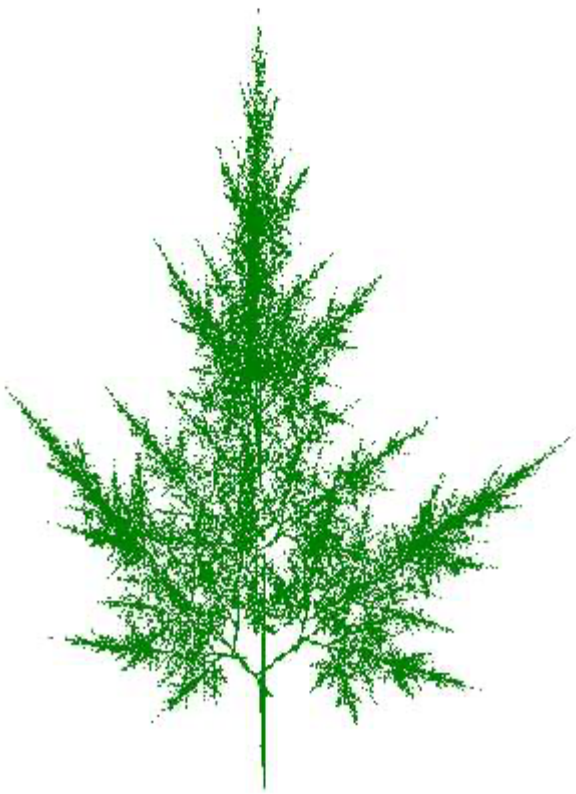
Calculation of the fractal dimension

Objective
To become aware of the fact (not very intuitive) that there are not only bodies in one, two or three dimensions, but others whose number of dimensions is decimal.

Required material
Tin foil, caliber, precision electronic balance, bilogarithmic paper, spreadsheet (e.g. MS Excel).

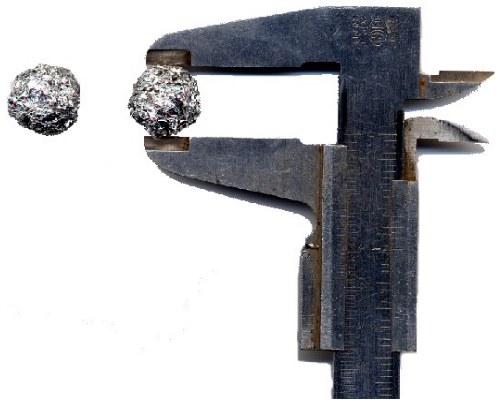
Theoretical premise
" Of all quantities, the one that has one dimension is the line, the one that has two dimensions is the surface, the one that has three dimensions is the body, and apart from these there are no other quantities », said Aristotle 23 and a half centuries ago.
Today it should change its mind, after the discovery of fractals by Benoit B. Mandelbrot.
A fractal is a structure such that a small part of it reproduces the whole structure on a reduced scale; this property is called self-similarity, and characterizes very different entities of the universe, from galaxies to trees, from human lungs to the frequency of wordusage in a newspaper.
To explain this unusual property, Mandelbrot generalizes some classic mathematical concepts, including that of the number of dimensions of a body. If it is calculated for regular objects (sheets, cylinders, spheres, etc...), it assumes the usual integer values referred to by Aristotle in the passage reported above; but, in the case of a structure such as the contour of the island of Sardinia, for example, one can realize that this is not the case. Indeed, it possesses a very large number of inlets and recesses, its length is therefore much greater than that which appears to the naked eye, and it has more than the single dimension associated with it by Aristotle. Thus, an irregular surface such as that of a sponge has more than just two dimensions. We can speak of 1.4 or 2.7 dimensions. Therefore, the difference in nature between different objects is traced back to the calculation of the difference between their dimensions.



The study of fractals has received great impetus in recent years, because only through the use of fractals is it possible to generate and make visible irregular and random objects endowed with self-similarity, just as it is only with computer simulations that it is possible to reconstruct landscapes and objects that do not exist starting from this basic property (see fir on previous page, non-existent in nature but recreated by computer simulation). The aim of this experience will be to show the calculation of the fractal dimension using poor materials, how can be tin aluminum foils, to demonstrate that this problem concerns not only mathematics scholars but anyone who wants to approach the investigation of reality in correct terms.

Observations

Take 10 fragments of tin aluminum foil of various sizes. Roll them up into a ball with the same degree of compression and obtain from them the same number of non-uniform spheres in both radius and density.
Then, measure the mass of these10 spheres with a precision electronic balance. By means of a slider caliber, measure their average diameter, making at least five measurements for each sphere and taking the arithmetic mean.



Treatment of experimental data

By plotting on the x-axis of a bilogarithmic chart the values of the mean radii, and on the y-axis of the same chart the values of the masses of the ten tinfoil spheres, it can be seen that the graph thus obtained is a straight line. It corresponds to a relationship between mass M and mean radius r of the type:

$$M = k \cdot r^{\alpha} \qquad (1)$$

where k is a constant dependent on the material used, in this case aluminum, and α is called the **fractal dimension**.
If, instead of aluminum foil balls, we had used solid and homogeneous aluminum spheres, their volume would be given by:

$$V = \frac{4}{3}\pi r^3$$

And, said ρ the density of aluminum, we would easily have:

$$M = \rho \cdot V = \frac{4}{3}\pi \rho r^3$$

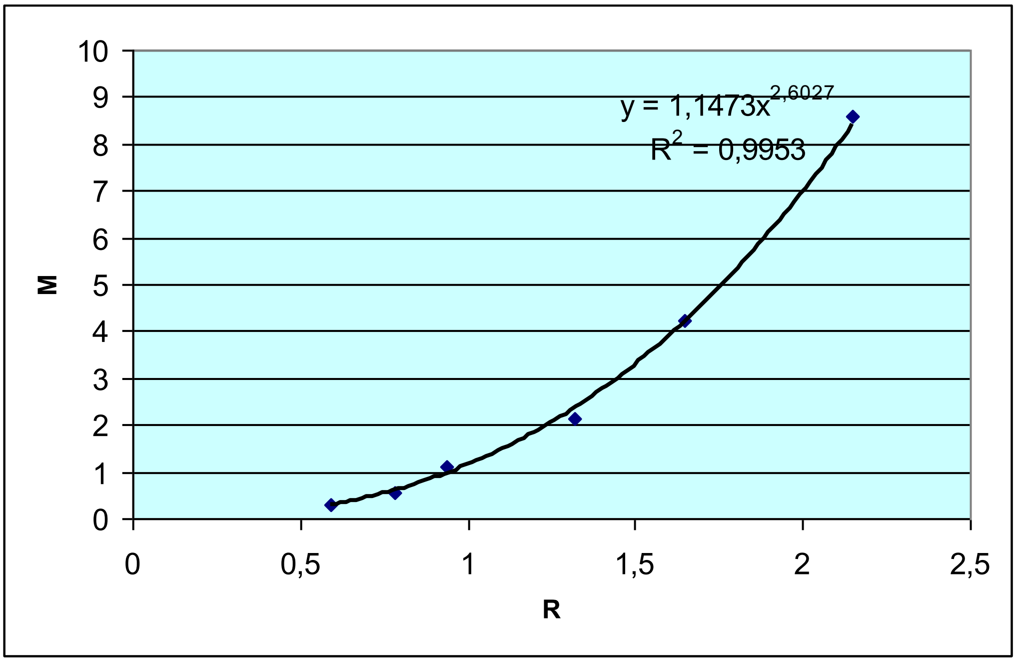
It follows from (1) that the fractal dimension of these full spheres is equal to 3.
If, on the other hand, the sphere were homogeneous but internally hollow, it would only count its surface $S = 4\pi r^2$, and then $M = 4\pi \rho r^2$; its fractal dimension would therefore be 2.
The purpose of this experience is to demonstrate that the fractal dimension of our aluminum tinfoil spheres, neither completely full nor completely empty, is between 2 and 3:

$$2 < \alpha < 3$$

α is calculated as the angular coefficient of the line drawn above, while *k* is the intercept of this line with the y-axis, because it is the value corresponding to ln x = 0, i.e. to x = 1.
Here are some values we obtained and tabulated by us (Mis.= measure; D.m.= average diameter; R.m. =average radius). R is express in cm, M in grams:

Ball	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5	D.m.	R.m.	Mass
# 1	1.20	1.15	1.17	1.21	1.18	1.182	0.591	0.30
# 2	1.52	1.47	1.53	1.62	1.70	1.568	0.784	0.55
# 3	2,35	2.40	2.28	2.47	2.21	1.423	1,871	0,936
# 4	2.48	2.36	3.00	2.82	2.53	2.638	1.319	2.15
# 5	2.91	3.46	3.32	3.22	3.55	3,292	1,646	4.25
# 6	4.30	4.01	4.47	4.36	4.35	4.298	2.149	8.60

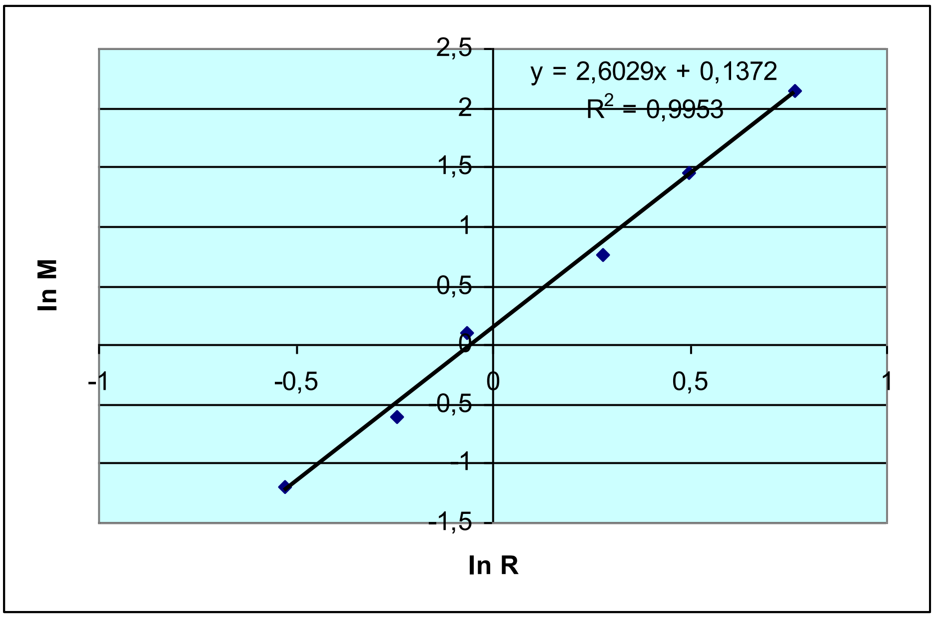
Instead of the bilogarithmic chart, data analysis can be advantageously conducted through the use of the spreadsheet. For this purpose, we report the data obtained above in MS Excel and directly calculate the diameter and radius using the corresponding, simple functions. Then we select the radius and mass columns and use them to build a graph (Insert + Graph). We choose the type Dispersion without lines; after making it, click on any of the points on the graph and choose the option Add Trend Line. We adopt the Power type and check, in the Options folder, the boxes Displays the equation on the graph and displays the value of R squared on the graph. Here's what we got:



R^2 is the parameter that measures how much the points deviate from the expected equation. The fact that here it is equal to 99.53% tells us that the match is excellent. Automatically, and without the need to go to logarithms, we immediately obtain that (1) in our case is:

$$M = 1{,}1473 \cdot r^{2{,}6027}$$

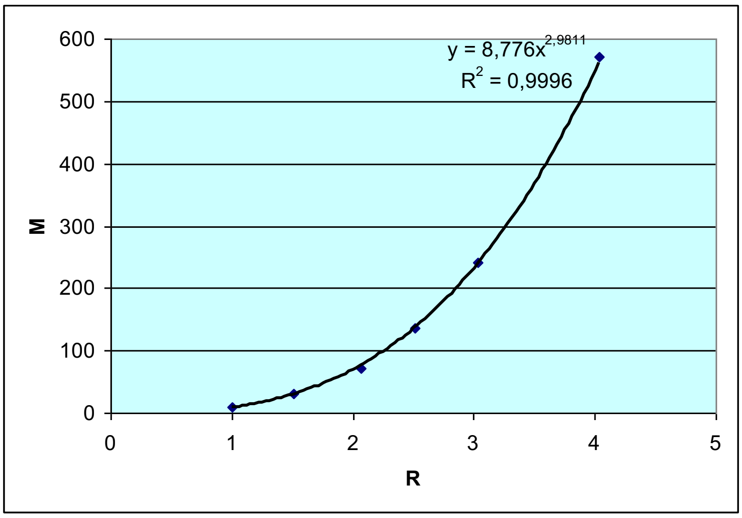
And therefore $\alpha = 2.6$. We have shown that indeed there are objects, even very common ones, whose size is intermediate between 2 and 3!
However, we have also built a bilogarithmic diagram by calculating the logarithms of R and M, and here is the satisfactory result obtained:



After having performed all these measurements and analyzes, it is convenient to repeat them also for full objects of defined shape.
In our case we can used copper cubes whose Edge and Mass are measured. Here are some data we obtained:

Cube	Measure 1	Measure 2	Measure 3	Edge	Mass
# 1	1.00	1.00	1.00	1,000	9
# 2	1.52	1.50	1.52	1.513	30
# 3	2.00	2.09	2.10	2.063	72
# 4	2.50	2.50	2.52	2.507	137
# 5	3.03	3.02	3.03	3.027	241
# 6	4.05	4.05	4.01	4.037	571

Here too the Mass is measured in grams and the average Edge in cm (but, being the regular figure, we have limited ourselves to three measures). The resulting graph is:



The (1) this time becomes:

$$\boldsymbol{M = 8{,}776 \cdot r^{2{,}9811}}$$

It can be seen that this time α is worth practically 3. The multiplication constant, on the other hand, is very close to the density of copper, which is 8,96 g/cm^3!

Extract from a lecture of
Prof. Franco Maria Boschetto
franco.boschetto@tin.it
www.fmboschetto.it

Experimental experience n° 1:

Title: Dimensionality of a body.

Date 17/10/2022

From 10:30 to hours 13:30

Purpose: **Verify the dimensionality of two bodies.**

Linear density : $$\mu = \frac{m}{L}$$

Surface density : $$\sigma = \frac{m}{S}$$

Volumic density : $$\rho = \frac{m}{V}$$

Being $S = L^2$ and $V = L^3$

We can write more generally the density as: $$\mathrm{k} = \frac{\mathrm{m}}{L^{\alpha}}$$

from which we can write: $$\boxed{m = kL^{\alpha}}$$

The variables are **m** (mass) and **L** (characteristic length) while k (density) and α (dimensionality) are the parameters to be defined.

Tools and materials to use:

- Precision balance
- Caliber
- Micrometer
- Drawing Rule and / or square
- Scissors
- Aluminum foil

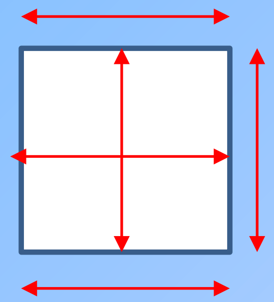
Remember to always write the value of a quantity together with its error!

$$X \pm \Delta X$$

Part I - Aluminum foil squares (how to proceed):

- Cut a square of aluminum foil (3cm x 3cm);
- Weigh its mass **m** with the precision balance;
- Repeat the measurement at least 3 times;
- Evaluate $\bar{m} \pm \Delta\bar{m}$

- Measure the sides **L** of the square and the two medians (as if they were two other sides);



- Evaluate $\bar{L} \pm \Delta\bar{L}$

REPEAT FOR 10 SQUARES WITH SIDE VALUES RANGE

FROM 3cm UP TO 25 ÷ 30cm

ALWAYS USE THE SAME ALUMINUM ROLL

Record all measurements in a tables

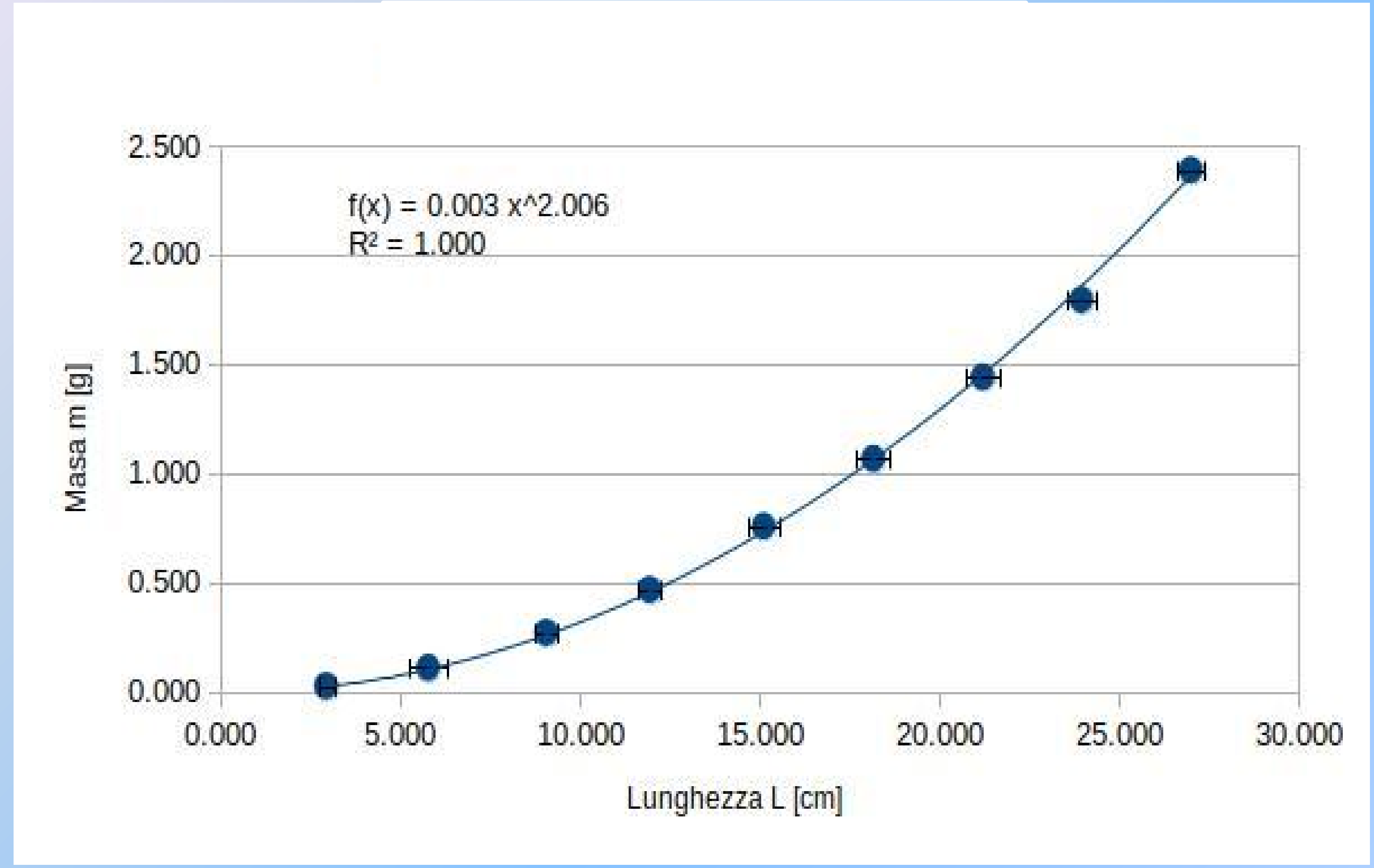
Square (cm)	$m_1(g) \pm \Delta m_1$	$m_2(g) \pm \Delta m_2$	$m_3(g) \pm \Delta m_3$
3 x 3	0,03 ± 0,01	0,03 ± 0,01	0,03 ± 0,01
6 x 6	0,11 ± 0,01	0,12 ± 0,01	0,13 ± 0,01
9 x 9	0,28 ± 0,01	0,29 ± 0,01	0,29 ± 0,01
12 x 12	…	…	…
15 x 15	…	…	…
18 x 18	1,13 ± 0,01	1,14 ± 0,01	1,13 ± 0,01
…	…	…	…

