Counterfactual Explanations on Robust Perceptual Geodesics

Anonymous authors, ICLR 2026 submission

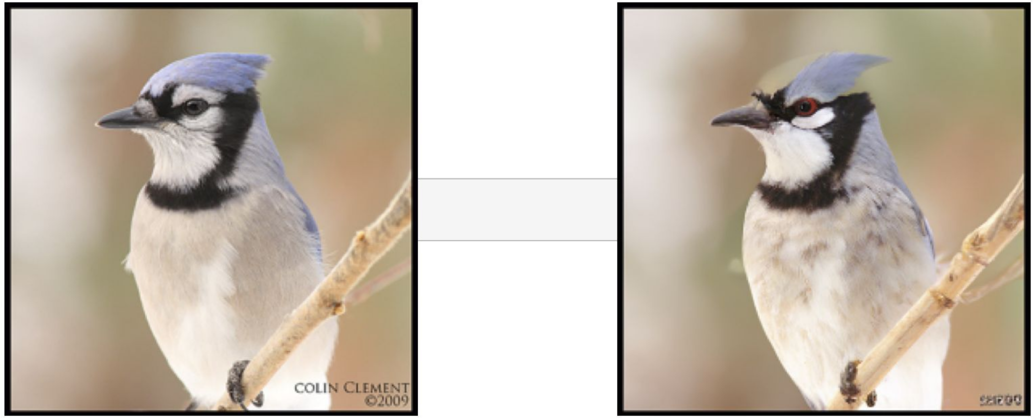
Visual Counterfactual Explanations

Visual Counterfactual Explanations (VCEs)

Identify **minimal semantic** change that alters the decision of a classifier to a certain **target** class

$f(\text{jay} \mid \mathbf{x}^*) = 0.98 \quad f(\text{bulbul} \mid \mathbf{x}_{\text{VCE}}) = 0.97$

VCE



$\mathbf{x}^*$ $\mathbf{x}_{\text{VCE}}$

VCEs formally

$$\min_x \quad \underbrace{r(x^\star, x)}_{\text{Similarity Distance}} \quad + \lambda \;\; \underbrace{\ell(f(x), y')}_{\text{Classification Loss}}$$

Adversarial attacks/examples

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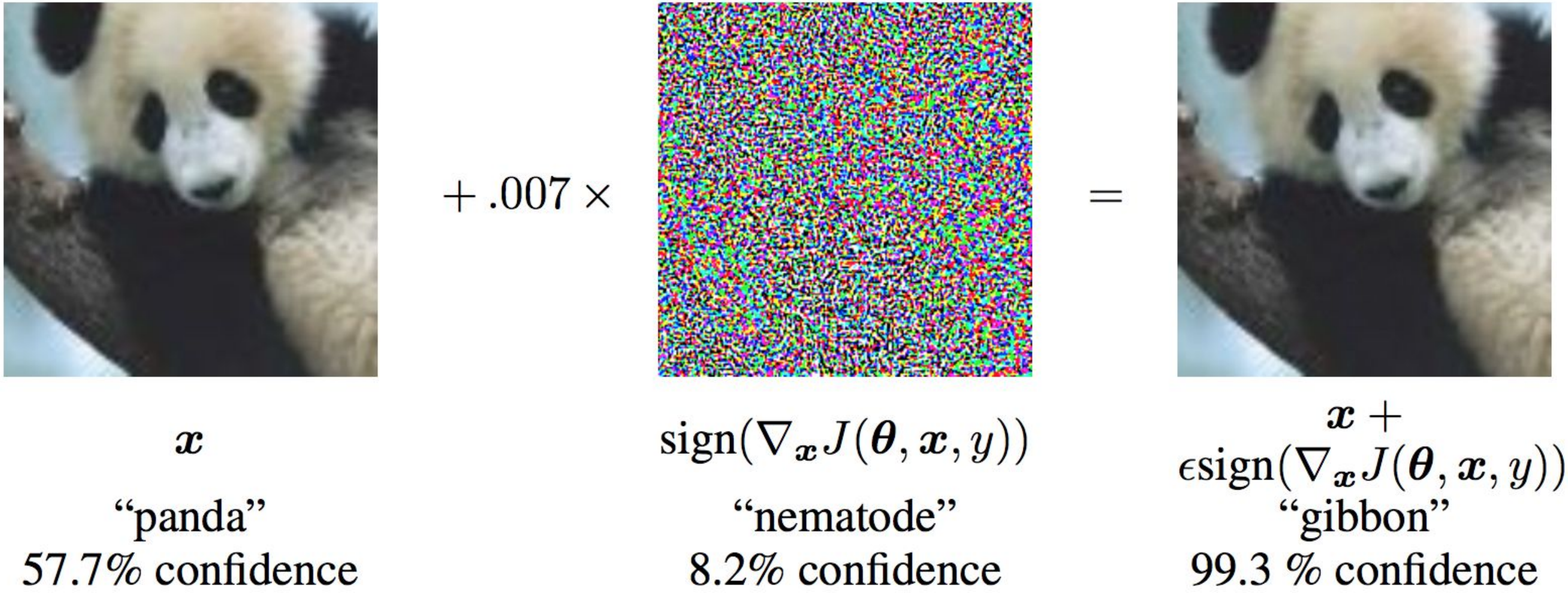

Adversarial attacks/examples formally

$$\min_{x} \quad \underbrace{r(x^\star, x)}_{\text{Similarity Distance}} \quad + \lambda \; \underbrace{\ell(f(x), y')}_{\text{Classification Loss}}$$

Crucial difference

Choice of similarity distance

Adversarial attacks

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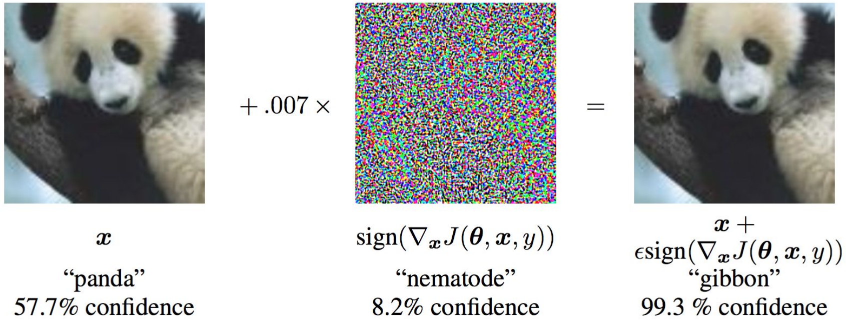

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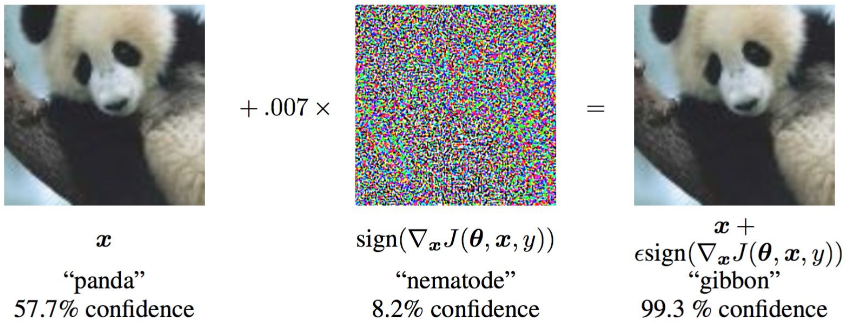
Counterfactual explanations

