Live Session - Week 3: Discrete Response Models Lecture 3

Professor Jeffrey Yau

#Required Readings:

BL2015: Christopher R. Bilder and Thomas M. Loughin. Analysis of Categorical Data with R. CRC Press. 2015.

- Ch. 2.2.5 – 2.2.8, 2.3

Topics covered in Week 3

- Variable transformation: interactions among explanatory variables
- Variable transformation: quadratic term
- Categorical explanatory variables
- Odds ratio in the context of categorical explanatory variables
- Convergence criteria and complete separation

Familiarity with the concepts and techniques coverd in this and last lecture are critical, as they will be used frequently in the next two lectures in situations that are more general (from two categorical to $J > 2$ categories and from unordered cateogrical variables to ordinal variables). With multinomial logistic regression models, the notation will be heavier.

The key objectives in this live session are to learn how to incorporate various transformation of variables (or, in machine learning terminology, "feature engineering") and interpret the results when these transformed variables are part of the model specification. Variable transformations (or feature engineering) are useful in real life statistical and machine learning modeling.

In general, the odd ratios answer the question "how much the odds of success have changed by k-unit increase?" The amazing feature of logistic regression model is that the odd ratios (of the odds of success before and after the k-unit increase in a particular explanatory variable) is simplified to the exponential of the product between k and the coefficient estimate associated with that variable. That is, "the odds of a success change by exp(kβ_j) times for every k-unit increase in x_j"

Review some concepts from w203

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + (\beta_3 x_1 \times x_2) + \epsilon$$

$$\frac{\partial y}{\partial x_1} = \beta_1 + \beta_3 x_2$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \epsilon$$

$$\frac{\partial y}{\partial x_1} = \beta_1 + 2\beta_2 x_1$$

Start-up code

```r
library(knitr)
opts_chunk$set(tidy.opts=list(width.cutoff=60),tidy=TRUE)

# Set working directory
#setwd("~/Documents/Teach/Cal/w271/LiveSessions/week03/")
#wd <- getwd()
#wd

# Start with a clean R environment
rm(list = ls())

# Load Libraries
library(car)
library(dplyr)
library(Hmisc)
library(skimr)
library(ggplot2)
library(stargazer)
library(mcprofile)
```

In this live session, we will practice binary logistic regression modeling, with a focus on the materials covered in week 3, using an autism screening dataset (for toddlers) obtained from kaggle

The dataset comes with a very detailed description in the included Word document. Below are some brief description provided on the aforementioned kaggle webpage:

Context: The dataset was developed by Dr Fadi Fayez Thabtah (fadifayez.com) using a mobile app called ASDTests (ASDtests.com) to screen autism in toddlers. See the description file attached with the CSV data to know more about the variables and the class. This data can be used for descriptive and predictive analyses such as classification, clustering, regression, etc. You may use it to estimate the predictive power of machine learning techniques in detecting autistic traits.

Brief Description of the Variables This data page on kaggle also provide some very basic descriptive graphs on the variables in this dataset.

`A1 - A10`: Items within Q-Chat-10 in which questions possible answers : "Always, Usually, Sometimes, Rarly & Never" items' values are mapped to "1" or "0" in the dataset.

