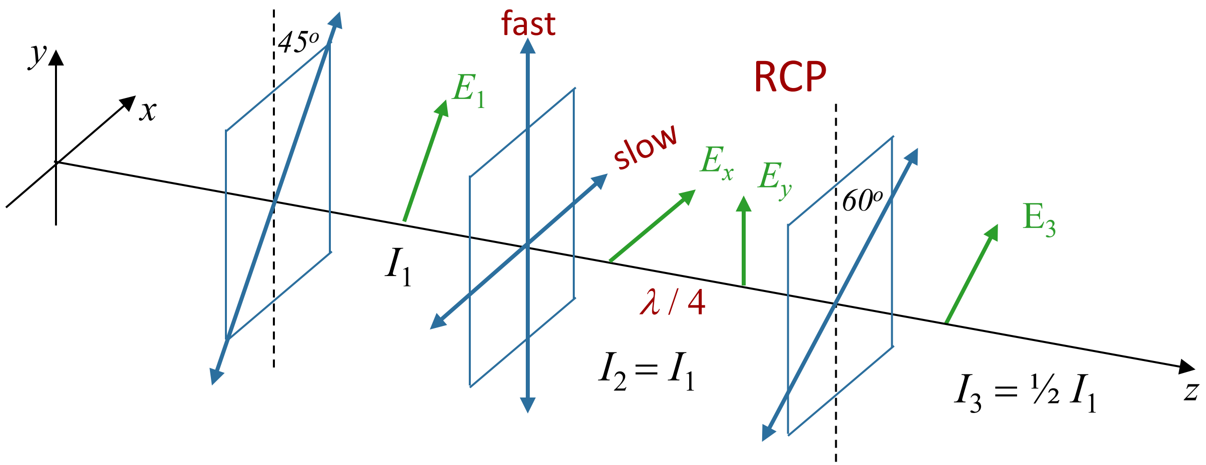
D) 

E) un-polarized

Absorption: only passes components of E parallel to TA $(\theta = 60^o)$

Calculation

Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.



What is the intensity I_3 of the light after the 60^o polarizer?

A) $I_3 = I_1$

B) $\boxed{I_3 = ½ I_1}$

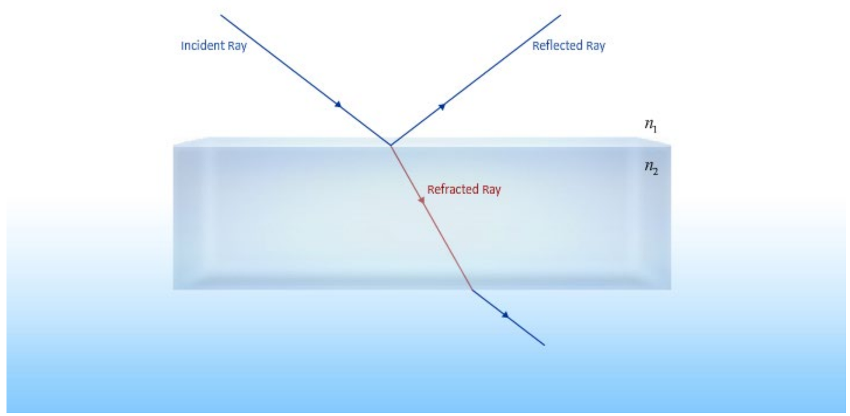
C) $I_3 = ¼ I_1$

Circularly polarized light through linear polarizer cuts intensity by ½. $\Rightarrow$ $I_3 = \frac{1}{2} I_1$

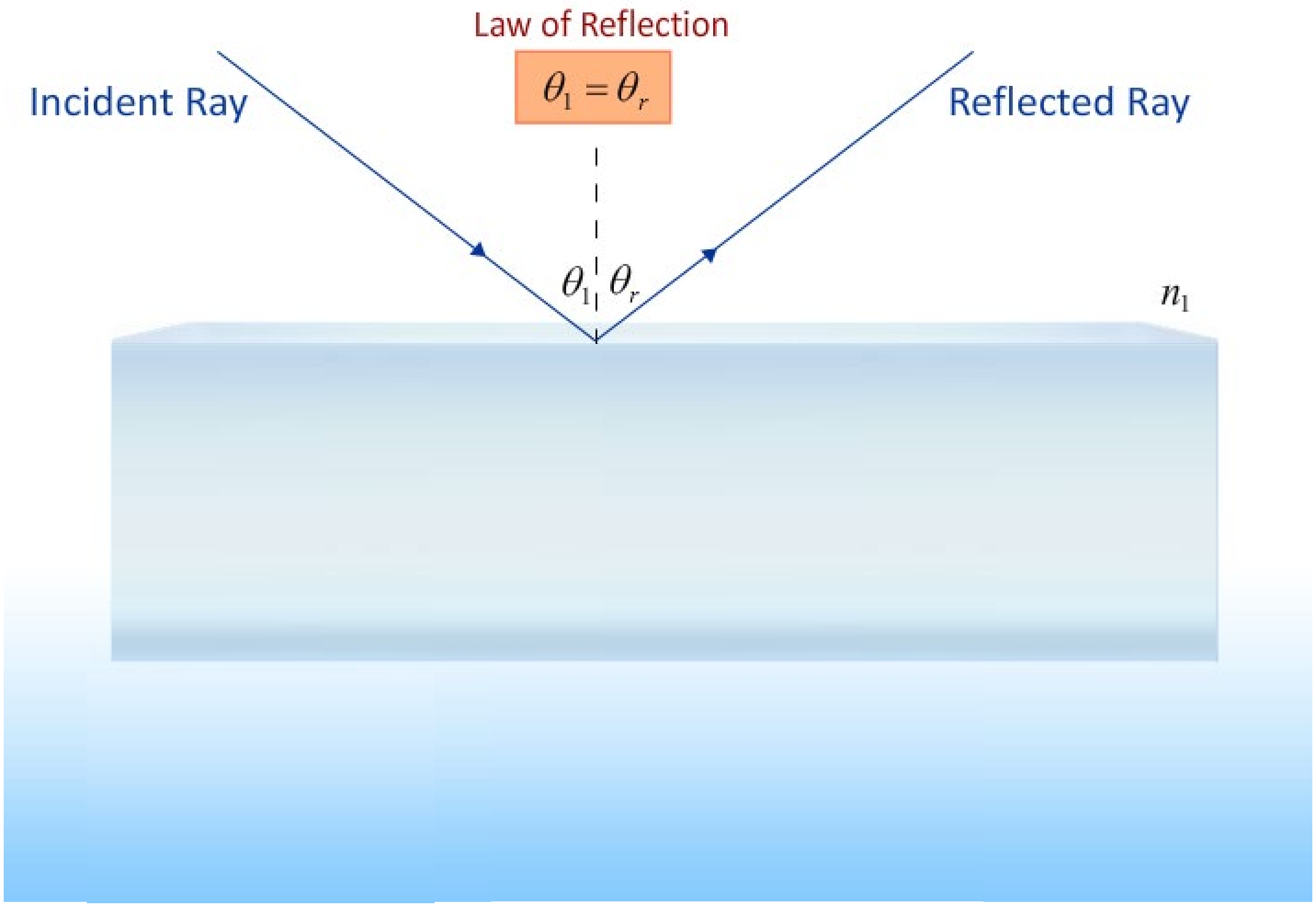
NOTE: This does not depend on θ !

Physics 212
Lecture 25

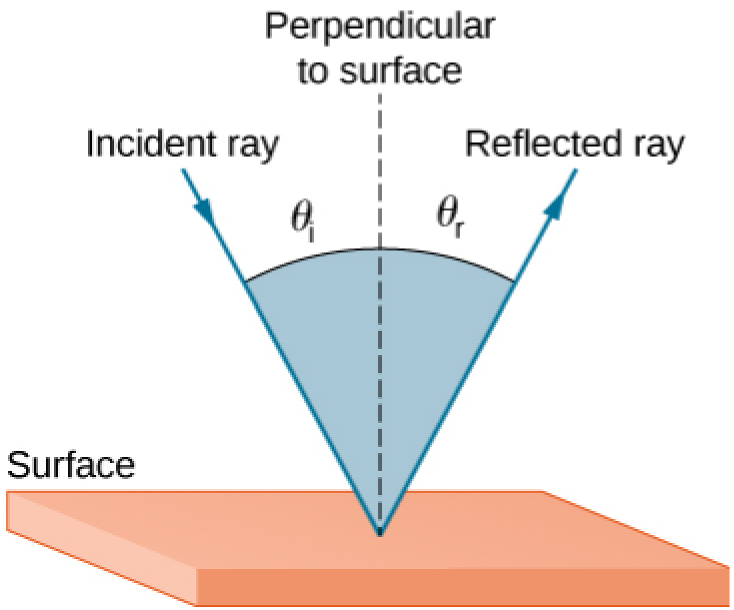
REFLECTION and REFRACTION



Let's Start with a Summary:



Law of reflection



$$\theta_r = \theta_i$$

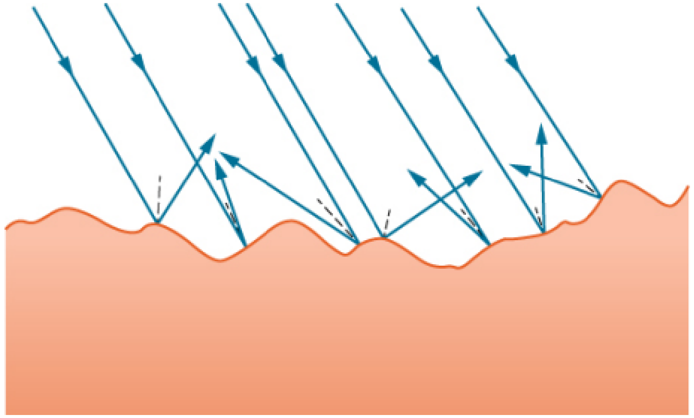


Figure 1.6 Light is diffused when it reflects from a rough surface. Here, many parallel rays are incident, but they are reflected at many different angles, because the surface is rough.

Law of reflection

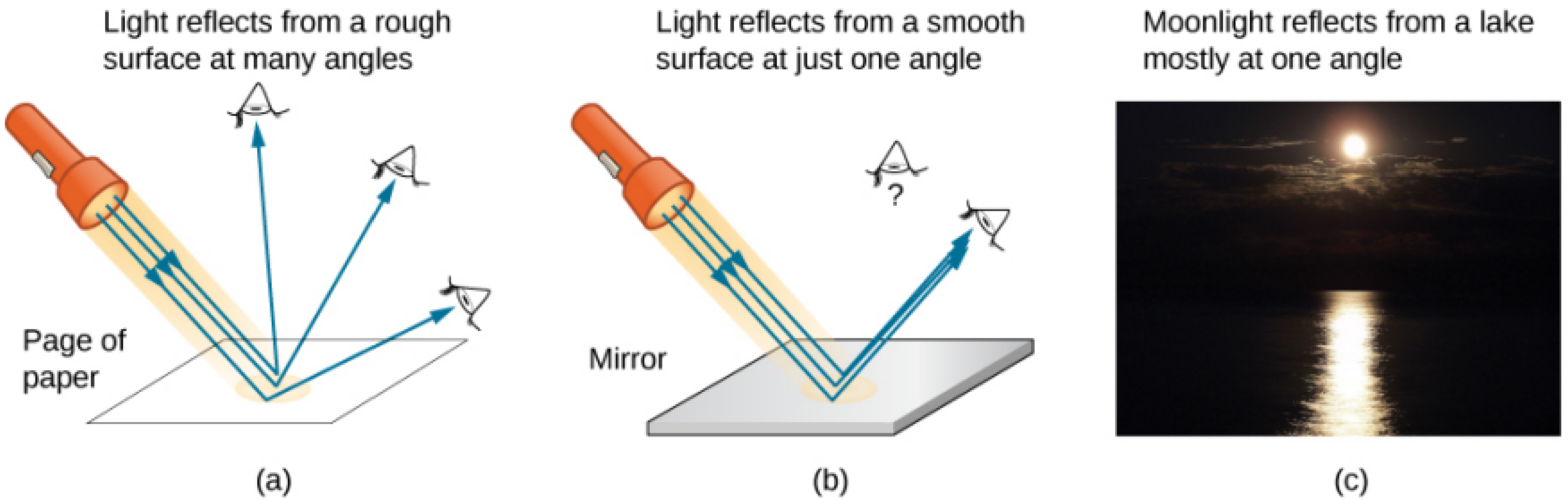


Figure 1.7 (a) When a sheet of paper is illuminated with many parallel incident rays, it can be seen at many different angles, because its surface is rough and diffuses the light. (b) A mirror illuminated by many parallel rays reflects them in only one direction, because its surface is very smooth. Only the observer at a particular angle sees the reflected light. (c) Moonlight is spread out when it is reflected by the lake, because the surface is shiny but uneven. (credit c: modification of work by Diego Torres Silvestre)

The speed of light in a medium is slower than in empty space:

Speed of Light

$$v = \frac{1}{\sqrt{\mu\varepsilon}} < \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

Index of Refraction

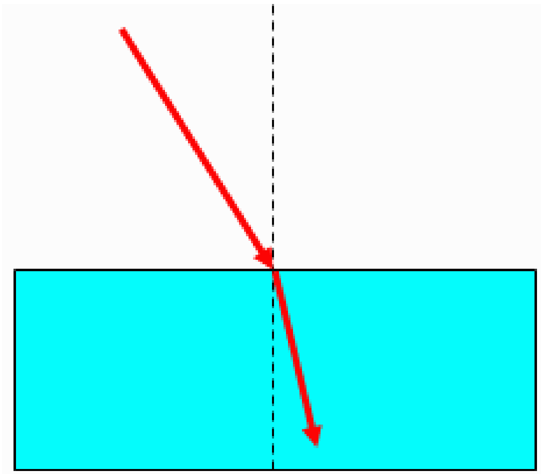
$$n \equiv \frac{c}{v}$$

κ is the dielectric constant

$$v_{\text{medium}} = c \, / \, n_{\text{medium}}$$

Check Point 1a

2) A ray of light passes from air into water with an angle of incidence of 30 degrees.



Which of the following quantities does NOT change as light enters the water?

answers.

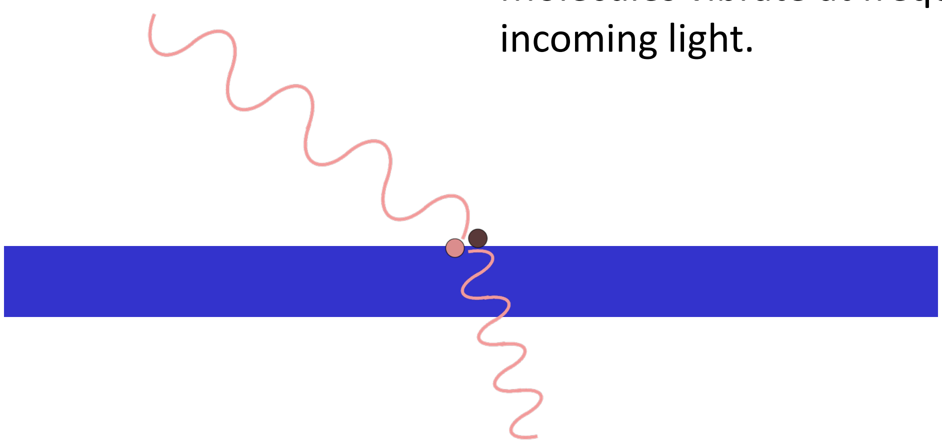
A wavelength

$\boxed{\text{B frequency}}$

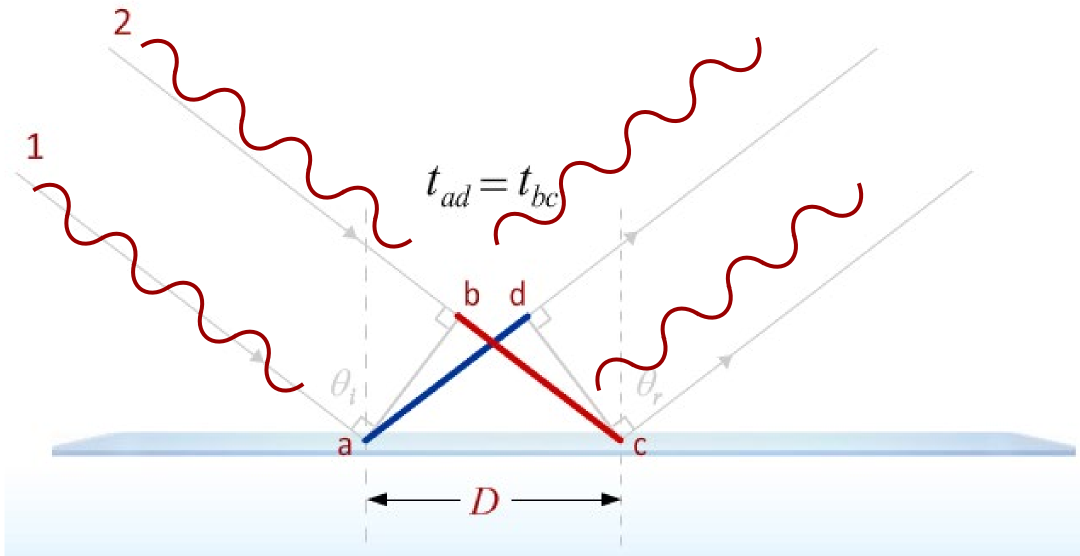
C speed of propagation

What about the wave must be the same on either side?

Molecules vibrate at frequency of incoming light.



Reflection

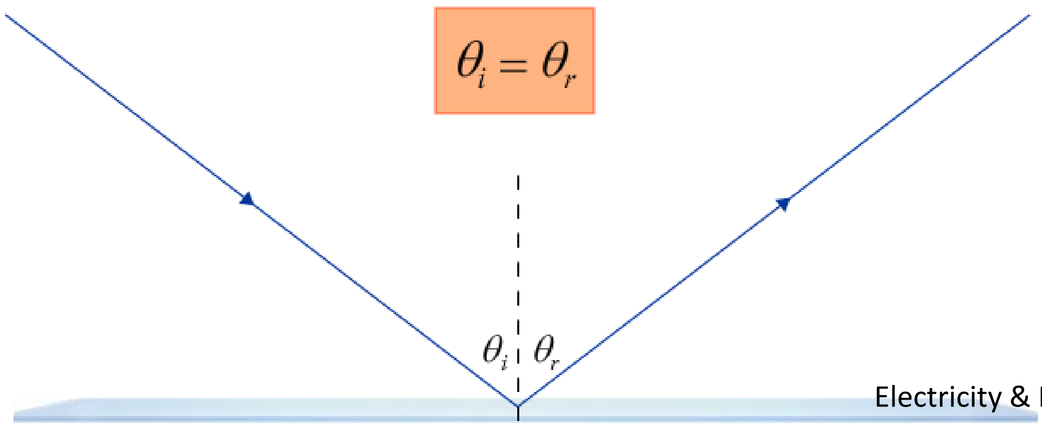


$$L_{ad} = L_{bc}$$

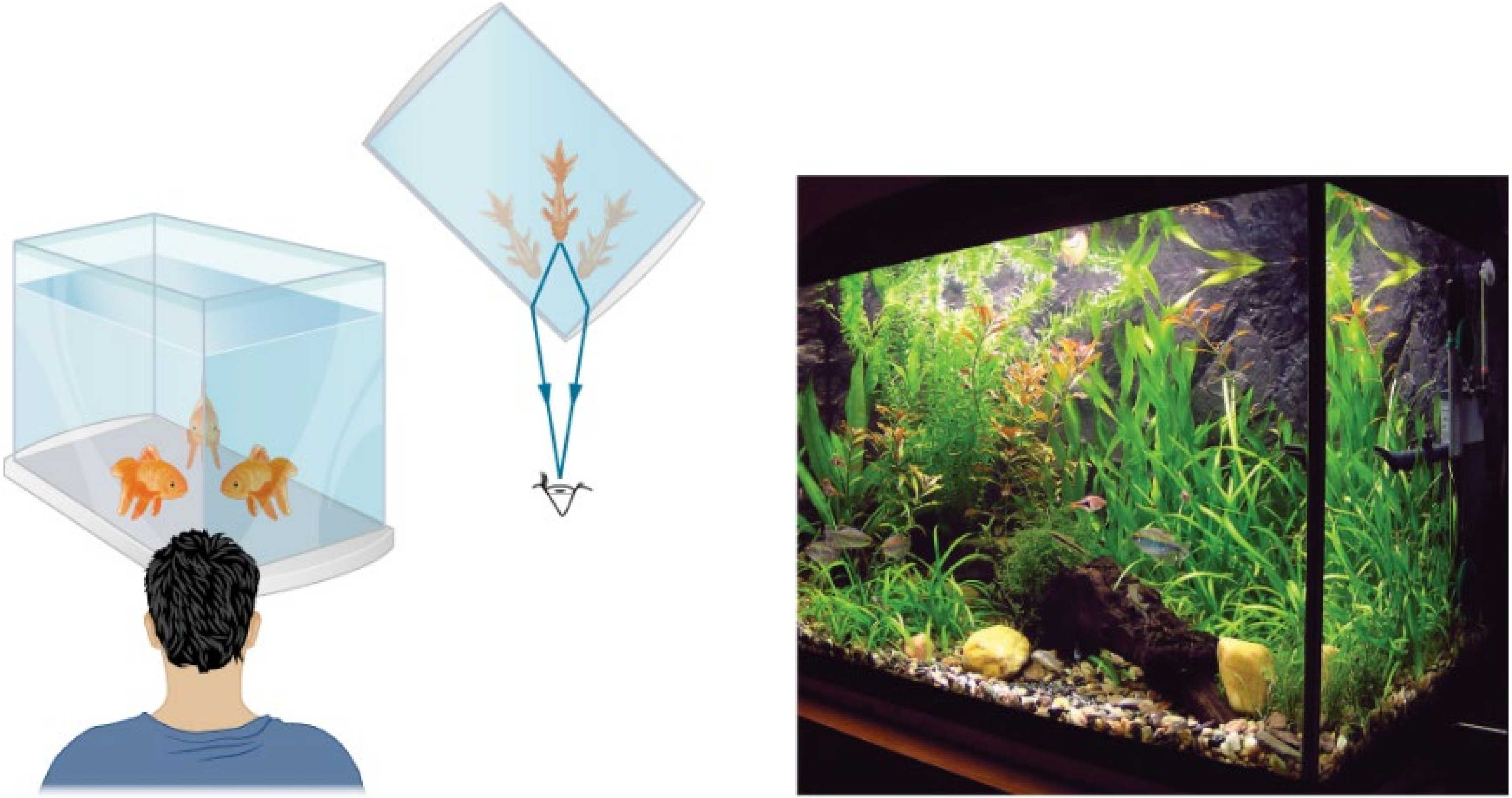
$$D\sin\theta_r = D\sin\theta_i$$

Law of Reflection

$$\boxed{\theta_i = \theta_r}$$



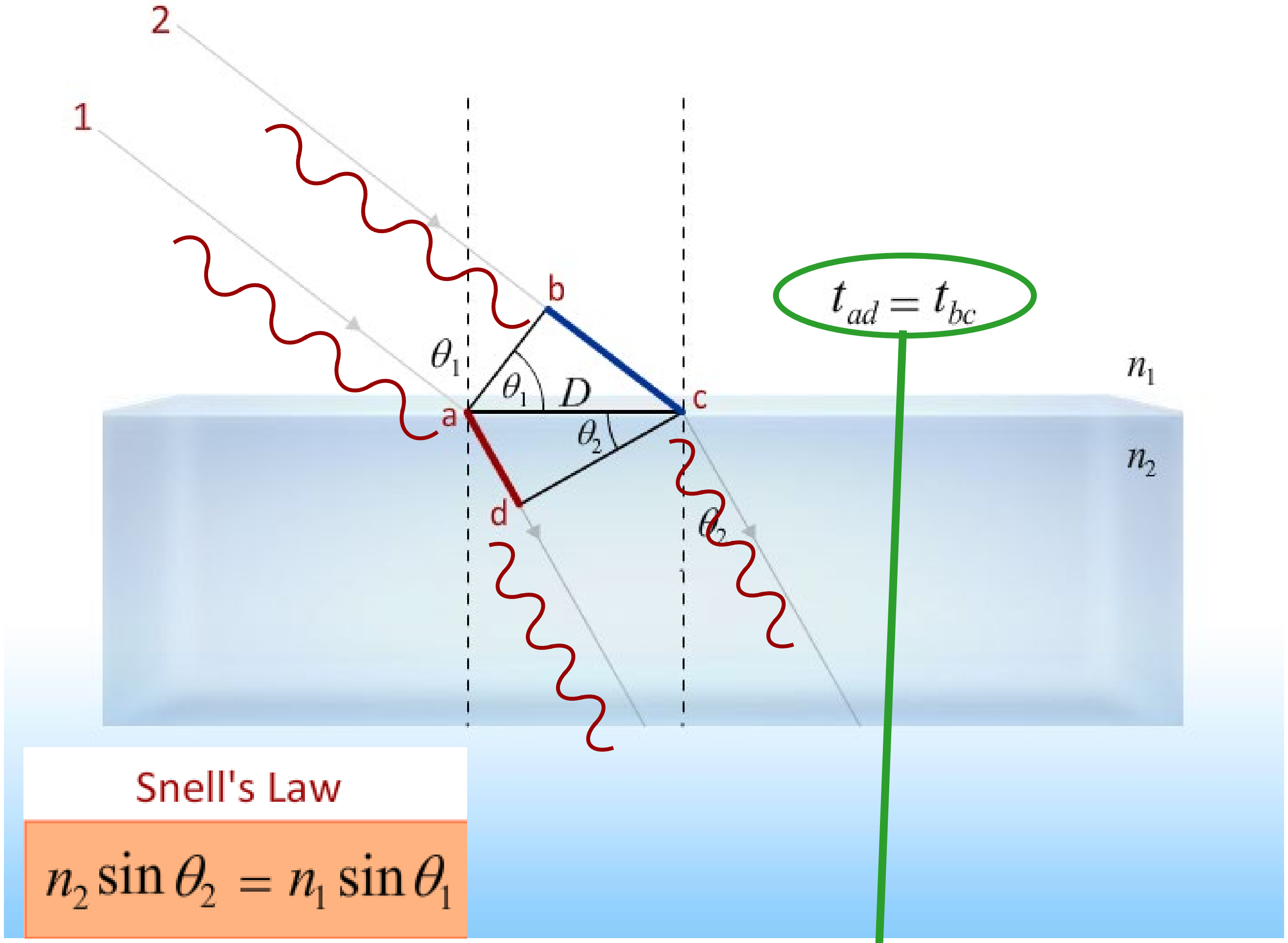
Refraction



(a) (b)

Figure 1.12 (a) Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, the light can reach the observer by two different paths, so the fish seems to be in two different places. This bending of light is called refraction and is responsible for many optical phenomena. (b) This image shows refraction of light from a fish near the top of a fish tank.

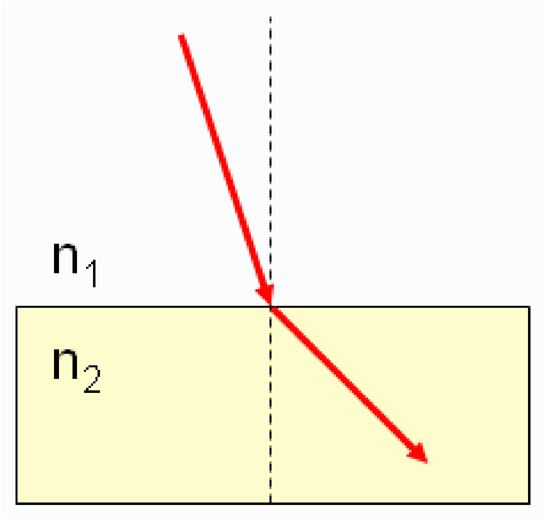
Refraction: Snell's Law



$$\frac{D\sin\theta_2}{c/n_2} = \frac{D\sin\theta_1}{c/n_1} \quad \Rightarrow \quad n_2 \sin\theta_2 = n_1 \sin\theta_1$$

Check Point 2a

The path of light is bent as passes from medium 1 to medium 2.



Compare the indexes of refraction in the two mediums.

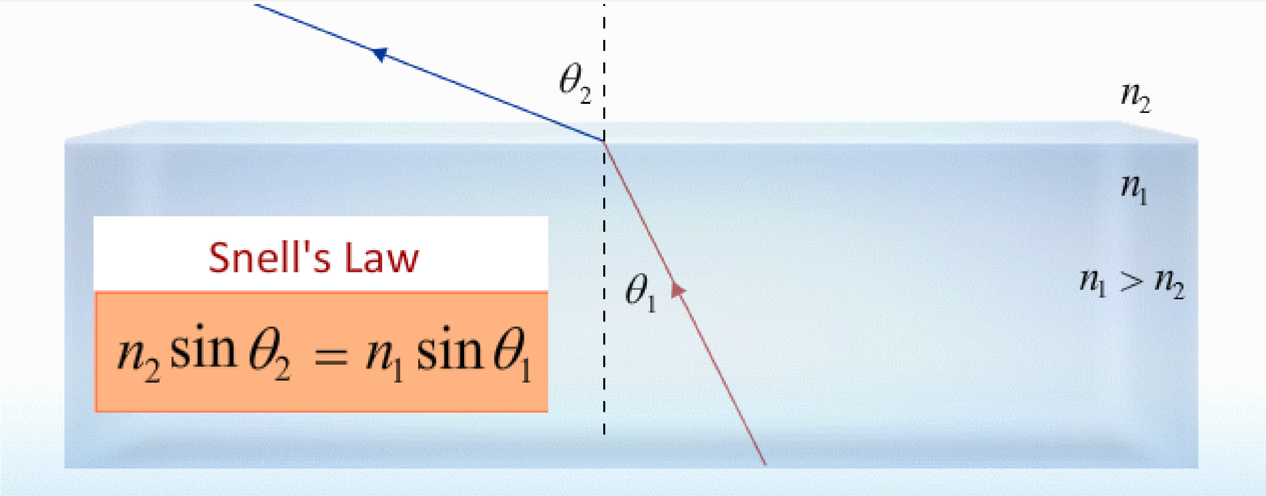
A $\boxed{n_1 > n_2}$
B $n_1 = n_2$
C $n_1 < n_2$

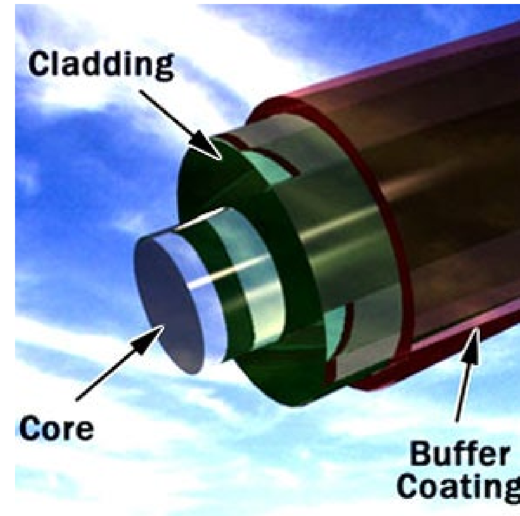
Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

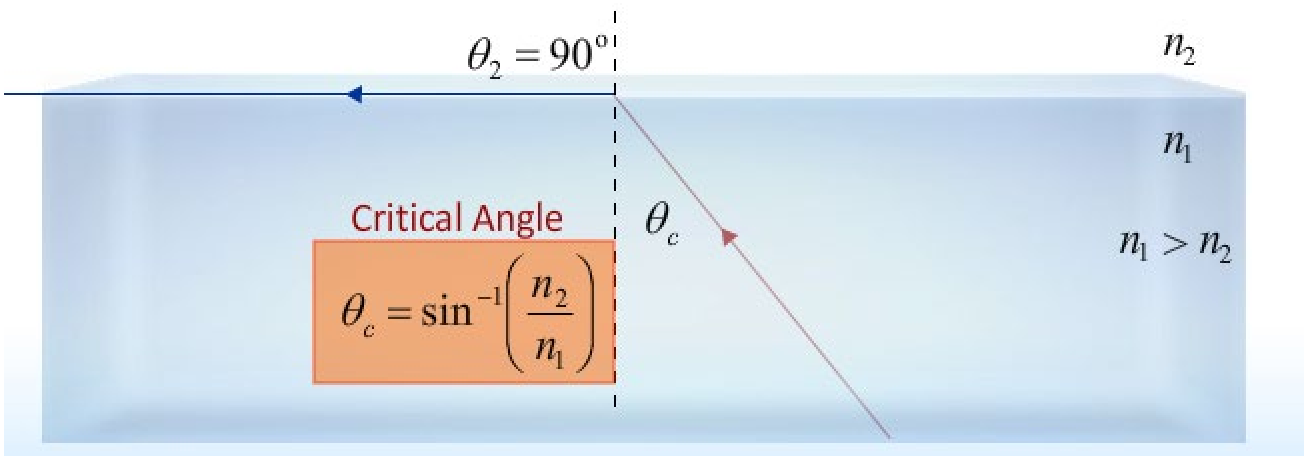
n decreases $\longrightarrow$ θ increases

Total Internal Reflection





NOTE: $n_1 > n_2$ implies $\theta_2 > \theta_1$ BUT: θ_2 has max value = 90^o!



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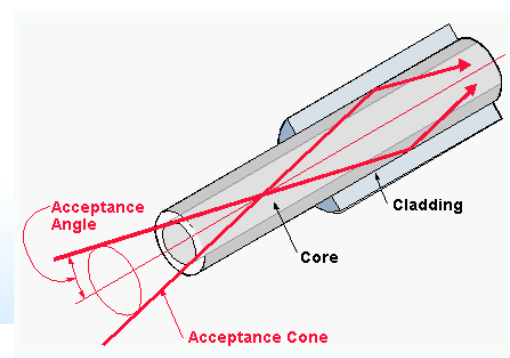


$\theta_1 > \theta_c$ → Total Internal Reflection

Total Internal Reflection

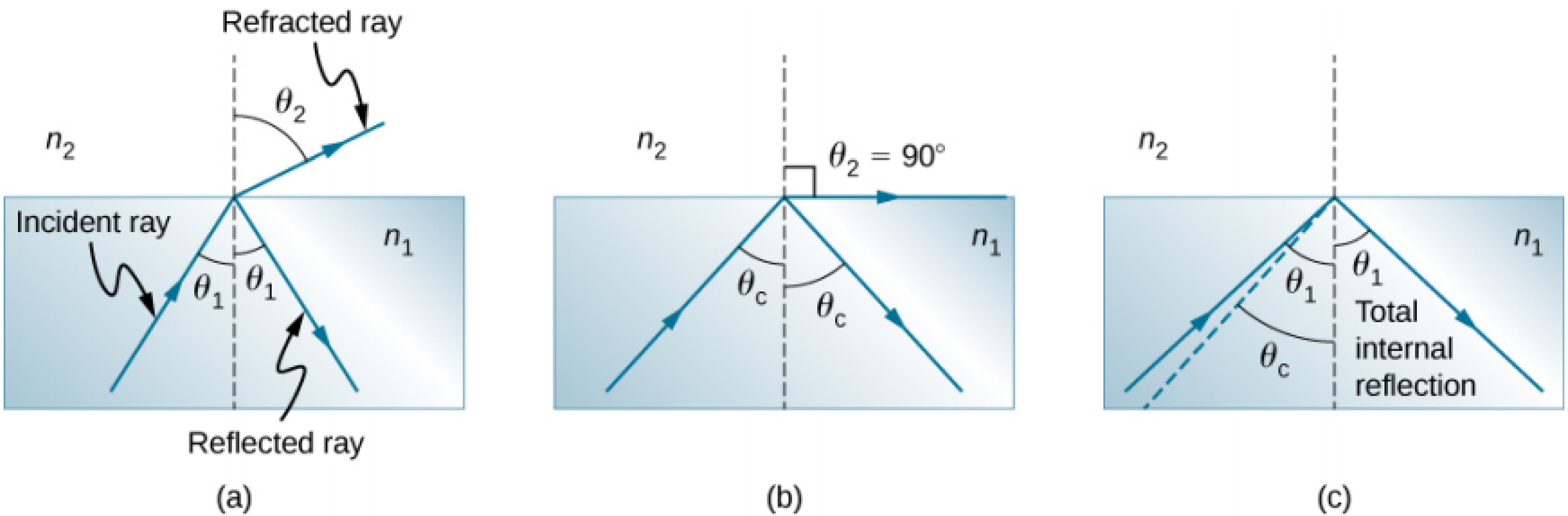
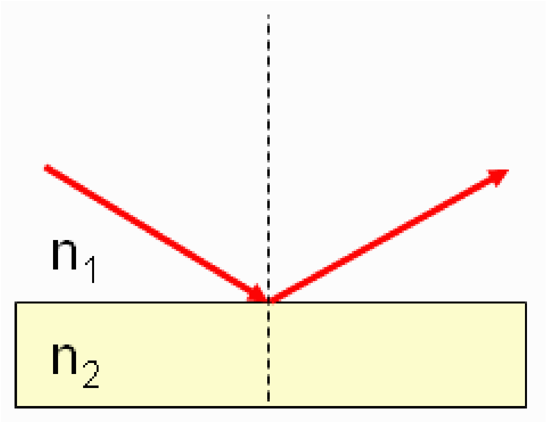


Figure 1.14 (a) A ray of light crosses a boundary where the index of refraction decreases. That is, $n_2 < n_1$. The ray bends away from the perpendicular. (b) The critical angle θ_c is the angle of incidence for which the angle of refraction is $90°$. (c) Total internal reflection occurs when the incident angle is greater than the critical angle.

Check Point 2b

A light ray travels in a medium with n_1 and completely reflects from the surface of a medium with n_2.



The critical angle depends on:

A ○ n_1 only

B ○ n_2 only

C ○ both n_1 and n_2

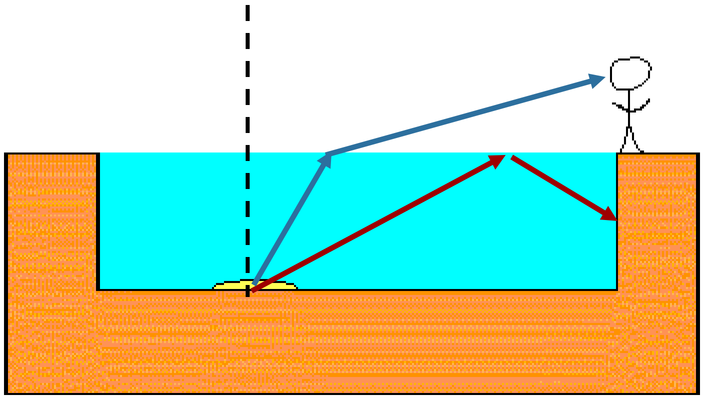
Critical Angle

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

θ_c depends on both n_2 and n_1

Check Point 3

A light is shining at the bottom of a swimming pool (shown in yellow in the figure). A person is standing at the edge of the pool.



Can the person standing on the edge of the pool be prevented from seeing the light by total internal reflection at the water-air surface?

A. Yes **B. No**

The light at the bottom of the pool will be shining in all directions. While there may be some light rays that are totally internally reflected, there will also be rays coming out of the pool.