- generative-model-based
- closely related to human perception



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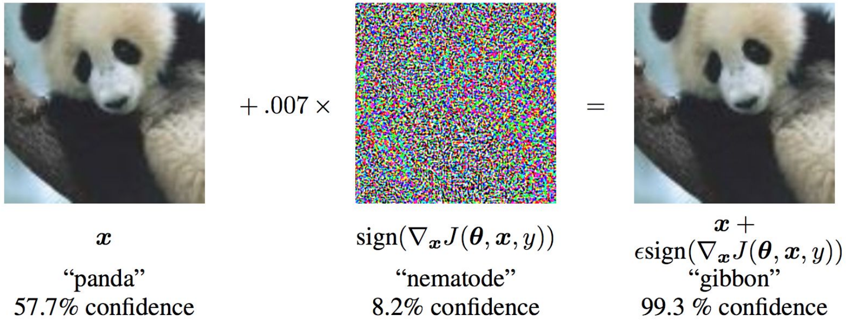
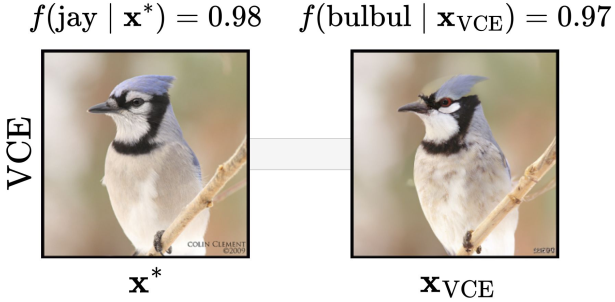
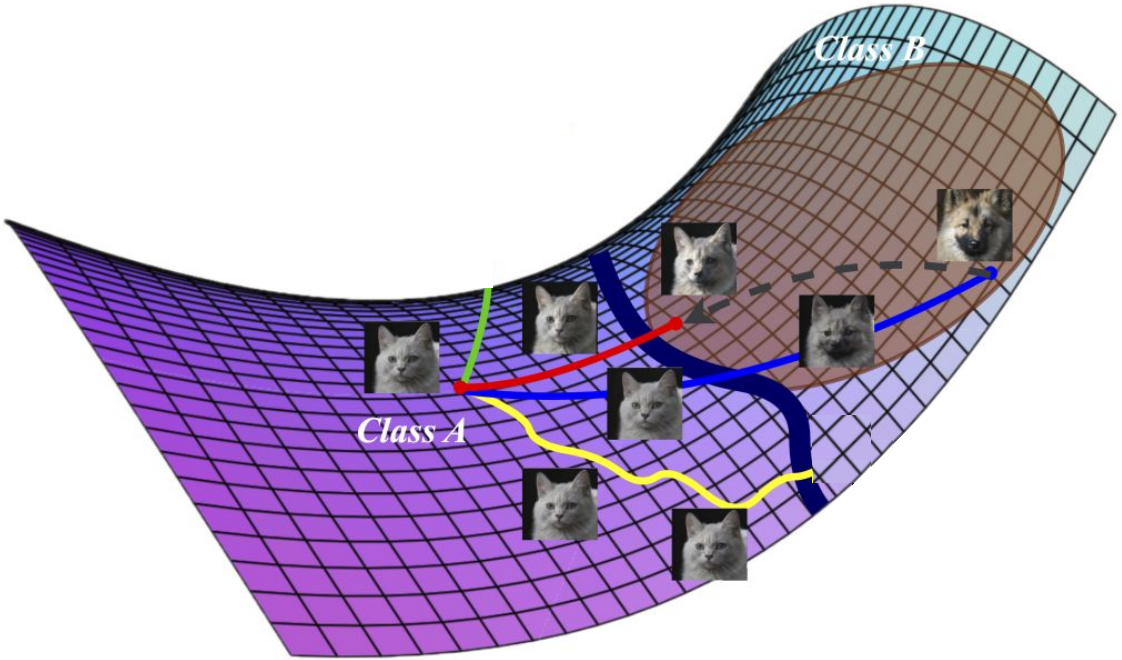




But is that enough?

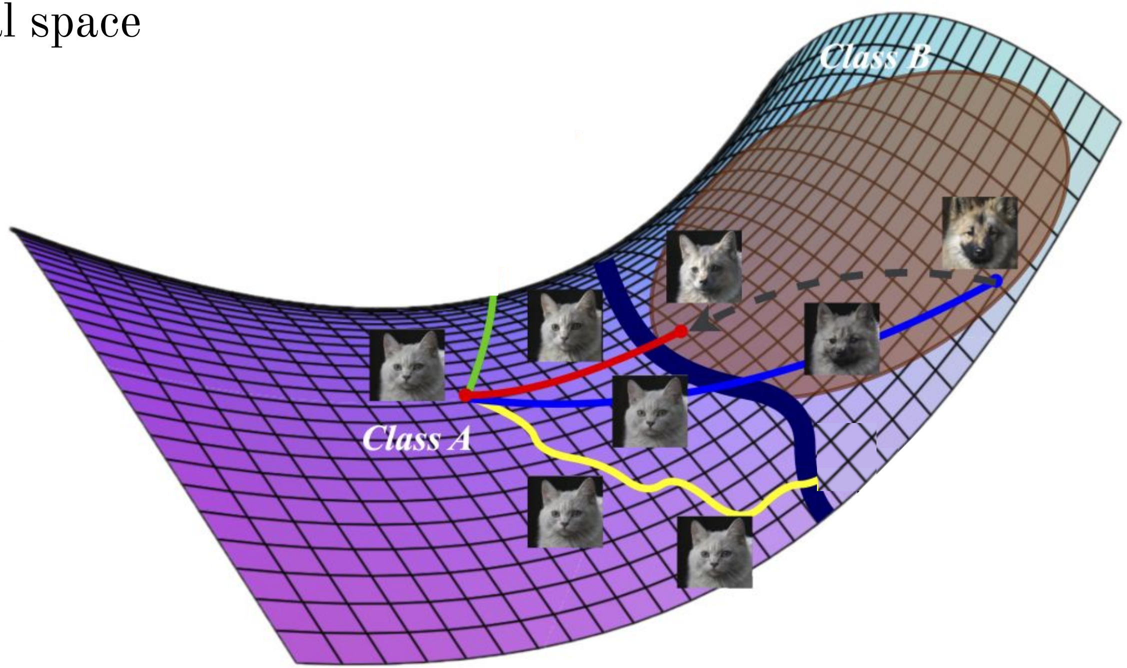
Main claim of this paper: not really

Manifold hypothesis



Manifold hypothesis

Real high-dimensional data lies on a low-dimensional manifold embedded in a high-dimensional space



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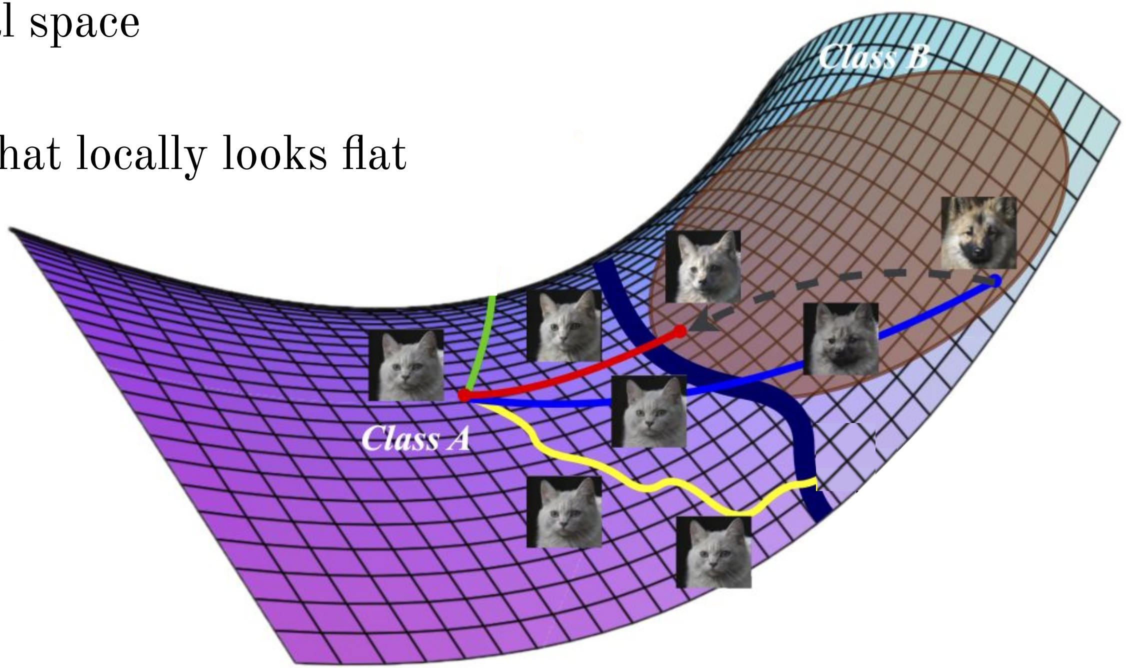
Manifold - a topological space that locally looks flat