Same table for the side measurements.

Record all calculations in a table

Square [cm]	$(\bar{m}_i \pm \Delta\bar{m}_i)$ [g]	$(\bar{L}_i \pm \Delta\bar{L}_i)$ [cm]
3x3	(0,027±0,006)	(2,933±0,047)
6x6	(0,110±0,000)	(5,773±0,256)
9x9	(0,270±0,000)	(9,050±0,111)
12x12	(0,467±0,006)	(11,934±0,095)
15x15	(0,757±0,015)	(15,104±0,189)
18x18	(1,067±0,006)	(18,152±0,223)
21x21	(1,440±0,000)	(21,200±0,239)
24x24	(1,793±0,012)	(23,942±0,186)
27x27	(2,387±0,050)	(27,017±0,160)

Plot m_i as function of L_i (draw the error bars)



You will notice that the graph represents a power law.

Linearization of $m = kL^{\alpha}$

For the calculation of **k** and **α** it is convenient to linearize the law that links **m** and **L**.

$$ln\, m = ln\, kL^{\alpha}$$

$$ln\, m = ln\, k + \alpha ln\, L$$

We can rewrite this relation as :

$$Y = AX + B$$

where is it:

$$Y = ln\, m \qquad A = \alpha \qquad X = ln\, L \qquad B = ln\, K$$

Calculate the logarithms of the data of the average values (m , L) with the relative errors and report them in a second list

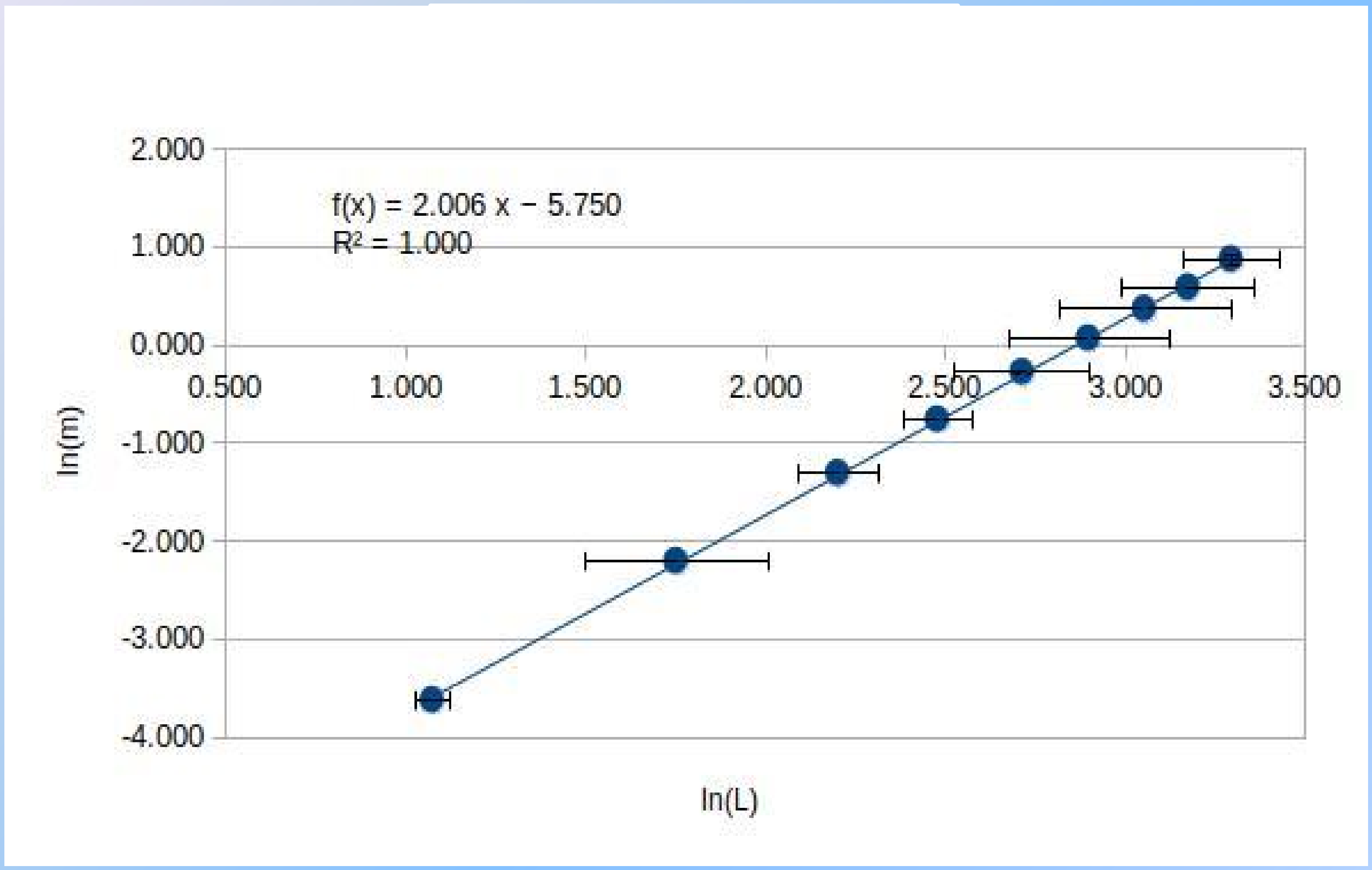
ln(m)=Y	ln(L)=X
-3,624	1,076
-2,207	1,753
-1,309	2,203
-0,762	2,479
-0,279	2,715
0,065	2,899
0,365	3,054
0,584	3,176
0,870	3,296

