Logistics Enhanced Blood Bank Problem

Logistics Project

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1 Introduction: The Problem

A blood bank wants to determine the least expensive way to transport available blood donations from Pittsburgh and Staunton to hospitals in Charleston, Roanoke, Richmond, Norfolk and Suffolk. The figure below shows the logistics network that can be used to transport blood donations and the unit cost of shipping along each arch.

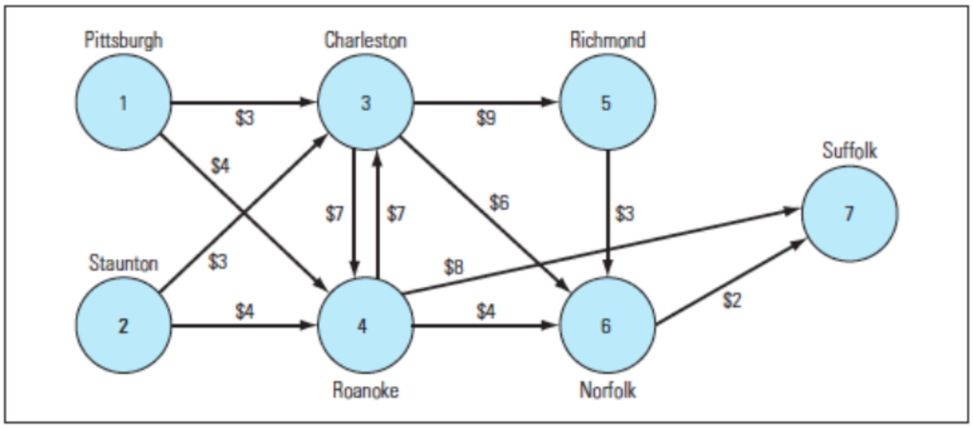


Figure 1: Logistics Network

The courier service used by the blood bank charges a fee of $125 each time it makes any of these bows trips, regardless of how many units of blood are transported. The van used by the courier service can carry a maximum of 1,000 units of blood. Suppose Pittsburgh has 600 units of blood type O positive (O+) and 800 units of blood type AB available, while Staunton has 500 units of O+ and 600 units of AB available. The number of units of each are reported the blood types needed in the various hospitals below:

Ospedale	0+(b_i^1)	$A_B(b_i^2)$
Pittsburgh	600	800
Stauton	500	600
Pittsburgh	600	800
Charleston	100	200
Roenoke	100	100
Richmond	500	300
Norfolk	200	500
Suffolk	150	250

2 Project Requirements

1. Propose an ILP model to formulate the problem of meeting hospital demands with minimal total cost.
2. Implement the model using the modeling AMPL language and solve it using the CPLEX optimization solver.
3. Consider the variant of the problem in which the blood bank has a budget of $24000 (for both transportation and fixed costs), and wants to meet the hospital requirements of using the minimum total number of arcs.
4. Implement the variant and compare the determined optimal solution to the one found in step 2

3 Project Development

3.1 Mathematical Formulation

The first step toward solving the presented problem is to write the mathematical model that correctly describes it. Let G=(N,A) be the oriented graph where N is the set of vertices and A is the set of arcs; in particular, N={Pittsburgh, Stauton, Charleston, Roenoke, Richmond, Norfolk, Suffolk} is the set of hospitals, while A is the set of possible pairs of hospitals where blood units are transported.

Each node in the network j∈N is associated with an activation cost cij .

- fij =125 for each (i,j) ∈N

- uij = 1000 for each (i,j) ∈N

Consider these additional input data:
1) K is the number of commodities
2) $B_{i,k}$ is the balance of node i for commodity k, for each I belonging to N, k=1,..., K
3) $C_{i,j,k}$ is the unit transportation cost along (i,j) for commodity k, $\forall (i, j) \in A$, k=1,...,K
We than define the following decision variables:

- $y_{i,j}$ =1 if the link (i,j) is opened, 0 otherwise

- $x_{i,j,k}$ ≥ 0 for each (i,j) $\in$N, $k = 1, 2...K$

Studying the objective function of the problem:

$$\boldsymbol{min \sum_{i,j \in A} f_{i,j} y_{i,j} + \sum_{i,j \in A} \sum_{k=1}^{k} c_{i,j}, x_{i,j}^{k}}$$

The mathematical model for solving the optimization problem is as follows.

