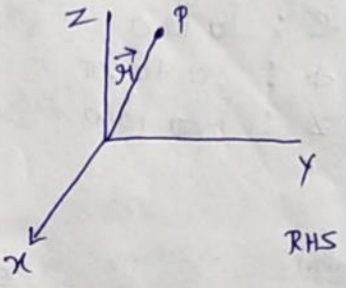
"Unit 1"

Coordinate system:-

1. Cartesian
2. spherical
3. cylindrical

1. Cartesian:-



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

★ limits:-

x : $-\infty$ to ∞

y : $-\infty$ to ∞

z : $-\infty$ to ∞

★ length element:-

$dl_x = dx\,\hat{i}$

$dl_y = dy\,\hat{j}$

$dl_z = dz\,\hat{k}$

★ surface element:-

$ds_x = dy\,\hat{j} \times dz\,\hat{k} = dy\,dz\,\hat{i}$

$ds_y = dx\,dz\,\hat{j}$

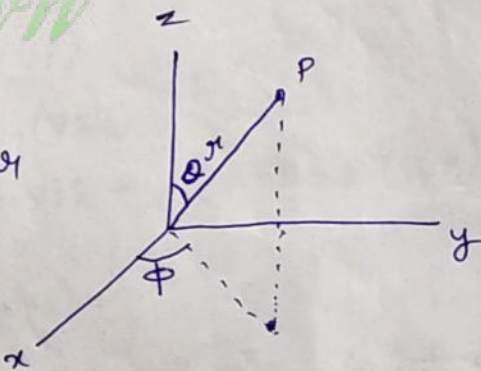
$ds_z = dx\,dy\,\hat{k}$

★ volume element:-

$dV = \vec{dx}\cdot(\vec{dy}\times\vec{dz})$

$$\boxed{dV = dx\,dy\,dz}$$

2. spherical:- (r, θ, ϕ)



r, θ, ϕ will be mutually prependicular

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$$\vec{r} = r\sin\theta\cos\phi\,\hat{i} + r\sin\theta\sin\phi\,\hat{j} + r\cos\theta\,\hat{k}$$

★ limit-

r : $0 \rightarrow \infty$

θ : $0 \rightarrow \pi$

ϕ : $0 \rightarrow 2\pi$

★ length element:-

$dl_r = dr\,\hat{r}$, $dl_\theta = r\,d\theta\,\hat{\theta}$,

$dl_\phi = r\sin\theta\,d\phi\,\hat{\phi}$

★ surface element:-

$(r \rightarrow \text{fix})$ $ds_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{r}$

$(\theta \rightarrow \text{fix})$ $ds_\theta = r\sin\theta\,dr\,d\phi\,\hat{\theta}$

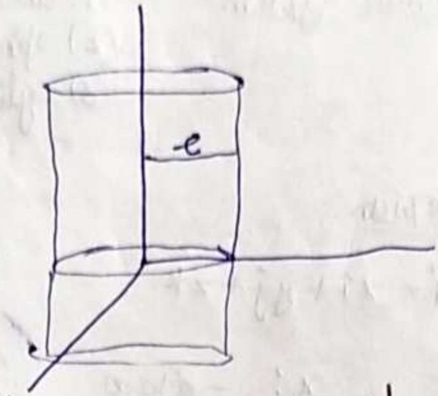
$(\phi \rightarrow \text{fix})$ $ds_\phi = r\,dr\,d\theta\,\hat{\phi}$

★ volume element:-

$d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

③ Cylindrical Coordinate system :- (ρ, ϕ, z) or (s, ϕ, z)

$x = \rho\cos\phi$
$y = \rho\sin\phi$
$z = z$



limit:- ρ : 0 to ∞
ϕ : 0 to 2π
z : $-\infty$ to ∞

length element:-

$dl_\rho = d\rho\ \hat{\rho}$
$dl_\phi = \rho d\phi\ \hat{\phi}$
$dl_z = dz\ \hat{z}$

surface element:-

$ds_\rho = \vec{dl_\phi} \times \vec{dl_z}$
$ds_\rho = \rho d\phi dz\ \hat{\rho}$
$ds_\phi = dz d\rho\ \hat{\phi}$
$ds_z = \rho d\rho d\phi\ \hat{z}$

volume element:-

$dv = \rho d\rho d\phi dz$

Gradient :-
① The gradient ∇T points in direction of max^m increase of function T.
& $|\nabla T|$ gives slope along this direction.

