TurtleBot Kinematic Model

Differential Drive Overview

A differential kinematic model describes how the velocities of a system evolve over time, focusing on the rates of change of position and orientation. In robotics, it's particularly useful for **non-holonomic systems (such as wheeled robots)** that have constraints on their motion, meaning they cannot move freely in all directions. ***The differential kinematic model expresses the relationship between control inputs (e.g., wheel velocities) and the resulting motion of the robot.***

The term "differential" refers to how small changes in the wheel velocities result in small changes in the robot's state. These changes are described using **differential equations**, which relate the velocities to the rate of change of the robot's position and orientation over time.

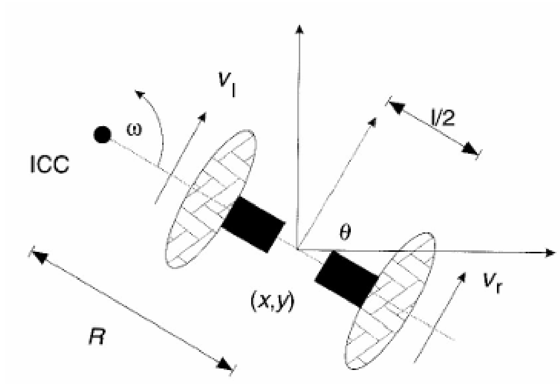
Differential Drive Robot

A differential drive robot typically has two wheels mounted on either side of the robot, and the robot's motion is determined by the speed of these two wheels.

Parameters Definition:

- v_r and v_l - linear velocities of the right and left wheels, respectively.

- v - linear velocity of the robot.
- w - the angular velocity (rate of change of orientation).
- L - the distance between the two wheels.
- R - signed distance from the Instantaneous Center of Curvature (ICC).
- v_x, v_y - the robot's velocity in its local frame.
- v_{gx}, v_{gy} - the robot's velocity in the global frame.
- θ - the orientation of the robot relative to the global frame.



Relationship Between Linear Velocity and Angular Velocity of Wheels:

$$v_r = r.w_r$$

$$v_l = r.w_l$$

Because the rate of rotation about the ICC must be the same for both wheels, we can write the following equations:

$$w.(R + L/2) = v_r$$

$$w.(R - L/2) = v_l$$

At any instance in time we can solve for R and w:

$$w = (v_r - v_l)/L$$

$$R = L/2.(v_r + v_l)/(v_r - v_l)$$

Deriving Equations to Solve for Robot’s Velocity in its Local Frame:

Since v_x is the direct moment,

$$v_x = w.R = \frac{v_r + v_l}{2}$$

Since the robot’s wheels are fixed in the forward direction, the wheels cannot exert any force in the sideways direction due to the non-holonomic constraint imposed by the wheels.

$$v_y = 0$$

Transformation to Global Frame:

Using the current orientation θ , the velocities in the global frame can be expressed as:

$$v_{gx} = v_x cos\theta - v_y sin\theta$$

$$v_{gy} = v_x sin\theta + v_y cos\theta$$

The angular velocity in the local frame is equal to angular velocity in the global frame.