Draw some rays

Polarization by Reflection

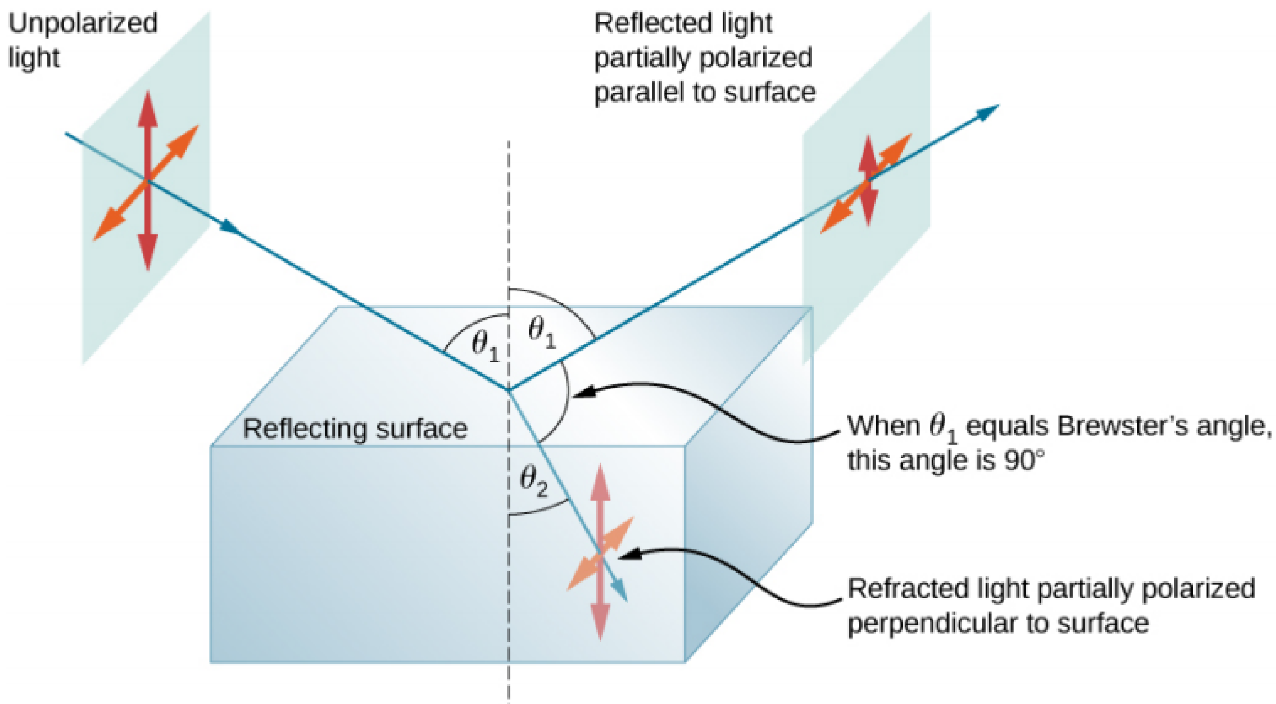
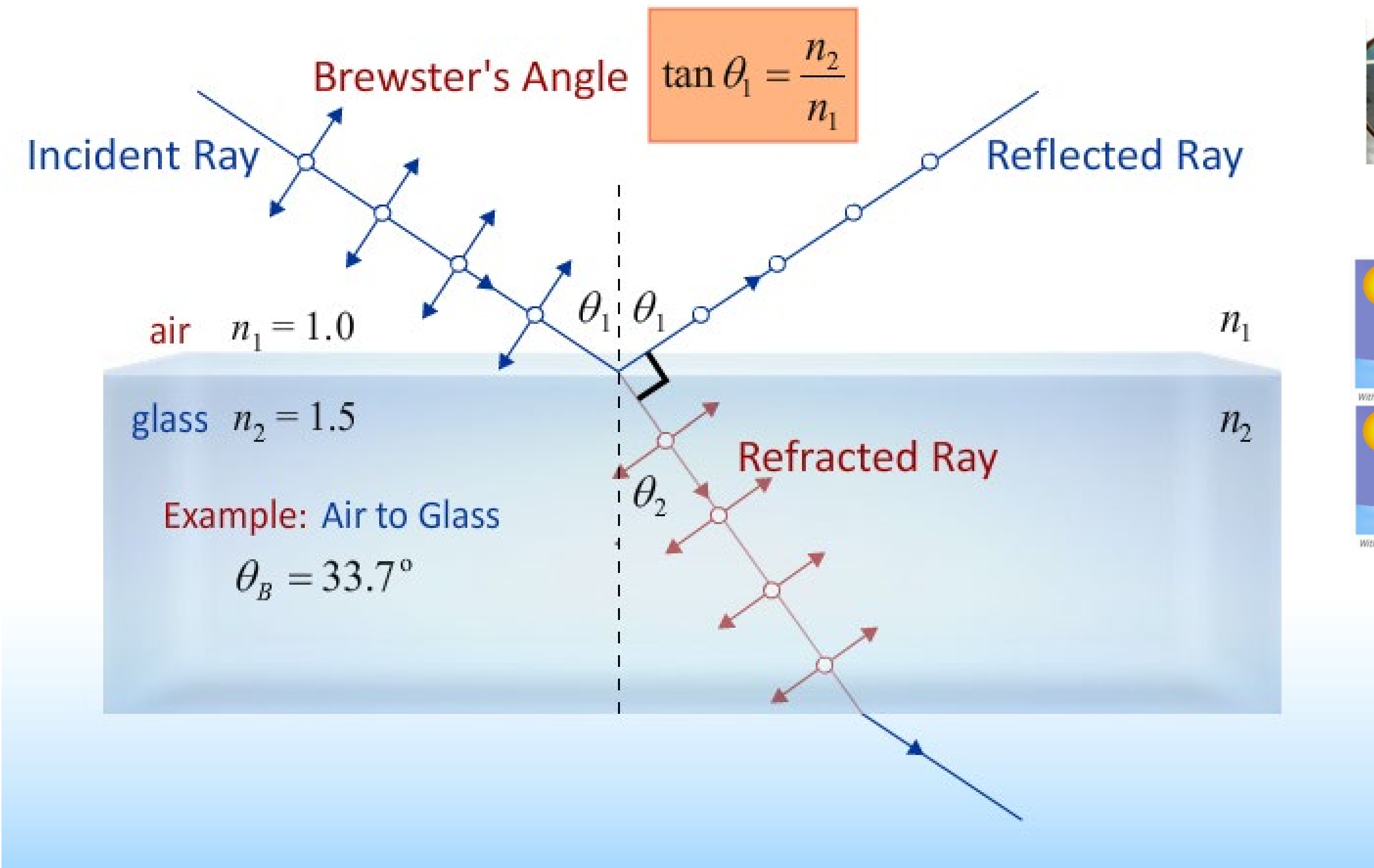
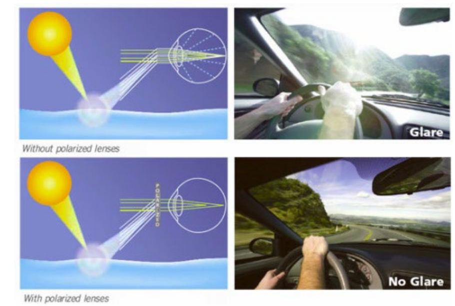


Figure 1.38 Polarization by reflection. Unpolarized light has equal amounts of vertical and horizontal polarization. After interaction with a surface, the vertical components are preferentially absorbed or refracted, leaving the reflected light more horizontally polarized. This is akin to arrows striking on their sides and bouncing off, whereas arrows striking on their tips go into the surface.

- Polarizing sunglasses cut the glare in reflected light, because that light is polarized.
- Vertically polarized light is preferentially refracted at the surface, so the reflected light is left more horizontally polarized.
- Vertical polarization is like an arrow perpendicular to the surface and is more likely to stick and not be reflected. Horizontal polarization is like an arrow bouncing on its side and is more likely to be reflected.

Polarization



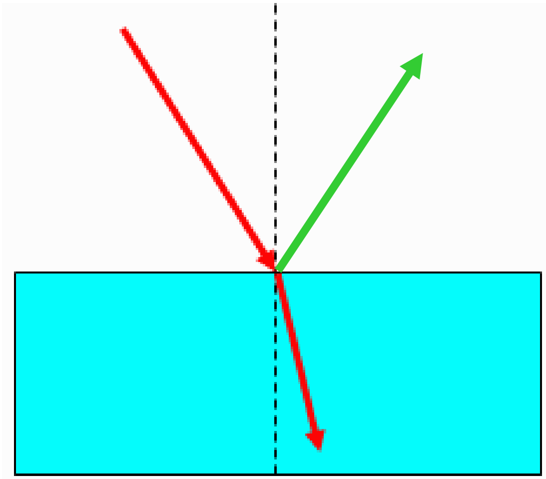


$$\theta_1 + \theta_2 = 90^\circ \quad \longrightarrow \quad \sin\theta_2 = \sin(90^\circ - \theta_1) = \cos\theta_1$$

Snell's Law: $n_2 \sin\theta_2 = n_2 \cos\theta_1 = n_1 \sin\theta_1 \quad \longrightarrow \quad \tan\theta_1 = \frac{n_2}{n_1}$

Check Point 1b

A ray of light passes from air into water with an angle of incidence of 30 degrees.

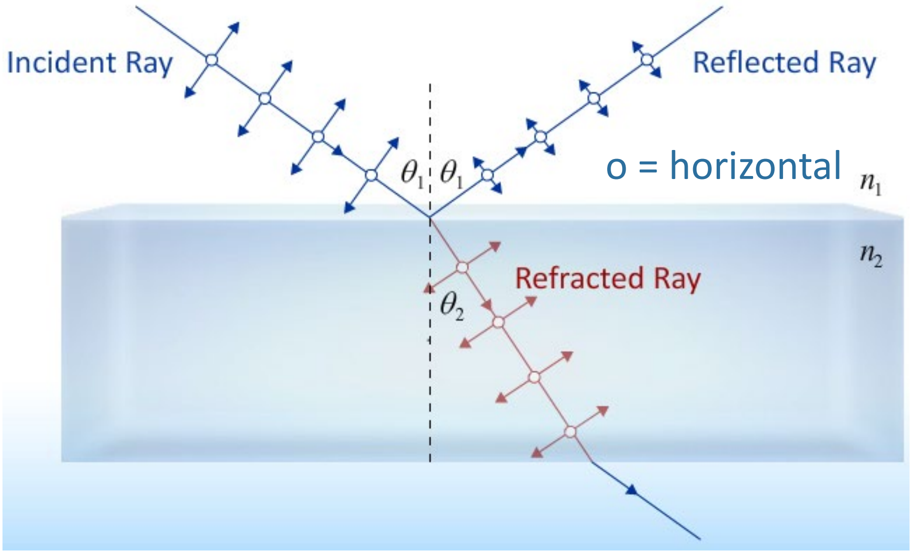


Some of the light also reflects off the surface of the water. If the incident light is initially unpolarized, the reflected light will be

A unpolarized

B $\boxed{\text{somewhat horizontally polarized}}$

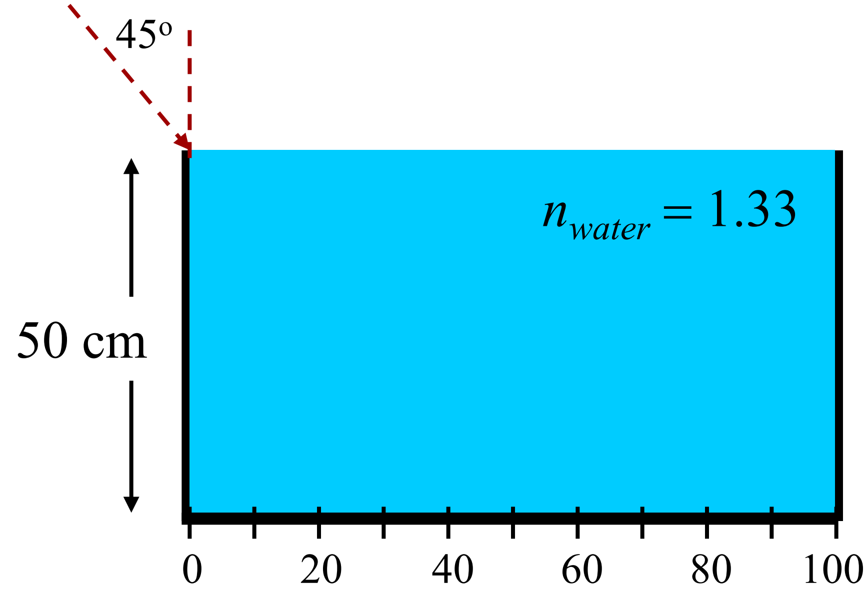
C somewhat vertically polarized



Exercise

A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of 45^{o} relative to vertical along a line that skims the top edge of the tank.

What is the smallest number on the ruler that you can see?



Conceptual Analysis:

- Light is refracted at the surface of the water

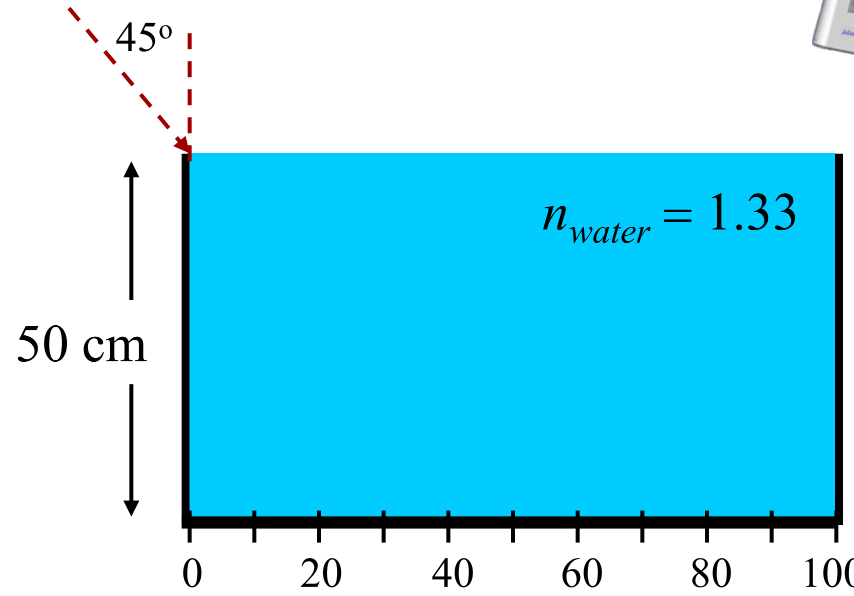
Strategy:

- Determine the angle of refraction in the water and extrapolate this to the bottom of the tank.

Exercise

A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of 45° relative to vertical along a line that skims the top edge of the tank.

What is the smallest number on the ruler that you can see?



If you shine a laser into the tank at an angle of 45°, what is the refracted angle θ_R in the water ?

A) $\theta_R = 28.3^o$ B) $\boxed{\theta_R = 32.1^o}$ C) $\theta_R = 38.7^o$

Snell's Law: $n_{air}\sin(45) = n_{water}\sin(\theta_R)$

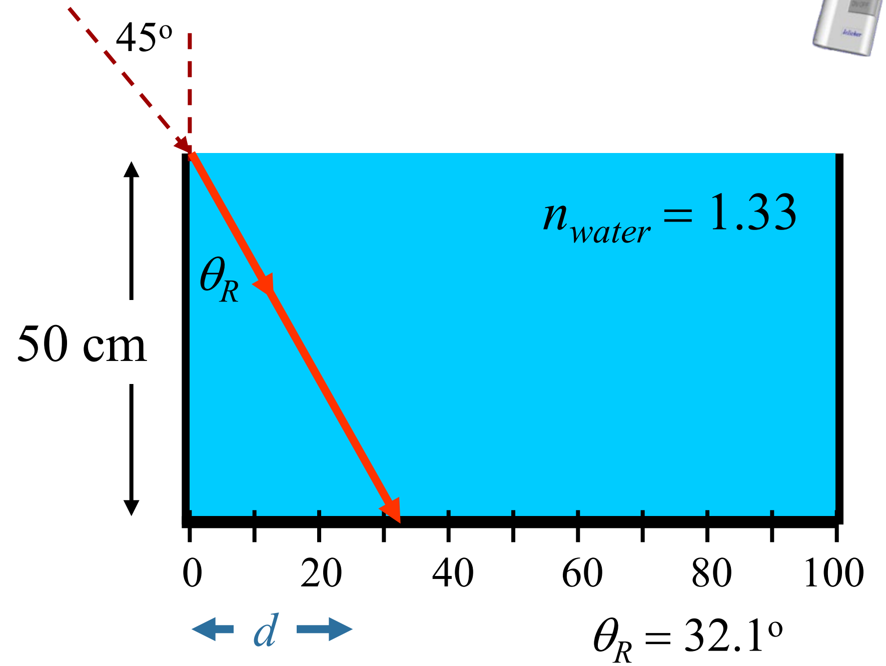
$\sin(\theta_R) = n_{air}\sin(45)/n_{water} = 0.532$

$\theta_R = \sin^{-1}(0.532) = 32.1^o$

Exercise

A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of 45° relative to vertical along a line that skims the top edge of the tank.

What is the smallest number on the ruler that you can see?



What number on the ruler does the laser beam hit?

A) 31.4 cm B) 37.6 cm C) 44.1 cm

$$\tan(\theta_R) = d/50$$

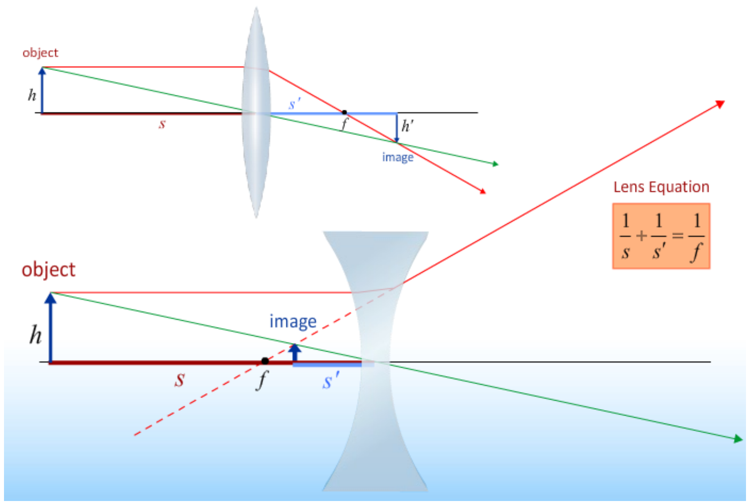
$$d = \tan(32.1) \times 50\text{cm} = 31.4\text{cm}$$

Bonus Point1

Physics 212
Lecture 26

Today's Concept:

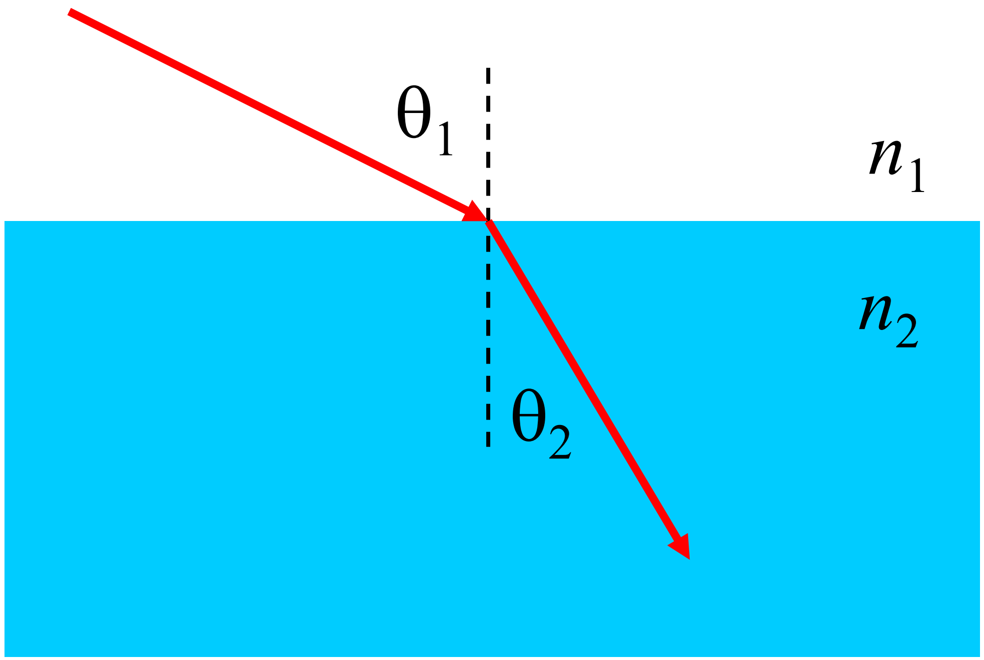
Lenses



Refraction

Snell's Law

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$



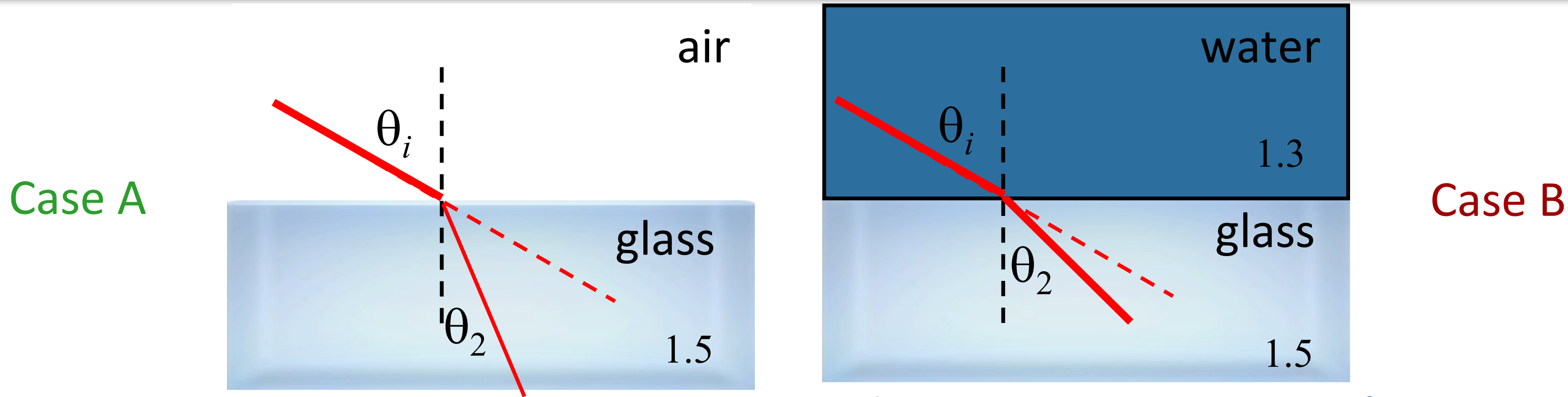
That's all of the physics –

everything else is just geometry!

For spherical lens

$$\frac{1}{f} = (n_1 - n_2)\frac{1}{R}$$

Check Point Warmup



In Case A light in air heads toward a piece of glass with incident angle θ_i
In Case B, light in water heads toward a piece of glass at the same angle.

In which case is the light bent ($\theta_1 - \theta_2$) most as it enters the glass?

A) $\boxed{\text{Case A}}$
B) Case B
C) Same

The angle of refraction is bigger for the water – glass interface:

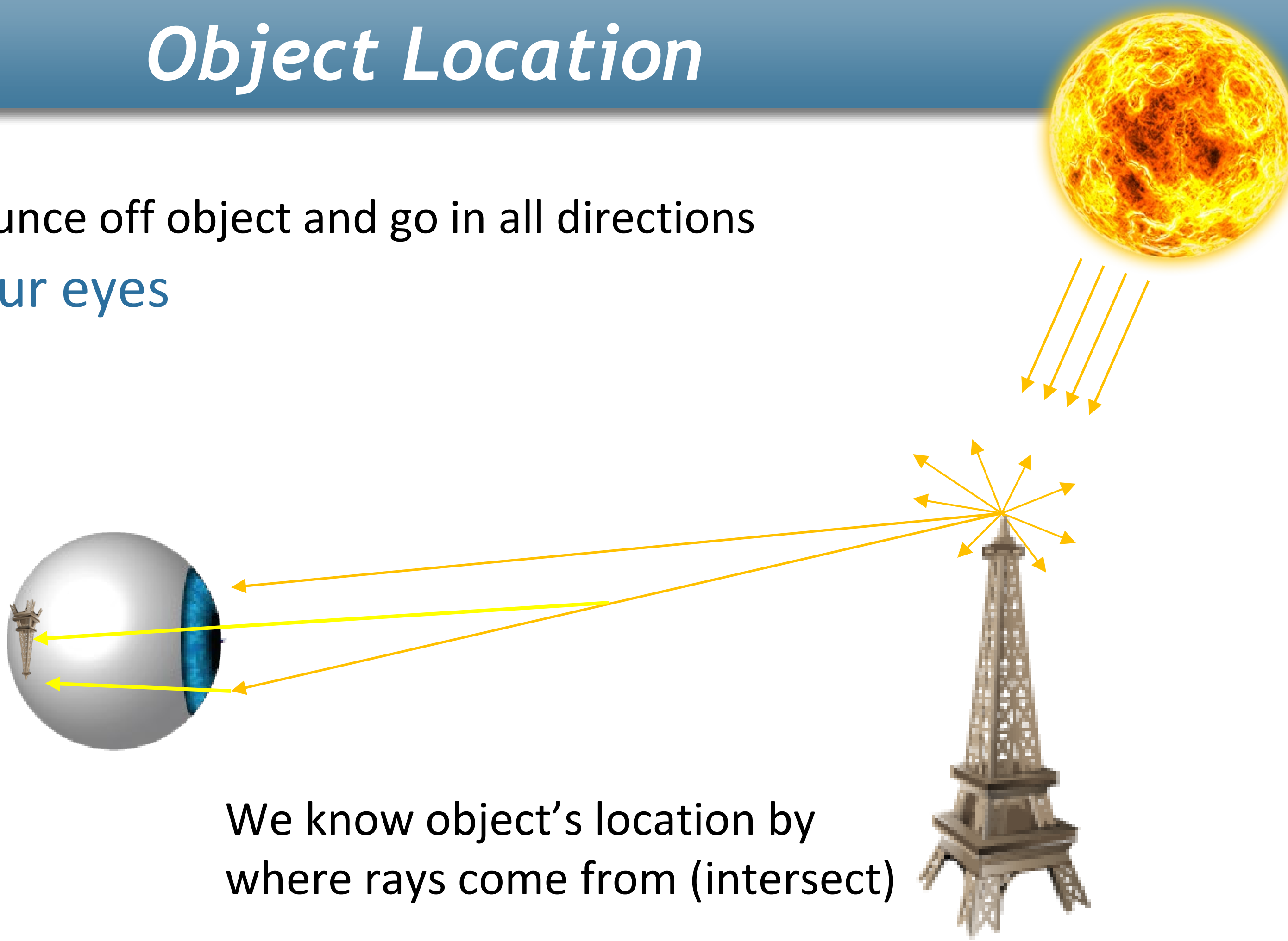
$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \quad \longrightarrow \quad \sin(\theta_2)/\sin(\theta_1) = n_1/n_2$$

Therefore the BEND ANGLE ($\theta_1 - \theta_2$) is BIGGER for air – glass interface

Object Location

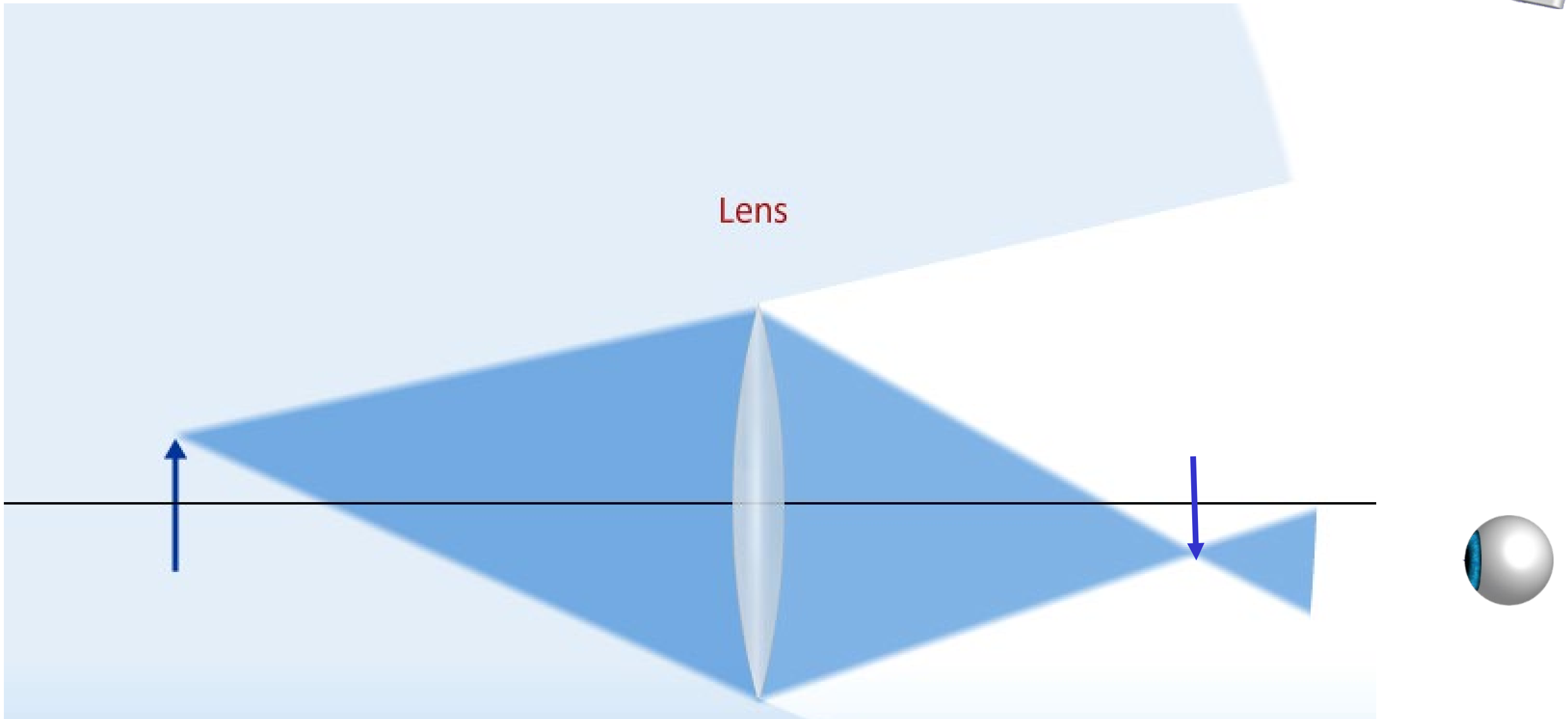
Light rays from sun bounce off object and go in all directions

- Some hits your eyes



We know object's location by where rays come from (intersect)

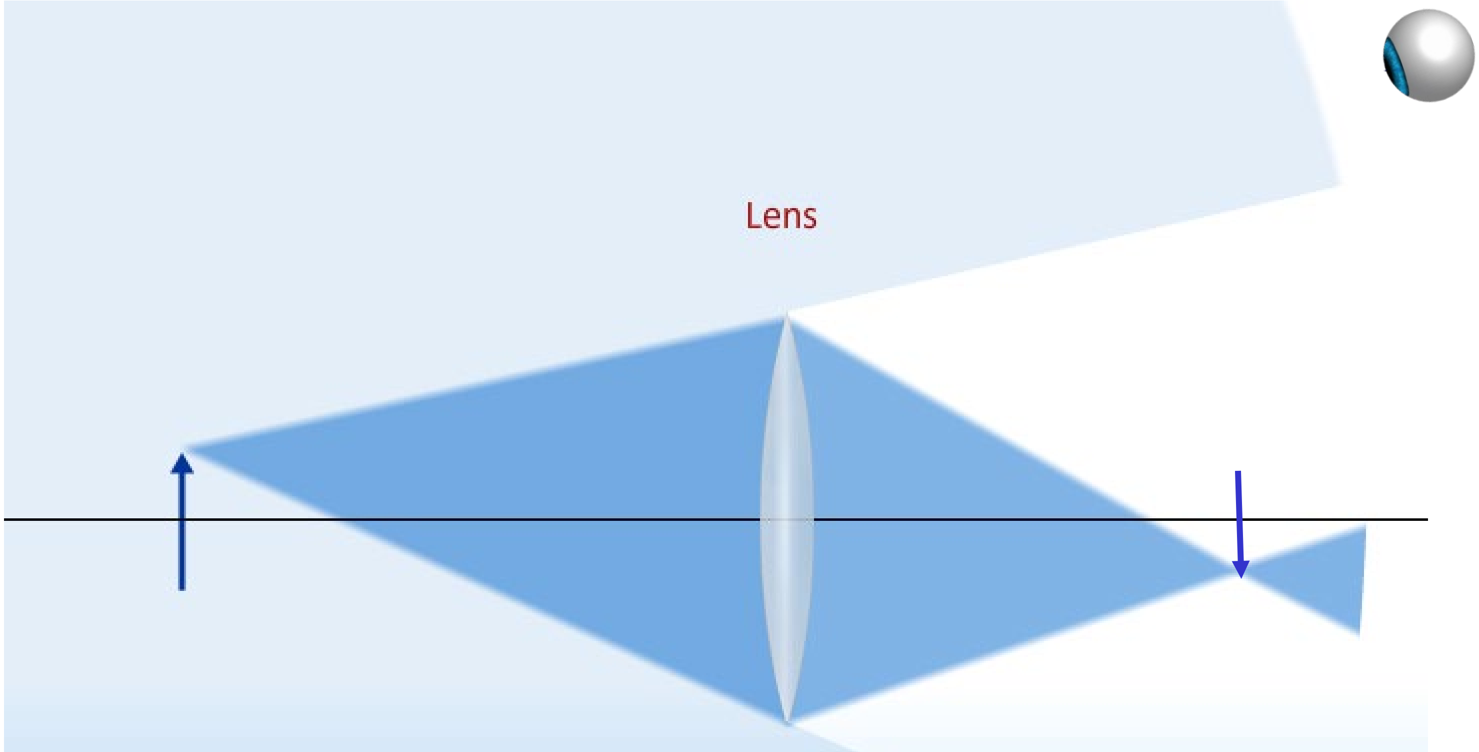
Waves from Objects are Focused by Lens



What would a person looking from location of eye see?

A) Actual object B) Image of object C) Both D) Neither

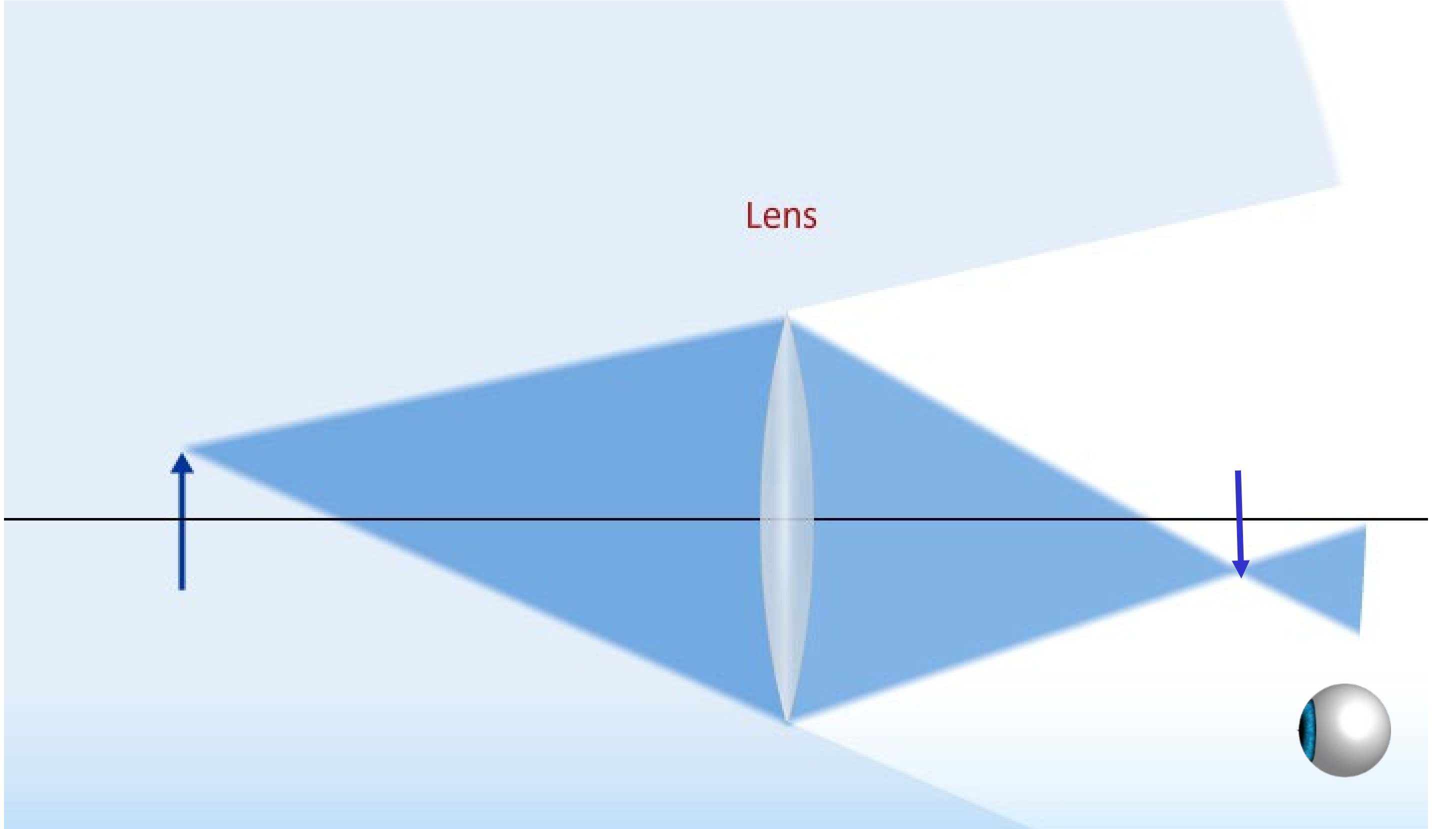
Waves from Objects are Focused by Lens



What would a person looking from location of eye see?

A) Actual object B) Image of object C) Both D) Neither

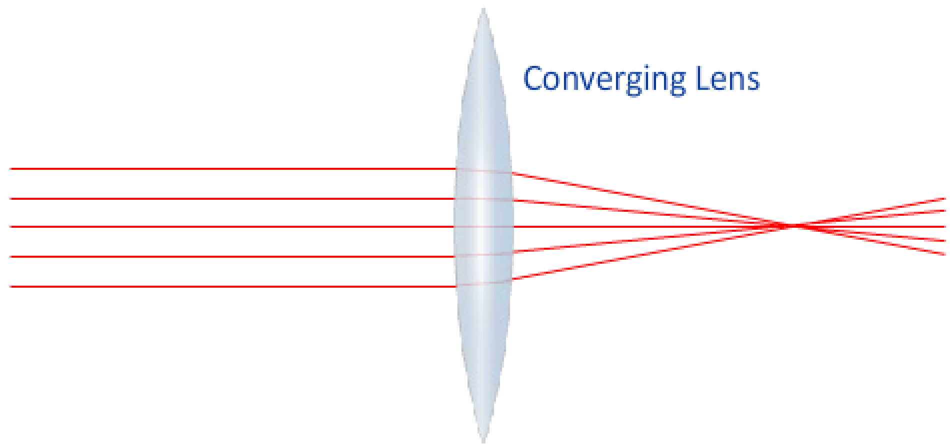
Waves from Objects are Focused by Lens

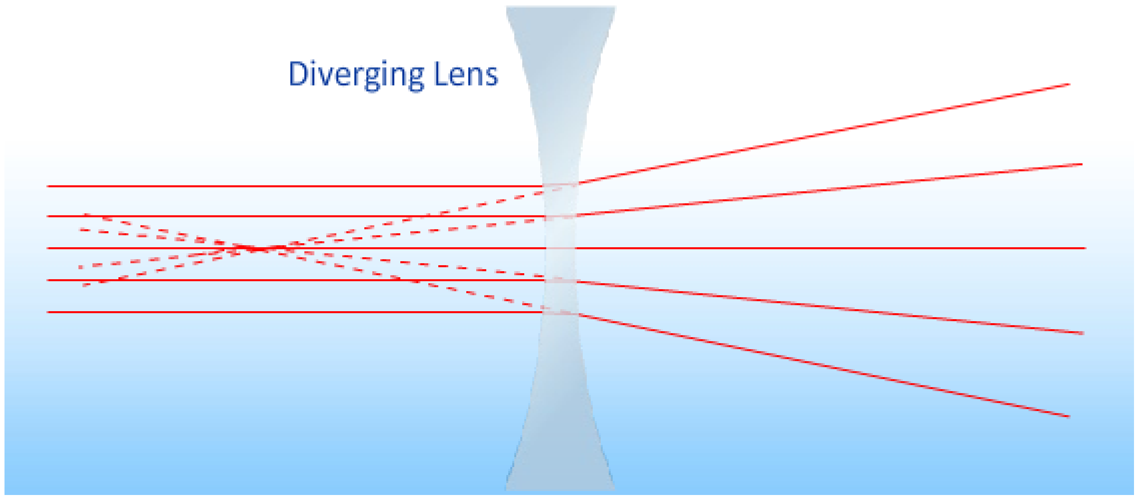


What would a person looking from location of eye see?

