**CSCE 586 - Design and Analysis of Algorithms**
**Date: Tuesday 16th October, 2018**

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**Algorithm 1** Pseudocode of Huffman Algorithm [1]

To construct a prefix code for an alphabet $S$, with given frequencies:
**if** $S$ has two letters **then** :
    Encode one letter using 0 and the other using 1.
**else**:
    Let $y*$ and $z*$ be the two lowest frequency letters
    Form a new alphabet $S'$ by deleting $y*$ and $z*$ and replacing them with a new letter $w$ of frequency $f_{y*} + f_{z*}$
    Recursively construct a prefix code $\gamma'$ for $S'$, with tree $T'$.
    Define a prefix code for $S$ as follows:
        Start with $T'$
        Take the leaf labeled $w$ and add two children below it labeled $y*$ and $z*$
**end if**

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**Algorithm 2** Proof of Termination

Let *S* be an alphabet, with given frequencies
Combine the two lowest frequency letters *x* and *y* into a meta-letter *z*.
Let *S'* be the updated alphabet containing *z*.
$\mid S' \mid = \mid S \mid -1$
Each iteration produces an alphabet with 1 fewer letter than before; thus the algorithm will terminate when there are only two letters

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$$ABL(\gamma) = \sum_{x \in S}^{n} f_x \mid \gamma(x) \mid \tag{1}$$

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**Algorithm 3** Proof of Optimality, by contradiction

Let *S* be the alphabet x, y, z, with given frequencies such that
    $f(z) < f(x) < f(y)$ and $Depth(x) < Depth(y) = Depth(z)$
Let *a* denote the meta-letter *x, y*.
Prior to the formation of *a*: *x, y, and z* are all leaves. Thus, $a$ would select the lowest two frequency letters. Because $f(x) < f(y)$, $a$ would select $x$, not $y$, a contradiction.

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