'Unit-1"

"Special Theory of Relativity"

1. ✓ Frame of Reference
2. ✓ Galilean Tranfor-mation
3. ✓ Inertial and Non-Inertial frames.
4. ✓ Postulates of special Theory of Relativity.
5. ✓ Michelson - Morley Experiment
6. ✓ Lorentz tranformation of space and time
7. ✓ Length Contraction.
8. ✓ Time dilation
9. ✓ Simultaneity in relativity Theory.
10. ✓ Addition of velocities.
11. ✓ Relativistic dynamics
12. ✓ Variation of mass with velocity.
13. ✓ Equivalence of mass and Energy.
14. ✓ Momentum - energy Transformation Equations.

~~15~~

Motion :- object is said to be in motion when object changes its position wrt time as well as surrounding.

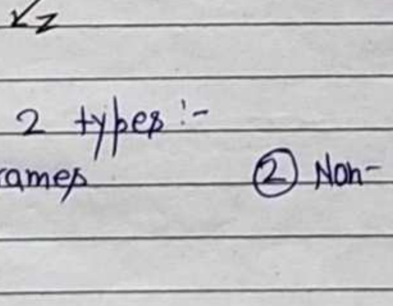
Terms :-

① **Classical velocity** :- Limit of velocity below which Mass, length and time are considered absolute

② **Critical value of velocity / Relativistic velocity** :-

velocity closed to velocity of light mass, length and time become no more absolute.

Frame of Reference :- 3-D dimension system used to locate object at any time t.

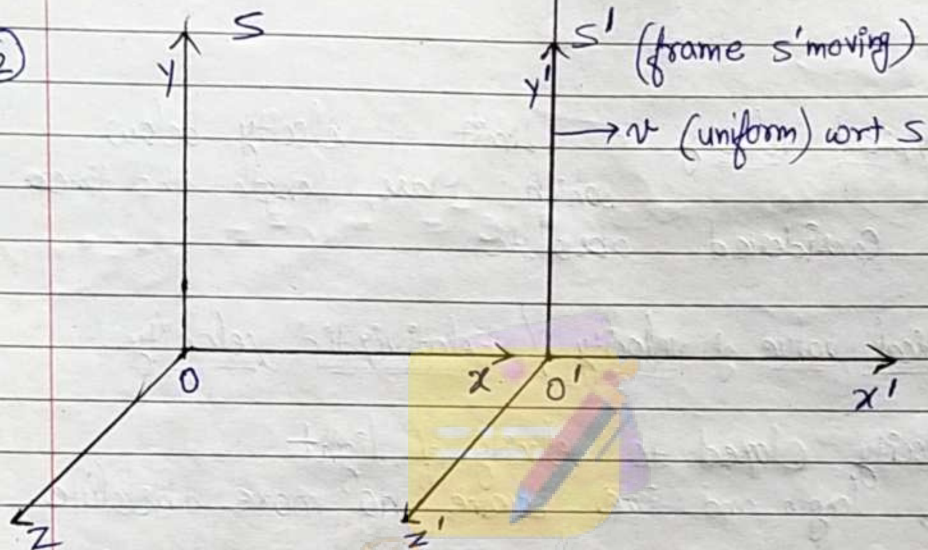


It is of 2 types :-

① Inertial frames ② Non-Inertial frames.

Inertial Frames	Non-Inertial Frames
① Newton's law of motion are valid / obeyed.	① Newtons law of motion are not valid

