A COURSE IN

**ELECTRICAL** AND ELECTRONIC ~/IEASUREMENTS AND

INSTRUMENrATIC>N

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*By*

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Preface to the· Fourtn 1:arc1on

. 'The author i~ pleased to brfogi out the Fourth Editiori of the book a.ncf is \_fhankful t~· both· teachers and students for their aff ecfi-0nate and warm receptio:n to the thlfd ed1t~n of the boo~ whJ9h has been sold out in a period of less than one year. The present edit.ion retai~s essentially the same subjectmatter as the third edition. However, the typographical and (\ther .errors, which had crept in earlier edition, have been corre.cted.

The book is intended as a stand~rd text for students studying for their first de8ree i•. Electrical, Electronics and lhstrumentation Engineering at ~ndian Universities (}nd abroad, qd· also for those appearing for A.M.l.E. section Band other profession~l examinations. The book is equally useful for postgraduate students as well as practising engineers involved in ihe iel4A of Measurements and Instrumentation. 1 · •

There have been significant changes in curriculum of almost all the upiversities in recent years. Electrical and Electronic In,strumentation is now offered· as separate paper in maay universitks. This has been necessitated on account of latest technological advances which put greater emphasis and reliance on Electronic Instrumentation. . . The contents of the book have been· drastically modified, . re-arrange~ an9 updated t9 · acquaint the reader of modern trends in the field: pf\_ Measurements and Instrumeµtation. \_ .

, The bobk has b.een divided into two parts. *-i* Part I d~aJs with Mea.surements .and Measuriq Instruments and Part II takes care of the Instrmnentat1on. There 1s an extensive coverage 4f Electrical and Electronic Instrumentation in this edition of the book as compared with the earlier ones. The coverage of Instrumentation portion is about 500 pages · out of a total of nearly 1200 pages. The subject of ·Instrumentation has been developed in logical steps. Separate ahapteu are devoted to topics like Generalized Measurement Systems, Transducers, Signal Conditioning, Data Transmission and Telemetry, Display Devices and Recorders, Measurement of Non· electrical,

Quantities and Data Acquisition Systems. Also there are additional chapters on static and dynamic characteristics of Measurement Systems. There is an increased emphasis on digital instrumftlnts and instrumentation which is constant with the present trends. · . ·

There are . three Appendices in the book. Appendix A deals with Number Systems, Appendix B with Logic gates and Appendix C with conversions from various system~ of units to S.I. The significant additions to the measurements· portion are Transformer ratio bridges (wliich are fast replacing the conventional four arm a.c. bridges), spectrum analyzer, vector impedance meter, vector meter, digital maximum demand indicator and Hall effect multiplier to name a few .

. The book , in fact covers a very wide spectrum of the\_Jield of Electrical and Electro1io Measurements ?.nd Instrumentation and .is a conlplete reference in. itself.

Another outstanding feature of the book is the i~clusion Qf over 400 1solveQproblems which· ill addition to linking the theory with actual applications gives an insight of the industrial practieo. Also about 300 unsolved problems (with answers) have been included to give the students practiee in solving problems.

. The author considers the inclusion of problems on Instrumentation (both· ''solved and uB.l ~~ved) as a speciality of this book. This is a pioneering effort which is the outcome of cuthor's, experience of teaching the subject for almost two decades.

The book though voluminous, covers two papers, *i.e.,* first on Electrical and Ele~tronie: Measurements and Instruments and second on Electrical and Electronic Instrull\_1entat\Qn .arid thus fully justifies its volume. SI units have been uniformly used in .the book throughout. 1 ·

The autho'r will feel highly obliged to·all the readers for their constructive suggestions aQd healthy criticism of the book which will go a long way in the improvement of the text. - ,· The author is thankful to his wife, Chander J for render,ing assistance in the compilation Qd editing of the work. · · · · . . ' · The author is grateful to his brothers, Ravish a~d Ajay, for their constant help durinatlae: p~eparatiQn of tqe text. - · · ·

I To ...... aut~or;s parents who have ~~en~ s~urce of encourascment and guidance. PATIALA 26-9-1.984 ' ~. A..K. S.A.WQlf

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1010-1073 LOGlC GATES

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PART I

ELECTRICAL AND ELECTRONIC MEASUREMENTS **A'ND.**

MEASURING INSTRUMENTS

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Measurements and Mea~u~ement Systems

1'1. Measurements

. The .measurement of a given quantity is essentially an act or the result of comparison between the quantity (whose magnitude is unknown) and a predefined standard. Since two quantities are compared the result. is expressed in numerical values. . . .

In order that the results of the measurement are meaningful, there are two basic requirements : (i) Th.e standard used for comparison purposes must be accurately defined and should be commonly acc\_epted, ·

and({;) The apparatus used and the method adopted must be provable.

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1 ·2. · Significance of Measurements

The advancement of Science and Technology is dependent upon a parallel progress in measurement techniques. It can be safely said that the quickest way to assess ·a nation's progress in Science and Technology is to examine the type of measurements that are being made and the way in which the data is acquired by measurements and is processed.

The reasons for this are abvious. As Science and Technology move ahead, new phenom~na and rela~ionships are discovered an~ these.~dvances make new types of measurements imperative. New discov~rtes are not of any practical utihty unless they are backed by actual measurements. The measurements, no doubt, confirm the validity Of·a hypothesis but also add to its understanding. This results in an unending chain which leads to new discoveries that require more, new ·and sophisticated measurement techniques. Hence modern Science and Technology are associated with sophisticated methods of measurement while elementary Scieuce and Technology require only ordinary methods of measurement.

There are two major functions " of all branches of engineering :

vJ Design of equipment and processes,

and (U) Proper operation and maintenance of equipment and· processes. . . . Both these functions require measurements. This is because proper and economical design, ..

operation and maintenance require a feedback .of . information. This information is supplied by making suitable measurements.

, 1'3. Methods of Measurement

The methods of measurement may be broadly classified into two categories.

1'3'1. Direct Methods. In these methods, the unknown quantity (also called the measurand) is directly compared against a standard. the result is expressed as a numerical number and a unit. The standard, in fact, is a physical embodiment of a unit. Direct methods are quite common f ?r the measurement of physical quantities like length, mass and time. - -~~,,

Suppose we want to measure the length of a bar. The unit of length is metre. A bar is so many times long because that many units on (}Ur standard have tne same length as. the bar. A humaq being can make direct length comparisons with a preciseness of,.i!bout 0·25 mm. Therefore, on account of human factors it is not possible to make very accurate measurements. The direct method for measurement ·of ·length can be utilized with a good degree of accuracy but when it comes to measuremenf of mass, the problem becomes much more intricate. It is just not possible for huma? beings to distinguish between :Wide margins of mass. · ·

2. .ELECTRICAL MEASUREMENTS AND MEASURING INSTRUMENTS  

1 ·3·2~ Imliiled Methods. Measurement by direct methods are not always possible, feasible and pracdcable. These methods in most of the cases, are inaccurate because they involve human factors. They are also less sen~itive. Hence direct methods are not preferred and are rarely used.

Jn engineering applications Measurement Systems are used. These measurement systems use indirect methods for measurement purposes.

A meaemrement system consists .. of a transducing element which converts the quantity to be measured in an analogous form.\ . The analogous signal is then processed by some intermediate means and is. then fed to the end devices\which present the results of the measurement. ·

1·4. Instromeimt

Measurement generally involves using an in.strume~t as a physical means of determining a quantity or variable. The. instrument serves as an· ex~ension of human faculties and enables the · man to determine the value of an unknown quantity which his unaided human faculties cannot measure.·

An instrument may be de.fined as a device for determining the· value or magnitude of a quantity or variable.

1·s. Mecbanicnl, Electrica' and Electronic Instruments

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The first instruments were mechanical in nature and the principles on which these instruments worked are even in vogue today. The earliest scientific instrumpnts used the same three essential elements as our modern instruments do. These elem~nt~ are,: .

(i) a detector, (ii) an in.termediate transfer device, and (iii) an indicator, recorder or a storage device.

The history of development of in~truments ~ncompasses three phases of instruments, *viz.* : {l) mechanical instrume~ts, (ii) electrical instrum~nts ~I;ld (i(i) tilCQtronic instruments.

1 '5'1. M~niml IMtnmeutl. ·These instruments are very reliable for static and stable conditions. But they suffer from a very major disadvantage. · They. are unable to respond rapidly to measurements of dynamic and transient conditions. This.fS-<due to the fact that th~se instruments have moving parts that are rigid, heavy and bulky and oonseqmmtly have a large mass. Mass presents inertia pr-0bmll)s aod hence. these instruments cannot faithfuUy foUow the rapid changes which are involved in dynamio1measurements. Thus it would be virtually impossible to measure a *50* Hz voltage by a mechanica] method, but it is r~latively . e~\_sy to, meapu~,e a. ~lqwJy v.arying pressure. Anotqer disadvantage of mechanical insttuments i! ehat 'most of them are ~ potential.sour~e of poise and cause pollution of silence.

rs~2. Electrleill Instrumetm. B~ctrfoal nttthods. of indicating the output of detectors are more rapid 'than m(iebanioal methods. It ii'. 1unfortunatt that~- eleetrioal system normally depends upon a mechanicru meter movement.as indicating. device.· This: mechanical movement has som~ inertia and therefore these instruments have a limited time (and hence, frequency): response. For.· example, some electrical recorders can give full scale response in 0·2 s, the majo1'itY of industrial reoorde,rs have responses of 0·5 to 24 s. Some g~lvan,otn~ters ~aq follow 50 Hz variationst bu.t ~en these are too slow for present day requirements of fast meastu:~ment. .

1 ·5·3. ElectroDic lnstrnmentll.. . These -days m0.$t of tie scientific and ind\lstrial measurements require very. fast· responses. The ·mechanical and ~l~ctr1:~ i.nstr~ts and systems cannot copo up with these requirements. The necessity to step UP, response;! · ~p~ ~nd also the detection of dynamic. changes in certain parameters,. which require .. the . m.onitorin2 tim~ of tbe order of ms and many a times, i.t8" ha.ve led to the desisn of today's el~~.troniQ 1 in,~ruttl.~ts and their .~ssociated ~ircuitey .. These instrutruints ~equire vacuum tubes or s"mi,cmnducto.r .devices .. R,ecent l>~ctice is to use· se.mi~ondu~tor devices as they have \_many ad'1antages over th~ir vaC\UJm tub~ , cc;~nterparts.. $in~.in elyctronic ~~vices, the only mov~nt~I)t iQvoJV'ed *is* that of "f~tion~ -'~h~ · se~p9n~.~ tjJAe i$ e'tr~e~y small o~

MEt\SUREMENfS AND MEASUREMENT SYSTEMS

account of very small inertia of electrons. P\>r example, a C.R.O. is capable ·of following dynamic and transient changes of the order of' a few ns (l0-9 s).

Electronically controlled power supplies are used to provide stable voltages for studies in the field of chemical reactions and nuclear instrumentation. Electronic instruments are steadily becoming more reliable on acco.unt of improvements in design and manufacturing processes of semi-conductor devfoes. Another a~va'ntage of using electronic devices is that very weak signals can be detected by using pre-amplifiers and amplifiers. The foremost importance of the electronic instruments is the power amplification provided by the electronic amplifiers. Additional power may be fed into the system to provide an increased power output beyond that of the input. This. has been only possible through the use of electronic amplifiers, which have no important mechanical counterpart This is particularly important where the data presentation devices use stylus type recorders, galvanometers, cathode ray oscilloscopes and magnetic tape recorJers.

It is a fact that hydraulic and pneumatic systems may be used for power amplification of signals. However, their use is limited tJ slow acting control applications like servo-systems, chemical processes and power systems. Electronic instruments find extensive use in detection of electro. magnetically produced signals such as radio, video, and microwave. Electrical and electronic instruments are particularly useful in the intermediate signal modifying stage. Electronic instruments are light compact and have a high degree of reliability. Their power consumption is very low.

Communications is a field which is entirely dependent upon the electronic instruments and associated apparatus. Space commuriications, especially, makes use of air borne transmitters and receivers and job of interpreting the signals is left entirely to the electronic instruments. .

Electronic instruments make it possible to build analog and digital eomputers without which the modern developments in science and technology are virtually impossible. Computers require a very fast time response and it is only possible with use of electronic instruments.

l '6, Classification of Instruments

There are many ways in which instruments can be classified. Broadly, instruments are classified into two categories : -

(1) Absolute Instruments, and (2) Secondary Instruments. 1. Absolute Instruments. These instruments give the. magnitude of .the quantity under measurements in terms of physical constants of the instrument. The examples of this class of instruments are Tangent Galvanometer and Rayleigh's current balance. . 2. Secondary Instruments. These instruments are SO' constructed that the quantity being measured can only be measured by observing the output indicated by the instrument. These instruments are calibrated by comparison with an absolute instrument or another secondary instrument which has already been calibrated against an absolute instrument.

Working with absolute instruments for routine work is time consuming since every time a measurement is made, it takes a lot of time to compute the magnitude of the quantity under measure ment. Therefore secondary instruments are most co~monly used. Absolute instruments are seldom used except in standards institutions while secondary instruments find usage almost in every sphere of measurement. A voltmeter, a glass thermometer and a pressure gauge are typicalexample.s of secondary instruments.

*1·1.* Analog an~.Digital Modes of Operation. Secondary instruments work in two modes : (i) Analog . mode, and (ii) Digital mode.

Signals that vary in a continuous fashion and take on an infinity of values in any given range are called analog signals .. The devices which produce ihese signals· are called analog. 4~vi~el!I .. In contrast, "the signals which vary in discrete steps and th1i~ 1~ke up ordy ~oite different vaiues in a given range are called digital signals. The devfoes the.t producQ such ·si$nals are called· di~ital devices. · ·

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4 ELECJRICAb MBASURBMBNTS AND MEASURING, lN5TRUMBNTf  Let us elabo.rate further on Digital and Analog instruments and systems. In an analogue

system the function varies continuously. A typical exam~le. of 10 variation is shown in Fig .. l' 1. On the other hand the d1g1tal . values are discrete and vary 9in equal steps. Each digital

number is a fixed sum of equal steps which is defined by the t . 8 number.

' In ord'1. to convert an analog quantity into a digital ~ 1 number, the vertical displacements must be divided into equal CJ 6

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parts. For exam pl~ in Fig. 1·1, the vertical quantities are · ~ divided into l 0 equal parts and each part has a length of 1 > S

~.,,

unit. When dealing with digital numbers, a quantity between 'E 4o to o·s is O while a quantity between 0·5 to 1 '5 is 1 and a ~

~ ~~-~~

-

the analogue curve is *5·5* from the origin but in digital system !' 2 it would be read as *5.* From *A* to *B* is 6 and from *B* to *C* is ·

quantity between 1 ·5 to 2'5 is 2. For example a point A on ; 3 .•

7. It apparentl~ seems t~at if we ado~t digit~l ~ystem, the

1

errors involved will be considerable. But tf we d1v1de each of

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the 10 steps into 10 equal parts, we get 100 steps instead of 10.

And if these 100 steps are fUrther divided into 10 parts each, 'f 2 3 4 5 6 *7* s 9 10 we will hav~ 1000 steps. This gives much better accuracy in lnd~penderrt variable \_.,... converting ~~a~ogue quantities ·into digit.al numbe!s· We can Fig. l '1. Representation of {ln go on subd1v1ding further and furth~rt1ll ~he destred ~c?uracy . analogue quantity. . is achieved. But it should be kept m mmd that a d1g1tal number 1s still a sum of equal umts.

In a digital system, magnitudes lying within one of these steps lose their identity and are all defined by the same number. For example, if we have ten steps, numbers lying between 2'5 to 3·5 *i.e.,* 2·6, 2·7t 2·s, 2·9, 3·0, 3·1, 3·2, 3·3, 3'4, would ~Ube read as 3. ·

From the above discussion we conclude that the difference between analog and digital information is .that the analog output is' a continuous function while the digital output is a discrete number of units. The last digit of any digital number is rounded to ±0'5 of the last digit. It should also be marked that the magnitude of the digital quantity is measured only at the instant the reading is taken. One reading persists till another reading is taken (unlike the analogue quantity which is a continuo\ls function).

The majority of present day instruments are analogue type. The impNtance of digital instru ments is increasing1 mainly because of the increasing use of digital computers in both data reduction a~d automatic control systems. Since digital computer works only with digital signals, any information supplied to it must be in digital form. The computer's output is also in digital form. Thu,s working with a digital computer at either the input or the outp~t, we must use digital.signals.

However, most of our present day· measurement and control apparatus produces signals of analog · nature, it is thus necessary to have b9th Analog to 'Digital (A/D) Converters at the input to the computer and Digital to Analog (D//1) Converters at the output of the computer. 1 ·s. Functiom of IMtnme!lltS ud Measurement Systems

· . There is another way in which instruments or measurement systems may be classified. This classification is based upon the functions they perform. The three main functions are explained below: '

' 1. Indicating Function. Instruments and systems use different kinds of methods for supplying

information con~rning the variable quantity under measurement. M-0st of the time this information

r~---

is obtained as a deflection of a pointer of a measuring instrument. In this way the instrument per forms a function which is commonly known as indicating function. For example, the deflection of  pointer of a ~pe~dometer indicates the speed of the automobile at that moment. . A pressu~ gauge is used for md1~t1ng pressure'. , ·

2. R~ording Function. In many cases the instrument·makes a written record, usually on paper, of the value ofthe quantity under measurement· against time or ~gainst some other variable. Jhus the

MEASUREMENtS AND MEllSUREMBNT SYStEMS *5*

instrument performs a recording function. F 011 example, a potentiometric type of recorder used fo ~ monitoring temperature records the inst~ntaneous temperatures on a strip chart recorder. 3. Controlling Function. This is one of the most important functions especially in the field of industrial control processes. In 1his case, the information is used by the instrument or the system to control the original measured quantity.

Thus there are three main groups of instruments. The largest group has the indicating function. Next in line is the group 1of instruments which have both indicating and or recording functions. The · last group falls into a special category and performs all the three functions, I.e., indicating, recording ·  and controlling.

In this text, main emphasis is laid upon instruments whose functions are mainly indicating auu recording, especially those instruments which are used for engineering analysis purposes. The control function will be analyzed in those cases where controlling enters as an integral part of the indicating and recording functions of instrumentation.·

The examples of controlling instruments are thermostats for temperature control and floats for liquid level control. '

1'9. Appli~ations of Me!lsurement Systems

In order to build u·p background for our later detailed study of measuring instruments and systems and their characteristics, it is useful to discuss, in general, the various ways these instruments are put in use. The way the instruments and measurement systems are used for different applications are as under :

I. Monitoring of processes and operations. 2. c(~drol of processes and operations. 3. Experimental Engineering analysis.

1. Monitoring of Processes· and Operations. There are certain applications of measuring instruments that have essentially a monitoring function. They simply indicate the value or condition of parameter under study and their readings do not serve any control functions. For example, an ammeter or a voltmeter indicates the value of current or voltage being monitored (measured) at a particular instant. Similarly, ·water and electric energy meters installed in homes keep· track of commodity used so that later on its cost may be computed to be realized from the user.

2. Control of Processel] Hd Operations. A very useful application of instruments is in automatic control systems. There has been a very strong association between measurement and control.

In order that process variables like temperature, pressure, humidity, etc. may be controlled, the prerequisite is that they can be measured at the desired location in the individual plants. Same is true of servo-systems, *i.e.,* systems connected with measurement of position, velocity and acceleration.

A block diagram of a. simple control systept *is* shown in Fig. 1 ·2. Let us assume that the output variable to be controlled is non-electrical a~d the control action is through electrical means,



I

Input Error signal Feedforward

·-+'- elements 1--~ Aduator  (Amplified

R~fefence

(Oesirfld ou1put

Measuring

Ou1put

1---411--.,...--.....,~ Conirolled

quant\tv'

lnstrumen1s ~--~....---'

- or

Transducer

Fig. 1'2. Block diagram of a simple control system. */.*

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The input is reference which corresponds to the desired value of the output. The input is com pared with the 'output with the help of a comparato'r. The output is a non-electrical quantity and 'is converted into a corresponding electrical form by a transducer connected in the feedback loop. In case the input and output differ, there is a resultant error signal. This error signal is amplified and then fed to an actutator, which produces power to drive the controlled circuitry.

The corrective action goes on till the outp.ut is at the same level as the input which corresponds to the desired output. At this stage, there is no error signal and hence there is no input to the actuator and the contrnl action stops.

Examples of this type of application are numerous .. A common one is the typical refrigera· tion system which employes a thermostatic control. A temperature measuring device (often a bimetallic element) senses the room temperature, thus providing the information necessary for proper functioning of the control system.

3. Expedme1mtal EJl]gioecring Analysis. For solution of engineering problems, tfieoretical and experimental methods are available. ·Many applications require application of both the methods. The relative affacability of the method depends upon the nature of the problem. Experimental engineering analysis has many uses and some are listed below :

1. Testing the validity of theoretical predictions.

2. Formulations of generalized. empirical relationships in cases where no proper theoretical backing exists. . · , · · 3. Determination of system parameters, variables artd performance indices.

4. For development in important spheres of study where there is ample scope of study. 5. Solutions of mathematical relationships with the help of analosies .. 

1'10. Ekmemts of a Generalized Measurement System

It is important to have a systematic organization and analysis of measurement systems. An instrument may be· defined as a device or a system which is designed to maintain a functional relationship between prescribed properties of physical variables and must include ways and means of communication to a human observer. The functional relationships remain valid only as long as the static calibration of sy&tem remains constant. On the other hand, the performance of a measurement system can be described in terms of static and dynamic characteristics.

It is possible and desirable to describe the operation of a measuring instrument or a system in a generalized manner without resorting to intricate details of the physical aspects of a specific instrument or a system. The whole operation can be described in terms of functional elements.

