"Laplace Transform"

Definition:- Let $F(t)$ be a f^n defined for $t>0$. The Laplace transform of $F(t)$ is denoted by $L\{F(t)\}$ or $f(s)$ and defined as :

$$L\{F(t)\} = f(s) = \int_0^\infty e^{-st} F(t)\,dt \qquad \text{---- (i)}$$

where (i) $s>0$
(ii) $e^{st}F(t) < M$, where M is a finite number
(iii) s is a real parameter.

Laplace Transform of elementary function-

① $F(t) = t^n$, $n > -1$

$$\boxed{L\{t^n\} = f(s) = \frac{\Gamma(n+1)}{s^{n+1}}}$$

for $n = 0, 1, 2, \ldots$

$$\boxed{L\{t^n\} = \frac{\lfloor n}{s^{n+1}}}$$

Proof!

$$f(s) = \int_0^\infty e^{-st} \cdot t^n\,dt$$

Let $st = x$
$s\,dt = dx$

$$f(s) = \int_0^\infty \frac{e^{-x} x^n}{s^{n+1}}\,dx$$

$$= \frac{1}{s^{n+1}} \int_0^\infty e^{-x} x^{n+1-1}\,dx$$

$$= \frac{\Gamma(n+1)}{s^{n+1}}$$

② $F(t) = e^{at}$

$$\boxed{L\{e^{at}\} = f(s) = \frac{1}{s-a}}, \ (s>a)$$

$$f(s) = \int_0^\infty e^{at} e^{-st}\,dt$$

$$= \int_0^\infty e^{-t(s-a)}\,dt$$

$$= \left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty$$

$$= \frac{-1}{(s-a)}[0-1]$$

$$= \frac{1}{s-a} \quad (s>a)$$

③ $F(t) = \sin at$

$$\boxed{L\{\sin(at)\} = f(s) = \frac{a}{s^2+a^2}}$$

$$f(s) = \int_0^\infty e^{-st} \sin at \, dt$$

$$= \frac{e^{-st}}{(-s)^2+a^2}\Big[-s\sin at - a\cos at\Big]_0^\infty$$

$$f(s) = \frac{1}{s^2+a^2}\left[0 - 1(0-a)\right]$$

$$= \frac{a}{s^2+a^2}$$

④ $F(t) = \cos at$

$$\boxed{L\{\cos at\} = f(s) = \frac{s}{s^2+a^2}}$$

$$f(s) = \int_0^\infty e^{-st} \cos at \, dt$$

$$f(s) = \frac{e^{-sx}}{s^2+a^2}\Big[-s\cos ax + a\sin ax\Big]_0^\infty$$

$$= \frac{1}{s^2+a^2}\Big[0 - 1\big[-s+0\big]\Big]$$

$$= \frac{s}{s^2+a^2}$$

⑤ $F(t) = \sin h(at)$

$$\boxed{L\{\sin h(at)\} = f(s) = \frac{a}{s^2-a^2}}, \quad s > |a|$$

$$f(s) = L\{\sin h(at)\} = L\left\{\frac{e^{at}-e^{-at}}{2}\right\}$$

using Linearity property

$$f(s) = \frac{1}{2}\Big[L\{e^{at}\} - L\{e^{-at}\}\Big]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left(\frac{\cancel{s}+a-\cancel{s}+a}{s^2-a^2}\right) = \frac{a}{s^2-a^2}$$