②

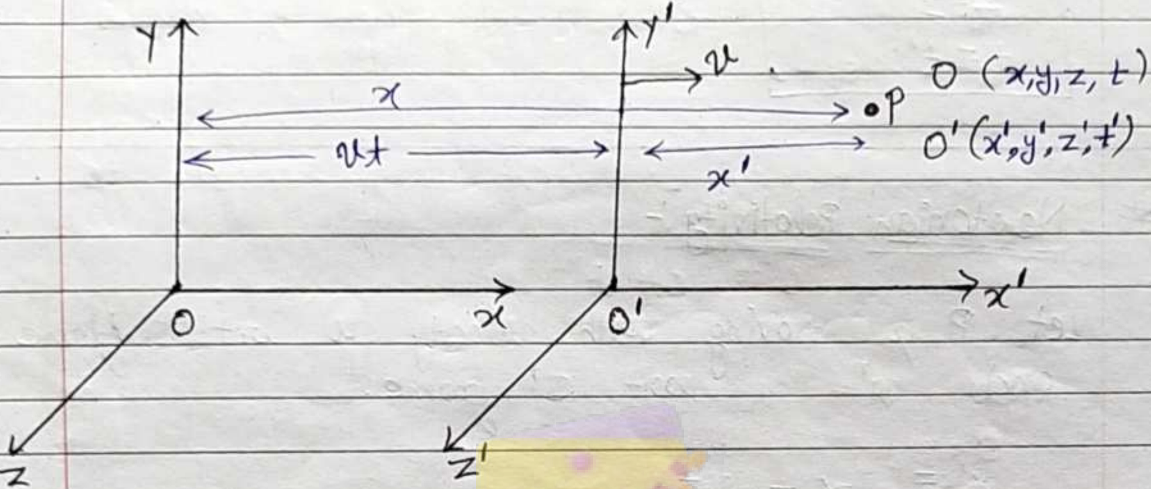


When the frame S' moves with uniform speed wrt S then the frames are said to be inertial frames.

when the v is not uniform both frames are said to be non-inertial frames of reference.

Galilean Transformation :-

Here $v <$ classical value of velocity



For simplicity Let $t=0$, $t'=0$ when O and O' of frames S and S' coincide.

for $v <$ classical value of velocity

$$\boxed{t = t'}$$

In time t distance covered by $S' = vt$ (along x-axis)

$$\boxed{x' = x - vt}$$

$$\boxed{y' = y}$$

$$\boxed{z' = z}$$

$$\boxed{t' = t}$$

Galilean Transformation

$$\boxed{\begin{aligned} x &= x' + vt' \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}}$$ $\rightarrow$ Inverse Gallilean Transformation

Newtonian Relativity :-

Let P is moving with velocity u wrt S frame and u' wrt S' frame.

$$x' = x - vt$$

differentiation wrt t gives

$$\frac{dx'}{dt} = \frac{dx}{dt} - v$$

but $t = t'$ so $\frac{d}{dt} = \frac{d}{dt'}$

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

$$\boxed{u_x' = u_x - v}$$

$$\boxed{u_y' = u_y}$$

$$\boxed{u_z' = u_z}$$

for acceleration transformation (i.e partical P in acceleration) :-

$$\frac{d u_x'}{dt'} = \frac{d}{dt}(u_x - v)$$

$$\frac{d}{dt'} u_y' = \frac{d}{dt}(u_y)$$

$$\frac{d}{dt'} u_z' = \frac{d}{dt}(u_z)$$

$$\Rightarrow \boxed{a_x' = a_x}$$

$$\boxed{a_y' = a_y}$$

$$\boxed{a_z' = a_z}$$

Michelson-Moorley Experiment :-

① Choosen frame = "Ether" Frame, in which speed of light is C.

② Properties of "Ether" medium assumed :-

ⓐ Zero density
ⓑ Perfectly Transparent
ⓒ Mass less
ⓓ Rigid
ⓔ Non-restive
ⓕ Invisible

