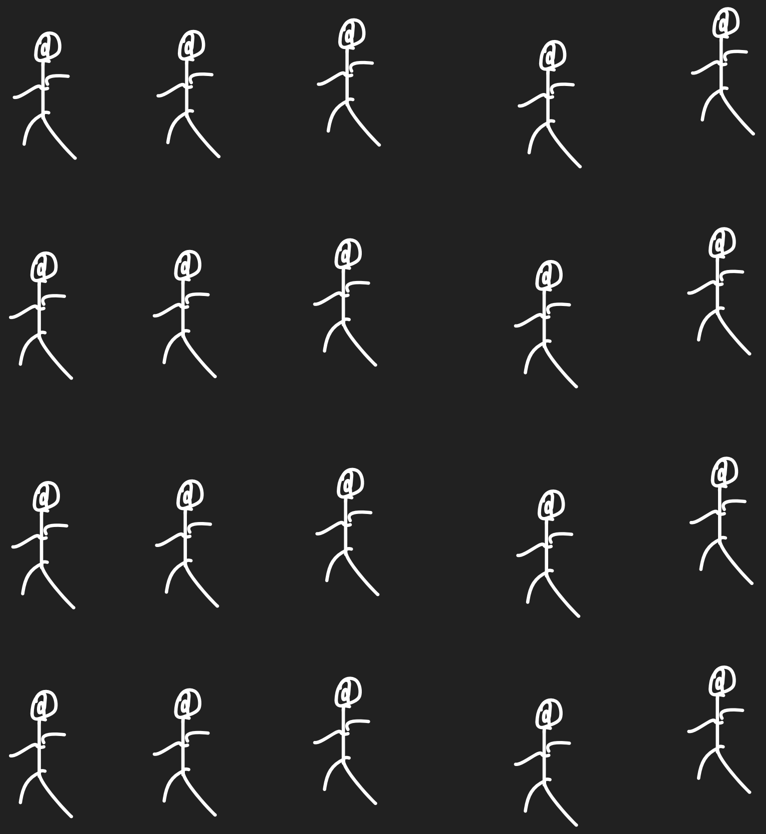
Mathematically Optimised Dating

Made in 30 minutes

The rules

- 100 potential partners and you must find your ideal partner
- You can speak to any number of potential partners you want
- However you can only speak to each potential partner once.
- After you speak with someone you must reject them or accept them as your ideal partner and you won’t be able to speak to anymore potential partners
- You can not go speak to someone you have rejected
- You have no prior knowledge about who you are speaking with you speak with potential partners in a random order.



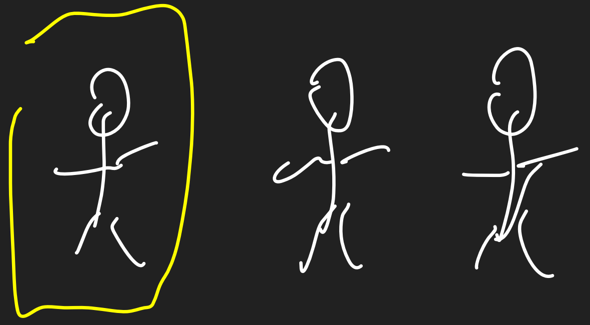
Mathematical framing

We can see each person as their compatibility rating (1-100) but we would not know the ratings of each person until after the game is over. We can note that we may find our most compatible person (100) very early on but we have no way of knowing this person is 100 and that no one better awaits us.