‘Seeing’ the manifold

through latent-space generative models

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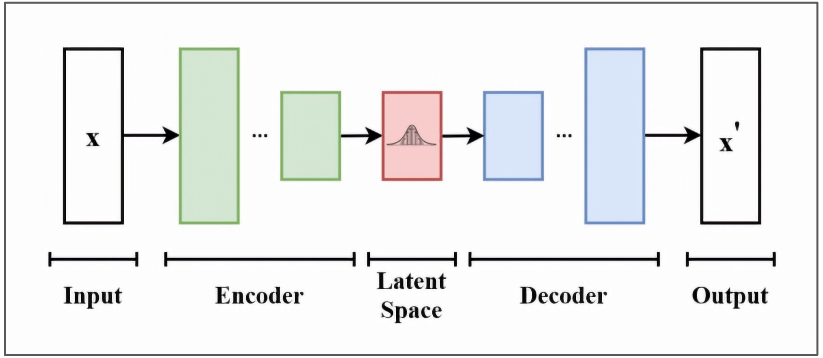

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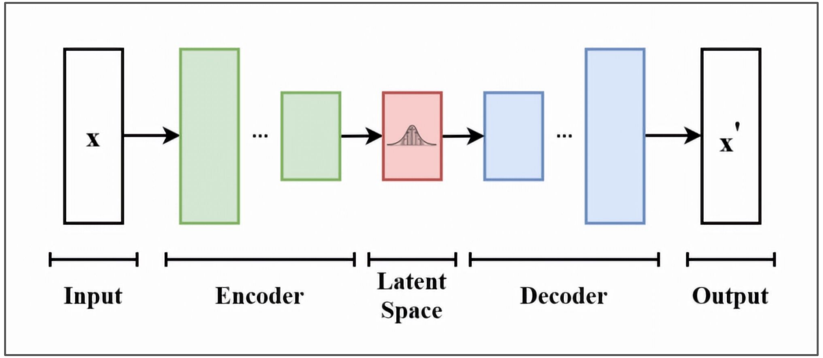

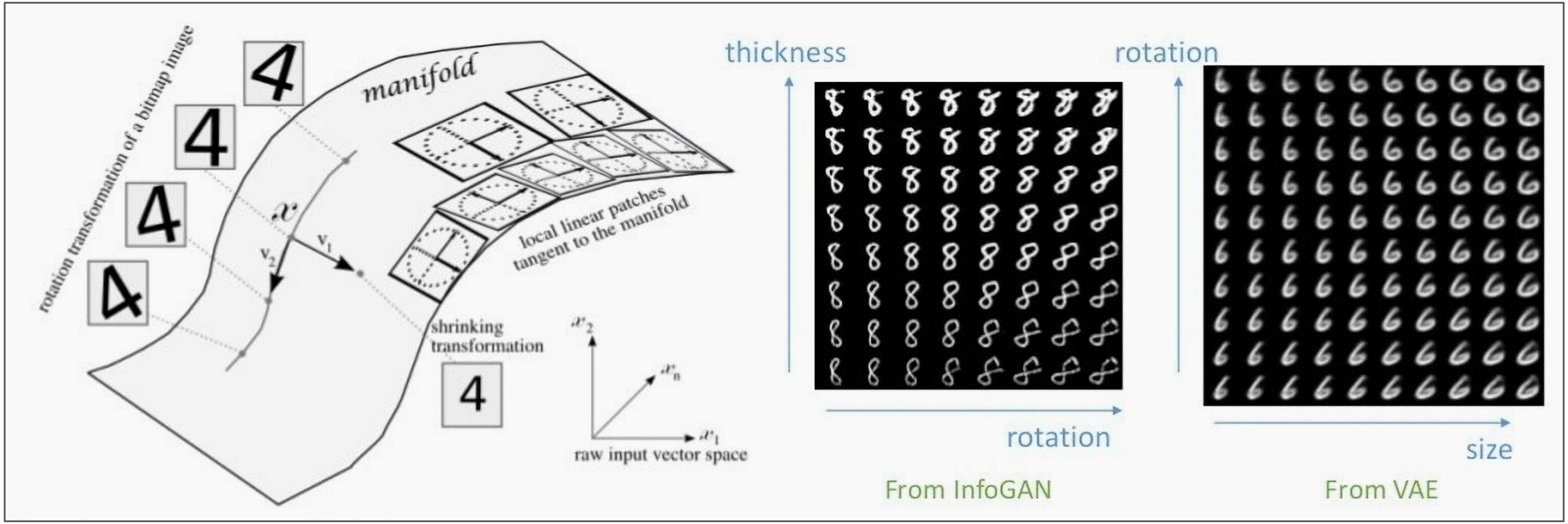

So why not enough?

Informal definitions

- on-manifold
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Informal categorization

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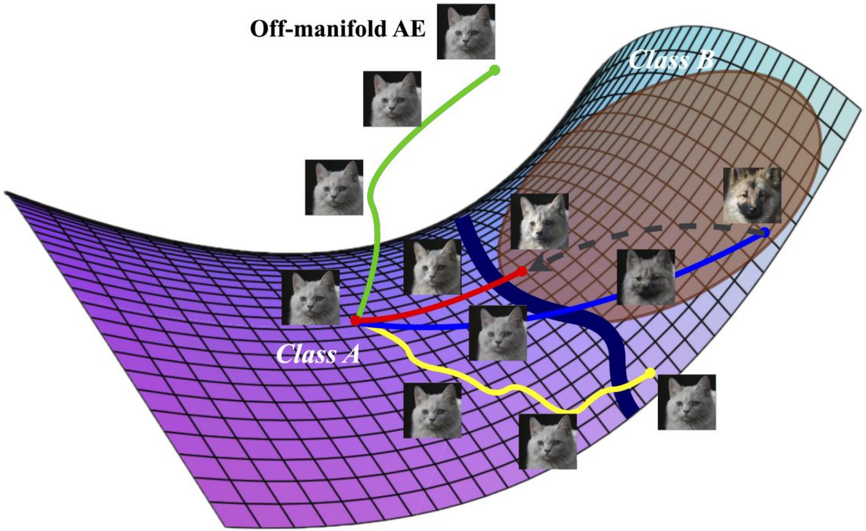
  - easy to distinguish
  - a failure to generate semantically coherent content



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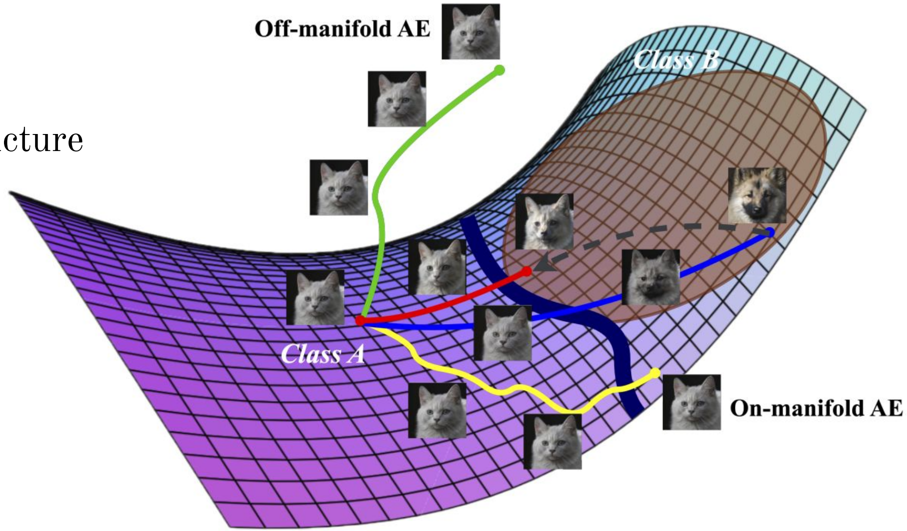