Most of the measurement systems contain three main functional elements. They are : 1. Primary Sensing Element, 2. Variable Conversion Element, and , 3. . Data Presentation Element. '

Each functional element is made up of a distinct component or groups of components which perfo,rm required and definite steps in the measurement. These may be taken as basic elements, whose scope is determined by their functioning rather than their construction.

t. \Primary Sensing Element. The quantity under measurement makes its first contact with the pri,mary sensing element of a measurement system. In other words the measurand is first detec· ted by primary sensor. This act is then immediately followed by the conversion of measurand into an analogous electrical signal. 1 his is done by a transducer. A transducer in general; is defined as a device which cotnerts energy from one form to another: But in measurement systems, this definition is limited in scope. A transducer is defined as a device which converts a physical quantity into an electrical quantity. The physical quantity to be measured, in the first place is sensed and dete.cted by an element which gives the output in a different analogous form This output is then converted into an electrical signal by a transducer. This is true of most ;br the cases but ,is not.true for all. In many .cases the physical quantity is directly con .. ver!ed .. i9'fo an elect:r!cal· quant1ty by a transducer. However, tbe first stage of a measurement system is knpwq as a ~~tedpr transducer stage. ·. .. · · / 1 • */*

MBASUlU!MENTS ANl> MB}...SUREM~NT S\'STEMS 7 2. Varil!!bJe ·Conversio~ Elemtmt. The output of the primary sensing element may be any kind of electrical signal. It may be a voltage, a frequency or some other electrical parameter. Some· times tbiS" output is not suited to the system. For the instrument to perform the desired function, it may be necessary to convert this output to some other suitable form while preserving the in- form~tion content of the original signal. We may cite an example. Suppose output is in analogue form and the next stage of the system accepts input signal only in digital form. Therefore we will haye to use an A/D converter.

Many instruments do not need any variable conversion element, while others need more . than one element. . Variable Mmwpulation Element. The . function of this element is to manipulate the signal presented to it preservinjZ the origiµal nature of the signal. Manipulation here means a change in numerical value of the signal. For example, an electronic amplifier accepts a small voitage signal

Quantity to be Primary m11osured sensing lamont

VcrioblQ Vor'1obla conv~rston manipulation . element cloment l, 

Fig, f 3. Functional elements of an instrumentation system.

as input and produces an output signal which is also voltage but of greater magnit~de. Thus voltage amplifier acts as a variable ma~ipulation element. H is not necessaJ:y that a variable manipulation element should follow the variable conversion element as shown in Fig. 1 ·3, It may precede the variable conversion element in many cases.

As discussed earlier, the output of tranducers contains information needed for further processing by the system and the output signal is usually a voltage or some other kind of electrical signal. The two most important properties of voltage are its magnitude and frequency though polarity may be a consideration in some cases. Many transd~cers develop low voltages of the order of m V and some even µ V. A fundamental problem is to prevent this signal being contaminated by unwanted signals like.noise due to an extraneous source which may interfere with the original output signal. Anot4er problem is that the signal may not be distorted by processing equipment. The signal after being . sensed cann'>t be directly transmitted to the next stage without removing the interferring sources, as otherwise we may·get highly distorted results which are.far from true. Many a times we have to perform certain operations on the signal before it is transmitted further. These processes may be linear like amplification, attenuation, integration, differentiation, addition and subtraction. Some non-linear processes like modulation, detection, sampling, filtering, chopping and clipping etc. are performed on the signal to bring it 'to the desired form. This is called Sigma! Conditioning. Tile term signal conditioning includes many other functions in addition to variable conversion and variable manipulation. In fact the element that follows the primary sensing element in anv instrument or instrumentation system should be called Signal Conditioning Element.

When the elements of an instrument are actually physically separated, it becomes necessary to transmit data from one to another. The element that performs this function is called a Data Transmission Element. For example space-crafts are physically separated from the earth where the control stations guiding their movements are located. Therefore control signals arc sent from these stations to space-crafts by a complicated telemetry systems using radio signals.

The signal conditioning and transmission stage is commonly known as Intermediate Stage. 3. Data Presentation Element. The information about

the quantity under measurement has to be conveyed to the

personnel handling the instrument or the system for monitol'ing,

control, or analysis purposes. The information conveyed must be in a form intelligible to the personnel. This function is done by data psesentation element. In case data is to be monitored, visual display devices are needed. These devices may be analogue or digital indicating instruments like ammeters. voltmeters etc. ln case the data is to be recorded, recorders like magnetic tapes, high speed camera and T.V. equipment, storage type C.R.T., . printers, analogue and digital computers may be used. For con trol and ar:ialysis purpose computers tnaY -be used. ·

·The final. stage. in a .:measurement system is known as

Closed end *ot* 

bourdon tubfl

Mechanical

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8 I ELECJ1UCAL MEASOREMBNTS AND MEASU1UNO 1NSTRlJMB~T$

terminating stage. As an example of a measurement ·system, ~nsi~er-the simple b~urdon .·tube pressure gauge as shown in Fig. 1·4 .. ·This gauge offers a good example of a measurement system. ID this case. the bourdon tube acts as the primary sensing element and a variable conversion element. It senses the input quantity (pressure in this case). On account of the pressure the closed end of the bourdon tube ~s displaced. Thus the pressure is converted into a small displacement. The closed end of the bourdon tube is connected through mechanical linkage to a gearing arrangement. The gearing arrangement amplifies the small displacement and makes the pointer to rotate through a large angle. The mechanical linkage thus acts as a data transmission element while the gearing arrangement acts as a data manipulation element. · ..

The final data presentation stage consists of the· pointer and dial arrangement, which when calibrated with known pressure inputs, gives an indication of the pressure signal applied to the bourdon tube. The schematic diagram of this measurement system-is given in Fig. 1 '5.

Bourdon lubt Poln!Qr g (IOI  

Oat a , \_ \_\_\_.... prqsentatlon

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Fig. 1'5, Schematic diagram of a bourdon tube pressure gauge . .

When a control device is used for the final measurement stage, it is necessary. to apply ·some feedback to the input signal to accomplish the control objectives. The .control stage compares the signal representing the measured variable with a reference signal of the same form. This reference signal has a value the measured signal should have and is presented by a con.troller. If the measured signal agrees with the reference value, the controller does nothing. However, if there is a difference between the measured value and the reference vnlue, an error signal is generated. Thus the controller sends a signal to a device which acts to alter the value of the measured signal. Suppose the measured variable is flow of a liquid, then the control device is a motorized valve placed in the flow system. In case the measured flow .rate is too low than the preset flow rate, then the controller would cause the valve to open, thereby increasing the flow rate. If on the other band, the flc;>w rate were too high, the valves are closed. The operation of closing or opt..iing of valve will cease when the output flow rate is equal to preset value of flow rate.

2-

Characteristics of Instruments and

Measurement Systems

2·1. Measurement System Performance

The treatment of instrument and measurement system characteristics\_ can be divided into two distinct categories *viz.* :.

(i) Static characteristics, and (ii} Dynamic characteristics.

Some applications involve the measurement of quantities that are either constant or vary very slowly with time .. Under these Circumstances it is possible to define a set of criteria that gives a meaningful description of quality of measurement without/interfering with dynamic descriptions that involve the use of differential equations. These criteria are called Static Characteristics.

Normally static characteristics of a measurement system are, in genera], those that must be cdhsidered when the. system or instrument is used to a condition not varying with time. - - 1· I •

However ro,any measurements are concerned with rapidly varying quantities and, therefore, fo1 such cases we must examine the dynamic relations which exist between the output and the input 1 This is normally done with the help of differential \equations. Performance criteria baseµ upor dynamic relations co.nstitute the Dynamic Charactersitics.

2·2. Static Calibration

All\he s~atic performance characteristics are obtained in one form or another by a proc./ ~~~~~~~. , I The calibration ofaU instruments is,\_important since it affords the opportunity to check ~ instrument against" a known standtird and subsequently to errors in accuracy. Calibration piocedu t involve a comparison of the particular in~trument with either (1) a primary standard, (l) a seconda1

standard with a higher accuracy tharl·the.!Psfrument to be calibrated, or (3) an instrument of know accuracy. :~

Actually all working instruments, *t.e.,* those instruments which are actually m~ed for measu~ ment work m:ust be calibrated against some refereace instruments which have a higher accuracy. Ttli ' reference instruments in turn must be calibrated against instrument of still higher grade of accurac or against primary standard, or against other standards of known accuracy. It is essential that a: measurement made·must ultimately be traceable to· the relevant primary standards.

2·3, Static Characteristics

, The main static characteristics discussed here are :

(i) Accuracy (ii) Sensitivity, (iv) Drift (v) Static error, and

(iii) Reproducibility (vi) Dead· Zone

The qualities (i), (ii) and (iii) ai:e desifable; while qualities (/v), ( v) and *(vi)* are undesiral · In addition to above characteristics, definitions of many other quantities have been given; It must be stated; however, that sf.ere are many definitions 'of the above characteristics, anc 9

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some cases the definitions are unrelated. Care has been taken to .select the most ge11erally accepted definitions so as to avoid confusion.

2·4, Errnrsin Measurements

Measurements done in a laboratory or at some other place always involve errors. No measurement is free from errorn. If the precision of the equipment is adequate, no matter what its

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accuracy is, a discrepancy will always be observed between *two* measured results. ·

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In ordinary usage, the word err()r may have certain unpleasant connotations. It may imply

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a mistake, a moral offence, or possibly a belief in something untrue. In its extreme, it may be a

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blunder. But errors are to be there in measurements and therefore there is nothing shameful· about them as it should be understood that no measurement is free from errors.

Since errors are a must in any measurement, it is imperative to interpret. the results of a quantitative measurement in an intelligent manner. An understanding and thorough,e'.rvaluation of the errors is essential.

2·s. True Value

The true value of quantity to be measured may be defined as tb.e average ofan infinite number of measured values when the average deviation due to the various contributing factors tends to zero. ·Such an ideal situation is impossible to realise in:practice and hence it is not possible to determine the ~'True ValueH of a quantity by experimental means. The rea;son for this is that the positive deviations from the true \'.alue do not equal the negative deviations and hence do not cancel each other.

· Thus, r.ormally an experimenter would never know that the value or quJntity being measured by experimental means is the "True Value" of the quanti~ or not: ·

In fact in practice, tbe term, ''Tr!ro Value", then, refers to a value that would be obtained if the quantity under consideration were measured by an "E111mpl1r Methed",. that is a method agreed upon by experts as being sufficiently accurate for the purposes to which the data will ultimately be put to use. ·

2'6, Static Eno.Ii ·

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The most important characteristic of an instrument qr measurement system is its accuracy, which is the agreement of the instrument reading with the trµe value of quantity being measured. Tbe accuracy of an instrument is measured in terms of its error.

We have mentioned earlier that it isimp-0ssible to tneasure the true value of a quantity. An approximation of the "true va~ue" obtained by sufficiently extended series of measurements and also taking into account parameters and conditions to which corrections may be applied, we obtain, what is called the best meatmred v~h1e ·of the quantity. WbiJe it is·never possible to measure the true or exact value of a quantity, it is nearly always possible to give· a best measured value. Static error is defined as the difference between the measured :value and the· true. value of the 'quantity. Then :

. *8A.* =Am~ At ... (2• l)

where *8A* = error,

*Am* =measme4 value of quantity,

and Ai=true value <:>f~quantity. ··

~A is also called the absolute static error of qua~tity A.. ·

We have Eo=8A ... (2•2) ..  vhere Eo=absolute static error of quantity *A* (under measurement).

The ab~olute ·value of ~A- does not indicate precisely the . accuracy of measurements. As an :xample, an error of +. 2 A is negligible when the current being·meas~d is of the order of 1000 A vhile the same error of ±2 A may be regarded as intolerable when the current under measurement i 10 A or so. Thus the quality of measurement is provided by the relative static error, *i.e.,* the ratio

CHARACJiRElICS-OF INSTRUMENIS AND MEAStJRlMBNT SYSTEMS

of absolute static error *8A* to the true value *At* of the quantity under measurement. relative static error *Er* is given by :

E =' abs· lute error = *SA* = \_io \_

r tr e value · *At At*

Percentage static error % E-r *=Er* X 1 CO -

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Therefore, the

... (2'3)

.. (2'4)

We have *At=Am-SA* (see Eqn. 2'1)

*=Am-Eo=Am-ErAt Am*

(see Eqns. 2·2, 2'3)

= l+Er~ ... (2'5)

However, when the absolute static error Eo *=8A* is small, which means that the di1ltrence between measured and true values is small.

fr~l-

*:.* Eqn. 2'5 may be written as *Ai=Am(l-Er) 2·1.* Static Correction

It is the differ~nce between the true value and the measured value of the quantity, or 1  SC= *At-Am* ... (2'7)

where 5C=static correction= *-SA* .. (2'8) Example 2'1. A meter reads 127'50 V and the true value of the voltage is 127'·13 *\:.* Determine :

(a) the static error, and (b) the static correction for this instrument.

Solution. From Eqn. 2· 1, the.error is

*8A=Am-At* =127'50-127'43= +0·07 V

Static correction 3C = - *SA=* -o·o 7 V.

Example 2·2. A thermometer reads 95·45•c and the static correction given in the correction curve is -o·os 0c. Determine the true value of the temperature.'

Solution. True value of the temperature *Ai=Am+SC=95'45-·0'08* =95 37°C.

Example 2·3, A voltage has a true value of l '50, V. An analog fodicating in~trument with a scale range of 07 2·50 V shows a voltage of 1 46 V. What are the values of absolute error and correction. Express the error as a fraction of tbe true value and the full sGale rleftection \f.s.d.}. Solution : Absolute error *3A =Am-At='* l 46- 1'50=-0·04 V

Absolute correction 3C=-8A=+o·o4 V

*3A* \_ -0'04 . %' Relative error *€r=At-:-* 1.50 x 100=-2 66 o·

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Relative error (expressed as a percentage of f.s~d.)

--0·04 . 2'5 x 100~\_-l '60%.

2·s. Scale R1mge and Scale Span

. In an a~alog . indicating instrument the value of measurand is indicated on a scale by a pointer while in a recording instrument it is indicated on a chart by a pen mechanism. The choice of proper range instruments is important in instrumentation. The Scale Range of an instrument is defined as the difference between the largest and the smallest reading of the instrument. Supposing the highest point of calibration' is *Xma:i:* units "'hile the lowest is *Xmin* units and that the calibration is continuous between the two points. Then ;we /can say that the .instrument range is between *Xm111* and *Xmax* (or many a times we say that ,the i?St'lu.ment range is *Xmax).*

12 BLECTRICAt MBAStJRllMENTS AND MEASURING INSTRUMENTS The instrul1leot sp111 as given by : Span .... *Xmaz-Xmtn* . · ... (2'9\ . For a pyrometer calibrated between 0 to I000°C, the range is lOOO"C and span is lOOO"C. For a thermometer calibrated between 200°C to 50YC, the range is 200°C to *soo·c* (or 500°C) but

the span is 500-200=300°C. .· . · The same is true· of digi~al instruments. ·  The accuracy of an instrument may be expressed in many ways. A common way is to specify "accurate to within x per cent." This means that the instrument is ''accurate to within +x percent of instrument span at all points on the scale unless and otherwise specified." However often accuracy is based upon instrument range and these two specifications, one based on span a~d the other on range differ greatly. , . . Th.er~ is another factor that ~ust ~e considered while det~rmining th.e range of the instru ment. · This ts the Frequency Range, which is defined as frequencies over which measurements can  be pe,rformed with a specified degree of accuracy. For example a moving . iron instrument may have

a 0-250 ~ range a~d 0-135 Hz frequen~y ra~ge .with an

accuracy pf o· *5* or I% of full scale reading.

Example 2 4. A thermometer is cahbrated 150 C to 200 C. The accuracy is specified within

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±0'25 percent What is the maximum static ~rror. · Solution : Span of thermometer=200-150=50~c.  M . . . ±0·2sxso ± 0 ,

*:.* axu!1um static error = 100 = 0' 125 C.

2 9-. Error Calibration Curve

Error calibfation means that an instrument has been cali brated against a suitable standard as discussed in Art. 2·2 and its static error determined at a number of points on its scale.

+2 r---r--r---.------

These data form an error curve, which can be used for correcting f +1 -r-·-"""l--+--+--+l-1 instrument readings. A typical error curve is shown in Fig. 2· 1. 

*i·10.* Reproducibility and Drift

, · Reproducibility. It is the degree of closeness with which

a given value may be repeatedly measured. It may be specified u

ii a

in terms of units for a given period of time. Perfect 1eproduci- *B* \_, r----t--r--1---1---.1 bility means that the instrument has no drift. N~;drift means vi

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. ·that with a giveQ inp11t • the measured values do not vary "1ith -20 \_ 20 40 time- 101)  

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Drift is an undesirable quality ·in industrial instruments Measured variable,% span because it is rarely apparent and cannot be easily compensated Fig. 2·1 Error calibration curve.

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for. • Thus it must be carefully guarded against by continuous prevention, inspection and

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· maintenance. For example, stray electrostatic and electromaguetic fields can be prevented from affecting the measurements by proper shielding. Effect of mechanical vibrations can be minimized a1

by having proper mountings. Temperature changes during the measurement process should be

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preferably avoided or othenyise be properly compensated for.

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2·u. Repeatability " . Reproducibility and Repeatability are a measure of closeness with which a given input may

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be measured over aud over again. The two terms cause confus;on.

Therefore, a distinction is made between the two terms. Reprodecitiility 

is specified in terms of scale readings over a given period of tinie. On

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the other hand, RepeatabllitJ is defined as the variation of scale reading

and is random :n nature. Fig. 2·2 shows th'is rapeatability.

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2·12. Noise. Noise may be defined as any signal that does. not

convey any useful information. Extraneous disturbances generated in

~A

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Input\_..

the measuring system itself or coming from outside, frequently con .. stitute a background against which a signal may be read. · There are many sources of .Q~. Noise may originate at the

Pig. 2·2. Input-output relation. pnmary sensing device, in a comurnnication channel or other inter sbip with± repeatability. mediate links. The noise maraiso be prod'(Jced by ir.uJicating elements

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of the system. ·

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CHARACTEHHSTIC$ OF INSTRUMENTS AND MBASUREMBNT SYSTllMS 13

The common sources of noise are given below : ·

(1) Stray electrical and magnetic fields present in the neighbourhood of the instruments produce extraneous signals which ten<! to distort the original signal. The effects of these stray .fields can be minimized by adequ\*te shielding or relocation of the components of the instruments.

*-(ii)* Mechanical shbcks and vibrations are another source of trouble. Their effect can be eliminated by proper mounting 9evices. . \_ ·

· . · (iii) -Resistors generate thermal agitation noise·· due to thermal · fotion of the electrons in their interior. The effect increases with increase4 temperature1 of the resistor. Th'is is caJJed Johnson or Th·ermal noise. The .ma~nitude of this qoise voltage 1is :

*V=2VkTR!Sf* volt : ... (2·10) ·where k=B<;)l~zmann.<:onsta~t=..1·3s x 10.::23 J/K, ·

· 7\=:a~s0lµte tem.peraturp of. reeistor .; K ,

' ' . ' . ~- . *:* ' . ' . . .. ' . ' '• . . . "1 ' ' . ' ' ' .. ' ,· '

R ..:...re$istat1ee~ 'O , · . · . · .· .• . ·

'\" . : *.:\_,* .•..... ' .. ·.···:... . "''I'·. . .

and 6/ =fr~que11qy, rangec over which m~as\Jrements .are being made ; Hz . . ,, This noise can assl:1¢e ala.rming proportions as explained in Example 2· 5.

(iv) The noise mayi also origina.t.e fr<>;m use of vacuum tubes;

*i* It is desfrable to ~e~p the signal to noise ratio *(SJN* ratio) as high as possible -~o as to accurately meas\lre the want~d sjgnal. · {It ampUfying system, the-sign~] to noise ratio sets. a.~ upp,-.· ·• limit to aQ1plificf).d6n.. 'I)utthe wanted', si$fittl ¢an~ot:be amplified to. the ~xtent as we\_ want it i~:

be on acc0unt of the fact th~i· noise is also atnp)ift~d by the same ra'tio as the original signal., Another · handicap is: that t~e signal ~~.$t9·1'.C~4 against' the b~ck~r.ound of amplified noise. ·  . ' .. '· .. · E;taQiple. rs .. ·. J>r~s~qre'ls ri)'a8Qred ·by. strain\_·,gauges. One strain gauge is act.ive ·and th!' ot~r is dummy., . \_These· *$ttMn* ga~ f9hn .the opposite arms of a Wheatstone bridge. The other two arms a,re formed· by equal resist.anccts "Of l20!l each at 300' K. The frequency bandwidth is . I 00,000 Hz~ The output of; tbe bridge is a votta-ge signa'~ . .

· . (;). Wheg a pre&surc' of70QQ kN/m• is applied· the'· output voltage i,s 0·12 mV; Find the ratio of the output (signal) voltal• tt'>" t.he noise voltage generated by the resistors. . .· . ..· (ii) Find the ratio 6r putplit'. (signal) voltage to the noise voltage if the applied pressure is 7 kN/m2• . : . .

The Boltzm~nn constant is l '38 *x* 10-23 J /K .

Comment upon the results ..

Solution.: (i) From Eqtt. 2·10, the noise voltage is:

--· r --··-. . . .· *V=2V kTIJ.!Jf* .,,,.2'11 '$8 x10-2a ~ 300)(J20XJOOOOO volt=0'466 ~v.

. . . i '

: -•• ·. ·• .. , ». . : '· . . 0'12 x l0-3 · . '. ·

· Sign.at to'noise,($/N) ratio- 0.446 x rn-u.=269;,

, · *in* this case tht noi~· VQltage ls negligible ai'. compared with the signal voltage and therefote. *SIN* ratio is higb:. Thus the interference due *(o* noise is ins1gnificant and hence does not distort the . signal and the result is unaff~cted. '

(ii) Assume a linear relationship between ~he output voltage of the bridge and the applied pressure. . . ! . . ,

. ,'. · -Output (sjgnaJ)\_voJ~~ge, when the ~pplied p~essure is, 7 k N/m2, is :

.·.,: ~ ·}xlOl3 xOiftt'k 10""8volt=0'12x10~11 =0'12 11.V.

: 7000 x ro . . .~'. .. · ' . . . . ·.... r·

1 • 0·12x 10-6 ' . . ,·. Signal to noise (S/N) rat10=·0.446 X 10.;:;6·=0 27.

,,,

' . . . -":.··

ep::craiCAL MEASUREMENTS AND. MEASURING lNSTRllMBNTS

This ind~cates that the noise has a · magnitude 'which is. about 3'75 times that of signal and hence the signal will be completely lost in the noise.

