Data Science - Assignment 2

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The following tasks should be solved, individually or in groups of 2. Write the answers to the questions in a readme. When reviewing, check that you got the same answers.

Vector Basics

Solve the following *by hand*.

(a) A point is given in a coordinate system: $B = (7, 8)$.
What is the vector going from $(0, 0)$ to B?

A vector between two points can be found by subtracting one point from the other. E.g. the vector going from Copenhagen (C) to Kolding (K) would be found by $K - C$.

(b) Two points, A and B, in a coordinate system are given by
$A = (4, 3)$, $B = (7, 8)$.
The vector $\vec{a}$ goes from A to B. What is $\vec{a}$?

(c) Given vector $\vec{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, what is its magnitude $|\vec{a}|$ (its length)?

Two guys are pulling a boat on a river, each on their side, as seen on Figure 1. The pulls from our heroes are given as $\vec{a}$ and $\vec{b}$ respectively.

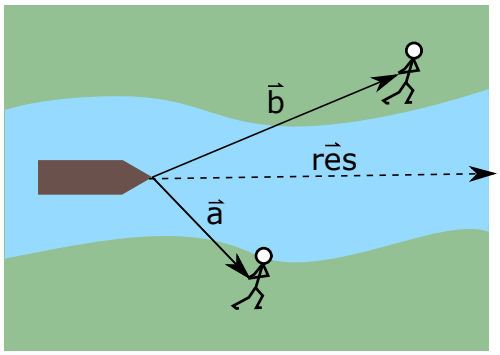

Figure 1: Two bachelor students pulling a boat

$$\vec{a} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \vec{b} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

(d) What is the resulting vector of their combined pull $r\vec{e}s$?

(e) Given three vectors, $\vec{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \vec{b} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \vec{c} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$ calculate

1. $\vec{a} + \vec{b}$
2. $\vec{b} + \vec{c}$
3. $(\vec{a} + \vec{b}) + \vec{c}$
4. $\vec{a} + (\vec{b} + \vec{c})$
5. $\vec{b} + \vec{a}$

Vector Decomposition

(f) In Figure 3, what is the x-component of the force applied?
What is the y-component?

(g) In Figure 3, what is the vector of the force $\vec{F}$?

(h) Having found the x-component (let's call it F_x) and the y-component (let's call it F_y), find $\sqrt{F_x^2 + F_y^2}$.

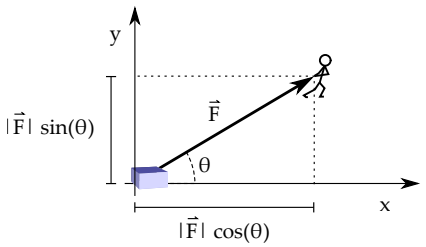

Figure 2: A vector can be decomposed into perpendicular vectors.

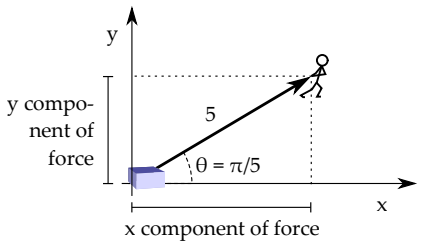

Figure 3: Find the x and y components.

Multiplication of a Vector with a Scalar

When multiplying with a scalar, each component is multiplied with the scalar.

(i) Given $\vec{a} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$ and $\vec{b} = 5\vec{a}$, what is $\vec{b}$?

(j) What is the length of $\vec{a}$?

(k) What is the length of $\vec{b}$?

(l) What is the result of $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot 4$?

(m) What is the result of $\begin{pmatrix} 1 \\ 3 \end{pmatrix} \cdot \frac{1}{2}$?

Unit Vectors

Unit vectors have a magnitude of 1. The unit vector of a vector $\vec{a}$ is denoted $\hat{a}$, and points exactly in the same direction as $\vec{a}$ – it just has the magnitude of 1.

Example Given $\vec{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, its unit vector would be $\hat{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Example Given $\vec{a} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, its unit vector would be $\hat{a} = \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$.

(n) Given any vector, how can you find it's unit vector?

(o) What is the unit vector of $\vec{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$?

(p) What is the unit vector of $\vec{a} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$?

Dot Product / Scalar Product

The product (multiplication) of two vectors is called the *dot product* or the *scalar product* (confusing – I know! We will stick to the term "dot product").

The dot product of two vectors $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

where θ is the angle between the two vectors. It is important to *not* use $\times$ to denote the scalar product. That is reserved for a different product – namely the *vector product* or *cross product*.

(q) Given two vectors, $\vec{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$, calculate $\vec{a} \cdot \vec{b}$. Notice, they are parallel!

(r) Given two vectors, $\vec{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, calculate $\vec{a} \cdot \vec{b}$. Notice, they are perpendicular!

Python

(s) Using Python and numpy, implement a `mag(vec)` function to return the magnitude (length) of a 2-dimensional vector (as a numpy array).

(t) Likewise, implement a `unit(vec)` function to return the unit vector of a 2-dimensional vector (as a numpy array).

(u) Rotating a 2D vector 90 degrees counter clockwise is done by swapping x and y, and negating the new x. Implement `rot90(vec)`. It must return a new numpy array, that is a 90 degree rotation of the `vec` input.

Solve the following *using Python*, given

$$\vec{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \vec{b} = \begin{pmatrix} 8 \\ 7 \end{pmatrix}, \vec{c} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

(v) $2 \cdot \vec{a}$

(w) $\vec{a} + \vec{b} - \vec{c}$

(x) Use `help(numpy)` (or online documentation) to find out how to find the `dot` product of two vectors. (hint: in the `help` prompt, you can search by pressing /, then typing text, then `enter`, then `n` or `shift-n` to navigate the search results!)

(y) Using numpy, find $\vec{a} \cdot \vec{a}$. Compare with `mag(a)*mag(a)`.

(z) Using numpy, find $\vec{a} \cdot \vec{b}$.

(æ) Using your `rot90` function, find the dot product of a and its rotation. (That is, given $\vec{a_r} =$ `rot90(a)`, find $\vec{a} \cdot \vec{a_r}$)