$$\begin{array}{lll}
\min \underbrace{\sum_{(i,j)\in A} f_{ij}\, y_{ij}}_{\text{fixed cost}} + \underbrace{\sum_{(i,j)\in A} \sum_{k=1}^{K} c_{ij}^{k}\, x_{ij}^{k}}_{\text{transportation cost}} & & \\
\sum_{(j,i)\in \mathrm{BS}(i)} x_{ji}^{k} - \sum_{(i,j)\in \mathrm{FS}(i)} x_{ij}^{k} = b_i^k & \forall i \in N, & k = 1, \ldots K \\
0 \leq x_{ij}^{k} \leq u_{ij}^{k} & \forall (i,j) \in A, & k = 1, \ldots K \\
\sum_{k=1}^{K} x_{ij}^{k} \leq u_{ij}\, y_{ij} & \forall (i,j) \in A & \circledast \\
y_{ij} \in \{0, 1\} & \forall (i,j) \in A &
\end{array}$$

Figure 2: Mathematical Model

Let us first start with the explanation of the objective function; it consists of two addends:

1. first part of the formula (sum of $F_{i,j}$*$Y_{i,j}$) represents the fixed cost, calculated by multiplying the fixed cost associated with each link (i,j) by the total arcs used (in fact $Y_{i,j}$ = 1 if the link (i,j) was used, otherwise 0)

2. second part of the formula (sum of $C_{i,j,k}$*xi,j,k) represents the total transportation cost, calculated by multiplying the unit cost associated with each (i,j) by the the amount of commodity k that will be sent along each (i,j) leg.

Since we have to minimize the total cost, this is a minimum problem with respect to the specified objective function, which sums the total transportation cost and the total fixed cost.
Let us now analyze the constraints one by one:

- flow conservation constraints: BS(i) is the "backward star" of node I while FS(i) is the "forward star" of node i. It represents the constraint for which :

  1. the entering flow minus the leaving flow of each node for commodity 1 must equal the balance of the node i for commodity 1 (i.e., the demand for K=1 in node i)
  2. the entering flow minus the leaving flow of each node for commodity 2 must be equal to the balance of the node i for commofity 2 /that is, the demand for K=2 in node i)for the two supply nodes, this constraint is an inequality.

- Third and Fourth constraints : these constraints are the ones linking the K commodities. These are capacity constraints and also link the transportation decisions ($X_{i,j,k}$) with the design decision ($y_{i,j}$). The sum of the amount of commodities (both) transported along each route (i,j) cannot exceed the global capacity $u_{i,j} = 1000$ multiplied by $y_{i,j}$ ($y_{i,j} = 1$ if (i,j) is used

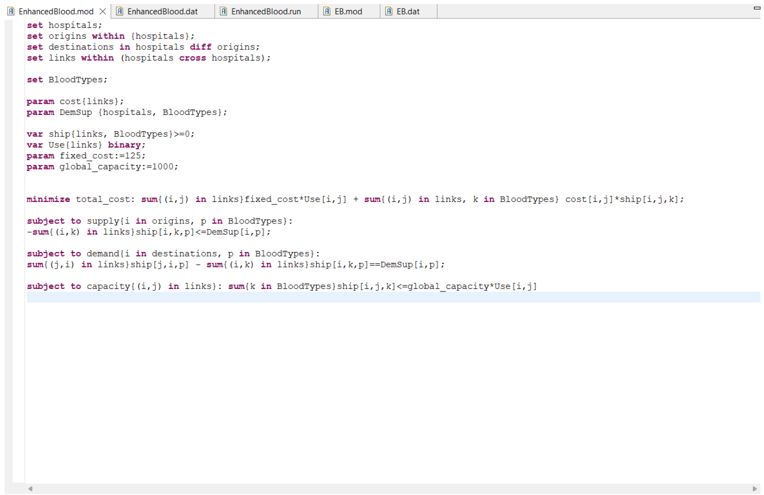
- Fifth constraint: $y_{i,j}$ has been defined as a binary variable

3.2 AMPL Implementation & Cplex Solver

The second requirement for our project was to implement the model with the modeling language AMPL and solve it with the optimization solver CPLEX solver.
We used the following functions to complete the request submitted to us.

```
EnhancedBlood.mod | EnhancedBlood.dat | EnhancedBlood.run | EB.mod | EB.dat

set origins:= Pittsburgh Stauton;
set destinations:= Charleston Roenoke Richmond Norfolk Suffolk;

set links:=
(Pittsburgh, Charleston)
(Pittsburgh, Roenoke)
(Stauton, Charleston)
(Stauton, Roenoke)
(Charleston, Roenoke)
(Roenoke, Charleston)
(Charleston, Richmond)
(Charleston, Norfolk)
(Roenoke, Norfolk)
(Roenoke, Suffolk)
(Richmond, Norfolk)
(Norfolk, Suffolk);

set BloodTypes:= Zero AB;

param cost:=
Pittsburgh, Charleston 3
Pittsburgh, Roenoke 4
Stauton, Charleston 3
Stauton, Roenoke 3
Charleston, Roenoke 7
Roenoke, Charleston 7
Charleston, Richmond 9
Charleston, Norfolk 6
Roenoke, Norfolk 4
Roenoke, Suffolk 8
Richmond, Norfolk 3
Norfolk, Suffolk 2;

param DemSup:
                 Zero  AB:=
Pittsburgh       600   800
Stauton          500   600
Charleston       100   200
Roenoke          100   100
Richmond         500   300
Norfolk          200   500
Suffolk          150   250;
```