2'13. Accuracy an,~ Precision

. In ordinary' usage, the distinction between words ~'Accuracy" 'and ''Precision'' is usually very vague. In fact even1 the diction~ties invariably link the definition of one \\ ith the other. But as far as field of measurements is concerned, there is a. big difference between the two terms as they r have sharp differ,eilces *in* !meanings. In the fiel~,of measurements, the two terms may be defined as :

~ccuracy. It is the closeness with which· an instrument reading approaches the true value of th~ quantity b~ing measured: Thus accuracy of a· measurement means confirmity to truth. · . · . . i>recjsion:. It is a measure of the reprodl1cibility of the measurements, *i.e.* given' a fixed va.lue of a quantity,·precision is. a measure · of·t~e degree of agreement within. a group of measure~ · merits:. The term 'Precis~': rµe~\_ns cleaily or s\_harplf defined. .As an example of the difference in tneani:tig of the two terms, suppose.that we have an amtµefer which possesses high degree of precision by virtue bf its clearly legible, finely· divide9, ·.distinct scale and a knife edge pointer with mirror

arrangements to remove parallax. Let us say that its, readings can be taken to 1/100 Of an ampere. At the same time, its zero adjustment is w'rrihg. Noi,y every time we take a reading, the ammeter i.s as precise as ever, we can take readings down to 1/100 of an: ampere, and the readings are consistent · · ~.,a1~1.;i."dear.ly *defiried'' .* . However, the.readings taken with thfa ammeter· are not accurate, since they <h nofconfirm to turth on accouutof,its faulty- zero ,adjustment. ' . .

· .. ·.Let us .. cite a not her exainpte. .. Consider th~ measurement of a known voltage of 100. V with a.meter. Fivereadingsare·takeji, andthe~indicated values ar~ 104, 103, 105, 103 and lOSV. From the~e values it is seen·· that the instruhi.ent. cannot pe depended on for · an accuracy better . · than 5% (5 V in thi~ .case), while a precision of'.± I%, is indicated sine~ the maximum deviation from · the ifl.ean reading of 104 V is only .1 ·o. V~ Thus we find that the instrument can be calibrated so that. it could b.e used to read ± 1 V depen9ably .. This example illustrates that accuracy can, be improved OP.on ~ut not the precision of the instrument by. calibration. Another point which is evident from ab,ove is tba~ although th~ readj~g& are ·close. t.ogeWer thfY . have ~, small scatter (or dis~e:sion) · and thµs have a h1gh degree of pr~c~ston , but ~he result~ are ·Jar from accurate. The precmon of

an instrument ~s usually dependentupon man,y factors and' requires many sophisticated techniques of . 8,,n.alysis. , · Thus we say fhat .a. set of readings sho\vs precision if the results agree among themselves. Agreement, however, 'is no guarantee, as there may be some systematic disturbing effect that causes all the values to be in error. . ''

2'14 Indication~ of Precision

·· J>recisivnis comp'bsed of two characteristics :

· {i) Conformity and (ii)· N\linher of significant figures.

Precision is used in measurements to d~scribe the *consistency or the reprodudbility* of results. A quantity catted predsioo ir>dex defined in chapter 3 describes the 1 spread, or dispersion of repeated . result about some central value. High precision means a tight cluster of repeated results while low:· precision indicates a broad scattering of r~sults. · But this may not lead us to the misconception that high preci~ion indicates high degree of accuracy since all the repetitions in result may be biased in the sarne way by some systematic effect that produces saCPe deviation of results from the true value. For example, a sp'ring scale used with a spring designed for a different spring scale may repeatedly· show the same value of weight. Thus the rea·Hngs may display exceptional agreement between them- selv~s, but they all would be inaccurat~ ·values of weight since use of a wrong spring introduces a systematic shift of all readings, . · · " . . . 'we ·m~y well cite a~othe,r ¢~llmple to illµstr~t~ another aspect of precision.' ' .*.. : .. "* . ' Consider, for ex~01ple, th~f *{* re~i~t0r ~hose true resistance i~ 1, 185,692 n, L'1:~'ein'g measured. by an Ohmmeter. The obmmetercon·sistently and repe~t\_edly indicates the true value. ·nut the obser"' ~er cannot read this value f~om t~e:~9ale: The observer~s estimates from the scale reading consistently . . •. '.1.'· • . . ' . 

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yield a value of 1·4 megohm (l '4 MO). This is as ·close to the true value as he can read the· scale by estimation. Although there are no deviations from the observed value, the error creakd by the limi tation of the scale reading is a precision error.

The above example illustrates that *conformity* is necessary, but not sufficient condition for precision because of lack of *significant figures* obtained. Similarly, *precision* is a necessary, but not sufficient condition for *accuracy.* , · · 

•. In critical work, good pr~ctice requires an independent set of measurements, using differen~ 'instruments or different measurement techniques which are not subject to the same systematic errors. Where this is not possible, the experimenter must take steps that insure proper functioning of instrn· ments and to discover and eliminate any systematic disturbing factor. Calibration against a known standard may be resorted to in order to achieve the above l?urpose.

2'15. Signfficnnt Figures, .

· · An indicaticm of the precision of the measurement is obtained from the number of significant figures in which it js·expressed. Significant figures convey actual· information regarding the magni· tude and the measurement precision Qf a. quantity.· The more the significant figures, the greater the precision of measurement. 1' , ,. · , . .·

Let us take an example. If a voltage is specified as 256 V its value should be taken as closer *to* 256 V than to either 257 Wot 255 V. If the value .of voltage is described as 256'0 V it means that the voltage is closer to 256'0 V than it is to 256' l V or 255'9 V. In 256 there are three significant figures whHe in 25~'0 there are four. The latter, with more sig~ificant figures, expresses a measurement of greater precision than the former.

Frequently iarge numbers with zeros before a decimal point are used for approximate popula~ tions or amounts of money. For example the population of a City is reported as 490,000. This may due to misconception, imply that the true value lies between 489,999 and 490,001, which is six significant figures. But in fact, what is meant, however, is that the population is closer to 4·,0,000 !ban to 480,000 or 500,000, Since the population can be reported only to two significant figures. How else large numbers be expressed ? A more technically correct notation uses powers of ten like 49 x 104 or 4·9 x I os. This indicates that the population figure is only accurate to two significant figures. Thus. reference to populatiOil of a city as 3,000,COO would be interpreted automatically as an approximate number while reference to velocity of light as 300,000,000 metre per second creates ·no confusion to a person with a technical background. Uncertainty caused by zeros to the left of decimal point is therefore usually resolved 'by scientific notation using powers of ten.

Exan,Df)le 2'6. Stat'' the number Of significant figures in each of the followinj! numn?.r.Q ; (a) 302 A(b) 302'10 V (c) 0'00030 !l (d) 0'0000300 Hl (e) 5·01x104 (/) sorno. Solution : (a}. The number is 302. This means'that it is more close to 302 that to either 30 l or. 303~ Thus this number.has 3 significant figures. ' .

(b) The number inv9lved h .302' IO, TheFefore it is .more close to 302' 10 than either to 301 ·09 or to l02'I l. Thus it has 5 significant figures. · -. . , · (c) The. number is 0'00030. This means tb~t it is more close to o·ooo 30 than either to 0'00029 or 0'00031. Thus it has 5 sigoificaoe figures.,

(cl) The resistance is 0'00003 MO. . This can be written as 30 !l Thus it is m.ore close to l{) thu to ~itbor 29 or 31. Therefore it bas 2 significant figures. The zeros to the left of 3 are due C i> large size of unit.

(e) The number under consideration is S'OJ x 104. Hence it.is more close to *s·o* 1 x 104 than to either s·oo x 10' or 5·02x104. Thus it bas 3 significant figures.

( f) · 1;be number is 50100. This is· a larg: n'uml?er, and a situation may arise that causes uncer· taint) Strictly ,S0,100 means thafthe.,iun>bet'is more close to 50,100. th!ln either to. 50,099, or .50; 101. This means ihat the number has 5 $~gnificant figur~s. ij.ow~ver, if this number is po~mlation of a town it could be written as 501x102• Under.this' situation we ·can say that the population is close to

~01 x J02 than to ~ither soox 102 Of $02 *x* IQ'. This·mQ&ns that nu~ber bas 3 significant figures.

'

16 ELECTRICAL MERSUREMENTS AND MBASURINO INSTRUMBN'fS 2'.16. Range of Doubt OJ' Possible Errors and Doubtful Figures . . . It is customary in measurement work to record the result with all the digits of which we . are sure, and final digit which is believed to be nearest to the true value. This usually implies that the reported result is good to ±I in the digit in the last place *i.e.,* the digit in doubt. For example, in rea.ding a. wattmeter the power may be read as 22'6 W. This simply indicates that the power read by the observer to the best estimation is closer to 22'6 W, than to either 22·5 W or -22'7 W. Another way of expressing this result indicates the 'range of doubt or possible error'. The way to express doubt regarding the Jast place, the dig'it in doubt is put in italics form. For example 22'6 W indicates that we know the power to better than unit and that we are not sure about the ·tenths but au the same believe the value of digit in the tentlis place is close to 6. A similar but definite method is to express the above power as 22'6±0'05 W indicating that. the power lies between·22·55 Wand 22'65 W giving the range of d<?ubt qr possible error as o· 1 W . 

. Wheµ 3. 'nq~ber of independent measurements are taken in ordel' to obtain the best measured value~. t.~e result is usuall¥ ex:pressed:·as aritbmetic mean of all readings. The range of doubt or possible error ts the largest deviation from the mean. · . · . · . : : .. ·· ',.· !' '. ' ··.' .. ·. . ' ' '' ,' / . .'

. Examp1e·\_2·1. .A set' of independent current measurements were recorded as 10'03, 10· 10~ 10· 11 and 10'08 A. Calculat~·(a) the· average current, and (b) the range of error.

Solution : 

(a). Avera,i~ cµrrent Ia~ 1i+~2tla+h .=!0'0~+101\_0~1Q.1l+to·os =l0.'08 A. *'··:',* ,·'

·< · .. '..>' . . •( . . , " . .

(b) · :'Ma·?tiri:J~m value of current.Imaai= 10· 11 A , .... " *,(·.•:·* .... .. '. . .. "· .

· · ~ang~ ~i~aai;..:.lav=10'11~10'08=0'03'A~

. :·.'':,1 ~ ':• .. ,. ·1· .• "' *:i* ' •, '

· · . MiniI.Jlum ·.value bf l~ngth /mt"~ 10·03 A.

. . ·lav.....:lmin~jo~os-10·03 .:::o·os A.

~· ' - .

I . ~" ;· ,: . . ·0.03+0'05 · · Therefor~ •v~rage range.of ,error is . 2 · =±0'04 A.

The ·null)ber of significant figures in a quantity is one measure of precision, though not as definite as a percentage statement. Suppose the range. of dou~t in a 101 0 resistor (the value has 3 signifioaqtitfigures) is t n. This means that the ·value lies between 100·5 Q and 101'5 n. This range . of doubt i.n p~rcentage terms is 1 perceJ?.t. The same range of doubt of I n in a 999 0 . resistor (again

the value has·~ ·sigQ.ific1n.t figure's) creates a percentage range of doubt as only o· 1 per cent; . . Therefore three significant .figure~ may cover a percentage range of doubt of o· 1 to 1 per cent and hence is indefinite. as a measure of precision.

1 ,. I

-· ~upefn:uous fi~ures are so~~~imes allowed ·to accumulate in ordinary arith~etic processes of addition, subfr~¢tfon~ rhultiplication'and ,division. In these computational· processes, the doubtful tigu~es are·;wrlt~en in italics; .arid al~o ·figures that ~esult from;t~eir use in 'th~ ptocesses and are there· by placed tn doubt. There is.anotber thing that 1s obvious 1t 1s useless to give more than one doubt· fur figure. · To illustrate the point :W.e: should not put the result 11T6 since the figure in the units place *(i.e•i* 7) is ht doubt, and th~refot'ei· it is· useless to put a figure in. tenths place. The correct way to ex prea the above re.suit is lf1.,:.: .. ' ·: ....

. . TO\fllUs~:·N~~er •.. t\V(>'.~~amples are given below : . , . · .•••11.~.·t~t .. :. :· Tbtfc r~'Jbrs ha!~ vatu~s of 72·3? ffa and 0'6.12 O respectively with. an uncertau,itf•·•Q~t~.i,t'ih,;.jo:~:·fasrig.llr~ ~pe~ch.~se; .. rmd:t~,e suiµ of three c·cnnected in ~eries.'. · Solutto• f ·· · · ' · . "

$in.cithe three ~sistanc~s are in series, theirsum is: RcR1+Ra+~3

17 . •

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'? • '•, '.

Now

*Ri* ==72'3 *Sl,.* R2= 2·73 0. *Rs= 0'612 Q,*

·~ ':,

' ' .

*R= 15'64)0.*

The result cannot be expressed as 65'642 .0 as even the figure in'theterith place *i.e.,* 6 is in doubt. Therefore the resultant resistance is 65'6 n witJ1~6 as first doubtful figure.

Exampl«J 2·9. The ~oltage and current recorded in a d.c. circuit are respectively 12'16 V and 1 ·34 A. calculate the power;

Solution: Power-12'16Xl'34 W.

Multiply by long hand, we have :

12'16

1'34

*4864*

3648

1216

*16'2944*

.. .;

· The power is expressed as 16'2 Was 2 is the first. doubtful figure. It would obv1ousJy be absurd to write the answer with the entire product obtained ..

·When two or more measurements with different ·degrees o(, accuracy are added, the result is only as accurate as the least 'accurate measurement. ·This is illustrated by the following ex'\mple : ·

Example 2·10. Tw() resistors. Ri and R2 are con~ect~d in serie~ Y'ith 'R1=28'7 0 and *R2* = 3 '624 n. Calculate the total resistance to the appropriate number 'Of s1gmficant figures.

Solution :

Ri=28'7 0

R2=3'624 0

Total .resistance *R=R1* + *R2* =132'324 !l

=32'3 0

(three significant lfii'es)

(four significantigures}

(five significant ftgures) ·

(three significant figures)

"

This is done because one of the. resistances is accurate only to three significant figures (or

I

tenth of an ohm in this case) and therefore the' result should also be reduced to three significant figures

~,,.,I

(or the nearest tenth of an ohm in this case) and hence the value 32'3 (l.

The number of significant figures in IQUltiplication may increase rapidly. but only the appro· priate figures are retained in the answer as is illustrated by the following example. · · · Example 2·11. In calculating voltage drop; a current of 4'37 A is. recorded in a resistance

' <

of 31 '27 n. Calculate the voltage drop across the resistor to the appropriate number of significant figures. ·

Solution:

Current /=4'37 A (three significant figures)

Resistance *R=* 31 '27 n . I (four significant figures) .

Voltage-drop E=/R • ..,4·31x31'27=l36'6499 V

(seven sisnificant fisures)



'.\ ~,+·

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Since there are three significant figures involved in the multiplication, the answer can te written orly to a maximum of three significant figures.

*:.* E~137 V.

t 17. Static Sensitivity

The static sensitivity of an inst! ument or an 'instrumentation system is the . ratio of the magnitude of the output signal or response to the magnitude of input. signal or the quantity being

measured. Its units are millimetre per micro-ampere, counts per volt etc. 1

of input and output. . i

depending upon the type

Sometimes the static seinsitivity is expressed as the ratio of the magnitude ~f the measured

quantity to the magnitude of the response. Thus the. sensitivi.ty expresse

d this way\_ has the units of

micro-ampere per millimetre or volt per count etc. as the case may be depending upon nature of input

1

and output. Thus it is reciprocal of the sensitivity as detined above. This ratio is defined as deflection · :factor or invrrse sensitivity. Many manufacturus define the sensitivity of /their instruments in terms of inverse sensitivity and still call it sensitivity. 1

When a calibration cui:ve is linear as in Fig. 2· 3 (a) the sensitivity. of the instrument can be defined as in slope of the caHbratiou curve. For this case the sensitivity is constant over the entire rnnge of ~he instr~me~~t.: However, if the :c.u~ve }s no~ nominall~/ a straight line the sensitivity varies with tbe mput as m Fig.:2'3 (b). The sens1ttv1tv m this case vart~s · i. '

*i *

f Output  <to

----A~z·· -·-· 

I

Sensitivity=~

lnput,q\-

t Output  %

i I ,•

,/

I . I

' i :.o.q,o 

•.•. J.l A ! . Li.ct, Cl Sens·1v1ty= *iiIT'*

lnput,q.t -

I

(a)

Fig. 2 '3. Definition of sensitivit,v.

(b)

In general, the static sensitivity at the operating point is defined as. :

St tic sensitivit \_ ~l}fip.~!~.s~m\_al change ~n ~utput..,. *6qo*

a · Y mfimtes1mal change 1ll mput b. *q,*

Similarly,

Inver£>e sensitivity or deflection factor=- *6.qi b.qo*

... (2' 11) ... (2' J2)

fbe sensitivity of an instrument should be high and therefore the instrument should not have a range greatly exceeding the value to be measured. However, some margin should be kept for any accidental overloads.

' Example 2·12. A-Wl.eatstone bridge requires a change of 7 *Q* in the unknown arm of the J bridge to produce a ckngc in deflection of 3 mm of the galvanometer. Determine the sensitivity. Also ·determine the defle~tion factor

Solution :

Sens.itiv:~y= magnitude. of outp~t response \_ 3'000 mm ... 0.429 /1.

1 magmtude·of mput . 7·000 Q mm "

\.

CHAR.ACTERiSTICS OF INSfR.UMaNTS AND.MSASUREMBNr SYSfBMS . 19 I . . ; . . '"} f: magnitude of input nverse sens1hv1ty or sea e actor=;: · 't d f · t · t . . , · magm u e o ou pu · resp~nse *1·00* .0

3'000 mm

= 2;33. O/mtl1. .

· Example 2'13. A mercury thennometer has a·capillary\ube of 0·25 mm diameter. If the -.,,, bulb is made of a zero expansion matedal what volume must it have if a sensitivity of 2'5 mm/°C is desired ? Assume that the operating temperature is 20°C and the c<,>-efficient of volumetric expansion oflmercury is 0'181x10-3/°C.

Solution : Let *:*

Lc=length of capillary tube which would be occupied by mercury contained in the bulb when it is not heated ; mm,

*Le+* 6 l.c=length of capillary tube which would be occ~pied by mercury contained in the bulb when heated; mm,

Ae=area of capillary tube; mm2,

~v=co-efficient of volumetric expansion ; J11m3/mm3·°C,

and 6T=change in temperature; QC, · . .

It should be noted that there will be only a change in length of mercury column since it is given that the bulb has a zero expansion. material and hence there will be no changes in its area and length. This is true of capillary tube as well.

Sensitivity S· = ~:: ·=(Le+~)-*Le* ~f~ =2'5 *mmrc.*

Now *Ac (Le+ l:,Lc)=Ac (Le+ iv* Le/::.T) ·

· Length of capillary tub~

L - l *·!::.Le* \_\_ l -~ X2'5-13'8X103 -13"8 *c-* ~~ *6.T.=* 0'181x10-s . . ~ mm- m.

TC ,

Hence area of bulb *Ah=AcLc=4-(0'25)2 x* 13'8X 103=680 mms.

·2' 18, In~tmment Efficiency

The efficiency of any instrument is defined as the ratio of the measured quantity at full scale to the power taken by the instrument at full scale. The introduction of an instrument used for measurements in a circuit should not affeet the existing conditions in the circuit. The efficiency of an instrument should be as high as possible, as the higher the efficiency the lesser the effect the instrument has upon the circuit under measurement.

Let us consider the case o!f voltmeter for which we have :

Rm=resistance of meter,

and· *Eta* =full scale vo~tage reading. ,· ~ .

. Current at full scale reading

*Eta lts=-Rm*

Power consumed at full scale reading *E,,2 Pts=Ets* Its=--~ · *Rm*

Efficiency of instrument



... (2'13} (2'14

20 ELECTRJIJAL MEASUREMENTS AND MBASURJNO iNSTRUMBNtS · ' ' . ""::,>::~'"··\ . ' . . : . . . . ' . ·~ .· H~nce the efficiency· of a volt1Ueter is the ratio of resistatice orthe 'meter to its voltage at full scale and is expressed in ohm per volt. . · · . ·. · l · ' .· · · • From Eqns. 2' 13 and 2·14 it iS clear that in order that a voltmeter has a high eflcien~v and it draws smaller values of current, the resistance of the voltmeter should' be high. High efficiency voltmeters are a prerequisite for measurements especially in electronic cir¢uits where the current. and the power are small. 

!llllf1

Example 2'14. A s·o ampere .ammeter bas a resistance of 0'01. 0; Determine the efficiency

I I

of the instrument.- · ·

Solution : Full scale reading of ammeter *Its= 5* A.

Power consumed at full scale Pts=lts2,Rm=(5)2X(0'01)=0'25 W

'· f *Its 5* 20 A/.W Efiknency o ammeter= *Pis* = 0.2f= .

2'19. Index Scale and Index Number ,

. An analogue instrument may be thought as a meqhanism which ha~ an input. 9f the quantity being mrnsured and an ·output which is usually displacement of a pointer ~v~r a scale. Divisions are marked on a si;ale, the set of marks or divisions form an md.~x scale and the divisions moved is the index reading. , ·' 1

As far as digital instruments are concerned same theory may be applied: Jn a spatial encoder the quantity being measured produces J displacement of the mechanism. This cli~placement is conver ted into a digital number. . · ;

Example 2·1s. An ammeter has 100 divisions on its index scale. Tl~e instrument is provided with range multiplier switches of 1, 10 and 100. Find the range of the instrument and the scale range. Solution : Highest multip~ier switch= 100 times.

*:.* Range of instrument= 100 x 100mA=10x103 mA=lO A

fodex range =0-100. · · :

2·20. Linearity. One of the best characteristies of an iristrumenti or a measur~,tneni systefu is considered ~o be li~earity.1. that ~s, the o~tp~t is li~early provor~ional . to; the input. !y!ost \_of\_ th~·

systems require a hnear behav10ur as 1t · is demable. This 1s · because the conversion from a scale reading to the corresponding rolasured value of input quantity is mo~t convenient if 110~ merely' has to multiply by ~fixed constant rather than consult a non-linear calibratfon curve or compute frolli non-linear caJibration equatio~Also when the instrument is part of a large data or co~1t9t system,

linear behaviour of the part often simplifies the design. and analysis of the whole system. therefore · relationships to the qegree of straight line relationship *(i.e.,* linearity) are c6mmon.

If for an instrument calibration curve (relating output to input) is. ~ot a straight line, it should not be concluded that the instrument is inaccurate. This is a misconceptiOn, a non-linear behaviour does not essentially lead to inaccuracy. Such an instrument may be highlY: accurate as ever.