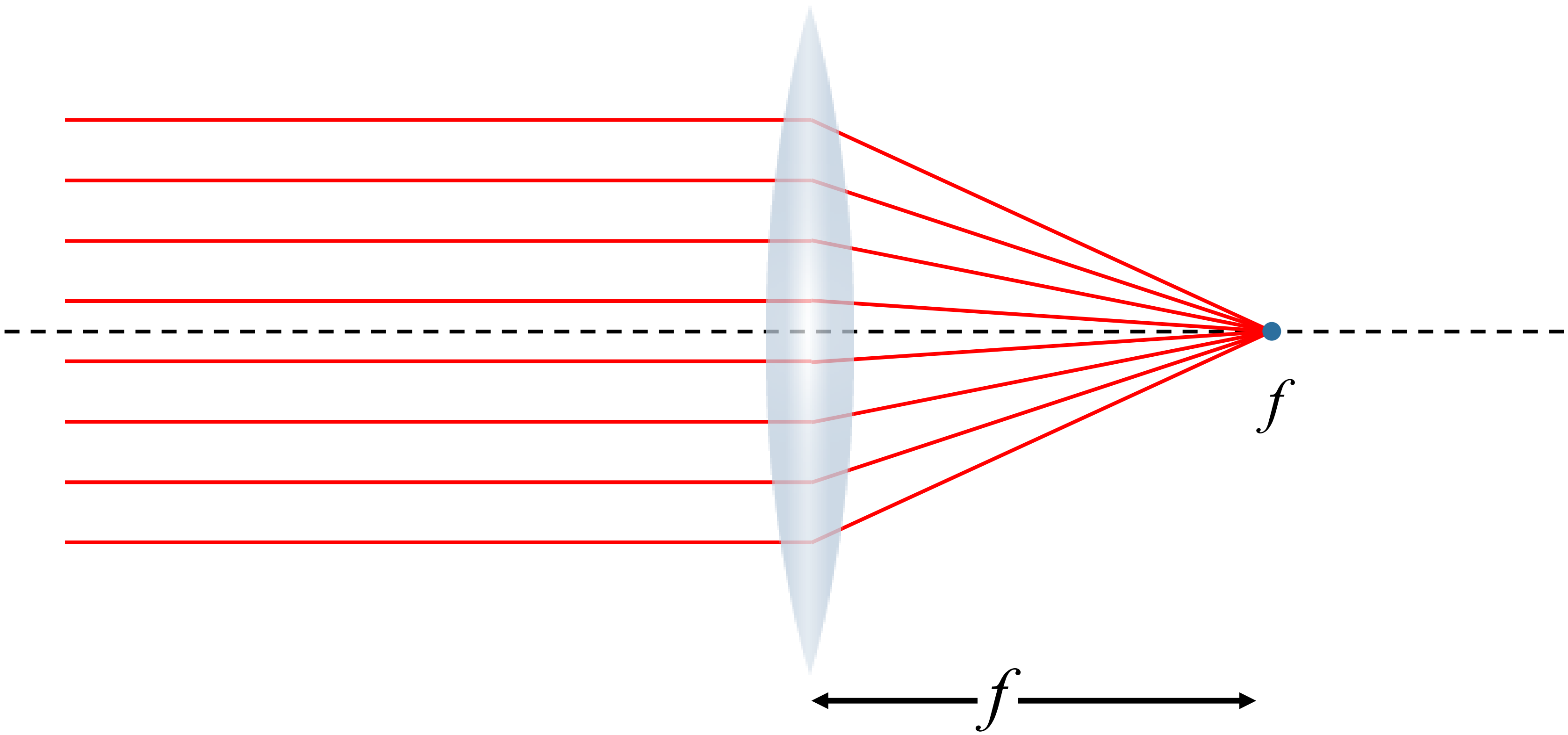
A) Actual object B) Image of object C) Both D) Neither

Two Different Types of Lenses





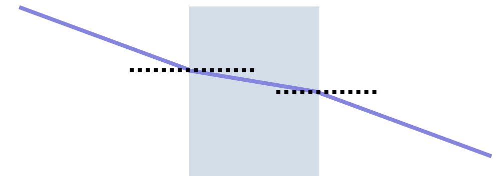
Converging Lens: Consider the case where the shape of the lens is such that light rays parallel to the axis of the mirror are all "focused" to a common spot a distance f behind the lens:



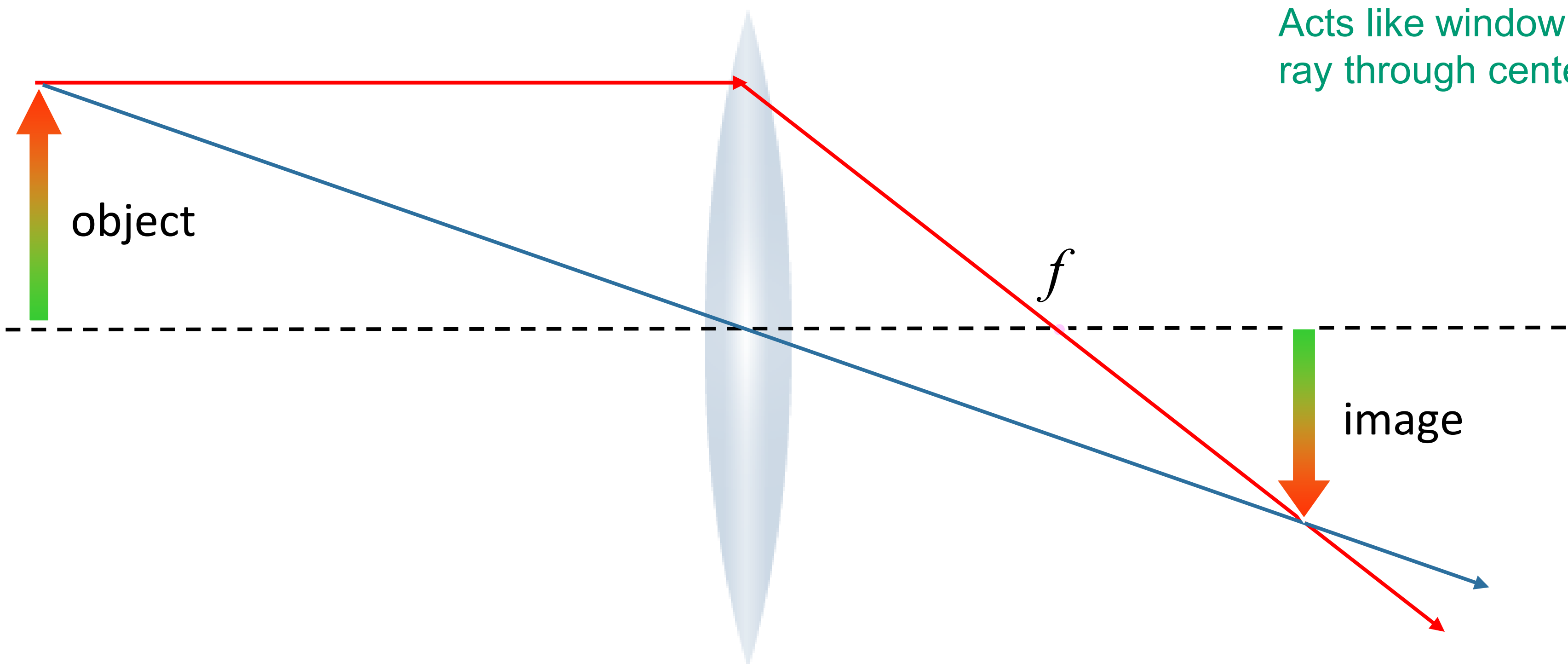
Recipe for Finding Image:

1) Draw ray parallel to axis — refracted ray goes through focus

2) Draw ray through center — refracted ray is symmetric

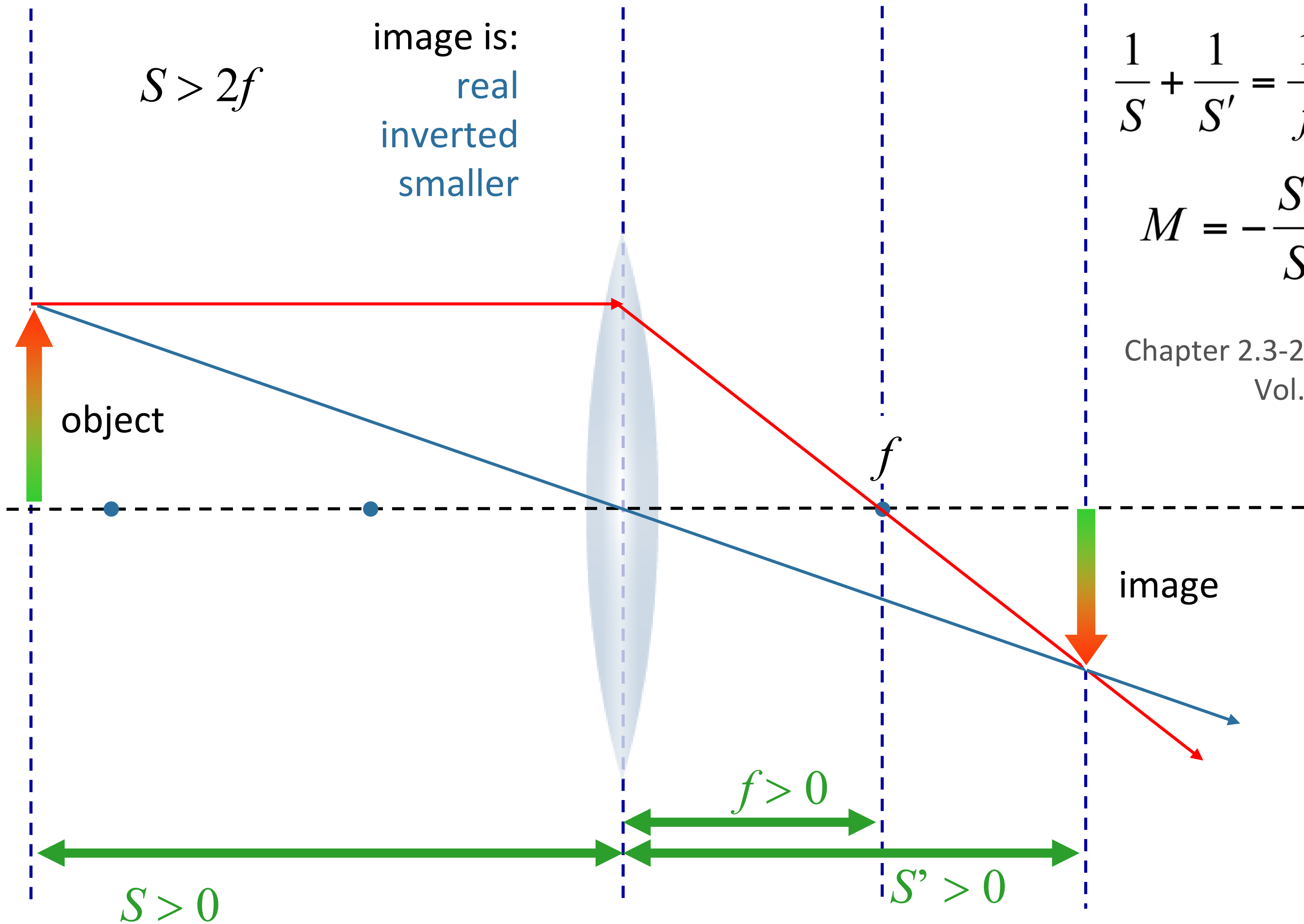


Acts like window for ray through center



You now know the position of the same point on the image

Example



$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$$

$$M = -\frac{S'}{S}$$

Chapter 2.3-2.4
Vol. 3

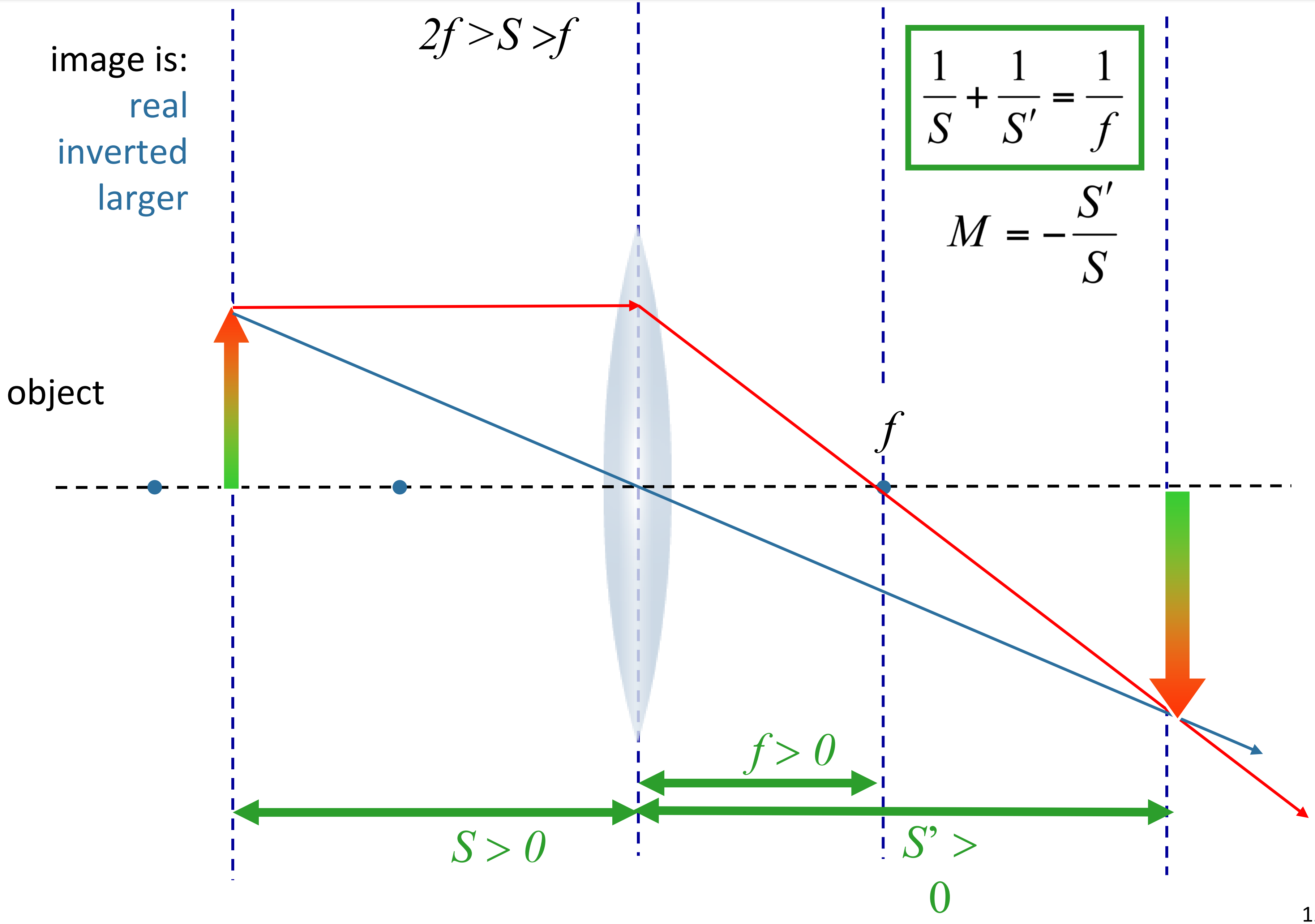
Example

image is:
real
inverted
larger

$2f > S > f$

$$\boxed{\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}}$$

$$M = -\frac{S'}{S}$$



Example

$0 < S < f$

image is:
virtual
upright
bigger

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$$

$$M = -\frac{S'}{S}$$

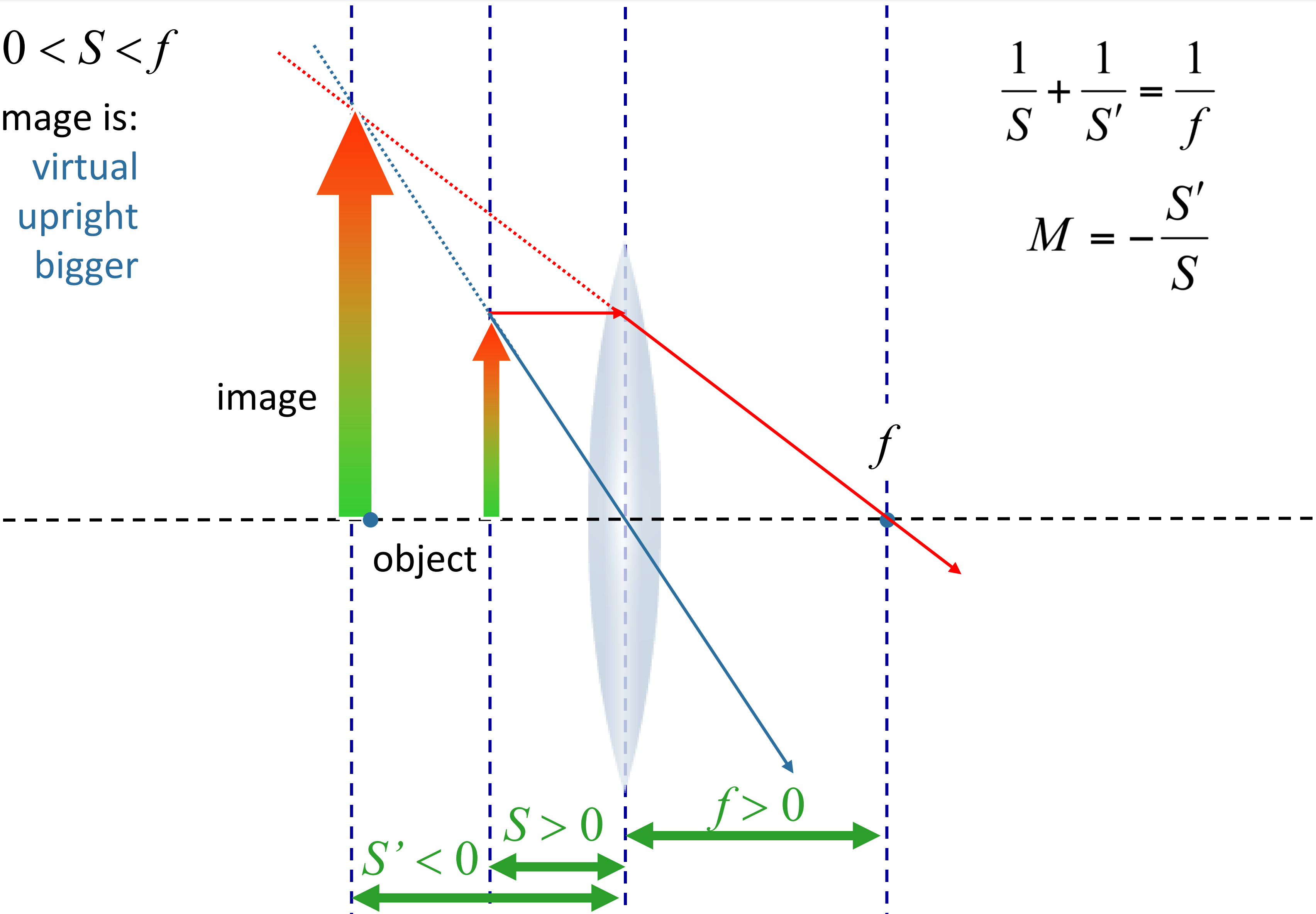


Image distance vs. Object distance

$$\frac{1}{S}+\frac{1}{S'}=\frac{1}{f} \Longrightarrow S'=\left(\frac{1}{f}-\frac{1}{S}\right)^{-1}$$

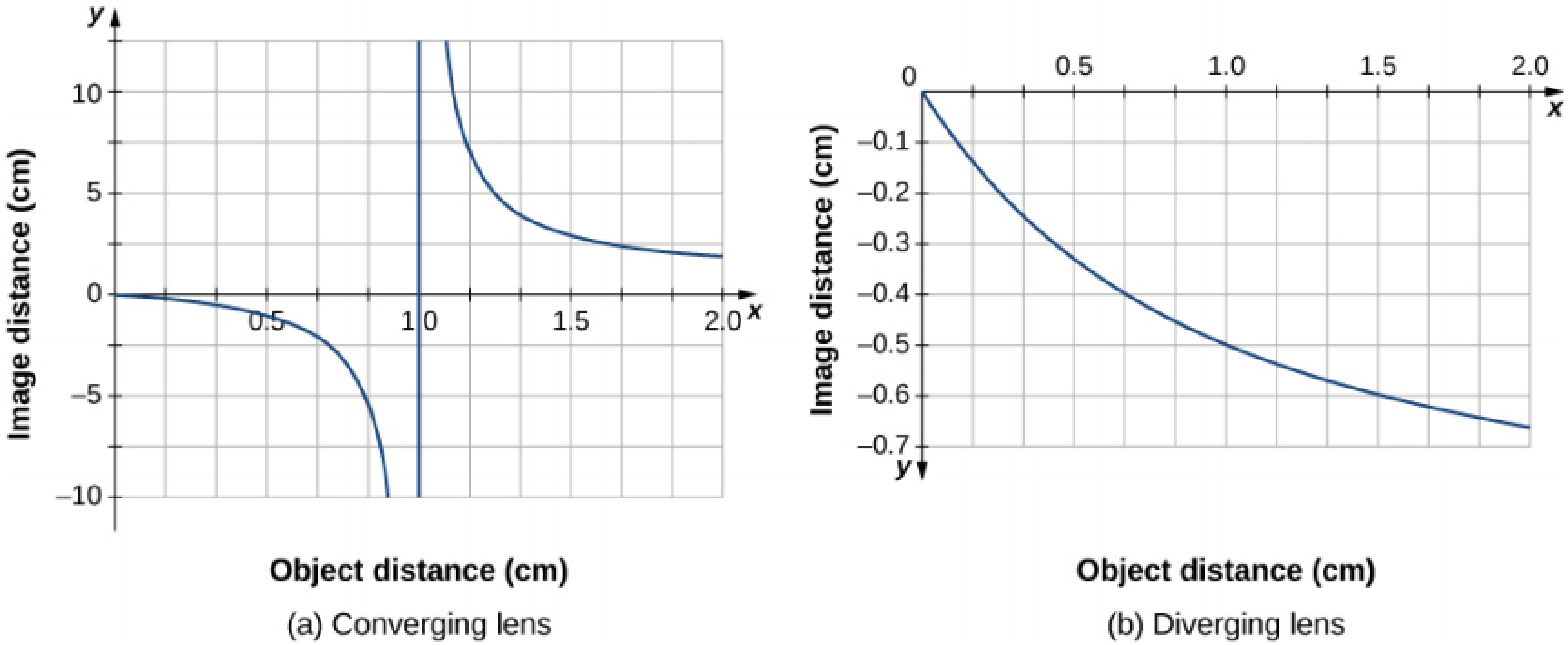


Figure 2.25 (a) Image distance for a thin converging lens with $f = 1.0$ cm as a function of object distance. (b) Same thing but for a diverging lens with $f = -1.0$ cm.

Image distance vs. Object distance

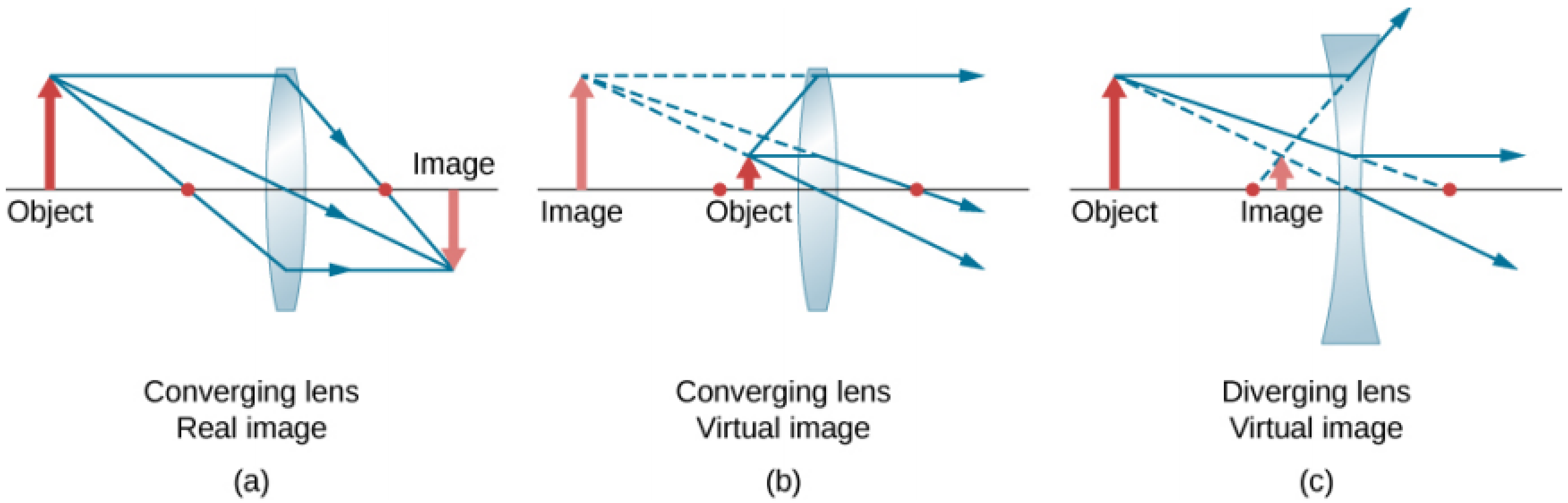
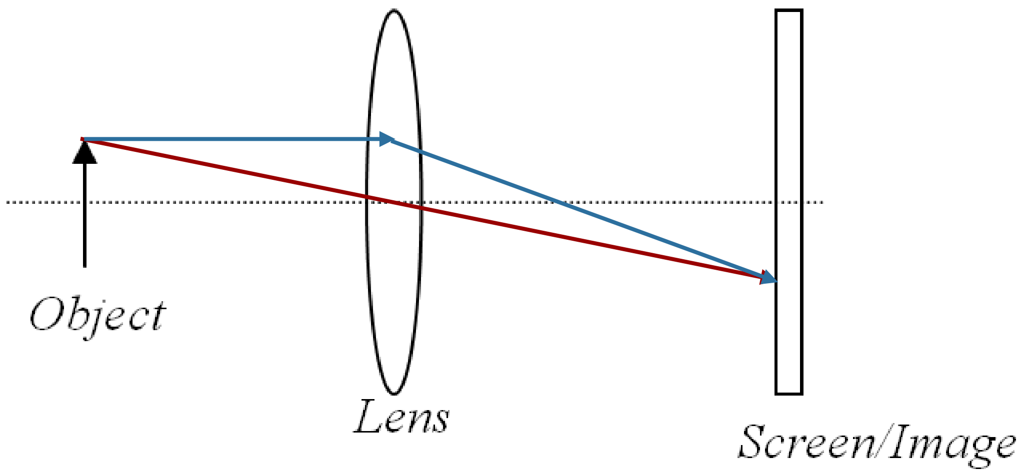


Figure 2.26 The red dots show the focal points of the lenses. (a) A real, inverted image formed from an object that is farther than the focal length from a converging lens. (b) A virtual, upright image formed from an object that is closer than a focal length from the lens. (c) A virtual, upright image formed from an object that is farther than a focal length from a diverging lens.

Check Point 1a



A converging lens is used to project the image of an arrow onto a screen as shown above

The image is:

A. Real
B. Virtual

The image is:

A. Inverted
B. Upright

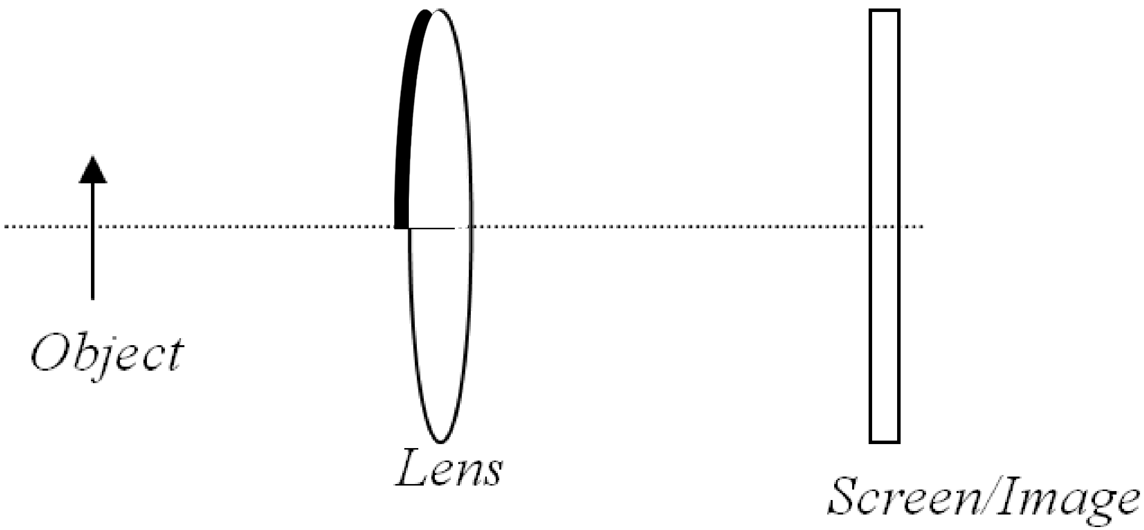
Image on screen

MUST BE REAL

$\rightarrow s' > 0$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad M = -\frac{s'}{s}$$

Check Point 1b



A converging lens is used to project the image of an arrow onto a screen as shown above

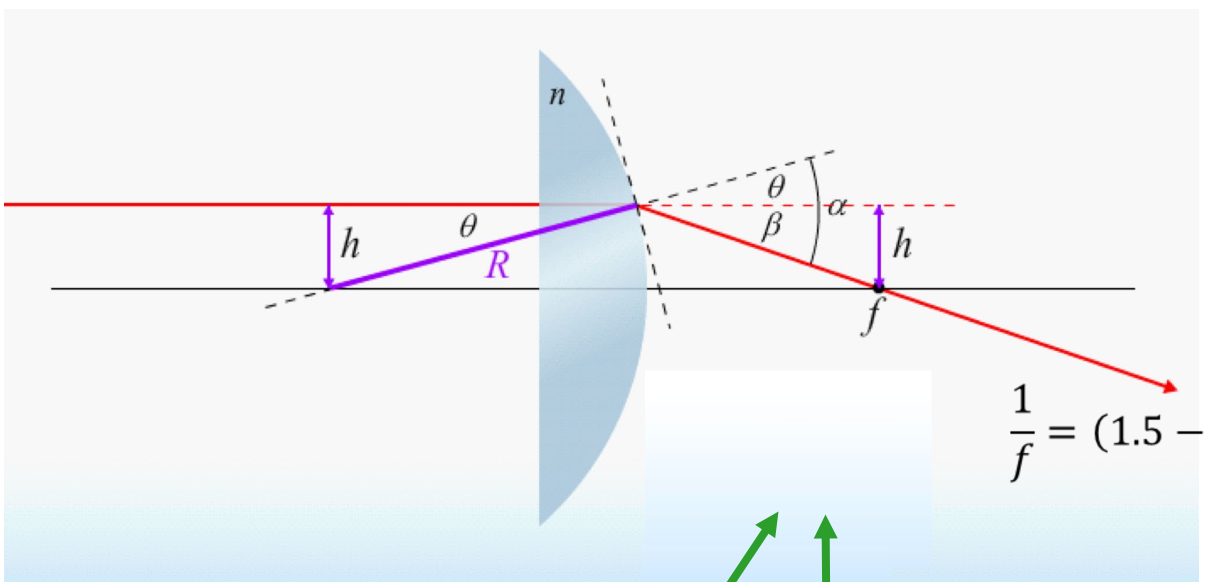
A piece of black tape is now placed over the upper half of the lens. Which of the following is true?

A. Only the lower half of the object will show on the screen
B. Only the upper half of the object will show on the screen
C. The whole object will show on the screen

Check Point 1c

What happens to the focal length of a converging lens when it is placed under water?

A. Increases
B. Decreases
C. Stays the same

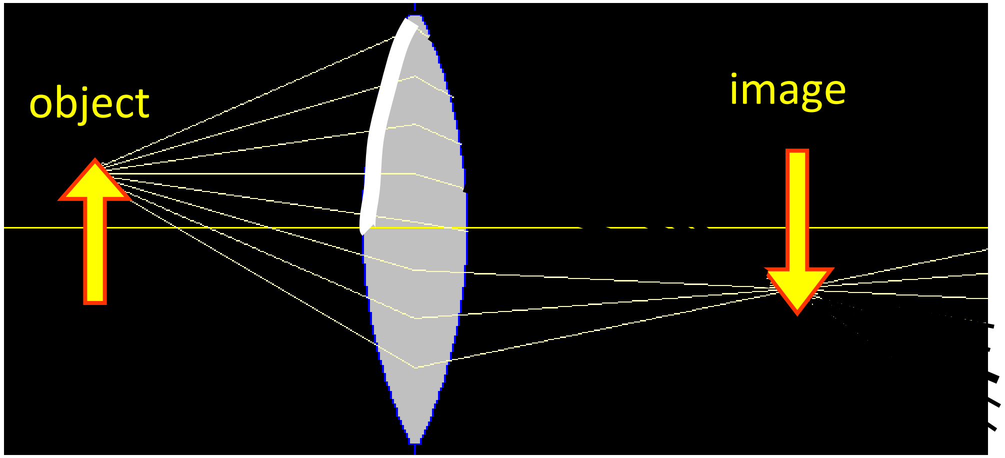


$$\frac{1}{f} = (1.5 - 1.1)\frac{1}{R} \Rightarrow \frac{1}{f} = (1.5 - 1.3)\frac{1}{R}$$

n_{lens} n_{air}

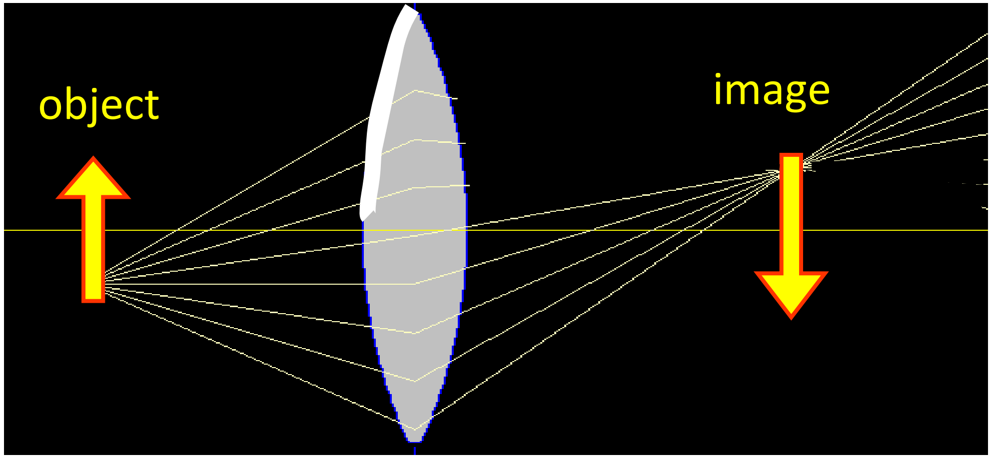
Cover top half of lens

Light from top of object



Cover top half of lens

Light from bottom of object

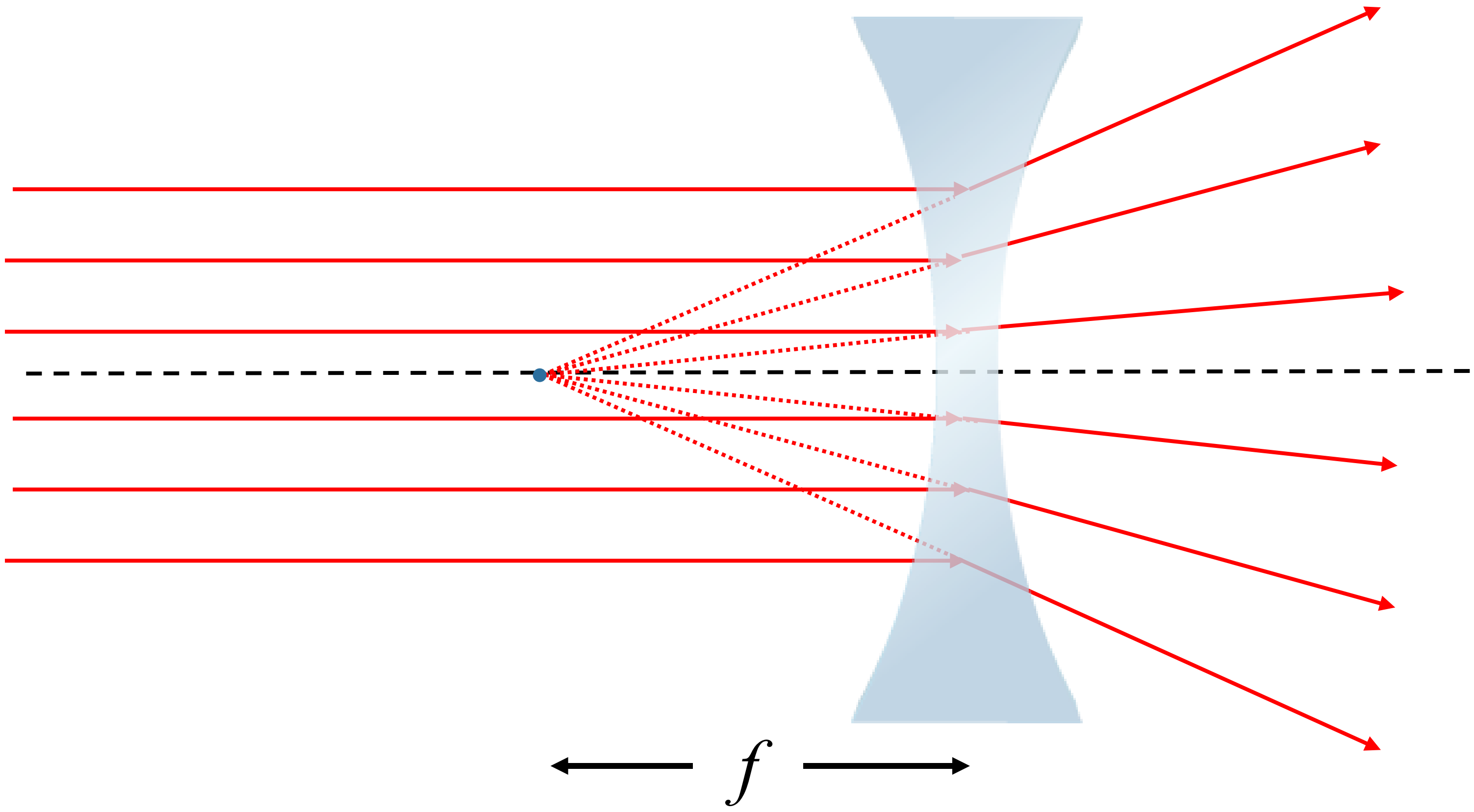


What's the Point?

The rays from the bottom half still focus
The image is there, but it will be dimmer!

