PANEL DATA ANALYSIS: LINEAR MIXED-EFFECT MODELS

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Introduction and Motivation

Random Intercept vs. Random Slope:

- Note that each scenario and each subject is assigned a different intercept. That's what we would expect, given that we've told the model with "(1|subject)" and "(1|scenario)" to take by-subject and by-item variability into account.

- On the other hand, the fixed effects (attitude and gender) are all the same for all subjects and items. This model is called a **random intercept model**.

- In this model, we account for baseline-differences in pitch, but we assume that whatever the effect of politeness is, it's going to be the same for all subjects and items.

```
> coef(politeness.lmm2)
$scenario
  (Intercept) attitudepol   genderM
1    243.3398   -19.72111 -108.5163
2    263.4292   -19.72111 -108.5163
3    268.2541   -19.72111 -108.5163
4    277.4757   -19.72111 -108.5163
5    254.9102   -19.72111 -108.5163
6    244.6724   -19.72111 -108.5163
7    245.8426   -19.72111 -108.5163

$subject
   (Intercept) attitudepol   genderM
F1    242.9386   -19.72111 -108.5163
F2    267.2654   -19.72111 -108.5163
F3    260.3348   -19.72111 -108.5163
M3    285.2283   -19.72111 -108.5163
M4    262.2248   -19.72111 -108.5163
M7    223.0857   -19.72111 -108.5163

attr(,"class")
[1] "coef.mer"
```

- However, it may not be a valid assumption. For instance, it is possible that some items would elicit more or less politeness. That is, the effect of politeness might be different for different items. Likewise, the effect of politeness might be different for different subjects.

- In this case, we need a **random slope model**, where subjects and items are not only allowed to have differing intercepts, but different slopes for the effect of politeness.

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: frequency ~ attitude + gender + (1 + attitude | subject) + (1 +
    attitude | scenario)
   Data: politeness

     AIC      BIC   logLik deviance df.resid
   814.9    839.1   -397.4    794.9       73

Scaled residuals:
    Min      1Q  Median      3Q     Max
-2.1947 -0.6691 -0.0789  0.5256  3.4252

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 scenario (Intercept) 182.082  13.494
          attitudepol  31.262   5.591   0.22
 subject  (Intercept) 392.474  19.811
          attitudepol   1.707   1.307   1.00
 Residual             627.880  25.058
Number of obs: 83, groups:  scenario, 7; subject, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)  257.989     13.529  19.069
attitudepol  -19.747      5.922  -3.334
genderM     -110.802     17.512  -6.327

Correlation of Fixed Effects:
            (Intr) atttdp
attitudepol -0.105
genderM     -0.647  0.003
```

- Note that the only thing being changed is the random effects: "$(1 + attitude|subject)$" means that you inform the model to expect differing baseline-levels of frequency (the intercept, represented by 1) as well as differing responses to the main factor in question, which is "attitude" in this case.

- Let's look at the coefficients of this updated model:

```
$scenario
  (Intercept) attitudepol    genderM
1    245.2603   -20.43832 -110.8021
2    263.3012   -15.94386 -110.8021
3    269.1432   -20.63361 -110.8021
4    276.8309   -16.30132 -110.8021
5    256.0579   -19.40575 -110.8021
6    246.8605   -21.94816 -110.8021
7    248.4702   -23.55752 -110.8021

$subject
   (Intercept) attitudepol    genderM
F1    243.8053   -20.68245 -110.8021
F2    266.7321   -19.17028 -110.8021
F3    260.1484   -19.60452 -110.8021
M3    285.6958   -17.91951 -110.8021
M4    264.1982   -19.33741 -110.8021
M7    227.3551   -21.76744 -110.8021
```

- Now, with the by-subject and by-item coefficients, the effect of politeness ("attitudepol") is different for each subject and item.

- **Interpretation:** Note the coefficients are always negative and quite similar to each other, meaning that despite individual variation, there is also consistency in how politeness affects the voice: for all of our speakers, the voice tends to go down when speaking politely, but for some people it goes down slightly more so than for others.

- The coefficients for gender have not changed, as we did not specify random slopes for the by-subject or by-item effect of gender.

- Let's obtain the *p-value* using likelihood ratio test:

- Note that the null model needs to have the same random effects structure. If the full model is a random slope model, then the null model also needs to be a random slope model.

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: frequency ~ attitude + gender + (1 + attitude | subject) + (1 +
    attitude | scenario)
   Data: politeness

     AIC      BIC   logLik deviance df.resid
   814.9    839.1   -397.4    794.9       73

Scaled residuals:
    Min      1Q  Median      3Q     Max
-2.1947 -0.6691 -0.0789  0.5256  3.4252

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 scenario (Intercept) 182.082  13.494
          attitudepol  31.262   5.591   0.22
 subject  (Intercept) 392.474  19.811
          attitudepol   1.707   1.307   1.00
 Residual             627.880  25.058
Number of obs: 83, groups:  scenario, 7; subject, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)  257.989     13.529  19.069
attitudepol  -19.747      5.922  -3.334
genderM     -110.802     17.512  -6.327

Correlation of Fixed Effects:
            (Intr) atttdp
attitudepol -0.105
genderM     -0.647  0.003
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: frequency ~ gender + (1 + attitude | subject) + (1 + attitude |      scenario)
   Data: politeness

     AIC      BIC   logLik deviance df.resid
   819.6    841.4   -400.8    801.6       74

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.09488 -0.64641 -0.08678  0.60655  3.00533

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 scenario (Intercept) 231.845  15.226
          attitudepol 410.122  20.251   -0.40
 subject  (Intercept) 378.499  19.455
          attitudepol   5.439   2.332   1.00
 Residual             628.654  25.073
Number of obs: 83, groups:  scenario, 7; subject, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)   253.37      13.44  18.856
genderM      -112.49      17.47  -6.439

Correlation of Fixed Effects:
        (Intr)
genderM -0.650
```

```
> anova(politeness.null,politeness.lmm3)
Data: politeness
Models:
politeness.null: frequency ~ gender + (1 + attitude | subject) + (1 + attitude |
politeness.null:     scenario)
politeness.lmm3: frequency ~ attitude + gender + (1 + attitude | subject) + (1 +
politeness.lmm3:     attitude | scenario)
                Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
politeness.null  9 819.61 841.37 -400.80   801.61
politeness.lmm3 10 814.90 839.09 -397.45   794.90 6.7082      1   0.009597 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

- What kind of random slope structure, if at all, should be imposed?

- It turns out that random slopes are very useful in practice. In experimental settings, experimental subjects often differ with how they react to an experimental manipulation, and the effect of an experimental manipulation differs for different items.

- **Assumptions:** We will study them in details in the next section. For now, we just want to highlight the importance of the *independence* assumption.

- Mixed models can still violate independence if important fixed or random effects are excluded in the model.

- In fact, missing some of the important variables may also lead to omitted variable bias, if these variables are correlated with the explanatory variables included in the model.

- For example, if we analyzed our data with a model that excluded the "subject-specific" random effect, then the model would not "know" that there are multiple responses per subject. This leads to a violation of the independence assumption. The punchline is that choose fixed effects and random effects carefully, and always try to resolve non-independencies when using data with repeated measurements.

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