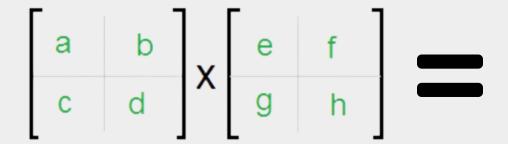
## Strassen Algorithm

## Naive Technique



How many operations were performed in this multiplication technique?

4 results with 2 multiplications and 1 addition each. So 8 total multiplications and 4 additions.

In the nxn case, n^2 results, n multiplications and n-1 additions each. So n^3 multiplications and n^2 - n additions.

## Strassen Algorithm

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} X \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

$$p5 + p4 - p2 + p6$$
  $p1 + p2$   $p3 + p4$   $p1 + p5 - p3 - p7$ 

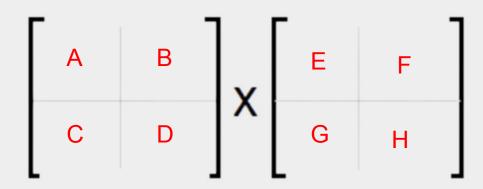
$$p_1 = (a+d)(e+h)$$
  
 $p_2 = (c+d)(e)$   
 $p_3 = (a)(f-h)$   
 $p_4 = (d)(-e+g)$   
 $p_5 = (a+b)(h)$   
 $p_6 = (-a+c)(e+f)$   
 $p_7 = (b-d)(g+h)$ 

We can see that there is now only 7 multiplications but 18 additions.

## Strassen Complexity proof by induction

Claim: You can multiply a nxn matrix (where n is  $2^k$  for some k) in  $n^{\log_2 7}$  multiplications.

Base case was shown on the last slide.



Where entries A-H are matrices of size 2<sup>(k-1)</sup>.

By our Induction Hypothesis each entry will take 7<sup>(k-1)</sup> multiplications. We know a 4x4 matrix takes 7 multiplications so therefore the matrix takes

$$7^k = 7^{\log_2 n} = n^{\log_2 7} = n^{2.80735492}$$

Through some optimizations by using tensors as of today the fastest known algorithm runs in. Discovered only in April 2023.

 $n^{2.37188}$