Multi-Step TD Target

Shusen Wang

Sarsa versus **Q-Learning**

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- TD target: $y_t = r_t + \gamma \cdot Q_\pi(s_{t+1}, a_{t+1})$.

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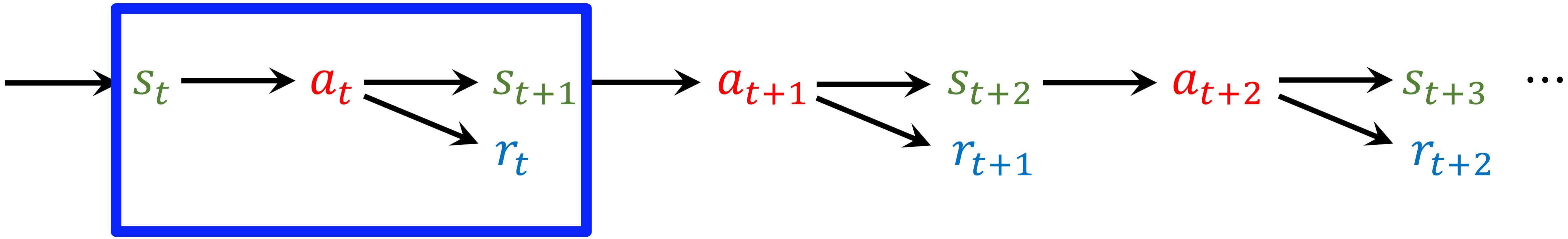
Using One Reward



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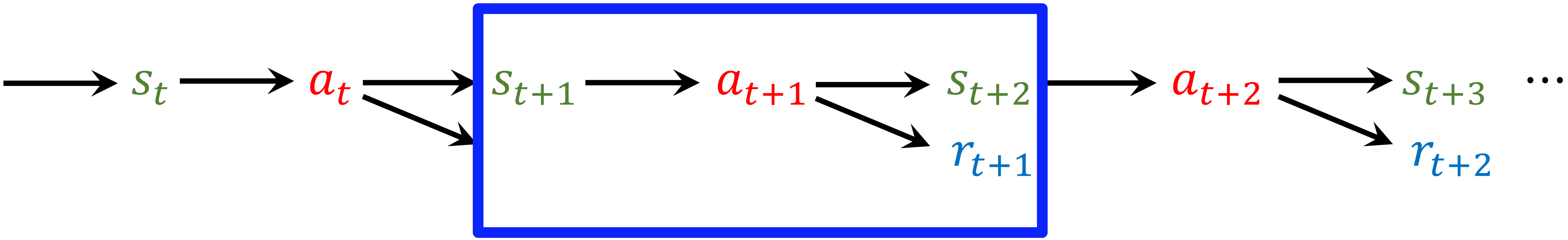

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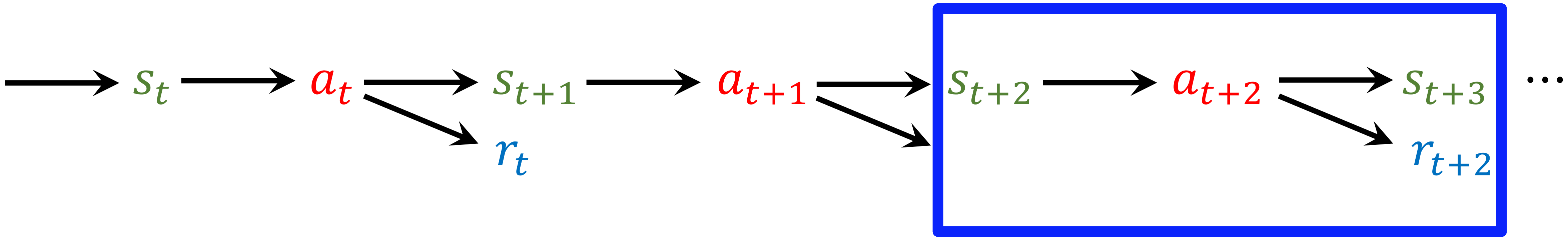
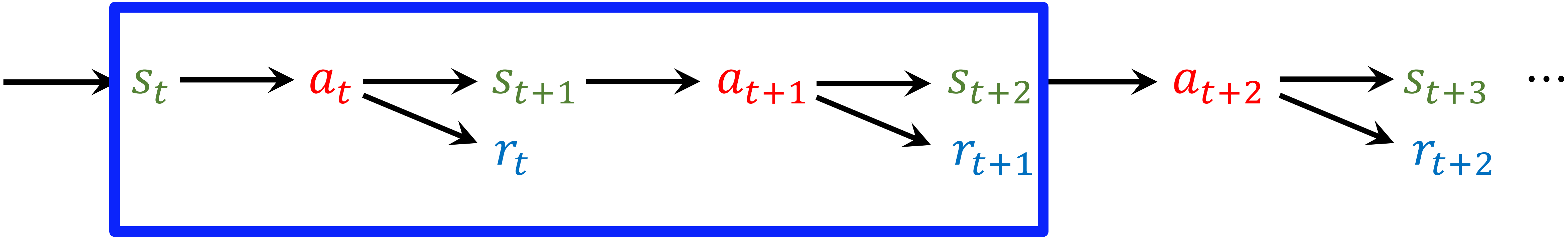
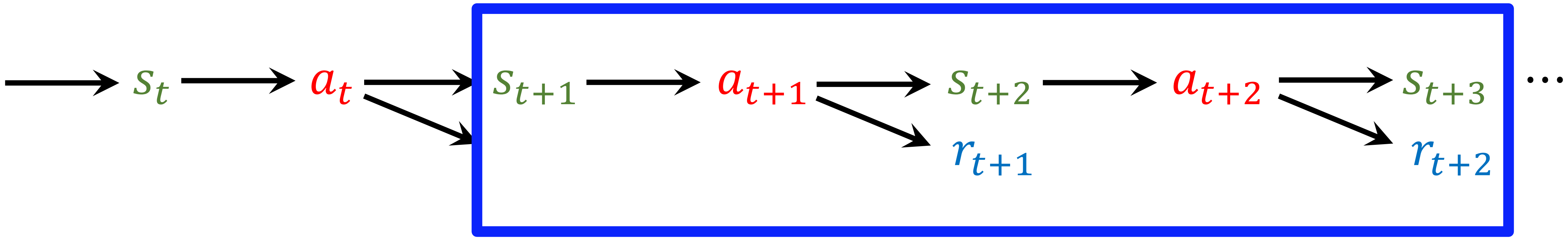


Using Multiple Rewards



Using Multiple Rewards



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One-Step versus Multi-Step

- One-step TD target uses only one reward: r_t.
- m-step TD target uses m rewards: $r_t, r_{t+1}, r_{t+2}, \cdots, r_{t+m-1}$.
- If m is suitably tuned, m-step target works better than one-step target [1].

Reference:

1. Hossel et al. Rainbow: combining improvements in deep reinforcement learning. In *AAAI*, 2018.

Thank you!