Technical Appendix — Torque & Gate Coupling (v 2.5)

See Buoyancy Algebra v 2.5 for overview.



Figure 1. Convergence Under Repair Interventions

When $\tau = \kappa \cdot I \cdot \theta$ exceeds $\sigma = E \cdot N \cdot W$, each small edit (R/C/F) guarantees a negative $\Delta \theta$ until gates pass and $\theta < \delta$.

Technical Appendix — 2T Torque Addendum (v2.4)

Definitions

- $0 = \{R, C, F, \rho\}$ with $\rho^3 = I$; $\beta \in [0,1]$; $\Pi = H/(1 + P)$.
- H: harm (evidence-weighted); P: provocation/agitation factor (0–10 default).
- $\theta \in [0, \infty)$ misalignment; $\tau = \kappa \cdot I \cdot \theta$; $\sigma = E \cdot N \cdot W$ with $E, N, W \in \mathbb{R}^+$.
- Repair step applies a small operator move $\Delta O \in \{\Delta R, \Delta C, \Delta F\}$.

Spiral Update & Convergence

- Define $V(\theta) = \theta$ as a Lyapunov-like measure. If $\tau > \sigma$, select ΔO that yields $\Delta \theta < 0$.
- Assume lower-bounded steps $\Delta\theta \le -\eta$ for $\theta \ge \delta$, with $\eta > 0$. Then θ decreases monotonically and reaches $\theta < \delta$ in at most $\lceil (\theta_0 \delta)/\eta \rceil$ steps.

Finite-Step Realignment (Theorem)

- If (i) $\kappa \cdot I \geq \sigma + \epsilon$ throughout the trajectory, and (ii) each ΔO satisfies $\Delta \theta \leq -\eta$ (task-dependent but bounded below), then $\theta_- t$ reaches δ in finitely many steps and the gate set G^* passes.
- Proof sketch: Monotone descent in V with step size $\geq \eta$ implies finite termination; gate feasibility follows from coupling bullets (operators reduce Π or E, N, or W along the path). (See Figure 2 in the One-Pager.) and Torque-Based Fix (p.5) for the convergence sketch and toy code.

Algorithms (2T-A ... 2T-D)

- 2T-A (Greedy): pick the operator with largest expected $|\Delta\theta|$ under current σ .
- 2T-B (Balanced): alternate R/C/F in fixed ratios until $\tau > \sigma$ ceases.
- 2T-C (Well-breaker): when W is high, favor C, then F to shrink σ before R.
- 2T-D (Budgeted): impose max_steps; if unmet, return counterfactual notes.
- Step-size selection: choose $\Delta 0$ to maximize expected $|\Delta \theta|$ subject to a small-step budget; if $\tau \leq \sigma$, prioritize $C \rightarrow F$ to reduce σ before R.

Measurement & Reporting

- Publish θ _min, θ _max, δ and sampling procedure; log (β , Π) and gate verdicts per step.
- Rater protocol: two raters + α/ICC ; target $\alpha \ge 0.67$ (or $ICC \ge 0.60$).
- Stability checks: verify ρ -mirror symmetry post-repair (e.g., $r_\rho = ||\rho(H^*) H||$) and log commutator residuals (e.g., $r_[R,C] = ||[R,C]H||$).

Torque-Based Fix (finite-step repair)

We treat editing as controlled realignment: a small operator set $O = \{R, C, F, \rho\}$ acts on text; torque $\tau = \kappa \cdot I \cdot \theta$ must exceed stickiness $\sigma = E \cdot N \cdot W$. When $\tau > \sigma$, each step reduces misalignment θ and the process terminates in finitely many steps, with outputs audited by gates (4/6/7/8/12/15), rater agreement targets $(\alpha \ge 0.67 \text{ or ICC} \ge 0.60)$, and symmetry checks $(\rho^3 = I)$.

Key parameters (defaults/notes)

Operators: R (Re-express), C (Contextualize), F (Frame), ρ (mirror/symmetry check).

Algorithm 2T-A (Greedy realignment)

Reporting note. Log (β, Π) and gate verdicts per step; verify $\|\rho(H^*) - H\|$ and record $\|[R,C]H\|$.

