# Counterfeit Coin Problem

### **Problem 1 Definition**



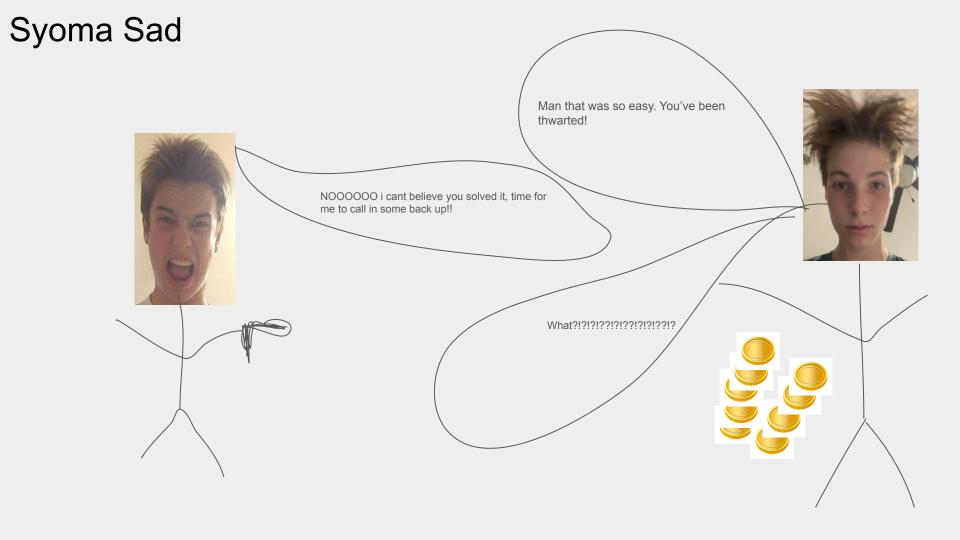
#### Solution

Let us see how many weighings we can do with 3 coins.

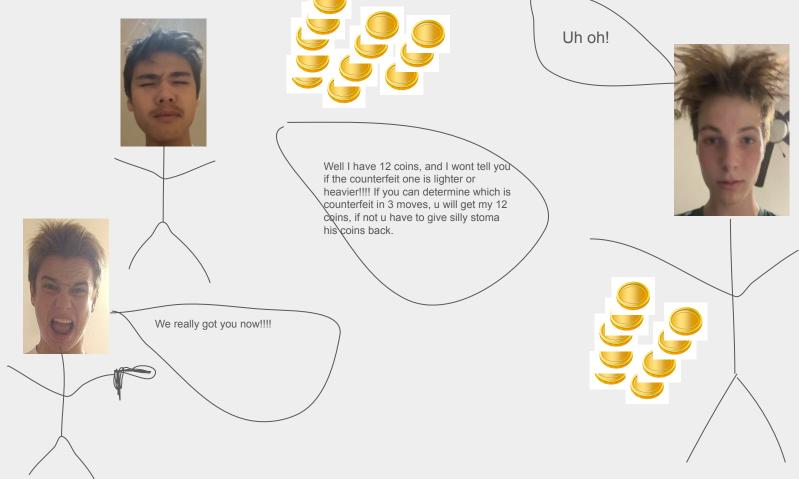
In the case of three coins we are able to compare any two coins, if they weigh the same we know it is the third, and if they do not weigh the same we know it is the lighter one.

We now know that if we have 3 coins we can find the counterfeit one in 1 weighing. How can this be applied to our 9 coin case?

We can weigh two sets of three of the coins in order to determine in which set of three the lighter coin is. The result is that we can determine which coin is lighter in 2 moves.



## Problem 2 Definition



#### How to do

## How many coins should we compare on the first weighing?

If we do 1 we are left with way too many coins if they are equal weight. Same issue with 2-3.

Let's try 4 coins.

If the scale results in an even reading, we know the counterfeit coin must be in one of the 4 unweighted coins.

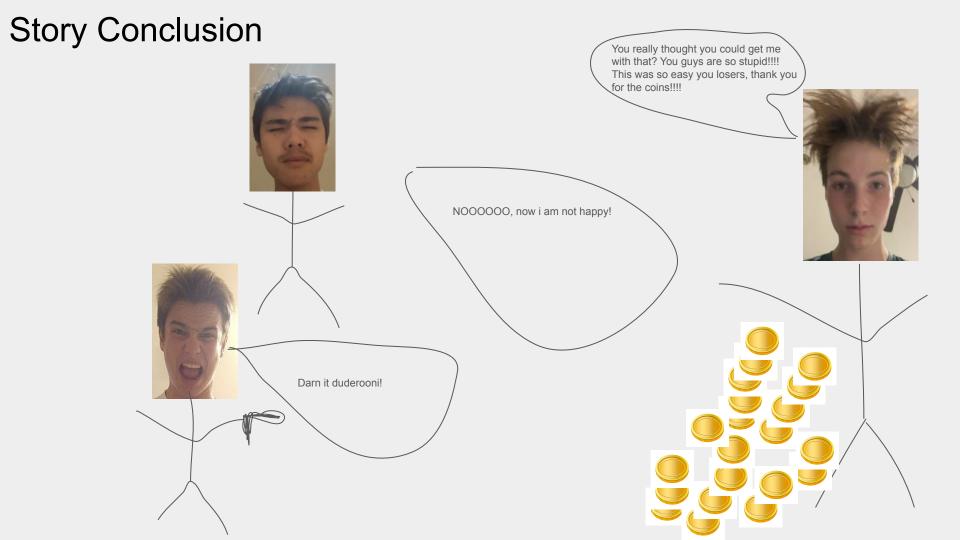
#### Same Weight Case.

Place 3 unknown coins on the scale with 3 known real coins. If the unknown coins side is lighter/heavier this is the same case with 3 coins from before. If they're the same weight we know the remaining unknown coin is the counterfeit one.

What if the scale was not even?

#### Different Weight Case.

Remove three coins from the heavier side, move three coins from the lighter side to the heavier side, replace these coins with coins that were not weighed the first time. If the result is the same, it must be one of the coins not moved, we can compare either one of them against a known coin to determine which of the two is counterfeit. If the scale tips the other way, we know one of the coins we moved to the other side is lighter. Again the same case with 3. If the scales are even we know one of the ones removed was heavier. Once again, the case with 3 coins.



### Patrick Fu Slide