③ Objective of ~~Expedition~~ Experiment :-

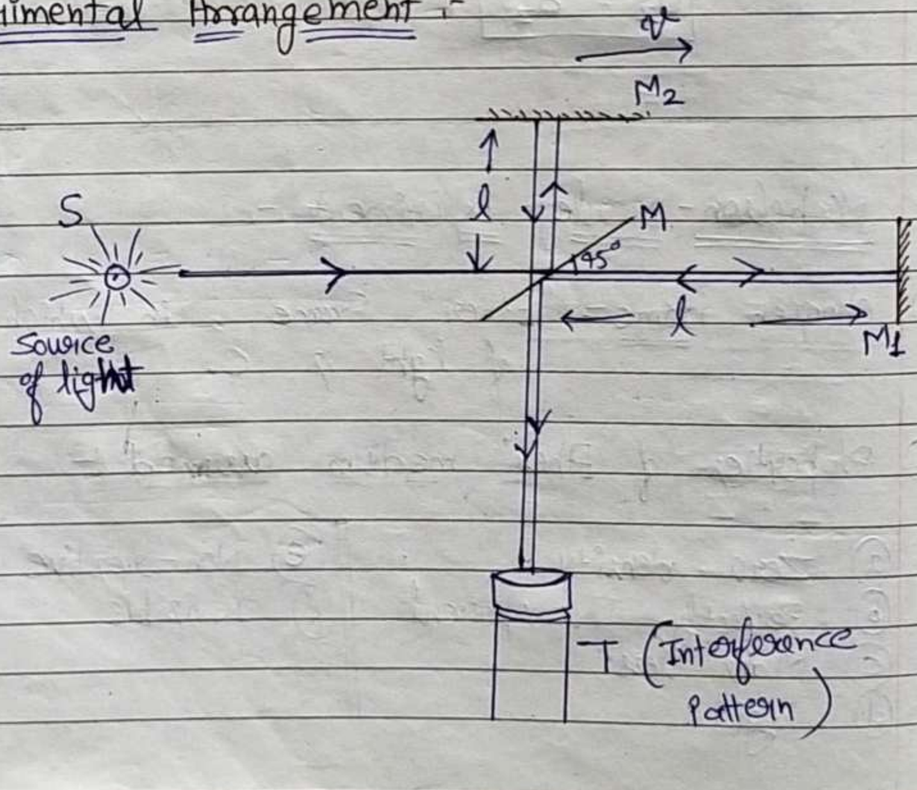
To determine the velocity of earth wrt a medium (say ether) which is always at rest.

Michelson - Morley Experiment was designed to test

$$\boxed{c' = c + v}$$

where v = velocity of observer moving through ether.
c' = velocity of light wrt observer moving through ether.
c = speed of light.

④ Experimental Arrangement :-



where,

$M =$ partially Polished Mirror

$M_1, M_2 =$ completely polished Mirrors

$v =$ velocity of earth

$c =$ velocity of light through ether.

Time taken for beam 1 to travel from M to M_1 and back is

$$t_1 = \frac{l}{c-v} + \frac{l}{c+v}$$

$$t_1 = \frac{l(c+v+c-v)}{c^2-v^2}$$

$$t_1 = \frac{2lc}{c^2-v^2}$$

$$\boxed{t_1 = \frac{2l}{c}\left(\frac{1}{1-v^2/c^2}\right)} \quad ---- \ (1)$$

The time taken by beam 2 to travel from M to M_2 and back, is a cross stream path

The transit time is given by

$$\cancel{(BM_2')^2} = \cancel{(BM_2)^2} + \cancel{(M_2M_2')^2}$$

$$\cancel{(ct_2)^2} = \cancel{l^2} + \cancel{(vt_2)^2}$$

$$(ct_2)^2 = 2\left[l^2 + \left(\frac{vt_2}{2}\right)^2\right]$$

$$t_2 = \frac{2l}{\sqrt{c^2 - v^2}}$$

$$\boxed{t_2 = \frac{2l}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

The difference in transit times is:- ~~[Instrument is rotated by 90°]~~

$$\Delta t = t_2 - t_1 = \frac{2l}{c}\left[\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right]$$

Using binomial theorem

$$\Delta t = \frac{2l}{c}\left[\cancel{1} + \frac{v^2}{c^2} - \cancel{1} - \frac{1}{2}\frac{v^2}{c^2}\right]$$

$$\boxed{\Delta t = \frac{l v^2}{c^3}}$$

(4) Path difference:-

$$\text{Path diff} = c\Delta t$$
$$= c \frac{l v^2}{c^3}$$

$$\boxed{\text{path diff} = \frac{l v^2}{c^2}}$$

If path diff is $\lambda \longrightarrow$ 1 fringe observed

when path diff is $\frac{lv^2}{c^2} \longrightarrow \frac{lv^2}{c^2\lambda}$ fringe shift will observed.