A. Only the lower half of the object will show on the screen
B. Only the upper half of the object will show on the screen
C. $\boxed{\text{The whole object will show on the screen}}$

Diverging Lens: Consider the case where the shape of the lens is such that light rays parallel to the axis of the lens all diverge but appear to come from a common spot a distance f in front of the lens:



Example

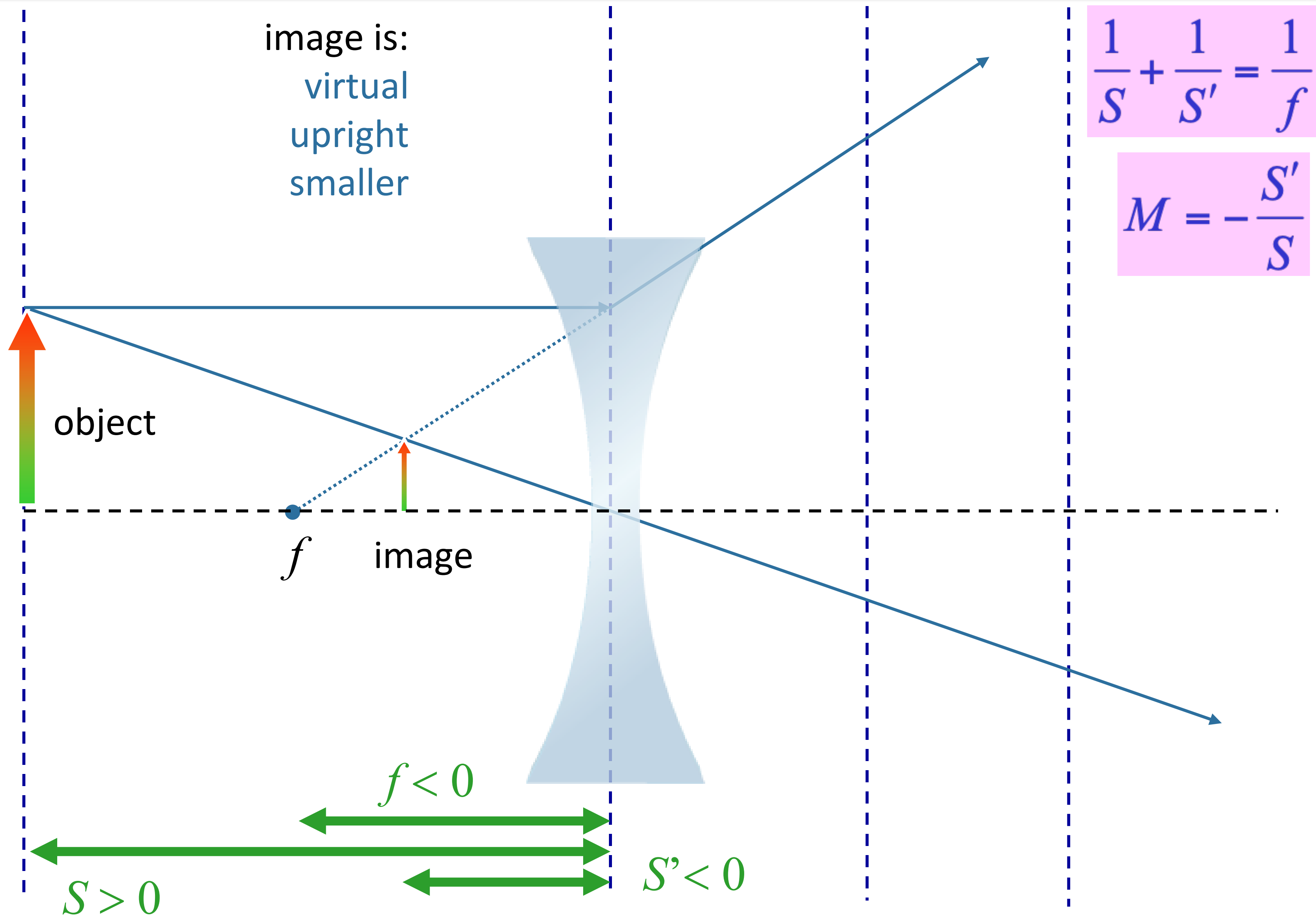
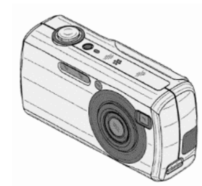


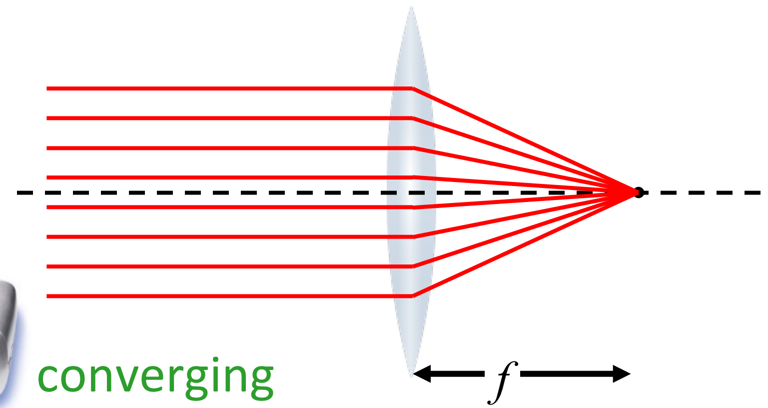
image is:
virtual
upright
smaller
$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$
$M = -\frac{S'}{S}$
object
f
image
$f < 0$
$S > 0$
$S' < 0$

Summary - Lenses

$S > 2f$

real
inverted
smaller





$2f > S > f$

real
inverted
bigger

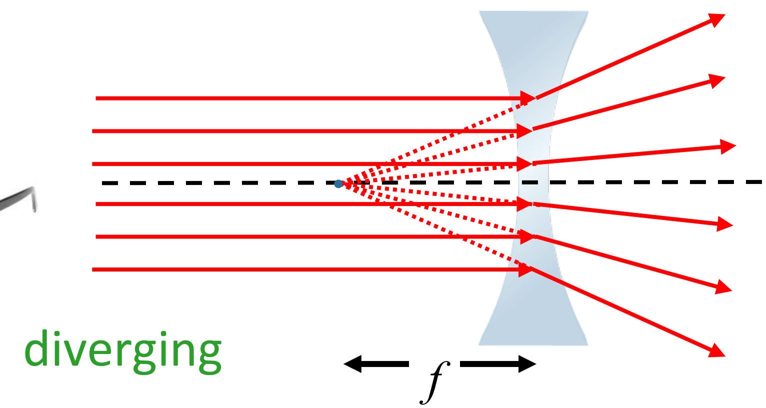
$f > S > 0$

virtual
upright
bigger

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad M = -\frac{s'}{s}$$

$S > 0$

virtual
upright
smaller



Summary - Lenses

(a) (b)

Figure 2.27 (a) When a converging lens is held farther than one focal length from the man's face, an inverted image is formed. Note that the image is in focus but the face is not, because the image is much closer to the camera taking this photograph than the face. (b) An upright image of the man's face is produced when a converging lens is held at less than one focal length from his face. (credit a: modification of work by "DaMongMan"/Flickr; credit b: modification of work by Casey Fleser)

It's Always the Same:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad M = -\frac{s'}{s}$$

You just have to keep the signs straight:

The sign conventions

S: positive if object is "upstream" of lens

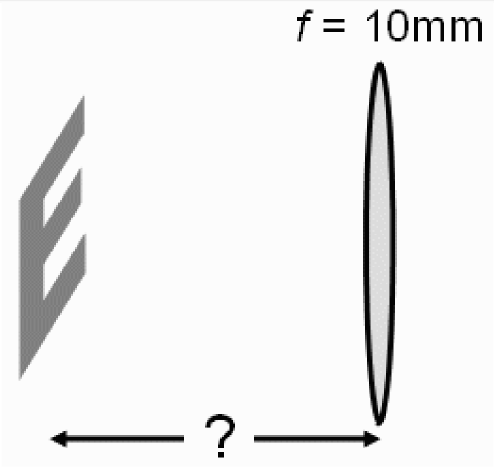
S': positive if image is "downstream" of lens

f: positive if converging lens

Calculation

A magnifying glass is used to read the fine print on a document. The focal length of the lens is 10mm.

At what distance from the lens must the document be placed in order to obtain an image magnified by a factor of 5 that is not inverted?



Conceptual Analysis

Lens Equation: $1/s + 1/s' = 1/f$

Magnification: $M = -s'/s$

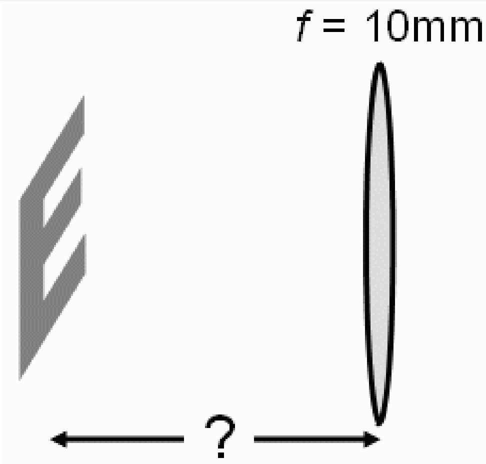
Strategic Analysis

Consider nature of image (real or virtual?) to determine relation between object position and focal point

Use magnification to determine object position

A magnifying glass is used to read the fine print on a document. The focal length of the lens is 10mm.

At what distance from the lens must the document be placed in order to obtain an image magnified by a factor of 5 that is not inverted?

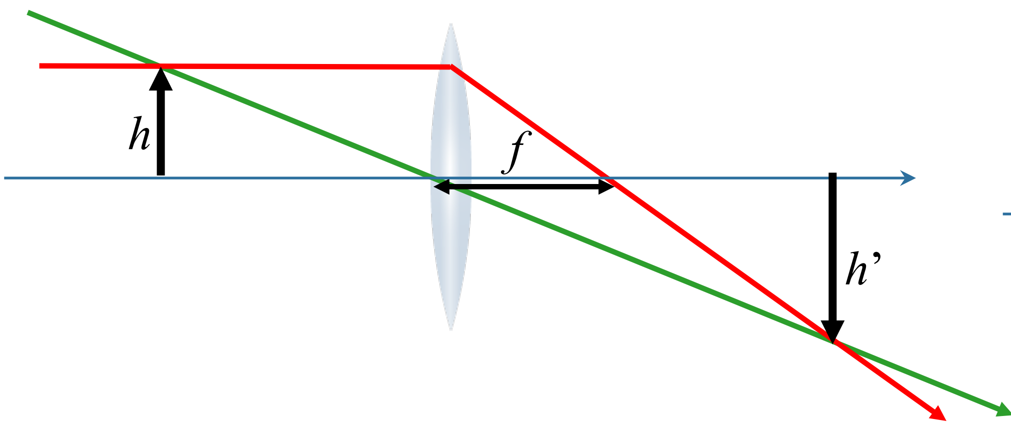


Is the image real or virtual?

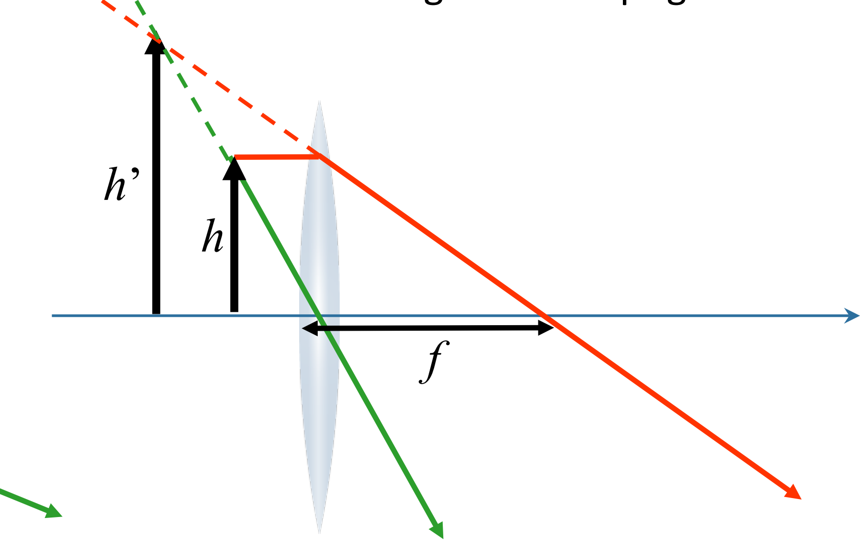
A) REAL

$\boxed{\text{B) VIRTUAL}}$

A real image would be inverted

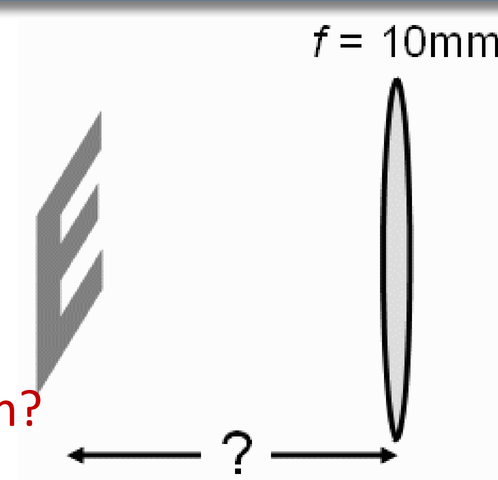


A virtual image will be upright



A magnifying glass is used to read the fine print on a document. The focal length of the lens is 10mm.

At what distance from the lens must the document be placed in order to obtain an image magnified by a factor of 5 that is not inverted?



How does the object distance compare to the focal length?

A) $|s| < |f|$

B) $|s| = |f|$

C) $|s| > |f|$

Lens equation

$$\frac{1}{S'} = \frac{1}{f} - \frac{1}{S}$$

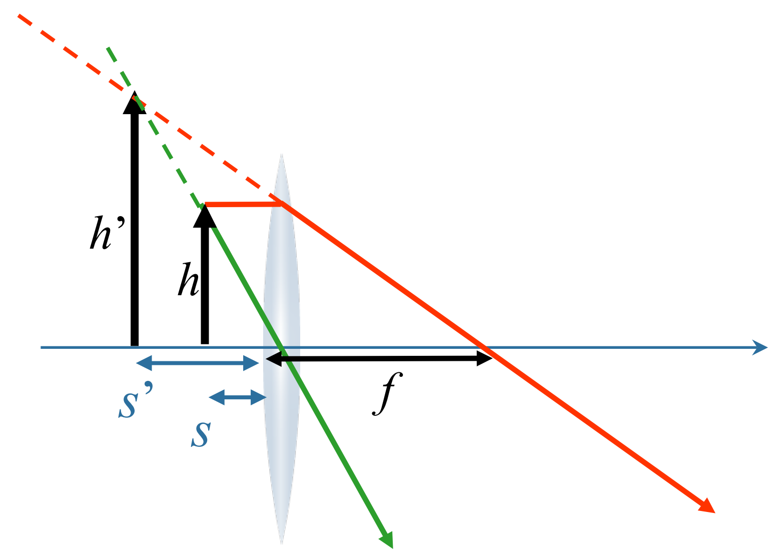
$$S' = \frac{f\,S}{S-f}$$

Virtual Image $\longrightarrow s' < 0$

Real object $\longrightarrow s > 0$

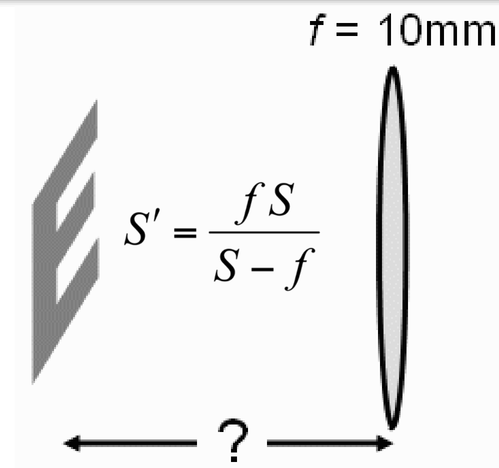
Converging lens $\longrightarrow f > 0$

$$S - f < 0$$



A magnifying glass is used to read the fine print on a document. The focal length of the lens is 10mm.

At what distance from the lens must the document be placed in order to obtain an image magnified by a factor of 5 that is not inverted?



What is the magnification M in terms of s and f?

A) $M = \dfrac{s-f}{f}$ B) $M = \dfrac{f-s}{f}$ C) $\boxed{M = \dfrac{-f}{s-f}}$ D) $M = \dfrac{f}{s-f}$

Lens equation:

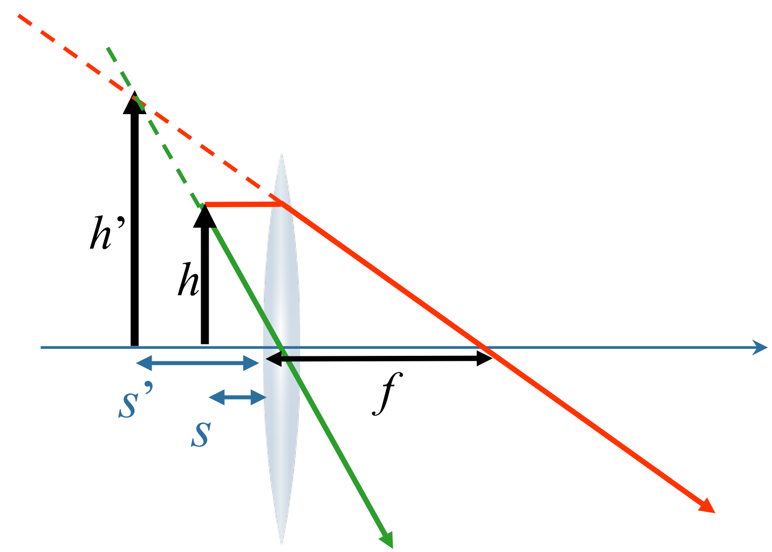
$$\frac{1}{S'} = \frac{1}{f} - \frac{1}{S}$$

$$S' = \frac{fS}{S-f}$$

Magnification equation:

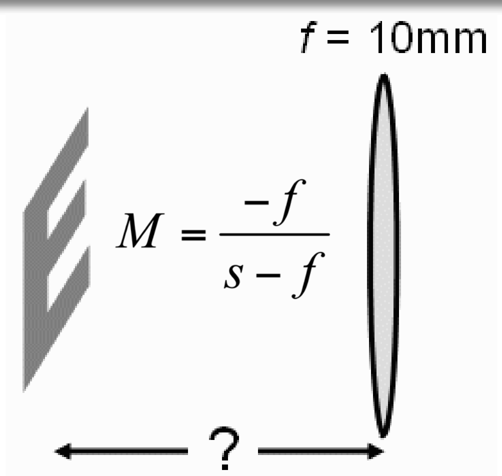
$$M = -\frac{s'}{s}$$

$$M = \frac{-f}{s-f}$$



A magnifying glass is used to read the fine print on a document. The focal length of the lens is 10mm.

At what distance from the lens must the document be placed in order to obtain an image magnified by a factor of 5 that is not inverted?



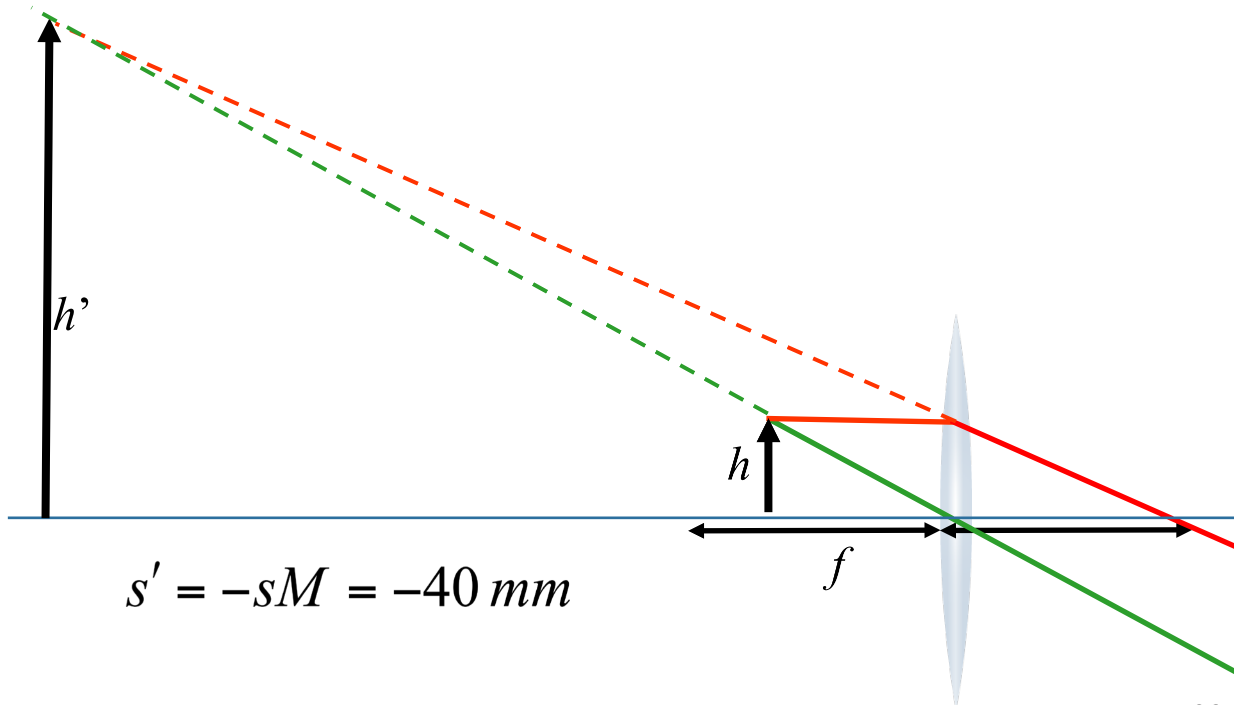
A) 1.7mm   B) 6mm $\boxed{\text{C) 8mm}}$ D) 40 mm   E) 60 mm

$$M = +5$$

$$f = +10\,mm$$

$$M = \frac{-f}{s-f} \longrightarrow s = f\frac{(M-1)}{M}$$

$$\longrightarrow s = \frac{4}{5}f = 8\,mm$$



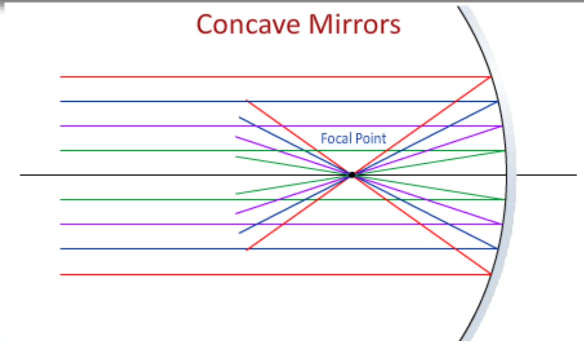
$$s' = -sM = -40\,mm$$

Quick note about exams

- Vast majority comply with honor code
- Some don't…
- Please do not post exam questions to Chegg or other sites!
- We are aware of these incidents and appropriate disciplinary action will be taken
- Best to admit to infraction and not do this again

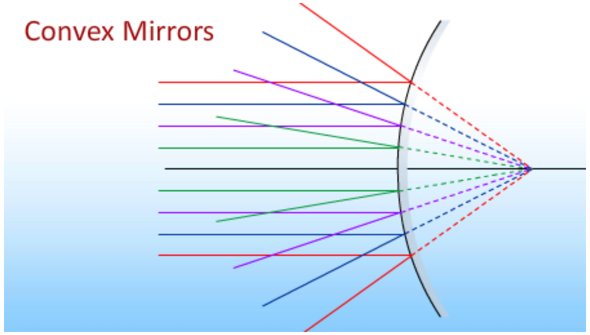
see you next week!

Physics 212
Lecture 27



Today's Concept:

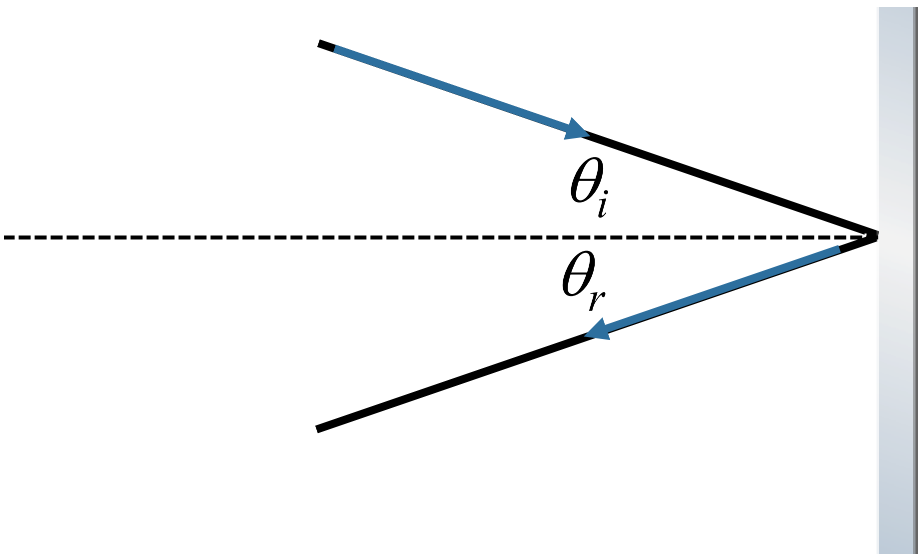
Mirrors



Reflection

Angle of incidence = Angle of reflection

$$\theta_i = \theta_r$$

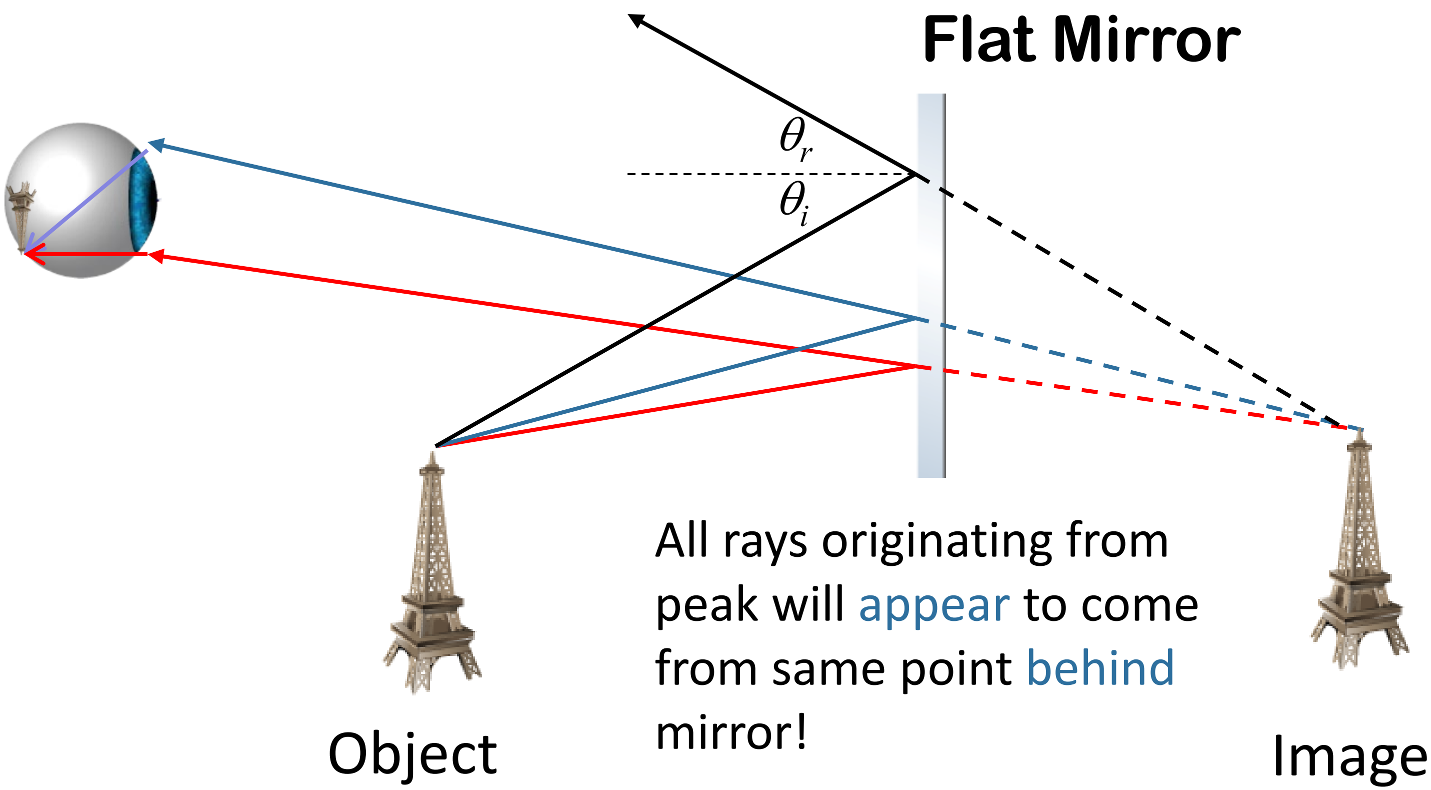


That's all of the physics – everything else is just geometry!

Flat Mirror

All you see is what reaches your eyes

You think object's location is where rays appear to come from.



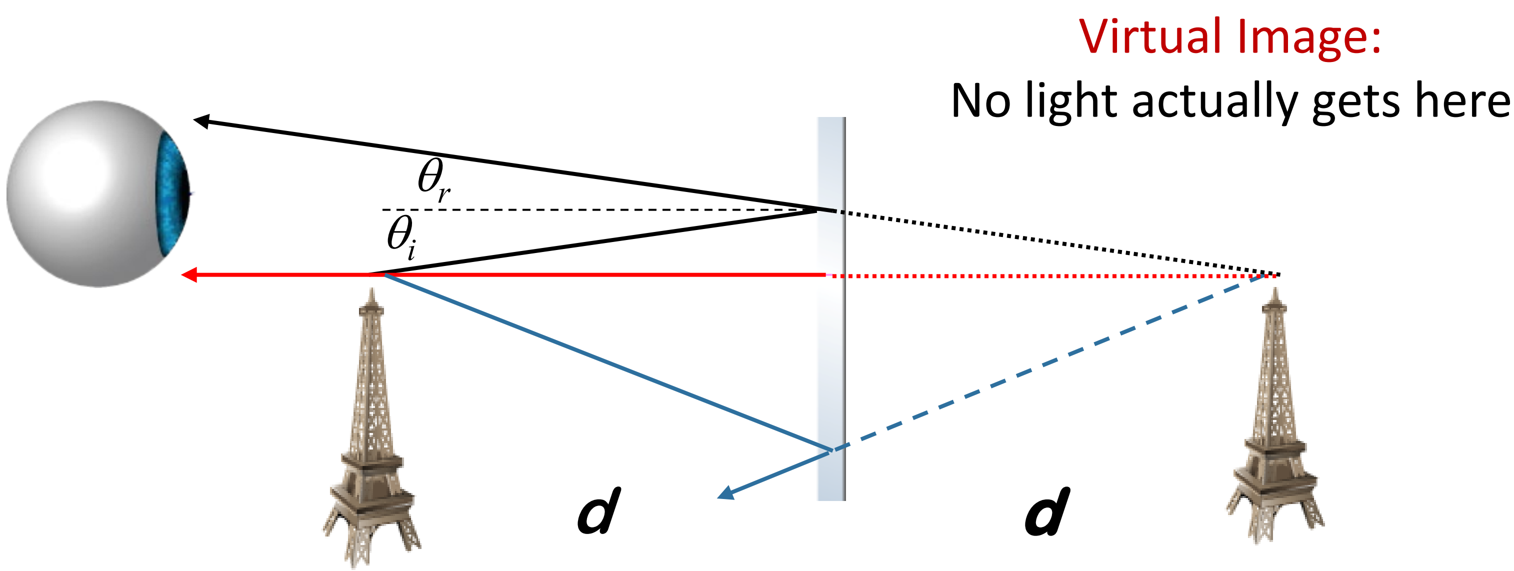
All rays originating from peak will appear to come from same point behind mirror!

Flat Mirror

1) Draw first ray perpendicular to mirror $0 = \theta_i = \theta_r$

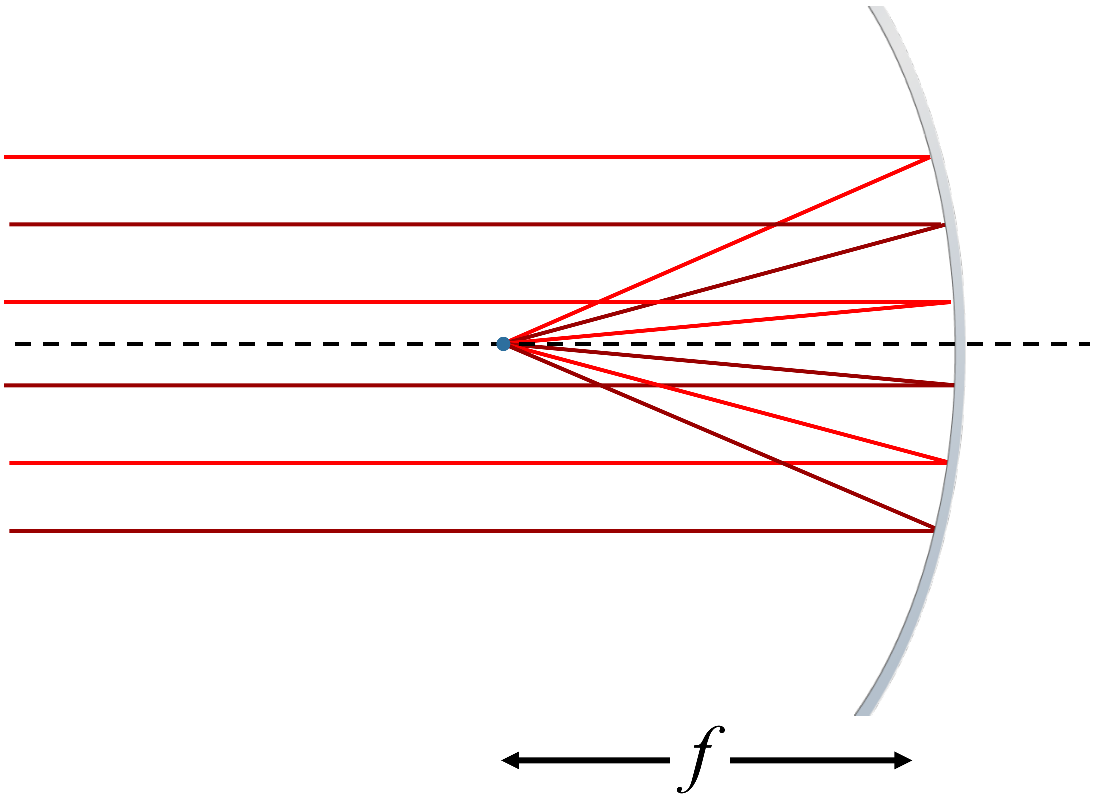
2) Draw second ray at angle. $\theta_i = \theta_r$

3) Lines appear to intersect a distance d behind mirror. This is the image location.



Concave: Consider the case where the shape of the mirror is such that light rays parallel to the axis of the mirror are all "focused" to a common spot a distance f in front of the mirror:

Note: analogous to "converging lens"
Real object can produce real image

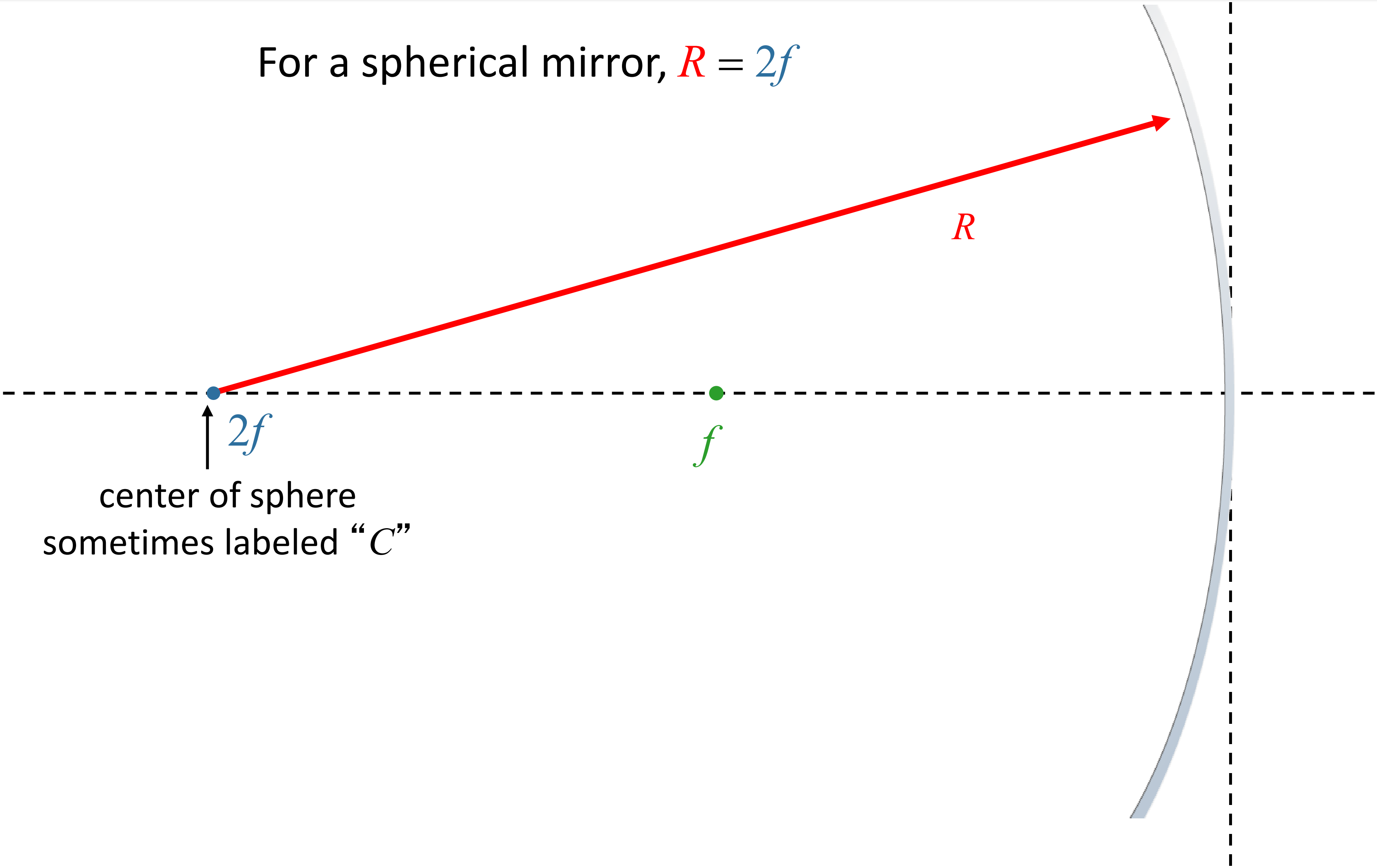


These mirrors are often sections of spheres (assumed in this class).

For such "spherical" mirrors, we assume all angles are small even though we draw them big to make it easy to see…

Aside:

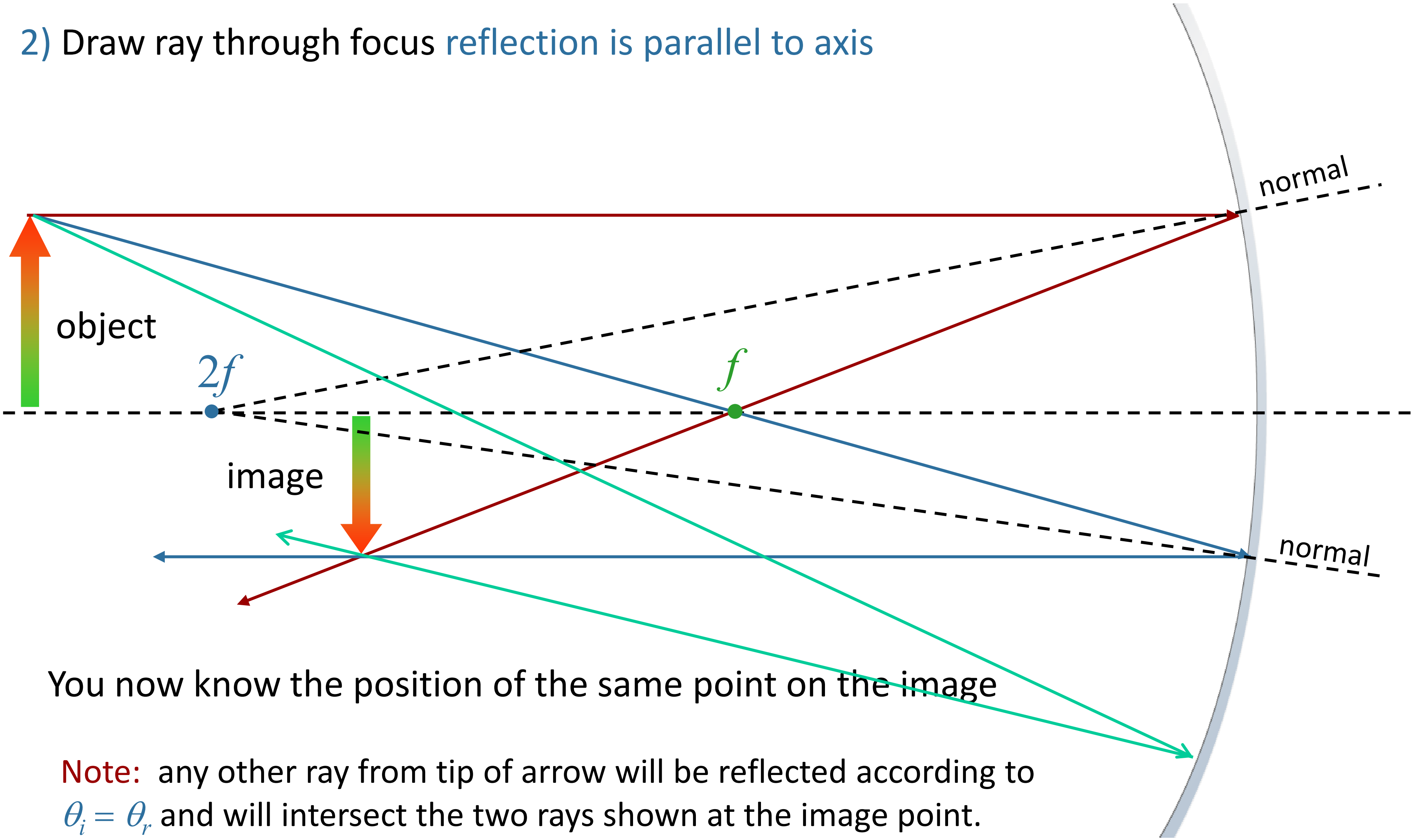
For a spherical mirror, $R = 2f$



Recipe for Finding Image:

1) Draw ray parallel to axis reflection goes through focus

2) Draw ray through focus reflection is parallel to axis



You now know the position of the same point on the image

Note: any other ray from tip of arrow will be reflected according to $\theta_i = \theta_r$ and will intersect the two rays shown at the image point.