1	17	5	12	8
3	19	14	6	10
2	11	7	20	9
16	4	15	13	18

Simplifying the problem

Let's say we have the situation with 3 potential partners. We use the strategy of using our first potential partner as our comparison person which we must always reject. For our second date if the second date is better than our first date we accept them, otherwise we reject them and accept the last person. To see why this is a good strategy despite being technically possible to result in any person we look at the mathematical formalization of the issue with representing the potential partners as ratings 1, 2, and 3, we can see that 50% of the time we choose the absolute best person.



comparison

= reject

= pick

Compability

1 2 3

least most

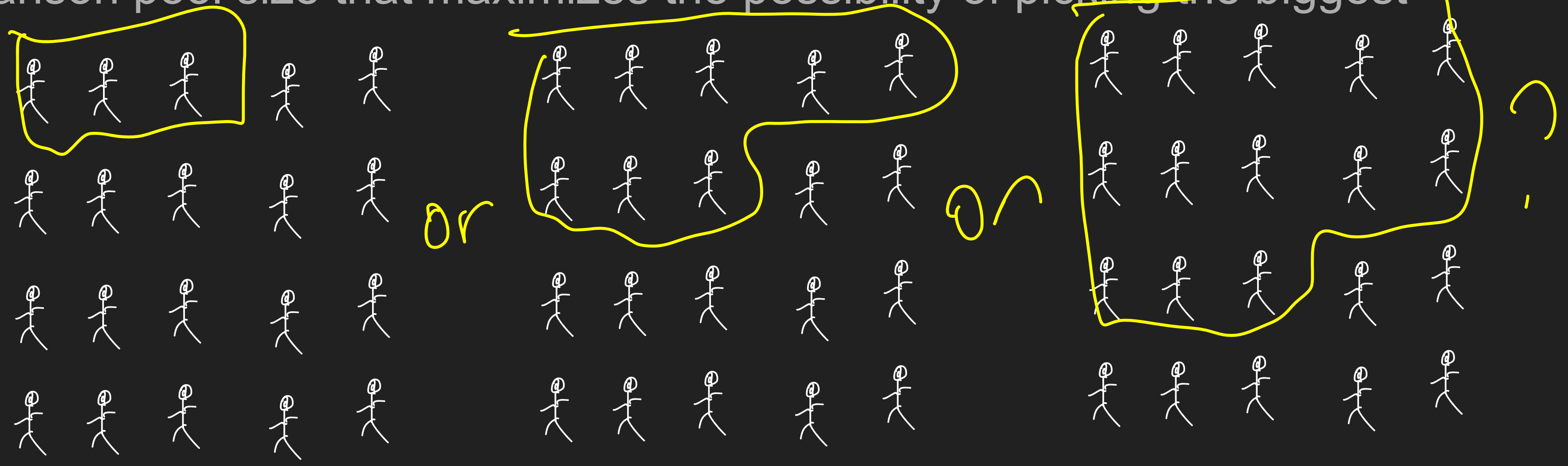
$\frac{3}{6}$ times we get ideal

all possibilities

1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

Generalizing our simple solution.

If we keep a comparison of one person it is very likely we have many people better than the one person and will settle for a non-ideal partner. Thus we must adjust our comparison pool size, and pick the first person that exceeds all potential partners in our comparison pool. Let's now use some mathematics to understand the comparison pool size that maximizes the possibility of picking the biggest partner.

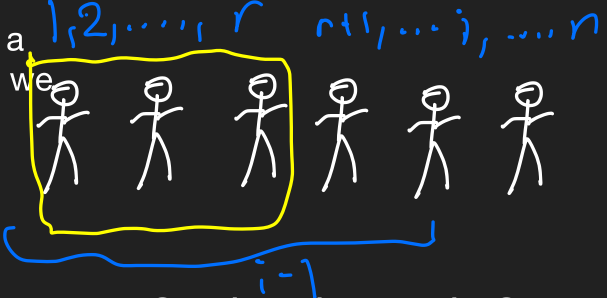


Using mathematics, let's say we have n potential partners

Let's say our comparison pool has r people. We want to maximize P(The Best Partner is Chosen) we can rewrite this P(Partner 1 is chosen AND Partner 1 is the best partner) + P(Partner 2 is chosen AND Partner 2 is the best partner) + P(Partner 3 is chosen AND Partner 3 is the best partner) + … + P(Partner n is chosen AND Partner n is the best partner). With some summation notation and the conditional probability formula we can rewrite this.