The whole apparatus is turned to $90°$

No of fringe shift $= -\frac{l}{\lambda}\frac{v^2}{c^2}$

so Now
Total Number of fringe shift $= \frac{lv^2}{c^2\lambda} - \left(\frac{-lv^2}{c^2\lambda}\right)$

$$\boxed{\Delta N = \frac{2l}{\lambda}\left(\frac{v^2}{c^2}\right)}$$

(5) Values by Michelson and Morley :-

$l = 11\,m$
$\lambda = 5.5 \times 10^{-7}\,m$
$c = 3 \times 10^{8}\,m/s$
$v = 3 \times 10^{4}\,m/s$

$$\Delta N = \frac{2 \times 22}{5.5 \times 10^{-7}}\left(\frac{3 \times 10^{4}}{3 \times 10^{8}}\right) = 0.4$$

i.e $\boxed{\text{a shift of four-tenths a fringe}}$

(6) Results of Experiment :-

(a) Hypothesis of existance of stationary medium around the earth is wrong.

(b) The velocity of light is Constant in all directions

(c) A new theory with different concept of space, time and mass is needed.

Einstein's postulate of special Theory of relativity :-

(1) ~~The~~ All the laws of physics are same in all inertial reference frames.

(2) The speed of light in ~~vacuu~~ vacuum is same (3×10^8 m/s) in all Inertial frames. regardless of motion of observer or source.

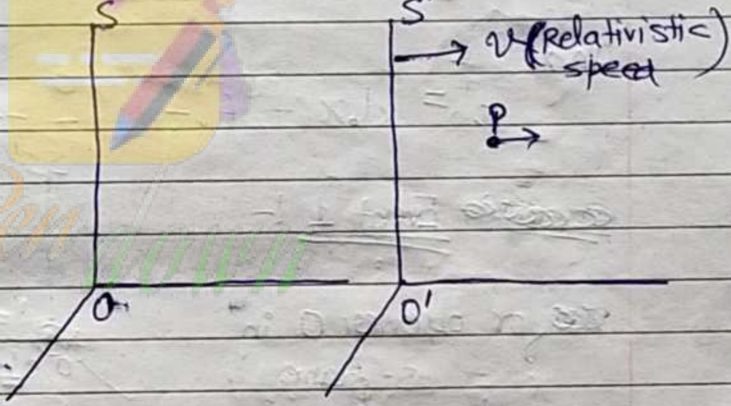
Lorentz Tranzformation :-

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$y' = y$

$z' = z$

$$\boxed{t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}}$$



Proof :-

we have,

$x' = ax + bt$

$y' = y$

$z' = z$

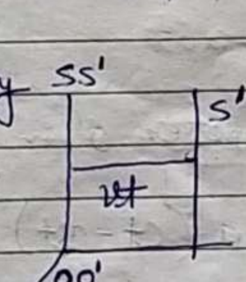
$t' = dx + ft$

where a, b, d, f are Constant Parameter.

we have to vizualize some events to find them.

