Aarhus University
Department of Electrical and Computer Engineering
Signals and Systems (Signaler og Systemer)
Course Coordinator: Henrik Karstoft

Final examination
Jun 1st, 2023
Time allowed: 180 minutes

INSTRUCTIONS:

- This paper contains **FIVE (5)** questions.
- Answer **ALL FIVE (5)** questions.
- Write down each step clearly.
- Questions carry equal marks.
- This is an **open book** examination. All aids, like computer, mathematical software, books, notes, and internet are allowed.

Question	Points
1	/20
2	/20
3	/20
4	/20
5	/20
Total	/100

Question 1: (20 points)

(a) (10 points) Determine whether or not each of the following signals is **periodic**. Justify all your answers with necessary calculations. If it is periodic, determine the **fundamental period**. (Computer plots are neither needed nor accepted as an answer.)

	periodic/non-periodic	fundamental period
$x(t) = 3\cos(3t + \pi/3)$		
$x(t) = 3\cos^2(3t + \pi/3)$		
$x(t) = \cos(\frac{\pi}{2}t) + \cos(\frac{1}{2}t)$		
$x(t) = e^{j\pi t}$		

(b) (10 points) Determine whether or not the following systems are **memoryless**, **causal**, **stable**, **time-invariant**, and/or **linear**. Y and N stand for Yes and No, respectively. Justify all your answers with necessary explanations.

	memoryless	causal	stable	time-invariant	linear
$y(t) = x(t)x(t+2)$	Y/N	Y/N	Y/N	Y/N	Y/N
$y[n] = \max\{x[n], x[n+1]\}$	Y/N	Y/N	Y/N	Y/N	Y/N

Question 2: (20 points)

Determine the Fourier series representation of the following signal with period **T=4**:

$$x(t) = \begin{cases} -t & \text{for } 0 \leq t < 1 \\ 1 & \text{for } 1 \leq t < 2 \\ 0 & \text{for } 2 \leq t < 4 \end{cases}$$

Question 3: (20 points) A linear time-invariant (LTI) system is described by the following differential equation:

$$\dot{y}(t) + 3y(t) = x(t)$$

where $x(t)$ represents the input signal and $y(t)$ represents the output signal.

(a) (5 points) Determine the frequency response of the system.

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

(b) (5 points) Sketch the Bode plot of the system.

(c) (10 points) Determine the output $y_1(t)$ when the input is:

$$x(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Question 4: (20 points)

(a) Consider a discrete-time linear time-invariant (LTI) system with the impulse response $h[n] = 2\big(u[n] - u[n-5]\big)$. The following input signal is considered: $x[n] = u[n] - u[n-3]$

(i) (3 points) Sketch the signals $h[n]$ and $x[n]$.

(ii) (7 points) Sketch the output signal $y[n]$ for the input $x[n]$. $\big(y[n] = x[n] * h[n]\big)$

$\big(u[n]$ is the unit step function in discrete-time.$\big)$

(b) Consider a continuous-time LTI system with the impulse response $h(t) = u(t) - u(t-5)$. The following input signal is considered: $x(t) = u(t) - u(t-3)$.

(i) (3 points) Sketch the signals $h(t)$ and $x(t)$.

(ii) (7 points) Sketch the output signal $y(t)$ for the input $x(t)$. $\big(y(t) = x(t) * h(t)\big)$

$\big(u(t)$ is the unit step function in continuous-time.$\big)$

Question 5: (20 points)

(a) (11 points)

A continuous-time causal LTI system is characterized by the differential equation,

$$\ddot{y}(t) - 3\dot{y}(t) - 4y(t) = \dot{x}(t) - x(t)$$

where $x(t)$ represents the input signal and $y(t)$ represents the output signal.

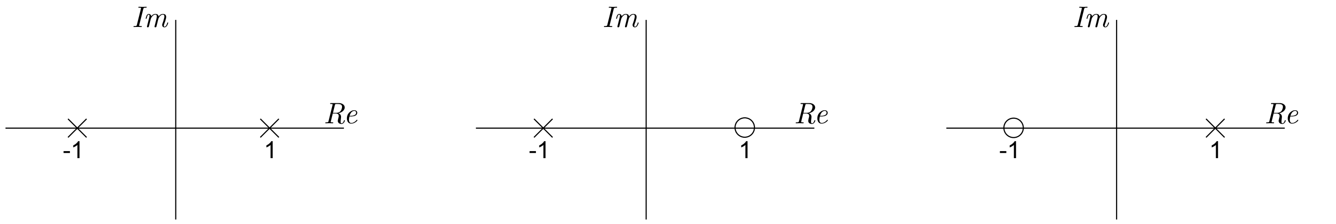
(i) Find $H(s)$, the Laplace transform of the impulse response, $h(t)$, of the system.

(ii) Specify the region of convergence (ROC), poles, and zeros of $H(s)$ on a plot.

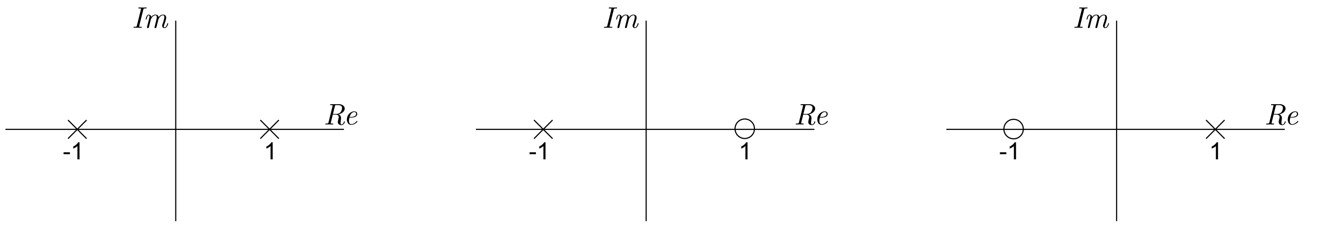
(b) (9 points)

Sketch the corresponding region of convergence (ROC) in each pole-zero plot below, considering the statements about $x(t)$.

(i) $x(t)$ is absolutely integrable.



(ii) $x(t) = 0,$ for $t < -2$. (I.e., the signal is right-sided.)



(iii) $x(t)e^{3t}$ is absolutely integrable. (Hint: Use the property of shifting in the s-domain)