

# Room Squares

Matthew Henderson

2020-09-16

## Bibliography

- Anderson, Ian. “On the construction of balanced room squares” [in en]. *Discrete Mathematics*, 16th British Combinatorial Conference, 197–198 (February 1999): 53–60. issn: 0012-365X, accessed September 12, 2020. doi:10.1016/S0012-365X(99)90039-0. <http://www.sciencedirect.com/science/article/pii/S0012365X99900390>.
- Archbold, J. W. “A combinatorial problem of T. G. Room.” *Mathematika* 7 (1970).
- Archbold, J. W., and N. L. Johnson. “A construction for Room squares and application in experimental design.” *Ann. Math. Statist.* 29 (1958).
- Berlekamp, E. R., and F. K. Hwang. “Constructions for balanced Howell rotations for bridge tournaments.” *J. Combinatorial Theory, Series A* 12 (1972).
- Bose, R.C. “On a resolvable series of balanced incomplete block designs.” Publisher: JSTOR, *Sankhyā: The Indian Journal of Statistics*, 1947, 249–256.
- Bruck, R.H. “What is a loop.” Publisher: Prentice-Hall, Englewood Cliffs, NJ, *Studies in modern algebra* 2 (1963): 59–99.
- Byleen, K. “On Stanton and Mullin’s construction of Room squares.” *Ann. Math. Statist.* 41 (1970).
- Chong, B. C., and K. M. Chan. “On the existence of normalized Room squares.” *Nanta Math.* 7 (1974).
- Constable, R. L. “Positions in Room squares.” *Utilitas Math.* 5 (1974).
- Conway, J.H., and R. Guy. *The Book of Numbers*. Springer New York, 2012. ISBN: 978-1-4612-4072-3. <https://books.google.co.uk/books?id=rfLSBwAAQBAJ>.
- Dillon, J. F., and R. A. Morris. “A skew Room square of side 257.” *Utilitas Math.* 4 (1973).

Dinitz, J. H., and D. R. Stinson. “A Hill-Climbing Algorithm for the Construction of One-Factorizations and Room Squares.” Publisher: Society for Industrial and Applied Mathematics, *SIAM Journal on Algebraic Discrete Methods* 8, no. 3 (July 1987): 430–438. ISSN: 0196-5212, accessed June 25, 2020. doi:10.1137/0608035. <https://epubs.siam.org/doi/10.1137/0608035>.

This paper was published in 1987 and describes an approach to constructing Room squares with hill-climbing. The authors make use of the fact that a Room square is equivalent to a pair of orthogonal one-factorizations of  $K_n$ . Two one-factorizations of the same graph are said to be orthogonal if any two factors, one from each factorization, only share at most one edge in common. The second section of the paper describes how to use hill-climbing to find one-factorizations of  $K_n$ . In the third section this approach is modified to find a one-factorization of  $K_n$  which is orthogonal to a given one-factorization of  $K_n$ . The basic idea behind hill-climbing is that you look for a solution by first finding a feasible solution and then making a small change to the feasible solution that decreases a cost function. The cost function measures how far you are from the solution, so as you continue making small steps, decreasing (or, at least, not increasing) the cost function you get closer and closer to the solution. In the fourth section they look at a more complicated problem, of constructing Room squares with subsquares. A subsquare here just means the intersection of some rows and columns of the Room square that also happens to be a Room square. Finally, in the fifth and final section they talk about some applications. One possible application is constructing large numbers of nonisomorphic one-factorizations and Room squares. Another is to use the same approach to look for previously unknown designs, like the Room squares with subsquares.

———. *Contemporary Design Theory: A Collection of Surveys*. Wiley Series in Discrete Mathematics and Optimization. Wiley, 1992. ISBN: 978-0-471-53141-8. <https://books.google.co.uk/books?id=nDiEFrSwvGgC>.

Gross, K. B. “Equivalence of Room designs I.” *J. Combinatorial Theory* 16 (1974).

———. “Equivalence of Room designs II.” *J. Combinatorial Theory* 17 (1974).

