Time Series Analysis
Lecture 2

Regression With Time Series, An Intro to Exploratory Time Series Data Analysis, and Time Series Smoothing

datascience@berkeley

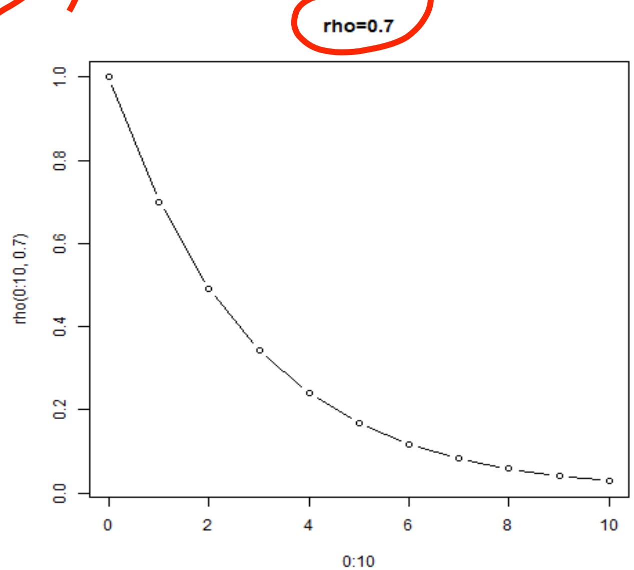
Autoregressive Models Part 2 Simulation and Empirical Properties of AR(1) Models

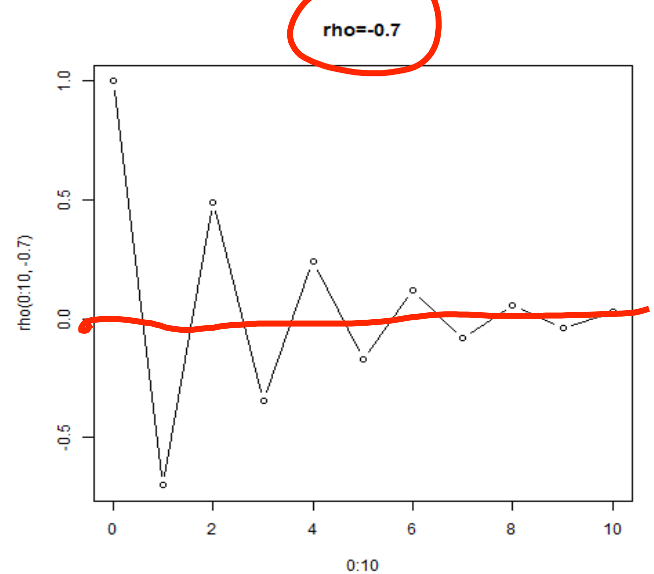
AR(1) Model: Theoretical Autocorrelation Function

The autocorrelation function of an AR(1) process, and in fact, AR(p) process in general, decay to zero gradually, either monotonically or in a oscillated fashion, although the general process allows for richer dynamics.

The damped oscillation comes from the switching signs at each successively longer time displacement.

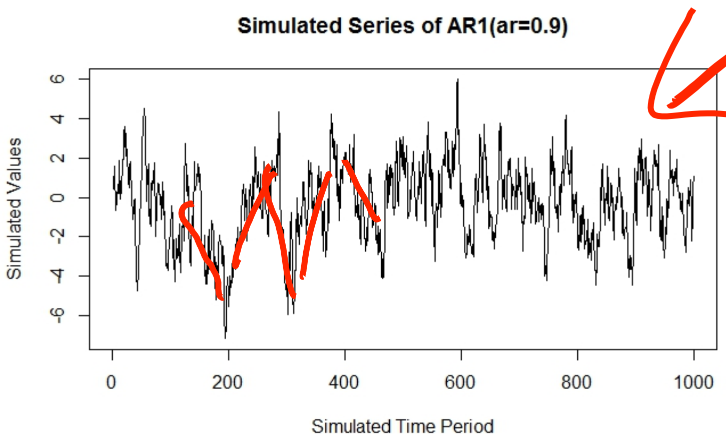
$$\rho_k = \phi^k \quad \text{(provided that } k \geq 0 \text{ and } |\phi| < 1\text{)}$$