$(|s| > |a|)$

⑥ $F(t) = \cos h(at)$

$$\boxed{L\{\cos h(at)\} = f(s) = \frac{s}{s^2-a^2}}, \quad s > |a|$$

Properties of Laplace Transform

① **Shifting Property:-** If $L\{F(t)\} = \bar{F}(s)$ then

$$\boxed{L\{e^{at}F(t)\} = \bar{F}(s-a)}$$

Eg: Find $L\{e^{2t}\cos(3t)\}$

Sol^n) we know that,

$$\boxed{L\{\cos(3t)\} = \frac{s}{s^2+9}}$$

using shifting property

$$L\{e^{2t}\cos(3t)\} = \frac{(s-2)}{(s-2)^2+9} = \frac{s-2}{s^2-4s+13}$$

② **Multiply by t property:** If $L\{F(t)\} = \bar{F}(s)$ then

$$\boxed{L\{t\cdot F(t)\} = -\frac{d}{ds}\bar{F}(s)}$$

Proof: we know that

$$\bar{F}(s) = L\{F(t)\} = \int_0^\infty e^{-st}F(t)\,dt \quad \text{--- (i)}$$

Now $$\frac{d}{ds}\bar{F}(s) = \frac{d}{ds}\left(\int_0^\infty e^{-st}F(t)\,dt\right)$$

$$= \int_0^\infty \frac{\partial}{\partial s}\left(e^{-st}F(t)\,dt\right) = \int_0^\infty -e^{-st}\cdot t\,F(t)\,dt$$

$$= -\int_0^\infty e^{-st}\,t\,F(t)\,dt$$

$$\boxed{-\frac{d}{ds}\bar{F}(s) = L\{t\cdot F(t)\}}$$

E.g.: Evaluate $L\{t \sin 3t\}$

Sol^n)

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L\{t \sin 3t\} = \frac{-d}{ds}\left(\frac{3}{s^2+9}\right) = -3 \, \frac{-1(2s)}{(s^2+9)^2} = \frac{6s}{(s^2+9)^2}$$

③ **Divide by t property:** If $L\{F(t)\} = \bar{F}(s)$ then

$$\boxed{L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty \bar{F}(s)\,ds}$$

Proof:

$$\bar{F}(s) = \int_0^\infty e^{-st} F(t)\,dt$$

integrating both sides wrt s from s to ∞

$$\int_s^\infty \bar{F}(s)\,ds = \int_s^\infty \int_0^\infty e^{-st} F(t)\,dt\,ds = \int_0^\infty \int_s^\infty e^{-st} F(t)\,ds\,dt$$

$$= \int_0^\infty \left[\frac{e^{-st}}{-t} F(t)\right]_s^\infty dt$$

$$= \int_0^\infty F(t) \left[\frac{e^{-st}}{-t}\right]_s^\infty dt = \int_0^\infty \frac{1}{t} e^{-st} F(t)\,dt$$

$$\boxed{\int_s^\infty \bar{F}(s)\,ds = L\left\{\frac{F(t)}{t}\right\}}$$

E.g.: Evaluate $L\left\{\frac{1-\cos t}{t}\right\}$

Sol^n)

$$L\{1-\cos t\} = L\{1\} - L\{\cos t\} = \frac{1}{s} - \frac{s}{s^2+1}$$

by divide by t property

$$L\left\{\frac{1-\cos t}{t}\right\} = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+1}\right) ds = \left[\log s - \frac{1}{2}\log(s^2+1)\right]_s^\infty$$

$$L\left\{\frac{1-\cos t}{t}\right\} = \frac{1}{2}\left[\underbrace{\lim_{s\to\infty} \log\left(\frac{s^2}{s^2+1}\right)}_{0} - \log\left(\frac{s^2}{s^2+1}\right)\right]$$

$$= -\frac{1}{2}\log\frac{s^2}{s^2+1}$$

④ Laplace Transform of derivative:- If $L\{F(t)\} = \bar{F}(s)$ then

$$L\{F'(t)\} = \boxed{L\left\{\frac{d}{dt}F(t)\right\} = s\bar{F}(s) - F(0)}$$

Proof: $L\{F'(t)\} = \int_0^\infty e^{-st}F'(t)\,dt = e^{-st}F(t)\Big]_0^\infty - \int_0^\infty e^{-st}(-s)F(t)\,dt$