Plot ln (m) as a function of ln (L) on a graph

(draw the error bars)

Now the graph represents a linear law.

Plot ln(m) as function of ln(L)



What to do now:

1. Apply the "method of least squares” to the data obtained from linearization to calculate the slope of the line, the intercept and the related errors (obtaining k and α).

2. Using the software of your choice, fit the data represented by the power law to obtain the parameters k and $\boldsymbol{\alpha}$**.**

3. Calculate the regression coefficient R^2 in both cases.

Comparison with the expected values:

- Discuss the obtained values of the parameter α in both cases and compare the obtained values with the expected value.

- Using the intercept value, obtain $k \pm \Delta k$

Part II - crumpled aluminum squares (how to proceed):

- Crumple, very compactly, with the same pressure, to obtain small balls, the aluminum squares previously used

Measure several times (at least 6 times) the typical dimension d (diameter) of each ball at different points and then calculate $\bar{d}_i \pm \Delta\bar{d}_i$

For the mass use the previously measured values $\bar{m}_i \pm \Delta\bar{m}_i$

As in the case of aluminum squares:

- Create list of the acquired data;
- Draw the graph (note the power law);
- Linearize and create list 2 of the data;
- Draw graph;
- Use the least squares method to derive the parameters of the line;
- Fit of the power law data;
- Calculate the regression coefficient R2 in both cases;
- Discuss the results obtained of the parameters α;

Experimental experience   n° 2:

Wired logic and programmed logic.

Date 07/11/2022

From 10:30 to hours 13:30

Purpose:

Compare wired logic
with the programmed logic.

PART 1[a]

Create a wired circuit that turns on an LED and keeps it on for a time τ.

Using a chronometer measure the time the LED stays on.

PART 2[a]

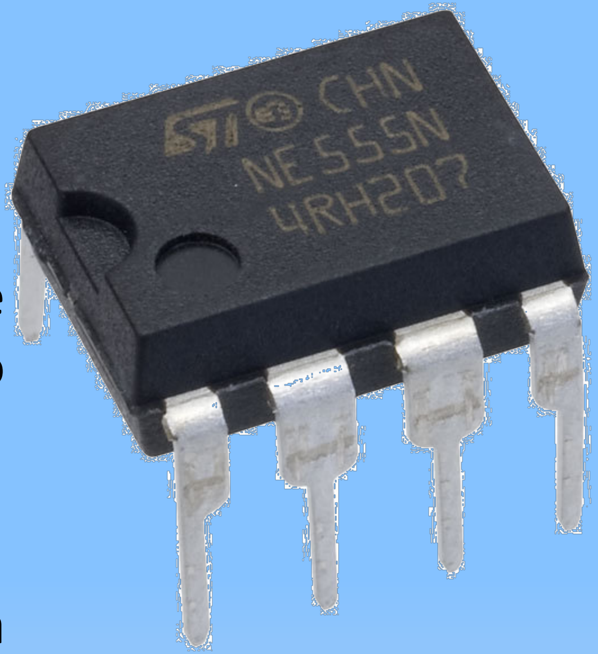
Turn on a Led using an Arduino board.

Measure in several ways the duration of LED on.

Instrumentation:

- Variable power supply
- Chronometer
- Miscellaneous electronic components
- Arduino UNO board
- Raspberry computer

Wired logic



Multivibrators are electronic circuits that have the characteristic of being able to be in one of two possible states.

The **NE555** is an integrated circuit containing a multivibrator that can be configured as:

- monostable (timer)
- astable (oscillator - generate a clock signal)
- bistable (flip-flop)

This integrated circuit is used in numerous applications.