There are two variables in the data that will not be used in our analysis: 1. `Case_No`: the individual case number; this is an identifier variable 2. `Qchat.10.Score`: the dataset document suggests that this variable not be used in a classification problem, as the score variable is used to defined the `Class.ASD.Traits`.

```
# Load data
autism <- read.csv("autism.csv", header = TRUE, sep = ",")

# Attach the dataframe autism
attach(autism)

# View(autism)

# Examine the structure of the data
str(autism)
```

```
## 'data.frame':    1054 obs. of  19 variables:
##  $ Case_No               : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ A1                    : int  0 1 1 1 1 1 1 0 0 1 ...
##  $ A2                    : int  0 1 0 1 1 1 0 1 0 1 ...
##  $ A3                    : int  0 0 0 1 0 0 0 0 0 1 ...
##  $ A4                    : int  0 0 0 1 1 0 1 0 0 0 ...
##  $ A5                    : int  0 0 0 1 1 1 1 1 0 1 ...
##  $ A6                    : int  0 1 0 1 1 1 1 0 0 1 ...
##  $ A7                    : int  1 1 1 1 1 1 0 1 1 0 ...
##  $ A8                    : int  1 0 1 1 1 1 0 1 0 1 ...
##  $ A9                    : int  0 0 0 1 1 1 1 1 0 1 ...
##  $ A10                   : int  1 0 1 1 1 1 0 1 1 1 ...
##  $ Age_Mons              : int  28 36 36 24 20 21 33 33 36 22 ...
##  $ Qchat.10.Score        : int  3 4 4 10 9 8 5 6 2 8 ...
##  $ Sex                   : Factor w/ 2 levels "f","m": 1 2 2 2 1 2 2 2 2 2 ...
##  $ Ethnicity             : Factor w/ 11 levels "asian","black",..: 5 11 5 3 11 2 1 1 1 10 ...
##  $ Jaundice              : Factor w/ 2 levels "no","yes": 2 2 2 1 1 1 2 2 1 1 ...
##  $ Family_mem_with_ASD   : Factor w/ 2 levels "no","yes": 1 1 1 1 2 1 1 1 1 1 ...
##  $ Who.completed.the.test: Factor w/ 5 levels "family member",..: 1 1 1 1 1 1 1 1 1 3 ...
##  $ Class.ASD.Traits.     : Factor w/ 2 levels "No","Yes": 1 2 2 2 2 2 2 2 1 2 ...
```

```
# Conduct some very basic EDA
describe(autism)
```

```
## autism
##
##  19  Variables      1054  Observations
## ---------------------------------------------------------------------------
## Case_No
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##     1054        0     1054        1    527.5    351.7    53.65   106.30
##      .25      .50      .75      .90      .95
##   264.25   527.50   790.75   948.70  1001.35
##
## lowest :    1    2    3    4    5, highest: 1050 1051 1052 1053 1054
## ---------------------------------------------------------------------------
## A1
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2    0.738      594   0.5636   0.4924
##
## ---------------------------------------------------------------------------
```

```
## A2
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2    0.742      473   0.4488   0.4952
##
## ---------------------------------------------------------------------------
## A3
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2    0.721      423   0.4013    0.481
##
## ---------------------------------------------------------------------------
## A4
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2     0.75      540   0.5123   0.5002
##
## ---------------------------------------------------------------------------
## A5
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2    0.748      553   0.5247   0.4993
##
## ---------------------------------------------------------------------------
## A6
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2    0.732      608   0.5769   0.4887
##
## ---------------------------------------------------------------------------
## A7
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2    0.683      685   0.6499   0.4555
##
## ---------------------------------------------------------------------------
## A8
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2    0.745      484   0.4592   0.4971
##
## ---------------------------------------------------------------------------
## A9
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2     0.75      516   0.4896   0.5003
##
## ---------------------------------------------------------------------------
## A10
##        n  missing distinct     Info      Sum     Mean      Gmd
##     1054        0        2    0.728      618   0.5863   0.4856
##
## ---------------------------------------------------------------------------
## Age_Mons
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##     1054        0       25    0.971    27.87    8.859       12       15
##      .25      .50      .75      .90      .95
##       23       30       36       36       36
##
## lowest : 12 13 14 15 16, highest: 32 33 34 35 36
## ---------------------------------------------------------------------------
## Qchat.10.Score
```

```
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##     1054        0       11    0.991    5.213    3.338        0        1