$$= 0 - F(0) + s\int_0^\infty e^{-st}F(t)\,dt$$

$$\boxed{L\left\{\frac{d}{dt}F(t)\right\} = s\bar{F}(s) - F(0)}$$

ⓐ $L\{F'(t)\} = s\bar{F}(s) - F(0)$

ⓑ $L\{F''(t)\} = s^2\bar{F}(s) - sF(0) - F'(0)$

ⓒ $L\{F'''(t)\} = s^3\bar{F}(s) - s^2F(0) - sF'(0) - F''(0)$

⑤ Laplace Transform of Integral! If $L\{F(t)\} = \bar{F}(s)$

$$\boxed{L\left\{\int_0^t F(t)\,dt\right\} = \frac{\bar{F}(s)}{s}}$$

Proof!- Let $G(t) = \int_0^t F(t)\,dt$ $\qquad G(0) = 0$

$G'(t) = \frac{d}{dt}\int_0^t F(t)\,dt = F(t)$ $\qquad$ if $F(0)=0$

$L\{G'(t)\} = s\,L\{G(t)\} - G(0)$

$$\boxed{L\{G(t)\} = \frac{L\{F(t)\}}{s}}$$

Eg: Evaluate $L\left\{\int_0^t \frac{e^{-t}\sin t}{t}\,dt\right\}$

Sol^n) $L\{\sin t\} = \frac{1}{s^2+1}$

$\cancel{L\{e^{-t}\sin t\}}$ $L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{1}{s^2+1}\,ds = \left[\tan^{-1} s\right]_s^\infty = \frac{\pi}{2} - \tan^{-1} s$

$L\left\{e^{-t}\frac{\sin t}{t}\right\} = \frac{\pi}{2} - \tan^{-1}(s+1)$

$L\left\{\int_0^t \frac{e^{-t}\sin t}{t}\,dt\right\} = \frac{\bar{F}(s)}{s} = \frac{\frac{\pi}{2} - \tan^{-1}(s+1)}{s}$

Ans

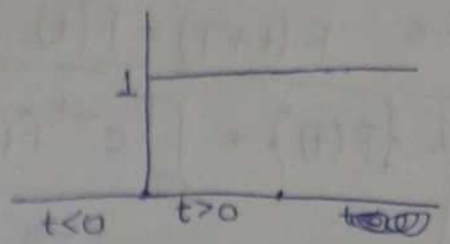
(6) $F(t) = \bar{F}(s)$

$$\boxed{L\, f(at) = \frac{1}{a} f\left(\frac{s}{a}\right)}$$

Laplace Transform of Unit step function:- (Heaviside USF)

① Unit step function:

$u(t)$ or $F(t) \rightarrow 0 \; , \; t<0$
$\qquad\qquad\quad \rightarrow 1 \; , \; t>0$

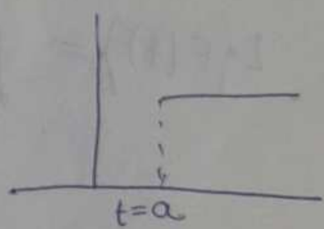


$$L\{u(t)\} = \bar{u}(s) = \int_0^\infty e^{-st}\, u(t)\, dt = \int_0^\infty e^{-st} dt = \frac{e^{-st}}{-s}\Big|_0^\infty =$$

$$\boxed{\bar{u}(s) = \frac{1}{s}}$$

② Unit Impulse function:-

$H(t-a)$ or $u(t-a) \rightarrow 0 \; , \; t<a$
$\qquad\qquad\qquad\quad \rightarrow 1 \; , \; t \geq a$



$$L\{u(t-a)\} = \int_0^\infty e^{-st}\, u(t-a)\, dt = \int_0^a e^{-st}(0)\, dt + \int_a^\infty e^{-st}(1)\, dt$$

$$= \frac{e^{-st}}{-s}\Big|_a^\infty = \frac{e^{-as}}{s}$$

$$\boxed{\bar{u}(s) = \frac{e^{-as}}{s}}$$

Laplace Transform of periodic function:

Let $F(t)$ be periodic function with period T

i.e $F(t+T) = F(t)$, then

$$\boxed{L\{F(t)\} = \frac{\int_0^T e^{-st} F(t)\,dt}{1-e^{-sT}}}$$

Proof: Since $F(t)$ is periodic f^n then

$F(t) = F(t+T) = F(t+2T) = - - -$

$$L\{F(t)\} = \int_0^T e^{-st} F(t)\,dt + \int_T^{2T} e^{-st} F(t)\,dt + - - -$$

put $t = u+T$, $t = u+2T$, $t = u+3T$ & so on

$$L\{F(t)\} = \int_0^T e^{-st} F(t)\,dt + \int_0^T e^{-s(u+T)} F(u+T)\,du + \int_0^T e^{-s(u+2T)} F(u+2T)\,du + - - -$$

$$= \int_0^T e^{-st} F(t)\,dt + e^{-sT}\int_0^T e^{-su} F(u+T)\,du + e^{-2sT}\int_0^T e^{-su} F(u+2T)\,du + - - -$$

$$= \int_0^T e^{-st} F(t)\,dt \left[1 + e^{-sT} + e^{-2sT} - - \infty\right]$$

$$\boxed{L\{F(t)\} = \frac{\int_0^T e^{-st} F(t)\,dt}{1-e^{-sT}}}$$

Initial & Final value Theorem

① **Initial value Theorem:** If $L\{F(t)\} = f(s)$ then

$$\boxed{\lim_{t\to 0} F(t) = \lim_{s\to \infty} sf(s)}$$

Proof: we know that

$$L\{F'(t)\} = sf(s) - F(0)$$

$$\int_0^\infty e^{-st} F'(t)\,dt = sf(s) - F(0)$$

taking limit $s\to\infty$ both sides

$$\lim_{s\to\infty} \int_0^\infty e^{-st} F'(t)\,dt = \lim_{s\to\infty} [sf(s) - F(0)]$$

$$\int_0^\infty \lim_{s\to\infty} e^{-st} F'(t)\,dt = \lim_{s\to\infty} sf(s) - F(0)$$

$$\int_0^\infty 0 \cdot F'(t)\,dt = \lim_{s\to\infty} sf(s) - F(0)$$

$$\lim_{s\to\infty} sf(s) = F(0)$$

$$\boxed{\lim_{s\to\infty} sf(s) = \lim_{t\to 0} F(t)}$$ h.p

② **Final value Theorem:** If $L\{F(t)\} = f(s)$

$$\boxed{\lim_{t\to\infty} F(t) = \lim_{s\to 0} sf(s)}$$

Proof: taking lim $s\to 0$ both side

$$\lim_{s\to 0} \int_0^\infty e^{-st} F'(t)\,dt = \lim_{s\to 0} [sf(s) - F(0)]$$

$$\int_0^\infty F'(t)\,dt = \lim_{s\to 0} sf(s) - F(0)$$

$$F(t)\Big|_0^\infty = \lim_{s\to 0} sf(s) - F(0)$$

$$\lim_{t\to\infty} F(t) - F(0) = \lim_{s\to 0} sf(s) - F(0)$$

$$\boxed{\lim_{t\to\infty} F(t) = \lim_{s\to 0} sf(s)}$$

"Inverse Laplace Transform"

Laplace Inverse: $L^{-1}\{\bar{F}(s)\} = F(t)$

① $L^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{\lceil n+1}$

② $L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

③ $L^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{1}{a}\sin at$

④ $L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$

⑤ $L^{-1}\left\{\frac{1}{s^2-a^2}\right\} = \frac{1}{a}\sinh(at)$

⑥ $L^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh(at)$

⑦ $L^{-1}\left\{\frac{1}{s}\right\} = 1$

⑧ $L^{-1}\left\{\frac{1}{s^2}\right\} = t$

⑨ $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{1}{2a}t\sin at$

⑩ $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\} = \frac{1}{2a^3}(\sin at - at\cos at)$