Wired logic

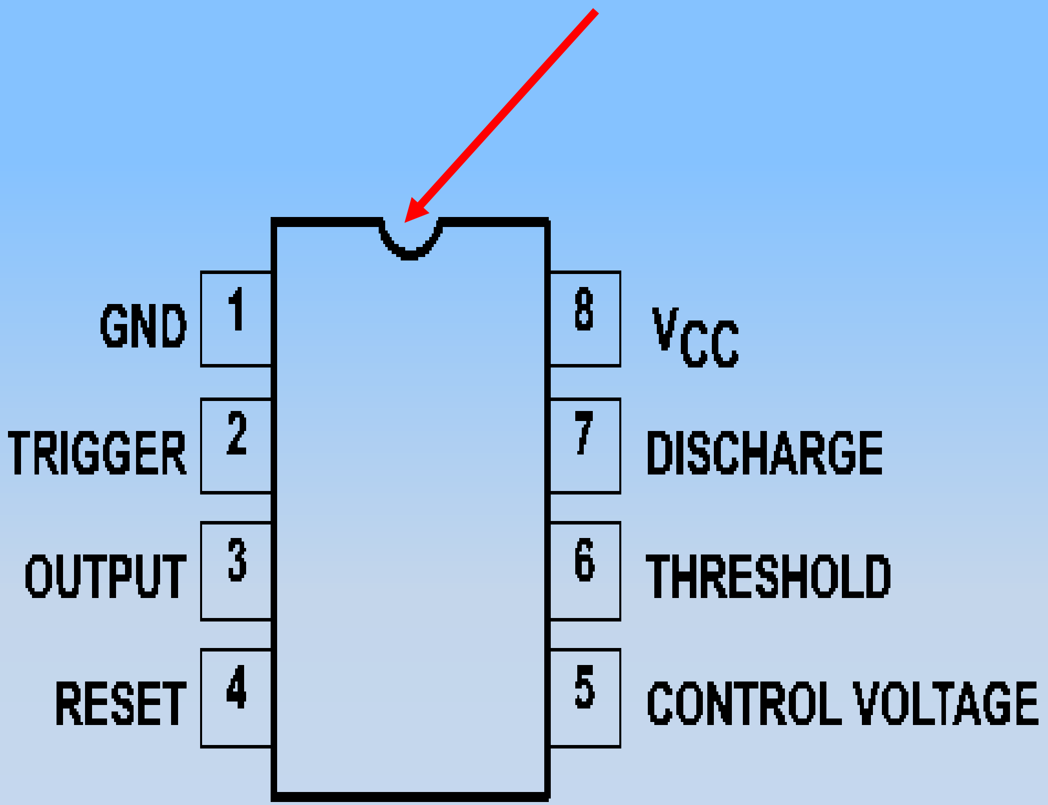
The NE555 integrated circuit

The heart of the wired circuit to be built is constituted by the NE555 used in monostable mode.

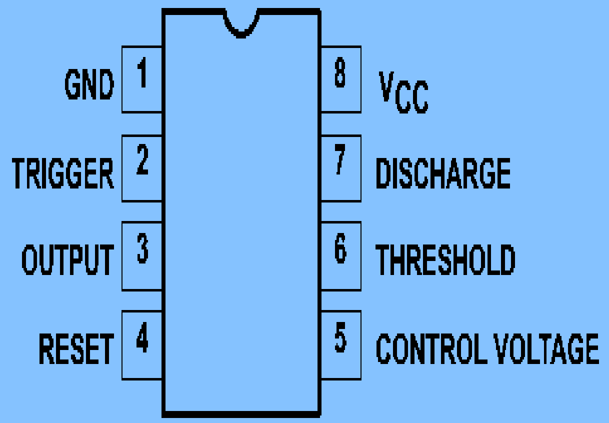
A monostable multivibrator which is in the stable state A, if perturbed, goes to the non-stable state B, keeps it for the time τ, then returns to the stable state A.

Wired logic

Pinout of the NE555 (note the curved recess at the top).