##      .25      .50      .75      .90      .95
##        3        5        8        9       10
##
## Value          0     1     2     3     4     5     6     7     8     9
## Frequency     54    88    88    96   110   120    96   135    97    95
## Proportion 0.051 0.083 0.083 0.091 0.104 0.114 0.091 0.128 0.092 0.090
##
## Value         10
## Frequency     75
## Proportion 0.071
## ---------------------------------------------------------------------------
## Sex
##        n  missing distinct
##     1054        0        2
##
## Value          f     m
## Frequency    319   735
## Proportion 0.303 0.697
## ---------------------------------------------------------------------------
## Ethnicity
##        n  missing distinct
##     1054        0       11
##
## asian (299, 0.284), black (53, 0.050), Hispanic (40, 0.038), Latino (26,
## 0.025), middle eastern (188, 0.178), mixed (8, 0.008), Native Indian (3,
## 0.003), Others (35, 0.033), Pacifica (8, 0.008), south asian (60, 0.057),
## White European (334, 0.317)
## ---------------------------------------------------------------------------
## Jaundice
##        n  missing distinct
##     1054        0        2
##
## Value         no   yes
## Frequency    766   288
## Proportion 0.727 0.273
## ---------------------------------------------------------------------------
## Family_mem_with_ASD
##        n  missing distinct
##     1054        0        2
##
## Value         no   yes
## Frequency    884   170
## Proportion 0.839 0.161
## ---------------------------------------------------------------------------
## Who.completed.the.test
##        n  missing distinct
##     1054        0        5
##
## family member (1018, 0.966), Health care professional (5, 0.005), Health
## Care Professional (24, 0.023), Others (3, 0.003), Self (4, 0.004)
## ---------------------------------------------------------------------------
## Class.ASD.Traits.
```

```
##        n  missing distinct
##     1054        0        2
##
## Value         No   Yes
## Frequency    326   728
## Proportion 0.309 0.691
## ---------------------------------------------------------------------------
```

```
summary(autism)
```

```
##     Case_No             A1               A2               A3
##  Min.   :   1.0   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.: 264.2   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median : 527.5   Median :1.0000   Median :0.0000   Median :0.0000
##  Mean   : 527.5   Mean   :0.5636   Mean   :0.4488   Mean   :0.4013
##  3rd Qu.: 790.8   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000
##  Max.   :1054.0   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##
##        A4               A5               A6               A7
##  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :1.0000   Median :1.0000   Median :1.0000   Median :1.0000
##  Mean   :0.5123   Mean   :0.5247   Mean   :0.5769   Mean   :0.6499
##  3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000
##  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##
##        A8               A9              A10            Age_Mons
##  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :12.00
##  1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:23.00
##  Median :0.0000   Median :0.0000   Median :1.0000   Median :30.00
##  Mean   :0.4592   Mean   :0.4896   Mean   :0.5863   Mean   :27.87
##  3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:36.00
##  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :36.00
##
##  Qchat.10.Score   Sex              Ethnicity   Jaundice
##  Min.   : 0.000   f:319   White European:334   no :766
##  1st Qu.: 3.000   m:735   asian         :299   yes:288
##  Median : 5.000           middle eastern:188
##  Mean   : 5.213           south asian   : 60
##  3rd Qu.: 8.000           black         : 53
##  Max.   :10.000           Hispanic      : 40
##                           (Other)       : 80
##  Family_mem_with_ASD              Who.completed.the.test Class.ASD.Traits.
##  no :884             family member           :1018       No :326
##  yes:170             Health care professional:   5       Yes:728
##                      Health Care Professional:  24
##                      Others                  :   3
##                      Self                    :   4
##
##
```

```
skim(autism)
```

```
## Skim summary statistics
##  n obs: 1054
```

```
##  n variables: 19
##
## -- Variable type:factor ---------------------------
##               variable missing complete    n n_unique
##       Class.ASD.Traits.       0     1054 1054        2
##              Ethnicity       0     1054 1054       11
##     Family_mem_with_ASD       0     1054 1054        2
##               Jaundice       0     1054 1054        2
##                    Sex       0     1054 1054        2
##  Who.completed.the.test       0     1054 1054        5
##                          top_counts ordered
##              Yes: 728, No: 326, NA: 0   FALSE
##  Whi: 334, asi: 299, mid: 188, sou: 60   FALSE
##              no: 884, yes: 170, NA: 0   FALSE
##              no: 766, yes: 288, NA: 0   FALSE
##                 m: 735, f: 319, NA: 0   FALSE
##     fam: 1018, Hea: 24, Hea: 5, Sel: 4   FALSE
##
## -- Variable type:integer --------------------------
##        variable missing complete    n   mean     sd p0    p25   p50    