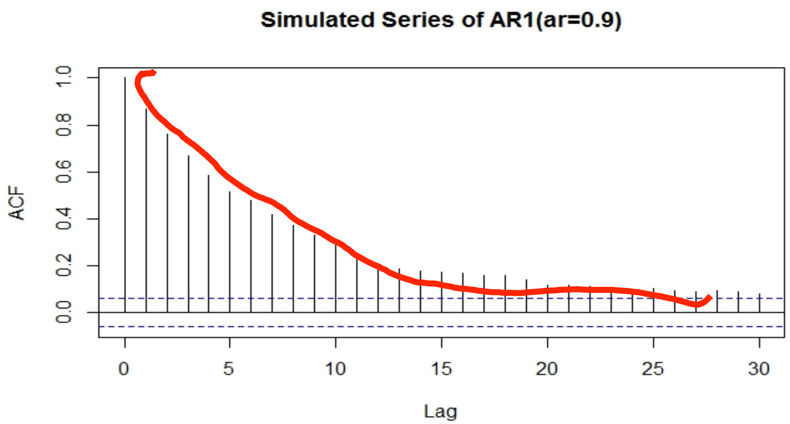




The ACF of AR(1) Model: Case 1

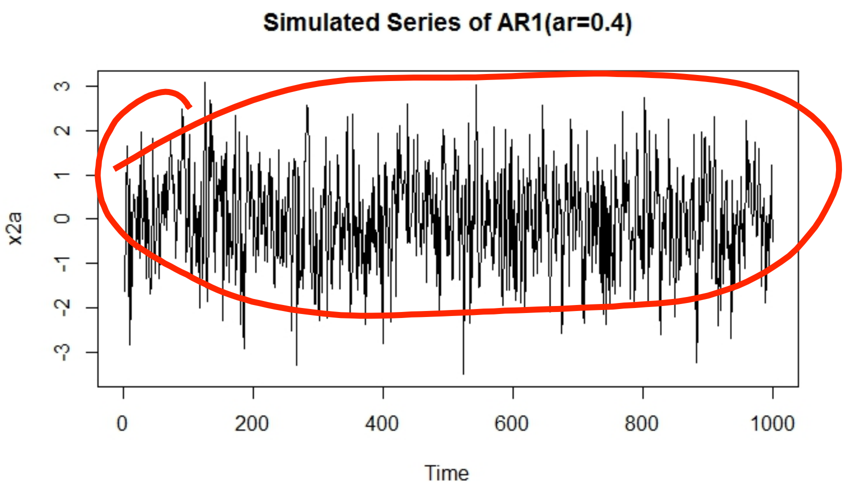
- The time plot of the simulated **AR(1) model with parameter = 0.9** appears to be persistent, meaning that there are time periods in which the series "tends" to go up or tends to go down. This is not surprising because with the AR parameter being 0.9, the simulated series has high autocorrelation.
- The correlogram decays gradually to zero, and the ACF is still statistically significant after 25 lags. In general, the autocorrelations converge to 0 in the limit as the time displacement approaches infinity. **The implication is that they do not "cut off" at zero abruptly.**
- Remember that we simulated 1,000 observations, so it is a pretty long time series and can compress the confidence interval.