Figure 3: 1. AMPL CPLEX Solver .dat data


EnhancedBlood.mod | EnhancedBlood.dat | EnhancedBlood.run | EB.mod | EB.dat

```
set hospitals;
set origins within {hospitals};
set destinations in hospitals diff origins;
set links within (hospitals cross hospitals);

set BloodTypes;

param cost{links};
param DemSup {hospitals, BloodTypes};

var ship{links, BloodTypes}>=0;
var Use{links} binary;
param fixed_cost:=125;
param global_capacity:=1000;


minimize total_cost: sum{(i,j) in links}fixed_cost*Use[i,j] + sum{(i,j) in links, k in BloodTypes} cost[i,j]*ship[i,j,k];

subject to supply{i in origins, p in BloodTypes}:
-sum{(i,k) in links}ship[i,k,p]<=DemSup[i,p];

subject to demand{i in destinations, p in BloodTypes}:
sum{(j,i) in links}ship[j,i,p] - sum{(i,k) in links}ship[i,k,p]==DemSup[i,p];

subject to capacity{(i,j) in links}: sum{k in BloodTypes}ship[i,j,k]<=global_capacity*Use[i,j]
```

Figure 4: 1. AMPL CPLEX Solver .mod data

Once the mathematical model has been defined, to obtain a solution to the problem it is necessary to implement it. For this purpose, the AMPL programming language was used, as required by the problem along with the CPLEX solver.

AMPL requires creating two files: a model (.mod) file in which the parameters, variables of the problem and the model describing it are given, and a data (.dat) file where the data characterizing that problem are specified.

After creating the files required for implementation, which can be consulted separately and of which we put only one example, we analyzed and studied the output of what was required.The following image depicts the output of AMPL when asked to solve the problem where the objective function is given which is 21000 i.e., the minimum cost obtained taking into account the constraints entered. The Mixed Integer Programming model (MIP) is also specified with the number of iterations performed by the system. It is also specified that no Branch and Bound nodes are used.:

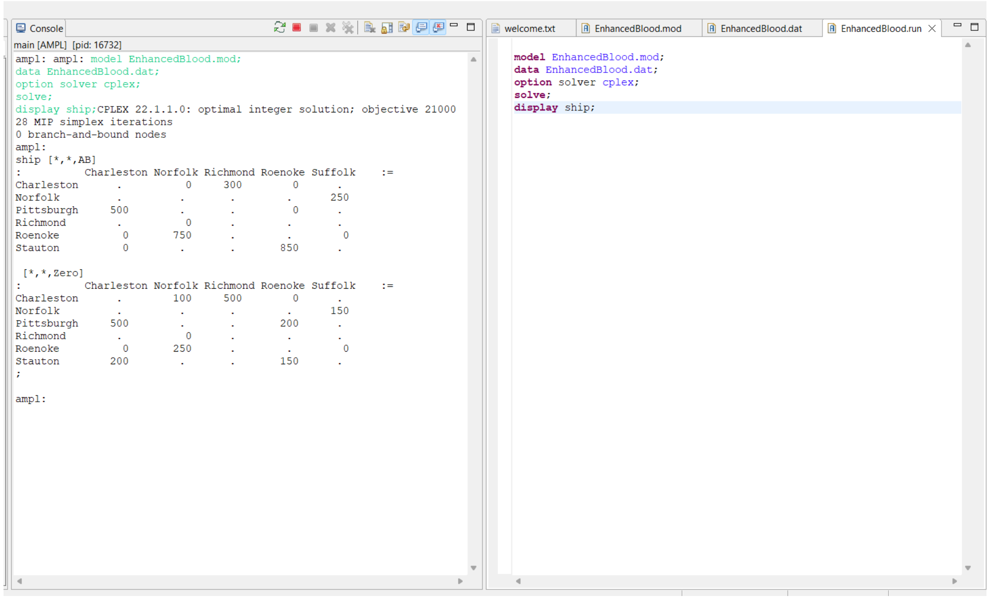


Figure 5: Output AMPL CPLEX Solver

Shown above is the output provided by AMPL where $x_{i,j,k}$ (in AMPL the ship variable) or the number of units transported from one hospital to another is specified for blood type 0+ and AB.

```
Use :=
Charleston  Norfolk     1
Charleston  Richmond    1
Charleston  Roenoke     0
Norfolk     Suffolk     1
Pittsburgh  Charleston  1
Pittsburgh  Roenoke     1
Richmond    Norfolk     0
Roenoke     Charleston  0
Roenoke     Norfolk     1
Roenoke     Suffolk     0
Stauton     Charleston  1
Stauton     Roenoke     1
;
```

Figure 6: Binary Variable

It is possible to see that the binary variable $y_{i,j}$ takes as its value 1 if the link is open, 0 if it is closed.