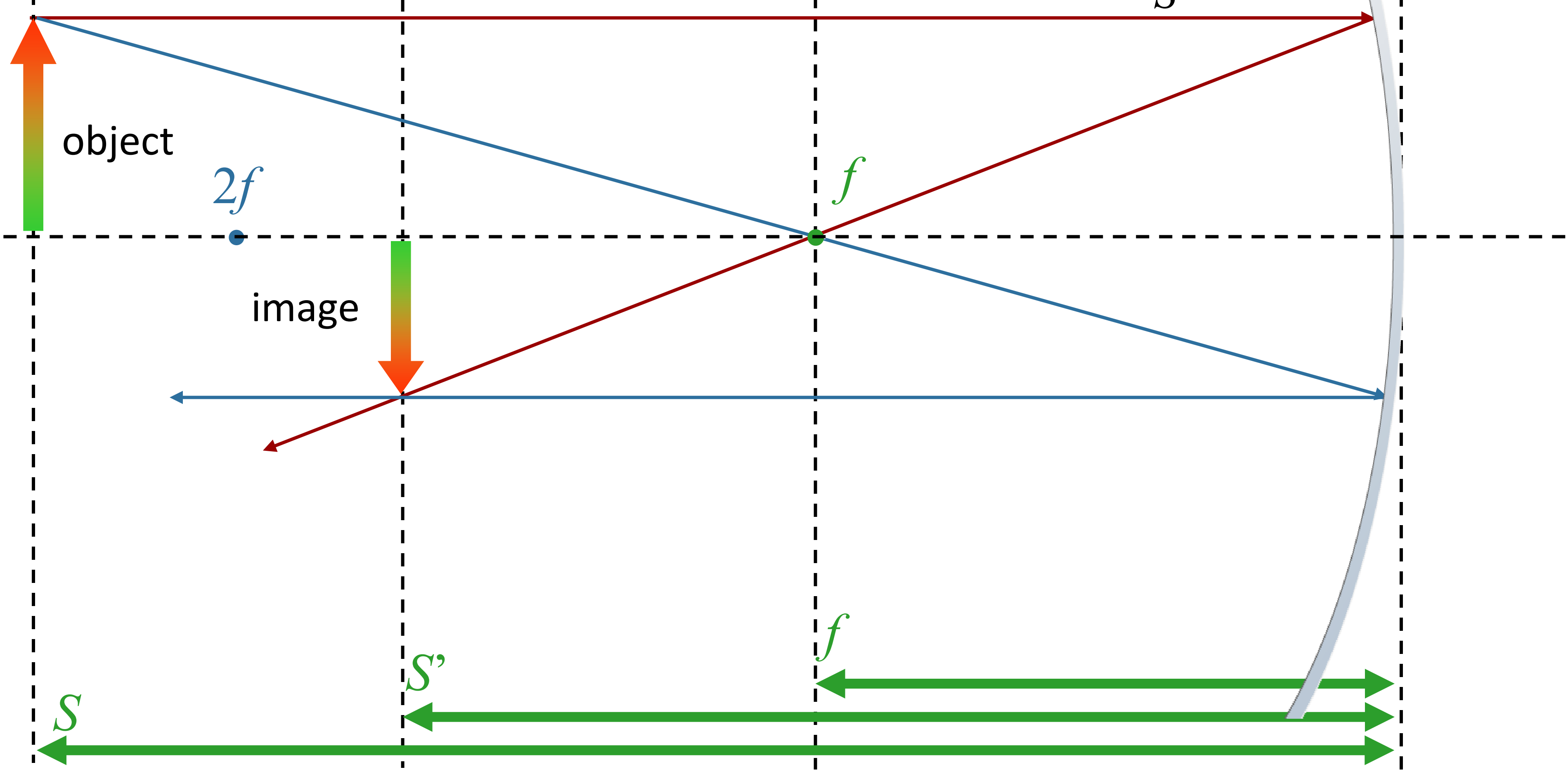
$S > 2f$

image is:
real
inverted
smaller

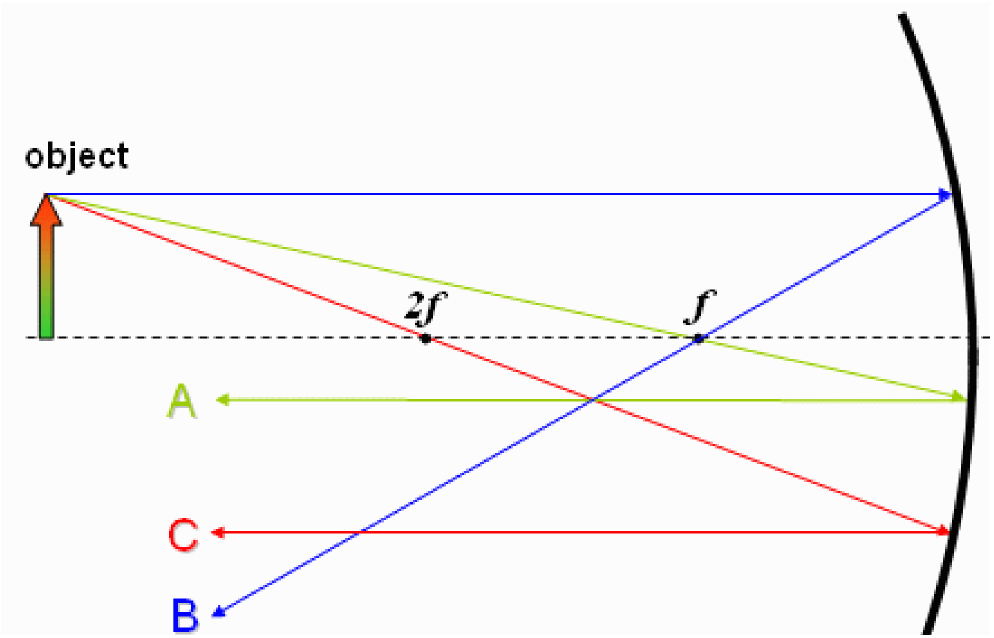
$f > 0$
$s > 0$
$s' > 0$

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$$

$$M = -\frac{S'}{S}$$



Check Point 1a

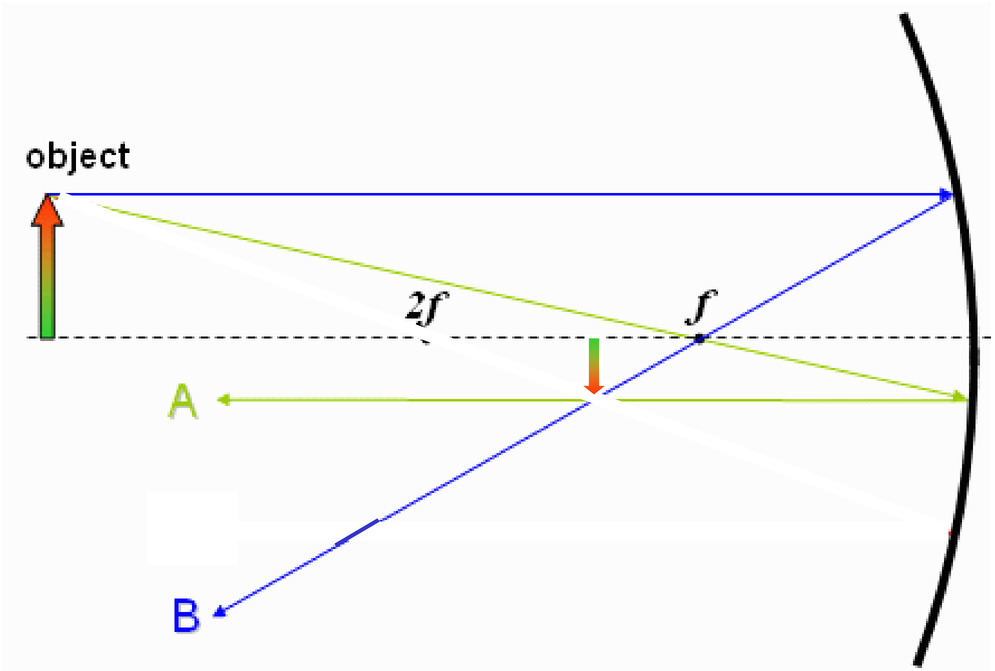


The diagram above shows three light rays reflected off a concave mirror. Which ray is NOT correct?

A ***B*** $\boxed{\textbf{\textit{C}}}$

The ray only bounces back horizontally if it passes through the focal point

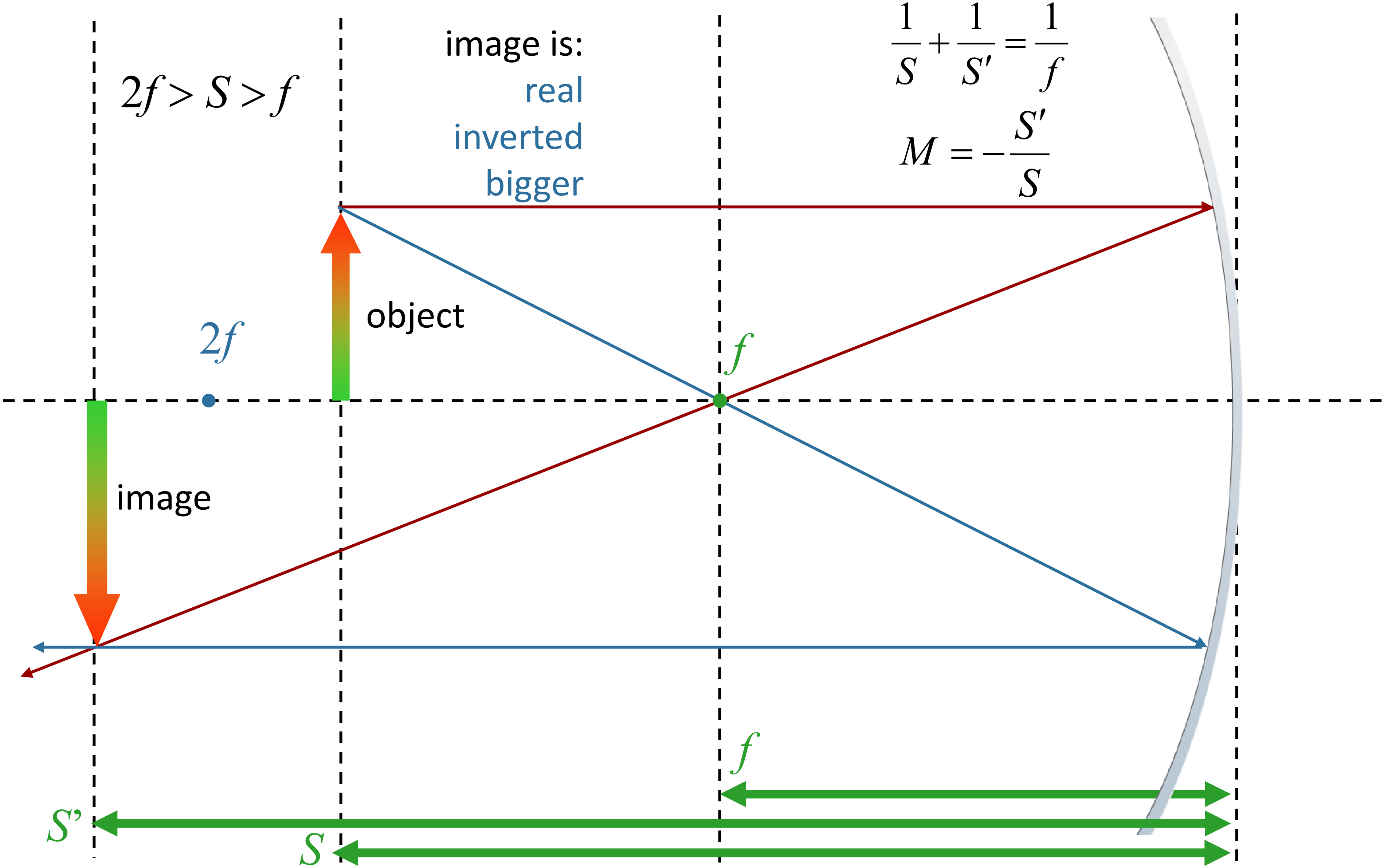
Check Points 1b



The diagram above shows two light rays reflected off a concave mirror. The image is

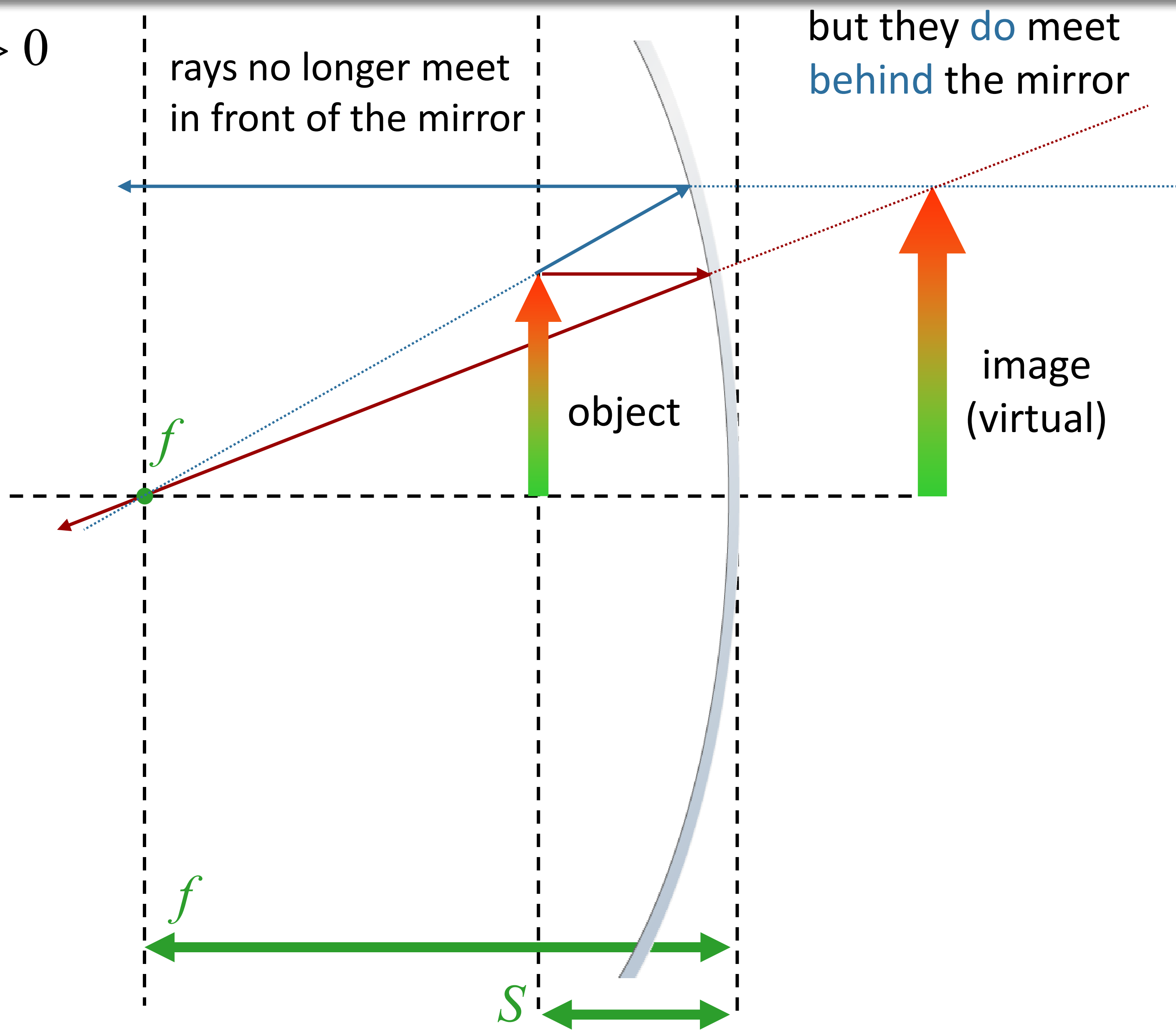
A. Upright and reduced ***B.*** Upright and enlarged

C. Inverted and reduced ***D.*** Inverted and enlarged



$2f > S > f$
image is:
real
inverted
bigger
$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$
$M = -\frac{S'}{S}$
object
$2f$
f
image
f
S'
S

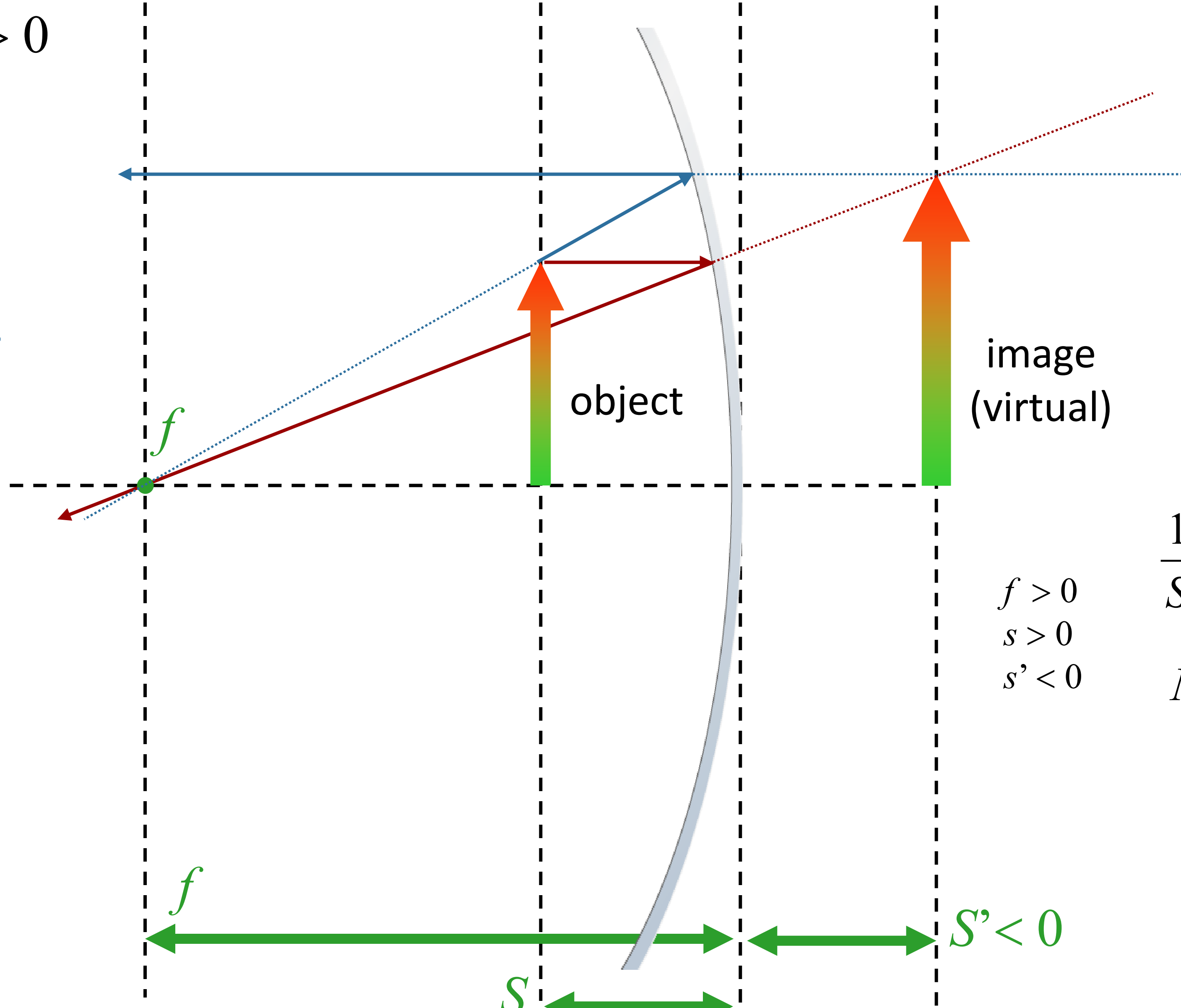
$f > S > 0$



rays no longer meet
in front of the mirror
but they do meet
behind the mirror
object
image
(virtual)
f
f
S

$f > S > 0$

image is:
virtual
upright
bigger



$f > 0$
$s > 0$
$s' < 0$

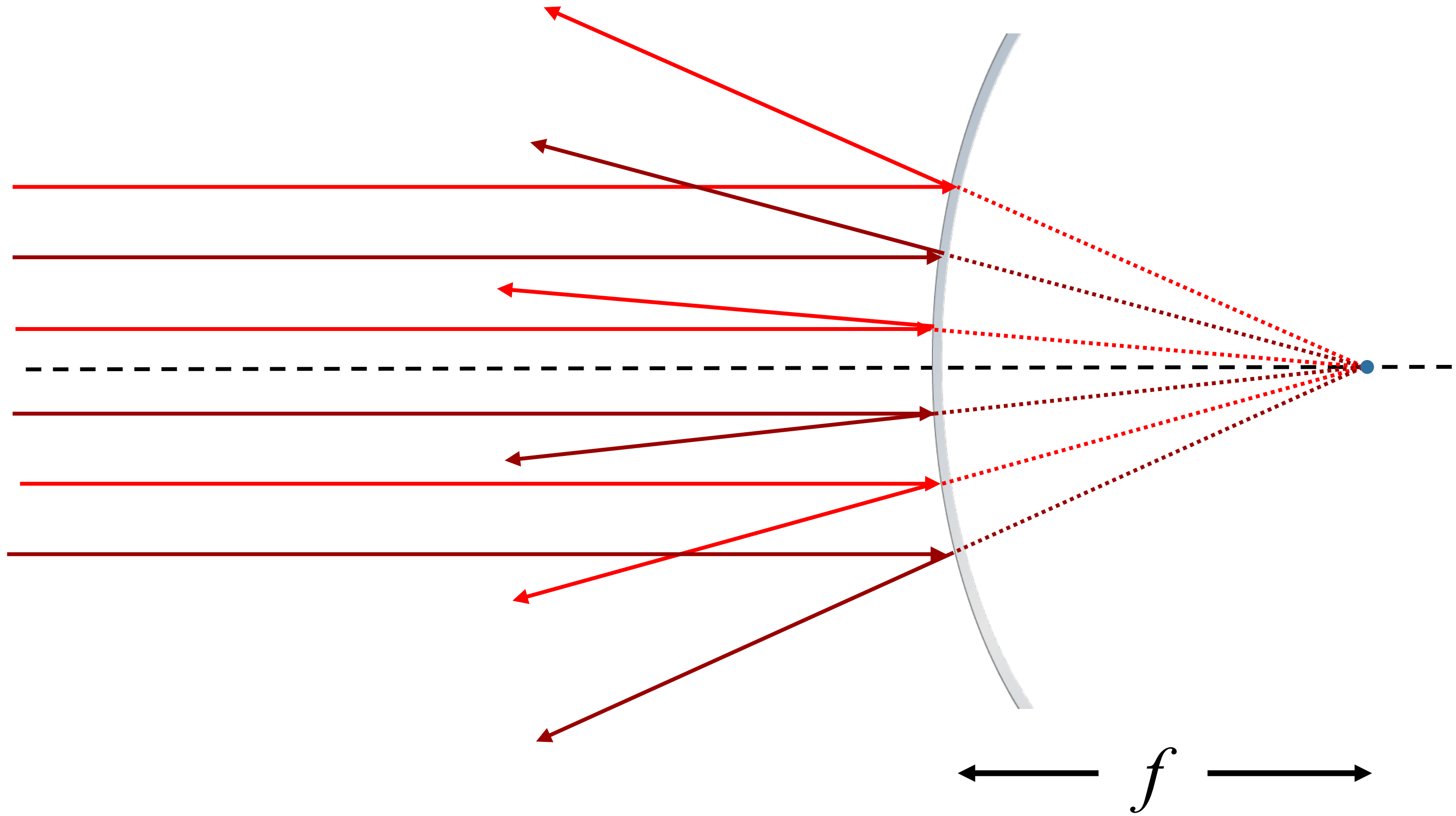
$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$$

$$M = -\frac{S'}{S}$$

Convex Mirror

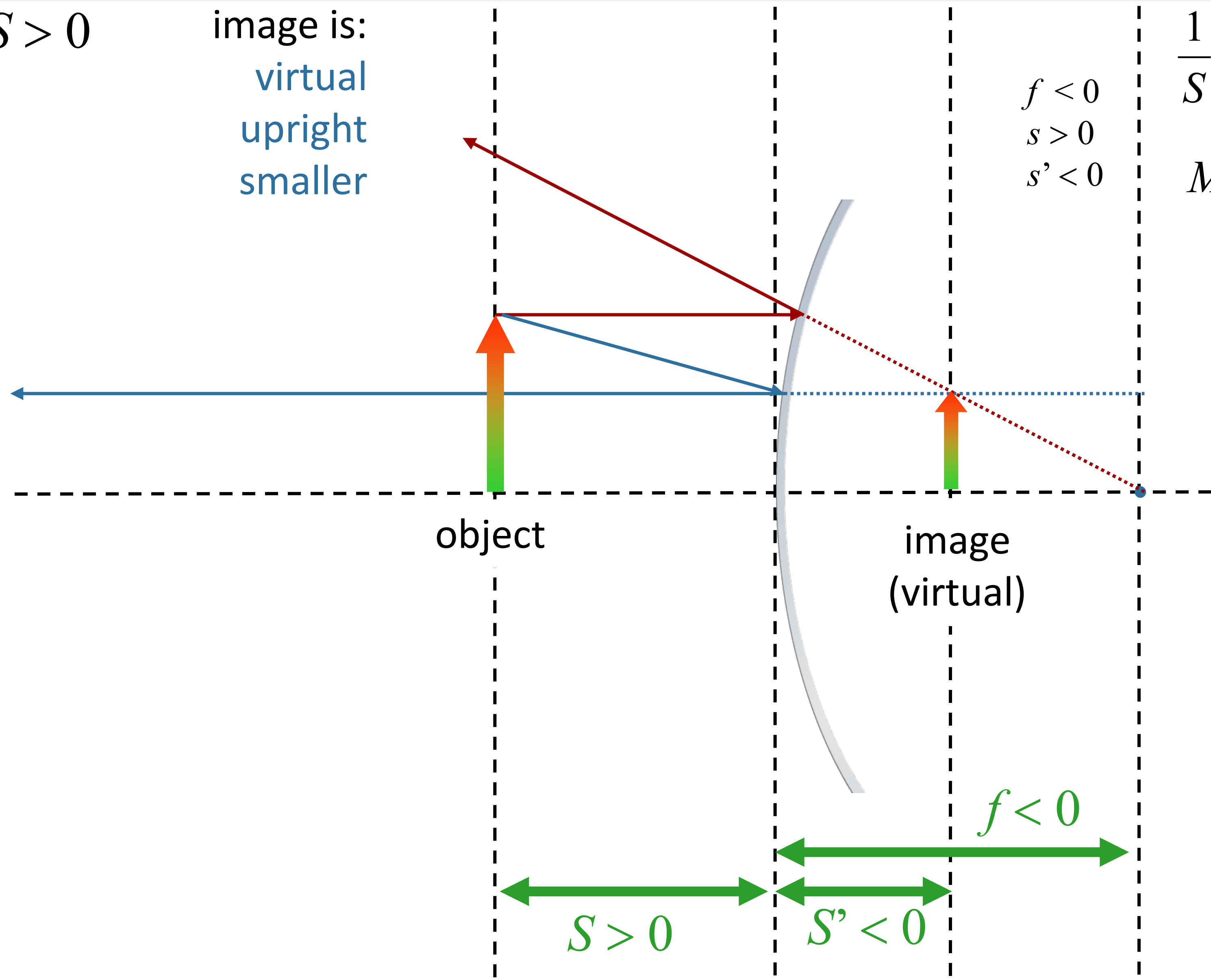
Convex: Consider the case where the shape of the mirror is such that light rays parallel to the axis of the mirror are all "focused" to a common spot a distance f behind the mirror:

Note: analogous to "diverging lens"
Real object will produce virtual image



$S > 0$

image is:
virtual
upright
smaller



$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$$

$$M = -\frac{S'}{S}$$

A famous convex mirror:

Summary - Mirrors & Lenses:

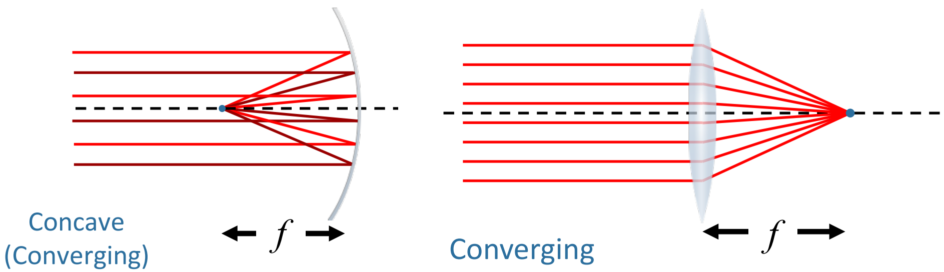
$S > 2f$

real
inverted
smaller

$2f > S > f$

real
inverted
bigger

$f > 0$

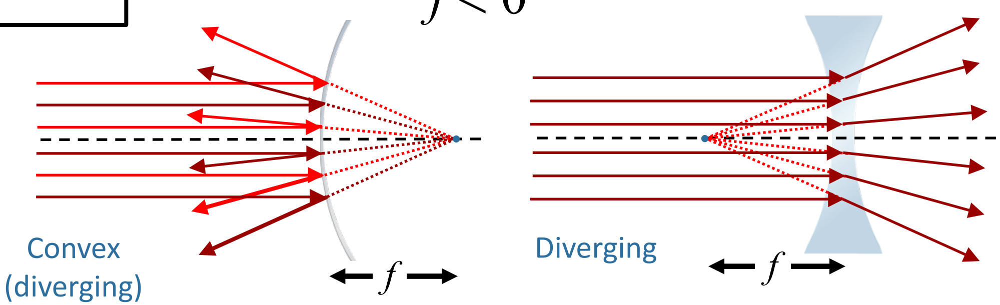


$f > S > 0$

virtual
upright
bigger

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \qquad M = -\frac{S'}{S}$$

$f < 0$



$S > 0$

virtual
upright
smaller

It's Always the Same:

$$\frac{1}{S}+\frac{1}{S'}=\frac{1}{f} \qquad M=-\frac{S'}{S}$$

You just have to keep the signs straight:

s' is positive for a real image

f is positive when focus is on side light "goes to" downstream

Upstream = Side light comes from
Downstream = Side light goes to

Lens sign conventions

S: positive if object is "upstream" of lens
S' : positive if image is "downstream" of lens
f: positive if converging lens

Mirrors sign conventions

S: positive if object is "upstream" of mirror
S' : positive if image is "downstream" of mirror
f: positive if converging mirror (concave)

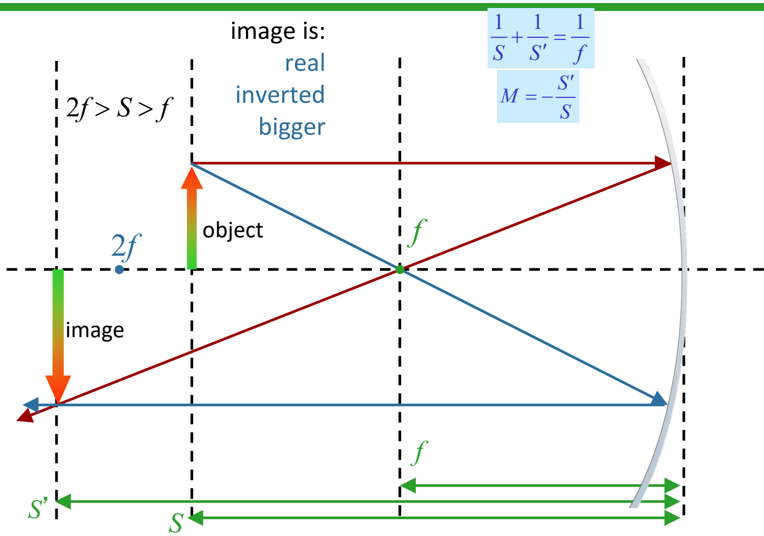
CheckPoint 2a

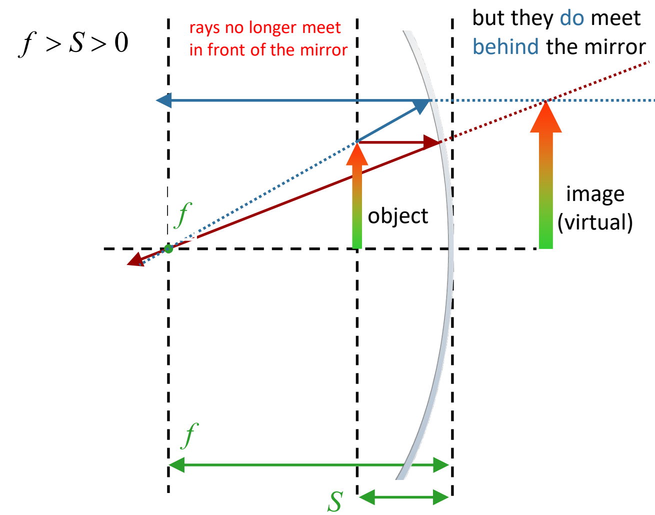
The image produced by a concave mirror of a real object is

A. Always upright

B. Always inverted

C. Sometimes upright & sometimes inverted



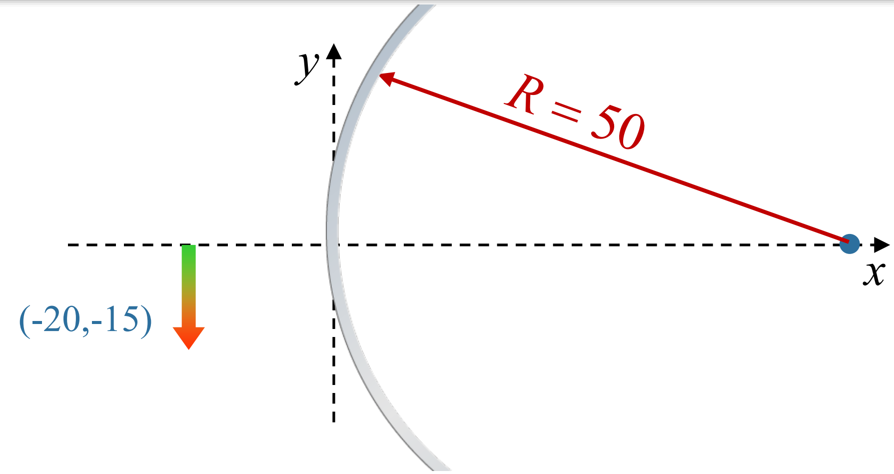


If the object is farther than focal length it will reflect an inverted image.

If the object is closer than focal length it will produce a virtual upright image.

Calculation

An arrow is located in front of a convex spherical mirror of radius $R = 50\text{cm}$. The tip of the arrow is located at $(-20\text{cm}, -15\text{cm})$.



Where is the tip of the arrow's image?

Conceptual Analysis

Mirror Equation: $1/s + 1/s' = 1/f$

Magnification: $M = -s'/s$

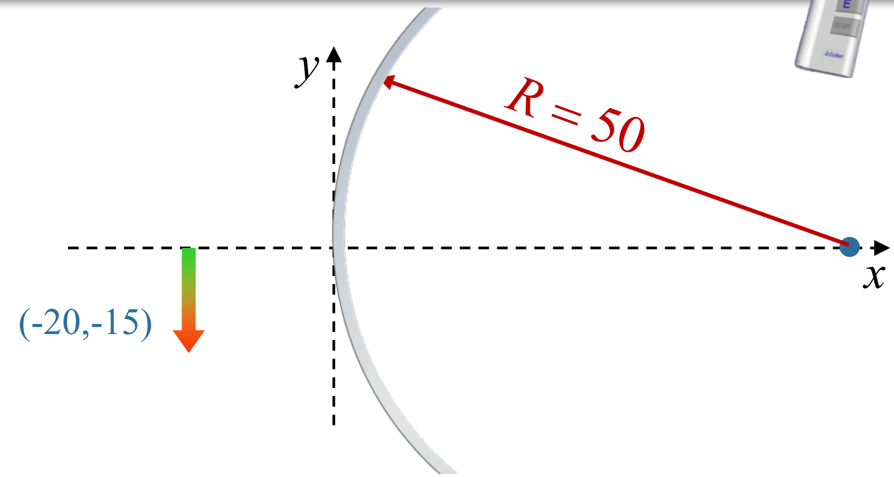
Strategic Analysis

Use mirror equation to figure out the x coordinate of the image

Use the magnification equation to figure out the y coordinate of the tip of the image

Calculation

An arrow is located in front of a convex spherical mirror of radius $R = 50\text{cm}$. The tip of the arrow is located at $(-20\text{cm}, -15\text{cm})$.

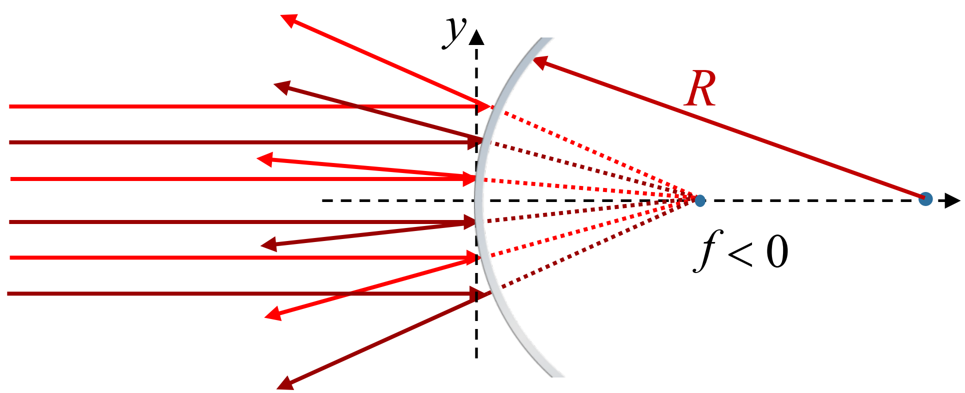


What is the focal length of the mirror?

A) $f = 50\text{cm}$ B) $f = 25\text{cm}$ C) $f = -50\text{cm}$ D) $\boxed{f = -25\text{cm}}$

For a spherical mirror $|f| = R/2 = 25\text{cm}$.