However~ most of the time.it is necessary . IOOOr----r--,---+--..-----..-~...,,.....-.---. h Id h Non- lineor that measurement system components s ou ave variatioris--t---i. .. / linear characteristics. For example, the resistance eooi--+--'--i-:c--+----'+------f,;L---+-- Line~

used in a potentiometer should very linearly with · g \tlrla on displacement of the sliding contact in order that the !! 600 ......\_--i-. 

displacement is directly proportional to the sliding ·~

contact voltage. Any departure from linearity would ·:

result in error in the read out system. ~ too r-1-T?l:;;f;;:::z~-:-t--t---r.....\_, Fig 2·4 shows the. variation of resistance § 

J

with displacement of sli,ding contact ... In case the

resistance to the sliding contact is P!Opo.rtion··~--to

the displacement, all the values of resistance w . 4 lie on a straight line (firm line as shown in Fig.~ ~4 . I-J owevei, in practice it may not always be poss·ible to have linearity, *i.e.,* linear variation. Suppose

0 ~~1or~~ 100 '200 300 •' 400 Units bf displacement -

. '



- Ctt\RACTERJSTJCS OF lNSfROMENTS AND MEASUREMENT SY~H!M'> · 2. \ i

that the resistance varies as shown by a curved dotted. line; in that ·case there is deviHticvo fron} linearity. This deviation from linearity ·may be expressed by a term ''Percent Linearity".

Percent linearity may be defined as 

P . 1. · . . ·(maximum resistance deviation·) X 10*r.* ercent mear1ty= ---- v . · full scale deviation ' •.. (215) for this particular case. In general percent linearity is :

\_:.(maximum displacemen~eviation) x 100 -- fuli scale deviation ... (2' 16)

It is desirable to keep the percent linearity as small as possible as it would, in thttt ca1c, rcsuh in small errors in ·the read out system. For example if a self-balancing potentiometer has a percent .. linearity i>fO'l per cent, its accuracy would be 1 part in 1000, With a 1000 count digital encoder

connected to the shaft of the potentiometer, it would produce an error of 1 count in 1000 counts . . ' *:* . . 'Example 216. A 10,000 .a variable resistance has a linearity of o· 1 % and the movement of contact arm is 320~. (a) Determine the maximum position deviation in degrees !ind the resistance de via- . tion'in ohm. (b) If this instrument is to be used as a pqtentiometer with a linear scale of 0 to l 6 V. determine the maximum voltage error. 

Solution: (a) From Eqn. 2'16,

Maximum displacement deviation percent linearity x fuH scale d~via~!~!l

100

= O'I x 320. 0.32•

• 100 I

. . · . . '. · d' I t o· 1 x 10,000 IO iJ

Srmtlarl~, maximum resistance .1sp acemen · = ---w0-:----· =

'(b) A qisplacement 320° corresf>onds to 1'6 V and therefore 0·32· corresponds to a voltage of;

. . Q:32 x1·1 = l '6 x 10.:.3 v

'320 '

*:.* Maximum voltage·error=1'6X 10"'."3 V=l'6 mV.

2·21 .. Hysteresis. Hysteresis effects show up in any physical, chemical or electric01l phe1rnme non. Hysteresis is a\phenomenon which depicts different output effects when loading and unl~C1.<iin.., whether-it is a mechanical system or an electrical system and for that matter any system. Hystere.\i~ is non-incedence o~.'1.oading'·and unloading curves, Hysteresis, in a system, arises due to th~ fact that all the energy put info the stressed parts when. loading is not I recoverable upon unloadin~. *This* is

·because the secondlaw of thermodynamics rules out any perfectly reversible process in the world . . ii·' . . . ' .

. ·Hysteresis effects are there in electrical phenomena. Orie of the examples . is the relationship :Mtwun ou'tpu~ voltage and field current in a d.c; generator. This fa due to magnetic hystw~5'is. This ~, ctirYe is of the shape shown in Fig. *2·5* (a). · · ·

In mechanical parts of a system, there may be internal friction, external sliding friction and ··coulomb friction. There may be a (ree PlilY or loDseness in the mechanism. In a giycn instrument a number of causes, such as listed above, combine to give an overall effect which may result in output~ -'input relationshipsuch as shown in Fig. *2·5* (b).

2·21: T~ahold. His c!e~r from above that if th.e instrument input is incr~~sed very gradually from zero there wdl be- some m101mum value below which no output c~ange can be detected. This

22

tJutputf

(a)

EtrctRICAL MEASUREMENTS AND MEASURlNO INStROMtlNTS OUfput f

Max.output 

Hysteresls '°"

...

(b)

Fig. 2·5, Hysteresis effects.

minimum value defines the threshold of the instrument. In specifying tbres~old, the first deteclible output change is often described as being any ''noticea~le measurable change ·'

2·23, Dead Time. Dead time is defined as the time required by a measurement. system to · ..... :

begin to respond to a change in the rneasuraod. Fig. 2'6 sho•vs the measured quantity and its value . as indicated by an instrument. Dead time, in fact,:,is the time before · 100 ..-----,--r-.----r----.

the instrument begins to respond after the measured quantity bas ' been changed. 

2'24. Dead Zone. It is defined as the largest change of input

quantity for which there i5 no output of the instrument (Fig. 2'6). For example the input applied to the instrument may not be sufficient to overcome the friction and will, in that case, not move at all. It will only move when the input is such that it produces a driving force which can overcome friction forces. As stated in Art. ·

2·20 other factors which produce dead zone are backlash or bystere· sis in the instrument.

The term *"dead zone"* is sometimes used interchangeably

with term hysteresis. Howe\'er, it may be defined as the total range Time t \_ of input values possible for a given output and may thus be num·

Dynamic error

ericalJy twice the hysteresis defined in Fig. 2'5 (b). Fig. 2'6, Dead time and dead zone.

Example 2·17. The dead zone in a certain pyrometer is 0'125 percent of span. The calibration is 400°C to 1000°C. What temperature change might occur before it is detected. Solution: Span=l000-400=600°C.

0'125 ,,.;1  Dead zone= 100 x 600=0'75°C.

A change of 0 7 5°C must occur before it is detected.

2·25. Resolution or Discrimination

If the input is slowly increased from. some ar~it:ary (non-.zero) input value, it will again be found that output does not change at all until a certam mcrement 1s exceeded. This increment is caJJed re~oJution or discrimi~ation of the instrument.. Thus ~he smalle~t increment in input (the quantity berng measured) which can be detected with certamty by an instrument is its resolution or discri·

minatkm. So resolution defines the smallest measurable input change while the threshold defines the smallest measurable input.

Example 2·rn. A moving coil voltmeter bas a uniform scale with JOO divisions He full scale . reading is 200 V and I /I 0 of a scale division can be estimated with a fair degree of certai~ty. Determine the resolution of the instrument in volt. ,

~--

1

. ClJARACTER!STfCS OF INSTRUMENTS AND MEASUREMENT SYSTEMS 23 Solution : 1 I d. . . 200. ·2 v sea e iv1s10n = 100 =

Resolution=& scale division=i~ x2=0'2 V.

Example *2)9.* A digital· voltmeter bas a read-out rang~ fr?m 0 to 9,999 counts. Determine the resolution of the instrument in volt when the full scale readmg is 9·999 V.

Solution : The resolution of this instrument is 1 or 1 count in 9,999.

Resohition= ---~---count= -1~. x9'99) volt=l0·-3 V=l mV. 9999 9999 '

2'26. Loading Effects. The ideal situation in a measurement system is that when an elemen!: used for any purpose may be for signal sensing, cor~ditioning, transmissioh or d€tection is introduced into the system, the original signal should remaiµ unmolested. This m~ans that the original signal should not be distorted in ·any form by introduction of any element 111 the measurement system. However, under practical conditions it bas been found that introduction of any element in a system results, invariably, in extraction of energy from the system thereby distorting the original signal. This distortion may take the form of attenuation (reduction in magnitude), waveform distortion, phase shift and many a time all these undesirable features put together. This makes ideal measurements impossible. The incapability of the system to faithfully measure, record, or control the input signal (measurand) in undistorted form is called the loading effect.

It may be recalled that a measurement system consists of three distinct stages *:* (i) Detector-Transducer stage,

(ii) Signal conditioning stage (including signal transmission stage),

and (iii) Signal presentation stage.

The loading effects do not occur only in the first stage but may occur in any of the two [subsequent stages while the first st~ge detector transducer lolds the input signal, the second stage !loads the first stage, and finally the third st1ge loads the secon(t stage.· In fact, the loading problem may be carried right down to the basic elements themselves.

i 2'27. Loading Effects due to Shont Connected Instruments i .

' In measurement systems, voltage measuring, displaying and recording instruments Jike :voltmeters, oscilloscopes and strip chart recorders are connected across the circuit in shunt foarallel) with the circrdt.

Consider a network consisting of linear bilateral impedances and generators with outpuu terminals. *A* and Bas shown in Fig. *2·1.* This is a blackbox with a Thevenin generator of voltage Eo and an output impedance Zo iil series. Supposing we are primarily concerned with the voltage of the output signal. Let Eo be .the ope~ circuit voltage *i.e.* the voltage that appears across the teril1inats *A* and *B* when the load (or any other measuring or recording. device) which *ls* a voltmeter in this case is .not connected to the terminals. .

f -r

l

I *A* A IL ~"'";' ~ 

oltoga o Eo

ource - \_J EL

I 9·

Fig. *2·1.* Voltage source and shunt connected instrument.

Ideally, when the load is connected across .terminals *A* and *B* the output voltage should remain

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the same. However, tJ1e load impedance is 'riot infinite and therefore when a voltmeter' with an input impedance ZL is connected across A an\_d B, a current IL flows. This causes a voltage drop ILZo.

;'. Qutput voltage under- loaded conditfons is :

EL=~-lr.Zo=ILZL or Eo=IL(ZL+Zo)

4

. . . *:* . Ratio of actual voltage appearing across the load (when the instrument is connected) to

1

I

the voltage under no load conditions (ideal in this case) is :

· Er, · I~Zi . 1

Eo: ~h(Z1+Zof= I +Zo/ZL ... (2'17) *},.* . Actual voltage measured, EL= 1 + ~~/ZL ... (2'18)

Thus the voltage which is measured is modified both in phase and magnitude. This means that the original voltage signal is distorted on account of con~.ection of measuring instrument across it.

It is clear from Eqn. 2'18 that in order that the original signal Eo should remain undistorted the value of input impedance of the instrument, ZL, should be infinite (or the value of output

l ........

impedance of the source, Zo should be equal to Iero which is not attained in practice). *i* '  In order to obtain as less distortion as possib1e ~he value of Zr., the input impedance of instrument, should be very high as compared with Zo, the outp..:.t impedance of the source. 

· To illustrate the loading effects of shunt devjpes a few examples are given below. First we will give examples connected with d.c. and then go ovefto a.c. applications.

Example 2·20. A multimeter having a sensitivity of ·2 ,000 O./V is used to measure the voltage across a circuit having an out~ut resistance of 10 kO.. The open circuit voltage of the circuit is ~ V. Find the reading of the multimeter when it is set to its 10 V scale. Find the percentage error. *//* Solution: Input impedance of,voltnieter ZL=20,000x10 n .... 20 kfi

Output impedance of circuit Zo= 10 k!l

Open circuit voltage of circuit under measurement *Eo=6* V.

From Eqn. 6' 18, reading .of voltmeter is

Eo 6

Er.= 14 ~fZL =I+ l0/20 = 4 V . -. .\

*:.* Percentage errol"'in \'.Oltage reading 466 x100=-33% .or 33% low.

The loadh1g problem given . in Example 2·20 is typical of cases where a common type of voltmeter such as a multimeter whose input impedance is.comparable with that of the impedance of the circuit under test is used for measuring voltage. Thus voltmeters having output impedance

comparable to that of circuit under test should not be used as they seriously modify the value of test . voltages. Such types of voltmeters are unsuitable for communication and electronic circuits where the impedance levels are high but they can be'used for low impedance circuits giving a good accuracy.

If we wish to achieve 99% accuracy in voltage measurement, th~ input resistance of the voltmeter should be greater 100 times the output re&istance. \_ For an accuracy of *95%* . the input resistance should be atleast 20 times the output resistanc'e. . .

:Example 2'2l. Suppose the voltmeter of Example 2·20 is used for measurement of voltage of circuit having an output impedance of I 000 n and an open circuit voltage of 6 V at its 10 V scale. find the error in measurement. ·

I . Solution : , Output resistance of circuit Zo = 1000 u = 1 kO.

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Reading of voltmeter Er,- --~-0 -:::-~ \_\_ \_§\_\_~ =5'97 V · l+Zo/ZL 1-1-1/200 ·

Percentage \_Error ~- 6 x 100=-0'5?~ or 0~5% low.

Thus when the output impedance of source is quite low as compared with input impedance of. voltmeter, the error involved in measurements is quite small. · . · . Example 2·22. A 50 V range voltmeter is connected across the terminals *A* and *B* of the 2ookn · circuit shown in Fig. 2·s'. Find the reading . of

.....J ~ the voltmeter under open circuit and loaded . + r conditions Find the accuracy !tnd the loading *:+L* 200 k!l ~~~~rnr error. The voltmeter has a resistance of 1000 k!l • • 100V Solution : Let us reduce the circuit to

'----\_\_\_,..1'------9 its Thevenin's. equivalent ~ircuit. Tho open .· Fig. 2'8 circ1,iit voltage, *Eo,* appearing across terminals *A* .. and B;is: ·

. 200

*Eo* = 100 x 400 = 50 V.

The output impedance (resistance in this case) of the sourc~ as looking into terminals *A* and Bis: . 200 X200

.Zo= 200+200""' lOO kq.

The Thevenin eauivale1lt circu'it under loaded conditions *is* shown in Fig. 2'9.

100 k!l. Voltag~ appearing across terminals *A* and *B* · ;.\ · · conditions is :  ~

under loading

J



· *io* 50 . 

EL=1+z--;1z~ = i +-ioo11000 =45 5 v.

L d. 45'5-50. 9%. 9% 1' oa mg ~rror= 50 =....,.. o = o ow.

Fig. 2'9 Accuracy= 100-% Joading error= 1.00-9 ==91 %. Operation with *A°* C. The anaJysis of loading effects with a.c. is

not so easy. It should be .borne in mind that both Zo and ZL are

dependent upon frequency. .Therefore .the indicated volt~ge will depend upon . ~he frequency of operation. On accou~t of the inp?t capacitance effects ~r the 1~struo;ient, the v~Iue of mput impedance ZL becomes low at .high frequencies with the result the mput signal 1s substantially attenuated at· high frequencies. ·. . .. ,. .

I I ,

It is not only the magriitupe of tlie signal that is effected)ut alsl) its· phase. Worst still; as is consequence of the shunt capacitance, the.non-sinusoidal signals are distorted in waveform a]so. · The rnagnit~de of th.e measured signal becomes-'substantiaJly smaller with increase in freqi1ency i?<S shown in Fig. 2·10. . *)* .

EL t 

~~\_;t~~ \_,

i

*'. ..* (. \_." • .. 1

""'

,; -:tnput outl>ut . '-., ... ,

(a) (b)

Fis. 2·11,' W~veform distortion, ' - .

Fig. 2· 10. Effect of frc:qg~nqy on output~ .

I

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The sharply changing non-sinusoidal waveforms are rounded off because of the finite . time. it · takes to charge a capacitor. This is shown in Fig. 2·11. ,' The effect of frequency on magnitude and phase shift of a signal is shown in the following example. . '

Example 2'23. An oscilloscope having an input resistance of 1 MO shunted by 50 pF capacitance is connected across a circuit having an effeGtive output resistance of 10 kO. If the open circuit voltage has 1 ·o V peak for a 100 kHz sine wave, what will be the voltage indicated on the oscilloscope when the frequency is (i) 100 kHz and *(ii)* 1 MHz ?

The equivalent circuit for the measurement system is

shown in Fig. 2· 12. Oscilloscope

1Qk!l

When frequency= 100 kHz :

The value of capacitivereactance at 100 kHz is 1 1

*Xo=* 2efC = 27t->aoo-x 1000-x·scfx 10-12 =32000 !~ The input impedan.ce of the oscilloscope is : *R(--jXo)* \_ 106 x(-j32xl03) ·. · Zr,= ·R=fx~- - -106 j32x-10~- ~ -]32x10a .a =32x103 L ....;.90°0.

, ·· . Eo fhe voltage across the Joad is EL=-----:--L·-·--: · . · 1 +Zo/iL

Volt age "" source

to

lMO. Opf Fig. 2·12

=l'OL0°X'--7 10xl~3L0° -1+0·3~3L90° l+j~'313 1+Tfx103 *L-90"* 

· *0·954* L -17· 4° V (peak) .

. This means that the magnitude of voltage indicated by the ! oscilloscope-is 0·954 of its original value. I · ·  *:.'* The error is (1-0'954)X 100=-=4'6 per cent. Also the voltage under loaded conditions lags the voltage under open circuit conditions by an angle of 17'4°. · ! Wfm1 frequency= 1 MHz

The value of capacitive reactance of o.scilloscope

. - 1

· *Xo=* 2; x 106x5o-·x 10-12~3200 n

T.he input·impedance of oscilloscope is :

Z "'"'\_R(-jXo) ~ 106(-j 3'2x10a) ~ - . 3200!l=32X10aL ~90" *Q*

r, *R-jXc* 106-j3'2X103 *J* .

. ·. The voltage across the load is EL l + ~o/ZL

1 . I =l'OLO"X ---·--· -:--= ·----- -------··· - IOX 1osL~o0 1+3'13L90?

l+ 3·2x193L-90°

=0·304L-n·3° v (peak).·

In this case the measured value is only 0'304 of its original value and the phase shift 4s 72·3•, Thus the output is considerably attenuated and is Jess than one third of its original value.

,.

,·

. ' ·.~ .. , .. ·

'! '

CHARACfElUSTICS OF INS1RtJMENtS ANb MEASUREMENT $YStbMS *21* ~ .

, This indicates. the effect of distortion of signal on account of increased shunting effect due to increase in frequency. · ·

2,'28. Loading Effects due to Series Connected Instruments

r

. . . Consider a network represented 'by a voltage source· having a voltage Eo and an output 1rupedance Zo. The output terminals are *A* and Bas shown in Fig. 2'13(a).

A 

A. 

lo

B

Ca) (b)

Fig. 2'13, Loading effect of ammeters.

{

. The value of current flowing between [terminals *A* and *B* under ideal ~onditions is 10• It is the current·that flows wben\_terminals *A* and Bare shorted. ·

Io=~ Zo 1

or Eo=IoZo.

However, when we· actually .measure the 'current, a· current measuring device has to be · introduced between terminals A and ·B. It is ·usually an ammeter. When an ammeter is pJaced between output terminalls, it adds to the impedance of cir~uit. This added impedance modifies the value of the current. · · · .

Suppose ZL . input impedance of ammeter ..

From Fig. 2'13 (/;)

. \

... (2'19) In order that the measured value of .current, be equal to the actual value of the current, *lo,* the value of Zo~ZL. This means that the input impedance .of the ammeter should be very small as compared with the output impedance of the source. . ·

. We can express the relationships of currents under loaded and unloaded conditions in terms of. admittances; · ·

From Eqn; 2·19;

... (2'20)

lo :::o---"--- ... (2'20 I+Yo/YL

· . . In oth¢r wofd~ tl,ie input admittance of the series elements should be very large as compared with the o~tptit, admittance 'of the source fa order to reduce ·loading effect.

' ·. . ' ,; ' - • . :· :, I • • ' i' ; ,. I

for achieving 99%. accuracy in measur.ements the output ·resista.nce should be at least 100 times the resistance of Che meter. Jn order to. have 95% accuracy the output resistance should be at . ]east 20 times the .resistance of metet

L

28 ELECTRICAL MEA~UREMENTS At'D MEAStJkJNG JNSTRUMBN't!i,  Ex·ampic 2'24. It is desired to measure 

the value of current in the 500 n resistor as

(b) me<J.sured value of current, and 1 ---i Amnn~rer . . 7 1000..

shown in Fig. 2·14 by connecting a JOO n

·ammeter. rlnd :

(a) the actual value of curi;e11t,

(c) the percentage error in measurement and the accuracy.

e

Fig. 2'14

Solution. (a) Let us reduce the actual circuit to an equivalent Thevenin's source. Open circuit voltage as applying at terminals. A and B is *:* . ' 10 . Eo=IO- ·.2000-X 1000=5 V.

Output imp~d ance of source as looking iflto terminals *A* and *B* is.:

' . .

1000x1000 . .

~0=1000-+-Tooo-+soo= moo !l

The Thevenin equivale.nt circuit is as shown in Fig. 2' 15.

Z0 ::'1000fi 

~--0A

tl\_ ..

·TEo'°sv . .. ·

·----00

fig;.2'15 Eo .:...\_; 5 · · · . Actual value ofcurrent lo:~ *-z;* ---;- 1000 A= 5 mA. . . .

(b) When the amm~ter is introduced into the.circuit the y~Ju.e of current is. modified. *:.* Measured value of curre11t: li.~ zo!°zr, 1000~ 100 A=4'55 *mA:*

( ) 4 -ss·-· 5 1',00. '9%. *9°*1 1 .. , *c* Error-~::-- --- x · = - o~ *10* ow.

Accuracy of measurement=--= 100-9 e:::9I%.

2'29. Impedance Matching aod Maximum Power Transfer

. In many applications it is desirable to match the impedance of the input device to the output impedance of the signal source·instead of rnakiog the impedance of the input device either too high or too low.

Typical c~ses of impedance matching are those involving applications of waveform generators like pulse generators and radio f requettcy {RF) ·generator's, which utilize a transmissio.n line to transfer energy from the source to the input ·cteviCe. However, many low frequ~ncy cases such a.if. audio amplifiers feeding loud11peaker·s and other electromechanical transducers require impedance matching for high power transfer.

In measurement systems, many a times we are concerned with the problem of maximum power transfer from the source to. the input device. The problem of

nrnximum power transfer1 js analyzed by first considering the source 1L <:md· also the input devfo,e \(load) to have orily pure resistances. (See Fig, 2'16}. . ' · . .· ' ' .. . -f • 

. ' . . I ...• ·· ·. . .. . : . ··.·. • . . I\ Let E'o =voltage dftne sourc~ under ~&.lo~d condit'ions;. · . 

. EL::=voltage uf the sol.tree t.i11ci~t' kiaded co~ditions,'.

*Ro* =output resistance of the sti1.irce, · and Rx.=inpufi'resistance of th~load.

~~Source~~· Loo

Fig. 2'16



CHARACTERISTICS OF INS~iRUM.ENTS .A~D ,ME.ASl!REMElNT· SY:STEMS. 29 - . . , '.· I • . ~ • •• • • • , .• • • \ • • ' ·'

· ' ·· . ·. > . , " · EL2 · · *Eo2RL* : ·. Power transferred to load· ts *P=* ..,....\_..:... = ,---···- · ··· · - ·-.. · . RL *.(Ro+* RL)2 · · .... (2'22) *dP* Maximum power transfer takes place when -dR~-= 0.