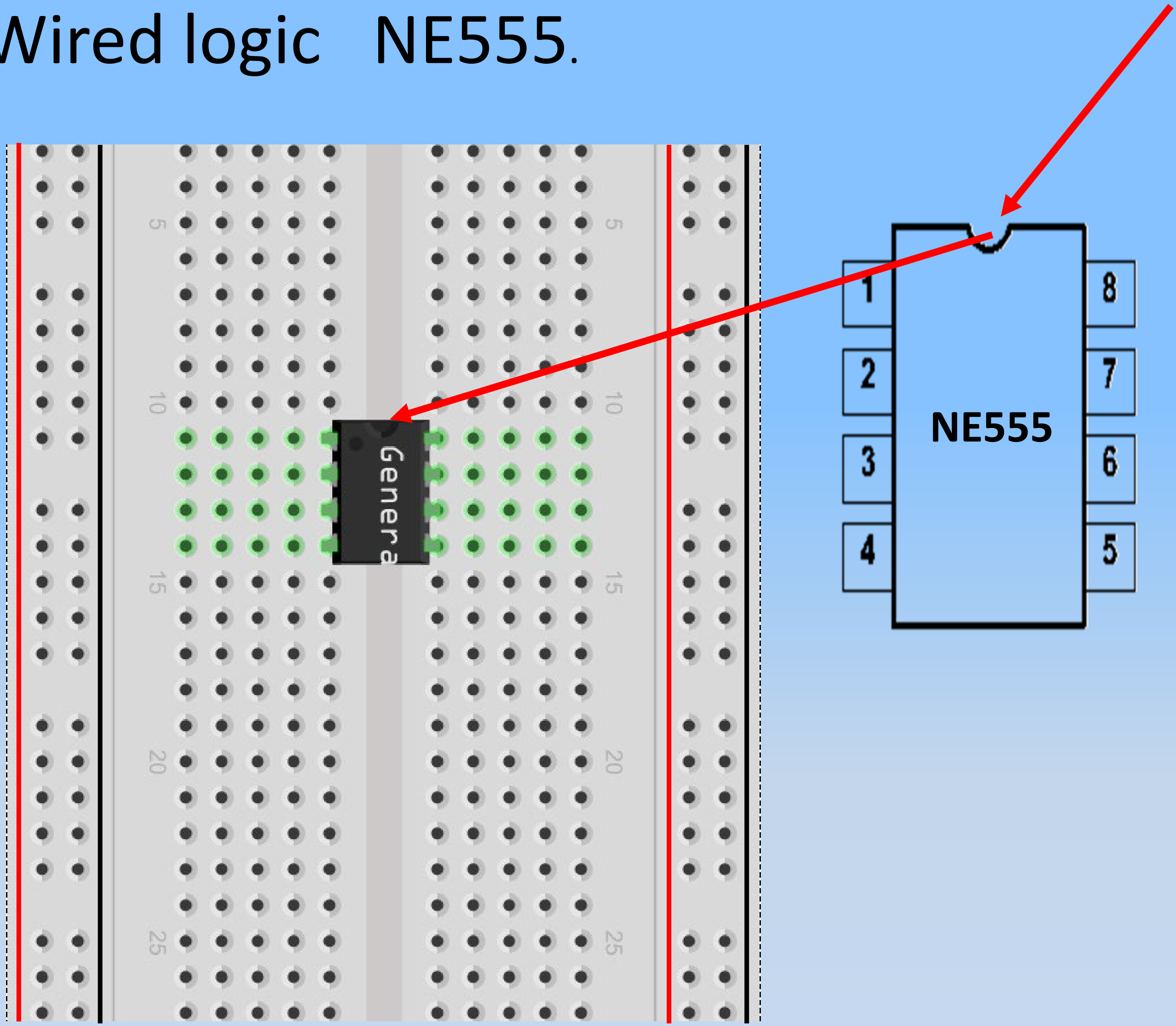


Pinout NE555



- **Ground:** To be connected to the power supply negative..
- **Trigger:** "start" terminal through which the output can be activated.
- **Output:** integrated output during the timing interval.
- **Reset:** reset if connected to ground. If you don't want to reset connect it to Vcc.
- **Control Voltage:** normally to be connected to ground through a capacitor.
- **Threshold:** "stop“, disable the output.
- **Discharge:** discharge pin of the timing capacitor.
- **Vcc:** power supply, can be between 4.5V and 18V (absorbs a current of 15mA with Vcc of 15V and no-load output).

Wired logic NE555.



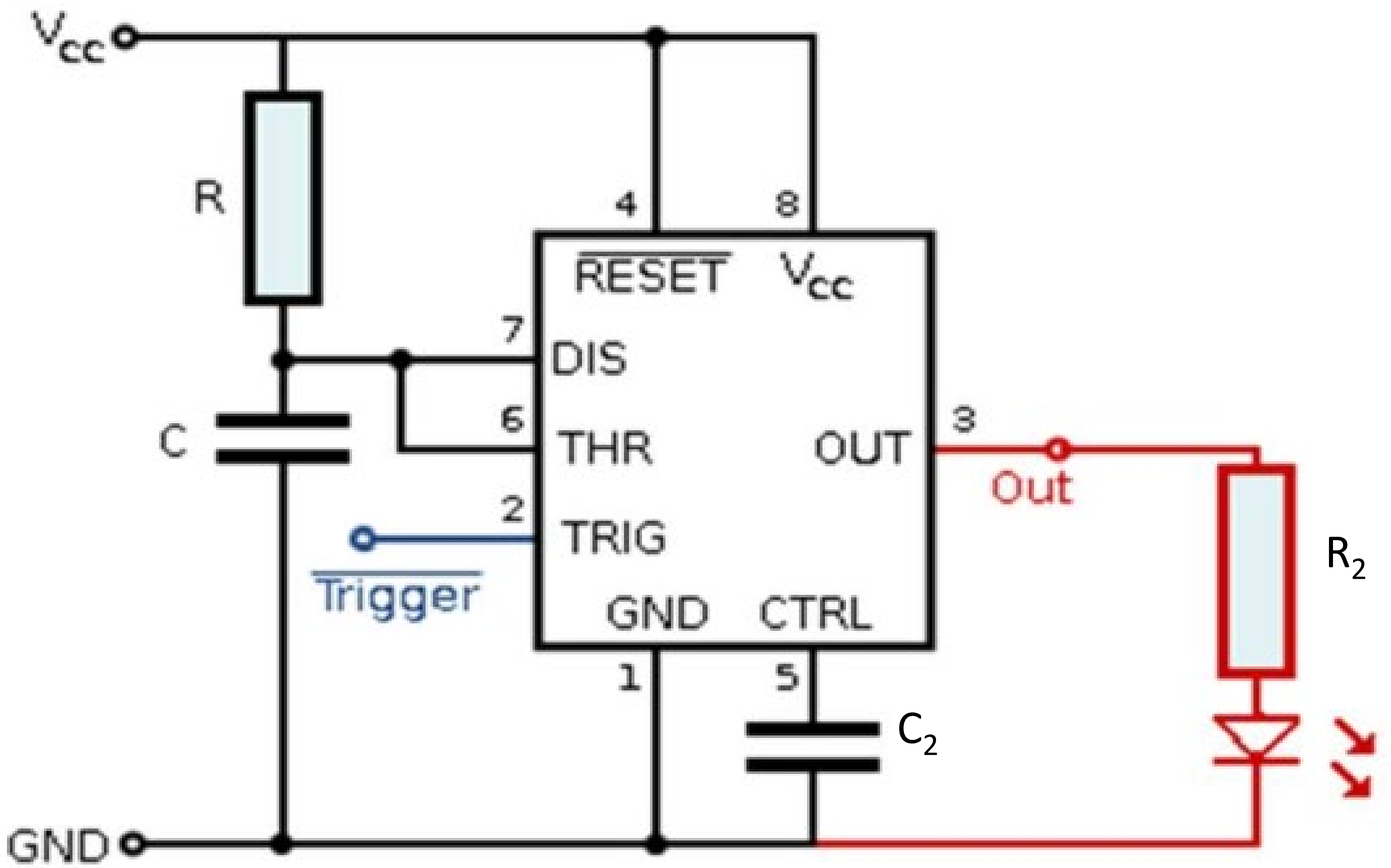
Components :

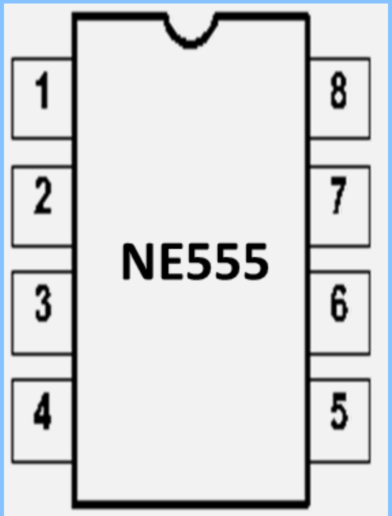
- Diode LED
- NE555
- $C = 100\ \mu F = 10^{-4}\ F$
- $R = 100\ k\Omega = 10^5\ \Omega$
- $R_2 = 1\ k\Omega = 10^3\ \Omega$
- $C_2 = 100\ nF = 10^{-7}\ F$
- Breadboard
- Nippers

- Vcc power supply : 15 V_{CC}

Wired logic

Circuit diagram to be created.





$$\tau = 1.1RC$$

Note:

τ = 1.1*R*C (In this case τ = 11s).