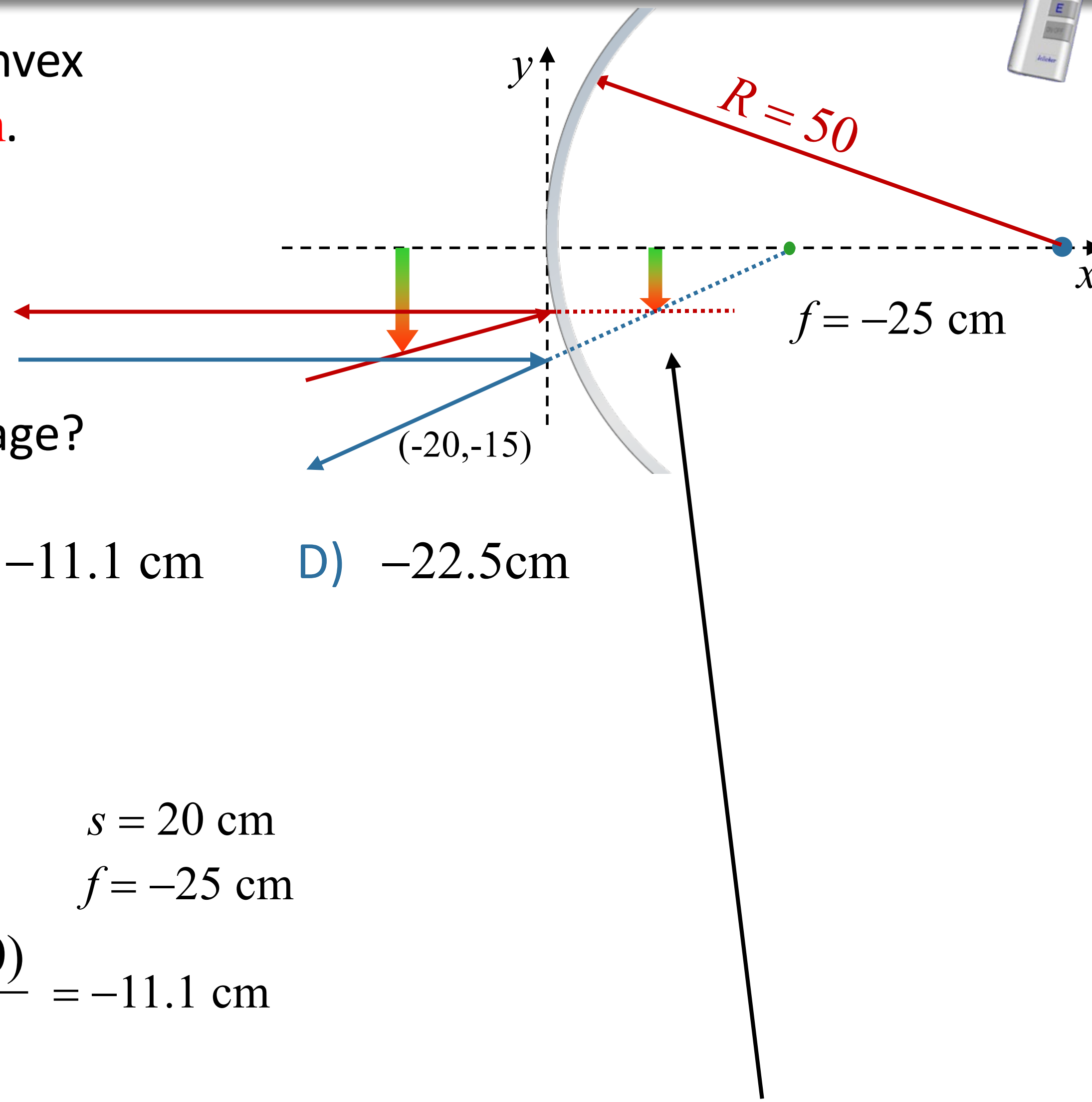
Rule for sign: Positive on side of mirror where light goes after hitting mirror



$f = -25$ cm

Calculation

An arrow is located in front of a convex spherical mirror of radius $R = 50\text{cm}$. The tip of the arrow is located at $(-20\text{cm}, -15\text{cm})$.



What is the x coordinate of the image?

A) 11.1 cm

B) 22.5 cm

C) –11.1 cm

D) –22.5cm

Mirror equation

$$\frac{1}{S'} = \frac{1}{f} - \frac{1}{S}$$

$$S' = \frac{fS}{S-f}$$

$s = 20$ cm

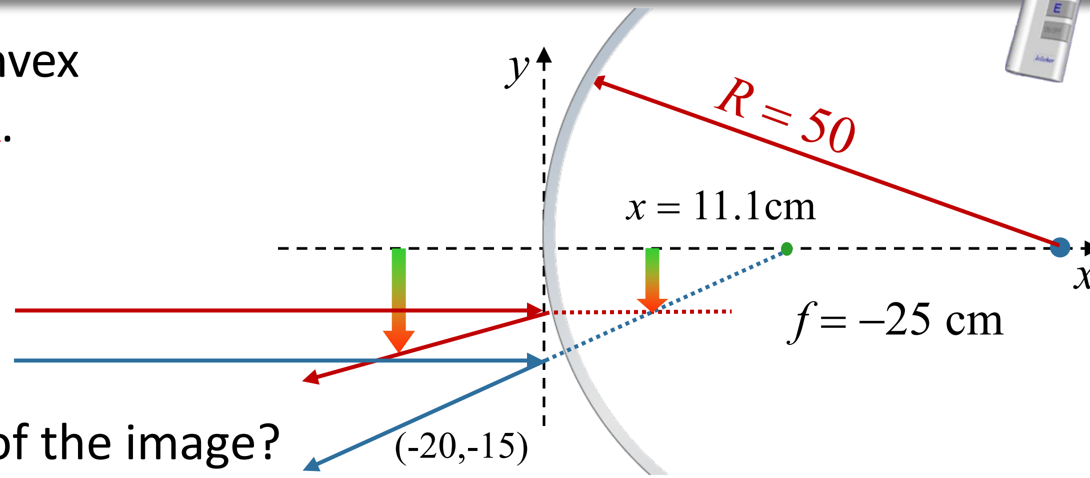
$f = -25$ cm

$$S' = \frac{(-25)(20)}{20+25} = -11.1 \text{ cm}$$

Since $s' < 0$ the image is virtual (on the "other" side of the mirror)

Calculation

An arrow is located in front of a convex spherical mirror of radius $R = 50\text{cm}$. The tip of the arrow is located at $(-20\text{cm},-15\text{cm})$.



What is the <u>y coordinate</u> of the tip of the image?

A) -11.1 cm B) -10.7 cm C) -9.1 cm D) $\boxed{-8.3\text{cm}}$

Magnification equation

$$M = -\frac{S'}{S}$$

$$s = 20 \text{ cm}$$

$$s' = -11.1 \text{ cm}$$

$$M = 0.556$$

$$y_{image} = 0.55\, y_{object} = 0.556*(-15 \text{ cm}) = -8.34 \text{ cm}$$

Thanks everyone for the interesting semester!

PHYS212
Electricity & Magnetism

Course Review

Coulomb's Law

Our notation:

$\vec{F}_{1,2}$ is the force by 1 *on* 2 (think "*by-on* ")

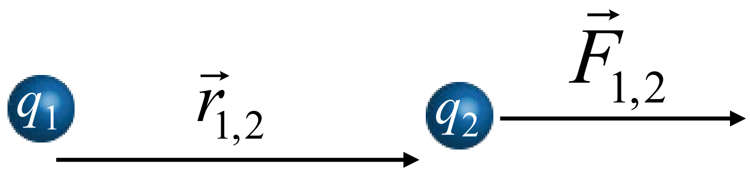
r_{12} is the unit vector that points *from* 1 *to* 2.

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$

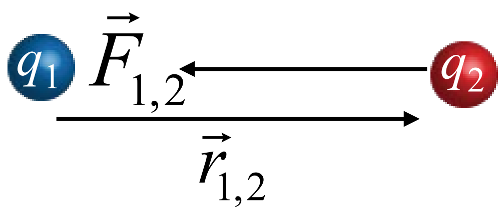
what is 'k'?

Examples:

If the charges have the same sign, the force **by** charge 1 on charge 2 would be in the direction of r_{12} (to the right).

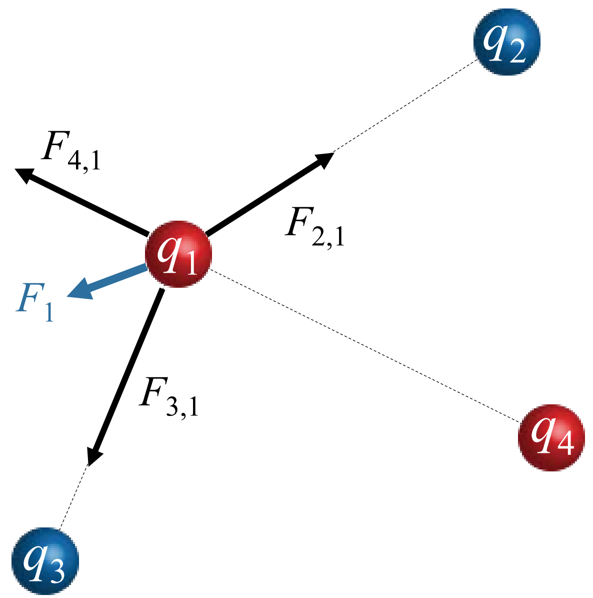


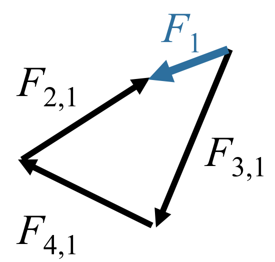
If the charges have opposite sign, the force **by** charge 1 on charge 2 would be opposite the direction of r_{12} (left).



Superposition:

If there are more than two charges present, the total force on any given charge is just the vector sum of the forces due to each of the other charges:





$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1} + \ldots$$

Electric Field

The electric field E at a point in space is simply the force per unit charge at that point.

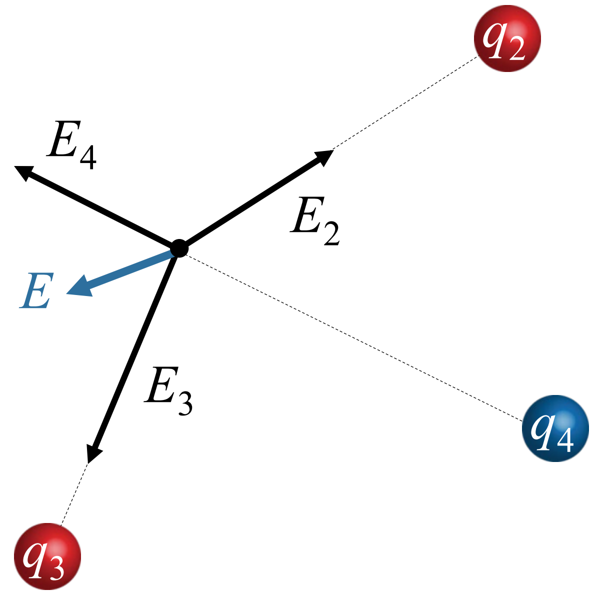
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle

$$\vec{E} = k\frac{Q}{r^2}r$$

Superposition

$$\vec{E} = \sum_i k\frac{Q_i}{r_i^2}r_i$$



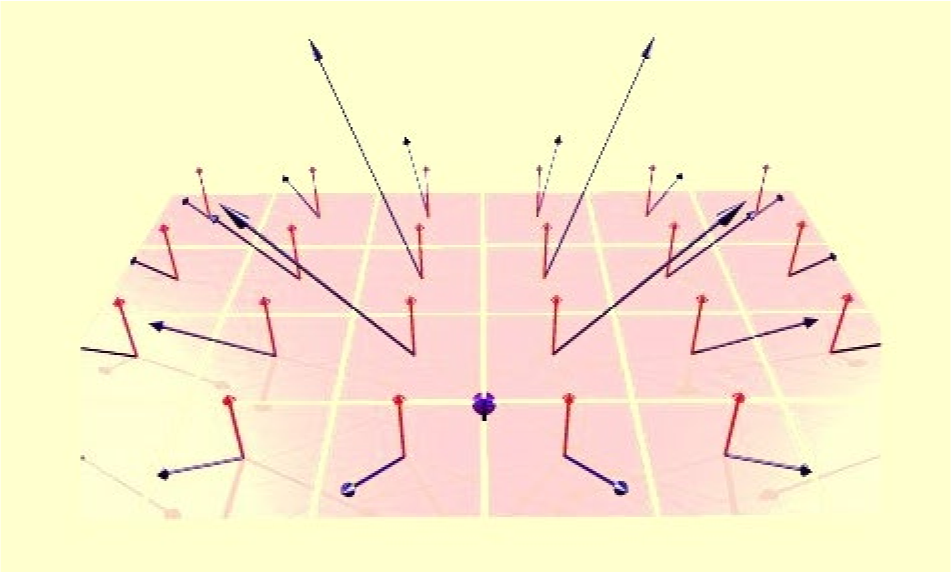
Field points away from positive charges.
Field points toward negative charges.

Electric Flux "Counts Field Lines"

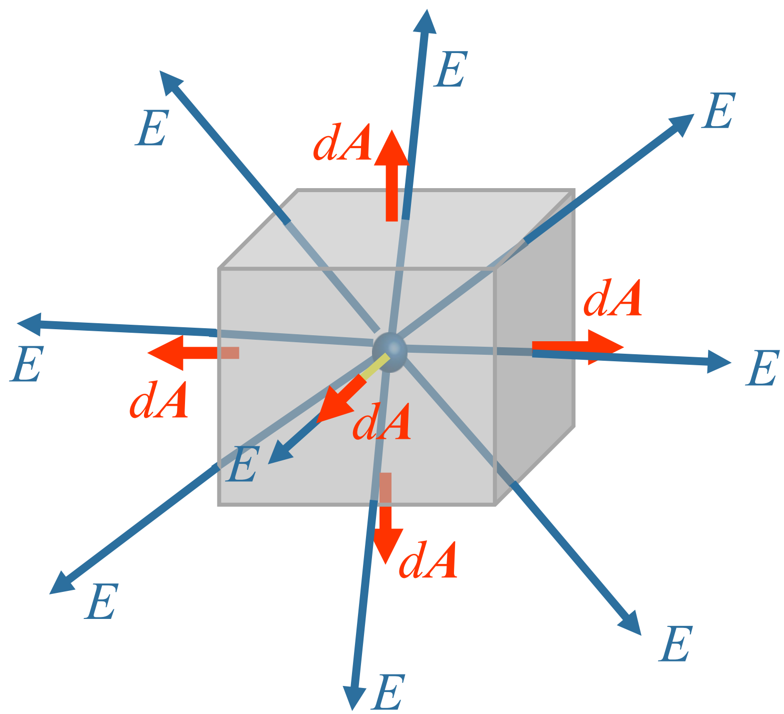
Can you give us a clear, simple definition of what flux is?

$$\underbrace{\Phi_S}_{\text{Flux through surface } S} \equiv \underbrace{\int_S \vec{E} \cdot d\vec{A}}_{\text{Integral of } \vec{E}\cdot d\vec{A} \text{ on surface } S}$$

Representing the area of a surface as a vector in order to take the dot product.



Gauss Law



$$\int_{closed-surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

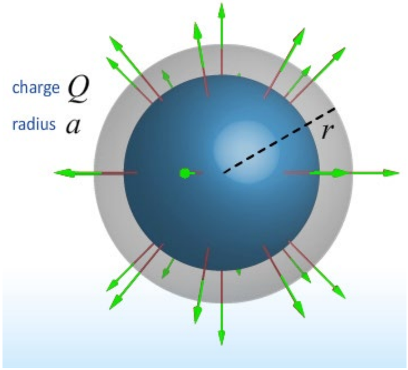
Gauss' Law Symmetries

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

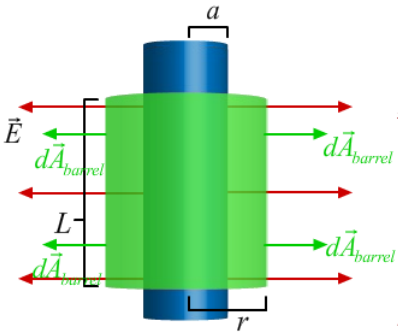
Spherical



$$A = 4\pi r^2$$

$$E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}$$

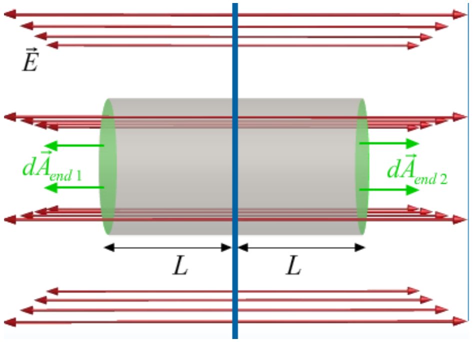
Cylindrical



$$A = 2\pi r L$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Planar



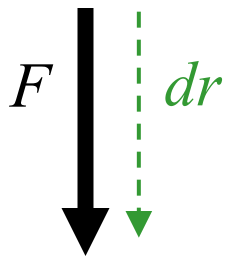
$$A = 2\pi r^2$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

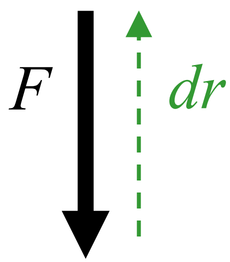
Work (Mechanics Review)

Recall from physics 211:

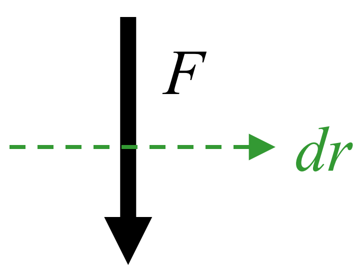
$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \qquad W_{TOT} = \Delta K$$



$W > 0$ (e.g. $W_{gravity}$ on object dropped)



$W < 0$ (e.g. $W_{gravity}$ on ball going up)



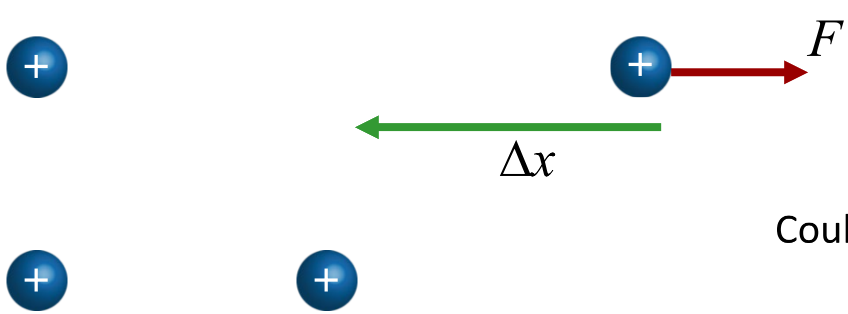
$W = 0$ (e.g. $W_{gravity}$ on moving horizontally)

Potential Energy

$$\Delta U \equiv -W_{conservative}$$

If gravity does negative work, potential energy increases!

Same idea for Coulomb force… if Coulomb force does negative work, potential energy increases.



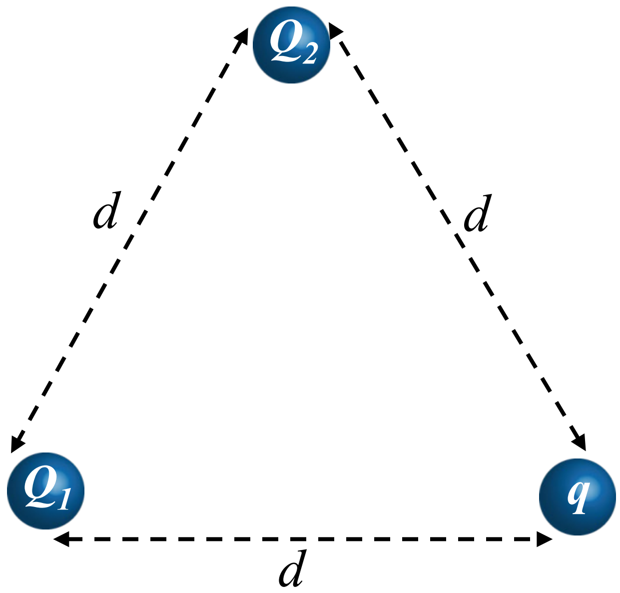
Coulomb force does negative work
Potential energy increases

Potential Energy of Many Charges

Two charges are separated by a distance d. What is the change in potential energy when a third charge q is brought from far away to a distance d from the original two charges?

$$\Delta U = \frac{qQ_1}{4\pi\varepsilon_0}\frac{1}{d} + \frac{qQ_2}{4\pi\varepsilon_0}\frac{1}{d}$$

(superposition)



"Can you go over in further depth, what electric potential energy actually means for a system of charges?"

Big Idea

Last time we defined the electric potential energy of charge q in an electric field:

$$\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{l} = -\int_a^b q\vec{E}\cdot d\vec{l}$$

The only mention of the particle was through its charge q.

We can obtain a new quantity, the electric potential, which is a PROPERTY OF THE SPACE, as the potential energy per unit charge.

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Note the similarity to the definition of another quantity which is also a PROPERTY OF THE SPACE, the electric field.

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

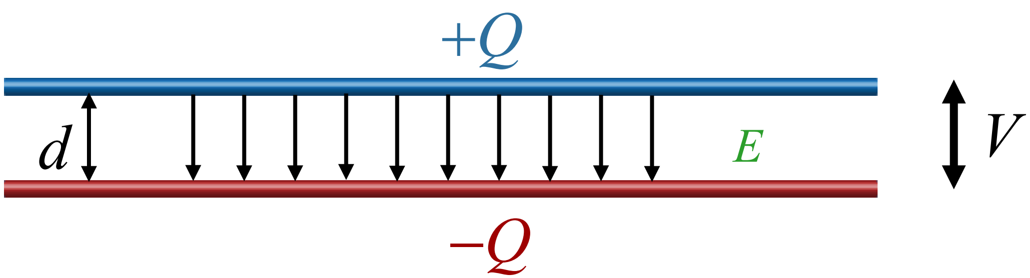
Capacitance

Capacitance is defined for any pair of spatially separated conductors.

$$C \equiv \frac{Q}{V}$$

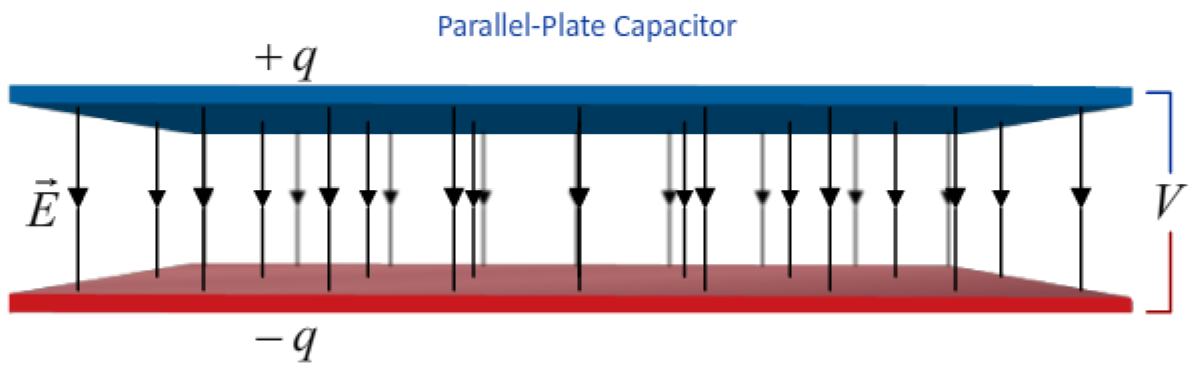
How do we understand this definition ?

- Consider two conductors, one with excess charge = $+Q$ and the other with excess charge = $-Q$



- These charges create an electric field in the space between them
- We can integrate the electric field between them to find the potential difference between the conductor
- This potential difference should be proportional to Q !
  - The ratio of Q to the potential difference is the capacitance and only depends on the geometry of the conductors

Main Point 3: Capacitors Store Energy in E

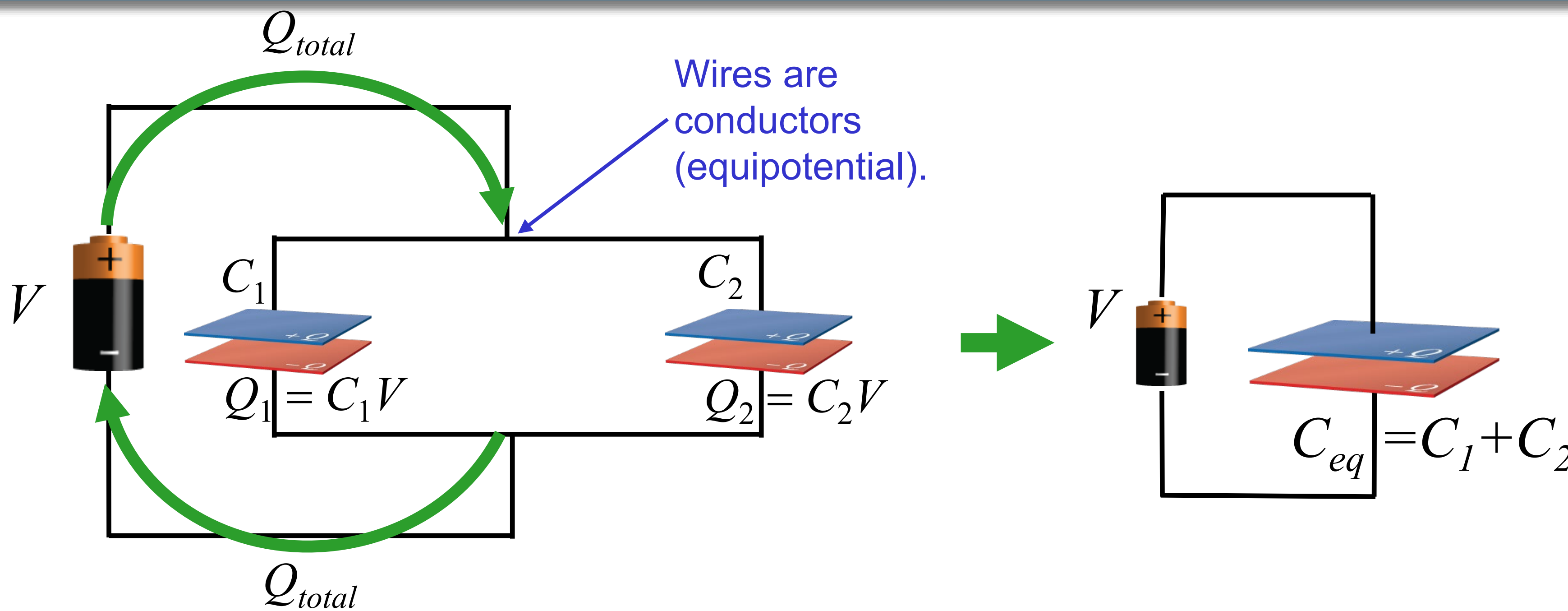


$$u = \frac{1}{2}\,\varepsilon_o E^2$$ Energy Density

Energy Stored in Capacitors

$$U = \frac{1}{2}QV$$ or $$U = \frac{1}{2}\frac{Q^2}{C}$$ or $$U = \frac{1}{2}CV^2$$

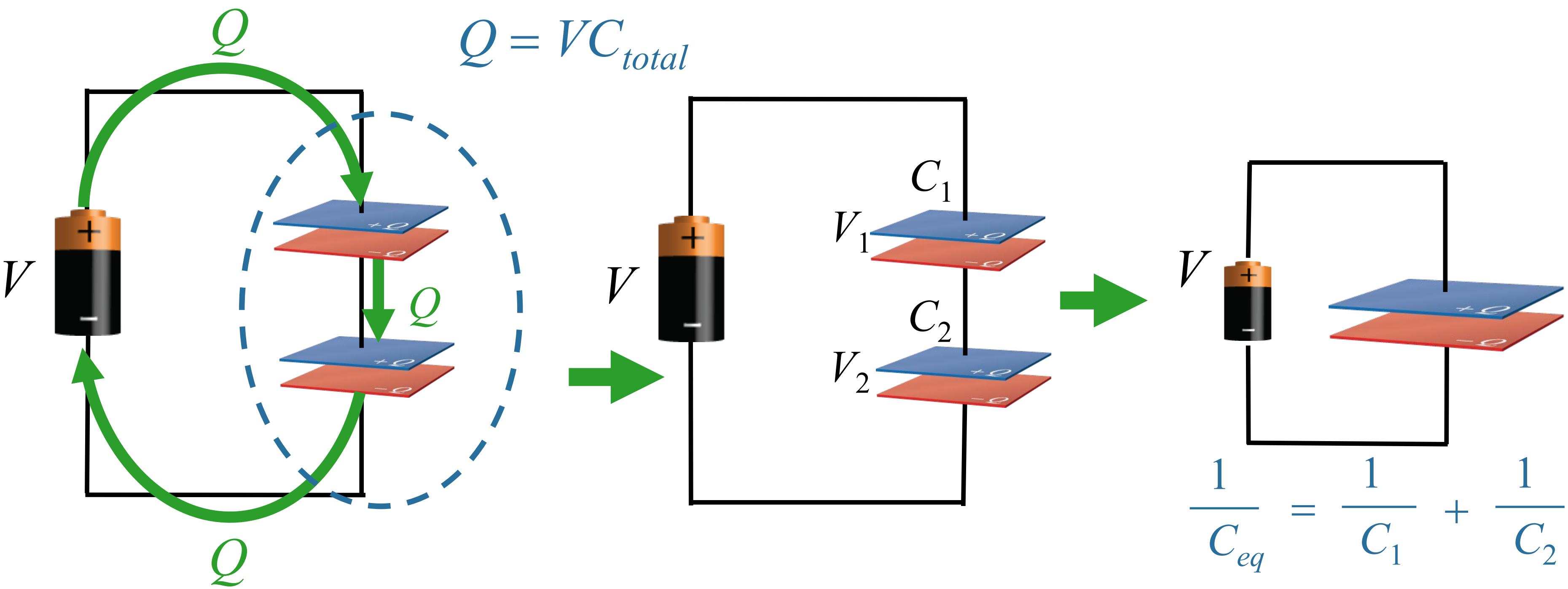
Parallel Capacitor Circuit



Key point: V is the same for both capacitors

Key Point: $Q_{total} = Q_1 + Q_2 = VC_1 + VC_2 = V(\underbrace{C_1 + C_2}_{C_{total} = C_1 + C_2})$

Series Capacitor Circuit

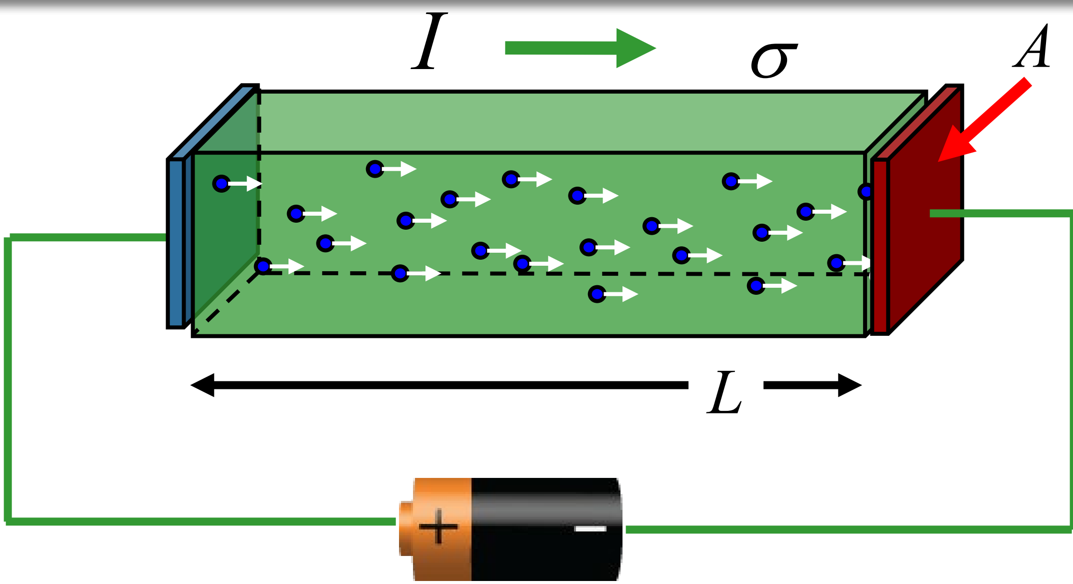


Key point: Q is the same for both capacitors

Key point: $Q = VC_{total} = V_1C_1 = V_2C_2$

Also: $V = V_1 + V_2$ $\longrightarrow$ $Q/C_{total} = Q/C_1 + Q/C_2$

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$



V

$$\boxed{\text{Ohm's Law: } J = \sigma E}$$

Conductivity – high for good conductors.

Observables:

$$V = EL$$
$$I = JA$$

$$\rightarrow \quad I/A = \sigma V/L \quad \rightarrow \quad I = V/(L/\sigma A)$$

$$R = \frac{L}{\sigma A}$$

$$\boxed{I = V/R}$$

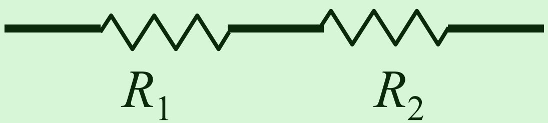
R = Resistance

$$\rho = 1/\sigma$$

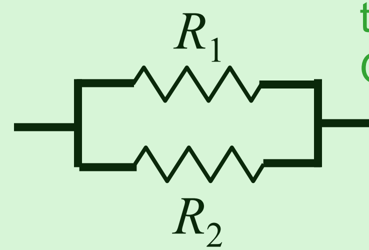
Resistor Summary

Series

Every loop with R_1 also has R_2



Parallel



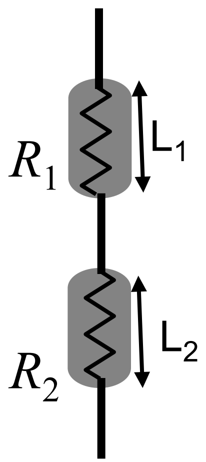
There is a loop that contains ONLY R_1 and R_2

	Series	Parallel
Wiring	Each resistor on the <u>same</u> wire.	Each resistor on a <u>different</u> wire.
Voltage	<u>Different</u> for each resistor. $V_{total} = V_1 + V_2$	<u>Same</u> for each resistor. $V_{total} = V_1 = V_2$
Current	<u>Same</u> for each resistor $I_{total} = I_1 = I_2$	<u>Different</u> for each resistor $I_{total} = I_1 + I_2$
Resistance	<u>Increases</u> $R_{eq} = R_1 + R_2$	<u>Decreases</u> $1/R_{eq} = 1/R_1 + 1/R_2$

Resistors and Capacitors

Can we go over why Capacitors and Resistors are inverses in series and parallel? Like more of a physical reason not just "the math works that way"

Series, you are adding lengths

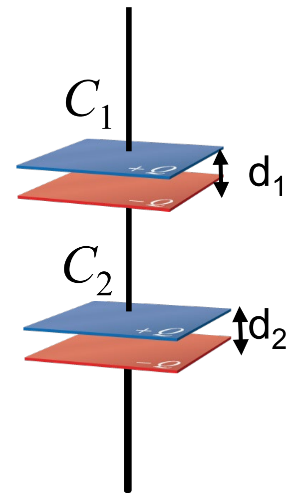


$$R = \rho\frac{L}{A}$$

$$R_{tot} = \rho\frac{L_1 + L_2}{A}$$

$$= \rho\frac{L_1}{A} + \rho\frac{L_2}{A} = R_1 + R_2$$

$$= R_1 + R_2$$



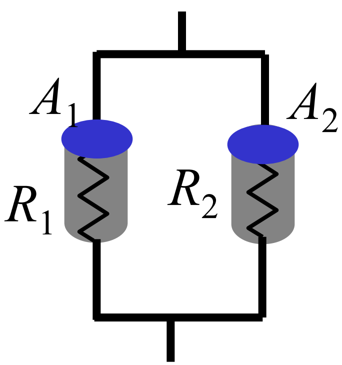
$$C = \varepsilon\frac{A}{d} \qquad \frac{1}{C} = \frac{d}{\varepsilon A}$$

$$\frac{1}{C_{tot}} = \frac{d_1 + d_2}{\varepsilon A}$$

$$= \frac{d_1}{\varepsilon A} + \frac{d_2}{\varepsilon A}$$

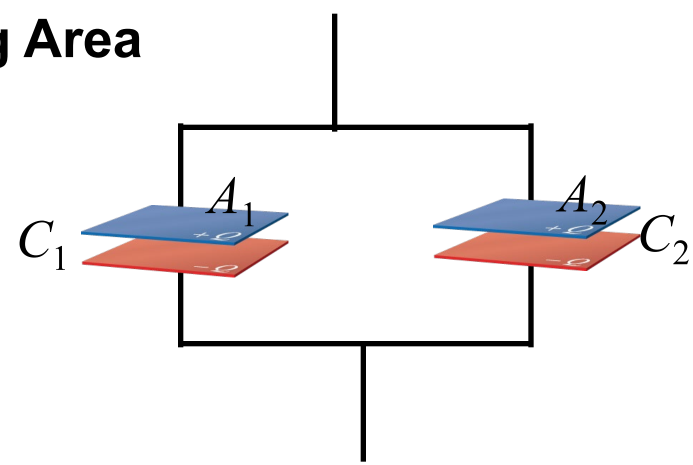
$$= \frac{1}{C_1} + \frac{1}{C_2}$$

Parallel, you are adding Area



$$\frac{1}{R} = \frac{A}{\rho L}$$

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$C = \varepsilon\frac{A}{d}$$

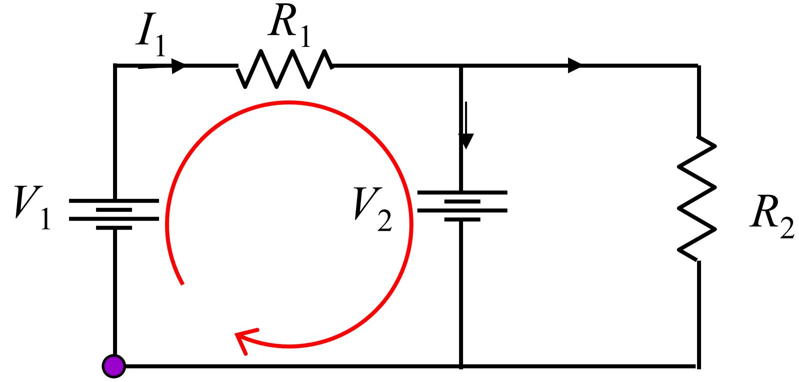
$$C_{tot} = C_1 + C_2$$

Kirchhoff's Voltage Rule

$$\sum \Delta V_i = 0$$

Kirchhoff's Voltage Rule states that the sum of the voltage changes caused by any elements (like wires, batteries, and resistors) around a circuit must be zero.

$$\sum \Delta V_i = -V_1 + IR_1 + V_2 = 0$$



WHY?

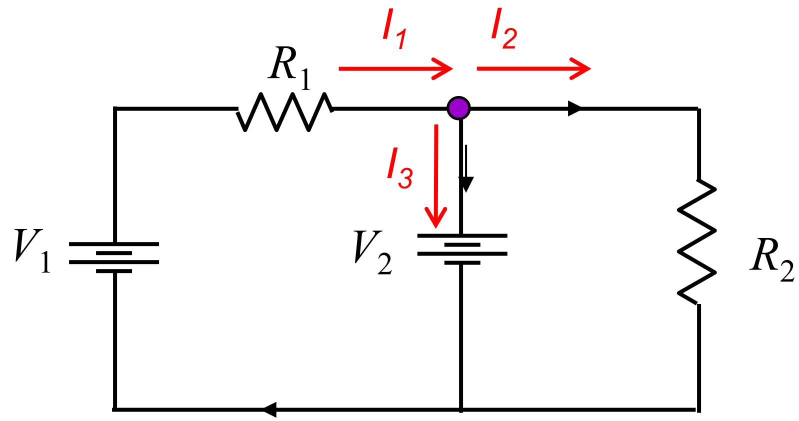
The potential difference between a point and itself is zero!

Kirchhoff's Current Rule

$$\sum I_{in} = \sum I_{out}$$

Kirchhoff's Current Rule states that the sum of all currents entering any given point in a circuit must equal the sum of all currents leaving the same point.

$I_1 = I_2 + I_3$



WHY?

Electric Charge is Conserved

Applying Kirchhoff's Laws in 5 easy steps

1) Label all currents

Choose any direction

2) Label +/– for all elements

Current goes + ⇒ – (for resistors)

Long side is + for battery

3) Choose loop and direction

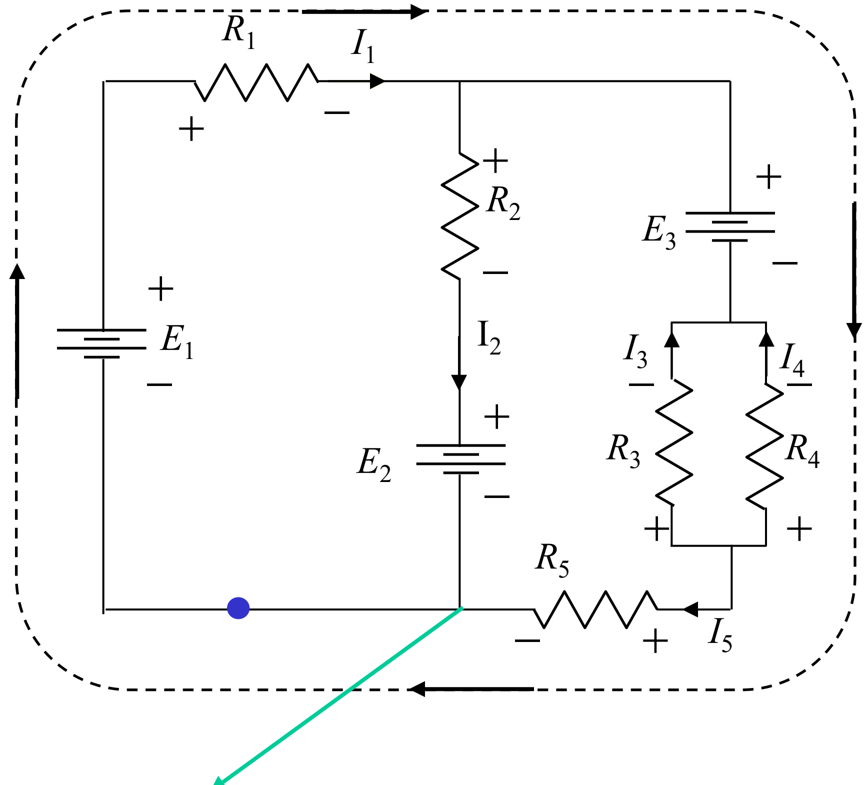
Must start on wire, not element.