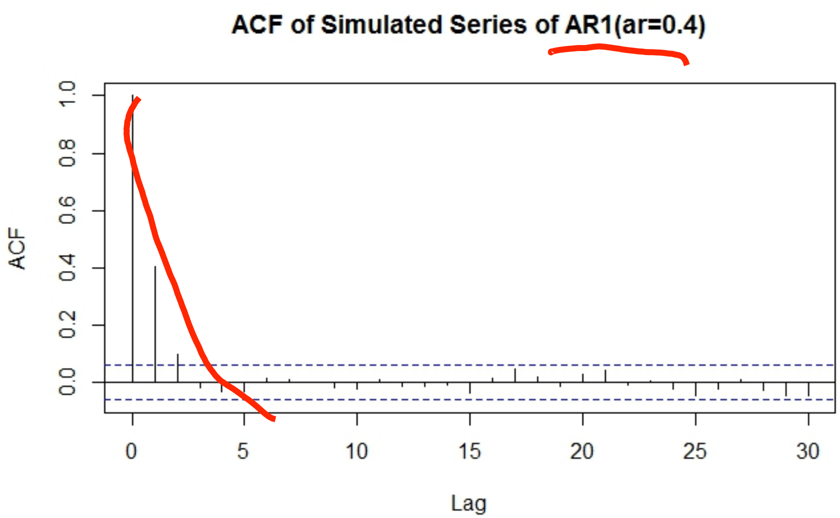




AR(1) Model: Case 2

- The time plot of the simulated AR(1) model with parameter = 0.4 looks very different from the previous AR(1) model. The persistence is much less apparent.
- The correlogram also decays to zero but in a much fast fashion. In fact, the ACF is no longer statistically significant after three lags.
- So, the correlogram of the ACF of AR models depends on the AR parameter.
- As we will see, this is one of the biggest different the correlogram ke a comment that this is in a sharp contrast with MA processes.





PACF of AR(1) Model

- Partial autocorrelation function for the AR(1) process, on the other hand, cuts off abruptly after the first time displacement:

$$\boxed{p(\tau) = \begin{matrix} \varphi, & \tau = 1 \\ 0, & \tau > 1 \end{matrix}}$$

- For AR(1) model, this is obvious, because the first partial autocorrelation is just the autoregressive coefficient, and the coefficients on all other longer lags, by construction, are zero.

- This is an important property of AR models because it helps identify the order of an autoregressive models.

- In fact, using the autocorrelation function alone is hard to identify the order of an autoregressive models.

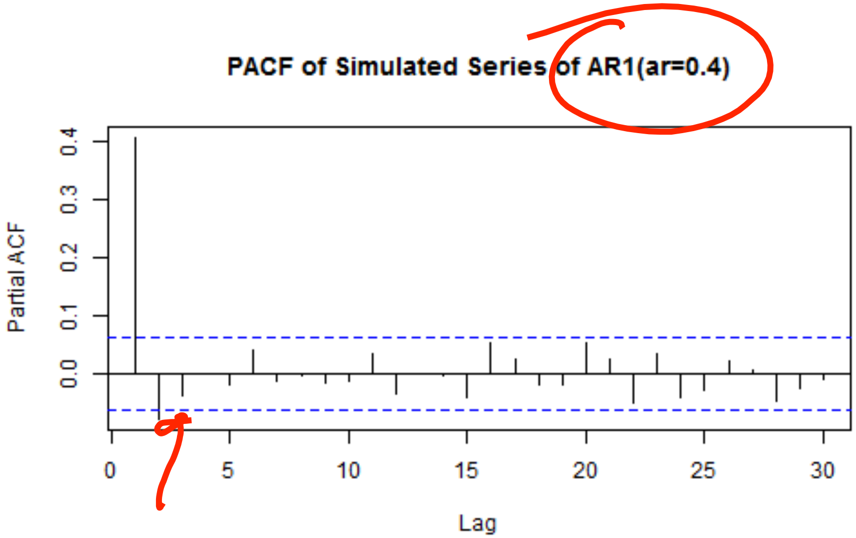
PACF of AR(1) Model

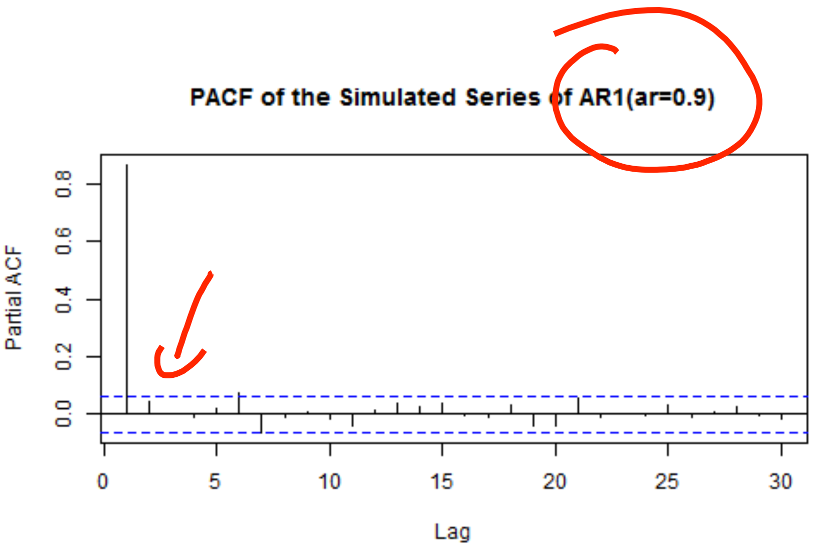
- Both of the partial autocorrelation function graphs indicate that the partial autocorrelations cut off to zero after lag 1.