Differentiating Eqn. 2'22, ai;id equating it to zerJ, we get *RL=Ro.*

Thus in order that maximum power m.ay be drawn from a source (when output impedance of the source is a resistance and also that of the load' is a pure resistance) is \\hen ,the external load resistance matches the internal resistance of the source.

Under conditions of maximum power transfer :

·. . . . *Eo2*

Power delivered to the load ls *Pma:u=* 4~.R~- •.• (2'23)

. For A.C. Circuits.·· The internal impedance (output impedance) ·of the ~ource and the input impedance 0.oa'd impe\_darice) may not be pure re.shtances. hut are usuall¥ complex impedances.

Let Zo=output (internal) impedance of the soµrce · *Ro+jXo,*

ZL=output impedance of the device=RL+jXL.

For maximum power transfer the impedance of the load

100

90

t 80

8 70 ~ 60 "-w so

" ~o

~E 30

0 ~~-~~001

~ 20

10

r\

\

~~'~~ I'\

/ '\ ~~"~~ ....

should be made equal to the complex conjugate of the The:venin equivalent impedance of the source.

ZL=Rt+jXL=Ro-JXo=Zo ... (2'24)  It is clear from Eqn. 2·22, if the no load voltage *Eo,* of the source and its output resistance *Ro* are given, the power depends upon the magnitude of load resistance RL; The power ~pproaches zero for either very small or very large values of load resistance RL .

01 ~~10~~ 10 100 We have derived that for maximum power transference *RL=Ro.* In ~./Rcr- other wordt maximum power can be transferred from a source to a Fig. 2'17. Impedance Matching load if the internal resistance of the source or the output resisfance Characteristics. · · of the preceding stage of an instrumentation system is equal to the load resistance or the input resistance of the succeeding source.

It is interesting to note that for maximum power transfer, the efficiency fg 50%.

The condition for lmpcdance matching is not critical. Fig. 2· 17 2hows the relative amount of power transferred from one system to another for different ratios of *RL/Ro.*

'For a .±10% deviation from the correct value of impedance matching, (RL/Ro=l'l or 0·9), the power transfer is still practically 100% For a 20% change, the power transfer· reduces to 99%. Even for a 100% change (ratio *.&/Ro=2* or 0'5), the power transferred is 89% of the maximum allowable power.

It is very often desirable to change or control the amount of power transferred from one ' system to another. T~is can be done by changing the voltage level applied to the load or the c,urrerit level in the circuit. A variation of. the voltage level is·· difficult with d.c. and generally requites the use of eJ;ctroni.c ~quip~ent. ~ con~rot (which. is' a red?cti?n) of /c~rrent is relatiy~~¥ ~iniple and !s accomphshed by msertmg a series resistance R& m · the circmt. A disadvantage of this method 1s consiqe~able Joss of power and a very low efficiency. With value of Ro=l *0;,R•* =98 .0 and RL = 1 .0, the us~ful power absorbe~ by the load is l W, while the amoun~ of power lost as I *2*R loss in both *Ra* and *Ro* put together is 99 W. Thus the total power furnished by the source is 100 W and that received ~y the load is 1 W giving an efficiency of 1%. 1 - ..



30 ELECIRICAL MBASURE~~N:rs "AND ·MEASURING INStrtifMBNTS  A very efficient w~y to control power transferred~to one.· Power

system from 1another is by pulse modulat.ion of power. Jn its simplest form this can be done b.y periodic o.pening and closing . Pmax of a switch located between both systems. '

Using the notation given in Hg. 2'18; the average power transfer is : · t>ov

on uft

·~ *Pav= Pm!!!J.* . *T*

where t=pulse width,

.. .<2·25) '

and .• , *T=* interval between successive pulses... Fig. 2· 1s. Pulse modulation of Power. ·By variation of~ between 0 and .r, the power can .be controlled between O and the maximum value of power. The power efficiency is maximum for· any value oft. However, it may· be noted that this method /s restricted to loads which are not affected by the periodic interruption- of current.

Power.efficiency may not be the foremost consideration in Instrumentation systems. Many other considerations and physical advantages like linearity, efficiency, or avoidance of loading of previous stage may be more important than the maximum power transfer.

Example 2·2 5. Measurements on a human nerve cell indicate an open circuit voltage of 80 mV, and a current of 5 nA through a 6 MO load .. What is maximum power available from the cell ? Solution : Let Ro be the internal (output) resistance of the cell, *IL* the load current and RL the load resistance. · *:.* Open circuit voltage of the cell Eo=/L(Ro+ RL) · . or so x 10-a=s x 10-9(Ro+6x106)

Hence, output resistance of cell *Ro=* 10x106 0= lO MLl · · Eo2

Maximum power availabJe from the cell *Pma:e= 4*Ro

= ~ ~~ 6:=160 x 1-0-:12 w o· I 6 n w.

Example 2'26. A voltage source has an open circuit voltage of 20 V and has an output impedance of *0·5 +j* 1 0. The voltage souree is connected to the load t.hrough a transmission net· work having an impedance of 1·s+j40. At what load will 11Daximum power transfer be.realized'/ Calculate the maximum power .. Also calculate the losses in the voltage source and the transmission network. ·What is the efficiency under maximum ,power transfer conditions ?

Solution. The transmission network is connected in series with the voltage source, . ·. Output impedance of source and the transmission network is : \_ =(O'Sfj o+o·s+; 4)=2+j *s* n. ·  

For maximum power transfer the load impedance should be complex conjugate of the·above impedance.

. . . Impedance of load for maximum power transfer *i.-e.* ZL = 2-j *5* n

Eo2 (20)2 Maximum power *Pina:1 4R* 4X2 -·50 W.

0 -

20 '

Load current IL=(l+)5)f(f-=Js)=5 A · 

Power loss. in source-(5)2 x0'5=12'5 W. ·

Power loss in transmission network=(5)2 x1'5=37'5 W

Total losses=J2'5+37 5=50 .W.

Bfficie~cy output x 100 output+ losses

50 '

so+so xrno. so%. I, j' 1 'I

CH A RAC'!ElUSTICS OE INSTRUMENTS AND MEASUREMENT SYSTEMS 31

Example *2·21.* The output voltage of. an audio amplifier is 10 V and 4 V when delivering powers, 25 Wand 16 W respectively. Find the open circuit voltage and output resistance of the amplifier. What is the maximum power that the amplifier can give ? ·

Solution : Let *Ro* and RL be the output resistaace and load resistance re:spectively. . '. EL1 2 (10)2 · Lo1ad resistance in the first case R\_Ll = *p;-* = 25 =4 0

. d • t • th d *R* EL22 (4)2 1 n Loa res1s ance m e secon case L2= p-;: = 4= . u

Eo RL

Voltage across load EL= Rof RL

:. In the first case 10= ]Sox 4 Ro-t-4

In the second case =Eo~  · Ro+l

From (t) arid (ii). we have~

Output resistance of amplifier . Ro·\_;4 n Open circuit voltage of amplifier Eo = 20 V.

or 10 Ro+40=4 *Eo* or 4Ro+4=Eo

... (i) ... (ii)

The m·aximum power that a source can give is when. its output 1·esistance is equal to load resist&nce .

. ·• The maximum power output of the amplifier is 2~.

2·30,,-· Dymunic Response

1 When an input is applied ·to an. instrument or a measurement system, the instrument or the system cannot take up its final steady stafe position immediately. On the other hand, the system go~ through a transie~t state before it finally settles to its final 'steady state• position.

1 Some measurements are made under conditions that sufficient time is. available for the instrument or the measuremen~ system to settle to its fi;ial · steady state conditions. Under such conditions the study of behaviour of the system under transient state,. called 'transient re'spoose' is not of much of importance ; only steady state resp.onse of the system need be considered.

However, in many areas of measurement system . appli< .. -ations it becomes necessary to study the response of the system under both transient as well as steady .state conditions. In many applica~ tions, the tnuisient response of the system *i.e.,* the way system settles down to its final steady state conditions is more important than the steady state response.

It bas been pointed out earlier that the instruments and measuring systems do not respond to the input imm~diately~ · '.fbis is on· account of the presence of energy storage elements in the sys tem, These energy storage elements are electrical inductance and capacitance, mass, fluid aml thermal capacitances etc. The systems exhibit a characteristic sluggishness on account of presence of these elements. Furthermore pure delay in .time is encountered when a syst~m "waits'' for some specific changes and reactions. to take place, . · 

Invariably measurement ~yetemsp 'especially in industrial, aecospace, a~ biological applications are subjected to inputs which sre not static· but d1namic in nat:ure. *i.e.* the inputs vary with time Since the input varies from instant to instant, so does the output. The behaviour of the system under such conditions is described by the dyumic response of the system.

The dy1flmic chi.tracterhltics of any measurement system are :

(I) Speed of response (ii) Lag

(iii) Fidelity (iv) Dynamic error

The qualities Ulto4)m.t the left side are . desirable fo a. dynamic. system while those on the tight. are undesira~le. 1

32 ELEC!RiCAt MEASUREMENTS AND MEASURING, INSIRUMENTs .

· Speed of Response. It is the rapidity with which an instrurnent responds to changes in the · measured quantity. ' . Re~ponsc Time. Jt is defined as the time required by instrument or system to settle to 1ts final steady position after the application of the input. For a step functi6n, the response timi;- rn .. J be defined as the time taken by the l.nstrument to settle to a specified percentage of the .quantity being measured after the application of the input. This percentage may be 90 to 99 percent depending upon the instrument. For portable instruments it is the time taken by the pointer to come to rest within ±0'3 percent of final scale length while for switch board (panel) type of instruments it is the tjme taken by the pointer to come to rest within ± 1 percent of *its* final scale length. l\·Ieasuri~g fag. An .instrument does not immediately react to . a change. Measu'dng lag is defined as the delay in the response of an instrument to a change in the measured quantity. This lag is usually quite small but it becomes highly important where high speed measurements are required. In these systems it becomes essential that the time Jag be reduced to minimum. · · · ·:Fidelity. Fidelity of a system is defined as the ability of ·the system to reproduce the output in the same form as the iupnt. Supposing if a linearly varying quantity is applied to a system and if the output is also a linearly varying quantity the system is said to have 100 percent fidelity. Ideally a syftem should have 100 percent fidelity and the output appears in the same form as the input and there is no distortion produced by the system. In the definit~on of fidelity any time lag or phase difference between output and input is not included. . · · . . Dynamic Error. It is the difference between , the true value of the quantity changing with time and the value indicated by the instrument if no static error is assumed.

However, the total dynamic error of the instrument is· the combination of its fidelity and the time lag or phase difference between input and output of the system.

2.31. Measuring Lag. Measuring Lag is of two types : , · (i) *Retardation Type.* ln this case the respoDse of the in- ! .. 1.0 ........ ~--T-..;.,-..,--r-"'.l strument begins immediate1y after a change in the mea.surand has ... ·cti 

occurred. · · ! cc 1-1-1~-.ld~-+--t--t ·-~ Q.6  

(ii) *Time Delay Type.* In this case the response of the system · ~ !

begins after a 'dead time' after the application of the input. This is· ='Co. 21--f-+-+-+--+-""'i-'-t---t

shown in Fig. 2'19. ~ e The measurement lags of this type are very small and are of the 2 3· order of a fraction of a second and can be ignored. But when these, lime, 1' -~ systems are subjected to periodically varying inputs~ the perf'orman~ of· Fig. 2'.19. dead time.in .. the instrument~ with dead time is usually not satisfactory. If the instruments. measurand varies at a fast rate. the dead time .has a severe adverse effect on the performance of the instrument. ·

2.32. Stft~da1·d Signals. The measurement systems may be subjected to any type of input. The type of input signals cannot be known fully ahead of time. In almost [al~ applications the

>. 

'O~t-----

c;

:~ 0 ' c; ;J

:>CT

0 0 Time \_\_..

f ~ l I bl

Step Ramp Sinusoidal ·

fig, 2·20. Various types of input functions.

CHARACTBRISTICS OF INSTRUMENTS ANI:> MEAStJREMENT SY.STEMS 33··

··signals are random in nature. Therefore, it becomes difficult to express the actual input signals mathematically by simple equations. Dynamic behaviour of measurement systems can be studied with the help of certain standard signals. These standard signals are :

(i) Step input, (ii) Ramp input, (iii) Parabolic input, and' (fv) Impulse input.

~ . ~

. The above signals are used for studying dynamic behaviour in the time domain. For studies

r

in frequency domain, steady state response to a\_ sinusoidal input signal yields a great deal of informata ion. This is because all actual inp~ts can be thought of c~nsisting of a band off requencies ranging from zero onwards (All types of signals c1n be broken mto sum of a series of sinusoidal signals according to Fourier series). · !1 \

,\ .

··When system stu~ies are carried out in time ,domain, the dynamic behavi~ut of the system depends upon the system pole~ and not on t~pe · of input. Therefore the system behaviour to any kind of inputs can . be predicted by studying its ·. response to one of! the, standard signals. T.he I

·standard input .·chosen for this \_purpose is a step input; ' ·· 

The step, ramp and sinusoidal signals are *J5* ,

shown in Fig. 2·20

2'3 *3.* Overshoot. Moving parts of instru~

ments have mass and·· thus possess iriertia. When

an input is applied to instruments, the. pointer does

ndt inimediat~ly come to rest at its . steady state

(or final deflected).7positiQn·1>U! goes beyond. i~ or

in other words "overslioots" its steady position.

The overshoot is defined as the maximum amount

by which moving system moves beyond the steady

, state position. (See Fig. 2'2C, · In many instru- Fig. 2'21. Overshoot.

Final st12ady p0s1tion

merits,. especially galvanometers it is desirable to have a little overshoot but an exc.essive overshooO is undesirable. ·

ExaJDple 2·2s. ·. A step Input of 5 A is applied to an ammeter. The p9inter swings.to a voltage ·· of 5• 18 A and finally comes toi re;st at 5.02 A. (a) Determine the overshoot of the reading in ampere and in percentage of final reading'. (b)'Determine the percentage error in the instrument. Solution. (a) Overshoot=5'J8-5'02=0'16 A ·

. . h.. 0'16% 0 Percentage overs oot.- 5.02 · x 100=3'2 o ..

· . . . s·o2-s·o , . o (b) Percentage error, ~ x 100~04%.

\

UNSOLVED PROBLEMS

1. An ammeter reads 6'7 A and the true value of current is 6'54 A. Determine the error an(tbe correction for this instrument. ·· , · \_ [Ans. 0·1~ A, -0'16 AJ 2. A voltmeter reads 109.5 V. The error taken from an. error curve is -0'37 V. Determine .the true ¥oltage; . , */* . [Ans. 109'87 VJ 3, The measured value of a voltage is 11 i V while its true is 110 V. Calculate the relative error. (An~. 0'91%1 4. A 0-100 V voltmeter has 200 scale divi:.;i,ms which can be read .to 1/2 divisiOn. Determine tbe resoJution of the meterin volt. · · . (A11s; o·~- VJ · 5. A diaphragm type pressure m·easuring instrument is calibrale~ .for absolute pressures of 6 to 76() mni ~f mer~ry. ·It has an acc~racy of ± 1%. Calculate the scale range, scale spen 1md maximum S~l}tic error, . , [Ans. 760 mm, 754 mm, *t 1·s4* mm] 6. State the number of significant figures 'in each of the.fQllowing : .

. ~) ~2.A, (b).l~65 V,. <.c.> ~t:is•\\r,;(d~-~QJll.{t) 4~~~.Jlll0.~2.. (,.). 0'34Sk.Q . .. . ,.- . *:* :. ,, .. ; . , , , '-· . , . . · .•~ <aH. t1'H. (c:) 4, *(d> Sa* (f}i; t /) J, (l)S] .. . ·\ .. . .. -, . .  ~ ..

*l )* .



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7. Four resistors are conneeted in series. The values of resistors are 28'4 .Q; 4.'25 Q, 56'605 Q,\_and 0'75 Q with a~ uncertainty of one unit in the last digit in each. Calculate the total series resistance, giving only significant figures in the answer. . [Ans. 90 Q].  . *S.* A curri!!nt of 2'56 A is flowing in a re5istance of 45'73 n. Each quantity has an uncertainty of one unit m the last digit. Find the value of voltiige drop to the appropriate number of significant figures. . [Ans. 117 V]

9. Determine the lineacity of a potentiometer to obtain an error not to exceed 1 part in 10,000. [Ans. 0'01%1 h 10. A recording instrument requires a current of 0'05 A to overcome initial friction and produce motion of t e movement. Define this effect and list factors which produce it. (Ans. Dead zone=O'OS AJ 11. The dead zone of a certain pyraometer is 0'125 per cent of the span. The calibration is 800°C to 1800°C. What temperflturn change must occur before it is detected? . . [Ane. 12'S'C] 12. Wb!lt i,IJ the true value of .voltage ~ross the SOO k{l resistor connected between terminals A and B as shown in I ig, *1'22* 7 What would a voltmeter with a sensitivity of 20 kQ/V read on tte foltowiog ranges : 50, 15, *5* volt ?

~M~O A

r-~~  

..:.. WV 500k0.

-l-.~~ t B~  I~\_J\_'o\_k\_°'\_~· ,...,1o:\_j

Fig. 2·22 fig. 2'23

13. What is the true value of cuaent in the 15 kQ resistor of Fig 2'23. If an ammeter of 2 kQ resistance is i1iied to measure the current in the 1 S k(J resistor, what will it read? If a loading accuracy of 99 percent is desired in measuring the current, what should the arome1er resistance be ? [Ans. 100 µ.A, 92'6 µA, 250 O]

14. A volt.Ilg~ source has an open circuit voltage of 100 V and ao output impedance of lO+J 20 n. It is connectea Hl n Joad through a transmission network. The impedance of tbe load is 20+J *XL Q* and that of transmis~ion r1etworlt Rrr:-J 15 o. Spe.cify tbe value of Rrr and XL for maximum power transfer to the load. If the variation in load resistance were permitted, what will t.e its value for maximum power transfer? . . · · . . (Ans. 10 0., 5 *0.* capacitive, 10 fl]

15. A step Volta~ of 20 V is applied to a voltmeter and it swings to a maximum value of 28'8 V and finally s1&ttJes to 20 V. Find tho percentage overshoot. . . [Ans. 4 0%] . 16. A self-balancing potentiometer is connected to spatial encoder required to read 2000. The lin~arity is O'OS%, find the accuracy of the read out system. (Ans. 1 part 10 2000]

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3"

Errors in Measurements and .th.air Statistical Anaf y.si~

3•1 Limiting Errors (Guarantee Errors)

; I

The accuracy and precision of an instrument depends upon its design, ·the material used and the workmanship that goes into· making the/ instrument. The .. choice of an instrument for a particular application depends upon the accuracy desired. If on'ly a fair degree of accuracy is desired, it is not economical to use expensive materials/and skill into the manufacture of the instrument. ·But an instrument used for an application req,uiring ~ high degree of accuracy has to use expensivt( material and a highly skill~d workmanship. The· economical production of any instrument requires the proper choice of material, design and skill. In. order ·to assur.e the purchaser of the quality of the instrument, the manufacturer guar aQtees a· certain accuracy. In most instruments the accu~ racy is guaranteed to. be within a certain. percentage of full scale reading. Cbmponents are guar~nteed to be within a certam percentage of the rated value; Thus the manufacturer has to specify the deviations from the nominal value of a particular quantity. The limits of these deviations. from the specified value are defined ~s Limiting Errors or Guarantee Errors. · . · ,

We can say that. the manufacturer guarantees or' promises that the error in the· item he is selling is no greater than the limit set. . · · The magnitude of a quantity having a nomfoal value *As* and a maximum error or limiting error of *±6A* must have a magnitude *Aa* between the limits *As-8A* and *As+8A* or · · · . Actual value *Aa =Aa±8A* . ...(3;1) , for example, the nominal magnitude of a resistor is 100 0 with a 'limiting error of ±10 il The magnitude of the resistor will be between the limits : . · .. A=lOO±lO Q ·or A)90 Q and A<;llO 0

In other words the manufacturer guarantees that the value of resistance of the resistor lies between 90 n and 110 n.

3·2. Relative (Fractional) Limiting Error

. . . The relative (fractional) error is defined . as the.· ratio of ~he error to' the specified (nominal), magnitude of a quantity .. Therefore.,

· · *BA.* E

relative limitillg error Er= A;= *A:*

or Eo=3A=E1A,

Then from Eqn. 3'1; limiting values are :

Aa~Aa±.Bi *Aa±ErAs=Aa* (l±Er}

Percentage limiting error %Er=Er X 100 ·

In the example considered in Art 3' l, we have *A,=* 100 0 and *3A* = ± t O n . . ' . l' . ' E SA io· .±· 0·1 . Relative 1m1tmg error . , . *As* :=r.± 100 .= •

Percentage limiting err~r %€r=O'l x 100=±10%

3S

·.;.(1'2) ••• (3'3)

••. (3'4) .~.(~



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and limiting values)>{ resistance are :

Aa=As(l±Er)=lOO(l±O'l)=lOO±lO !l.

In limiting errors the specified quantity *Aa* is taken as the true -quantity, and the quantity . which ~as the maximum deviation from *Aa* is taken as the erroneous quantity. Thus, we have . .

3A·-A~-A.

R ] t• 11. . . *Aa-As*

e a tve 1m1tmg error, E,. = *Aa*

actual value-nominal value

=--nominal value

,. ,· .~"· , . I'

'··-: ... (J'6) ... (3'7)

... (3 '8)

Example 3'1. Th~ value· of capacitance of a' capacitor is specified as 1 ~·F:b5% by the manufacturer.' !find the'fiinits between which the value of the capacitance is guaranteed. • . Solution': <~:T4tjuarariteed·value·of the cap~eit4'11c,e' lio within:;the limits:" . · · · A·~ *Aa(l* ±Er)=i= 1 x (l ±0·0~)..:..0·9·5 to t,·os µF. ·. · .· " · ; . '  Note; *The. same. idea (Jf a* guarante~ *·11mlting the worst possible case applies* t~ *electrical measurements. The meas.urements may involve several* ~omponents, *each of which may be delimited by a guarantee error. Thu8:>,Jhe same treatment is to be followed for quantities under measurement as is followed for specified quantities.* '. . · . . : . : · Example 3;2. A 0-150 V voltmeter bas a guaranteed accuracy. of 1 percent c>f full scale ·.reading. The voltage measured by this instrument is 75 V. Calculate the limiting error in percent .. Comment upon .the result. . . · ·, ·

Solution : The magnitµde of limiting error .6finstrutl\~,t,

. *3A* ! Er *As* · ·. ·

· 8A=O'Ol X150=1'5 V.