② $\nabla T = 0$ for max^m point (highest), min point (lowest) or a saddle point.

③ scalar $\longrightarrow$ vector

Divergence :- $\nabla \cdot \vec{V}$

① vector $\rightarrow$ scalar

② point with +ve divergence = source
" " -ve " = sink

Curl :-
① It is measure of how much a vector v swirls around a point.

Imp. Rules :-

① $\nabla(fg) = f\nabla g + g\nabla f$

② $\nabla(A \cdot B) = A \times (\nabla \times B) + B \times (\nabla \times A) + (A \cdot \nabla)B + (B \cdot \nabla)A$

③ $\nabla \cdot (fA) = f(\nabla \cdot A) + A \cdot (\nabla f)$

④ $\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$

⑤ $\nabla \times (A \times B) = (B \cdot \nabla)A - (A \cdot \nabla)B + A(\nabla \cdot B) - B(\nabla \cdot A)$

for second derivatives (5 possibilities) :-

① Divergence of gradient $= \nabla \cdot (\nabla T) = \nabla^2 T$ (called as Laplacian of scalar T)

for vector v,

$$\nabla^2 v = (\nabla^2 v_x)\hat{i} + (\nabla^2 v_y)\hat{j} + (\nabla^2 v_z)\hat{k}$$

(To calculate Laplacian of a vector)

where

$$\nabla^2 v_x = \left(\frac{d^2 v_x}{dx^2} + \frac{d^2 v_x}{dy^2} + \frac{d^2 v_x}{dz^2}\right)$$

$$\nabla^2 v_y = \frac{d^2 v_y}{dx^2} + \frac{d^2 v_y}{dy^2} + \frac{d^2 v_y}{dz^2}$$

$$\nabla^2 v_z = \frac{d^2 v_z}{dx^2} + \frac{d^2 v_y}{dy^2} + \frac{d^2 v_z}{dz^2}$$

② Curl of gradient :- $\boxed{\nabla \times (\nabla T) = 0}$

③ Gradient of divergence :- $\boxed{\nabla(\nabla \cdot v) \neq \nabla^2 v}$

④ divergence of curl : $\boxed{\nabla \cdot (\nabla \times v) = 0}$

⑤ curl of curl : $\boxed{\nabla \times (\nabla \times v) = \nabla(\nabla \cdot v) - \nabla^2 v}$

Integral Calculus :-

① Line integral :- $\int_a^b v \cdot dl$ (Line integral is independent of path so called as conservative.)

$(dl = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

② surface integral :- $\oint_a^b v \cdot da$ (represents total mass per unit time passing through surface) called as flux).

$da\,\hat{i} = dy\,dz\,\hat{i}$
$da\,\hat{j} = dx\,dz\,\hat{j}$
$da\,\hat{k} = dx\,dy\,\hat{k}$

③ Volume integral :- $\int_v T\,d\tau$, $(d\tau = dx\,dy\,dz)$

Fundamental Theorem of Calculus :-

① $\int_a^b F(x)\,dx = f(b) - f(a)$

② Theorem for Gradients :-

for a function T

$$\boxed{\int_a^b (\nabla T)\cdot dl = T(b) - T(a)}$$

ⓐ It is ~~fd~~ independent of path taken

ⓑ if $a=b \Rightarrow T(b) - T(a) = 0$

③ Theorem of divergence :-

$$\boxed{\int_v (\nabla\cdot v)\,d\tau = \oint_s v\cdot da}$$

Gauss' theorem
Green theorem
divergence theorem

$d\tau = dx\,dy\,dz$

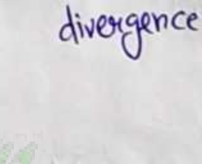
④ Theorem of curls :-

$$\boxed{\int_s (\nabla\times v)\cdot da = \oint_p v\cdot dl}$$

Stoke's theorem

ⓐ $\int(\nabla\times v)\cdot da$ depends only on the boundary line not on particular surface used

