**Homework 1 Problems**

Due: Wednesday, 3 Apr 19

1. A communication network receives messages from *R* sources with mean arrival rates λ1, λ2, …, λ*R*. On average, there are E[*Nk*] messages from source *k* in the network, for *k* = 1, 2, …, *R*.

a) What is the average time E[*Tk*] spent in the network by customers from source *k*?

b) Let λ denote the total arrival rate into the network. Find an expression for the mean time E[*T*] spent in the network by customers (from all sources) in terms of E[*Nk*].

c) Obtain an expression for E[*T*] in terms of E[*Tk*].

1. (Problem 2.6 from the Robertazzi, *Computer Networks and Systems*, 3rd edition.) Suppose a single server queue does not have a Poisson process input and does not have an exponential server. What variables are necessary to describe the state of the system at any given instant of time?
2. Number of customers in the system (or queue)
3. Inst. arrival rate or interarrival time
4. Inst. service rate or time
5. (Problem 2.11 from Robertazzi, *Computer Networks and Systems*, 3rd edition.)
6. Very briefly explain why it is not a good idea to operate a queuing system under a heavy load (arrival rate close to service rate).

When the arrival rate is close to the service rate, the queue will grow to infinity.

b) Very briefly explain why an infinite buffer queue is said to be “unstable” when the mean arrival rate is greater than the mean service rate.

It is unstable because it will grow to infinity because more customers arrive than can be serviced.

c) On advising day a professor schedules one student appointment every ten minutes. This is based on the assumption that he or she can advise the average student in ten minutes. Will the professor have a long line outside of his or her office?

Yes, with , the queue size will increase indefinitely

1. A communication line is divided into two identical channels, each of which will service a packet traffic stream where all packets have equal transmission time *T* and equal interarrival time *R*, where *R* > *T*. An alternative scheme is to statistically multiplex the two packet traffic streams by combining the two channels into a single channel with transmission time *T*/2 for each packet.

a) Show that the statistical multiplexing scheme will decrease the average time in the system from *T* to a value between *T*/2 and 3*T*/4.

**Best Case: packets arrive separated by T=R:**

* Both packets spend T/2 time in the system because there is no queue

**Worst Case: packets arrive at the same time**

* Packet 1 transmits 🡪 in the system for T/2
* Packet 2 transmits after Packet 1 🡪 total time = T/2 + T/2
* Average time = (T/2 + T)/2 = 3T/4

**Thus, the average time is somewhere between the best/worst case scenario, giving an average time between T/2 and 3T/4**

b) Show that the statistical multiplexing scheme will increase the variance of waiting time in queue from 0 to as much as *T*2/16.

Original: E[T] = T

**Worst case:**

E[T] = 3T/4

(E[T])2 = 9T2/16

E[T2] = T2/16

Var = E[T2] – (E[T])2 = 9T2/16 – 8T2/16

Var = T2/16

**Best case:**

E[T] = T

E[T2] = T2

(E[T])2 = T2

Var = E[T2] – (E[T])2 = 0

**Thus, the variance will be somewhere between 0 and T2/16**

1. A DS1 line with a usable capacity of 1.536 Mbps[[1]](#footnote-1)\* will be used to accommodate 24 sessions, each generating Poisson traffic at a rate of 1500 packets/min. Packet lengths are exponentially distributed with mean 1280 bits.

For the system, find the average number of packets in queue, the average number in the system, and the average delay per packet under the following two assumptions. (Note that the “system” includes all 24 sessions.)

(i) Time division multiplexing is used to divide the DS1 line into 24 equal capacity channels, with one session being assigned to one channel. [Hint: Expected number in the system is 24.]

L = 1280 bits/packet, m = 24 channels

The number of packets in the system is the summation of the expected number of packets in each channel

packets

The number of packets in the queue is 24 times the number of packets in each channel

The expected delay of the system is 24 times the expected delay of each channel:

(ii) Statistical multiplexing is used so that all 24 sessions share the full capacity of the DS1 line. [Hint: Expected number in the system is 1.]

**Number of packets in the system:** **Number of packets in the queue:**

**Expected Delay:**

1. *M* traffic sources send packets to *M* destinations across a network path with *N* intermediate channels (*N*+1 intermediate nodes), as shown below.



Each channel *i*, *i* = 1, ..., *N*, has capacity *Ci* = *C*. Message lengths are exponentially distributed with average length *L*. Times between departures at each source *j*, *j* = 1, ..., *M*, are exponentially distributed with parameter *j* = *r*/*M*. Assume that the sources are independent and that *r* < *C*/*L*.

a) Assume that the network is operated in circuit-switched mode. An end-to-end connection with capacity *C*/*M* is established for each of the *M* source-destination pairs. Determine the end-to-end delay *TC* through the network.

b) Assume that the network is operated in packet-switched mode. Determine the end-to-end delay *TP* through the network. (For this problem, model each of the *N* links as an independent M/M/1 queue.)

c) Given *N*, for what values of *M* is each mode superior to the other with respect to delay?

TP was multiplied by L/L to achieve the same denominator as TC

This gives the following difference:

Per the definition of r, . This proves that

Thus:

🡪 Circuit switched is better for M < N

🡪 Equivalent for M = N

🡪 Packet switched is better for M > N

1. \* When referring to communication capacity or transfer rate, 1K = 103 and 1M = 106. When referring to data volume, e.g., the length of a packet or file, 1K = 210 and 1M = 220. [↑](#footnote-ref-1)