Th~ magnitude of the voltage being measured is 75 V.

' h' 1 . E *SA* l '5 0'02 The relative error at t ts vo tage is *r* = *Aa* = 7S -

The.refore, the voltage being measured is between the limits of

A. 11=A.(l±Er)

.,:,,75(1±0'02) V=7:~±1'5 V;,,

The'.percent~ge limiting error is : % Er"\* 1;; ·x 100~ 2 percent.

1

< '

Comments. ~tis important to note that this .meter is guaranteed to have an error of less than

1 percent offull scale or t~e l\_i.t;niting err?r is I percent at the fall sc~IeAe.flecHori of ~50 V. But w~en the meter reads 75 V, the hm1tmg error is 2 percent. The percent hm1t10g error wlll be greater 1f a smaller voltage is measured. If' the meter reads , 37'S V~ the percentage limitieg error is :::1'5-5x 100 · 4'0 percent. . · . 37' . .  This increase in the percentage limiting error as small voltages. are 'measured occurs ,because the magnitude of the limiting error *8A* is based upon the fuIJ scale reading of the meter and is a fixed quantity, while the actual voltage readio,gs can be of·any Illagnitude frorir Q to ISO V. We infer from here that the percentage ~rror increases as the voltage. b,eiog mea$P,red decreases. \_. · . . - . . . ·.,···.. r . : . . . : - *.1* / •  . . Thus while sel~cting iristru.~e~ts, pa~tipular c~re sh~uld ~e t~Jcen ·a~-f~gar~s. the range.' The . values to be meas~re~ shoU;ld n~t lie m the Jo'tV~r third ofrtl1e f:ange., ,Tbts tS, partlcu1artv imnortaot  if the.meter accuracy 1s specified 10 tenns of the full sc11Ie defiecttonf.s.d. (another name for f.s.d. is .

' )

ERRORS IN MEASUREMENTS ANO THEIR STATISTICAL ANAtYSis *31*

fudicial value) as considerable error, (as a percentage ?factual value), may occur as is·.· Step in. Example 3·2. Thus mete~s which read well up their scale ~hould be selected as far as possible·. · Another example is given below which is illustrative of the .comments given above:: Example 3·3, A wattmeter having a range 1000 W has an error of± l % of full scale deflection. If the true power is 100 Vf, what would be the range of readings ? · . · · · . · Suppose the err'or is specified as percentage of true value, what would ~ .. the range·of the readings.· · ·

Solution : When the error is specified as a percentage of full scale deflection, ·the magnitude of limiting error at full. scale ·

=±1~0 x 1000~±10 w.

1

. . .. Thus the wattmeter reading when the true ~eading i~ 100 W may be 100±10 vJ, i~e., between

90 to 110 w. ,. . . . . . . . . ·

+10 The relative error = lOO x 100- ± 10 % .

-'Now ·suppose the error is specified as percentage of true value;.~

. 1 .. The magnitude of error=±100 X100=±1 W. · Therefore the meter may read 100± 1 W or between 99 to- 101 W.

3'3. Combination of Quantities with Limiting Errors. Wh~~ two or more· quantilfos, each having a limiting error, are combined, it is advantageous to be able to compute the limiting error of the combination. The limiting error can be easily found by considering the relative increment of the function if the final result is in the form of an algebraic equation.

(i) Sum of two Quantities. Let *y* be the final result which is the sum of measured. quantities *u* and *v.* ·

*y=u+v.*

The relative increment of the function is given by

*.dY* = *d(u+v) =du +dv.*

*y y y y*

Expressing the result *in* terms of relative increment of the component quabtities ' . *dy* =.!:!\_\_ *du* +~ . *dv.* ·,. . . y *y u* ·y *v*

If the err9rs i~ t~e co~ponent quantities are represent~d. by *±8u* and · ± *Bv* then co~respondiilg limiting error *8y* m *y* 1s given by : . · *8y* =±(\_!!.... . *\_8u +!\_\_* . *8v* ) · *y y u y v* ... (3 '9)

The ab.ove equation shows that the resultant relative error is equal to the sum of the products' fonl;led by' mu!tiplying the individual re~ative errors by the ratio of each term to the function. (ii) Difference of two Quantities.

Let *y=u-v*

*dy du dv*

.--=---·

y y y ·,

-

38 ELECTRICAL MERSlJREMENTS.Al\D MEASURING INSTRUMENTS 1 *!* '  Expressing the result in terms of relative increments of component quantities

*dy u du v dv* -=- .----· *y y u y v*

If the errors in *u, v* are *±au* and *±av* respectively, the signs n:iay be interpreted to give the· worst possible discrepancy *i.e.* when the error in *u* is +~u, the error in *v* 1s *-8v* and vice versa, then the corresponding relative limiting error *8y* in *y* is given by

*S'y* = ±(·!!.. . *au* -t!\_ . *av* )

*y y u y v* ... (3'10)

Eqn. 3' l O is the same as Eqn. 3 ·9, It may, however, be mentioned that in this case when *u* and *u* are almost equal ia magnitude *i.e.* when *y= u-v<(u* and also v~v then the relative error in *y* would be very large.

(iii) Som or Difference of more tlum two Quantities. The sum Of difference of more than two quantities may be treated in a similar way.

If we have *y= ±u±v±w*

Then the limiting error is given. oy :

BJ!=±(Jl-. tiu +.-E-. \_av~---'!.~). *y y u y v* Yw

(fr) Product of two Components.

Let *y=uu*

loge *y =loge* u+ loga *v*

Differentiating with respec;t to *y*

\_l=l.~+~.~ or~=~+~· *y u* ~ ~ ~ *y u* ~

. .. (3'11)

Representing the errors in *u* and *v* a'.l *±Su* and ±~v respectively, the error *Sy* in *y* is given by: *8y* = ±(? ..!!.. + ~-) *y* . *u* 0 ... (3'12)

Thus from above we conr'~u<le that the' relative limiting error of product of terms is equal to the sum of the relative errors of terms.

(v) Quotient *u* Let *y=--·* ll

loge *y=loge* u-logc *v,*

Differentiating with respect LO y, we have

1 1 *du* l *dv*

*y-=U* 'dy- *V· dy dy du. dv* or ·--=;::: \_\_ -·\_,

*y u v*

Representing the errors in *u* an~ Pafl -~8u and :1:ov respectively, the relative error in *y* is *a·y* ~~t ~. '8v ~- == ± .---,- '.:f---.

y, ·Jil v· . .. -(3' 13)

Thus maximum possible err.pr occurs whe-n ~'j + ve and *av* is -ve or vice versa. ~ v . -

i

' <:- *("'* • • • • • • 0 ..... \ ' •\_j *jJ.l* ,", Relative hm1tmg error ltl is l' .. =""-J: fhs.4'·-·) *y* \ lt . 'l ... (3'14)

. " ... ,

The above result is the sante '•US the ct>nespon-i;kPl bu.It for the 1roduct of two o.ua~tities.

i' I.

ERROR3 IN MEASUREMENTS ANO 1HElR STATISTICAL ANALYSIS

(vi) Product or Quotient of more than two Quantities.

*u* 1 Let *v=uvw* or y=- or y=-· ' . *vw uvw*

Considering Eqns. 3 12 and 3'14, we have relative limiting error for *y* ~y = ±(~~+?~+ *Sw)*

*y u v w*

*(vii)* Power of a Factor

Let *y=u·* ' log~ y..:\_11 logs *u.*

Differentiating with respect to y,

1 1 *du dy du*

*--·-=n.-··-* or -·=n--"

*y u dy y u*

1 . 1. . . f . *8y* ±. 8u Hence, the re at1ve 1m1tmg error o *y* is - = · *n* - *y u*

(viii) Composite F'a.ctors

lcl y=~.~

loga *y=n* loge *u+m* loge *v*

l *n du m dv* --·-=-. -+---

*y u dy v dy* or

or *\_dy =-n qu +m dv*

*y u v*

. . . . f . 8y ±. ( *Su* + . *Sv)* Relative hrmtmg error o *y* is - = - *n* - *m* - *y U· V*

Example 3'4. Three resistors have the following ratings :

Ri=37 12±5%, R2=75 0±5%, Ra==50±5%.

Determine the magnitude and limiting error in ohm and in percent of the resistance resistances connected in series. ·

Solution : The values of resistances are :

R1=~7± x37=37±1'85\0

Rz=75± x75=75±3'75 n

Ra=SO:-l:r~ xso=50:l:2'5 n

The limiting value of resultant resistanc.e.

R=(J1+1s+so)±(l·ss+3·1s+2·S)=t62±s·1 ,l,

*:* . Magnitude of resistance= 162 0

Error in ohm =±8'1 0

. Percent limiting error =:I::~~~ X 100=±5%.

... (3'15) ... (3'16)

... (3'17) of these

· Example 3' 5. The resistance of a circuit is found by measuring current ff owing and the power fed into the circuit. Find the limiting error in the measurement of resistance wllen the limiting errors '1  in the measurement of power aud current are respectively ±1'5% and ±l'Q%.

. 4o .. . ELECTlUCAl ·MBASURBMENIS AND MBASURING IN 1TRUMliNTS . .. . ' ' . . .• . . .  · ·solution l Resistance.· *R* . ((poweia= *P12 =Pi-2 •* . . . · . · . curren ·. ·

Prom Eqfi; · 3'16, telati.ve limiting error in meaure.ment of resistance is

· . 3R: ±(3P.+2· *81* 1 ..

· R"~ · ·p . T J

=±O-s-t2x 1·0)=±3'5%. . ; · .. ·. . .

E:umple "3:5 .. The solution for the unknown resistance for a Wheatstone ~ridge is 1 . ·R~=R2Ra · . ' ..

R,1 ., . ' '

where . . .. . ·Ri=100±0'5% o, Ra. 1.000:±:0·5% n, . Ra~842±0'5% o · ; '·.. .I .. • • : • \·'' • . • ' . ; ' ' . Deternih1e.the rnagnirude of the1 uilkni>wn reSfstance·and the.'Hmiti'ng error in. percent and in ohm fort.he unknown ·resistilnee R~. . · · .- ·. · Solution : Unknowi:i resistance

. ' . ' '

*Ra1=R2Ra* =1Q~0-~42.:\_\_s 420 n

R1 100 ..

Relative limitin·g error of unknown resistance is :. .

8 8 . . .

BR~=± *..:&* + *Ra+* 8R1 )=±(o·s+o·s+o:s)==±l.5%.. , R~ R2 . Rs . Ri .. . ·

(

Limiting error in ohm is : . 1'5 . . =±lOO X8420-± 126'3 !1.

Guaranteed· values of resistance are between.

s420-126·3=s293·1 o, · s420+i.26:3 · 'ss46'3 n

: Example 3·7. A 4-dial decade. box has

decade *a* of lOX 10000±0'1%

decade *b* of 10 X 1000±0'1%

. decade *c* of 10~ 100±0'5%

decade d of 10 x 1n:r1'0%

· It is set at 4619· 0 .. Find the pe!centage li~iting error an~ the range of resistance value ... Sol~tion : D~cade a is s'et at '4000 0

I ..

. 0·1' ... Therefore. err?r . ±4000 x 100 =. ~4 n Decad~ bis set at 600' n.

Therefore~ error== ±600 x ~~ = ±0'6 *Q* Similarly · . '0'5 . . . error in decade *c=* ±30 x 100 = ±0' 15 0

. . . 1 . . ,. error fa decade *d* == ~9 X 100 = ±0"09 O . · Total .error · ==i ·±'(4+0·6fO'l 5t0'09)=4'84 Q

. - . ~ . - . . . .

tRR.ORSI.N tv1J.lASUREMENTS AND JHl:.IR sti\TISTtCAL ANALYSiti.

R, 1 t' 1· · · - E + 4·s4 .+o·oo· 1· o.4. e a ive lQlltmg error . *r* = - 4639 = - .

Percentage Hmfring error % Er= ±co··oot 04XlOO) = ±0' 104% .

Limiting values of resistan~~ A.a.=4639(1±0'00104) .

=4639±5 0~4634 n 4644 n. ·

· ·Thus we conclude from ·th~ above examples that the guarantee vaJues are obtained by taking a direct sum of the po~sible errors, adopting the algebraic signs that give. the worst possible case. In fact setting of guarantee limits is necessarily a pessimistiC process. ·This is t~ue from manufacturer's viewpoint as regards his promise to the buye.r, . and it· is also true of the·. user in setting accuracy limits ill results of measurement. · ·

. 3·4 Known Errors. When the error of a quantity or an instrument is knowri\the effect or this . err~r, .~hen combined with oq1er error~, can ?e computed in a manner si~i!ar -to -th\ combinations .of hm1tmg errors. But the difference is that m case of koown errors the signs of relative errors are

given and must be preserved in the calculations. . \ Example 3·s. A resistance is fated at 3200 u and the current flowing thro.ugh thi~;Js 64 mA •. • *(* q), Compute the power foss in the resistor. (b) It wa:s later found that the resistance· of the resistor was 0'2 percent greater than the specified resistance.and the ammeter read 0·75 percent more than the true current. Determine the known error in the computed power in part (a).

Soluti~n : *(a}* Power consumed *P=I2R=(64x* l0\_;3)2x3200=13'1 w. ·

*(b).* Relative error in power

*8P* (28/ . *8R)* , · . r . *-1 +-R* =(2 XO 75t0'2)=1'7% more.

Example 3·9, Current was measured during .a test .as 30'4 A, flowing· in a·iesistor of . *0·105* n. It was discovered later that the ammeter reading was low by 1 '2 percent and the marked . ·resistance was high by 0'3.percent. Find the true power as a perc~ntage of the power that· was  originally calculated. · · Solution : True value of /=30'4(1-0'012)=30'035 A

True value of *R=0'* 105(1 t0'0003):::;:0'1053 0

True power ==/2R-=(30'035)2X(0'10S3)=95 W

Originally measured power =(30'4)2 x 0'105= 97'04 W

True power = *\_2i\_* . \_ . . . 11 d x 100 97 04 X 100-97 9 percent Ongma y measnre powei:  ·~ "

We arrive at the same results by using the following wethod : · . '1•

Power. *P= 12 R* .

Totalrelativeerror=8: . 1 ~=2X(-0'012)+o·oo3=-0'021

• . . True power -='t-0·021=0'979=97'9'%.

• • Ongmally measured power . ·.,

Example3'10. - Three 250 0, a 500 Q and a 375 ~resistors are c~nnected ~n parallel, The 250 n resistor has a +0·025 fractional error, the, 500 .a r~s1stor has a -0 0;36 fractional .error, and the 375 n resistor has a +0·014 fractional error. Determme (a) the total res1&tance neglectmg errors, (b) total resistance considering the error of each. resistor and (c) the fractional error of the total resistance based upon rated values. . ,



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. Soh~tkm :

(a) Total rn:iistance of resistors connected in parallel and neglecting their errors is :

*R-·* 1/R1+1ka+ *11R2* -1;~cf+Tisoo+ I/375-- =t 15·4 0 ·

(b). The fractional error in *Ri* =250 n is +0·025

8R1=(0 025 ><250)= +6'25 .Q

Hence Ri =25Q,\_15·z5 .. .,, 256'25 11

Similarly

aR2=(-0'036x500)=-18 n

R:a=S00-18=482 .Q

*8Rs=(* +0·014 x 375)=5'25 n

Rs=375+s·25=380'25 n.

Therefore the resultant resistance of three resistances in parallel

1 1

*R=* -·-l/R-1+\_1\_/R-2+I/R3 -·· l/256'25-t··i/4sfF1/38o'i5 =llG'J *a.*

(c) The fractional error of the parallel resistance based on the rated values is ;

~R ll6'3--115'4 *·R* - 115.4 +o·oo776=+o·n6~~ . . '

3'5 1 Types of Ert"iH'S

No measurement can be made with perfect accuracy but it is important to find out what accuracy actually is and how different errors have .entered into the measurement. A study of errors is il' first step in finding W'!YS *to* reduce them. Errors may arise from· different sources· and are usually classified as under : ·

1. Gross Errors. 2. Systematic Errors. 3. Random Errors,

.3'6 Gross Errors. This class of errors mainly covers human mistakes in reading instruments and recording and calculating measurement results. The responsibility of the mistake normally lies with the experimenter. The experimenter, may grosdy misread the scale. For example he. m·ay, due to an oversight, read the temperature as 31'5°C while the actual reading may be 21 TC. He may transpose the reading while recording. For example he may read 25'8°C and record 23·5·c. But as long as

t

human beings are involved, some gross errors will definitely be committed. Although complete dirni~ nation of gross errors is probably impossible, one should try to anticipate and correct them. Some gross errors arc.easily detected while others may be very difficult to, detect.

Gross errqrs may be of any amount and therefore their mathematical analysis\_ is impossible. However, they can be avoided by adopting two means. They are· :

1. Great care should be taken in reading and recording the data.

2. Two, tln·~i~ or even more readings should be taken for the quantity under measurcimen.~. These readings should be taken preferably by . different experimenters and the teadings should be taken at a diffe.ren~ reading point to avoid re-reading with the same error. It should he understood mhat no re.Hance be placed on a single reading. It is always advisable to take a large number of read· ings as a close agreement between readings assures that no gross error has been com.mitte d,

*3'1* Systieum~k Enm·1~

These types of c1Tors are divided into three categories :

I. Instrumental Errors. 2. Environmental Errors. · 3. Observational Errors.

b *r* I

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ERRORS IN MEASUREMBN'fS AND THEIR STATISTICAL ANALYSIS

3'7'1. Instrumental Enors

fhese errors arise due to three main reasons :

(i) Due to inherent shortcomings in the instrument)

(ii) Due to misuse of the instruments,

and (iii) Due to loading effects of instrµments.

1. Inherent Shortcomings of Instruments. These errors are inherent in instruments because · of their mechanical structure. They may be due to construction, calibration or operation of tile instruments or measuring devices. These errors may cause the instrument to read too low or too high. For example, if the spring (used for producing controlling torque) of a permanent magnet instrument has become weak, the instrument will always read high. .

While making precision measurements, we must recognize the possibility of such errors as it is often possible to eliminate them, or at least reduce them to a great extent , by using the following methods:

(i) The procedure of measurement must be carefoJly planned. Substitution methods or cali bration against standards may be used for the purpose. '(ii) Correction factors should be applied after determining the instrumental errors. (iii) The instrument may be re-calibrated carefully.

2. Misuse of !nshuments. There is an old saying that instruments are better than the peop1e who use them. Too often, the errors caused in measurements are due to the fault of the operator than that of the instrument. A good instrument used in an unintelligent way, may give erroneous results. · Examples which may be cited for this misuse of instrument may be failure to adjust the zero of instruments, poor initial adjustments, using leads of too high a resistance and so on.

No doubt the above malpractices may not cause a permanent damage to the instrument but . all the same they cause errors. However, there are certain ill pra~tices, which in addition to produc ing errors cause permanent damage to the instruments as a result of overloading and overheating which may ultimately result in failure of the instrument and sometimes the system itself.

. 3. Loading eft'ects. One of the most common errors cqmmitted by begfoners, is the imp~·o'~ per use of an instrument for measurement work. For example, a wen calibrated voltmeter may give a misleading voltage reading when counected across a high resistance circuit (See Ex.ample 3·11 *).* The . same voltmeter~ when connected in a low resistance circuit, may give a more dependable reading .(See Example 3'12). These examples illustrate that the voltmeter has a loading effect on the circuit, aHerM ing the actual circuit conditions by the measurement process.

Example 3'11. A voltmeter having a sensitivity of 1000 D./V reads 100 Von its 150 V scale when connected across an unknown resistor in' series with a milliammeter.

When the milH-ammeter reads s· mA, calculate

(a) apparent resistance of the unknown resistor,

(b) actua1 resistance of the unknown resistor,

(c) error d·ue to the loading effect of voltmeter.

Solution *:* Total circuit resistance

Eir 100 .

RT~= 1*T* =~~>no==a "'<wx 103 n=20 k!l

Neglecting the resistanc\_e of milli-ammeter, the value of unknown resfotm fa :

*Rw=20* k!l.

(b) Resistance of voltmeter *Rv=* 1000 X 150= 150 x 1 oa n = *150* kf>..

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'

I

*Rv+Roo'*

I

I

k · . . · R *RTRv* 20 X 150 23 .. 077 ki1

t!I

or un nown res1stq.nce *oo=.Rv-R.-:;* = 150\_ 2p = •

. (c) Percenta$e errol1 erroneous quantity~.tr~e quantity x 100 · true quantity .

20~~3·077x100= 13'33% . 23 077 . 0

Example 3'12. Repeat example 3'11 if the milli-amm\_eter reads\ 800 mA and the voltmeter reads 40 V on its 150 V scale.

*ET* 40 .·. Solution: (a) *RT="[-;* =sooxT0-3 =50 n.

(b) . *Rv=* 1000 x:15o n .. 1 150 ki1. • R~ = *RTRv* = 5\_~~XJ50 ~-10 = 50'017 n

*Rv-RT* 150 x 103-50 '·

. 50'0-:-60'017 . *01* (c) Percentage error - 50 .. 017 x 100=-0 034,10 •

Errors caused by loading effects of the meters can be avoided by using them intelligently. For example when measuring a low resistance by ammeter-voltmeter method a high resistance volt meter should be used. ·

: In planning any measurement, the lo~ding. effect of instruments should be considered and corrections for these effects should be made, 1f needed, or more suitable instruments should be used. Preferably tpose methods should be used which result in. negligible or no loading effects.

3·7·2. Environmental Errors

These errors are due. to conditions external to the measuring device including conditions in the area surrounding the instrument. These may be.effects of temperature pressure, humidity, dust, vibra-. tfons or of external magnetic or electrostatic fields. The corrective measures employed to eliminate or to reduce these undesirable effects are : *:*

1. Arrangements are made to keep the conditions as nearly as constant as possible. Por example, temperature can be kept constant by keeping the equipment in ·a temperature controlled enclosure. ·

, , 2. Using equipment which is immune to these effects. For example, variations in resistance 'with temperature can be minimized by using resistance materials which have a vety low resistance temperature co-efficieµt.

3. Employing techniques which eliminate the effects of these disturbances. For example, the effect of humidity dust etc. can be entirely eliminated by hermetically sealing the eq9ipment. · 4. Applying computed corrections: Efforts are normally made to avoid the use:of'application of computed corrections, but where these corrections are . needed and are necessary, they are incorpo·. rated for the computations of the results.