4) Write down voltage drops

First sign you hit is sign to use.

5) Write down node equation $I_{\text{in}} = I_{\text{out}}$

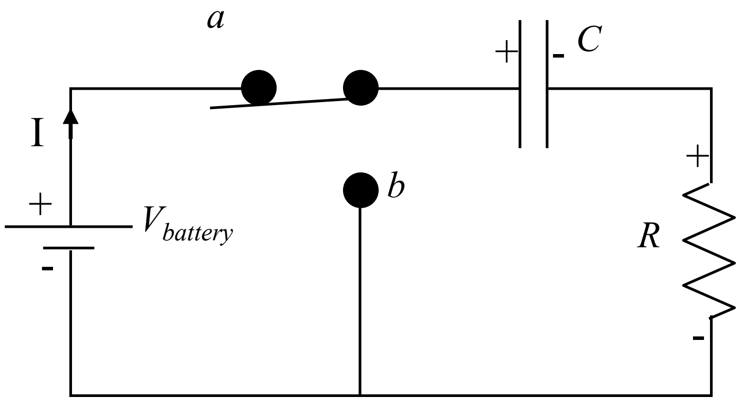
$-E_1 + I_1R_1 + E_3 - I_4R_4 + I_5R_5 = 0$



$I_2 + I_5 = I_1$

RC Circuit (Charging)

Capacitor uncharged, Switch is moved to position "a"



Kirchoff's Voltage Rule

$$-V_{battery} + \frac{q}{C} + IR = 0$$

Short Term $(q = q_0 = 0)$

$$-V_{battery} + 0 + I_0 R = 0$$

$$I_0 = \frac{V_{battery}}{R}$$

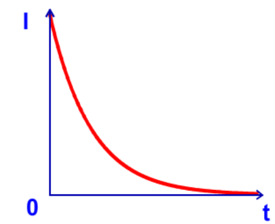
Long Term $(I_c = 0)$

$$-V_{battery} + \frac{q_\infty}{C} + 0 \cdot R = 0$$

$$q_\infty = CV_{battery}$$

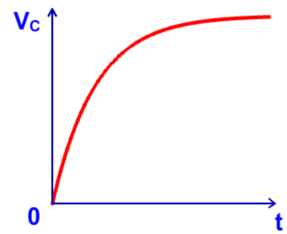
Intermediate

$$-V_{battery} + \frac{q}{C} + \frac{dq}{dt}R = 0$$



$$q(t) = q_\infty\left(1 - e^{-t/RC}\right)$$

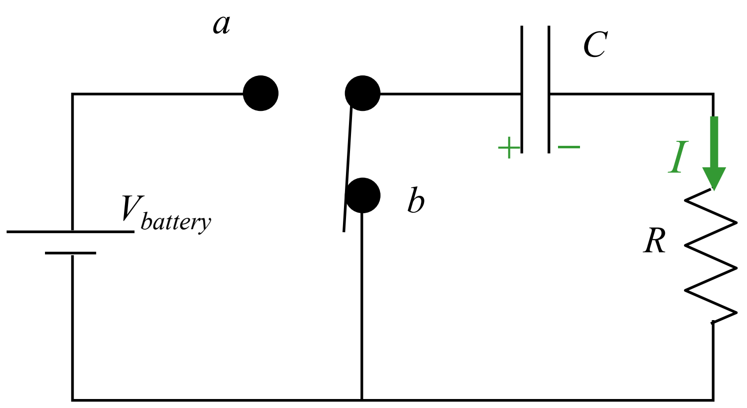
$$I(t) = \frac{dq}{dt} = I_0 e^{-t/RC}$$



RC Circuit (Discharging)

Capacitor has $q_0 = CV_{battery}$, Switch is moved to position "b"

Kirchoff's Voltage Rule



$$+\frac{q}{C} + IR = 0$$

Short Term $(q = q_0)$

$$V_{battery} + IR = 0$$

$$I_0 = \frac{-V_{battery}}{R}$$

Long Term $(I_c = 0)$

$$\frac{q_\infty}{C} + 0 \cdot R = 0$$

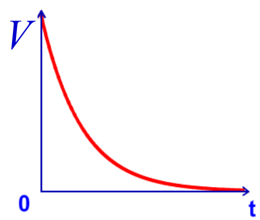
$$q_\infty = 0$$

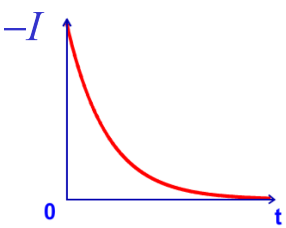
Intermediate

$$+\frac{q}{C} + \frac{dq}{dt}R = 0$$

$$q(t) = q_0 e^{-t/RC}$$

$$I(t) = I_0 e^{-t/RC}$$





Magnetism & Moving Charges

All observations are explained by two equations:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Today

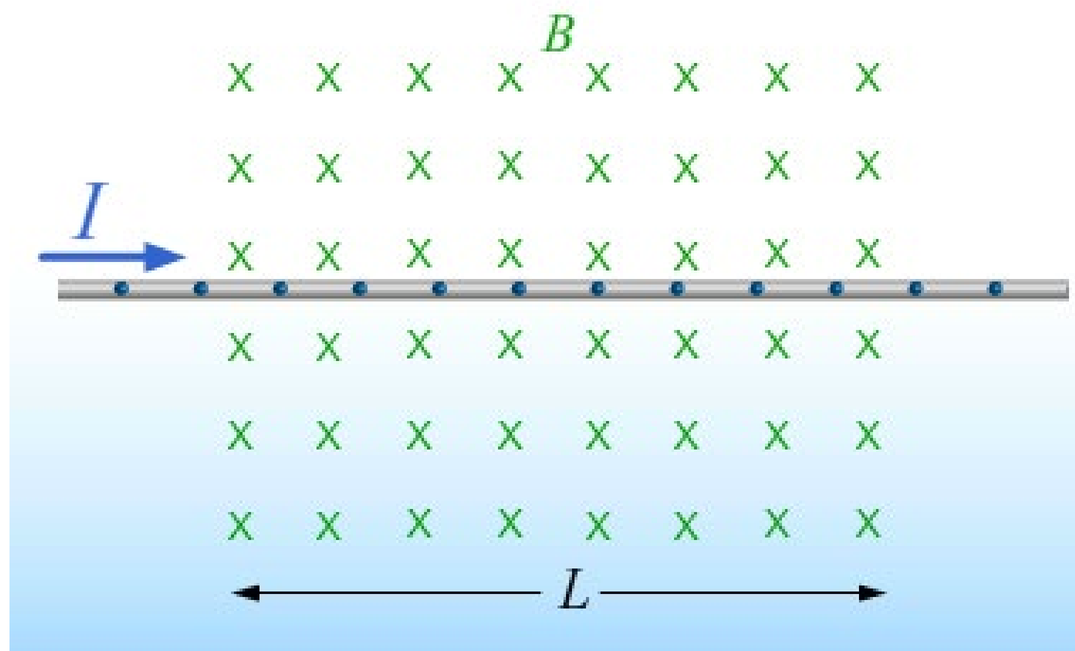
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times r}{r^2}$$

Next Week

Magnetism & Moving Charges

Last Time:

$$\vec{F} = q\vec{v} \times \vec{B}$$

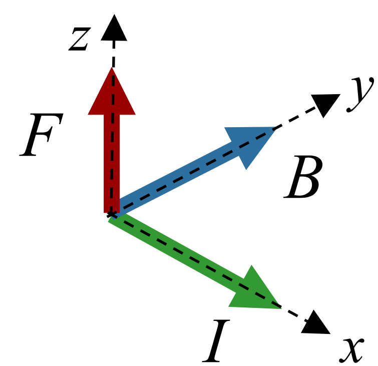


This Time:

$$\vec{F} = q\sum_i \vec{v}_i \times \vec{B}$$

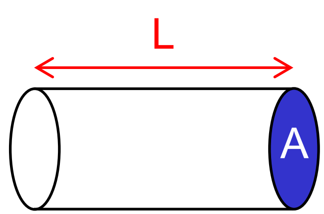
$$\vec{F} = qN\vec{v}_{avg} \times \vec{B}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$



$$N = nAL$$

$$I = qnAv_{avg}$$



N: total charge carriers
n = N / volume: charge carriers per unit volume

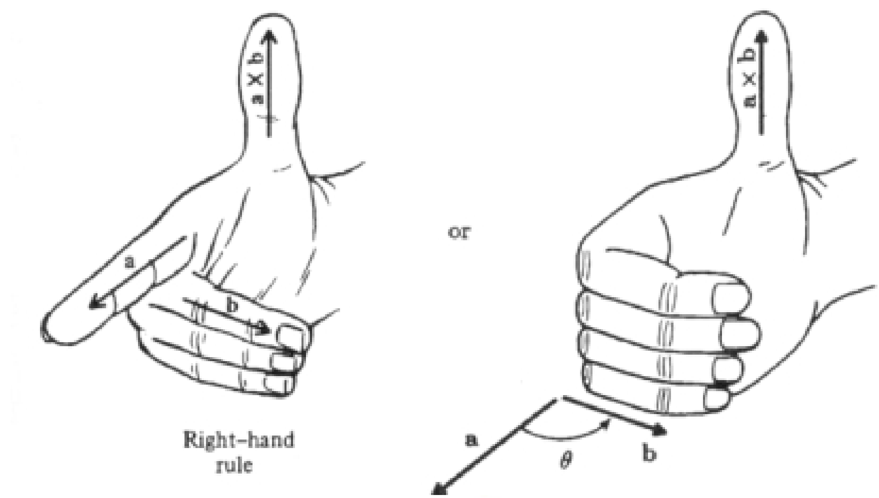
Right Hand Rule Review

1. ANY CROSS PRODUCT

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = I\vec{L} \times \vec{B}$$

$$\tau = \vec{r} \times \vec{F} \qquad \tau = \vec{\mu} \times \vec{B}$$

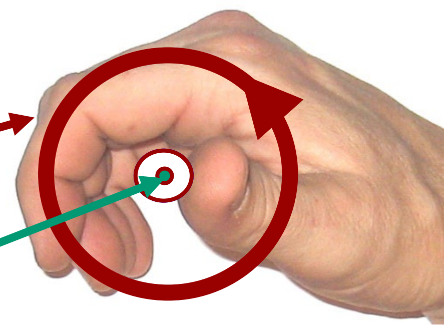
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times r}{r^2}$$



2. Direction of Magnetic Moment

Fingers: Current in Loop

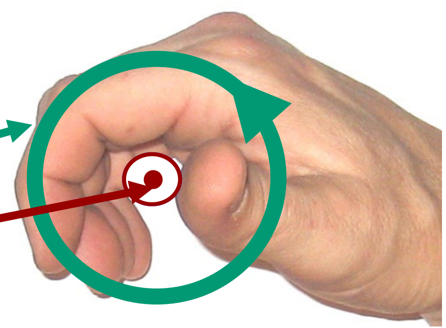
Thumb: Magnetic Moment



3. Direction of Magnetic Field from Wire

Fingers: Magnetic Field

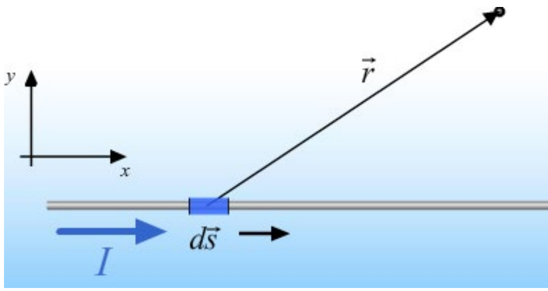
Thumb: Current



Biot-Savart Law:

What is it?

Fundamental law for determining the direction and magnitude of the magnetic field due to an element of current

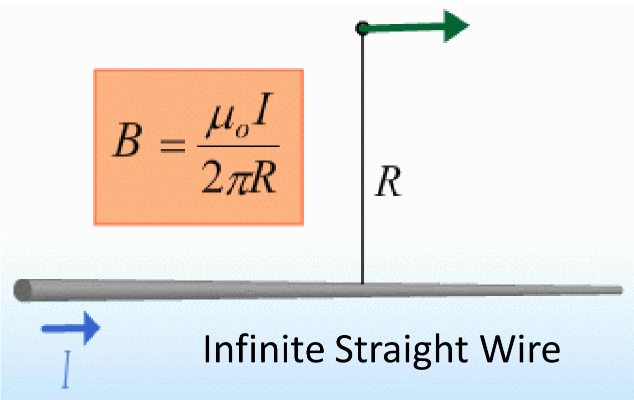


$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$

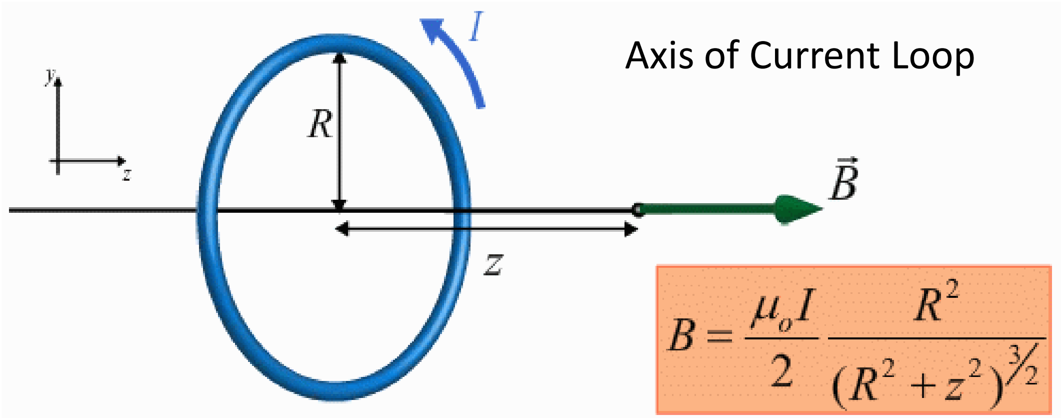
We can use this law to calculate the magnetic field produced by ANY current distribution

BUT

Easy analytic calculations are possible only for a few distributions:



$$B = \frac{\mu_o I}{2\pi R}$$



$$B = \frac{\mu_o I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$$

Plan for Today: Mainly use the results of these calculations!

GOOD NEWS: Remember Gauss' Law? Allowed us to calculate E for symmetrical charge distributions

NEXT TIME: Introduce Ampere's Law Allows us to calculate B for symmetrical current distributions

B from Infinite Line of Current

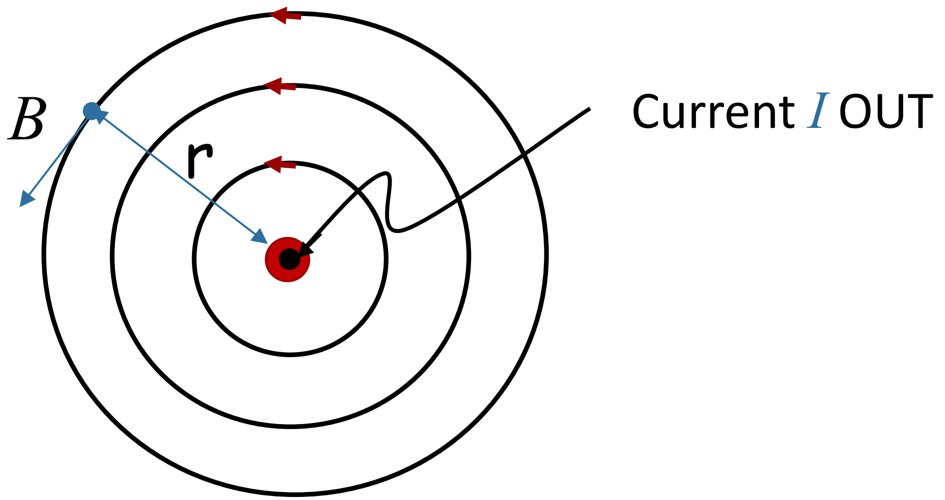
Integrating $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s} \times r}{r^2}$ gives result

Magnitude:

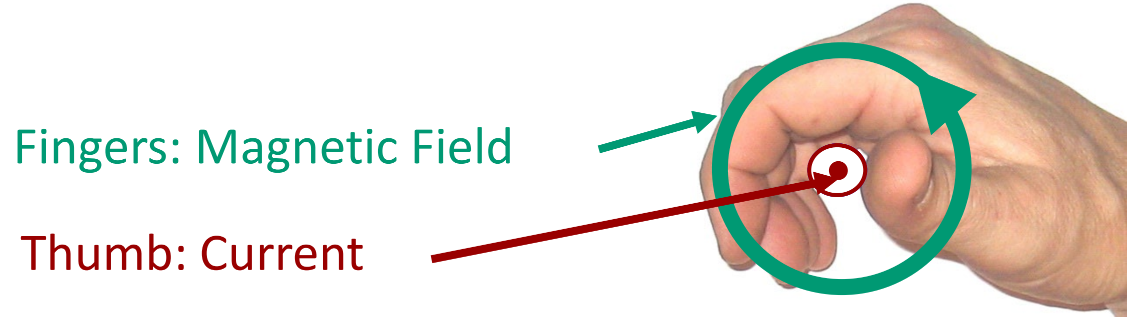
$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mu_0 = 4\pi \times 10^{-7}\, Tm/A$$

$r =$ distance from wire

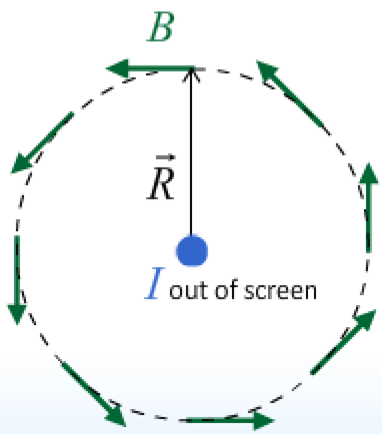


Direction:



Ampere's Law

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_o I}$$



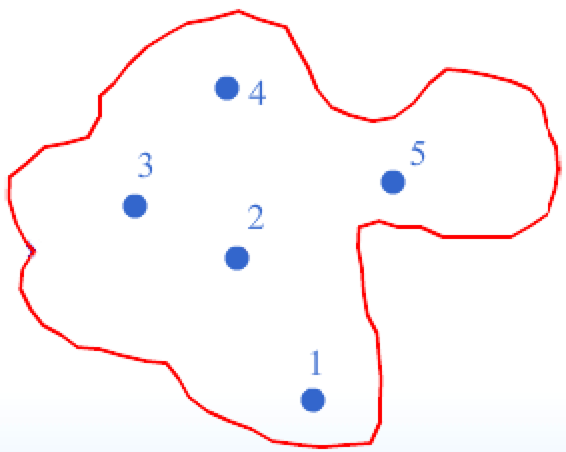
Infinite current-carrying wire

LHS: $\oint \vec{B} \bullet d\vec{\ell} = \oint B d\ell = B \oint d\ell = B \cdot 2\pi R$

RHS: $I_{enclosed} = I$

$$\longrightarrow \quad B = \frac{\mu_o I}{2\pi R}$$

General Case



Ampere's Law

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}}$$

Faraday's Law

Faraday's Law: $\boxed{emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$ where $\boxed{\Phi_B \equiv \int \vec{B} \cdot d\vec{A}}$

Looks scary but it's not – its amazing and beautiful!

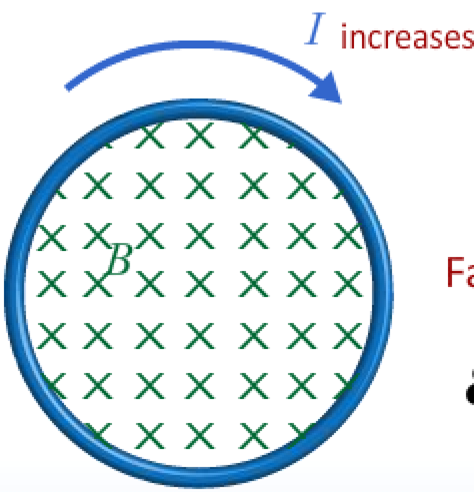
↓

A changing magnetic flux produces an electric field.

↓

Electricity and magnetism are deeply connected.

From the Prelecture: Self Inductance



Self-Inductance

$$L \equiv \frac{\Phi_B}{I}$$

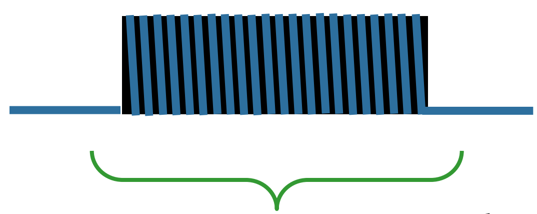
SI Unit

H = T-m²/A

Faraday's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

Wrap a wire into a coil to make an "inductor"…



$$\mathcal{E} = -L\frac{dI}{dt}$$

Energy in an inductor

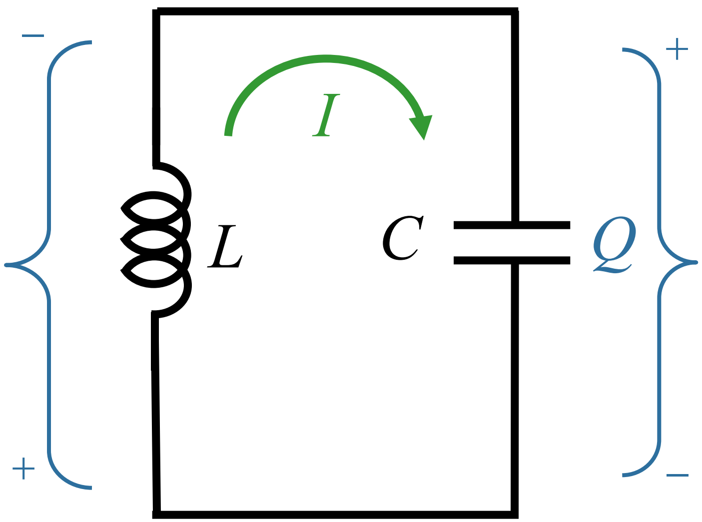
$$V = L\,\frac{dI}{dt}$$

$$P = IV = IL\frac{dI}{dt}$$

$$\text{Energy} = \int_0^t P\,dt = \int_0^I IL\,dI = \frac{1}{2}LI^2$$

LC Circuit

$$V_L = L\frac{dI}{dt}$$



$$V_C = \frac{Q}{C}$$

KVL Circuit Equation: $$\frac{Q}{C} + L\frac{dI}{dt} = 0$$

$$I = \frac{dQ}{dt} \rightarrow \frac{d^2Q}{dt^2} = -\frac{Q}{LC} \rightarrow \frac{d^2Q}{dt^2} = -\omega^2 Q$$

where

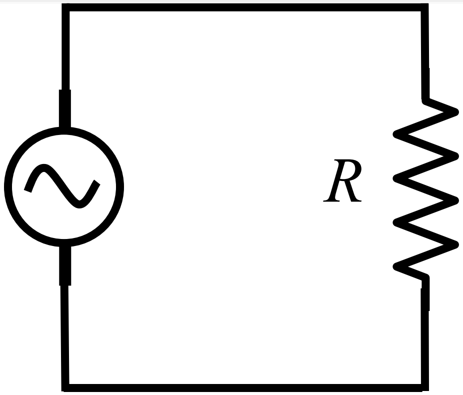
$$\omega = \frac{1}{\sqrt{LC}}$$

Solution:

$$Q(t) = Q_{max}\cos(\omega t + \phi)$$

$$I(t) = -\omega Q_{max}\sin(\omega t + \phi)$$

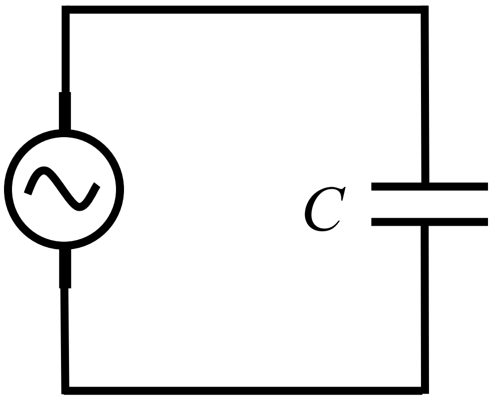
Summary



$$I_{max} = V_{max}/R$$

V_R in phase with I

Because resistors are simple



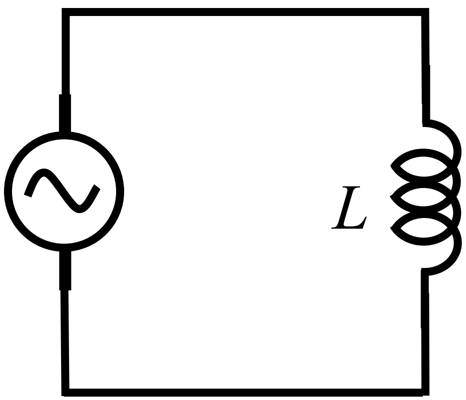
$$I_{max} = V_{max}/X_C$$

$$X_C = 1/\omega C$$

V_C 90^{o} behind I

Current comes first since it charges capacitor

Like a wire at high ω



$$I_{max} = V_{max}/X_L$$

$$X_L = \omega L$$

V_L 90^{o} ahead of I

Opposite of capacitor

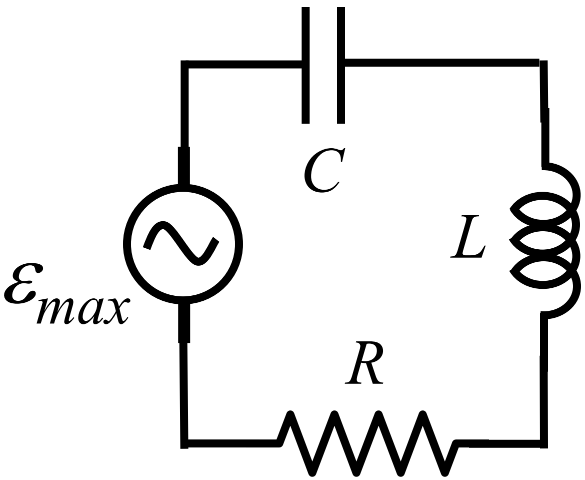
Like a wire at low ω

Driven RLC Circuit

Makes sense to write everything in terms of I since this is the same everywhere in a one-loop circuit:

$$V_{max} = I_{max} X_C$$

V_C 90° behind I



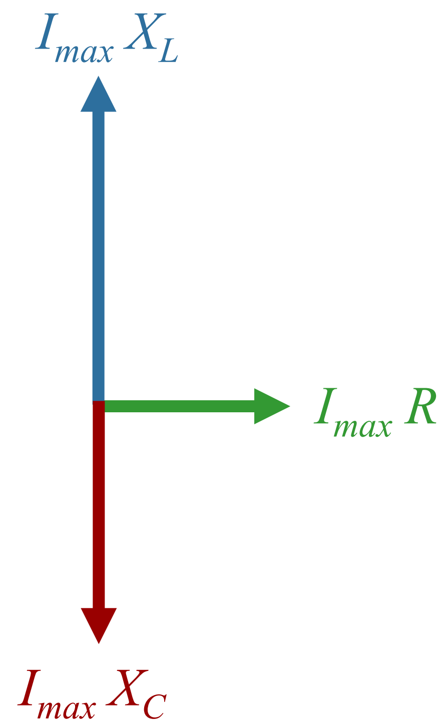
$$V_{max} = I_{max} X_L$$

V_L 90° ahead of I

$$V_{max} = I_{max} R$$

V_R in phase with I

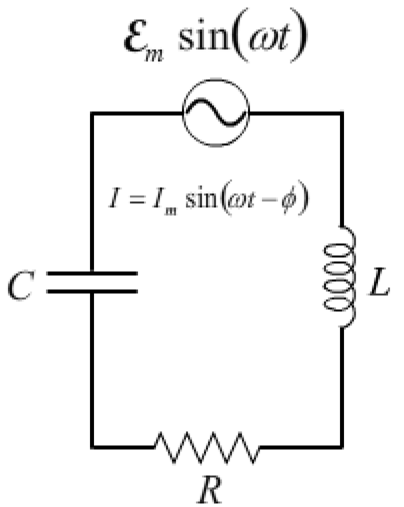
Phasors make this simple to see



L,R,C always looks the same,
Only the lengths will change

Looks intimidating, but is OK w/ practice

The Driven LCR Circuit



Frequency Dependence of Maximum Current

$$I_m = \frac{\mathcal{E}_m}{R} \frac{1}{\sqrt{1 + Q^2 \frac{(x^2-1)^2}{x^2}}}$$

Average Power per Cycle

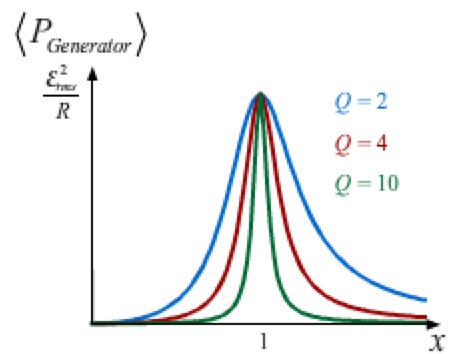
$$\langle P_{Generator} \rangle = \frac{\mathcal{E}_{rms}^2}{R} \frac{x^2}{x^2 + Q^2 (x^2-1)^2}$$

where $x \equiv \frac{\omega}{\omega_o}$ & $Q^2 = \frac{L}{R^2 C}$

Quality Factor

$$Q \equiv 2\pi \left[\frac{U_{max}}{\Delta U} \right]_{cycle}$$

evaluate at $\omega = \omega_o$



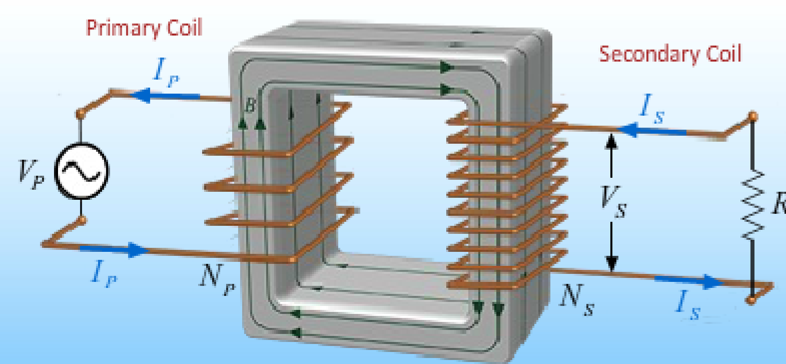
Transformers

Voltage Relation

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Current Relation

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$



Resonance

Frequency at which voltage across inductor and capacitor cancel

R is independent of ω

X_L increases with ω

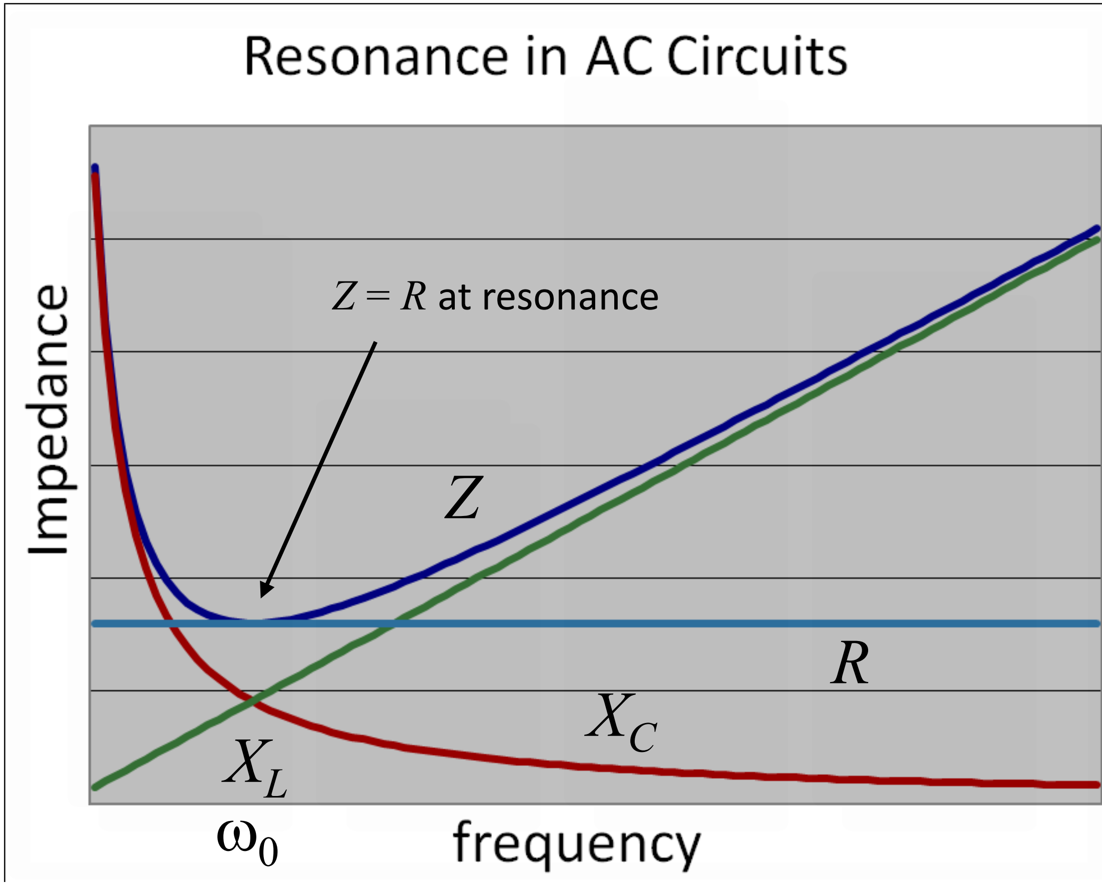
$$X_L = \omega L$$

X_C increases with $1/\omega$

$$X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

is minimum at resonance

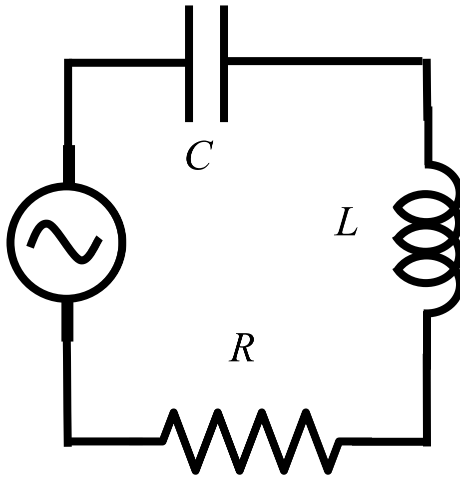


Resonance: $X_L = X_C$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

Power

$P = IV$ instantaneous always true

- Difficult for Generator, Inductor and Capacitor because of phase
- Resistor I,V are always in phase!



Average power

$$P_{\text{ave}} = \frac{1}{T}\int_0^T p(t)dt$$

where $T = 2\pi/\omega$ is the period of the oscillations. With the substitutions $v(t) = V_0 \sin \omega t$ and $i(t) = I_0 \sin(\omega t - \phi)$, this integral becomes

$$P_{\text{ave}} = \frac{I_0 V_0}{T}\int_0^T \sin(\omega t - \phi)\sin \omega t\, dt$$

Using the trigonometric relation $\sin(A - B) = \sin A \cos B - \sin B \cos A$, we obtain

$$P_{\text{ave}} = \frac{I_0 V_0 \cos\phi}{T}\int_0^T \sin^2 \omega t dt - \frac{I_0 V_0 \sin\phi}{T}\int_0^T \sin \omega t \cos \omega t dt$$

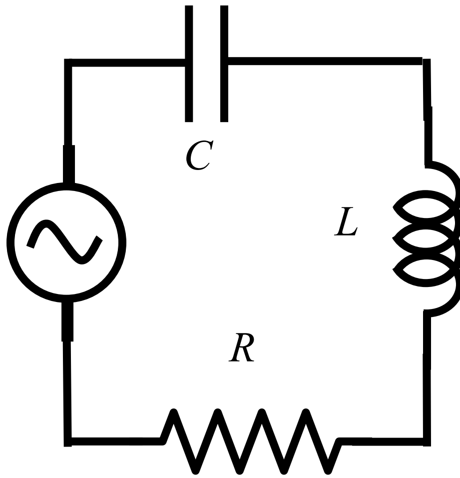
$$\frac{1}{T}\int_0^T \sin^2 \omega t dt = \frac{1}{2} \qquad \frac{1}{T}\int_0^T \sin \omega t \cos \omega t dt = 0$$

$$\boxed{P_{\text{ave}} = \frac{1}{2} I_0 V_0 \cos\phi}$$

Average Power

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0 \cos\phi$$

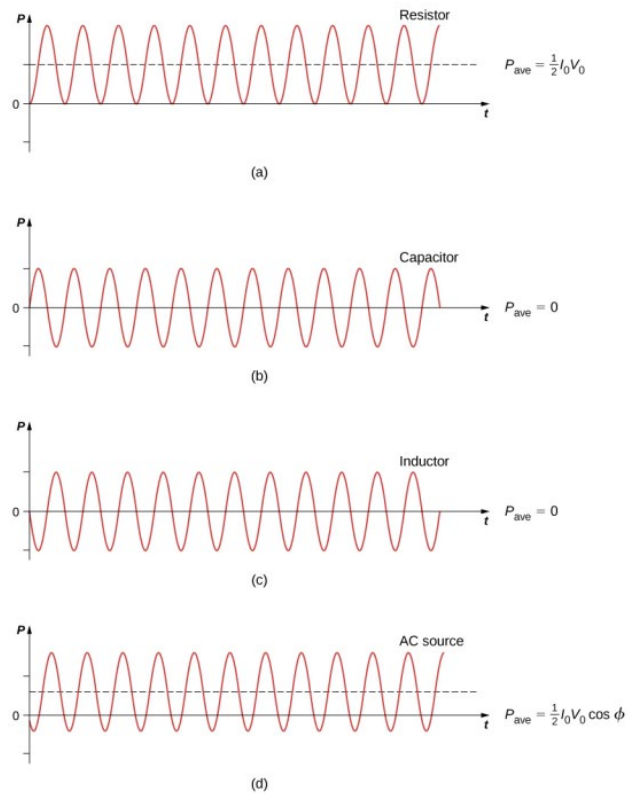
$\cos\phi$ is known as the **power factor**



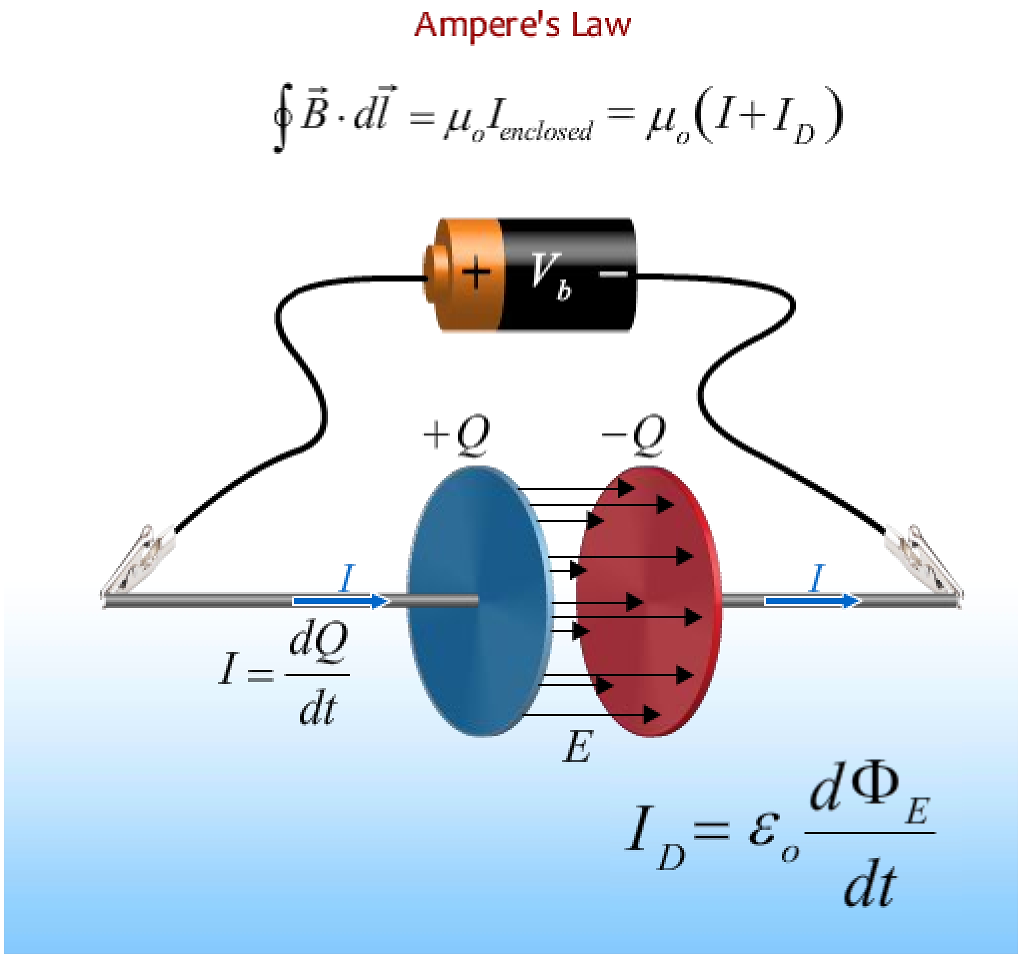
For a resistor, $\phi = 0$, so the average power dissipated is

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0$$

For capacitor or inductor, $P_{\text{ave}} = 0$!