ⓑ $\oint(\nabla\times v)\cdot da = 0$ for any closed surface



$da = dy\,dz\,\hat{i}$ (for y-z plane)

$da = dx\,dy\,\hat{k}$ (for xy plane)

$da = dx\,dz\,\hat{j}$ (for xz plane)

② Spherical Coordinates :-

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$\hat{r} = \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}$$
$$\hat{\theta} = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}$$
$$\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$$

$$\left.\begin{aligned} dl_r &= dr \\ dl_\theta &= r\,d\theta \\ dl_\phi &= r\sin\theta\,d\phi \end{aligned}\right\} \quad \boxed{d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}}$$

① Gradient :-

$$\nabla T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\phi}$$

② divergence :-

$$\nabla \cdot V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{(r\sin\theta)}\frac{\partial}{\partial \theta}(\sin\theta\, V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial \phi}$$

③ curl :-

$$\nabla \times V = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, V_\phi) - \frac{\partial V_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial \phi} - \frac{\partial}{\partial r}(r V_\phi)\right]\hat{\theta}$$
$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(r V_\theta) - \frac{\partial V_r}{\partial \theta}\right]\hat{\phi}$$

OR

$$\nabla \times \vec{A} = \begin{vmatrix} \frac{1}{r\sin\theta}\hat{a}_r & \frac{1}{r\sin\theta}\hat{a}_\theta & \frac{1}{r}\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 1 \cdot A_r & r A_\theta & r\sin\theta A_\phi \end{vmatrix}$$

☆☆☆

$$\vec{\nabla}\times v = \begin{vmatrix} \frac{1}{r^2\sin\theta}\hat{r} & \frac{1}{r\sin\theta}\hat{\theta} & \frac{1}{r}\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ V_r & rV_\theta & r\sin\theta\, V_\phi \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\hat{r}\left(\frac{\partial}{\partial\theta}(r\sin\theta V_\phi) - \frac{\partial}{\delta\phi}(rV_\theta)\right) - \frac{1}{r\sin\theta}\hat{\theta}\left(\frac{\partial}{\partial\phi}V_r - \frac{\partial}{\partial r}r\sin\theta V_\phi\right) + \frac{1}{r}\hat{\phi}\left(\frac{\partial}{\partial r}rV_\theta - \frac{\partial}{\partial\theta}V_r\right)$$

$$= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta V_\phi) - \frac{\partial V_\theta}{\partial\phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{\partial}{\partial r}rV_\phi\right)\hat{\theta}$$

$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial\theta}\right)\hat{\phi}$$

④ <u>Laplacian</u> :-

$$\boxed{\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}}$$

<u>Trick</u>: from divergence again multiply "D" by $1, r, r\sin\theta$

③ Cylindrical Coordinate :- $(\hat{s}, \hat{\phi}, \hat{z})$

$x = s\cos\phi$
$y = s\sin\phi$
$z = z$

$$\left.\begin{aligned} dl_s &= ds \\ dl_\phi &= s\,d\phi \\ dl_z &= dz \end{aligned}\right\} \quad \boxed{dl = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}}$$

① Gradient :-

$$\boxed{\nabla T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}}$$

② Divergence :-

$$\boxed{\nabla \cdot V = \frac{1}{s}\frac{\partial}{\partial s}(s V_s) + \frac{1}{s}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}}$$