$\boxed{L^{-1}\{\text{Constant}\} = 0}$

Properties of Laplace Inverse:-

① **Shifting Property:**

If $L^{-1}\{\bar{F}(s)\} = F(t)$ then $\boxed{L^{-1}\{\bar{F}(s-a)\} = e^{at}F(t)}$

② If $L^{-1}\{\bar{F}(s)\} = F(t)$

then $\boxed{L^{-1}\left\{\frac{d}{ds}\bar{F}(s)\right\} = -tF(t)}$

③ $L^{-1}\left\{\int_s^{\infty}\bar{F}(s)ds\right\} = \frac{F(t)}{t}$

④ $L^{-1}\{s\bar{F}(s)\} = \frac{d}{dt}F(t)$, if $F(0)=0$

⑤ $L^{-1}\left\{\frac{\bar{F}(s)}{s}\right\} = \int_0^t F(t)dt$

Examples

① $L^{-1}\left\{\frac{1}{(s+2)(s-3)}\right\}$

Sol^n)

$$\frac{1}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3} \Rightarrow A(s-3) + B(s+2) = 1$$

put $s=3$ and $s=-2$

we get $\boxed{B = \frac{1}{5}}$ & $\boxed{A = \frac{-1}{5}}$

put A & B

$$L^{-1}\left\{\frac{-1}{5}\frac{1}{(s+2)}\right\} + L^{-1}\left\{\frac{1}{5(s-3)}\right\} = \frac{-1}{5}e^{-2t}\cdot 1 + \frac{1}{5}e^{3t}\cdot 1$$

$$= \frac{1}{5}\left(e^{3t} - e^{2t}\right)$$

② $L^{-1}\left\{\log\frac{s+1}{s-1}\right\}$

Sol^n) $\boxed{L^{-1}\left\{\frac{d}{ds}\bar{F}(s)\right\} = -tF(t)}$ (use always in this type ques)

Now $\bar{F}(s) = \log(s+1) - \log(s-1)$

$$\frac{d}{ds}\bar{F}(s) = \frac{1}{s+1} - \frac{1}{s-1} = \frac{s-1-s-1}{s^2-1} = \frac{-2}{s^2-1}$$

$$L^{-1}\left\{\frac{d}{ds}\bar{F}(s)\right\} = L^{-1}\left\{\frac{1}{s+1}\right\} - L^{-1}\left\{\frac{1}{s-1}\right\}$$

$$-tF(t) = e^{-t} - e^{t}$$

$$\boxed{F(t) = \frac{e^{t} - e^{-t}}{t}}$$

③ $L^{-1}\left\{\log\frac{s^2+1}{s(s+1)}\right\}$

Sol^n) $\bar{F}(s) = \log(s^2+1) - \log(s) - \log(s+1)$

$$\frac{d}{ds}\bar{F}(s) = \frac{2s}{s^2+1} - \frac{1}{s} - \frac{1}{s+1}$$

$$L^{-1}\left\{\frac{d}{ds}\bar{F}(s)\right\} = 2\cos t - 1 - e^{-t}$$

similarly

$$\boxed{F(t) = \frac{e^{-t}+1-2\cos t}{t}}$$

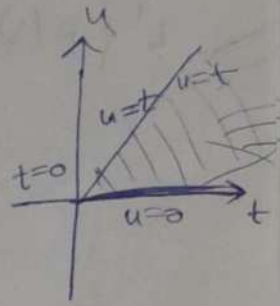
Convolution Theorem: If $L^{-1}\{\bar{f}(s)\} = F(t)$ and $L^{-1}\{\bar{g}(s)\} = g(t)$

then $$\boxed{L^{-1}\{\bar{g}(s)\cdot\bar{f}(s)\} = \int_0^t F(u)\, g(t-u)\,du = \int_0^t g(u)\, F(t-u)\,du}$$