Modify Ampere's Law



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$\downarrow$$

$$\Phi = EA = \frac{Q}{\varepsilon_0}$$

$$\downarrow$$

$$Q = \varepsilon_0 \Phi$$

$$\downarrow$$

$$\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi}{dt} \equiv I_D$$

The **displacement current** is analogous to a real current in Ampère's law, entering into Ampère's law in the same way. It is produced, however, by a changing electric field. It accounts for a changing electric field producing a magnetic field, just as a real current does, but the displacement current can produce a magnetic field even where no real current is present.

Displacement Current

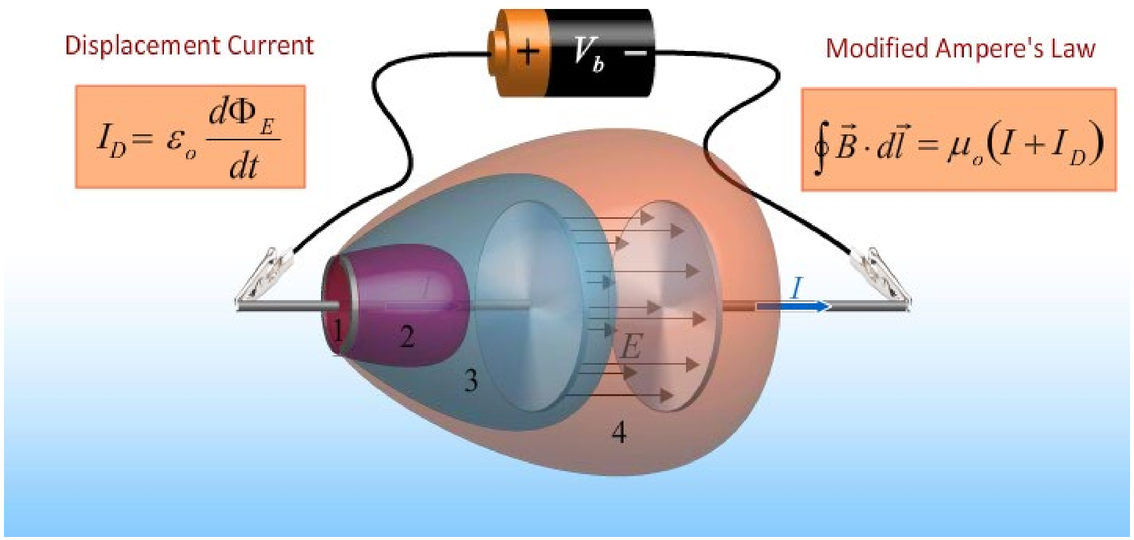
Real Current: Charge Q passes through area A in time t:

$$I = \frac{dQ}{dt}$$

Displacement Current: Electric flux through area A' changes in time

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

DISPLACEMENT CURRENT and EM WAVES



Faraday's Law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

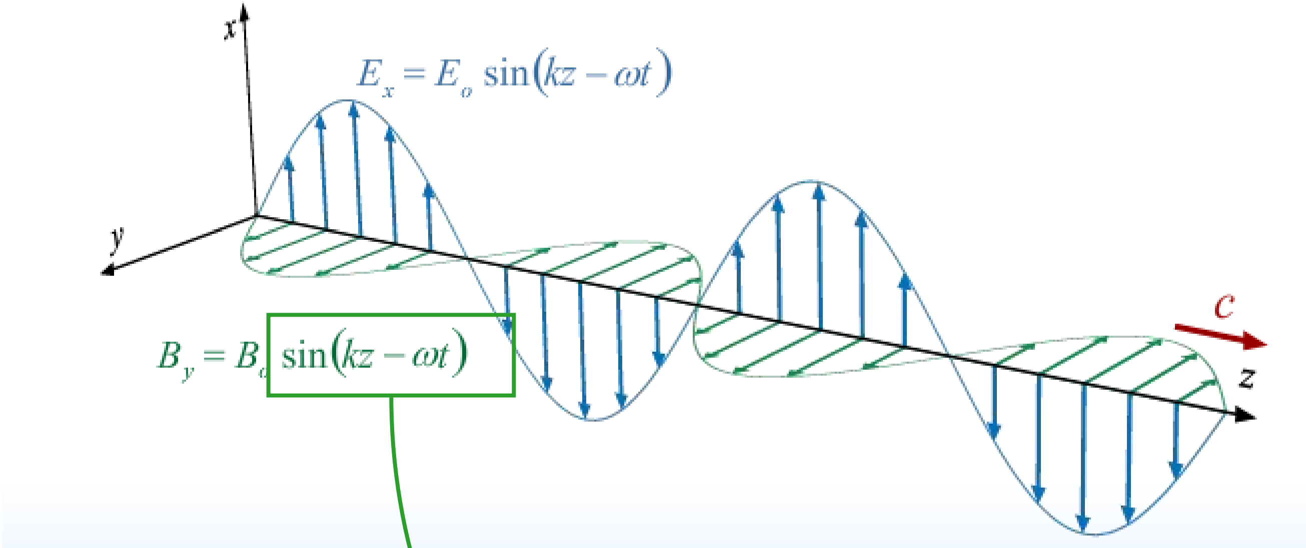
Modified Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_o \varepsilon_o \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

Free space

Plane Waves

Velocity $c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = \frac{E_o}{B_o} = 3\times 10^8 \text{ m/s}$



E and B are perpendicular and in phase

Oscillate in time and space

Direction of propagation given by E X B

$E_0 = cB_0$

Argument of sin/cos gives direction of propagation

Waves Carry Energy

Total Energy Density

$$u = \varepsilon_o E^2$$

Average Energy Density

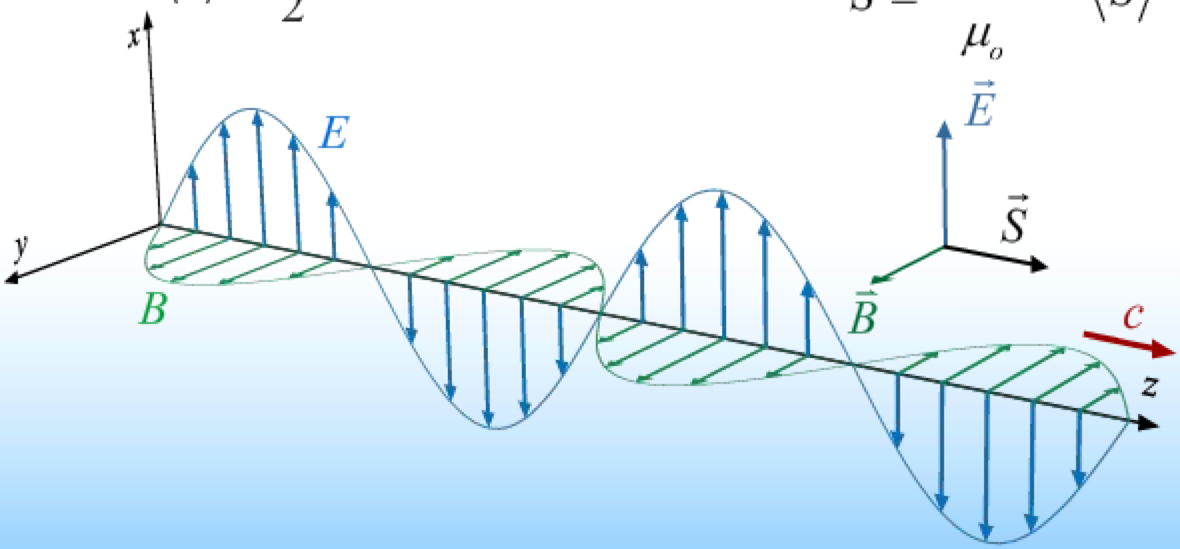
$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\ 2}$$

Intensity

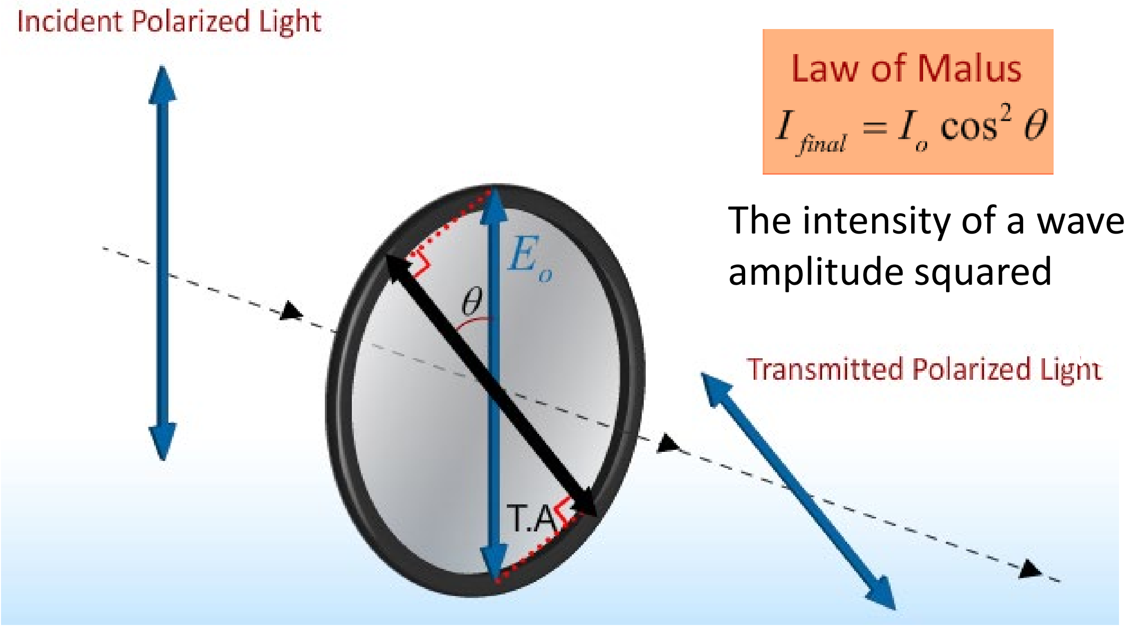
$$I = \frac{1}{2} c\,\varepsilon_o E_o^{\ 2} = c\,\langle u \rangle$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E}\times\vec{B}}{\mu_o} \qquad \langle S \rangle = I$$



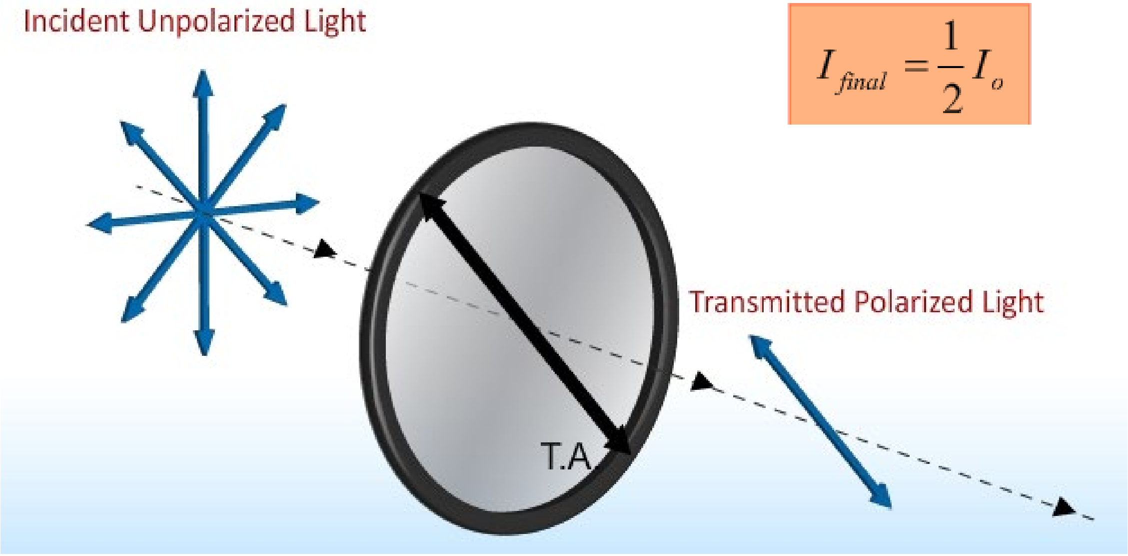
Linear Polarizers



Law of Malus

$$I_{final} = I_o \cos^2 \theta$$

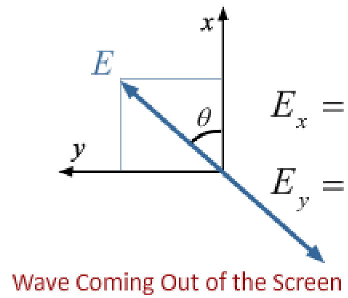
The intensity of a wave is proportional to its amplitude squared



$$I_{final} = \frac{1}{2} I_o$$

Circularly Polarized Light

There is no reason that ϕ has to be the same for E_x and E_y:



$$E_x = E_o \cos\theta \sin(kz - \omega t + \phi_x)$$
$$E_y = E_o \sin\theta \sin(kz - \omega t + \phi_y)$$

Satisfies Wave Equation

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$
$$\frac{\partial^2 E_y}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

Making ϕ_x different from ϕ_y causes circular or elliptical polarization:

Example:

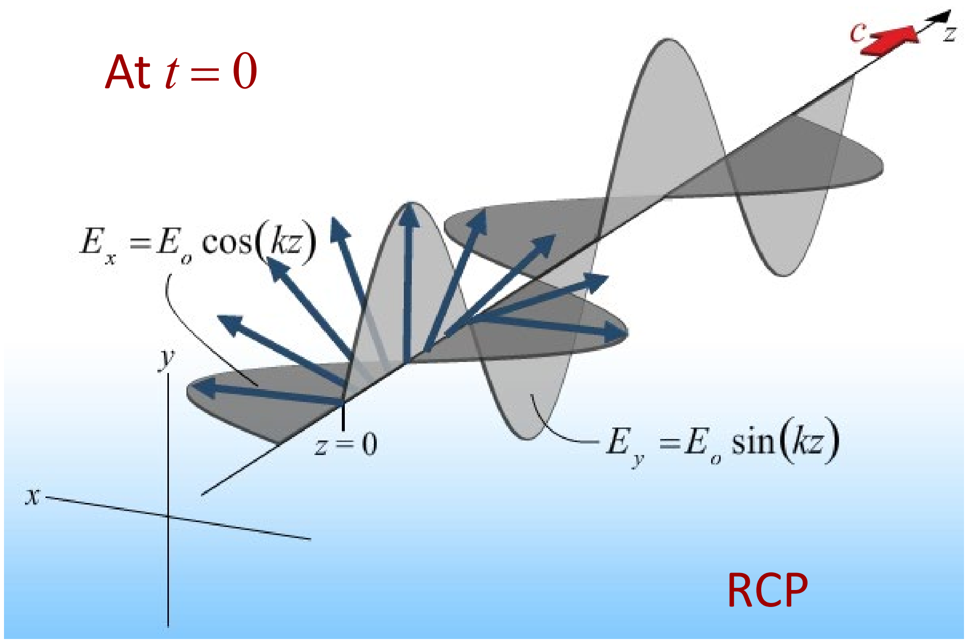
$$\phi_x - \phi_y = 90^\circ = \frac{\pi}{2}$$

$$\theta = 45^\circ = \pi/4$$

$$E_x = \frac{E_0}{\sqrt{2}}\cos(kz - \omega t)$$

$$E_y = \frac{E_0}{\sqrt{2}}\sin(kz - \omega t)$$

At $t = 0$

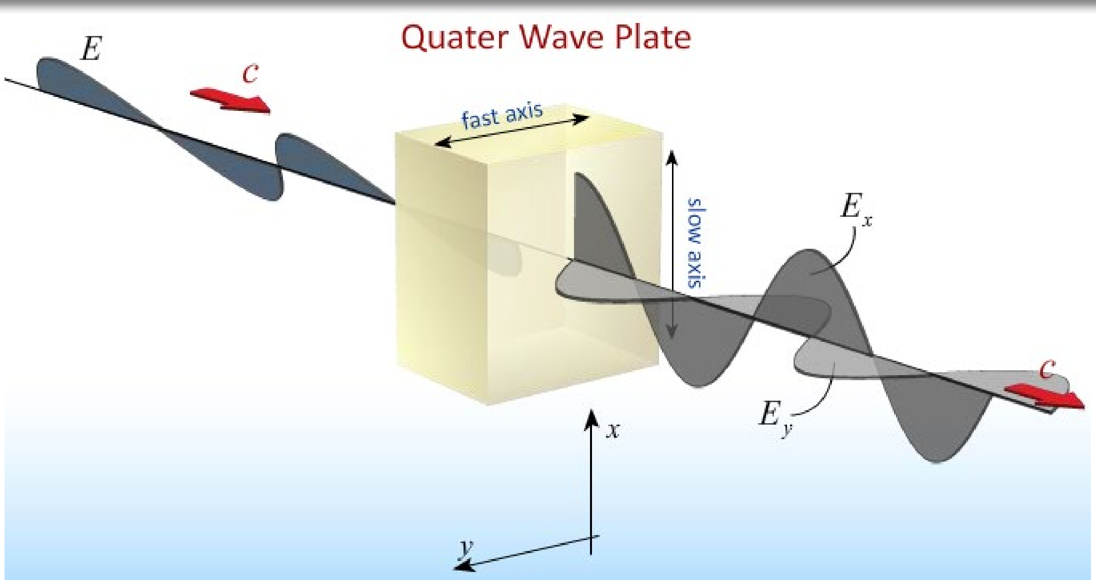


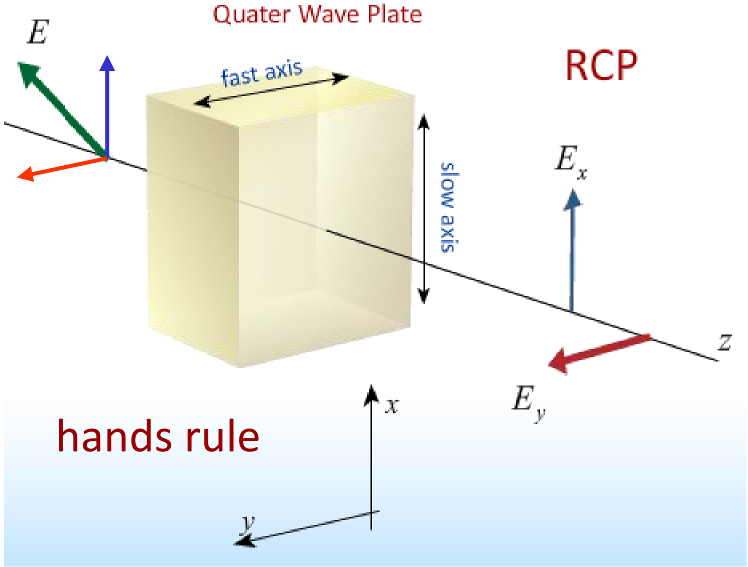
(Right-Circularly Polarized)

Quarter Waveplates

Q: How do we change the relative phase between E_x and E_y?

A: Birefringence



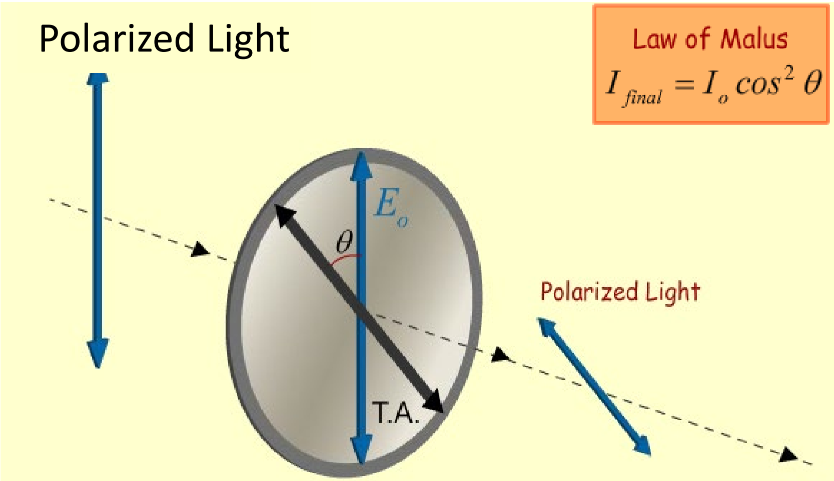


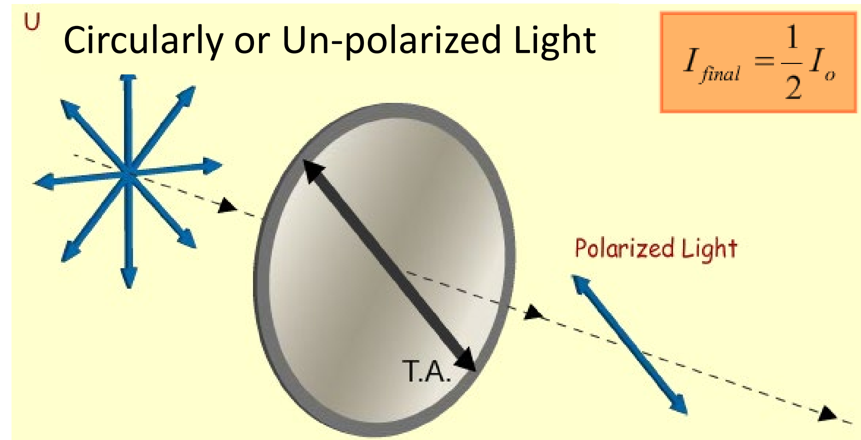
By picking the right thickness we can change the relative phase by exactly 90^o.

This changes linear (if @45°) to circular polarization and is called a *quarter wave plate*

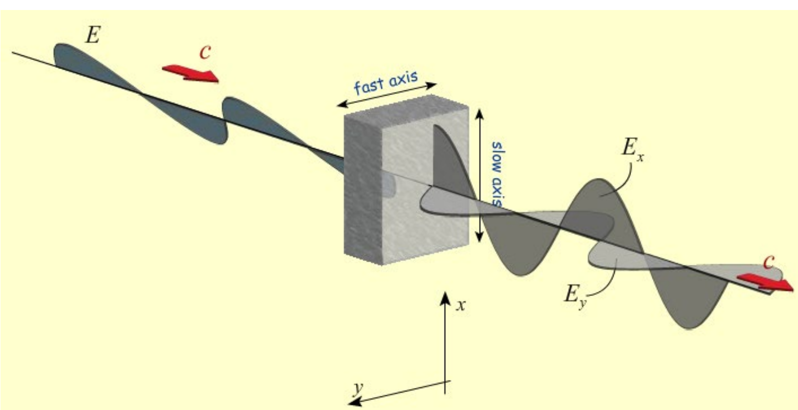
Summary:

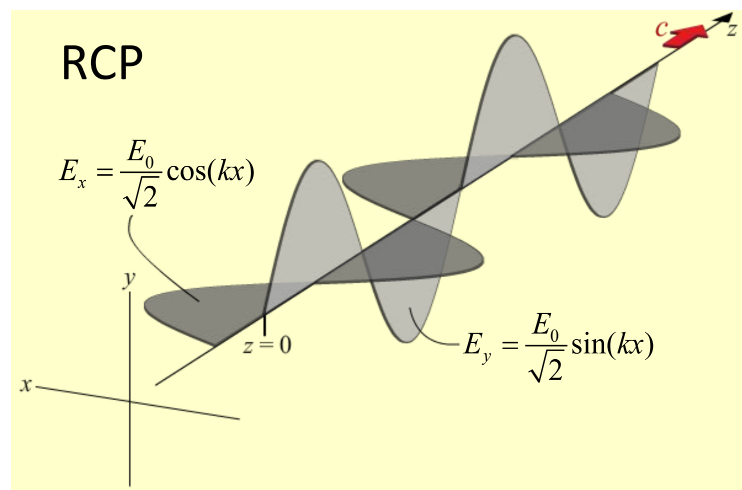
Polarizers & QW Plates:



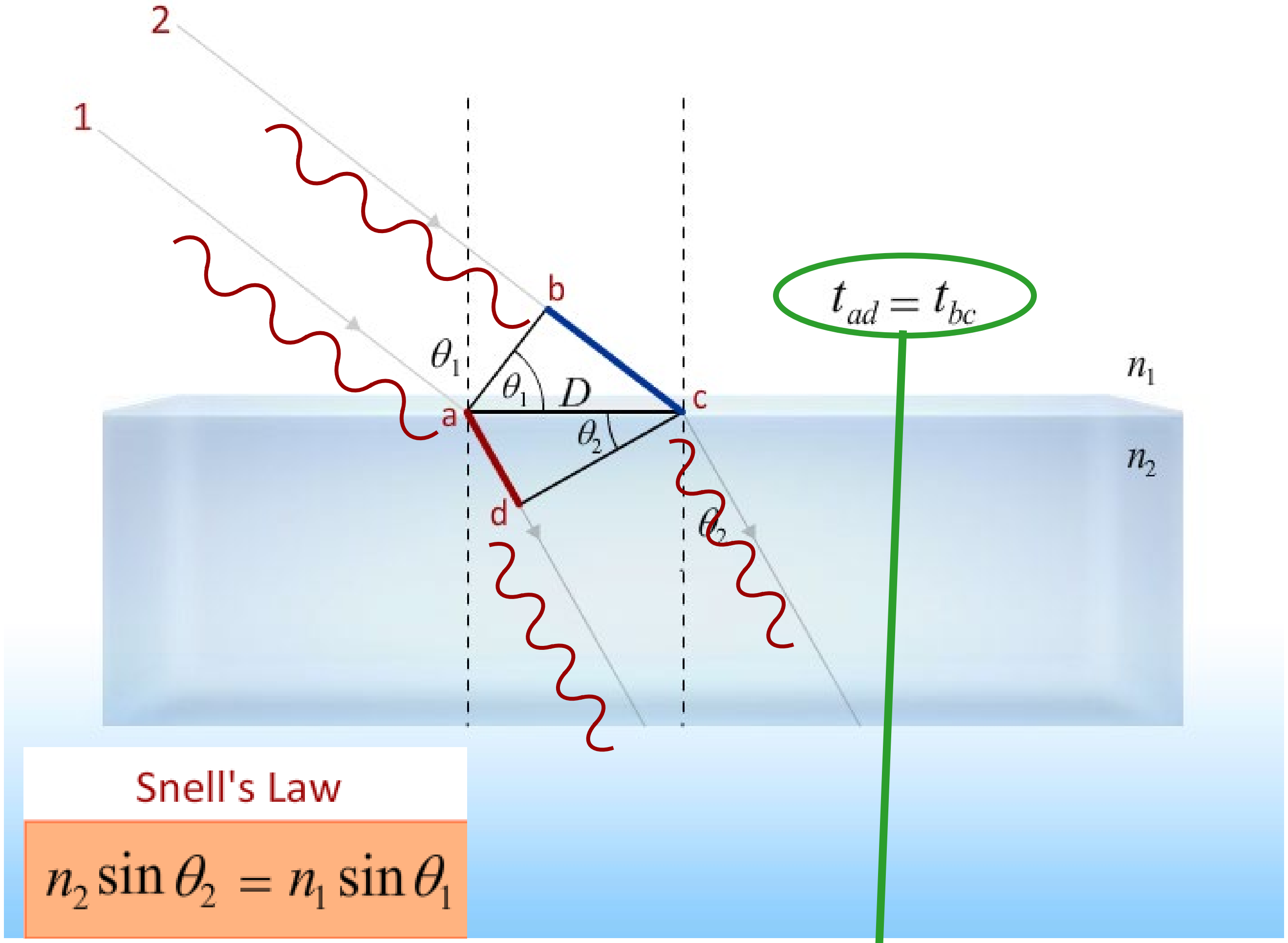


Birefringence



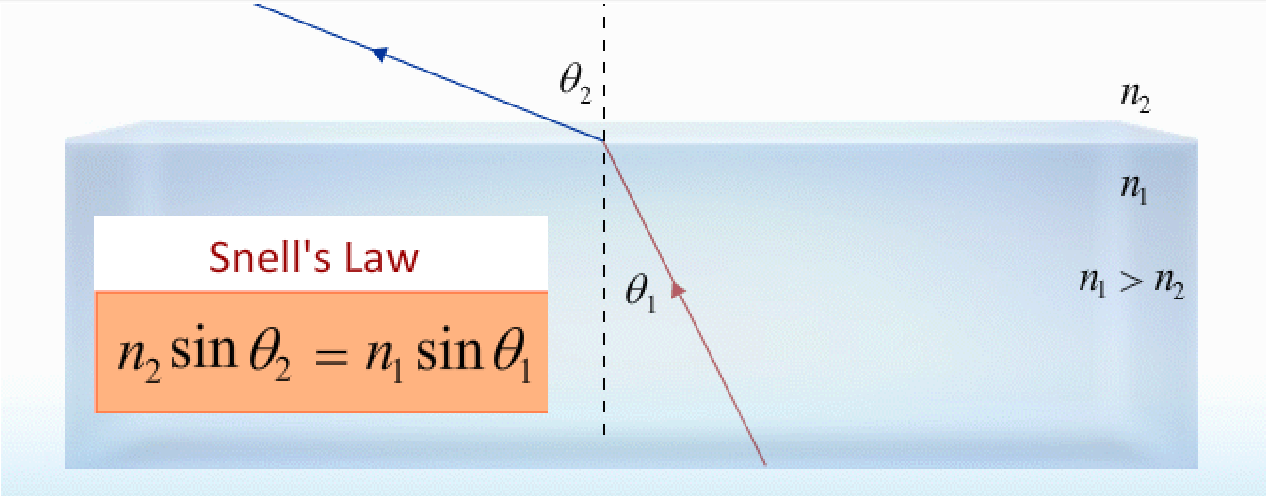


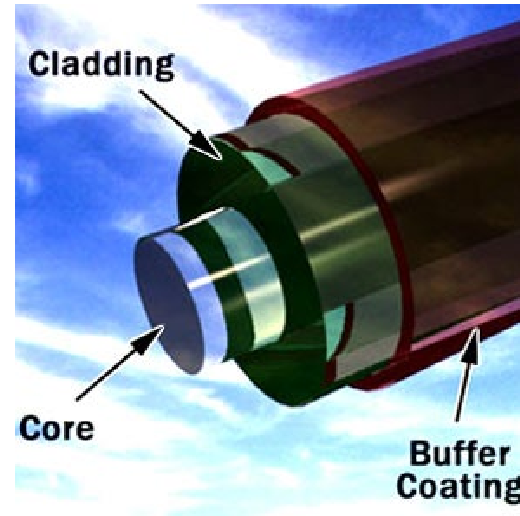
Refraction: Snell's Law



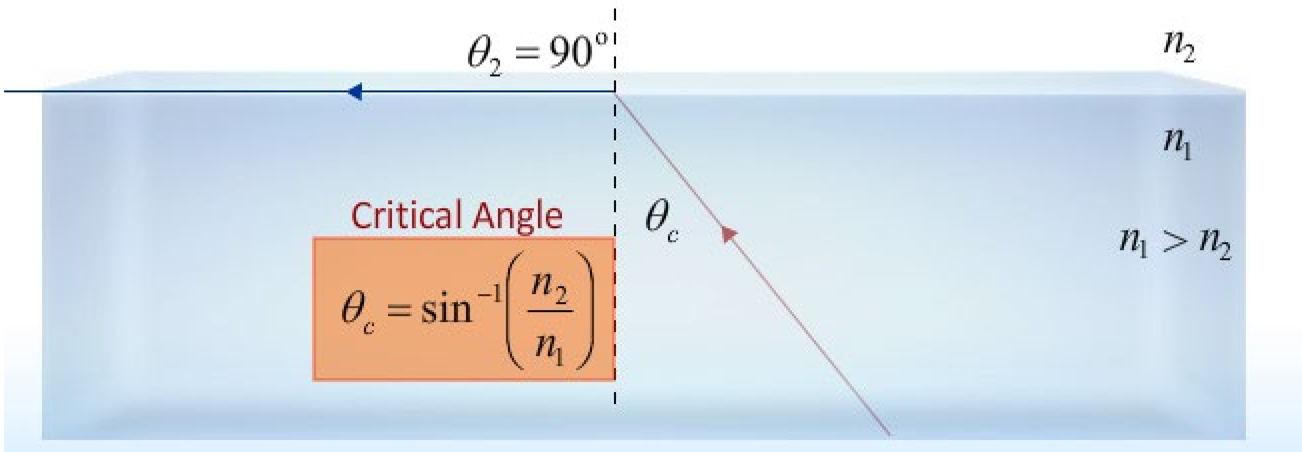
$$\frac{D\sin\theta_2}{c/n_2} = \frac{D\sin\theta_1}{c/n_1} \rightarrow n_2 \sin\theta_2 = n_1 \sin\theta_1$$

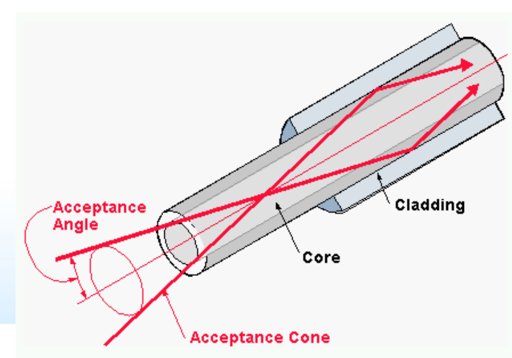
Total Internal Reflection





NOTE: $n_1 > n_2$ implies $\theta_2 > \theta_1$ BUT: θ_2 has max value = 90^o!





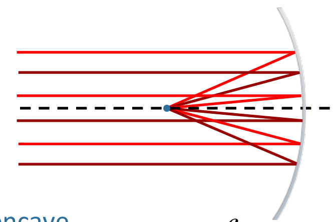
$\theta_1 > \theta_c$ → Total Internal Reflection

Summary - Mirrors & Lenses:

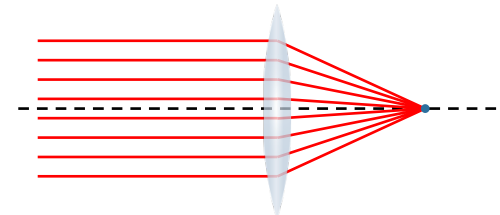
$S > 2f$ — real, inverted, smaller

$2f > S > f$ — real, inverted, bigger

$f > 0$



Concave (Converging) $\leftarrow f \rightarrow$



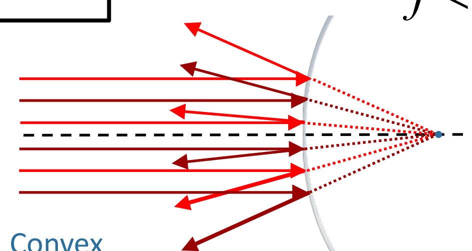
Converging $\leftarrow f \rightarrow$

$f > S > 0$ — virtual, upright, bigger

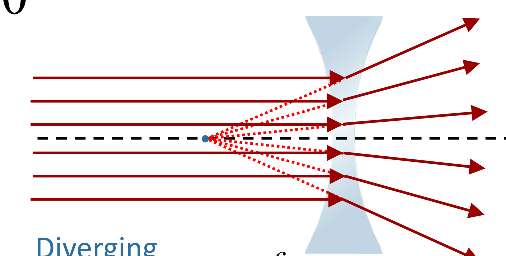
$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \qquad M = -\frac{S'}{S}$$

$S > 0$ — virtual, upright, smaller

$f < 0$



Convex (diverging) $\leftarrow f \rightarrow$



Diverging $\leftarrow f \rightarrow$

It's Always the Same:

$$\frac{1}{S}+\frac{1}{S'}=\frac{1}{f} \qquad M=-\frac{S'}{S}$$

You just have to keep the signs straight:

s' is positive for a real image

f is positive when focus is on side light "goes to" downstream

Upstream = Side light comes from
Downstream = Side light goes to

Lens sign conventions

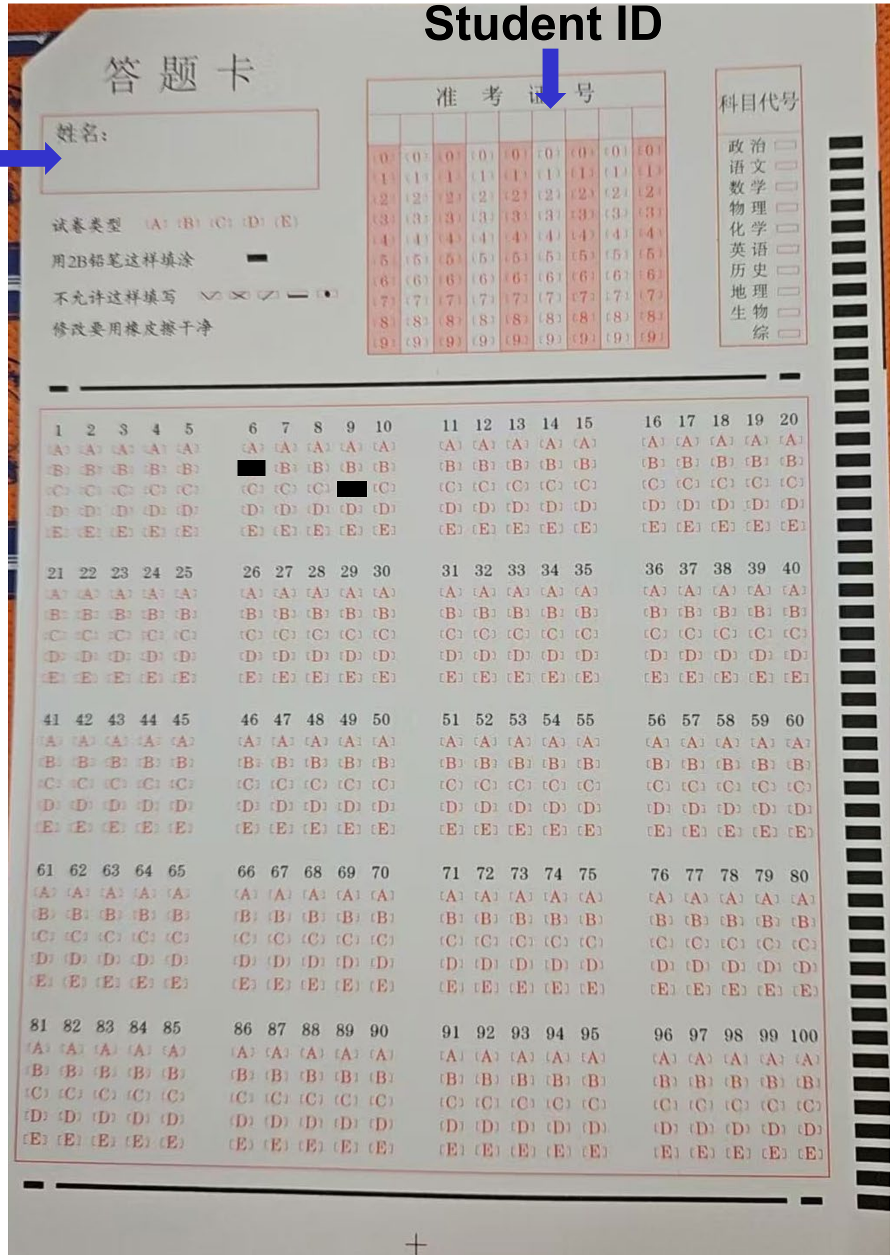
S: positive if object is "upstream" of lens
S' : positive if image is "downstream" of lens
f: positive if converging lens

Mirrors sign conventions

S: positive if object is "upstream" of mirror
S' : positive if image is "downstream" of mirror
f: positive if converging mirror (concave)

Answer Sheet

Name



Please bring a **pencil (2B)**, an eraser, a calculator to the exam

Good Luck!