- So, we know that if the underlying data-generating process is a class of autoregressive models, then these PACF graphs indicate that the process should be of order 1.

- When analyzing real-world data, it is never that clear cut. As such, we need to use combine various graphical techniques and statistical tests to identify the order of an AR model.

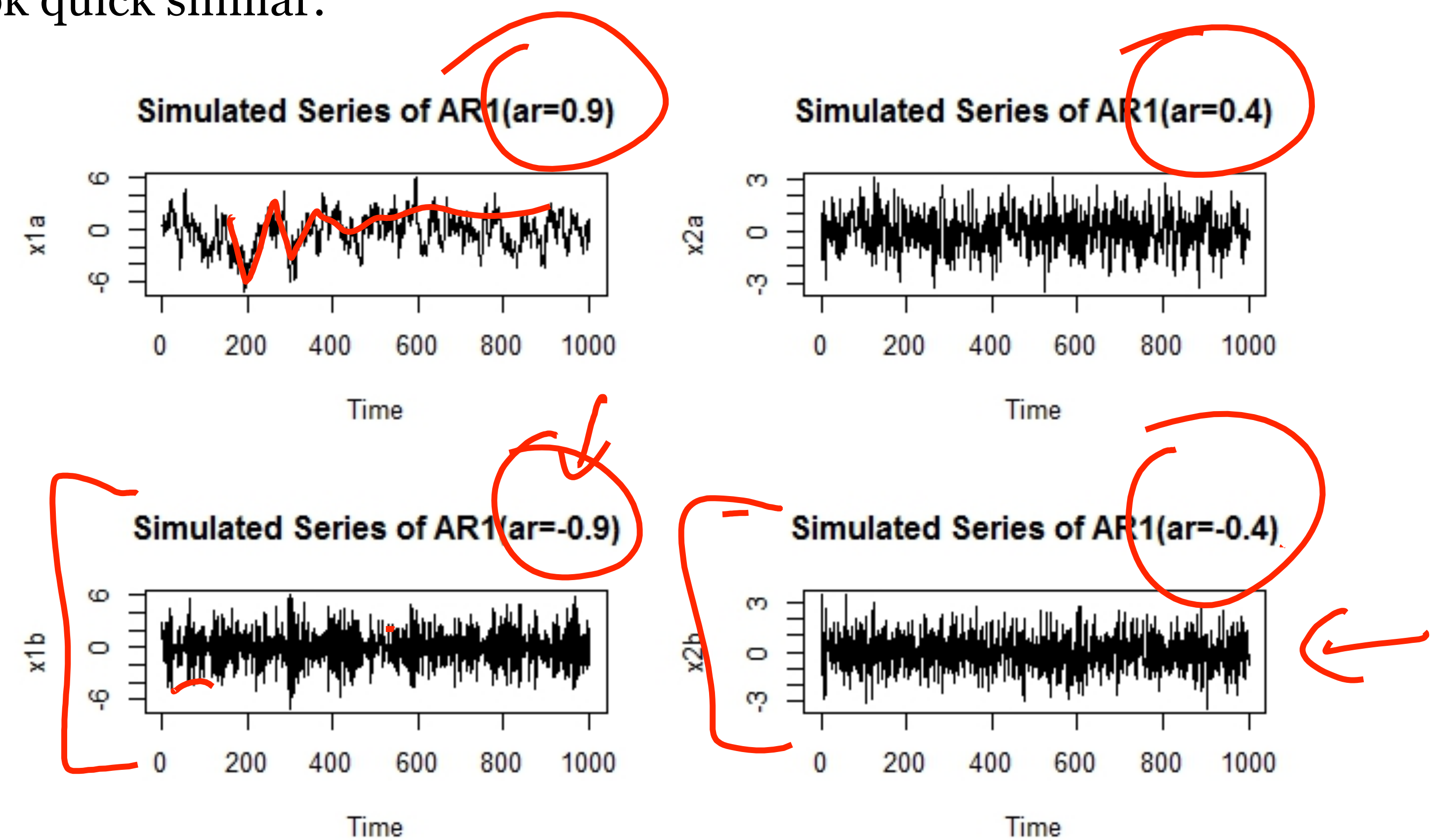
- In fact, we may be able to narrow down only to a few models and use other statistics, such as AIC and BIC, to choose a model.





