RIS - LAB 2 Prof. Dr. Fanging Hu

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Lab 2 - (Task 1.4 - 1.9)

Task 1.4

We are given the following equations in the task to estimate K_t the motor torque constant:

$$\tau_m = \hat{k}_t i$$

$$\tau_g = \eta \, \hat{k}_t i \triangleq k_t i$$

 T_m is the torque generated outside shaft without gears. Given stall current i = 5A and small torque t_m = 0.5329m we can calculate that:

$$k_{\rm t} = \frac{0.5932Nm}{5A} = 0.11864NmA^{-1}$$

Task 1.5

We are given the upper limit for K_eto be;

$$k_e \leq \frac{v_a}{\omega_0}$$

Given w_0 = 52.36rad/sec and V_a = 12V, we can calculate that

$$k_{\rm e} \leq \frac{12V}{52.36 radsec^{-1}}, k_{\rm e} \leq 0.22918 V.sec.rad^{-1}$$

With this, it is eminent that maximum $K_e = 0.22918V.sec/rad$

Task 1.6

We are given the equation:

$$b = \frac{k_t i_0}{\omega_0}$$

From the previous tasks, we have k_t = 0.11864Nm/A, the no-load speed w_o = 52.36rad/s and the no-load current i_o = 0.3A so, we can make the calculation for b as follows:

$$b = \frac{0.11864Nm.A^{-1} \cdot 0.3A}{52.36rad.s^{-1}} = 6.798 \cdot 10^{-4} Nms.rad^{-1}$$

Task 1.7

We are given the equation from 1.19 to be:

$$R_a = \frac{v_a}{i_s}$$

Given i_s = 5A and V = 12V in Table 1.1,

$$R_{\rm a} = \frac{12V}{5A} = 2.4\Omega$$

We are given the equation from 1.18 to be:

$$v_a - iR_a - L_a \frac{di}{dt} = e \stackrel{\text{(1.6)}}{=} k_e \dot{\theta}_g.$$

Given $\Theta = w_0$, $i=i_0$ and di/dt=0, we can change the equation above as follows for k_e :

$$k = \frac{v_{\rm a} - i_{\rm o} \cdot R_{\rm a}}{w_{\rm o}}$$

Using V = 12V, i = 0.3A, w = 52.36rad/s and R = 2.4 Ω , we can calculate k to be:

$$k_{e} = \frac{12V - 0.3A \cdot 2.4\Omega}{52.36rad.s^{-1}} = 0.21543sec.rad^{-1}$$

which is more accurate than the previous value k_e = 0.22918V.sec/rad from Task 1.5

Task 1.8

Using the Equation 1.17 in Laplace Domain and solving for I(s), we get the new equation to be as follows:

$$I(s) = \frac{(Js^2 + bs) \cdot \Theta g(s) + T_L(s)}{k_t}$$

Using the Equation 1.20 and solving for I(s), we get the equation to be:

$$I(s) = \frac{V_{a}(s) - k_{e}s \Theta g(s)}{R + L_{a}(s)}$$

Using the equations above from Eq. 1.17 and 1.20, we can re-arrange the equations as follows:

$$s.\Theta g(s) \cdot ((Js + b) \cdot (R_a + L_a s) + (k_e \cdot k_t)) = V_a(s) \cdot k_t - T(R_a + L_a s)$$

Given Equation 1.21a,

$$D(s) = (R_a + L_a(s)) \cdot (Js + b) + k_e \cdot k_t$$

Substituting Equation 1.21a into the cross multiplied Equations 1.17 and 1.20, we can solve and simplify $\Theta g(s)$ by dividing it over sD(s) to get the final equation as follows:

$$\Theta g(s) \cdot sD(s) = V_a s \cdot k_t - T_L(R_a + L_a s)$$

$$\Theta g(s) = \frac{V_a(s)k}{sD(s)} - \frac{T_L(s) \cdot (R_a + L_a s)}{sD(s)}$$

Hence, this derivation is equal to the equation in 1.21b which was to be proved.

Task 1.9

To prove Equation 1.23 and finding parameters a and K in terms of other variables, we first take into account the Equation from 1.22a and 1.22b for $H_{wv}(s)$ and Equation 1.21a for D(s)

$$H_{\text{wv}}(s) = \frac{k_{\text{t}}}{D(s)}$$

$$D(s) = (R_a + L_a(s)) \cdot (Js + b) + k_e \cdot k_t$$

Substituting D(s) and considering L_a s = 0, we get:

$$H_{\text{wv}}(s) = \frac{k_{\text{t}}}{(R_{\text{a}} + L_{\text{a}}s) \cdot (Js + b) + k_{\text{e}} \cdot k_{\text{t}}}$$

$$= \frac{\frac{k_{t}}{RJ}}{s + \frac{b}{J} + \frac{k_{e} \cdot k_{t}}{RJ}}$$

Comparing the equation above with the original Equation from 1.23, we get K and a to be as follows with their parameters:

$$K = \frac{k_{\rm t}}{R_{\rm a}J}$$
 and $a = \frac{b}{J} + \frac{k_{\rm e} \cdot k_{\rm t}}{R_{\rm a}J}$