p75
##              A1       0     1054 1054   0.56   0.5   0    0      1      1
##             A10       0     1054 1054   0.59   0.49  0    0      1      1
##              A2       0     1054 1054   0.45   0.5   0    0      0      1
##              A3       0     1054 1054   0.4    0.49  0    0      0      1
##              A4       0     1054 1054   0.51   0.5   0    0      1      1
##              A5       0     1054 1054   0.52   0.5   0    0      1      1
##              A6       0     1054 1054   0.58   0.49  0    0      1      1
##              A7       0     1054 1054   0.65   0.48  0    0      1      1
##              A8       0     1054 1054   0.46   0.5   0    0      0      1
##              A9       0     1054 1054   0.49   0.5   0    0      0      1
##        Age_Mons       0     1054 1054  27.87   7.98 12   23     30     36
##         Case_No       0     1054 1054 527.5  304.41  1 264.25 527.5 790.75
##  Qchat.10.Score       0     1054 1054   5.21   2.91  0    3      5      8
##  p100     hist
##     1
##     1
##     1
##     1
##     1
##     1
##     1
##     1
##     1
##     1
##    36
##  1054
##    10
```

```
# skim(Age_Mons)

# Define a function to examine factor variables:
exam_cat_var = function(var.names) {
    table(var.names)
    round(prop.table(table(var.names)), 2)
```

```r
}
apply(autism[, 14:19], 2, table)
```

```
## $Sex
##
##   f   m
## 319 735
##
## $Ethnicity
##
##          asian          black       Hispanic         Latino middle eastern
##            299             53             40             26            188
##          mixed  Native Indian         Others       Pacifica    south asian
##              8              3             35              8             60
## White European
##            334
##
## $Jaundice
##
##  no yes
## 766 288
##
## $Family_mem_with_ASD
##
##  no yes
## 884 170
##
## $Who.completed.the.test
##
##            family member Health care professional Health Care Professional
##                     1018                        5                       24
##                   Others                     Self
##                        3                        4
##
## $Class.ASD.Traits.
##
##  No Yes
## 326 728
```

```r
apply(autism[, 14:19], 2, exam_cat_var)
```

```
## $Sex
## var.names
##   f   m
## 0.3 0.7
##
## $Ethnicity
## var.names
##          asian          black       Hispanic         Latino middle eastern
##           0.28           0.05           0.04           0.02           0.18
##          mixed  Native Indian         Others       Pacifica    south asian
##           0.01           0.00           0.03           0.01           0.06
## White European
##           0.32
```

```
##
## $Jaundice
## var.names
##   no  yes
## 0.73 0.27
##
## $Family_mem_with_ASD
## var.names
##   no  yes
## 0.84 0.16
##
## $Who.completed.the.test
## var.names
##            family member Health care professional Health Care Professional
##                     0.97                     0.00                     0.02
##                   Others                     Self
##                     0.00                     0.00
##
## $Class.ASD.Traits.
## var.names
##   No  Yes
## 0.31 0.69
```

```r
# Age describe(Age_Mons) summary(Age_Mons)
skim(Age_Mons)
```

```
##
## Skim summary statistics
##
## -- Variable type:integer --------------------------
##  variable missing complete    n  mean   sd p0 p25 p50 p75 p100     hist
##  Age_Mons       0     1054 1054 27.87 7.98 12  23  30  36   36
```

```r
# Crosstab
xtabs(~Sex + Class.ASD.Traits.)
```

```
##    Class.ASD.Traits.
## Sex  No Yes
##   f 125 194
##   m 201 534
```

```r
round(prop.table(xtabs(~Sex + Class.ASD.Traits.), 1), 2)
```

```
##    Class.ASD.Traits.
## Sex   No  Yes
##   f 0.39 0.61
##   m 0.27 0.73
```

```r
xtabs(~Ethnicity + Class.ASD.Traits.)
```

```
##                 Class.ASD.Traits.
## Ethnicity         No Yes
##   asian           87 212
##   black           14  39
##   Hispanic        10  30
##   Latino           6  20
##   middle eastern  92  96
```

```
##   mixed             3   5
##   Native Indian     0   3
##   Others            6  29
##   Pacifica          1   7
##   south asian      23  37
##   White European  84 250
```

```
round(prop.table(xtabs(~Ethnicity + Class.ASD.Traits.), 1), 2)
```

```
##                   Class.ASD.Traits.
## Ethnicity           No  Yes
##   asian           0.29 0.71
##   black           0.26 0.74
##   Hispanic        0.25 0.75
##   Latino          0.23 0.77
##   middle eastern 0.49 0.51
##   mixed           0.38 0.62
##   Native Indian   0.00 1.00
##   Others          0.17 0.83
##   Pacifica        0.12 0.88
##   south asian     0.38 0.62
##   White European 0.25 0.75
```

```
xtabs(~Jaundice + Class.ASD.Traits.)
```

```
##         Class.ASD.Traits.
## Jaundice  No Yes
##      no  253 513
##      yes  73 215
```

```
round(prop.table(xtabs(~Jaundice + Class.ASD.Traits.), 1), 2)
```

```
##         Class.ASD.Traits.
## Jaundice   No  Yes
##      no  0.33 0.67
##      yes 0.25 0.75
```

```
xtabs(~Family_mem_with_ASD + Class.ASD.Traits.)
```