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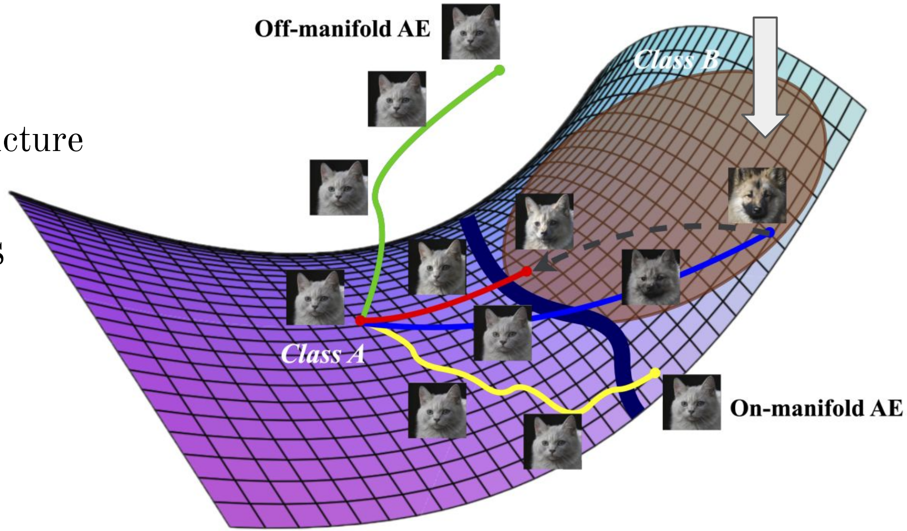

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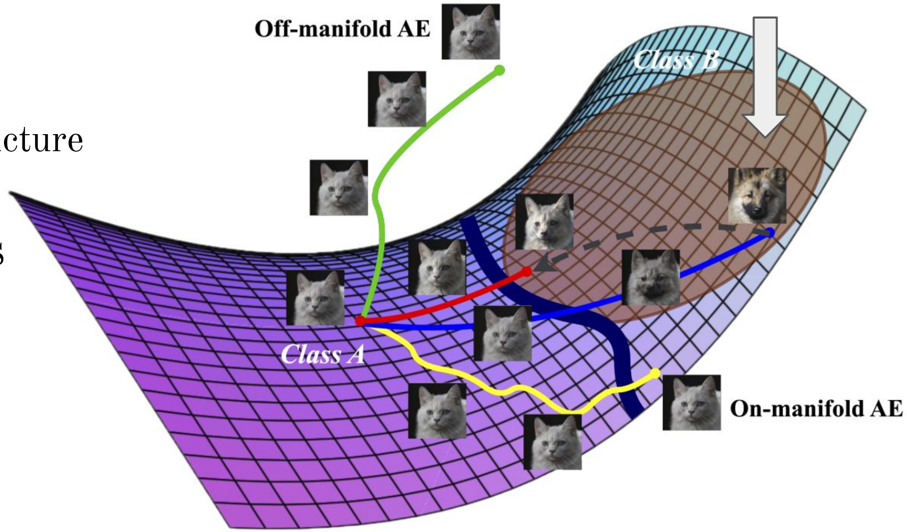
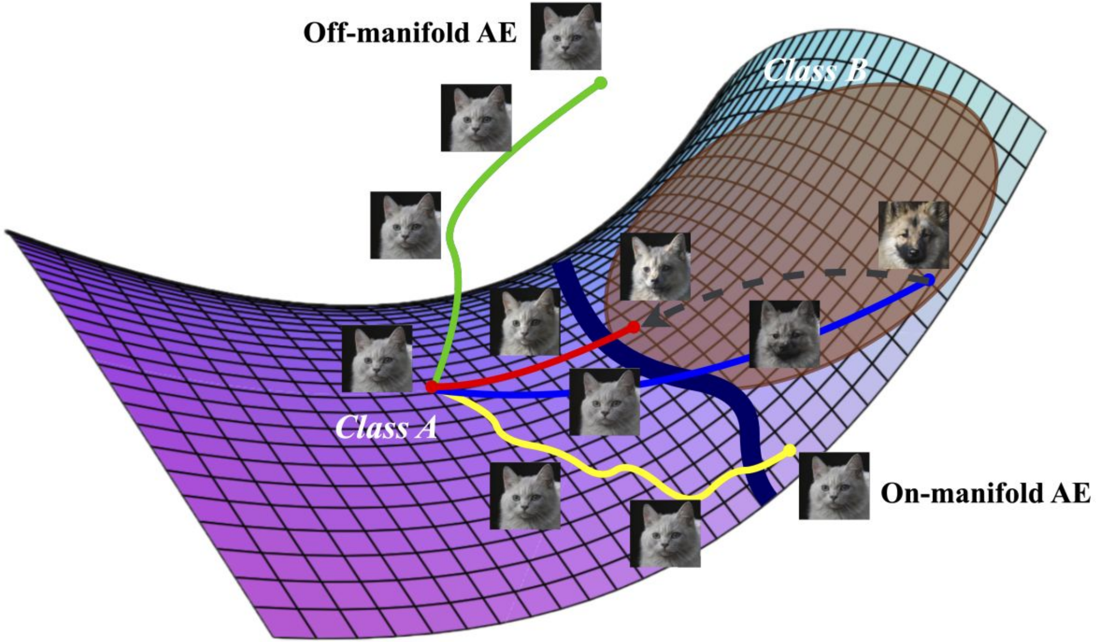
How to enforce on-manifold VCEs?