③ Curl :-

$$\vec{\nabla} \times V = \begin{vmatrix} \frac{1}{s}\hat{s} & \hat{\phi} & \frac{1}{s}\hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ V_s & s V_\phi & V_z \end{vmatrix}$$

$$= \frac{1}{s}\hat{s}\left(\frac{\partial V_z}{\partial \phi} - \frac{\partial (s V_\phi)}{\partial z}\right) - \hat{\phi}\left(\frac{\partial V_z}{\partial s} - \frac{\partial V_s}{\partial z}\right) + \frac{1}{s}\hat{z}\left(\frac{\partial}{\partial s}(s V_\phi) - \frac{\partial}{\partial \phi}V_s\right)$$

$$\vec{\nabla} \times V = \hat{s}\left(\frac{1}{s}\frac{\partial V_z}{\partial \phi} - \frac{\partial}{\partial z}V_\phi\right) + \hat{\phi}\left(\frac{\partial V_s}{\partial z} - \frac{\partial V_z}{\partial s}\right) + \frac{1}{s}\hat{z}\left(\frac{\partial}{\partial s}(s V_\phi) - \frac{\partial V_s}{\partial \phi}\right)$$

④ Laplacian :-

$$\boxed{\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial T^2} + \frac{\partial^2 T}{\partial z^2}}$$

for

1. Maxwell's Equations :- (4 equ^n)

① $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ (Gauss law of electrostatics)

② $\vec{\nabla} \cdot \vec{B} = 0$ (" " of Magnetostatics)

③ $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$ (faraday law of Mutual Induction)

④ $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \underbrace{\epsilon_0 \frac{\partial \vec{E}}{dt}}_{\text{correction factor (displacement current)}} \right)$ (Modified Ampere's Circuital law)

Notations: $\vec{D} = \epsilon_0 \vec{E}$ (displacement density)

$B = \mu_0 H$ ~~$\vec{H} = \mu_0 \vec{B}$~~ (magnetic field intensity)

$\vec{J} = \sigma \vec{E}$ (current density)

2. In Integral form of these 4 equ^n is -

① $\oint_s \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \int_v \rho dV$ [Gauss Law]

② $\oint_s \vec{B} \cdot \vec{ds} = 0$

③ $\int_l \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \oint_s \vec{B} \cdot \vec{ds}$ [Faraday law]

④ $\int_l \vec{B} \cdot \vec{dl} = \mu_0 \oint_s \left(\vec{J} + \epsilon_0 \frac{\partial E}{\partial t} \right) \cdot \vec{ds}$

③ Derivation of Maxwell's equations -

① $\oint_s \vec{E} \cdot \vec{ds} = \frac{1}{\epsilon_0}\int_v \rho \, dv$

Proof: we know that

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

apply volume integral both sides

$$\int_v (\vec{\nabla} \cdot \vec{E}) dv = \frac{1}{\epsilon_0}\int_v \rho \, dv$$

apply gauss divergence Theorem -

$$\boxed{\oint_s \vec{E} \cdot \vec{ds} = \frac{1}{\epsilon_0}\int_v \rho \, dv} \text{ h.p}$$

② $\oint_s \vec{B} \cdot \vec{ds} = 0$

Proof:-

$$\vec{\nabla} \cdot \vec{B} = 0$$

apply. V I both sides

$$\int_v (\vec{\nabla} \cdot \vec{B}) dV = 0$$

$$\boxed{\int_s \vec{B} \cdot \vec{ds} = 0} \text{ h.p}$$

③ $\int_l \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t}\oint_s \vec{B} \cdot \vec{ds}$

Proof:- we know that $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$

apply surface Integral -

$$\oint_s \vec{\nabla} \times \vec{E} \, ds = -\frac{\partial}{\partial t}\oint_s \vec{B} \cdot \vec{ds}$$

apply stoke's theorem -

$$\boxed{\int_l \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t}\oint_s \vec{B} \cdot \vec{ds}} \text{ h.p}$$

④ $\int_l \vec{B} \cdot \vec{dl} = \oint_s \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right) ds$