Case(i) :-

both frame coincide initially and s' has velocity = v



O will see

$x = vt$, $y = 0$, $z = 0$, t

O' will see

$x' = 0$, $y' = 0$, $z' = 0$, t'

in equ^n (given)

$x' = ax + bt$

$0 = aut + bt$

$t(au+b) = 0$

$\Rightarrow t \neq 0$

$\Rightarrow au + b = 0$

$\boxed{b = -au}$

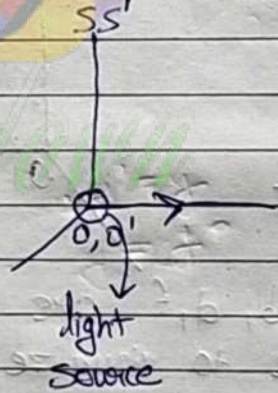
in equ^n

$x' = ax - aut$

$\boxed{x' = a(x - ut)}$ ------ (i)

and

$t' = d.x + f.t$ ------ (ii)



Event 1 :-

for observer O in S-frame

$x = ct$

$y = 0$

$z = 0$

t

for observer O' in S'-frame

in equ^n (i)

$x' = a(ct - ut)$

$x' = at(c - u)$ ------ (b)

then $t' = d(ct) + ft$ (from ii)

$\boxed{t' = t(dc+f)}$ ---- (c)

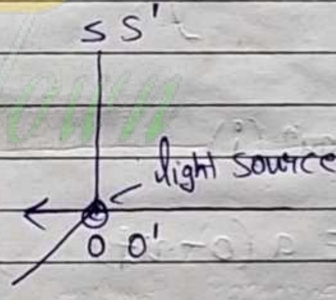
dividing equ^n (b) / (c)

$$\frac{x'}{t'} = \frac{at(c-v)}{t(dc+f)}$$

we Know that $x' = ct'$

$$\frac{c\cancel{t'}}{\cancel{t'}} = \frac{a\cancel{t}(c-v)}{\cancel{t}(dc+f)}$$

$\boxed{dc^2 + cf = a(c-v)}$ ---- (A)



Event-2 :-

for oberver O

$x = -ct$

$y = 0$

$z = 0$

t

in equ^n (i)

$$x' = a(-ct - vt)$$

$$x' = -at(c+v)$$

$$-ct' = -at(c+v)$$

$ct' = at(c+v)$ ---- (d)

$t' = d(-ct) + ft$ (in equ^n ii)

$t' = t(-dc+f)$ ---- (e)

dividing equ^n (d)/(e)

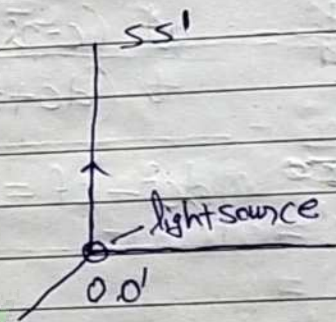
$$\frac{c\cancel{t'}}{\cancel{t'}} = \frac{a\cancel{t}(c+u)}{\cancel{t}(-dc+f)}$$

$$\boxed{-dc^2 + cf = a(c+u)} \quad \text{-- (B)}$$

Event-3:-

on-y-axis

$$\left.\begin{array}{l} x=0 \\ y=ct \\ z=0 \\ t \end{array}\right\} \text{observer is O}$$



in equ^n (i)

$x' = a(0-ut)$, as $y'=y$, $y'=ct$, $z=0$

$x' = -aut$

in equ^n (ii)

$t' = d.0 + ft$

$$\boxed{t' = ft}$$

distance from O' in frame s'

$$d^2 = x'^2 + y'^2 + z'^2$$

$$d^2 = a^2u^2t^2 + c^2t^2 + 0$$

Page No.: ______
Date: ___/___/___

$$c^2t'^2 = a^2u^2t^2 + c^2t^2$$

$$c^2f^2t^2 = a^2u^2t^2 + c^2t^2$$

$$\boxed{c^2f^2 = a^2u^2 + c^2} \quad \text{------} \quad (C)$$

from equ^n (A), (B), (C)

$$dc^2 + fc = a(c-u) \quad \text{---} \quad (A)$$

$$-dc^2 + fc = a(c+u) \quad \text{---} \quad (B)$$

$$a^2u^2 + c^2 = f^2c^2 \quad \text{---} \quad (C)$$

(A) + (B)