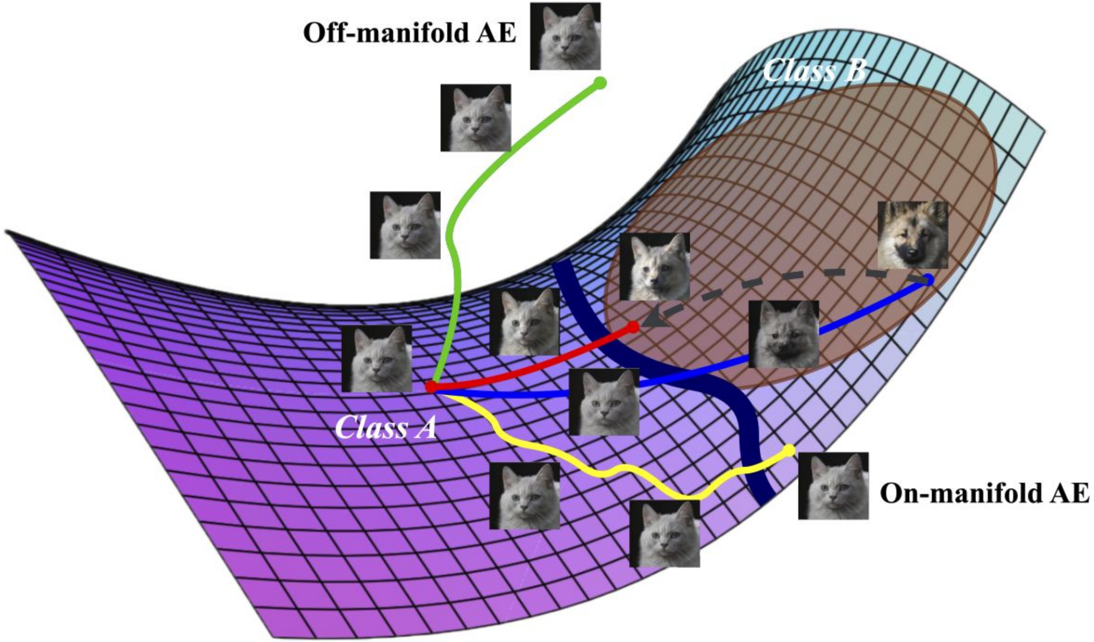


Problem restatement



Problem restatement

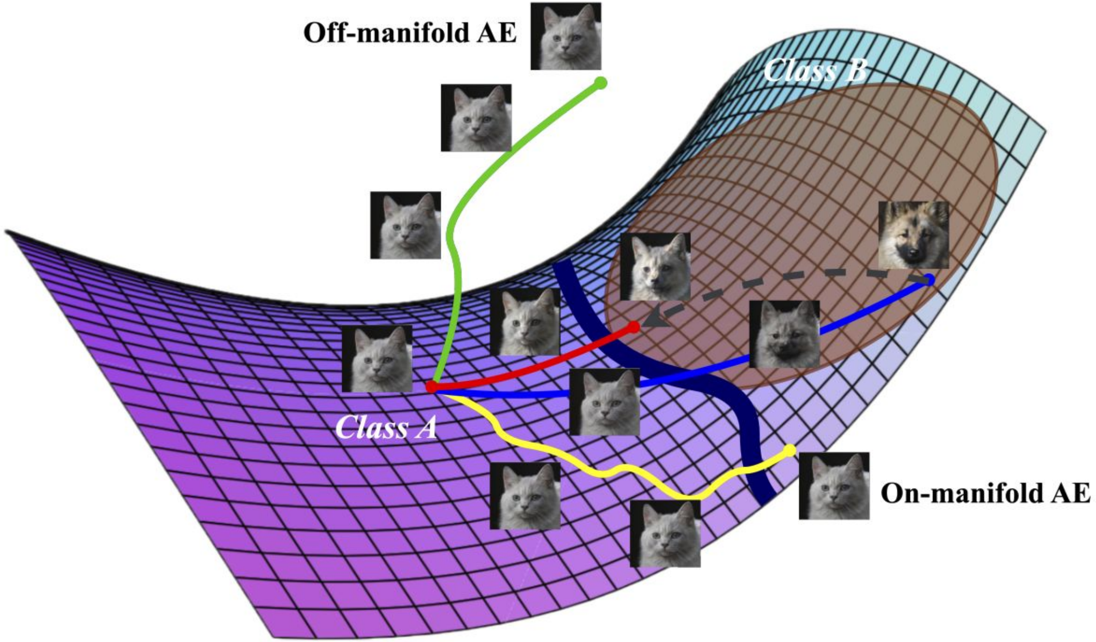
We are walking in a correct space



Problem restatement

We are walking in a correct space

We just don’t know how to walk



(Small) primer on differential geometry

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to find out how to walk (= to enforce a proper geometry)

How to borrow geometry?

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$\operatorname{rank}(J_g) = d$ full-rank

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$(\mathcal{M}, G)$ which leads to a Riemannian manifold

How to borrow geometry?

$$\langle u, v \rangle_z := \langle J_g(z)u, J_g(z)v \rangle_{G_X(g(z))}$$

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comparing u and v from tangent space of z

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$$G_Z(z) = J_g(z)^\top G_X(g(z)) J_g(z)$$

explicit metric on Z pulled from X

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- X's geometry is suboptimal - we already know

Picking *the right* geometry

$h \Leftarrow$ robustly trained model

Picking *the right* geometry

$$G_R(x) = \sum_{k=1}^{K} w_k J_{h_k}(x)^\top J_{h_k}(x), \quad w_k = \frac{1}{N_k}$$

h ⇦ robustly trained model

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robust

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kth layer number of elements

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Why?

Robustness via curvature regularization, and vice versa

Seyed-Mohsen Moosavi-Dezfooli, Alhussein Fawzi, Jonathan Uesato, Pascal Frossard, CVPR 2018

Which Models have Perceptually-Aligned Gradients? An Explanation via Off-Manifold Robustness

Suraj Srinivas, Sebastian Bordt, Hima Lakkaraju, NeurIPS 2023

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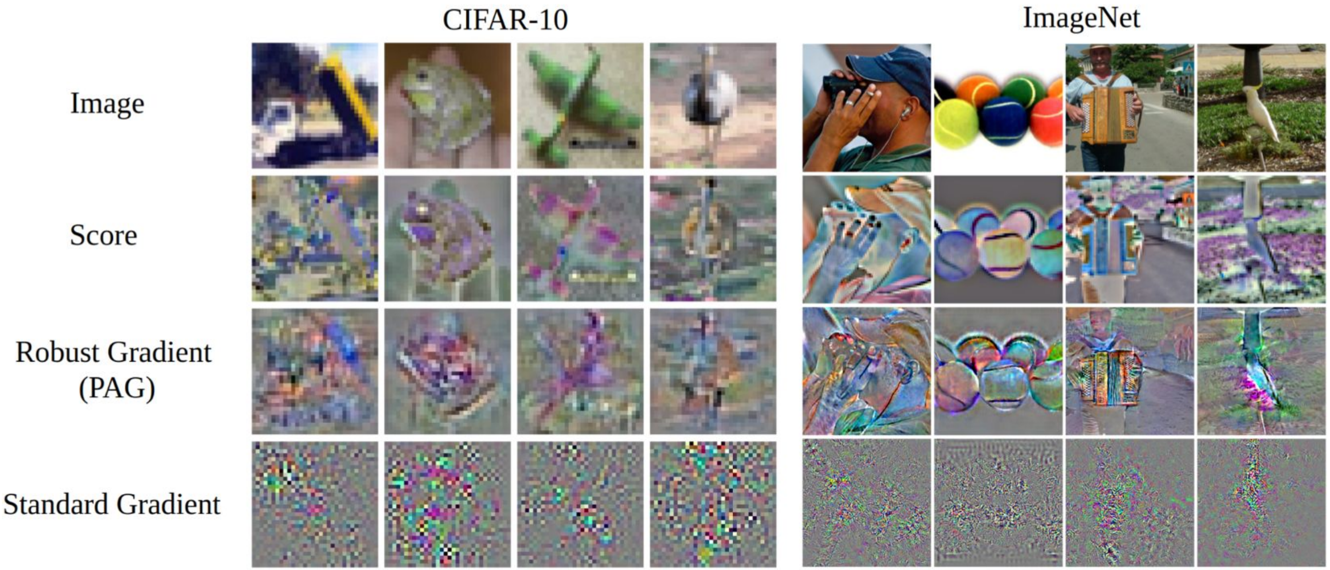

Figure 1: A demonstration of the perceptual alignment phenomenon. The input-gradients of robust classifiers ("robust gradient") are perceptually similar to the score of diffusion models [10], while being qualitatively distinct from input-gradients of standard models ("standard gradient"). Best viewed in digital format.

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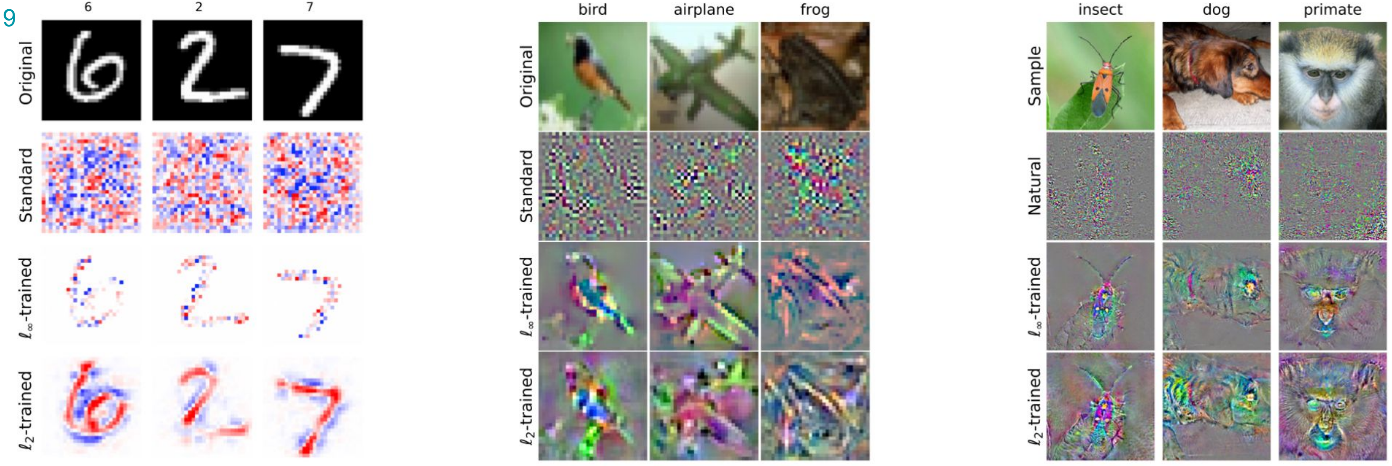

(a) MNIST (b) CIFAR-10 (c) Restricted ImageNet

Figure 2: Visualization of the loss gradient with respect to input pixels. Recall that these gradients highlight the input features which affect the loss most strongly, and thus the classifier's prediction. We observe that the gradients are significantly more human-aligned for adversarially trained networks – they align well with perceptually relevant features. In contrast, for standard networks they appear very noisy. (For MNIST, blue and red pixels denote positive and negative gradient regions respectively. For CIFAR-10 and ImageNet, we clip gradients to within ± 3 standard deviations of their mean and rescale them to lie in the $[0, 1]$ range.) Additional visualizations are presented in Figure 10 of Appendix G.

