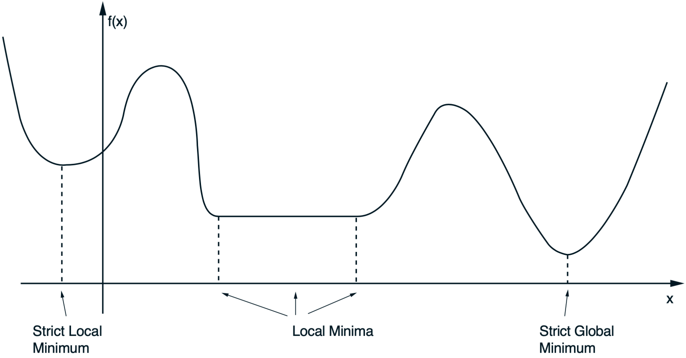
MIE424 (2021 Winter) Tutorial 3

Feb 8, 2021

Local Minima and Global Minima

Unconstrained local and global minima in one dimension.



By definition, any global minima must be a local minima.

FONC & SONC

First Order Necessary Condition (FONC)

Suppose $f(x)$ is differentiable at $\bar{x}$, if $\bar{x}$ is a local minimum, then

$$\nabla f(\bar{x}) = \mathbf{0}$$

Second Order Necessary Condition

Suppose $f(x)$ is twice differentiable at $\bar{x}$, if $\bar{x}$ is a local minimum, then

$$\nabla f(\bar{x}) = \mathbf{0}$$

$$H(\bar{x}) \succcurlyeq 0 \text{ i.e. } H(\bar{x}) \text{ is positive-semidefinite.}$$

Note: These two are all necessary conditions!

FONC & SONC

When we say $\bar{x} \in \mathbb{R}^n$ satisfy FONC or SONC, we refer to the conditions listed above, but not referring that $\bar{x}$ itself is a local optima.

In general, sufficient condition for local optima is difficult, let alone global optima. Even for continuous function, both local minimum and local maximum might not even exist.

But for global convex function ($H(x) \succcurlyeq 0, \forall x \in \mathbb{R}^n$), there exists only one local minimum.

Locally being convex could guarantee a local minimum (second order sufficient condition, SOSC).

Second Order Sufficient Condition

Second Order Sufficient Condition (SOSC)

Suppose $f(x)$ is **twice differentiable** at $\bar{x}$, if $\nabla f(\bar{x}) = \mathbf{0}$ and $H(\bar{x}) \succ 0$ i.e. $H(\bar{x})$ is **positive-definite**, then

$\bar{x}$ is a **strict** local minimum.

Note: SOSC guarantees only local minimum, not global minimum (in fact global minimum might not even exists).