$$2fc = 2ac \Rightarrow \boxed{f = a}$$

in (C)

$$a^2u^2 + c^2 = a^2c^2$$

$$\boxed{a = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}} \qquad \boxed{f = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}}$$

in (A)

$$dc^2 + ac = a(c-u) \qquad \text{as } f = a$$

$$\boxed{d = -\frac{u}{c^2\sqrt{1-\frac{u^2}{c^2}}}}$$

hence we get,

$$\boxed{x' = \frac{x-ut}{\sqrt{1-\frac{u^2}{c^2}}}}, \quad \boxed{y' = y}, \quad \boxed{z' = z}$$

$$t' = \frac{-vx}{c^2\sqrt{1-\frac{v^2}{c^2}}} + \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{vx}{c^2}\right)$$

$$\boxed{t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

hence proved

Note:-

$$\left.\begin{aligned} &\text{when } v << c \\ &x' = x - vt \\ &y' = y \\ &z' = z \\ &t' = t \end{aligned}\right\} \text{Gallilian Transformation}$$

Inverse Lorentz Transformation:-

$$\boxed{x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}} \quad , \quad \boxed{y = y'} \quad , \quad \boxed{z = z'}$$

Replace v by −v for inverse

$$\boxed{t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

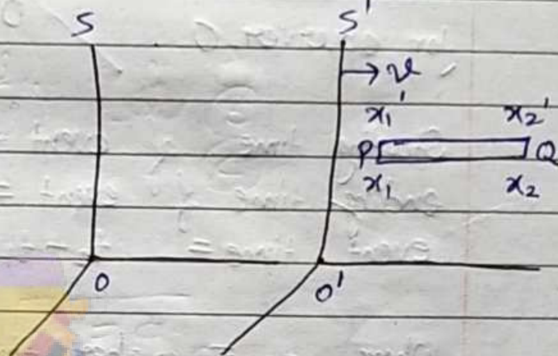
Application of Lorentz Transformation :-

(1) Length Contraction :-

Let Rod is in s' frame
then

original length of Rod

$$\boxed{l_0 = x_2' - x_1'}$$



apparent length of Rod

$$\boxed{l = x_2 - x_1}$$

by lorentz Transformation

$$x_2' = \frac{x_2 - ut}{\sqrt{1 - u^2/c^2}} \qquad x_1' = \frac{x_1 - ut}{\sqrt{1 - u^2/c^2}}$$

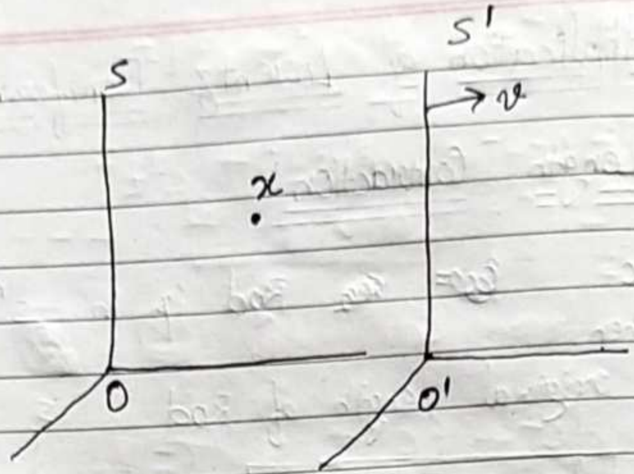
$$l_0 = \frac{x_2 - ut - (x_1 - ut)}{\sqrt{1 - u^2/c^2}}$$

$$l_0 = \frac{l}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\boxed{l = l_0 \sqrt{1 - \frac{u^2}{c^2}}} \Rightarrow \boxed{l < l_0}$$

i.e length is Contracted for the observer who is at rest by a factor of $\sqrt{1 - \frac{v^2}{c^2}}$

