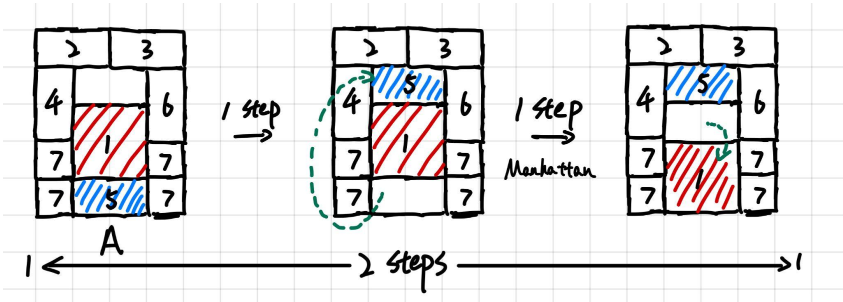
Huarongdao Rule: A piece can move from position A to position B if

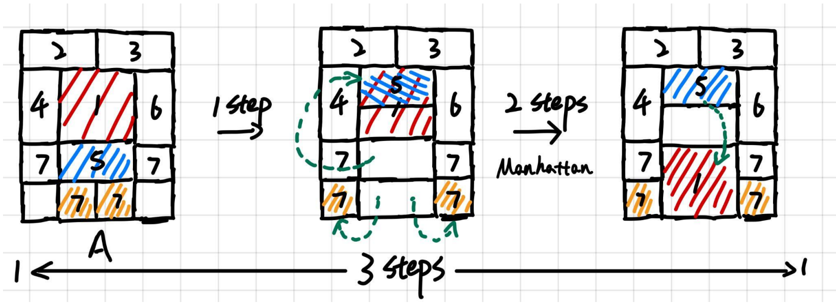
- Position B is empty.
- A and B are adjacent.

Relaxation:
The relaxation is based on the Manhattan heuristic: the pieces can still overlap, but except at the goal positions ((3, 1), (3, 2), (4, 1) and (4,2)), such that if there are some pieces on (in other words, overlapping with) the goal positions (except the 2x2 block), we need one extra step to move them away for the 2x2 block to move to the goal state. After we move the pieces away, we then use the regular Manhattan method to move the 2x2 block to the goal. Please see below for examples:

Example 1: State A has a heuristic $h(A) = 2$



Example 2: State A has a heuristic $h(A) = 3$



Heuristic Calculation:
At a given time, if the top left corner of Caocao is at position (a, b), the heuristic will be:

- $h(n) = (3 - a) + |1 - b|$, if there is no other piece (except Caocao) on the goal positions.
- $h(n) = (3 - a) + |1 - b| + 1$, if there is at least one piece (except Caocao) on the goal positions.

Proof of Admissible:

- Let $h_1(n)$ be the Manhattan heuristic value for a state, and $h(n)$ be the heuristic value of our heuristic.
- It is clear that:
    - $h(n) = h_1(n)$, if there is no other piece (except Caocao) on the goal positions.
    - $h(n) = h_1(n) + 1$, if there is at least one piece (except Caocao) on the goal positions.
- We have known that the Manhattan heuristic is admissible, meaning that $0 \leq h_1(n) \leq h^*(n)$, and therefore, we only need to analyze the case when there is at least one other piece on the goal position.
- In the original rule, to reach the goal state, Caocao must wait for the goal positions to be empty. If there is(are) some piece(s) on the positions, we have to take at least one step to move them away. In addition, although the goal positions are cleaned up, Caocao must take at least the Manhattan Distance to move to the goal state. All in all, Caocao must take at least 1 + Manhattan Distance ($1 + h_1(n)$) to reach the goal state in this case.
- Therefore, $0 \leq h_1(n) \leq h(n) \leq h^*(n)$, QED.

Proof of Domination:

- $h(n) = h_1(n)$, if there is no other piece (except Caocao) on the goal position.
  $h(n) = h_1(n) + 1 > h_1(n)$, if there is at least one piece (except Caocao) on the goal position.
- Therefore,
    - $h(n) >= h_1(n)$ for every state n.
    - $h(n) = h_1(n)$ when there is no other piece (except Caocao) on the goal position.
- QED