Proof:

$$\vec{\nabla} \times \vec{B} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

apply S.I

$$\oint_s \vec{\nabla}\times\vec{B}\, ds = \oint_s\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) ds$$

$$\boxed{\int_l \vec{B}\cdot\vec{dl} = \oint_s\left(\vec{J} + \epsilon_0\frac{\partial \vec{E}}{\partial t}\right) ds}$$

④ Maxwell's 3rd equ^n in differential form $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}}$

Proof:- We know that by faraday's law of mutual induction

$$\boxed{emf = -\frac{d}{dt}\phi_B} \quad \text{---(i)}$$

also $\quad emf = \oint_l \vec{E}\cdot\vec{dl} \quad$ ---(ii)

using equ^n (i) & (ii)

$$\oint_l \vec{E}\cdot\vec{dl} = -\frac{d}{dt}(\phi_B) \quad \text{----(iii)}$$

Now mag flux $= B\cdot A$

$$\phi_B = \oint_s \vec{B}\cdot\vec{ds}$$

putting in equ^n(iii)

$$\oint_l \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\left(\oint_s \vec{B}\cdot\vec{ds}\right)$$

by stoke's theorem

$$\oint_s(\vec{\nabla}\times\vec{E})ds = \frac{-\partial}{\partial t}\left(\oint_s \vec{B}\cdot\vec{ds}\right)$$

$$\oint_s\left[\vec{\nabla}\times\vec{E} + \frac{\partial}{\partial t}(\vec{B})\right]\vec{ds} = 0$$

$$\boxed{\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}}$$

(5) Maxwell's 4th equ^n :- $\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$

Proof:- by ampere ckt law

$$\int_l \vec{B}\cdot\vec{dl} = \mu_0 I \quad \text{---} ①$$

$$\int_l \vec{B}\cdot\vec{dl} = \mu_0 \int_s \vec{J}\cdot ds$$

using stoke's theorem to LHS

$$\int_s (\vec{\nabla}\times\vec{B})\vec{ds} = \mu_0 \int_s \vec{J}\cdot\vec{ds}$$

$$\oint_s \left[\vec{\nabla}\times\vec{B} - \mu_0\vec{J}\right]\cdot\vec{ds} = 0$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}} \quad \text{-----(i)}$$

Now checking validity of equ^n (i) -

using equ^n of Continuity i.e $\vec{\nabla}\cdot\vec{J} = \frac{-\partial e}{\partial t}$ --- (i)

Taking divergence of equ^n (i) [$\vec{\nabla}\cdot\vec{J}$ cannot be equal to zero]

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0(\vec{\nabla}\cdot\vec{J})$$

but from vector identity div (curl $\vec{B}$) = 0

so $\boxed{\vec{\nabla}\cdot\vec{J} = 0}$ but equ^n of continuity state that div $(\vec{J}) \neq 0$

hence we can say equ^n (i) is not a valid equ^n.

hence maxwell modified the ampere's ckt law →

maxwell introduced the concept of disp. current

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0(\vec{J} + \vec{J_d})} \quad \text{----(ii)}$$

disp. current density.

Now ~~???~~ checking validity of equ^n —

$$\text{Div}\,(\text{Curl}\ \vec{B}) = \mu_0(\vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{J_d})$$

$$0 = \mu_0(\vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{J_d})$$

$$\boxed{\vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\vec{J_d}} \quad ---n$$

that means $\vec{\nabla}\times\vec{J} \neq 0$ for equ^n (ii)

hence equ^n (ii) is a valid equ^n.