Gross, K. B., R. C. Mullin, and W. D. Wallis. “The number of pairwise orthogonal symmetric Latin squares.” *Utilitas Math.* 4 (1973).

Horton, J. D. “Quintuplication of Room squares.” *Aequationes Math.* 7 (1971).

———. “Variations on a theme by Moore.” *Proceedings First Louisiana Conference on Combinatorics, Graph Theory and Computing*, 1970, 146–166.

Horton, J. D., R. C. Mullin, and R. G. Stanton. “A recursive construction for Room designs.” *Aequationes Math.* 6 (1971).

Hwang, F. K., Qin De Kang, and Jia En Yu. “Complete balanced Howell rotations for  $16k + 12$  partnerships” [in en-gb]. Publisher: Published by Elsevier Inc. 1984. ISSN: 10.1016/0097-3165(84)90078-5, accessed June 25, 2020. doi:10.1016/0097-3165(84)90078-5. <https://core.ac.uk/display/82821272>.

Hwang, F.K. “Some more contributions on constructing balanced Howell rotations.” *Proc. Second Chapel Hill Conf. on Combin. Math. and its Appl.*, 1970, 307–323.

Lawless, J. F. *Pairwise balanced designs and the construction of certain combinatorial systems. Proceedings of the Second Louisiana Conference on Combinatorics*. Baton Rouge: Graph Theory / Computing, 1971.

Lindner, Charles C. “An algebraic construction for Room squares.” *SIAM J. Appl. Math.* 22 (1972).

- Lindner, Charles C., and N. S. Mendelsohn. "Construction of perpendicular Steiner quasigroups." *Aequationes Math.* 9 (1973).
- Mendelsohn, N. S. "Latin squares orthogonal to their transposes." *J. Combinatorial Theory, Series A* 11 (1971).
- . "Orthogonal Steiner systems." *Aequationes Math.* 5 (1970).
- Mullin, R. C. "On the existence of a Room design of side  $F_4$ ." *Utilitas Math.* 1 (1972).
- Mullin, R. C., and E. Nemeth. *A construction for self-orthogonal Latin squares from certain Room squares. Proceedings of the First Louisiana Conference on Combinatorics*. Baton Rouge: Graph Theory / Computing, 1970.
- . "A counter-example to a multiplicative construction of Room squares." *J. Combinatorial Theory* 7 (1969).
- . "A counterexample to a direct product construction of Room squares" [in en-gb]. Publisher: Published by Elsevier Inc. 1969. Accessed June 25, 2020. doi:10.1016/S0021-9800(69)80021-9. <https://core.ac.uk/display/81200860>.
- . "An existence theorem for Room squares." *Canad. Math. Bull.* 12 (1969): 493–497.
- . "On furnishing Room squares" [in en-gb]. Publisher: Published by Elsevier Inc. 1969. Accessed June 25, 2020. doi:10.1016/S0021-9800(69)80022-0. <https://core.ac.uk/display/82097424>.
- . "On the non-existence of orthogonal Steiner triple systems of order 9." *Canad. Math. Bull.* 13 (1970).
- Mullin, R. C., and W. D. Wallis. "On the existence of Room squares of order  $4n$ ." *Aequationes Math.* 6 (1971).
- . "The existence of Room squares" [in en]. *aequationes mathematicae* 13, no. 1 (February 1975): 1–7. ISSN: 1420-8903, accessed June 25, 2020. doi:10.1007/BF01834113. <https://doi.org/10.1007/BF01834113>.

This paper was published in 1975 and gives an overview of the work done to settle the existence question for Room squares. It's only seven pages long but it gives a very clear outline of the proof and all of the results needed to build the proof. The main components of the proof are the starter-adder approach, two results about the existence of starters and adders for prime power and Fermat prime order, and two multiplicative constructions. With this results established the proof itself is only one paragraph. The authors points out, however, that other such proofs have actually failed to properly establish the main result. Quite likely I made this mistake in my own version of the proof in my report. There is a section at the end of related problems. One is the existence of skew Room squares. I honestly don't know if this was eventually settled or not. Another related problem is the existence of MOLS. The authors point out that a standardized Room square is equivalent to a pair of MOLS. They mention the problem of finding as large a set of MOLS as possible and refer to some results of Gross. They end by reminding us that Room squares were first introduced by Howell in 1897 as Howell rotations.