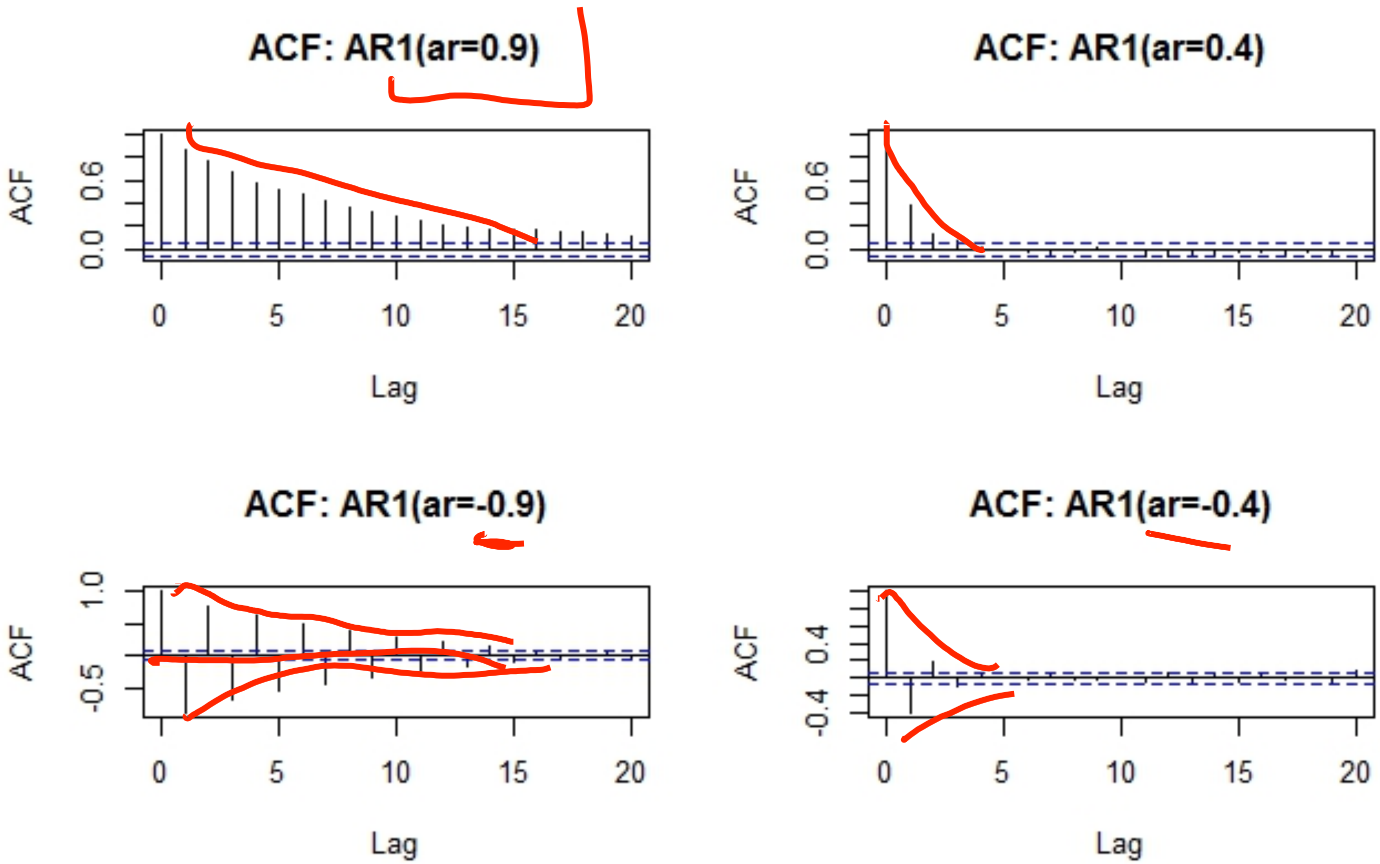
PACF of AR(1) Model

- Let's look at two other AR(1) models that have AR parameters being negative.
- Notice the difference among these time plots:
    - ➢ For the models with positive autocorrelation, the time series plots exhibit much more persistence for the series with high correlation.
    - ➢ For the models with negative autocorrelation, however, the time series plots look quick similar.