Robustness via curvature regularization, and vice versa

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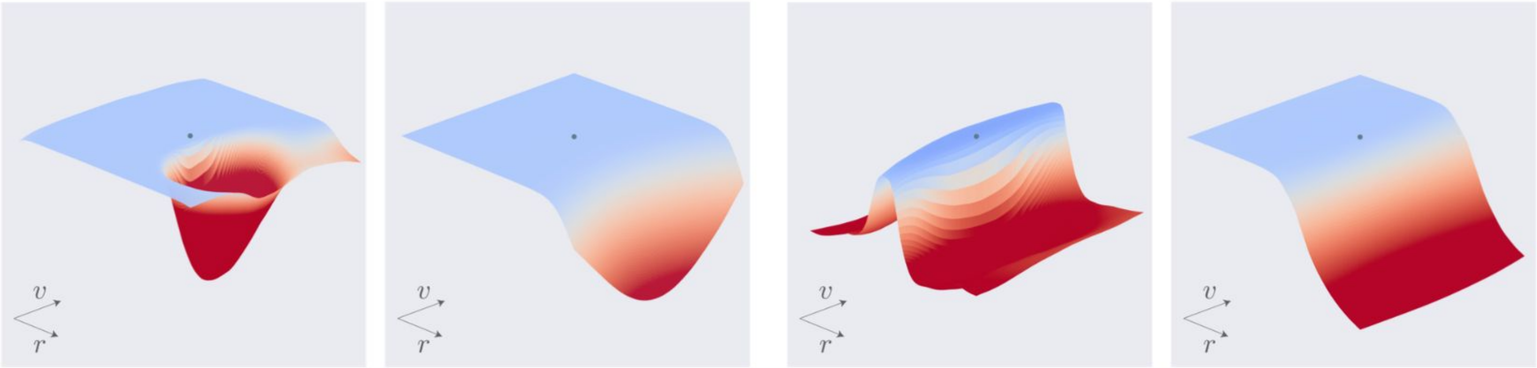

(a) Original (CIFAR-10) (b) Fine-tuned (CIFAR-10) (c) Original (SVHN) (d) Fine-tuned (SVHN)

Figure 3: Illustration of the negative of the loss function; i.e., $-\ell(s)$ for points s belonging to a plane spanned by a normal direction r to the decision boundary, and random direction v. The original sample is illustrated with a blue dot. The light blue part of the surface corresponds to low loss (i.e., corresponding to the classification region of the sample), and the red part corresponds to the high loss (i.e., adversarial region).

Adversarial Examples Are Not Bugs, They Are Features

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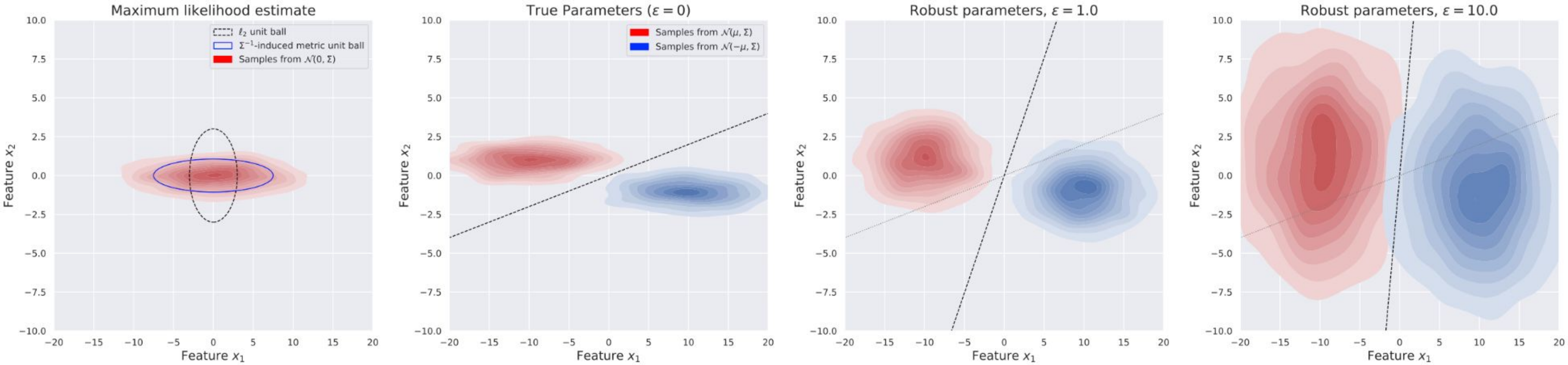

Figure 4: An empirical demonstration of the effect illustrated by Theorem 2—as the adversarial perturbation budget ε is increased, the learned mean μ remains constant, but the learned covariance "blends" with the identity matrix, effectively adding more and more uncertainty onto the non-robust feature.

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Perceptual Counterfactual Geodesics (PCG)

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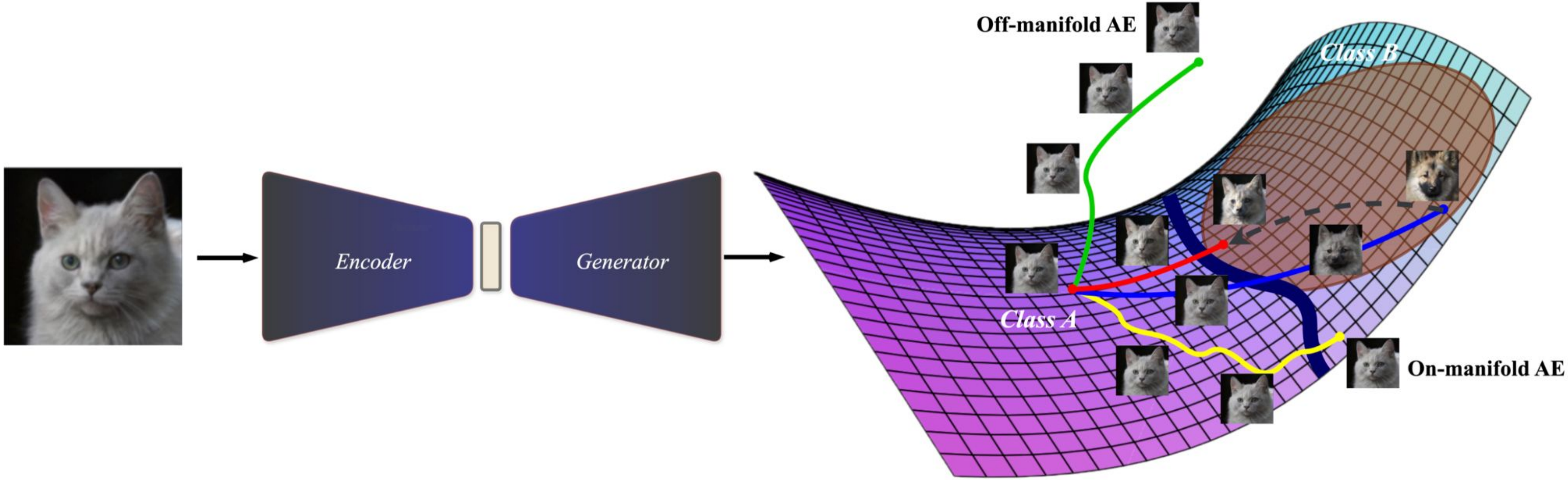
$$\mathcal{L}(z) = E_{\text{robust}}(z) + \lambda \cdot \ell\left(f(g(z_T)), y'\right)$$