- Nemeth, E. "Study of Room squares." PHD Thesis, University of Waterloo, 1969.
- O'Shaughnessy, C. D. "A Room design of order 14." *Canad. Math. Bull.* 11 (1968).
- . "On Room squares of order  $6m+2$ ." *J. Combinatorial Theory, Series A* 13 (1972).

- Parker, E. T., and A. N. Mood. "Some balanced Howell rotations for duplicate bridge sessions." *Amer. Math. Monthly* 62 (1955).
- Room, T. G. "A new type of magic square." *Math. Gazette* 39 (1955).
- Schellenberg, P. J. "On balanced room squares and complete balanced howell rotations" [in en]. *aequationes mathematicae* 8, no. 1 (February 1972): 196–197. ISSN: 1420–8903, accessed September 14, 2020. doi:10.1007/BF01832751. <https://doi.org/10.1007/BF01832751>.
- . "On balanced room squares and complete balanced Howell rotations" [in en]. *aequationes mathematicae* 9, no. 1 (February 1973): 75–90. ISSN: 1420–8903, accessed September 14, 2020. doi:10.1007/BF01838192. <https://doi.org/10.1007/BF01838192>.
- Shah, K. R. "Analysis of Room's square design." *Ann. Math. Statist.* 41 (1970).
- Stanton, R. G., and J. D. Horton. "A multiplication theorem for Room squares." *J. Combinatorial Theory* 12 (1972).
- Stanton, R. G., and R. C. Mullin. "Construction of Room squares." *Ann. Math. Statist.* 39 (1968).
- . "Room quasigroups and Fermat primes." *J. Algebra* 20 (1972).
- . *Techniques for Room squares*. . Proceedings of the First Louisiana Conference on Combinatorics. Baton Rouge: Graph Theory / Computing, 1970.
- Stanton, RG, and Joseph Douglas Horton. "A multiplication theorem for Room squares." Publisher: Elsevier, *Journal of Combinatorial Theory, Series A* 12, no. 3 (1972): 322–325.
- Wallis, W. D. "A doubling construction for Room squares." *Discrete Math.* 3 (1972).
- . "A family of Room subsquares." *Utilitas Math.* 4 (1973).
- . "Duplication of Room squares." *J. Austral. Math. Soc.* 14 (1972).
- . "On Archbold's construction of Room squares." *Utilitas Math.* 2 (1972).
- . "On one-factorization of complete graphs." *J. Austral. Math. Soc.* 16 (1973).
- . "On the existence of Room squares." *Aequationes Math.* 9 (1973).
- . "Room squares of side five." *Delta* 3 (1973).
- . "Room squares with sub-squares." *J. Combinatorial Theory* 15 (1973).
- . "Solution of the Room square existence problem." Publisher: Elsevier, *Journal of Combinatorial Theory, Series A* 17, no. 3 (1974): 379–383.
- Wallis, W.D., A.P. Street, and J.S. Wallis. *Combinatorics: Room Squares, Sum-Free Sets, Hadamard Matrices*. Lecture Notes in Mathematics. Springer Berlin Heidelberg, 2006. ISBN: 978-3-540-37994-2. <https://books.google.co.uk/books?id=cTN8CwAAQBAJ>.
- Weisner, L. "A Room design of order 10." *Canad. Math. Bull.* 7 (1964).
- Yu, J. E., and F. K. Hwang. "The Existence of Symmetric Skew Balanced Starters for Odd Prime Powers" [in en-gb]. Publisher: Academic Press Limited. Published by Elsevier Ltd. 1988. Accessed June 25, 2020. doi:10.1016/S0195-6698(88)80040-4. <https://core.ac.uk/display/82505808>.