3.3 Problem's variant

We then analyzed a variant of our problem in which it was necessary to figure out the minimum number of arcs of travel that needed to be accomplished in the case where the blood bankhas a budget of \$24,000 (for both transportation and fixed costs) and wants to satisfy the hospital's has a budget of \$24,000 (for both transportation and fixed costs) and wants to meet the hospital's requirements.
We then put the previous objective function as a constraint in our model, imposing that the total cost (transportation + fixed cost) cannot exceed the budget of \$24000.
There are no changes in the other constraints.
We used the following analysis to optimally analyze the request:

$$\boldsymbol{min \sum_{i,j} \sum_k c_{i,j} x_{i,j}^k}$$
$$\boldsymbol{\sum f_{i,j} y_{i,j} + \sum_{i,j} \sum_k c_{i,j} x_{i,j}^k \leq budget}$$
$$\boldsymbol{\sum_{BS} x_{i,j}^k - \sum_{FS} x_{i,j}^k = b_i^k} \text{ ; k=1,2 ; } \forall \text{i} \in \text{N}$$
$$\boldsymbol{\sum_k x_{i,j}^k \leq u_{i,j}} \ \forall i_{i,j}$$
$$\boldsymbol{y_{y,j} \in 0, 1; \forall (i,j) \in A}$$

3.4 AMPL variant implementation

In the last part of our project, we analyzed the results obtained and compared them with previous analyses in order to verify the different scenarios that were possible.
The AMPL variant implementation analysis within this project emphasizes that in step 3 the objective function is changed:from $\sum(\boldsymbol{f}_{\text{i,j}}\boldsymbol{y}_{\text{i,j}}) + \sum(\boldsymbol{c}_{\text{i,j}}\boldsymbol{x}_{\text{i,j}}{}^{\boldsymbol{k}})$
to $\sum(\boldsymbol{c}_{\text{i,k}}\boldsymbol{x}_{\text{i,j}}{}^{\boldsymbol{k}})$.
The previous objective function is also placed as a constraint < 24000 which would be the budget constraint in step 3.
We also represented the desired output given by $\text{x}_{\text{i,j}}$, representing the units of blood transported from one hospital to another.

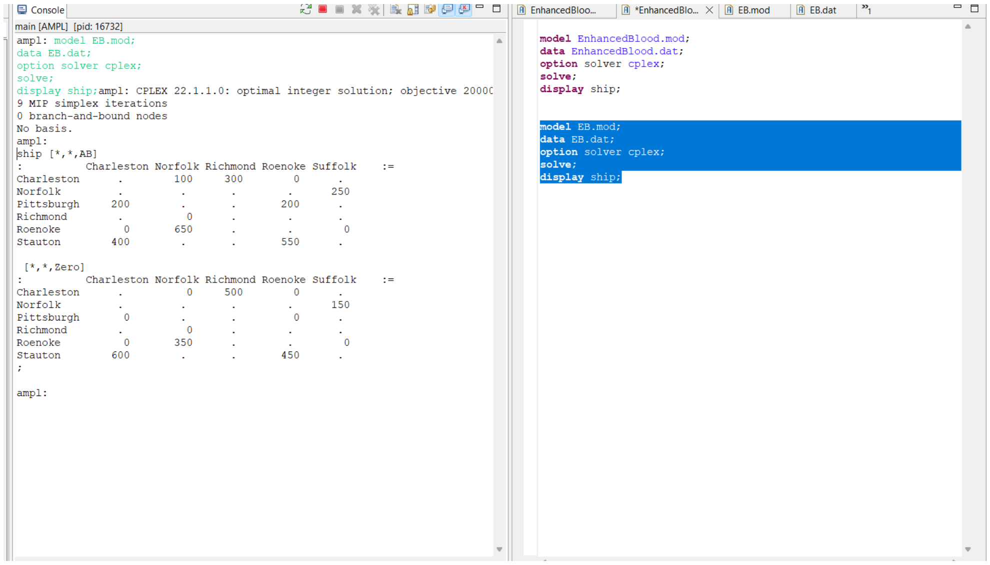


Figure 7: AMPL variant implementation output

The system outputs the minimized objective function that equals 20000 by performing 9 iterations. With display ship; the number of blood units transported are output taking into account the new objective function and constraints. Differ the number of units transported per node.