② Time Dilation :-



for observer O

Starting time of event = t_1
ending time of event = t_2
event time = $t_2 - t_1 = t_0$

where t_0 = proper time of event

for observer O'

Starting time of event = t_1'
ending time of event = t_2'

event time = $t_2' - t_1' = t$
wher t = apparent time

by Lorentz Transformation -

$$t_1' = \frac{t_1 - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad t_2' = \frac{t_2 - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

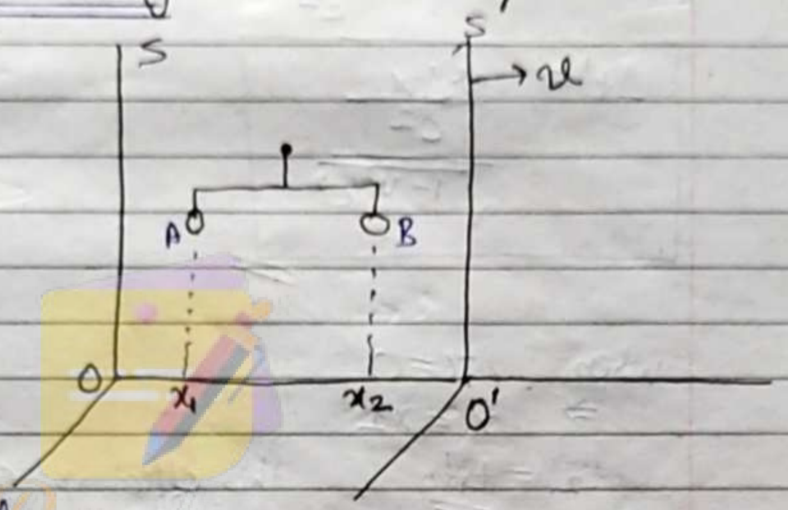
$$t = t_2' - t_1'$$

$$\boxed{t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

i.e $\boxed{t > t_0}$

t is increased (dilated time) by factor of $\sqrt{1-\frac{v^2}{c^2}}$.

③ Simultanity of Relativity :-



for observer O
both bulb will
glow simultaneously.

Time of glow of both bulb = t_0

for observer O'
both will not glow simultaneously.

Time of glow of bulb A = t_1'
Time of glow of bulb B = t_2'

by Lorentz Transformation

$$t_1' = \frac{t_0 - \frac{v x_1}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} , \quad t_2' = \frac{t_0 - \frac{v x_2}{c^2}}{\sqrt{1-v^2/c^2}}$$

$$t_2' - t_1' = \frac{t_0 - \frac{u x_2}{c^2} - t_0 + \frac{u x_1}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\Delta t' = \frac{u(x_1 - x_2)/c^2}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\boxed{\Delta t' = \frac{\frac{v \Delta x}{c^2}}{\sqrt{1 - u^2/c^2}}}$$

when $\Delta x = 0$

$\Rightarrow x_1 = x_2$

$t_2' = t_1'$ this effect is called simultanity of relativity.

Addition of velocities:-

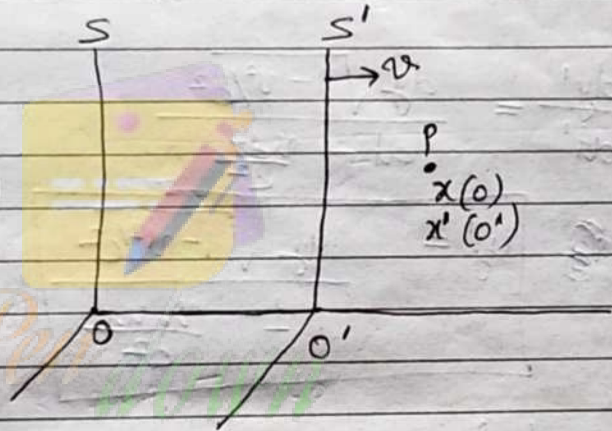
$$\boxed{u_x' = \frac{u_x - v}{1 - \frac{u_x \cdot v}{c^2}}} \quad , \quad \boxed{u_y' = \frac{u_y\sqrt{1 - v^2/c^2}}{1 - \frac{u_x \cdot v}{c^2}}}$$

$$\boxed{u_z' = \frac{u_z\sqrt{1 - v^2/c^2}}{1 - \frac{u_x \cdot v}{c^2}}}$$

Proof:-

$u_x = \frac{dx}{dt}$ (O)