Gloss: Unequal mass (m_e via \dagger) raises $\Delta\beta$; apply directed correction until proportionality aligns (Π drops into pass range).

$$\{\bigcirc A \mid low ; \bigcirc B \mid HIGH \} \cdot (\Delta\beta\uparrow) \rightarrow (A \mid A \rightarrow correction) \rightarrow \# (A \mid A \rightarrow$$

C. Symmetry-break \rightarrow justice vector

Gloss: Time steps by rows; human/AI roles by columns; audit channel closes the run.

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B. Dual-node matrix (roles as columns)

Gloss: Run mirror check; apply small edits; re-test β . Under $\tau > \sigma$, θ decreases (finite-step guarantee). (see Theorem above; see Algorithm 2T-A).

A. Mirror loop test

$$\beta \rightarrow \textcircled{\longrightarrow} \bullet \qquad \Pi = H/(1+P) \rightarrow \textcircled{\square} \quad \tau > \sigma \rightarrow \rightarrow \quad C \rightarrow \textcircled{\bigcirc} \diamondsuit \quad F \rightarrow \textcircled{\bigcirc} \heartsuit \qquad \text{Audit channel} \rightarrow \#$$

Operator mapping (recap).

6. Emoji Math Examples (spec-conformant)

Encode & audit: compute β , Π , θ ; run gate checks.

Compute forces: $\tau = \kappa \cdot I \cdot \theta$, $\sigma = E \cdot N \cdot W$.

If $\tau \le \sigma$: reduce σ (e.g., $C \downarrow N$, $F \downarrow E$) and/or raise κ , I.

If $\tau > \sigma$: apply a small edit $\Delta O \in \{R, C, F\}$ guaranteeing $\Delta \theta < 0$.

Iterate: re-audit gates; stop when $\theta < \delta$ and required gates pass (budgeted variant 2T-D returns counterfactuals if not achieved).

Finite-step realignment (sketch)

Let $V(\theta) = \theta$. Assume (i) torque margin $\kappa \cdot I \ge \sigma + \epsilon$ and (ii) bounded progress per edit $\Delta \theta \le -\eta$ < 0 while $\theta \ge \delta$. Then θ decreases monotonically to < δ in at most $\Gamma(\theta_0 - \delta)/\eta$ steps.

Example (memo repair)

Zero-tolerance memo fails Gate-4 (high Π = 1.8; θ = 0.41). C: add scope/context $\rightarrow \Pi \downarrow$, N \downarrow . F: balance trade-offs \rightarrow E \downarrow . R: calibrate language. Result: Π : 1.8 \rightarrow 0.9, θ : 0.41 \rightarrow 0.19 in \sim 3 steps; gates pass.

Python stub (toy demo)

import math

```
E=3.0, N=4.0, W=2.0, eta=0.10, delta=0.20, max steps=50):
"""Greedy 2T-A toy: lowers theta until theta < delta or steps exhausted.
H,P -> Pi; E,N,W -> sigma; tau = kappa*inertia*theta."""
Pi = H / (1.0 + P)
theta = float(theta0)
kappa = float(kappa); inertia = float(inertia)
E = float(E); N = float(N); W = float(W)
sigma = E * N * W
steps = 0
while theta > delta and steps < max steps:
steps += 1
tau = kappa * inertia * theta
if tau <= sigma:</pre>
# Reduce stickiness and/or raise gain
N = max(0.0, N - 0.5) \# add context (C)
E = max(0.0, E - 0.3) # reframe (F)
sigma = E * N * W
kappa *= 1.10 # modest gain increase
# Guaranteed minimum improvement
drop = eta + (0.05 if tau > sigma else 0.02)
theta = max(0.0, theta - drop)
return {
"steps": steps,
"theta final": round(theta, 3),
"converged": theta < delta,
```

def torque repair(H, P, theta0=0.62, kappa=2.0, inertia=1.0,

```
"Pi": round(Pi, 3),

"tau_last": round(kappa * inertia * theta, 3),

"sigma_last": round(sigma, 3)
}

# Example:
# print(torque_repair(H=0.8, P=10, theta0=0.62))
```