$$** \boxed{\vec{\nabla}\times\vec{B} = \mu_0(\vec{J} + \vec{J_d})} **$$

(6) Now maxwell modified the equ^n as —

~~by equ^n of continuity~~

$$\vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\vec{J_d} \quad \text{(by equ}^n\text{ of continuity)}$$

$$-\frac{\partial e}{\partial t} = -\vec{\nabla}\cdot\vec{J_d}$$

$$\vec{\nabla}\cdot\vec{J_d} = \frac{\partial e}{\partial t}$$

(using maxwell 1st equ^n $\vec{\nabla}\cdot\vec{E} = \frac{e}{\epsilon_0}$)

$$\vec{\nabla}\cdot\vec{J_d} = \epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})$$

$$\vec{\nabla}\cdot\vec{J_d} = \epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})$$

$$\vec{\nabla}\cdot\vec{J_d} = \epsilon_0\,\vec{\nabla}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\cdot\left(\vec{J_d} - \epsilon_0\frac{\partial\vec{E}}{\partial t}\right) = 0$$

$$\boxed{\vec{J_d} = \epsilon_0\frac{\partial\vec{E}}{\partial t}}$$

putting value of $\vec{J_d}$ in equ^n (ii)

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\left(J + \epsilon_0\frac{\partial\vec{E}}{\partial t}\right)}$$

Hence proved

"Electrostatics"

Coulomb's law :-

$$F = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r} \quad , \epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$$

Electric field :-

$$F = QE$$

$$\boxed{E(r) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i}$$

For Continuous charge distributions -

① for a line charge :- $dq = \lambda dl$

$$\boxed{E(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(r)}{r^2} \hat{r} \, dl}$$

② for a surface charge :- $dq = \sigma da$

$$\boxed{E(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(r)}{r^2} \hat{r} \, da}$$

③ for a volume charge :- $dq = \rho d\tau$

$$\boxed{E(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r)}{r^2} \hat{r} \, d\tau}$$

① Gauss law :- / divergence of E :-

① $\boxed{\oint E \cdot da = \frac{1}{\epsilon_0} Q_{enclosed}}$

and

$\boxed{\oint_V (\nabla \cdot E) d\tau = \int E \cdot da}$ (Gauss law)

so in differential form

$$\oint_V (\nabla \cdot E) d\tau = \frac{1}{\epsilon_0} Q_{enclosed} = \frac{1}{\epsilon_0} \int_V \rho \, d\tau$$

$$(\nabla \cdot E) - \frac{\rho}{\epsilon_0} = 0$$

★★ $\boxed{\nabla \cdot E = \frac{1}{\epsilon_0} \rho}$ (divergence of E = $\frac{1}{\epsilon_0}$ (volume charge density))

② Curl of E :-

$\boxed{\nabla \times E = 0}$

Note: if a $\nabla \times E \neq 0$ → impossible electric field

③ Potential :-

$\boxed{V(r) = -\int_{O}^{r} E \cdot dl}$

$\boxed{V_b - V_a = -\int_a^b E \cdot dl}$

from theorem of gradient

$$\int_a^b (\nabla V) \cdot dl = -\int_a^b E \cdot dl$$

★★ $\boxed{E = -\nabla V}$

(4)

$$\boxed{E = -\nabla V}$$

$$\boxed{\nabla \cdot E = \frac{e}{\epsilon_0}}$$

$$\boxed{\nabla \times E = 0}$$

using these 2 equ^{n}s $\nabla \cdot (-\nabla V) = \frac{e}{\epsilon_0}$

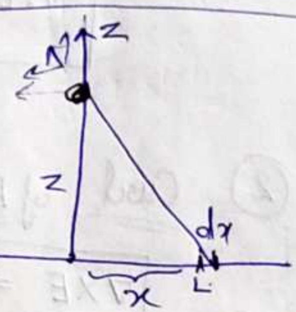
$$\boxed{\nabla^2 V = -\frac{e}{\epsilon_0}}$$

