# Element (mathematics)

In mathematics, an **element**, or **member**, of a set is any one of the distinctobjects that make up that set.

## Contents



## Sets

Writing $A = \{1, 2, 3, 4\}$ means that the elements of the set $A$ are the numbers 1, 2, 3 and 4. Sets of elements of $A$, for example $\{1, 2\}$, are subsets of $A$.

Sets can themselves be elements. For example, consider the set $B = \{1, 2, \{3, 4\}\}$. The elements of $B$ are *not* 1, 2, 3, and 4. Rather, there are only three elements of $B$, namely the numbers 1 and 2, and the set $\{3, 4\}$.

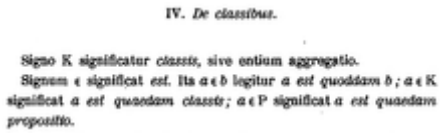
The elements of a set can be anything. For example, $C = \{\text{red}, \text{green}, \text{blue}\}$, is the set whose elements are the colors red, green and blue.

## Notation and terminology

The relation "is an element of", also called **set membership**, is denoted by the symbol "∈". Writing

$$x \in A$$

means that "$x$ is an element of $A$". Equivalent expressions are "$x$ is a member of $A$", "$x$ belongs to $A$", "$x$ is in $A$" and "$x$ lies in $A$". The expressions "$A$ includes $x$" and "$A$ contains $x$" are also used to mean set membership, however some authors use them to mean instead "$x$ is a subset of $A$".[1] Logician George Boolos strongly urged that "contains" be used for membership only and "includes" for the subset relation only.[2]



IV. *De classibus.*

Signo K significatur *classis*, sive entium aggregatio.
Signum ϵ significat *est*. Ita a ϵ b legitur *a est quoddam b*; a ϵ K significat *a est quaedam classis*; a ϵ P significat *a est quaedam propositio*.

First usage of the symbol ϵ in the work Arithmetices principia nova methodo exposita by Giuseppe Peano.

Another possible notation for the same relation is

$$A \ni x,$$

meaning "$A$ contains $x$", though it is used less often.

The negation of set membership is denoted by the symbol "∉". Writing

$$x \notin A$$

means that "$x$ is not an element of $A$".

The symbol ∈ was first used by Giuseppe Peano 1889 in his work Arithmetices principia nova methodo exposita. Here he wrote on page X:

> "Signum ∈ significat est. Ita a ∈ b legitur a est quoddam b; ..."

which means

> "The symbol ∈ means *is*. So a ∈ b is read as a *is a* b; ..."

The symbol itself is a stylized lowercase Greek letter epsilon ("ε"), the first letter of the word ἐστί, which means "is".

The Unicode characters for these symbols are U+2208 ('element of'), U+220B ('contains as member') and U+2209 ('not an element of'). The equivalent LaTeX commands are "\in", "\ni" and "\notin". Mathematica has commands "\[Element]" and "\[NotElement]".

# Cardinality of sets

The number of elements in a particular set is a property known as cardinality; informally, this is the size of a set. In the above examples the cardinality of the set $A$ is 4, while the cardinality of either of the sets $B$ and $C$ is 3. An infinite set is a set with an infinite number of elements, while a finite set is a set with a finite number of elements. The above examples are examples of finite sets. An example of an infinite set is the set of positive integers = { 1, 2, 3, 4, ... }.

# Examples

Using the sets defined above, namely $A$ = {1, 2, 3, 4 }, $B$ = {1, 2, {3, 4}} and $C$ = { red, green, blue }:

- $2 \in A$
- $\{3,4\} \in B$
- $3,4 \notin B$
- {3,4} is a member of $B$
- Yellow $\notin C$
- The cardinality of $D$ = { 2, 4, 8, 10, 12 } is finite and equal to 5.
- The cardinality of $P$ = { 2, 3, 5, 7, 11, 13, ...} (the prime numbers) is infinite (this was proven by Euclid).

# References


1. Eric Schechter (1997). *Handbook of Analysis and Its Foundations*. Academic Press. ISBN 0-12-622760-8. p. 12
2. George Boolos (February 4, 1992). *24.243 Classical Set Theory (lecture)* (Speech). Massachusetts Institute of Technology, Cambridge, MA.


# Further reading


- Halmos, Paul R. (1974) [1960], *Naive Set Theory*, Undergraduate Texts in Mathematics (Hardcover ed.), NY: Springer-Verlag, ISBN 0-387-90092-6 - "Naive" means that it is not fully axiomatized, not that it is silly or easy (Halmos's treatment is neither).
- Jech, Thomas (2002), "Set Theory", *Stanford Encyclopedia of Philosophy*
- Suppes, Patrick (1972) [1960], *Axiomatic Set Theory*, NY: Dover Publications, Inc., ISBN 0-486-61630-4 - Both the notion of set (a collection of members), membership or element-hood, the axiom of extension, the axiom of

separation, and the union axiom (Suppes calls it the sum axiom) are needed for a more thorough understanding of "set element".

# External links

- Weisstein, Eric W. "Element". *MathWorld*.