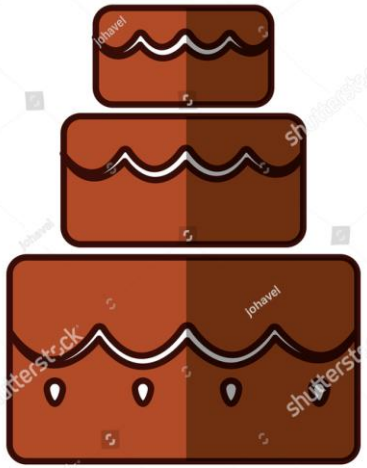

Fair cake cutting

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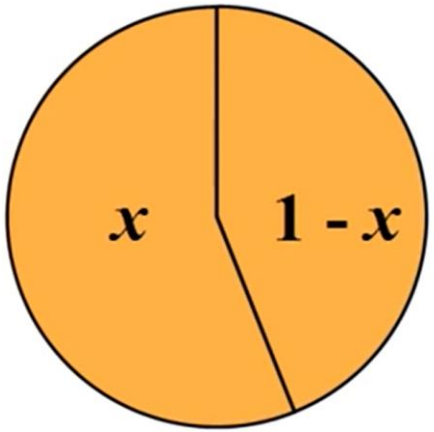
How do we split the cake between 2 people?

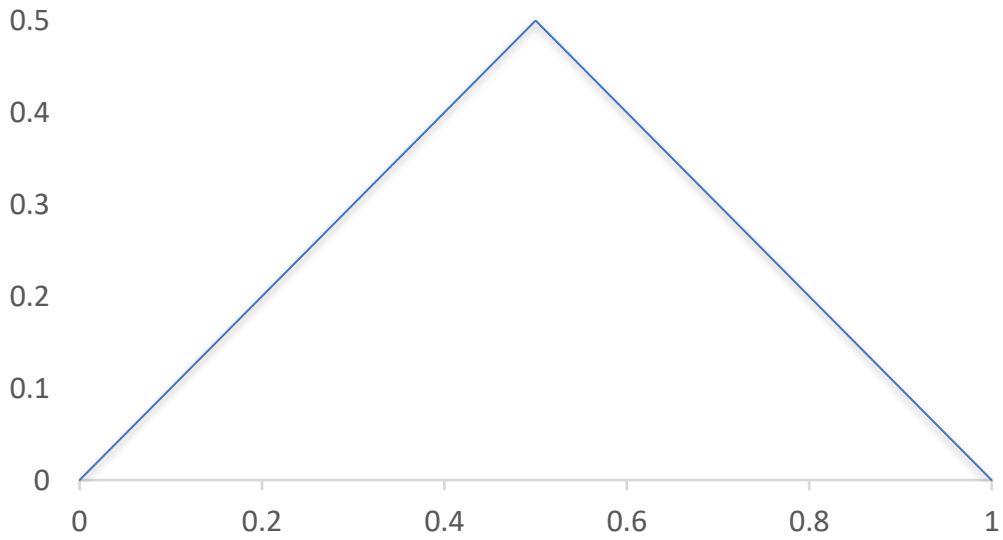
“I cut you choose”

- Blue cuts cake into 2 pieces
- Red chooses which of the pieces he get
- Blue gets remaining piece

proof

1) Suppose Cake is equal to 1
2) x = fraction of cake
3) Blue cuts cake into $(x, 1-x)$
4) Red chooses bigger
5) Blue gets min $(x, 1-x)$





How do we split the cake between 3 people?

Last Diminisher algorithm

- Blue cuts a slice
- Red has option to reduce the slice
- Green has option to reduce the slice
- Last person to cut cake gets it
- Remaining 2 perform “I cut you choose”

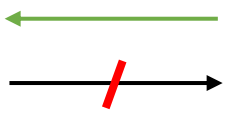
Solution for *N* people

- Person 1 cuts a slice
- Person 2,3…, *N* have a choice to reduce the cake
- Last person to cut cake gets the slice
- Repeat procedure with remaining cake and people
- 2 final people perform “I cut you choose”

Defining the notion of fairness

Proportionality

- each player feels that he received at $\frac{1}{n}$ or more fraction of the cake
- Don't care about what other people have



Envy-freeness

- each player feels that he received the best slice
- i.e no player envies anyone else's slice

envy-freeness implies Proportionality
but Proportionality does not imply Envy-freeness

checking our existing algorithms for envy-freeness

I cut you choose:

is envy free 

Last diminsher:

is not envy free 

Selfridge-Conway Algorithm

Player 1's Turn:

Divides the cake into three pieces.
Lets Player 2 and Player 3 choose first to avoid getting the smallest piece.

Player 2 and Player 3's Turn:

If they choose different pieces, the problem is solved.
If they both choose the same piece, a modification is needed.

Modification:

Player 2 cuts a small piece (D) from the chosen piece (C).
Player 3 chooses from the modified piece (C) and another piece (B).

Rules for Modified Piece C:

If Player 3 doesn't choose the modified piece, Player 2 must.

Resolution:

If Player 3 chooses the modified piece, Player 2 cuts D into three pieces (D1, D2, D3).
Players pick up their pieces in the following order: Player 3, Player 1, Player 2.

Proof of envy-freeness

Player Perspectives:

Player 1: Gets 1/3 of the cake plus one of D1, D2, or D3.

Player 2: Gets B plus one of D1, D2, or D3.

Player 3: Gets modified C plus one of D1, D2, or D3.

Envy Avoidance:

Player 1 doesn't envy others as he gets a fair share.

Player 2 doesn't envy as B and modified C are perceived as larger than A.

Player 3 doesn't envy as B and modified C are considered larger than A.

Cake Time!

- Person 1 cuts a slice
- Person 2,3…, *N* have a choice to reduce the cake
- Last person to cut cake gets the slice
- Repeat procedure with remaining cake and people
- 2 final people perform “I cut you choose”