Mathematics of Quantum Mechanics
Draft version

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Abstract

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Introduction

This text is largely inspired from professor Ramamurti Shankar's Principles of Quantum Mechanics[1].

1 Linear Vector Spaces

Let's recall the basic definition of a vector space :

Definition 1.1. (κ-vectorial space)
Let X be a non empty set, which forms a κ-vectorial (linear) space, having :

- An inner product :

$$\begin{aligned} f : X \times X &\to X \\ (u, v) &\to u + v \end{aligned}$$

- An extern product :

$$\begin{aligned} \kappa \times X &\to X \\ (\lambda, u) &\to \lambda.u \end{aligned}$$

X verifies the following rules :

- $\forall u, v \in X,\ u + v = v + u$ (associative).
- $\forall u, v, w \in X,\ u + (v + w) = (u + v) + w$.
- There exist a neutral element 0_X, such as $0_X \in X,\ u + 0_X = u, \forall u \in X$
- Each $u \in X$ has a symetric element $u' \in X$, such as $u' + u = 0_X : u' = -u$
- $\lambda.(\mu.u) = (\lambda\mu).u,\ \forall \lambda, \mu \in \kappa, u \in X$
- $\lambda.(u + v) = \lambda.u + \lambda.v,\ \forall \lambda \in \kappa$ and $u, v \in X$
- $(\lambda + \mu).u = \lambda.u + \mu.u,\ \forall \lambda, \mu \in \kappa$ and $u \in X$

In quantum mechanics, we shall use a similar concept of such vectorial spaces, deprived of concepts such as direction..

Definition 1.2. Let $\varkappa$ be a linear vector space, and $|1\rangle, |2\rangle, |3\rangle, ... |V\rangle$ .. be elements of such vector space.
We will see that such elements obeys laws similar to such of a κ-vectorial space.

Draft :: All of the equations will be modified later on.

Linear indepenence of a series of vectors :

$$a_1 |1\rangle + a_2 |2\rangle + a_3 |3\rangle + ... + a_n |n\rangle = |0\rangle = 0 \tag{1.1}$$

Can be expressed such as :

$$\sum_{i=1}^{n} a_i |1\rangle = |0\rangle \tag{1.2}$$

Expression of vectors in a **basis** :

$$|V\rangle = \sum_{i=1}^{n} \nu_i |i\rangle \tag{1.3}$$

Linear addition of 2 vectors :

$$|V\rangle = \sum_{i=1}^{n} \nu_i |i\rangle \tag{1.4}$$

and

$$|W\rangle = \sum_{i=1}^{n} \omega_i |i\rangle \tag{1.5}$$

$$|V\rangle + |W\rangle = \sum_{i=1}^{n} \nu_i |i\rangle + \sum_{i=1}^{n} \omega_i |i\rangle = \sum_{i=1}^{n} (\omega_i + \nu_i) |i\rangle \tag{1.6}$$

Linear superposition :

$$\langle aW + bZ | V\rangle = \langle V | aW + bZ\rangle^*$$

References

[1] **Principles of Quantum Mechanics, second edition**
`Springer US, DOI 10.1007/978-1-4757-0576-8`
`Ramamurti Shankar, Yale University, New Haven, United States`