Proof: Let $\phi(t) = \int_0^t g(u)F(t-u)\,du$

$$L\{\phi(t)\} = \int_0^\infty e^{-st}\left[\int_0^t g(u)\,F(t-u)\,du\right]dt$$

$$= \int_{t=0}^{\infty}\int_{u=0}^{t} e^{-st} g(u)\,F(t-u)\,du\,dt$$



after new limits

$$L\{\phi(t)\} = \int_{u=0}^{u=\infty}\int_{t=u}^{t=\infty} e^{-st} g(u)\,F(t-u)\,dt\,du$$

$$= \int_{u=0}^{\infty} g(u)\left[\int_{t=u}^{\infty} e^{-st}F(t-u)\,dt\right]du$$

put $t-u = x \Rightarrow dt = dx$ and limits are $x=0$ to $x=\infty$

$$L\{\phi(t)\} = \int_0^\infty g(u)\int_0^\infty e^{-s(x+u)}F(x)\,dx\,du = \int_0^\infty g(u)\left[\int_0^\infty e^{-sx}e^{-su}F(x)\,dx\right]du$$

$$= \int_0^\infty e^{-su}g(u)\,du \int_0^\infty e^{-sx}F(x)\,dx$$

$$L\{\phi(t)\} = \bar{g}(s)\,\bar{f}(s)$$

$$\boxed{L^{-1}\{\bar{g}(s)\bar{f}(s)\} = \phi(t) = \int_0^t g(u)\,F(t-u)\,du}$$

hence proved

Q) $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$ using Convolution Theorem.

Sol^n) Let $\bar{f}(s) = \frac{1}{s^2+1}$ $\quad \bar{g}(s) = \frac{1}{s^2+9}$

$L^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin t = f(t)$ $\quad L^{-1}\left\{\frac{1}{s^2+9}\right\} = \sin 3t = g(t)$

By Convolution Theorem

$$L^{-1}\{\bar{f}(s)\,\bar{g}(s)\} = \int_0^t F(u)\, g(t-u)\,du$$

$$= \int_0^t \sin u \,\sin 3(t-u)\,du$$

$$= \frac{1}{2}\int_0^t \left[\cos(u-3t+3u) - \cos(u+3t-3u)\right]du$$

$$= \frac{1}{2}\int_0^t \left[\cos(4u-3t) - \cos(3t-2u)\right]du$$

$$= \frac{1}{2}\left[\frac{\sin(4u-3t)}{4} + \frac{\sin(3t-2u)}{2}\right]_0^t$$

$$= \frac{1}{2}\left[\frac{\sin t - \sin 3t}{4} + \frac{\sin t - \sin 3t}{2}\right]$$

$$\boxed{L^{-1}\{\bar{f}(s)\,\bar{g}(s)\} = \frac{1}{8}\left[3\sin t - \sin 3t\right]}$$

Solving ODE by Laplace Method

① Solve by Laplace Method

$$\frac{d^2y}{dt^2} - \frac{2dy}{dt} + y = e^t \qquad \begin{array}{l} y(0)=2 \\ y'(0)=-1 \end{array}$$

Sol^n) Taking Laplace Transform both sides

$$L\left\{\frac{d^2y}{dt^2}\right\} - 2L\left\{\frac{dy}{dt}\right\} + L\{y\} = L\{e^t\}$$

$$s^2\bar{y}(s) - sy(0) - y'(0) - 2\left[s\bar{y}(s) - y(0)\right] + \bar{y}(s) = \frac{1}{s-1}$$

$$\left(\text{where } \bar{y}(s) = L\{y(t)\}\right)$$

$$s^2\bar{y}(s) - 2s + 1 - 2\left[s\bar{y}(s) - 2\right] + \bar{y}(s) = \frac{1}{s-1}$$