PCG



Encoder
Generator
Off-manifold AE
Class B
Class A
On-manifold AE

Prior approaches

PCG

1. Consider a semantically meaningful space

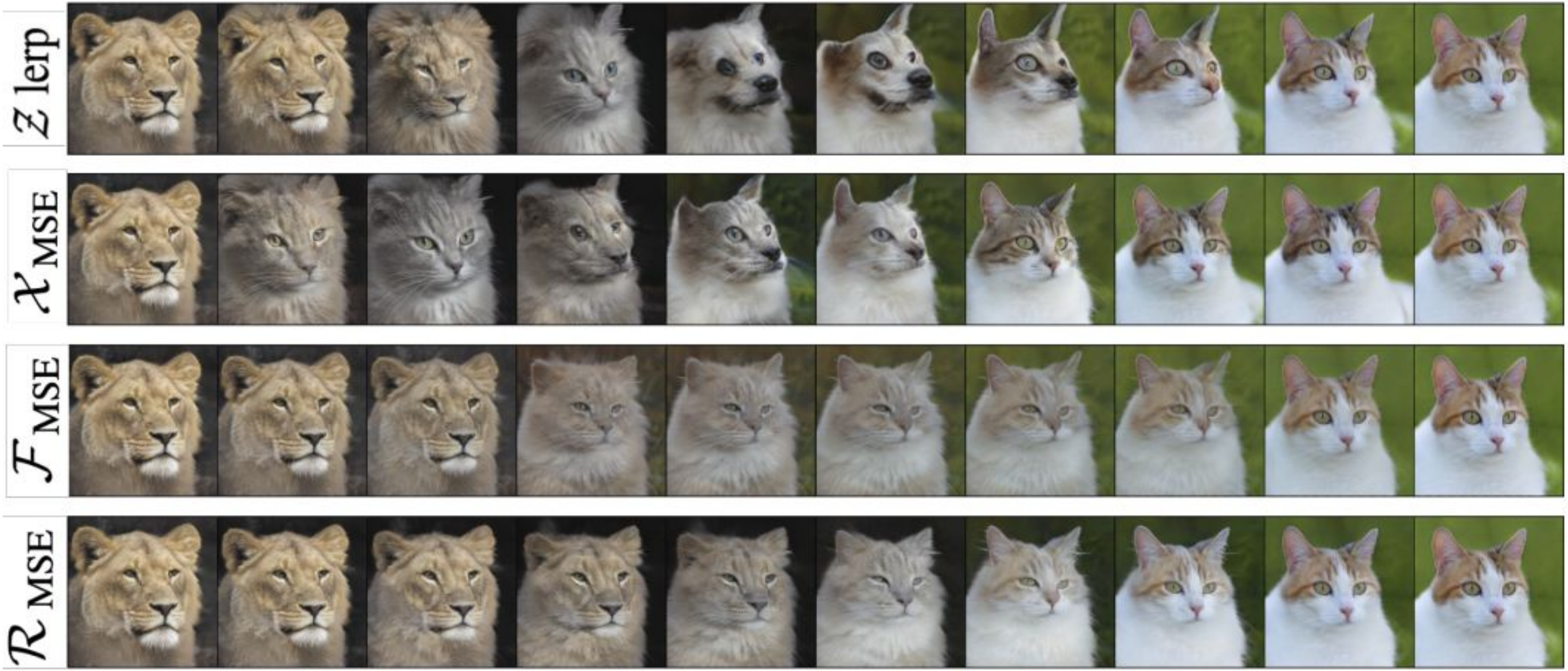
2. But **move with perception in mind**

Question: what replicates human perception well?

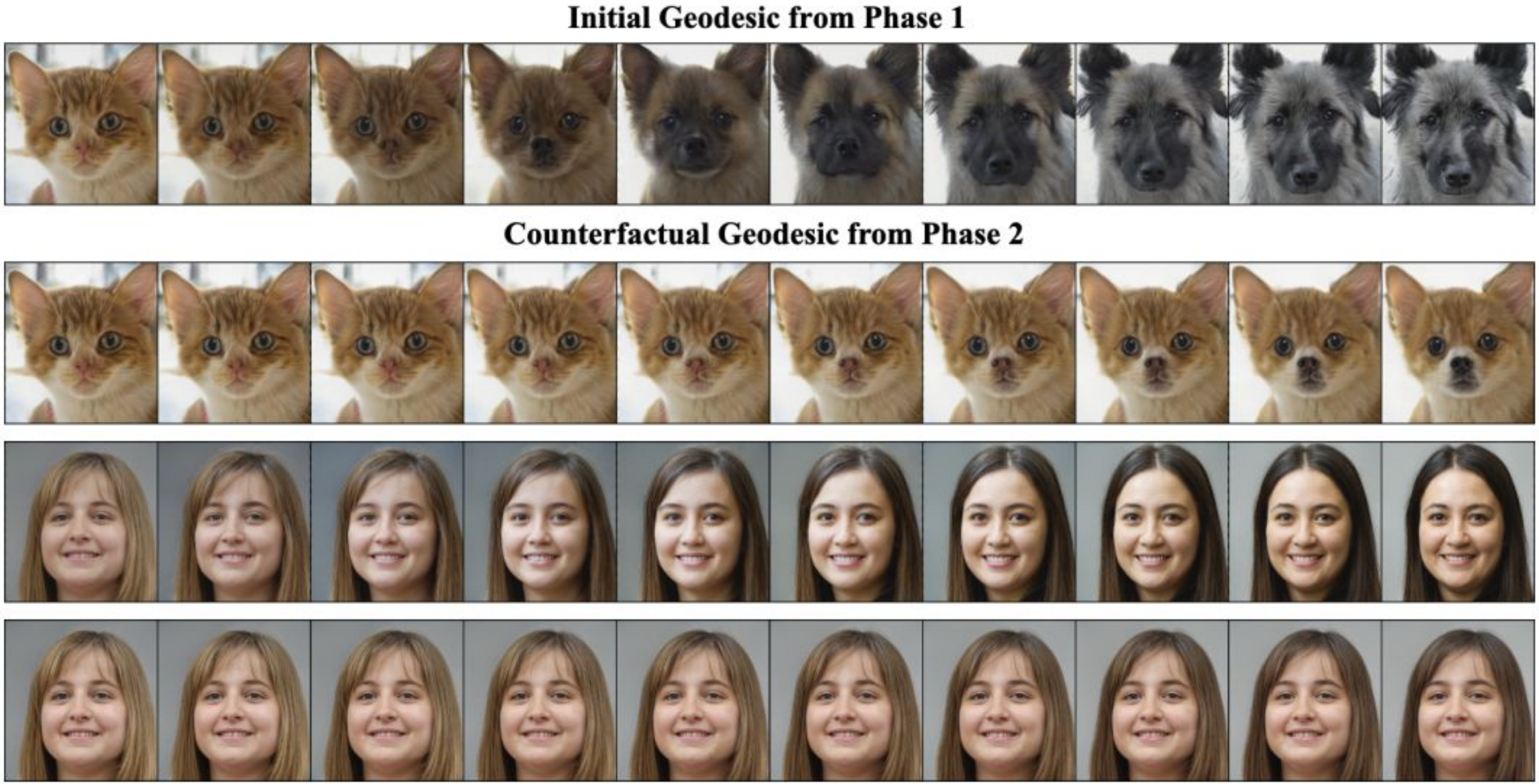
   1. quite good: LPIPS, since it aggregates convolution-based, often human-interpretable features
   2. even better: adversarial models, as they additionally incorporate robustness to perturbations, which improves representation geometry