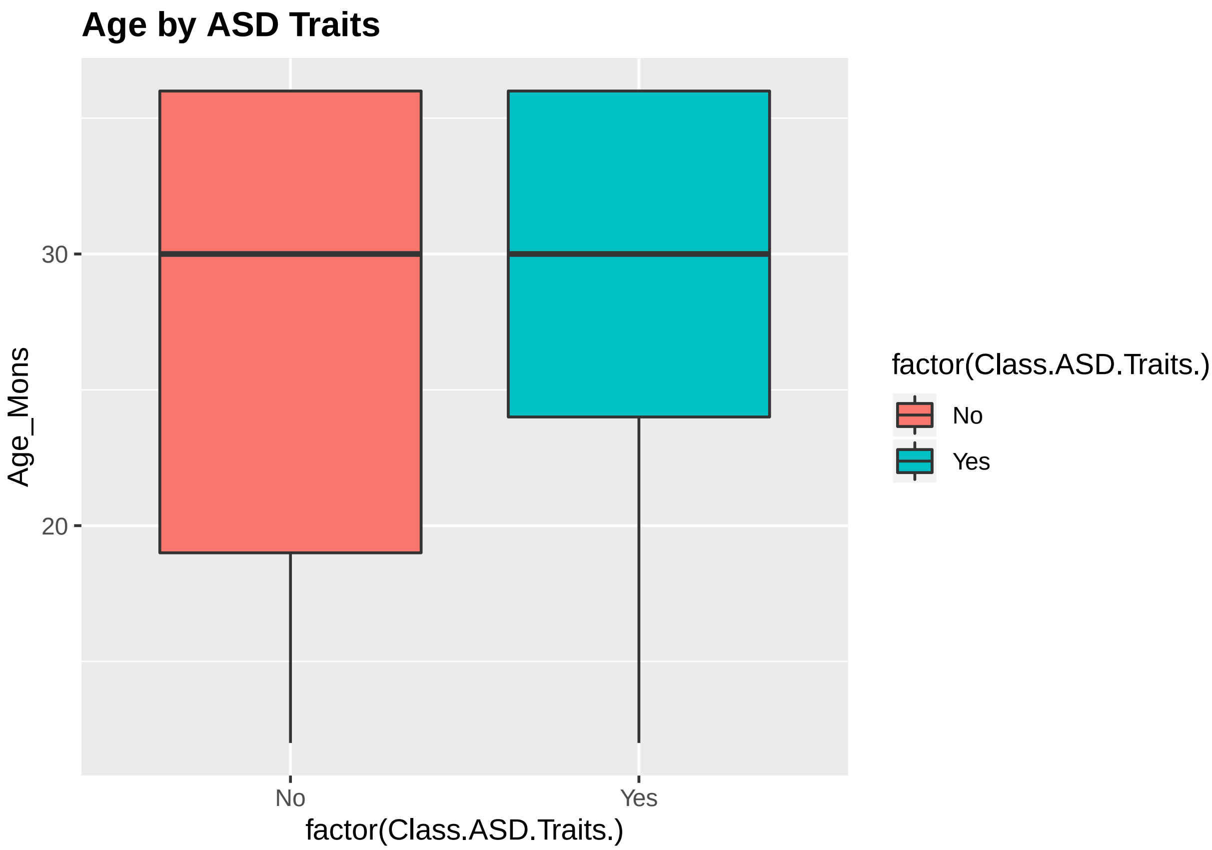
```
##                    Class.ASD.Traits.
## Family_mem_with_ASD  No Yes
##                 no  271 613
##                 yes  55 115
```

```
round(prop.table(xtabs(~Family_mem_with_ASD + Class.ASD.Traits.),
    1), 2)
```

```
##                    Class.ASD.Traits.
## Family_mem_with_ASD   No  Yes
##                 no  0.31 0.69
##                 yes 0.32 0.68
```

```
# Distribution of the Toddlers' Age by ASD Traits

ggplot(autism, aes(factor(Class.ASD.Traits.), Age_Mons)) + geom_boxplot(aes(fill = factor(Class.ASD.Tra
    ggtitle("Age by ASD Traits") + theme(plot.title = element_text(lineheight = 1,
    face = "bold"))
```