3·7·3, Observational Errors

There are many sources of observational errors. As an example, the pointer of a voltmeter rests slightly above the surface of the scale. Thus an error on account of PARALLAX will be in~ ·

ErtRORS IN MflASUidlM.BNTS AND THEIR STA"HSTICAL ANALYSIS *4S*

curred unless the line of vision of the observer is exactly above the pointer. To minimize parallax errors, highly accurate meters are provided with mirrored scales, as shown in Fig. 3'1.

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Mirror I

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toned ;;os·1tion

f.IO PARALLAX E.RROtk

Fig. 3'1. Errors due to Parallax,

WRONG

When the pointer's image appears hidden by the pointer, observer's eye is directly in line with the pointer. Although a mirrored scale minimizes parallax error, an error is necessarily present though it may be very smalL

Since the parallax errors arise on 11ccount of pointer. and che scale not 

being in the same plane, we can eliminate this error by having the pointer and ~~­ the scale in th~ same plane as shown in Fig. 3·2. ~ I There are human factors involved in measurement. The sensrng capa bilities of individual observers effect the accuracy of measurement PolnM  No two persons observe the same situation in . exactly the same way

where small details are concerned, For exampfo .. there ~re observational enors in measurements involving timing of an event. One observer may t·end to anti- Fig. 3·2. Arrangementr cipate the signal and read too soon. Different experimenters ma:y produce showing scale and pointe different results, especially when sound and light measurements are involved in the same plane. since no two observers possess the same capabilities.

Modern electrfoal instruments have digital display of output which completely eliminates the errors on account of human observational or e.emsing powers as the output \_is in the form of digits.

3'8. R.udom (Renidmd) Errors

It has been consistently found that experimental results show variafo.m from one reading to another, even after all systematic errors have been accounted for. These error'; are due to a multitude of small factors which change or fluctuate from otw measurement to anotb:;-~· and are due surely h· chance. The quantity being measured is affected by many happ1mings tbroug1:out the uuiverse. W.- are aware of and account for some of the factors influencing the measuremen~, bat about the res\ w, are unaware. The happenings or disturbance;; about which we are unaw~re are lumped \Gi~~1hc• . :

called *"Random"* or *"Residual0 •* Hence the errors caused by thes~ hapn~n:ng::. ·'.:: \.,,'>.~ *\_:* J>:~~.\on. (or Residual) Errors. Since these eriors remain even after the systematic errors l.:.:<:. l\_,;::e~· . */.* 1.'!l~.: of, we call these errors as Residmd (Random) Fnors.

3·9, Central Value; As stated above, t1-rn random error<; Ge Gaused *by* :1 :aqi.e *;* . 1'...1bcr (it small effects, each one being a variable. These varl.'tbks may be adcl:uive in iiOni,:· .: , )es :i •. J ~t: litral'· tive in some cases in their effect on the quantity being measured. lri many lliC:i:'.~1·H~:.m:,)1·: rI1r pnq1-

tive and negative effects are nearly equaJ, so that the resdbx~t ~~rror is sma n : ,·we in~ 1· •.: <>. lM~c number of measurements and the p1us effects are equal *to* tk m:rrns effects, : · : ·· .' .,:i1!d .:.ttcrl c.1cli other and we would obfain the scatter round a Central V:::lr,:\_ ~::oce this contj:c.. · fh--. ~.1• ', ;~ • ..: in practice, we are justifi"d in usin~ this concept as a basis o~· ~,\_., s:udy of errcis Y1 :.\-,.f\ -+"~ ,;;)kOL'W

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to us. Thus mathematical laws or "Probability'' can be applied for the study of randollf errors. There is no other way as the random errors are unknown and only statistical study c..rn lead· us to the best approximation of the true value of the quantity under study.

3·10. Statistical Treatment of Data

The experimental data is obtained ic. two forms of tests :

(i) Multisample test) and (ii) Single-sample test.

Multisamp1e Test. In this test, repeated measurement of a given quantity are done using different test conditions such as employing different instrumentsi different ways of measurement and by employing different observers. Simply making measurements with the same equipment, procedure technique and same observer do not provide multisample results. '

Single Sample Test. A single measurement (or succession of measurements) done under identical conditions excepting for time is known as single-sample test.

In order to get the exact value of the quantity under measurement, tests should be done using as many different procedures, techniques and experimenters as practicable. It should be borne in mind that the statistical means which help us to arrive at correct res11lts are only valid for multi· sample tests.

3' 10' t. Histogram.

When a number of multisample observations are taken experimentally there is a scatter of the data about ~ome central value. One method presenting test results in the form of a Histogram. The technique is ilJustrated in Pig. 3·3 representing th~ data given in Table 3' I. This table shows a set of fifty re~dings of a length measurement. The most probable or central value of length is 100 cm and the data are taken and recorded to the nearest o· 1 cm.

Table 3'1

-I *Length cm. Number of readings*

i *99'1* 1 I . 99'8 4

99·9 12 I 100·0 19

100'1 10

100·2 3

100·3 1

Total number of readings= SO.

This histogram of Fig. 3·3 represents these data where the ordinate indicate the number of observed readings (frequency of occurreni.;e) of a particular value. A histogram is also called a frequency distribution curve. At t be central value of 100 cm is a large number of readings, J 9 in this case, with other values placed '2

almost symmetrically on either side. If smaller incremental steps, ~ ~

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I l

I :

I I • I I

I

I

say 100 readings at o·os cm intervals are taken, the general form ~~ of the histogram will be almost the same but since the steps have 0 t smaller increments and we get a smoother curve. c

~ With more and more data taken at smaller and smaller increments the histogram would finally change into :a smooth curve, as indic~ted by the dashed line in Fig. 3'3. ·

I

1 /

I

I

*t./*

I

I . I

•10

The smooth curve is symmetrical with respect tp the central value. Many physical cases have been found whic4 give experimental data agreeing fairly well with the smooth symmetriw cal curve. · ·

lQnqth

Fig. 3' 3. Histogram.

ERRORS IN MEASURtlMBNTS AND THEIR STAfrSfICAL ANALY:m

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3·10·2. Arithmetic MeH

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The most probable value of measured variable (variate) is the arithmetic mean of the number of readings takea. The best appcoximation is made when the number of readings of the same quan- ~ity are yery Jarge. ~heoretic&lly, an infinite number of readings would give the best result, although m practice, only a fimte number of measurements can be made.

The arithmetic mean is given by :

.Y *x1+x2+xa+·x4+ ......* x11 Zx · *n n* , .. (3'17)

where *X* . arithmetic mean,

x1, x2 ... Xn=readings or variates or samples,

and n=number of readings.

3·10·3, Measure of Dispersion from the Me~n

Dispersion. The property which denotes the extenc to which ~he values are dispersed about the central value is termed as dispersion. The other names used for dispersion are §l~re~d ll}K'Scatter. Fig. 3 ·4 shows two sets uf data. ln one *v*

case (curve 1) the values vary from x1 to x2 and t .Curve l-Great~r prnc1s1bn

in other case (curve 2) the values vary from *x3*

to x4, though their central value is the same.

Clearly stt of data ,represented by curve l has a

smaller dispersion than that of the data repre~

sented by curve 2.

It is very important to have a measure

*<if* the dispersion from the central value as it is

an indication of the degree of consi~tency (pre.. /"' cision) and regularity of the data. / *;/* A large dispersion indicates that some -~~ ·+ h ... 1<1 factors involved in *t* e measurement process are

Curve 2-·low~r prncisfon

'n' for curve 1 Is greater

r. thon tor curve 1 ,.. vUrve l 

...,, .. '

·-~~-~""" ,/Curve 2 , l \ ~,

L-~. l<;i X1,

not under close control and therefore it becomes ....\_.,.,.)( difficult to estimate the measured quanthy with Fig. 3'4. Curves showing different ranges and confidence and definiteness. For example, if we precision indices. ( compare two sets of data and find that one set has less dispersion than the other set, *we* can .cer-· tainly pla~e more reliance on it and can definitely regard it as a superior set as regards random errors.

There are certain terms which must be defined as they form the basis of defining the measure of dispersion of data.

3'10'4, Range. The simplest possible measure of dispersion is the riunge which is the difference between greatest and least values of data. For example in Fig. 3 ·4 the range of curve 1 is (x2- x1) and that of curve 2 is (X4 -xaL.

3·1o·s. Deviation. Deviation is departure of the observed reading from the arithmetic mean of the group of readings. Let the deviation of reading x1 be d1 and that of reading x2 be *d2,* etc.

Then ancf ·

*ch==x1-X*

*d2=x2-}(*

•I• **IQl,tlflltl•**

*dn=Xn-X*

:v 'Z(xn- dn) *A=* -- - -~--· ------ *n* .

... (3'18) ... (3'19)

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""~(xd-x2+xa ... ·+-x,}--n,,Y ""'O as x1 +x:r+·xa+ ... *+xn=n* X.

Therefore the algebraic, sum of deviations is zero.

*,.*

3·10·6. Average Deviation. The average deviation is an indication of the precision of the instruments used in making the measurements. Highly precise instruments yield a low average deviation between readings. . Average deviation is defined as the sum of the absolute values of deviations divided by the number of readings. The absolute value of deviation is the value without respect to its sign. Average deviation may be expressed as :

*D=* I *di* I + I *d2* I + 1 *ds* I + ... + I *dn* I = ~\_L!l\_\_L\_

*n n* ... (3'20)

3·10·1. Standard Deviation (S.D.). Another important term in the analysis of random errors is the stand~rd deviation or the root mean square deviation. The Standard Deviation of ·an infinite number of data is defined as the square root of the sum of the individual deviations squared, divided by the number of readings. ·

Thus standard deviation is :

... (3'21)

In practice, however, the number of observatton' is finite. When the number of observations is greater than 20, S.D, is denoted by symbol cr while if it is less than 20, the symbol used is *s.* The Standard Deviation of a finite number of data is given by ;

s=Jcl12+422~.:~i·-t\_:\_::"ll\_J;1 -=~ J :~r- ... (3'22)

3'10'8. Variance. The variance is the mean square deviation, which is the same as S.D., except that square root is not extracted.

Variance *V* =(Standard Deviation)2

=(S.D.)2:::a2= d12+d22+da2+ ... *+dn2* .I ... (3'23)

*n*

*'2d2* =--*n*

But wLen the number of observations is Jess than 20 d2

Variance *V* =~ --- n--1

... (3'24) ... (3'25)

Example 3'13. A set of independent current 1ne:.isurements were taken by six observers and were recorded as 12 8 A, 12'2 A, 12'5 A, 13 1 A. 12'9 A, and 12·4 A. Calculate (a) the arithmetic mean, (b) the d\_eviations from the me~n, (c) the avcni.ge devi.ation, (d) the· standard deviation, and (e) variance.

Solution. ·(a) From Eqn. 3·17 the a;:':fhme\ic· mean. is

}l.= }:x =l~J-f:]2'2+12'5-!J}Jf:J.?'9+12'4 \_ 12.65 A.

n 6 .

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(b) Fro~ Eqn. 3'18, the.deviations are: 

*di* =x1-X=12·s-12·6s=+o·1s A

d2=x2-X=l2'2-12'65=-0'45 A

da=xa-X=12'5-12'65=-0'15 A

d4=X4-X=lll--12'65 . +0·45 A

d5=x5-X=t2'9·-12'6S=+o·2s A

d6=X6-X=l2'4-·12'65=--0'25 A.

(c) From Eqn. 3'20, average deviation:

D= ~-1 41 = 0·1s+0'4S+o·1sto·4s+o·2s+o·2s =0.283 A.

n .

(Note that for average deviation we have not to consider the signs).

(d) Since we have observations whose number is less than 20 therefor" we use Eqn. 3'25 for determining the value of standard deviation

*s= J Id*2 *=J* (0' 15) +:F·~0.4S) -H-0'15)2+W·4s) +(0'25)2+(-0'25)2

..,,,

n-1 6-1

I

=0'399 A.

(e) Variance V=s2=(0'339)2~-0·11s A2.

3'U)'9. Normal or Gaussian Curve of Errors. The Normal or Gaussian law of errors is the basis for the major part of study of random effects. This type of distribution is most frequently met in practice. .

The law of probability states the normal occurrence of deviations from avc(age value of an infinite number· of measurements or observations can be expressed by : ·

*h -h2x2*

Y=~ *e* .•• (3'26)

I Note : The student here is cautioned. not to confuse *x* with I

I magnitude of a quantity. Here *x* meuns Deviation. *{*

where x=magnitude of deviation,

y=number of readings at any deviation *x,*

(the probability of occurrence of devia·

tion *x)t*

and

h=a constant called pl'ecision index. Eqn. 3 '28 leads to curve of type ·shown in

Fig. 3'5 and this curve showing *y* plotted against xis 

called "Normal or Gaussian Prob~bility Curve,,,

This curve is symmetrical about the arithmetic

mean value, and area under the curve is unity. Under

the conditions specified bere the total number of

readings taken is represented by L This can be

explained as foll~~s : Suppcise for the time befog Hiat w \_\_ ,... we consider h/\/ ~ to be unknown and reph1;;;f.; ~~: b? 3'5, Normal r'rohat\il!ty {'.urve. the symbol A. If we have a large number of t~~:;1(~H'fsS; · *n,* the probable number *6.ni* with deviation betw.1~-;;n *x* and x+ *{.xis* given by :

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A A • ·~- *t,2x2* 1\_in=nyc:,.x=nA e '• .f:.,,x.

If we ioltcg~~t~; the above expression for ~oo to + ~ we shall have all the cases, or

00



·-00

Thus:

r ~J;2x2 A! *e dx=l* 

00

••• (3'27)

"

-oo

Thus the i.u.tegrnJ of *y* from .-oo to +oo is equal to unity. The fracuon of the total number of readings oc-Gvin:ing betw~©tn the values .X'l and x2 will be equal to the area under the curve between these values o.f *x.*

*x2*

*h* r --h'Ax9

r~1-3= vrr. J *e dx* m(3'28)

*Xi*

when:

m.-2"'--=munber of readi11gs occurring between the values xi and *x2•*

If the ueb'. ooiwoon x1 and X?J is o· *5,* then 50 per cent of the deviations fall between x1 and x2• fo general ·~iw p;mfr.lillbmty for iinding a deviation in an interval between *xi* and x:a becomes *x2* xs

' *h* f *-h?.xJt* ~{ .,1 <.•o' •'"1 *y* ,-f.,,,""' *--;::::::-* ,:P *d'11'* \.A. 11 .,~';("/ M.4 .... , I '6; ,/~  

"' *'\t* i1: '

*Xl* .:\.1

The fraction of the total number of deviations falling between zero a.nrl *x* are : '

••• (3·Jo)

3'10'10. Pred:ili~rm 1ltl0J\\,

From Eqn, 3'26 when *x=O,* we have *h*

*y=-=-*

v~

••• (3'31)

Thus it fa tkar from above that the maximum value of *y* depends upon *h.* The larger the iiidue of *h,* the slmfl'.!~;ir the curve. Thus the value or *h* determines the sharpness of the curve since. the ·t.:iirve drops .sh.arply ow~ng tv ~he term (-h2) being in the exp.one~t.. A sharp curve evidently indicates ~hat the dev1atwllls am mon~ r;fosely grouped together around devutt10n *x=O.*

Fig. 3·4 shows two cmvcs having difforent vaJw;s of h. Curve I bas a large value of h while Curve 2 bas a smaUe:r. valu~ of h.

*1t* is dear that the probability that n variate *ues* m a tp.ven rang0 becomes less as the deviation of the range becomesi greater. For a given deviation *x,* the probabili~y *is* less greater the h and vice versa. Thus the name llrecisfon inde!' for *h* is reason;ibfo. A large. value of *h* represents high precisiOJ\. of the data because the probability of occurrence of variates in a given .range falls off rapidly as the deviation increMes because the variates tend to cluster (bocome closer) into a n,arrow range. On the other hand, a small value of *h* represents low precision because t~e · probability of occurrence,s of variates in a given range falls off gradually as the deviation increases ; this is because the variates are spread oyer a wide range.

El.tRORS IN MBASlJRBMBNfS AND IHBIR STATISTICAL ANALYSIS. 51

It is evident therefore that Curve 1 of Fig. 3 ·4 represents a data of greater precision than that of Curve 2 since the value of *h* for Curve 1 is greater than that of Curve 2. we have

*h=·* 1

y20" ... (3'32).

3·10·u. Probable Error: We .have observed above that the most probable or best value of a Gaussian distribution is obtained by taking arithmetic mean of the various values of the variate. In addition iithas been indicated that the confidence in this best value (most probable value) is connected with the sharpness of the distribution curve.

~Let us consider the two points -rand *+r* marked in Fig 3'5. These points are so located that the area bounded by the curve, the *x* axis1 and the ordinates erected at *x=* -rand *x* ~+ *r* is

equal to half of the total area under the curve. That is half the deviations lie between *x* Dll *±r .* ..

A convenient measure of precision is the quantity *r.* It is called Probable Error or simply P.E. The reason for this name is the fact mentioned above that half the observed values lie between the limits *±r.* If we determine *r* as the result of *n* measurements and then make an additional measure~ ment, the chances are 50-50 'percent that .the new value will lie between-rand tr. That is, the cbances are even that any one reading will have an error no greater than *±r.*

The location of point r can be found from Eqn. 3'28, by putting



This gives 0'4764 r= ------ *h* ... (3'33) 3·10·12. ·Average Deviation for the Normal Curve

The average deviation may be computed when more than one reading is present at a given deviation by' multiplying the amount of the deviation by the number of points on the deviation. Then this product is added to other similar products (without regard to sign) until all readings are taken into account ; then divide by the number of readings.

In the case of normal curve,

tcx:i

Average deviation *D* = J I x I y dx

- ex:>

+oo

= -~'l\_J *e -h2x2x dx vrc* = ~~j] ... (3 '34)

0

From Eqn. 3'33, *h=* .9.'4764 . Putting this value in Eqn. 3'35, we have, average deviation : *r*

- r

D=0-8453 ... (3'3S)

3'10'13. Standard Deviation for the Normal Curve

The standard deviation is given by *"'d2* •) ""' CJ"=-*n* (See Eqn. *3'1* l'i

S2 

Follo'wing a method siroil~r to that foll.owed above1 we have : +oo

*2h* r -h~~x2 . 1

cr= \7 n:-.: j *e x2d.x* .-~ -- ~

0

or standard deviation for normal curve

1

*a=-;--- V* ~h

*r*

*=0·614s*

From Eqns. 3·35 and 3'37, we have P.E.=r:=0'8453 D . =0'6745 tT

3'10'14. ProbaJ,fo Error of~ Finite ~fo:It11bfr of.Reftdmgs

... (336)

•.. (3'37)

... (3'38) ... (3'39)

In the analysis of the normal Gaussian error curve we have assumed that an infinite number of • readings were taken. All the formulae derived above are based upon this assumption. With a. finite number of readings, there is a slight difference between the computed values given above and the values obtained with a finite set of readings. For example~ substitution of Eqn. 3'.21 into Eqn. 3·39 gives the probable error as

... (3'40) for an infinite number of deviations forming the normal probability curve, where *n* is infinite. But for a finite number of dc:viations, the probable error for one reading is :

rl=0'6745 II di2+a22+{laz+ ... ..:~:\_+d~2 ·. "=0'6745 /1I I d I a

" n--1 *N* n-1 ... (J'41)

Thiis in fact means that, Zor a computed probable error r obtained from n readintR. one more reading would have an even chance of being above or below r1.

With a finite number of readings, the average reading has a probable error of: 

*rm= A* I l\_ fl =0'6745 J-d12fd2~:~!2\_+ .. :+d11~ *v* 1t ' nvi-1)

=0'6745 J~·l: I 'L! -~

*,t* n(n·-U

The above equation 1neans that for n finite readings, the probable error fa *rm.* If we have n>l then n~-l~n



and (I

rm=0°6745 -::1=-

v fl

... (3'42)

... (3'43) ... (3'44)

3'10' 15. Studard Devimtioe ·~if t\1e:m. When we have a multiple sample data, it is evident that the mean of various *sets* of data can be analvzed. by statistical means. This is done by taking standard deviation of the mean.

*(i*

The standard deviation of the wean fa gxven by crm = -~;~- .,,(3'45)

ERRORS IN M.f!ASUIU!MBNTS AND Timm. STATISTiCAL ANALYSIS 53

3'10'16.· Sforadard Deviation of Standard Deviation

Fol' a multiple sample data, the standard deviation of the standard deviation is : *(J* ... (3'46) <ra= *V2n !  *... (3'47)

E:iuimple 3'14. The following 10 observations were recorded when measuring a voltage: 41 '7, 42'0, 41'8, 42'0, 42'1, 41 '9*9* 42'0, 41'9, 42'5 and 41 '8 volt. Find (I) the mean (ii) the standard deviation (iii) the probable error of one reading (iv) the probable error of mean and (v) range.

Sobdi®n : For the sake of ease in caJcuJations, the observations are tabulated and manipulated as under:

*x a*

---------------

41'7 42'0 41'8 42'0 42'1 41'9 42'0 41'9 42'5 41'8

-0·27 +0·03 -0'1'1 +0'03

+0'13 -o 07 +0·03 -0'07 +0·53 -0'17

0'0729 0'0009 0'0289 0·0009 0'0169 0'0049 0'0049 0·0049 0'2809 0'0289

I:xc:::419'7

~~I~~ :Ed~=0'44! I

~--------~~~~~-------·---~-~~

· u };x 41 '97 . (i) Mean length X= n-= 1o=4197 volt.

. ' d d d . . . Jd2 y0'441 *(ii)* The value of stan ar evtatton 18 cr=. - = --=0'21 volt · . A *l'l* 10 (See Eqn 3'21)

if the data is considered to be a set of infinite readings. However, the number of observations is nnRy 1 O and therefore the standm:d deviation is :

~ */(12-* ... *J* 0·441 . *s=* 'V n~ 1 = 'V (lO-l) =0 22 volt

(iii) Probable error r1=0'6745 s=0'15 volt.

(ill) Probable error of mean *rm=. 1* r1 ·=:-?·- ~=::(/'(15 volt. *'vn-1 v.9* (v) Range'--=42'5-41 '7 w.0'8 volt,

(See Eqn, 3'25) (See Eqn. l '41) (See Eqn. 3'42)

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Ex2mple 3'15. In a test temperature is measured ~00 times with variations in apparatus and procedures. After applying the corrections, the results are :

----------------~----,------------- ..

402 *403 404 405*

16 4 2 2

Calculate (a) arithmetic mean, (b) mean deviation, (c) standard deviation, (d) the probable error of one reading, (e) the standard deviation and the probable error of the mean, (/) the standard deviation of the standard deviation.