$u_x' = \frac{dx'}{dt'}$ (O')



we have

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad t' = \frac{t - \frac{v \cdot x}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$dx' = \frac{dx - v\,dt}{\sqrt{1 - v^2/c^2}} \qquad dt' = \frac{dt - \frac{v\,dx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$\frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - \frac{v\,dx}{c^2}}$$

$$u_x' = \frac{\frac{dx}{dt} - v\frac{dt}{dt}}{\frac{dt}{dt} - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u_x - v}{1 - \frac{u_x \cdot v}{c^2}}$$

$$\boxed{u_x' = \frac{u_x - v}{1 - \frac{v \cdot u_x}{c^2}}}$$

$$t' = \frac{t - \frac{v \cdot x}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$
$$dy' = dy$$

$$dt' = \frac{dt - \frac{v\,dx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$\frac{dy'}{dt'} = \frac{dy\sqrt{1 - v^2/c^2}}{dt - \frac{v\,dx}{c^2}}$$

$$\boxed{u_y' = \frac{u_y\sqrt{1 - v^2/c^2}}{1 - \frac{v \cdot u_x}{c^2}}}$$

similarly

$$\boxed{u_z' = \frac{u_z\sqrt{1 - v^2/c^2}}{1 - \frac{v \cdot u_x}{c^2}}}$$

☆ <u>for inverse Transformation</u> :-

Replace $v \rightarrow -v$

$$\boxed{u_x = \frac{u_x' + v}{1 + \frac{v . u_x'}{c^2}}}$$

$$\boxed{u_y = \frac{u_y' \sqrt{1 - v^2/c^2}}{1 + \frac{v . u_x'}{c^2}}} \quad , \quad \boxed{u_z = \frac{u_z' \sqrt{1 - v^2/c^2}}{1 + \frac{v . u_x'}{c^2}}}$$

<u>Some special Cases</u> :-

(1) when Particle P is moving with $u_x' = c$ (O')

$$u_x = \frac{c + v}{1 + \frac{c . v}{c^2}} = c$$

(2) when S' start moving with c

$$u_x = \frac{u_x' + c}{1 + \frac{u_x' . c}{c^2}} = c$$

(3) when $v = c$ and $u_x' = c$

$$u_x = \frac{c + c}{1 + \frac{c^2}{c^2}} = c$$

⇒ velocity doesnot exceed than c.

Relation of acceleration:-

we have

$$u'_x = \frac{u_x - v}{1 - \frac{v \cdot u_x}{c^2}} \qquad u'_y = \frac{u_y \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{u_x \cdot v}{c^2}} \qquad u_z' = \frac{u_z \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{u_x \cdot v}{c^2}}$$

$$du'_x = \frac{du_x - 0}{1 - \frac{du_x \cdot v}{c^2}} \quad , \quad t' = \frac{t - \frac{v \cdot x}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$dt' = \frac{dt - \frac{v\,dx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$a'_x = \frac{du_x'}{dt'} = \frac{du_x}{1 - \frac{v \cdot du_x}{c^2}} \times \frac{\sqrt{1 - v^2/c^2}}{dt - \frac{v\,dx}{c^2}}$$

on solving

$$\boxed{a'_x = \frac{a_x \left(1 - v^2/c^2\right)^{3/2}}{\left(1 - \frac{v \cdot u_x}{c^2}\right)^3}}$$