Interactions between explanatory variables are needed when the effect of one explanatory variable on the probability of success depends on the value for another explanatory variable. From these graphs, interactions between some of the explanatory variables seem to be warranted.

In `R`, there are several ways to implement interaction terms in a logistic regression model:

- `formula = y ~ x1 + x2 + x1:x2`
- `formula = y ~ x1*x2`
- `formula = y ~ (x1 + x2)^2`

To include a quadratic term in a logistic regression model, use the following:

- `formula = y ~ x1 + I(x1^2)`

For factor variables, either turn them into factor variables and enter them into a logistic regression model, which is my preferred method, or use the `factor()` function inside a logistic regression: `formula = y ~ x1 + factor(x2)`, if `x2` needs to enter the regression as a factor variable.

Binary Logistic Regression Modeling

```
# Model 1 (Base Model)
mod.glm1 <- glm(Class.ASD.Traits. ~ Age_Mons + Sex + Ethnicity +
    Jaundice + Family_mem_with_ASD, family = "binomial", data = autism)
summary(mod.glm1)
```

```
##
## Call:
```

```
## glm(formula = Class.ASD.Traits. ~ Age_Mons + Sex + Ethnicity +
##     Jaundice + Family_mem_with_ASD, family = "binomial", data = autism)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.0941  -1.2292   0.7224   0.8317   1.4565
##
## Coefficients:
##                          Estimate Std. Error z value Pr(>|z|)
## (Intercept)              0.281885   0.304062   0.927  0.35389
## Age_Mons                 0.008482   0.008859   0.957  0.33834
## Sexm                     0.448503   0.147869   3.033  0.00242 **
## Ethnicityblack           0.177787   0.342323   0.519  0.60351
## EthnicityHispanic        0.247918   0.392078   0.632  0.52718
## EthnicityLatino          0.423043   0.488840   0.865  0.38682
## Ethnicitymiddle eastern  -0.768519   0.202754  -3.790  0.00015 ***
## Ethnicitymixed          -0.330292   0.746778  -0.442  0.65828
## EthnicityNative Indian   13.396350 508.130189   0.026  0.97897
## EthnicityOthers          0.696988   0.471729   1.478  0.13954
## EthnicityPacifica        1.080882   1.079130   1.002  0.31653
## Ethnicitysouth asian    -0.425479   0.299045  -1.423  0.15480
## EthnicityWhite European  0.207318   0.186398   1.112  0.26604
## Jaundiceyes              0.353792   0.163423   2.165  0.03040 *
## Family_mem_with_ASDyes  -0.276408   0.187558  -1.474  0.14056
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1303.9  on 1053  degrees of freedom
## Residual deviance: 1243.4  on 1039  degrees of freedom
## AIC: 1273.4
##
## Number of Fisher Scoring iterations: 13
```

```
# Model 2 (Model with both interaction and Non-linear effect)
# interaction of age and sex a quadratic term on age

# mod.glm2 <- glm(YOUR FORMULA HERE, family = 'binomial',
# data = autism) summary(mod.glm2)

# Display the models together stargazer(mod.glm1, mod.glm2,
# type = 'text')
```

Testing Model Differences

```
# CODE HERE (1 line)
```

Based on the test result, we will use `mod.glm2`.

Our model:

$$log(\frac{\pi}{1-\pi}) = \beta_0+\beta_1 Age\_Mons+\beta_2 Age\_Mons^2+\beta_3 Sexm+\beta_4 Ethnicityblack+\beta_5 EthnicityHispanic+\beta_6 EthnicityLatino+$$

The odds ratio for an increase in age by c months is expressed in the following formula:

$$OR = exp(c\beta_1 + c\beta_2(2 \times age + c))$$

which depends on the level of age.

Model Interpretation

We need some questions, such as

- What is the effect of being a 30-month old boy on the odds of having ADS traits?

```
c = 1
Age_Mons = 30
# YOUR CODE HERE (1 line)
```

- What is the effect of an one month increase in age (measured in months) of a 24 months old `female` toddler on the odds of having ADS traits? (Hint: use the formula above.)

```
c = 1
Age_Mons = 24
Sexm = 0

# YOUR CODE HERE (1 line)
```