Sohltioun : The computations are done in a tabular form as under :

~--------------- ----·- ----- - - ·-------------

•!

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*Temperature Frequency of TXf Deviation fxd* a2 *r·c occurrence,* f d

---- - ··---~·-- -·---------- - ----- , \_\_ .  397 397 -3'78 -3\,'78 14'288 398 3 1194 -2'78 -s·3·4 7'728  399 12 4788 -1'78 -21'36 3' 168 400 23 9200 -0"78 + 17'94 0'608 401 37 14837 +0·22 +8·14 0'048 402 16 6432 '+ 1'22 + 19'52 1'488 403 4 l () ! 2 +z·22 f8'88 4'928 404 2 808 +3'22 +6'44 10'368 405 2 810 +4'22. +s·44 17'808

I

*fxd2*

'

l 4'288

23'185

38'020

13'99.~

1'708

2~'814

19 '714

20·737

35'618

I ---- ---------·---- -- -------- -- - --- -------- - ---~--~--- I

Total 100 I 40078 }; I *fxd* I

---- -- -----------'--- -- -\_\_\_\_\_\_\_ I

I =102'8

(a) Mean temperature= i6r}~ =400'780°C

(b) Mean deviation *15=* ~~'?\_= 1 ·02s·c

( ) d d d . . .. I 191 ·os :' 1.38-·-0-o·c

*c* Stan ar ev1at10n cr = 'V -100= .

,

(d) Probable error of one reading r1=0'6745 cr=~0'67,15X1'38=0'93°C

-------- - *--.·i*

*Ifd2=*

191 '08

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BJUWU IN MEASUIUJMBNTS AND THBlR STATi&'T!CAL ANALYSIS *55* " 0'93

(e) Probable error of the mean *tm=-.;;* £oo ~p·o93Qc

. \

'•,)

. (;:~) \:')\~ 

Standard deviation of the mean am=.\/ 100~=0'138°~

(f) Standard deviation of the standard deviation

0'111 0'138 *ua=vI-=v* 2 =0·0196°c.

3'11. S~ilylng ODDS

The probability of occurrence can be stated in terms of ODDS. Odds is the . number of chanees that a particular reading will occur when the error limit is specified. For example~ if the errnr limits are specified as ±0'6745 rJ• the chances are that *50%* of the observations will lie between the above limits or in other words we can say that odds are 1 to 1.

The odds can be calculated as under

odds b b'l't r odds+ 1 =pro a 1 t yo occurrence

The odds that th.e observation lies between ± cr limits are : odds . ----=06828 odds+l or odds are 2'15: 1. Table 3'3 gives the deviationst the probability of occunenc~ and the odds.

... (3'48)

TABLE 3·3

-

*Deviation Probability Odds*

*±0'6145a* 0'5000 i to J *±a* 0'6828 2·1s to l ± *2a* 0'9546 21 *to* 1 ± *3a* 0·~974 *256to1*

Specifying Measuremest Data

3·12.

',After doing the itatistfcal analysis of the muJtisampie dat:a, we muse specify the results The

resuHs are expressed as deviations about.a mean value. The deviations ru-e expressed as: • (i) *Standard deviation* : The result is expres~ed as *Jl* ±('.J

The error limit in this case is the standard deviation. This means that 0'6828 (about 68%) of the readings are within.the limits a=±l and the odds are 2'15 to 1. Thus there is approximately a 2 to l possibility that a hew observation will 'fall beyond this Jimit..,

(ii) *Probable error:* The result is CXPJ.'OS~d as J:'±0'6745 *a.*

This means that *SO%* of the readings lie within this limit and the od4s are 1 to I. This means that i&ere is ttn even possibility that ll now readins wiU lie within those limi~. ·

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(iii) *:i2a limits:* In case we went to increase our prnbability range we specify the results as :

Thus we assume tha11 0'9546 (or about 95%) of readings fall within these limit;. These odds Ii±~. *1.* 1n this case are 21 to 1. , *(iv) :±3a limits* : The results in this case are expressed as : i ,Y ±3cr,

The maximum or boundary error limit is ±3cr. The probability in this case is 0·9974, This means that 99·74 ~,~ of the observations will fall within this limit. In other words we can say that there is a p.ossibility of only 26 readings out of 1000 to fall beyond these limits. Thus practically a11 the observations are included in this limit. The odds of any observation falling out of this limit are 256 to 1. 

3'l3. Vmrlr.,itl'lC© l'gl1£iffi St~rill{ifaud Deviations of Combination of Components

Suppose Xfa n fuctlon of ~everal component variables, reach of which is subject to random effects, thus we llave :

*X -o.-=f(x1, x2,* :~3, *... ,xn).*

Now if *x1,* x2, ... , *xn* are independent variables, then for. small variations in *xi, x2, ... ,* Xn from their mean value, denoted by 3x1, 3x2, ... , *3xn,* tht resulting variations of *X* from its mean value for any one determhrntion is given by :

~)f= ~x · ax1+-~X \_\_\_ · *Sx2+...* . .. (3'49) uXl uX2·

(Eqn. 3'49, ignores the higher differentials).

Squaring Eqn, 3'49, we obtain

csx)2=(-~~ )2 (ox1)2+ (-E! )2 (Sx2)2+ ... +2 ( ~! )( %! ) (ax1 ax2)+... ...(lso)

Now, if the variatfons of *xi, x2* etc., are independent, as assumed1 positive values of one increment are equally likely to be associated with positive or negative values of other increments. Thus the sum of the cross product terms tends to be zero for repeated measurements. By definition, variance Vis the mean square error. . Thus the mean of (8X)2 becomes the variance·

of X for repeated measurements. This variarice of Xis denoted by Vil) and hence we can write : Vx=(8X)2

... (3'51) as in repeated meas~rements (8x1)2 tends to become mean value of variance of *xi,* i.e.~ *Vx*1. *:.* Eqn. 3'51 may be written as *Vx= Vx1* + *Vx2+* .. , + *Vxn* ... (3'52) This shows the component variances are addative with weighing factors (8X/8x1)2• The weighted v~riance x1 can be written as ·

Vx1 ::.c: (-~!\_) 3 V;J1 ox1 . ~.

The standard deviation of *X* may be found from Eqn. 3'51. The standard qe.vfation of Xis o-rc and fa equal t:J .



... (3'53) . .. (3'54)

ERRORS iN lillASUREMENTS AND THEIR STA1W1CiCAL ANALYSIS 57 . *-J--(-ai-){---:-·-(·* ax--)2·- 1. ---- ( *~~ax~~* ~~)2~~ . <1x= -- Jix:i.+ ,- *Vxz+ ...* + -"-- Voo•i ox1 .ox2 *oxn* J

... (3 '55) 

I

*11111*

It is clear from Eqn. 3·55 that }both component standard' deviation~; are addative with weighing factors ( *!X* )2 , etc. which express the relative influence of the various components on the *UX1* I

combined function.

Therefore we can write <>x = \/ a::x:12+cr::x:22+ ... +rJx0 2 where a:&:1 is the weighted standard deviation of x1. a:q = *\_o\_* 0':111 ( ~x )2

ox1

. .. (3'56) ... {3' 57)

I

~

.1

It is important to note that the above expressions ·are valid only if component quantities x1, x2, ... etc. are independent of each other and also that the increments are small so that the terms of higher order than the first may be neglected. ActuaHy in engineering applications, the increments are small as it is generally possible to keep the random effects under control,

3'13'1. Probable Error of Combination of Components

Suppose Xis a function of several component variables x1, .\·2, ... xu, each of which is an inde~ pendent variate, ~

Then we have the sf:andard devirition as

*1('* 2 y" )2 ( *BX* )2 I 3 *x* )2 ~ *0 X* = I -i:- CJ:~1 + -;::.,--- u,ez2+·,,., + ( -~~ *arm'!'* N \ L'A) ox2 ' oXn J

But from E~n. 3'39, the probable: error *r=0'6145* a,

or proabable error is .Proportional to standard deviation.

Hence we can write the probable error in X as

... (3'58) where roo1, roo2, etc. are the probable errosion x1, x2 etc. The contribution of probable error of x1 to

the total error in Xis ( *:x:* )2 rx12 and this contribution may be written in another form as rx12. Therefore Eqn. 3'52 becomes *r:v=* y'rx12+rx22+ ... +rxn2 ... (3'59) ( *ax* )2 where the weighted probable error of Xn becomes as rxn = *oxn rx1* ... (3 '60)

Example 3'16. We have a parallel circuit having two branches. The current in one branch is Ji::.-:::100±2 A an<l in the other is /z='.200:::1:5 A. Determine the value of the total current *l=h+I2,*

(a) considering the errors in *h* and *l2* as limiting errors,

and (b) considetfog the errors as st~vidurd cfoviations.

Comment upon the result.

Solutioo : (a) Now l = *h* + *h*

. , . ~I *{* 11 *E!J lo* o/2 \ .'. fractional error m J:=:.:-.. ;c-· 0=::!::! -1:: · ---;.-+·~ ~ · )' 11

*i* , . 11 ~s

58 ELECTRICAL MEASUREMENTS AND MEASURING INSTRUMENTS but ~!!\_- - 2-- . 0·02 and· 012 - ~-=0'025 Ii - 100 *h* - 100

Now I =200+ 100=300 A.

Hence fractional error E\_= ± *(* 1 OO x 0·02+ 20Q\_ x u·o2s ) = ±0'0233 *[* \ 300 300 -

~

Hence I can be written as . /=300(1~1:0'0233)=300±7 A.

Standard deviation of I cr1=~(;;[=) ~crn;-;\_(--~:: r cr122 (See Eqn. 3'55)

r

(b) Now the errors are standard deviations.

= v{2)2~t--(5) = 5'38 A

Sinc1.- *a1* .... ~£.=1 oft *012*

/=300±5'38 A.

Standard deviation in I expressed as a fraction is 5'38/300 =0'01~

It is clear from above calculations that limiting .errors of 2 per cent in h and 2'5 per cent in /2 combine in this case, to give a~ limiting error of 2·3 per cent in their sum r While these very errors, when they are standard deviations, combine to give an error of only 1 ·s per cent.

The use of standard deviation rather than limiting errors gives a more optimistic result. This is reasonable since the probability that both Ii and *12* are [far from their respective means is small.

· Example 317. A resistance is determined by voltmeter ammeter method. The voltmeter reads 100 V with a probable error of ±12 V and ammeter reads 10 A with a probable error of ±2 A. Determine the probable error in the computed value of resistance. ·

Solution : . We have resistance *R* = *i* = *vr1.*

· Weighted probable error in the resistance due to voltage is,

l'Rv=a2; *rv=r1 rv=* '; =± ~~~ =±1'2 n (See Eqn. 3'6P'

Weighted probable error in resistance due to current

-. 0It *V* 100 , (±2) ±2 A

*rm= -fl* rr=-12 rr=-no)2 x . u

From Eqn. 3' 59, probable error in computed resistance is

rR= y(rRv)2+rRI)2= v(l '2)2t(2)2=2'33 Q -

Example 3'18. The law .. of deflection of a galvanometer is l= K6/cos 0, where I is the r.urrcnt; Ka constant and 0 is the deflection. If the angle of deflection 0 is known to be within

±OT (standatd deviation) of 1 s·, what is the percent standard deviation of current, l ? (\_.

Solution : Now *I= KSe =KS* (cos er1

cos

~: = K[(cos or1+e(cqs Bt*2* sin 0]

*59*

ERRORS iN MEASUREMENTS AND THEIR STAIISIICAL ANALYSIS

!

~ K[o.;66+1;0x 15 x (0'916)2x~ss}1'11 *K.*

Standard deviation of /is 0"1= ~f *aO* <=> ±(1 'll *K)* ( o· L< l ~O) rad

Percentage standard deviation of I is

= *crf* x 100= ± ~--·Li.! l K)(O 1 x rt/180) x I 00= -1:0·71 %

I - Kx(n/180)Xl5Xcos 15° - o·

3'14. Uncertainty Analysis and Treatment of Single Sample Data

Many a times the data available is a single sample data and therefore the statistical method.s discussed earlier cannot be applied directly. On account of the single sample nature of the data, it is not possible to observe their scatter by plotting a frequency distribution curve. Hence, it becomes essential to modify our approach.

Kline and McClintock have proposed a method based upon probability and statistics which analyses the data employing uncertainty disttibution rather than frequency distribution. They have defined the uncertainty distribution as the error distribution the experimenter believes would exist if

!Ill

the situation permits multi-sampling.

I

Kline and McClintock suggest that a single sample result may be expressed in terms of a mean value and an uncertainty interval based upon stated odd.,.

The result may be written as follows :

*X=X±w (b* to 1) ... (3'61) where *X* =the value if only one reading is available on the arithmetic mean of several readings.

w-uncertalnty interval.

b=odds or the chance tba'tl the true value lies within the stated range, based upoa the opinion of the experimenter.

The concept of uncertainty may be explained by the following example.

The results of a temperature measurement may be expressed as T=l00°C±l°C

. . T~is me~ns that there is an uncertainty of± l"C in the result. In other words the ~xperime~ter is statmg m precise terms the accuracy of results with which they have heen made accordmg to\_ him. This bring~ about another dimension in measurements and that is, how far the experimenter ts s~re that his measurement falls within the specified limits. Therefore the need for a further specificat1~n arises. As mentioned earlier, Klinf. and McClintock proposed that the experimenter specify certam

odds for the uncertainty. The aforesaid results may be given as

T=1oo·c±1°c (20 to 1)

Now the results expressed in the above form become more specific in nature. This is because the experimeter is willing to bet 20 to 1 odds ttat the temperature measurement which he has made

are within ± i·c of l00°C. ' ·

\his approach is of a particular value in setting up an experimen.t, especially when it involves expensts m terms of man-power, time, and equipment. ·It provides a basis for establishing basis ~01· predetermined estimates of the reliabilily of results through a study of propagation of uncertainties (discussed below in Art. 3' 14' l). In this way evaluation of the test results can be made even before the test is physically done.

3'14'1. Propagation of Uncertainties

The uncertainty analysis in measurements when many variates are involved is done on the same basis as is done for error analysis when the results are expressed as standard deviations or probable errors,

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··t~j

Suppose Xis a .WI1.Gtion of several var1i'tles, *X-f(x1, x2,* X3, ... , *xn)*

where *xi,* x2, xa, ... *xn* are independent variables with the same degree of odds.

Let *w:u* be the resultant uncertainty and Will1, Wi1:2, wills, ... W:o11 be the uncertainties in the independent variables *xi, x2,* xa, ... *Xn* respectively. The uncertainty in the result is given by :

... (3'62)

Example 3'19. A certain resistor has a voltage drop of 110'2 V and a current of 5·3 A. The uncertainties in the measurements are ±0'2 V and **±0'06** A respectively. Calculate the power dissipated in the resistor and the uncertainty in power.

Solution: Power P=voltage x current= *VI=* 110·2 x 5'3=584 W

Now *.P=* VI

~~=1=5'3 and~;= V=l10'2

Wv = ±0'2 and WI= ±0'06.

= y (~~ r wv2t { ~ r wr2 =v'(5'3)2 x (0'2)2+(110'2)2t(0'06):il

**il**

· Uncertainty in power

=±6'7 W=±i7 x 100=±1'15%. 584

Example 3·20. Two resistors Ri and Ra are connected in series and then in parallel. The values of resistances are ;

*Ri* = 100'0±0' l Q~ .R2=50±0'03 Q

Calculate the uncertainty in the combined resistance for both series and parallel arraqgements. Soh'Rtion : When the two resistances are connected in series the resultant resistance is : R=R1-f-R2

()R *o.R*

:. -=1 and·-- ==1 *0R1* . R~

Hence) uncertainty in the total resistance is\_

Wa = ± ... f( oR )2 Wn11+ ( oR )i WR21 'V 8R1 0R1

= ± vo>~ x (o·n2+0>2 x (0·03)2= ±o· 1044 n

The totaJ resistance is R = 1OOt50=150 0

and can be expressed as R-1~J±O'I044 a.

When the two resistances are oonnected in pa1·aJ.lel, the resultant resistance is :

*R=* R1R2 ..., 100 x 50 ,...33.33 .Q

*Ri+R2* 10o+so

Now R=(R1R11)(R1tR2)-1

£~ =(R2)(R1 t .R2)-1- R1R~(R1 + R2)-2

\_ Rs · RiRr. 50 lOOX50 .

-R1+R2 (R1+Ji~im 150 -· (150)2 **=O Hl**

oR Ri *RiR2* \_\_ ,100. 1.00X 50 .

*0R2* = *Ri+Ri* (R1+l~2)~-l50- (150):'.·=0 *444·*

t!RRORS IN MBASU!lEMBN1S AND THEIR STATISTICAL ANALYS•S

Hence uncertainty in totaf resistance is :

V( *8R* )2 ( *()* R )2 WR=±· - WR12+ -- wa2

*0R1 8R2*

=±v (0:111)2 x (0'1)2+(0~444)2 x (J'03)2=0'01734 n.

The total resistance can be written as R=33'33±0'01734 n.

I

Ex1mple 3'21. A plot of land has measured dimensions of 50 by 150 m. The uncertainty in the 50 m dimension is ±0'01 m. Calculate the uncertainty with which the 150 m dimension must be measured to ensure that the total uncertainty in the area is not greater than 150 per cent of that value it would have if 150 m dimension were exact.

Solution : Let

L=length of the plot= 150 m

B=width of the plot= 50 m

and Now

A=area of the plot=LXB=l50x50=7500 m2. *A=LB*

oA 8A . --=Band -=L *ai* ·. *oB* ·

Uncertainty in area

*WA·=± ,J* (it *y* ~~~+- ( ~1 r- w~2 = ± yB2tt'~?+f2~~n-2

·-----

Case I. When there is no uncertainty in measurement of *L.*

*WL=O*

Uncertainty in measurement of area

*WA==±* V wr. +L wn~=±\IL wn =±Lwn= 150XO'Ol=1·5 m2.

Case II. When there is uncertainty in measurement of L.

The uncertainty in area is not to exceed l '5 x l '5 = ±2'25 m~.

*WA=v* iJ2w~2+£2;~2

or 2'25=y(50)2wx.2+(150)2(o·o1·)2

Hence uncertainty in measurement .of *Lis w1* = :10'0335 m.

I ,,

Example 3'22. A resistor has a nominal value of 10 .Q ±0' l % .A voltage is applied across the resistor and the power consumed in the resistor is calculated in two ways :

(i) from P=E~/R, .and (ii) from *P=El.*

Calculate the uncertainty in the power determination in each case when the measured values of *E* and *I* are ; . ·

*E=IOO* V±1% and *1=10* A±1%

Comment upon the result.

o--L--·0-~ . Ammeter l~

Voltmeter

Fig. 3'6

Solution : The circuit diagram is shown in Fig. 3'6. £2

(i) P~o=- -*R*

. oP \_2E DP E2

.. *-oE-R* and *8R* =-R2

··61 · It£C1RJCAt MFASURF.ME:NtS AND MMSlJRINO lNS1'RUMEl.lltS  . *, ..* { Hence uncertainty in power measurement.

w = *JT87;--)z-* WE-2.-+-:-(8-P )2-~R~ =~J(?:\_l{)2wE2+f\-~= )2 *w:;-*

*v* >J \ oE *8E* R R*2* .

]i x 1 GO= J 4 ( ~ t +(1 r x 100 , .

Percentage uncertainty in me~surement of power is calcuJated by putting *P=E2/R.*

=4v' 4(0·01)2+(0·002x100,,:;±2·236%.·

(ii) *P=El*

aP o.P *oE=l* and *·a1=.E.* ; ....  

Percentage uncertainty in power measurement

·~~~ X lOO=r::J(oP ..)2 win2+·(BP)2w12X100= v'I2wE2tE2w11-x 100

P oEJ f)l

= *J(w;* r:+(w}J'~ x 100=y(0'01)2+(0'0I)f x 100=±1'414%.

Tbe second method of power determination gives a much lower uncertainty than the first one •~n -tp0ugh the basic uncertainties in each quantity are the same. We conclude from here that a . ·'jndici:Qus selection uf method of measurement is important in order to reduce the uncertainty in the  :fb1ill\_-¢.0mputed results. . 

UNSOLVED PROBLEMS

. " .. ' · · 'l~ The value of a resistor is specified as .500 *Q* ± · 10% by a manufB.cfurer Find the limits of resist a nee between *"::* ·~.';:)vti~lt~"value is guaranteed. [Ans, 450 .Q to 550 !lJ - *:.;:* . I... The limiting errors for a four dial l'e!listance box are : 

Units ... ±0'2% hJmlteds ... 0'05%

Tens ... ±0'1% thotuJaoos ... ±0'02%

If tbe resistance ~?lne is set. at 3425 Q. calculate the limiting error In the resistance value, [Ans. ±0'83 OJ II

3. A flowm~ter is calibr~t<:d from ,Oto 100 ro3/s.. Tbe accuracy is< i;pecified as within ±0'75 per cent abo·ve 20 ! I,

per cent of scale reading, What is the s!1H1c error if the instrumt·nt indh::'.tes 80 m8/s. fAuo. ±0'6 m8/s]

. . . . 4. A 0·10 A ammeter has a guanmteed ac\~1.uacy oft *·5* per ceut of rnh scale reading. The current measured by this 1nst.nm1rnt is *7'S* A, Calculate the limitiJOg values or current and the percentage limiting error. · [ADii, *2'S±* 0·1sA; ±6%]

~0

, . . *5.* A liquid flows through a pi!,Je h1tviog a diameter of 100 mm with a velocity of 1 ill/fl. Calculate the rate flow • . · ...••.. tr·~ dia~eltr i~ guaranteed within ± c,~ iiild the velodty is known to be within ± 3% of measured value, find the ··:·~tt·Withm wh1~h rate of.flow can be sl>@Cified. [Ans. 7'S5x 10-• m•/s ±5%]

, · .. · . 6. The resistance of an unknown resistor i:J d~termined by Wheatstone ruidge. The solution fo1 tbt un~ · ·k.1mwn ree.istance is stated as 

\11/bere·limitin~ vafoes of various resistances are

1<1=50011 ±1%, R:i=61S fl±1%1 Ra=100±~»S%

· Calculate (a) the nominal value of the unkn<Jwn resistor, (h) the limiting erro1 of the unknown resnuor in ohm• and (c) the limiting error in per cent of unknown resistor, [Ans. (a) 3075 Q1 (b) ±76'88 Q and (c) ±2'S°/uJ. *1.  *