<u>Poisson's equation</u>:- $\boxed{\nabla^2 V = -\frac{e}{\epsilon_0}}$

<u>Laplace equation</u>:- for there is no charge

$$\boxed{\nabla^2 V = 0}$$

\# St. wire of $l = 2L$



$$E = \frac{1}{4\pi\epsilon_0}\int_{-L}^{L} \frac{\lambda dl}{(z^2+x^2)} = k\lambda\int_{-L}^{L} \frac{z\hat{z} - x\hat{x}}{(z^2+x^2)^{3/2}} dx$$

$$E = k\lambda\left[z\hat{z}\int_{-L}^{L}\frac{1}{(z^2+x^2)^{3/2}}dx - \hat{x}\int_{-L}^{L}\frac{x}{(z^2+x^2)^{3/2}}dx\right]^{L}$$

$$= k\lambda\left[\frac{z \cdot x}{z^2\sqrt{z^2+x^2}}\Big|_{-L}^{L} - \hat{x}\left(\frac{-1}{\sqrt{z^2+x^2}}\right)_{-L}^{L}\right]$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}}}$$

$$dl = (z\hat{z} - x\hat{x})\,dx$$

"Magnetostatics"

① Magnetic force =

$$\boxed{F_{mag} = Q(v \times B)}$$

② Lorentz force :-

$$\boxed{F = Q\left(E + (v \times B)\right)}$$

③ Cyclotron formula: $P = QBR$

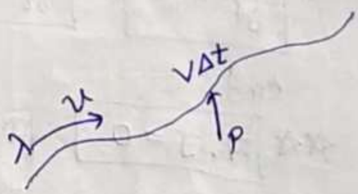
$$\boxed{R = \frac{mv}{QB}}$$

④ Magnetic forces do not work!

$$W_{mag} = F_{mag} \cdot dl = Q(v \times B) \cdot v\,dt = \underline{\underline{0}}$$

⑤ Currents :- A line charge λ travelling down a wire at speed v constitutes a current

$$\boxed{I = \lambda v}$$



The magnetic force on segment of current carrying wire is :-

$$F_{mag} = \int (v \times B)\,dq = \int (v \times B)\,\lambda\,dl = \int (I \times B)\,dl$$

$\frac{\lambda}{l} = q$

$I = \frac{q}{t} = \lambda$

when I = constant

$$\boxed{F_{mag} = I \int (dl \times B)}$$

⑥ ⓐ Mag. force due to line current:-

$$\boxed{F_{mag} = I \int \vec{dl} \times \vec{B}}$$

ⓑ Mag. force on surface current -

$$\boxed{F_{mag} = \int (K \times B)\, da}$$

where $K = \sigma V$
(σ = charge density, V = velocity)

ⓒ Mg. force on a volume current -

$$\boxed{F_{mag} = \int (v \times B) \rho \, d\tau = \int (J \times B)\, d\tau}$$

[J = current per unit area]

⑦ Biot - Savart Law:-

for steady currents, ⇒ Constant mag. fields : magnetostatics.

$$\boxed{\frac{\partial \rho}{\partial t} = 0}, \quad \boxed{\frac{\partial J}{\partial t} = 0}$$

and

☆☆ $\boxed{\nabla \cdot J = 0}$ for steady currents

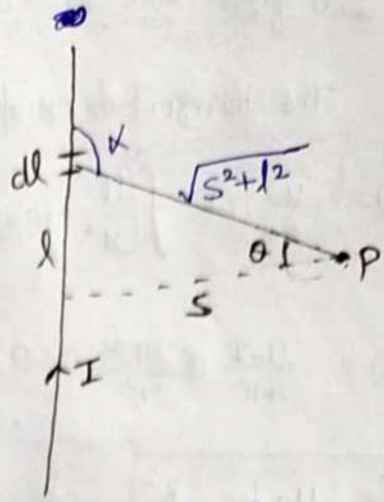
$$\boxed{B(r) = \frac{\mu_0}{4\pi} \int \frac{I \times \hat{r}}{r^2}\, dl' = \frac{\mu_0}{4\pi} I \int \frac{dl \times \hat{r}}{r^2}}$$

$$\left[\frac{\mu_0}{4\pi} = 10^{-7}\, N/A^2\right]$$

$\boxed{\text{unit of } B = \text{Tesla (T)}}$

(8) Mag. field due to wire :-

due to finite wire –



$$B(P) = \frac{\mu_0}{4\pi} I \int \frac{dl \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \left(\frac{\cos^2\theta}{s^2}\right)\left(\frac{s}{\cos^2\theta}\right) \cos\theta \, d\theta$$