$$\bar{y}(s)\left[s^2 - 2s + 1\right] - 2s + 1 + 4 = \frac{1}{s-1}$$

$$(s-1)^2\bar{y}(s) = \frac{1}{s-1} + 2s - 5 = \frac{2s^2 - 7s + 6}{s-1}$$

$$\boxed{\bar{y}(s) = \frac{2s^2 - 7s + 6}{(s-1)^3}}$$

$$L\{y(t)\} = \frac{2s^2 - 7s + 6}{(s-1)^3}$$

Taking Inverse Laplace both side

$$L^{-1}\left\{\frac{2s^2 - 7s + 6}{(s-1)^3}\right\} = y(t)$$

using partial fraction

$$\frac{2s^2 - 7s + 6}{(s-1)^3} = \frac{2}{s-1} - \frac{3}{(s-1)^2} + \frac{4}{(s-1)^3}$$

$$y(t) = L^{-1}\left\{\frac{2}{s-1}\right\} - L^{-1}\left\{\frac{3}{(s-1)^2}\right\} + L^{-1}\left\{\frac{4}{(s-1)^3}\right\}$$

$$= 2e^t - 3e^t \cdot t + 4e^t \cdot \frac{t^2}{\lfloor 2}$$

$$\boxed{y(t) = 2e^t - 3e^t \cdot t + 2t^2 e^t}$$

Laplace Transform of some special functions:

(1) Error function:

$$\boxed{\operatorname{erf}(\sqrt{x}) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{x}} e^{-t^2}\,dt}$$

Sol^n)

$$\operatorname{erf}(\sqrt{x}) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{x}}\left(1-\frac{t^2}{\lfloor 1}+\frac{t^4}{\lfloor 2}-\frac{t^6}{\lfloor 3}+\frac{t^8}{\lfloor 4}-\cdots\right)dt$$

$$= \frac{2}{\sqrt{\pi}}\left[t-\frac{t^3}{3}+\frac{t^5}{5.2}-\frac{t^7}{7.6}+\cdots\right]_0^{\sqrt{x}}$$

$$\operatorname{erf}(\sqrt{x}) = \frac{2}{\sqrt{\pi}}\left[x^{1/2}-\frac{x^{3/2}}{3}+\frac{x^{5/2}}{5.2}-\frac{x^{7/2}}{7.6}+\frac{x^{9/2}}{9.24}-\cdots\right]$$

by def. of laplace Transform

$$L\{\operatorname{erf}(\sqrt{x})\} = \frac{2}{\sqrt{\pi}}\left[\frac{\lceil 3/2}{s^{3/2}}-\frac{\lceil 5/2}{3.s^{5/2}}+\frac{\lceil 7/2}{5.2.s^{7/2}}-\frac{\lceil 9/2}{7.6.s^{9/2}}\cdots\right]$$

$$= \frac{2}{\sqrt{\pi}}\left[\frac{\frac{1}{2}\sqrt{\pi}}{s^{3/2}}-\frac{\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}}{3.s^{5/2}}+\frac{\frac{5}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi}}{5.2.s^{7/2}}-\frac{\frac{7}{2}\cdot\frac{5}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi}}{7.6.s^{9/2}}\right]$$

$$= \left[\frac{1}{s^{3/2}}-\frac{1}{2s^{5/2}}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{s^{7/2}}-\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{1}{s^{9/2}}+\cdots\right]$$

$$= \frac{1}{s^{3/2}}\left[1-\frac{1}{2.s}+\frac{1.3}{2\ 4}\cdot\frac{1}{s^2}-\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{1}{s^3}+\cdots\right]$$

$$= \frac{1}{s^{3/2}}\left[1+\frac{1}{s}\right]^{-1/2} = \frac{1}{s^{3/2}}\left(\frac{s+1}{s}\right)^{-1/2} = \frac{1}{s^{3/2}}\left(\frac{s}{s+1}\right)^{1/2}$$

$$\boxed{L\{\operatorname{erf}(\sqrt{x})\} = \frac{1}{s(s+1)^{1/2}}}$$