Experiments

Effect of latent geometry on interpolation



Two-phase visualization

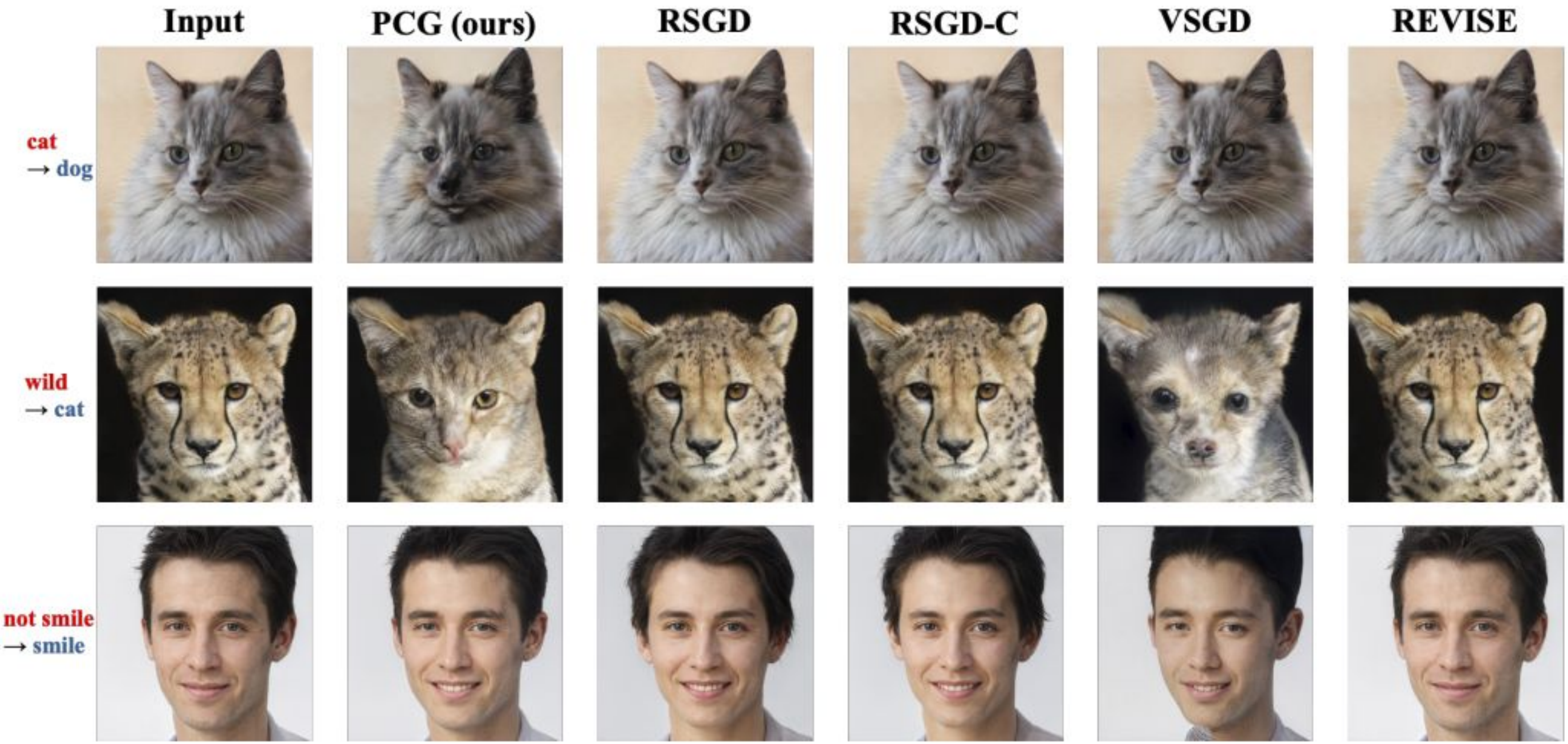


Quantitative comparison

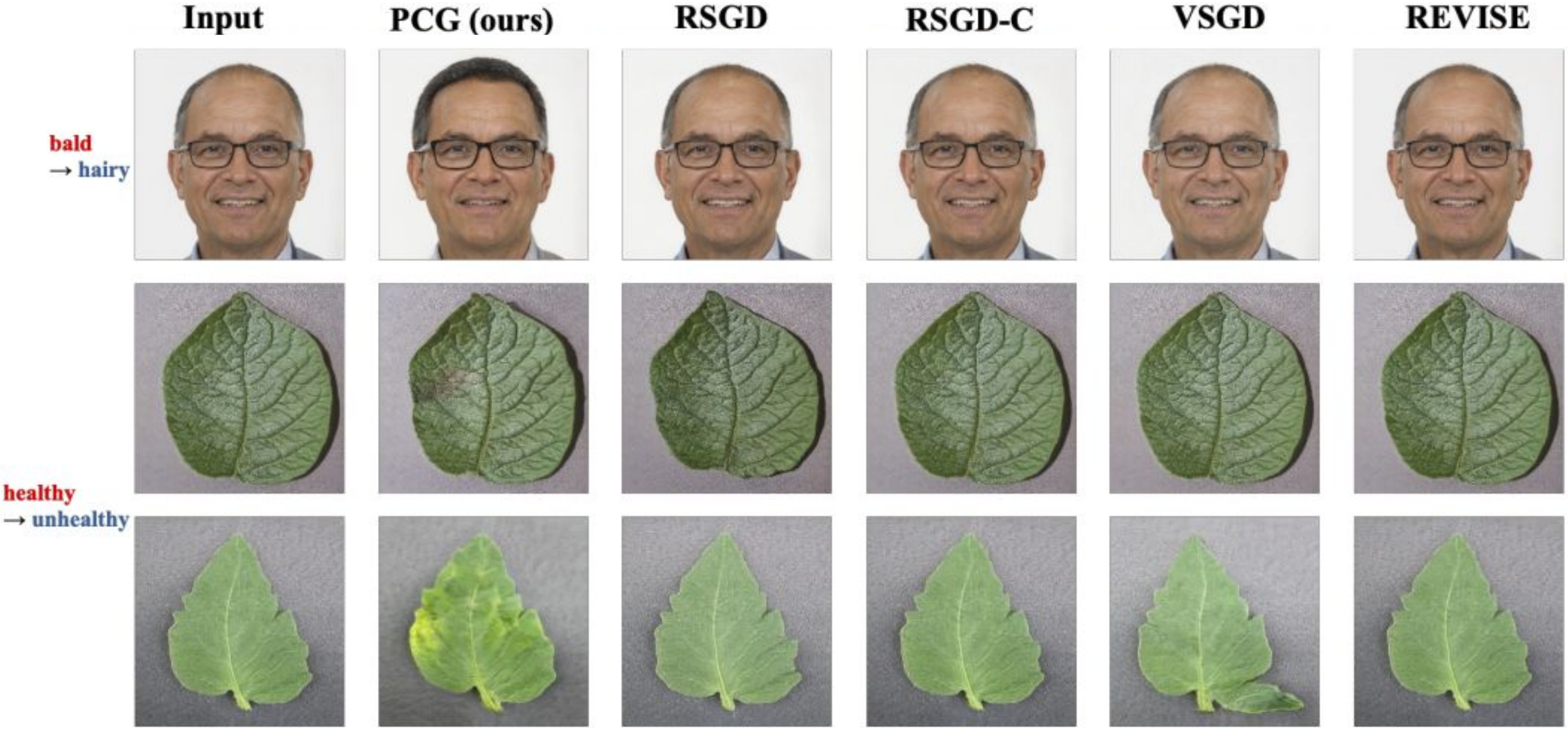
1. fair comparison is difficult
2. baselines adapted from tabular data

Method	AFHQ				FFHQ				PlantVillage			
	$\mathcal{L}_1$	$\mathcal{L}_2$	$\mathcal{L}_\mathcal{F}$	$\mathcal{L}_\mathcal{R}$	$\mathcal{L}_1$	$\mathcal{L}_2$	$\mathcal{L}_\mathcal{F}$	$\mathcal{L}_\mathcal{R}$	$\mathcal{L}_1$	$\mathcal{L}_2$	$\mathcal{L}_\mathcal{F}$	$\mathcal{L}_\mathcal{R}$
REVISE	1.20±0.12	**0.73**±0.18	1.08±0.10	2.70±0.05	0.82±0.08	**0.32**±0.13	0.82±0.08	2.78±0.06	0.50±0.13	**0.38**±0.15	0.96±0.06	2.87±0.07
VSGD	1.31±0.11	1.49±0.15	1.60±0.09	2.90±0.08	0.79±0.11	0.96±0.10	1.50±0.12	2.86±0.07	0.83±0.13	0.94±0.17	1.18±0.07	3.01±0.09
RSGD	0.85±0.08	1.32±0.09	0.70±0.07	1.85±0.05	0.61±0.05	0.84±0.07	0.61±0.04	2.41±0.05	0.78±0.08	0.82±0.11	0.54±0.05	2.28±0.04
RSGD-C	0.93±0.10	1.45±0.17	0.65±0.08	1.75±0.06	0.68±0.06	0.93±0.09	0.48±0.04	2.11±0.04	0.80±0.10	0.86±0.13	0.45±0.05	2.03±0.06
PCG (ours)	**0.79**±0.07	1.14±0.10	**0.53**±0.06	**0.31**±0.02	**0.42**±0.03	0.72±0.09	**0.39**±0.05	**0.22**±0.06	**0.36**±0.03	0.56±0.05	**0.34**±0.04	**0.20**±0.05

Qualitative comparison



Qualitative comparison



What if I reviewed it?

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3. Adaptation of tabular baselines

Weaknesses

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3. Lack of explanandum-related metrics
4. Some claims based purely on single examples
5. (they don’t cite us)

Thank YOU