

The Math of Jenga



Jenga rules

- Players take turns taking a block
- Players cant take from the top most completed level
- Player who makes the tower topple loses

Assumptions

- 2 players
- Player r perfect
- Tower falls when only side block is remaining
- Tower falls when lone middle block is removed

2 layers

3 choices what block to take

3 choices where to place block

$$3 \times 3 = 9$$

~~9 move options~~

3 options

N position – Next player to move win



P position – Previous player to move to win



3 layers

~~$6 \text{ blocks} \times 3 \text{ spots} = 18 \text{ options}$~~

~~$6 \text{ blocks} \times 1 \text{ type of spot} = 6 \text{ options}$~~

~~$4 \text{ types of blocks} \times 1 \text{ type of spot} = 4 \text{ options}$~~

~~$2 \text{ types of blocks} \times 1 \text{ type of spot} = 2 \text{ options}$~~

3 level jenga is a **P position**

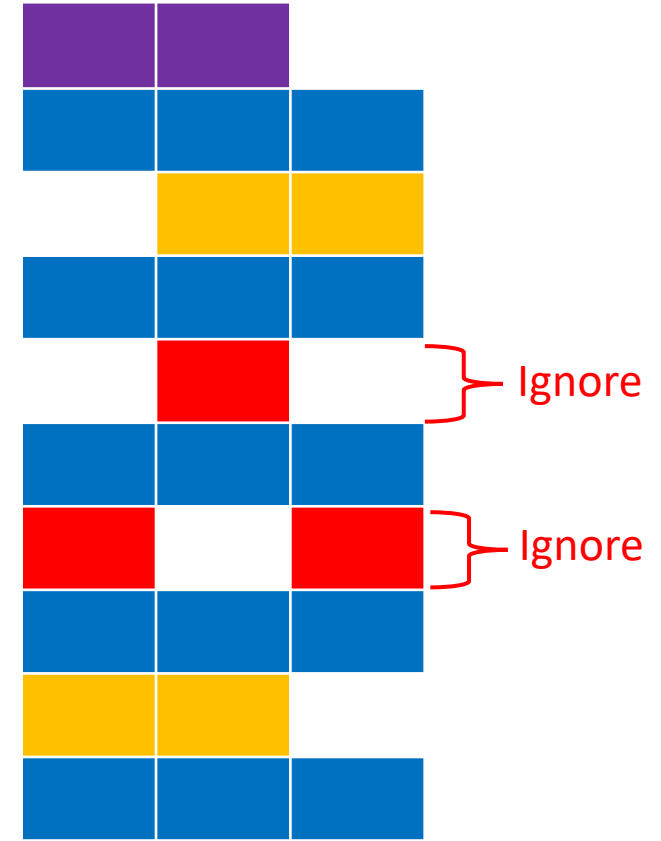
i.e 2nd player always wins

assigning ordered pairs

keep track of:

- 1) full levels – player can take middle or side
- 2) half levels – players can only take side block
- 3) how many blocks on top

$$\left\{ 5 + \frac{2}{3}, 2 \right\}$$



5 full levels

2 half levels

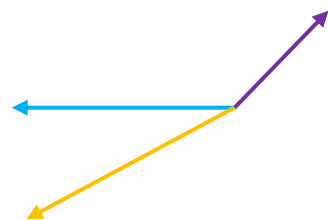
2 blocks on top

Analyzing change in ordered pairs

removing middle block from **full**: Net change $\left(-\frac{2}{3}, 0\right)$

removing side block from **half**: Net change $\left(+\frac{1}{3}, -1\right)$

removing side block from **full**: Net change $\left(-\frac{2}{3}, +1\right)$



$$\left(-\frac{2}{3}, 0\right), \left(+\frac{1}{3}, -1\right), \left(-\frac{2}{3}, +1\right)$$

	0	$0\frac{1}{3}$	$0\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2	$2\frac{1}{3}$	$2\frac{2}{3}$
0	P	P	P	N	N	N	P	P	N
1	N	N	P	P	N	N	N	N	N
2	P	N	N	N	N	P	P	N	N
3	P	P	P	N	N	N	P	P	N
4	N	N	P	P	N	N	N	N	N
5	P	N	N	N	N	P	P	N	N

	0	$0\frac{1}{3}$	$0\frac{2}{3}$	1	$1\frac{1}{3}$
0	P	P	P		
1					
2					
3					

	0	$0\frac{1}{3}$	$0\frac{2}{3}$	1	$1\frac{1}{3}$
0	P	P	P		
1	N				
2					
3					

	0	$0\frac{1}{3}$	$0\frac{2}{3}$	1	$1\frac{1}{3}$
0	P	P	P	N	N
1	N				
2	P				
3					

$h \backslash f$	0	$0\frac{1}{3}$	$0\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2	$2\frac{1}{3}$	$2\frac{2}{3}$	3	$3\frac{1}{3}$	$3\frac{2}{3}$	4	$4\frac{1}{3}$	$4\frac{2}{3}$	5	$5\frac{1}{3}$	$5\frac{2}{3}$
0	P	P	P	N	N	N	P	P	N	N	P	P	N	N	N	P	P	N
1	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N	N	N	N
2	P	N	N	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N
3	P	P	P	N	N	N	P	P	N	N	P	P	N	N	N	P	P	N
4	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N	N	N	N
5	P	N	N	N	N	P	P	N	N	N	N	N	N	N	N	P	P	N
6	P	P	P	N	N	N	P	P	N	N	P	P	N	N	N	P	P	N
7	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N	N	N	N
8	P	N	N	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N
9	P	P	P	N	N	N	P	P	N	N	P	P	N	N	N	P	P	N
10	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N	N	N	N
11	P	N	N	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N
12	P	P	P	N	N	N	P	P	N	N	P	P	N	N	N	P	P	N
13	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N	N	N	N
14	P	N	N	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N
15	P	P	P	N	N	N	P	P	N	N	P	P	N	N	N	P	P	N
16	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N	N	N	N
17	P	N	N	N	N	P	P	N	N	N	N	N	N	N	P	P	N	N

- 1) Start out on an N
- 2) Find vector that keeps the opponent on P
- 3) Do the corresponding move to that vector
- 4) Repeat until opponent has no moves