$$\sum_{i=0}^{n} P(\text{Partner } i \text{ is chosen AND Partner } i \text{ is the best partner}) = \sum_{i=0}^{n} P(\text{Partner } i \text{ is chosen given Partner } i \text{ is the best}) \cdot \underbrace{P(\text{Partner } i \text{ is the best})}_{\frac{1}{n} \text{ trivially}}$$

To make this clear if our best partner among the first i-1 is not among the first r we would pick a partner of r+1-i-1 before i. So we want our best partner out of the first i-1 to be in the first r, so we pick i GIVEN partner i is the best in our condtional statement



To understand our conditional statement

P(Partner i is chosen given Partner i is Best). We first note that for partners 0-r the chance is 0 since they are in the auto-reject comparison group. Then for an ith partner in the r+1 to n, we require that our rating is higher than our rating for the 0-r partners and that the people after the comparison but before the ith partner are worse than our ith partner. If you think about this requires that someone in our comparison group is good enough where they reject everyone but not our ith person. Note that the ith person is our ideal person is on our given conditional probabilitt statement. Aka the chance P(Partner i is chosen given Partner i is Best) = 0 for i in 0-r and the chance P(Partner i is chosen given Partner i is Best) = P(The best of the first i-1 partners is in the first r partners given Partner i is Best) so our sum becomes

$$\underbrace{\left[\sum_{i=0}^{r} 0 + \sum_{i=r+1}^{n} P(\text{The best of the first i-1 partners is in the first r partners given Partner i is Best})\right]}_{\sum_{i=0}^{n} P(\text{Partner i is chosen given Partner i is best})}\frac{1}{n} = \frac{1}{n}\sum_{i=r+1}^{n}\frac{r}{1-i}$$

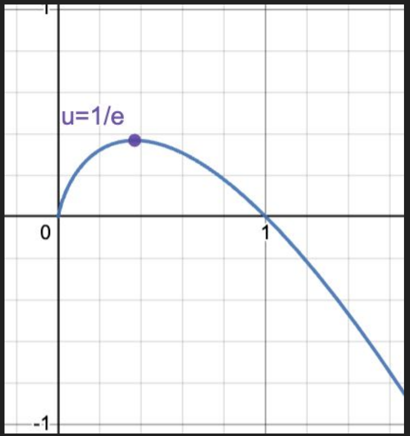
Using calculus for an approximation.

Writing this quickly in latex

$$
\begin{aligned}
P(success) &= \frac{r}{n}\sum_{i=r+1}^{n}\frac{1}{1-i} \\
&= \frac{r}{n}\sum_{i=r}^{n-1}\frac{1}{i} \\
&= \frac{r}{n}\left[\frac{1}{r}+\frac{1}{r+1}+\frac{1}{r+2}+\dots\frac{1}{n-1}\right] \\
&\approx \frac{r}{n}\left[\int_r^n \frac{1}{x}dx\right] \quad \text{Approximation gets better for big n like 100 see picture} \\
&= \frac{r}{n}ln\left(\frac{n}{r}\right) \quad \text{Let } u=\frac{r}{n} \text{ or percentage of total rejected} \\
&= uln(u^{-1}) \\
&= -uln(u)
\end{aligned}
$$

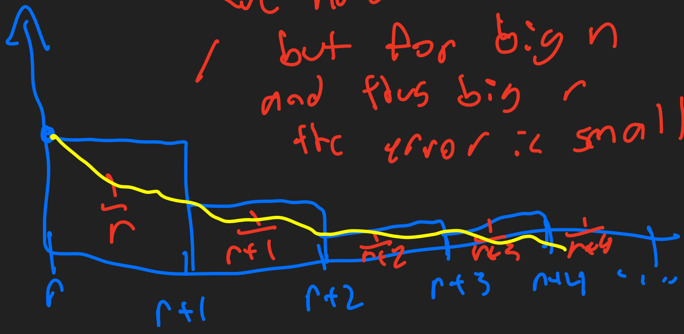
area under curve

As we can see on desmos this has maximum at $1/e$ or approximately 37% for our compariosn pool



$f(x) = \frac{1}{x}$

we have an error but for big n and thus big r the error is small



area = m