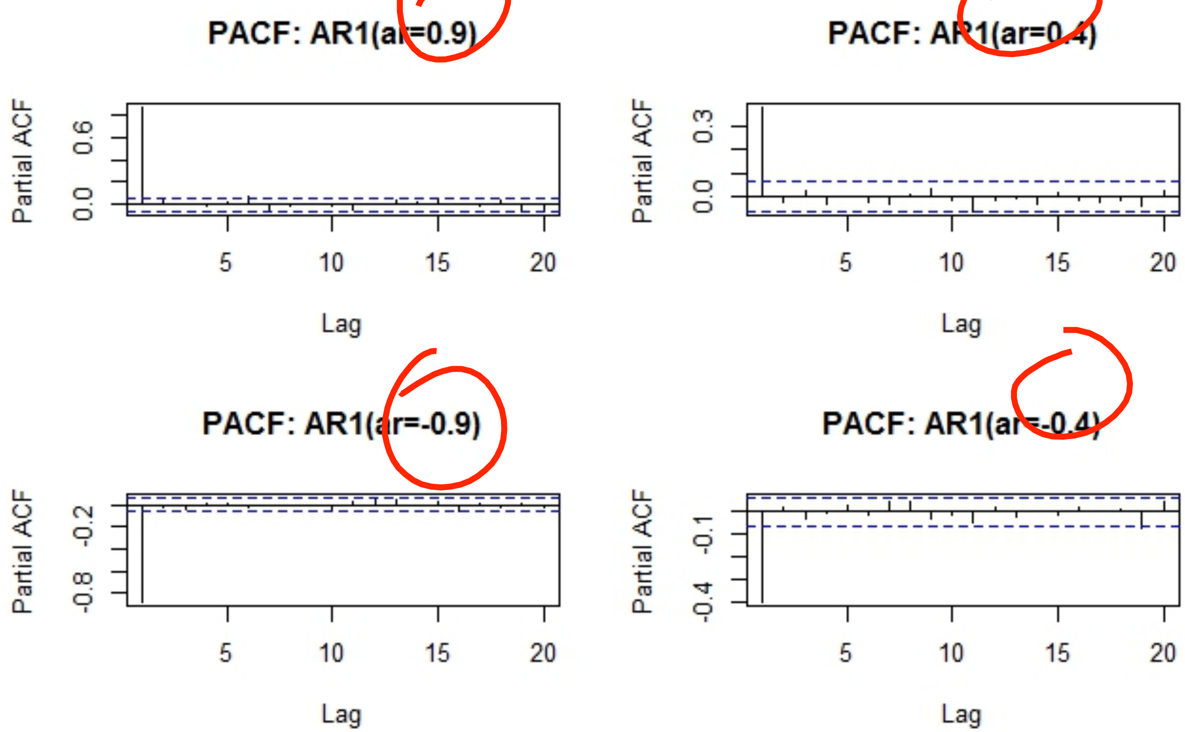
PACF of AR(1) Model

- To distinguish the two, we can rely on autocorrelation function because
  - ➢ The autocorrelations of AR(1) models with positive AR parameter decay to zero gradually and monotonically, and
  - ➢ The autocorrelation of AR(1) models with negative AR parameters decay to zero in an oscillated fashion.



PACF of AR(1) Model

- The PACFs of AR(1) models with and without positive AR parameters behave very different.
- Therefore, PACF can be used to help learning about the sign of the parameter of an AR(1) model. Remember that it is possible because we assume that the underlying data-generating process is an AR(1) model.
- In general, given a data series, we will have to plot the time series plot, ACF, and PACF as a starting point to choose candidate models for the data.



Berkeley SCHOOL OF INFORMATION