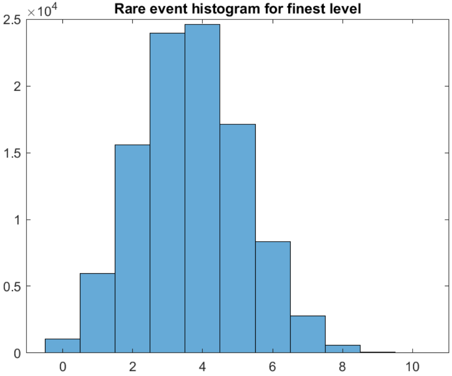
Using MLMC for Rare Event sampling

Manuel Holschbach

January 23, 2025

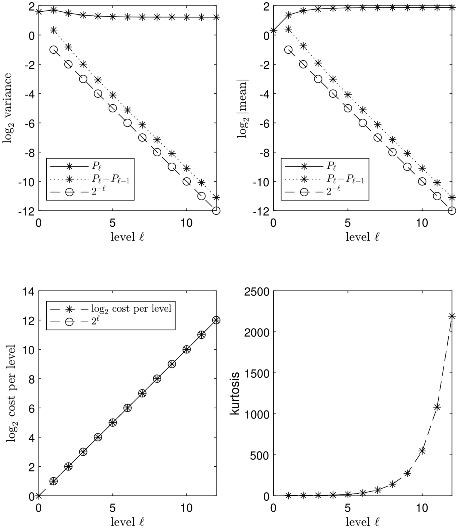
Abundance of species for decay example

- Using a trial run with $N = 100000$, we can estimate the probability $\mathbb{P}(X = x)$ at final time T
- We obtain
  - $\mathbb{P}(X > 3) \approx 0.53472$
  - $\mathbb{P}(X > 7) \approx 0.00654$



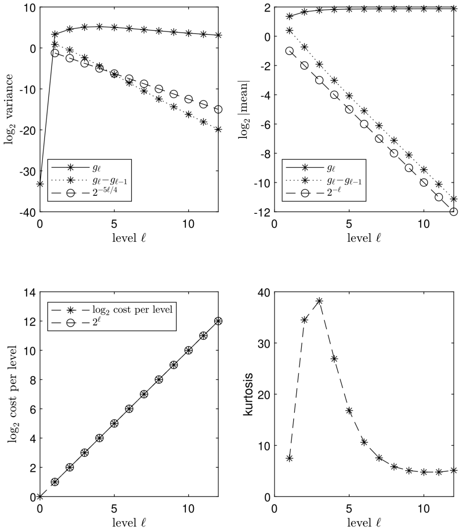
Numerical results

- As a reference, we can consider the estimation of $\mathbb{E}[X]$ without importance sampling
- We can see that the mean and variance of the differences decrease at a rate of $2^{-\ell}$
- We can also see that the kurtosis increases exponentially with the levels ℓ



Numerical results

- When using importance sampling the variance of the differences decreases at a rate faster than $2^{-\frac{5\ell}{4}}$
- Regarding the kurtosis, we observe that the maximum is reached at level $\ell = 3$ at a much lower value than without importance sampling



- We now consider the estimation of $\mathbb{P}(X > 3)$
- We can see similar behaviour to before for the mean, variance and kurtosis
- Additionally, the estimator estimates $\mathbb{P}(X > 3) \approx 0.53$, which is consistent with the a priori estimate

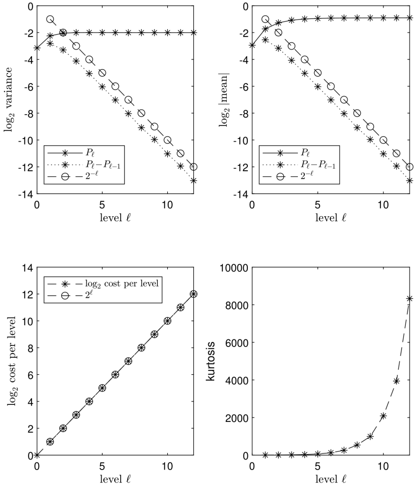


Figure: Estimation of $\mathbb{P}(X > 3)$ without importance sampling

Numerical results

- When using importance sampling, we again observe the same behaviour as in the estimation of $\mathbb{E}[g(X)]$
- Again, the estimator converges to approximately $\mathbb{P}(X > 3) \approx 0.53$
- We also observe a reduction in the variance

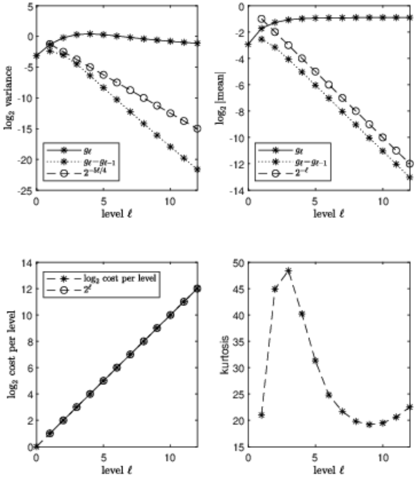


Figure: Estimation of $\mathbb{P}(X > 3)$ with importance sampling

Numerical results

- We now attempt to estimate $\mathbb{P}(X > 7)$
- Without using importance sampling, the estimator fails for high ℓ
- For example, we obtain $P_{11} - P_{10} = 0$ and $P_{12} = 1 * 10^{-6}$

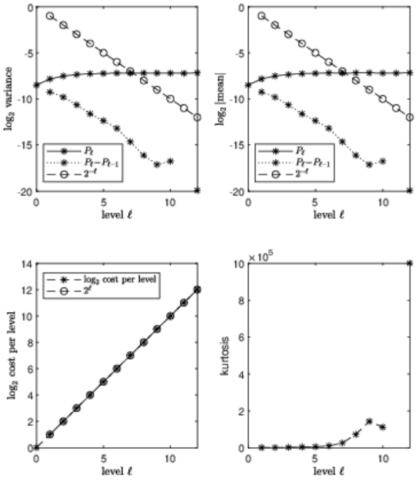


Figure: Estimation of $\mathbb{P}(X > 7)$ without importance sampling

Numerical results

- We now attempt to estimate $\mathbb{P}(X > 7)$ with importance sampling
- In this case, the estimator does not run into the same problems as before and we observe similar behaviour to the previous cases for the mean and variance
- However, the kurtosis does not follow the same behaviour as before, though we still see a significant reduction

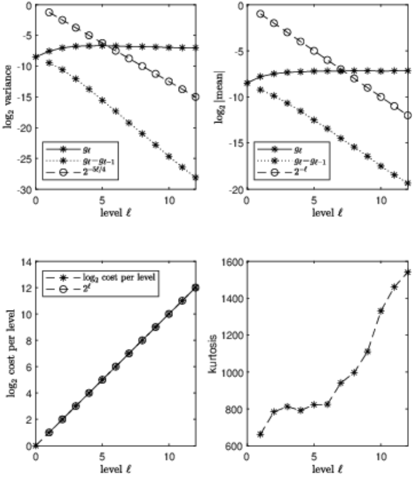


Figure: Estimation of $\mathbb{P}(X > 7)$ with importance sampling