$l = s\tan\theta$

$$\boxed{dl = \frac{s}{\cos^2\theta} d\theta}$$

$$B = \frac{\mu_0 I}{4\pi s} \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta$$

$$\boxed{B = \frac{\mu_0 I}{4\pi s} \left(\sin\theta_2 - \sin\theta_1\right)}$$

$s = r\cos\theta$

$$\boxed{\frac{1}{r^2} = \frac{\cos^2\theta}{s^2}}$$

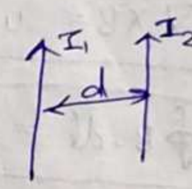
$$\boxed{dl \sin\alpha = dl \cos\theta}$$

for infinite wire :

$\theta_1 = \pi/2 \,, \quad \theta_2 = -\frac{\pi}{2}$

$$\boxed{B = \frac{2\mu_0 I}{4\pi s} \hat{\phi}}$$

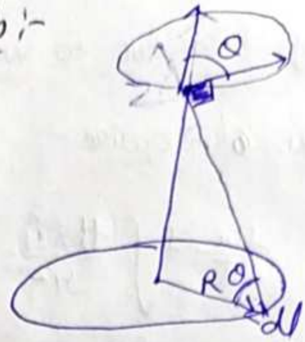
(9) Force b/w^n 2 infinite parallel wires :-



$F_{12} = I B \int dl$

$$\boxed{\frac{F}{l} = \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{d}}$$

(10) Mag. field due to current carrying loop :-



The horizontal components cancel and

$$B(z) = \frac{\mu_0}{4\pi} I \int \frac{dl}{r^2} \cos\theta$$

$$B(z) = \frac{\mu_0 I}{4\pi} \times \frac{2\pi r}{r^2} \cos\theta = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$$

as $\vec{dl} \times \hat{r} = dl$ as $\vec{dl} \perp r$

& $\left[\int dl = 2\pi R\right]$

$$\boxed{B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}}$$

B at centre of loop $= \frac{\mu_0 I}{2} \left(\frac{R^2}{R^3}\right)$

$$\boxed{B_c = \frac{\mu_0 I}{2R}}$$

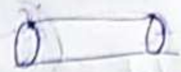
(11) Ampere's Law :-

☆ $\boxed{\nabla \times B = \mu_0 J}$ is called ampere's law (in differential form)

☆ $\boxed{\oint B \cdot dl = \mu_0 I_{enc}}$

Proof: $\left[\int (\vec{\nabla} \times B) \cdot da = \int B \cdot dl = \mu_0 \int J \cdot da\right]$

(14) Vector potential of an infinite solenoid with n turns per unit length



$$\oint A\cdot dl = \int(\nabla\times A)\cdot da = \int B\cdot da = \Phi$$

$$\oint A\cdot dl = A(2\pi s) = \oint B\cdot da = \mu_0 n I(\pi s^2)$$

$$A = \frac{\mu_0 n I \pi s^{\cancel{2}}}{2\pi \cancel{s}} = \frac{\mu_0 n I}{2} s\,\hat{\phi} \quad \text{for } s \le R$$

for outside loop, flux $\boxed{\int B\cdot da = \mu_0 n I(\pi R^2)}$

$$A = \frac{\mu_0 n I R^2}{2s}\hat{\phi} \quad \text{for } s \ge R$$

so

$$A \rightarrow \begin{cases} \dfrac{\mu_0 n I}{2} s\,\hat{\phi}, & s \le R \\[2ex] \dfrac{\mu_0 n I R^2}{2s}\hat{\phi}, & s \ge R \end{cases}$$