R_2 used to limit the current that flows through the diode LED.

C_2 reduces noise that could affect the timer behavior.

C it is electrolytic type, pay attention to polarity.

LED : longer electrode = positive; short electrode = negative.

Trigger : **s**hortly touch wire connected to pin 2 (TRIG) with a wire connected to GND.

How to proceed :

1. Assemble the circuit.

2. Using a chronometer, measure the time that the LED lights up for at least 20 times.

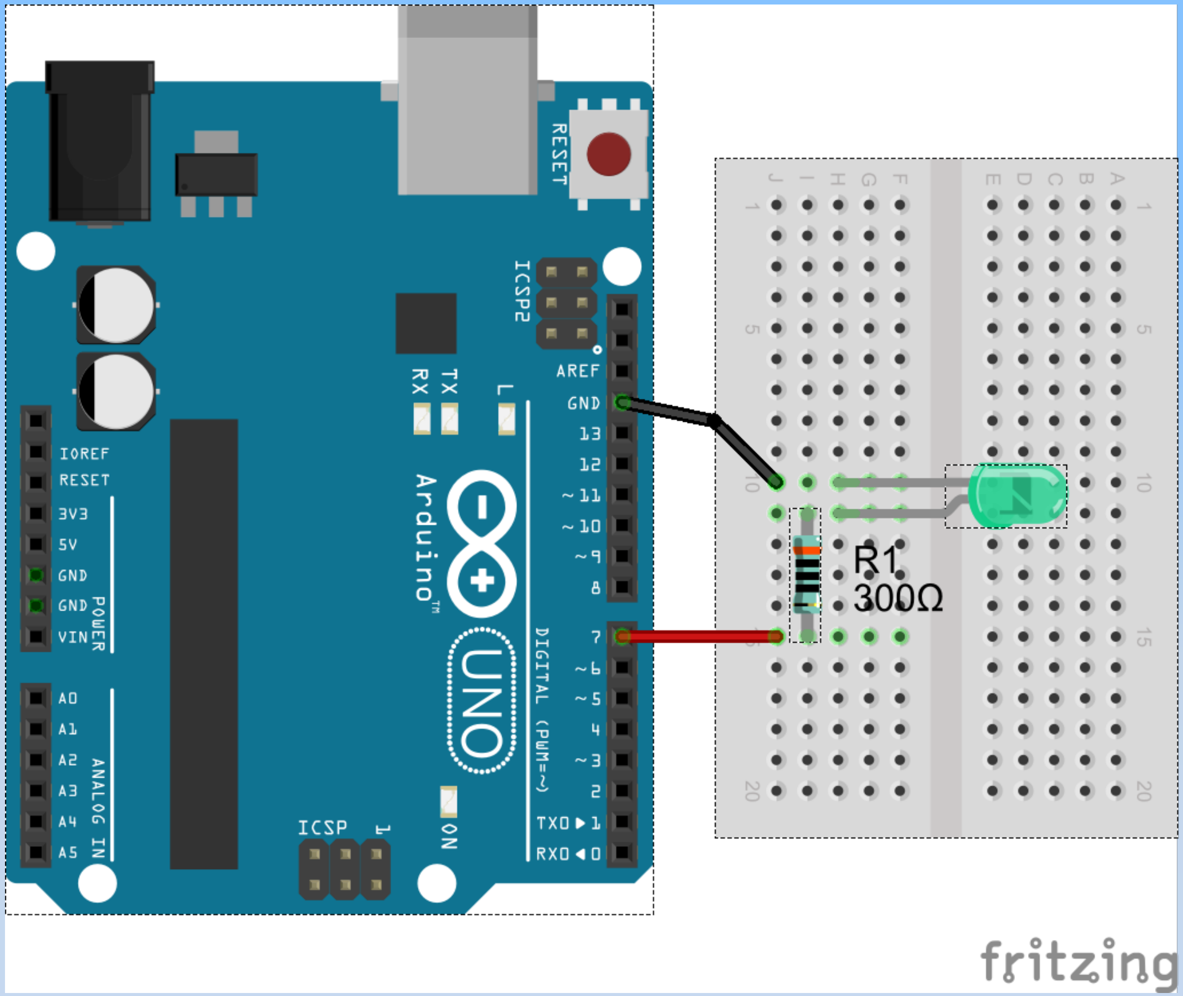
3. Calculate the average value.

4. Calculate the standard deviation.

The values obtained will be compared with those obtained using the circuit built in programmed logic.

Programmed logic

Circuit to be realized :



Led1 | Arduino 1.8.16

File Modifica Sketch Strumenti Aiuto

Led1

```
void setup() {
// put your setup code here, to run once:
  pinMode(7, OUTPUT);
  digitalWrite(7,LOW);
}

void loop() {
// put your main code here, to run repeatedly:
  digitalWrite(7, HIGH);          // TURN ON THE LED
  delay(11000);                   // WAIT 11 SECONDS
  digitalWrite(7, LOW);           // TURN OFF THE LED

  delay(10000);                   // WAIT 10 SECONDS
}
```

Using the chronometer, measure the LED lighting time for at least 20 times

Measurement of LED ignition time via Arduino

Acquire the time in ms (milliseconds) at least 20 times.

Led2 | Arduino 1.8.16

File Modifica Sketch Strumenti Aiuto

Led2

```
long t1,  t2;

void setup() {
// put your setup code here, to run once:
  Serial.begin(9600); // Sets the data rate in bits per second (baud)
                      // for serial data transmission
  pinMode(7, OUTPUT);
  digitalWrite(7,LOW);
}

void loop() {
// put your main code here, to run repeatedly:
  t1 = millis();                 // Number of milliseconds passed since the program started
  digitalWrite(7, HIGH);         // TURN ON THE LED
  delay(11000);                  // WAIT 11 SECONDS
  digitalWrite(7, LOW);          // TURN OFF THE LED
  t2 = millis();                 // Number of milliseconds passed since the program started

  Serial.println(t2-t1);         // CALCULATES AND SHOW t2-t1

  delay(10000);                  // WAIT 10 SECONDS
}
```

Measurement of LED ignition time via Arduino

Acquire the time in µs (microseconds) at least 20 times.

Led3 | Arduino 1.8.16

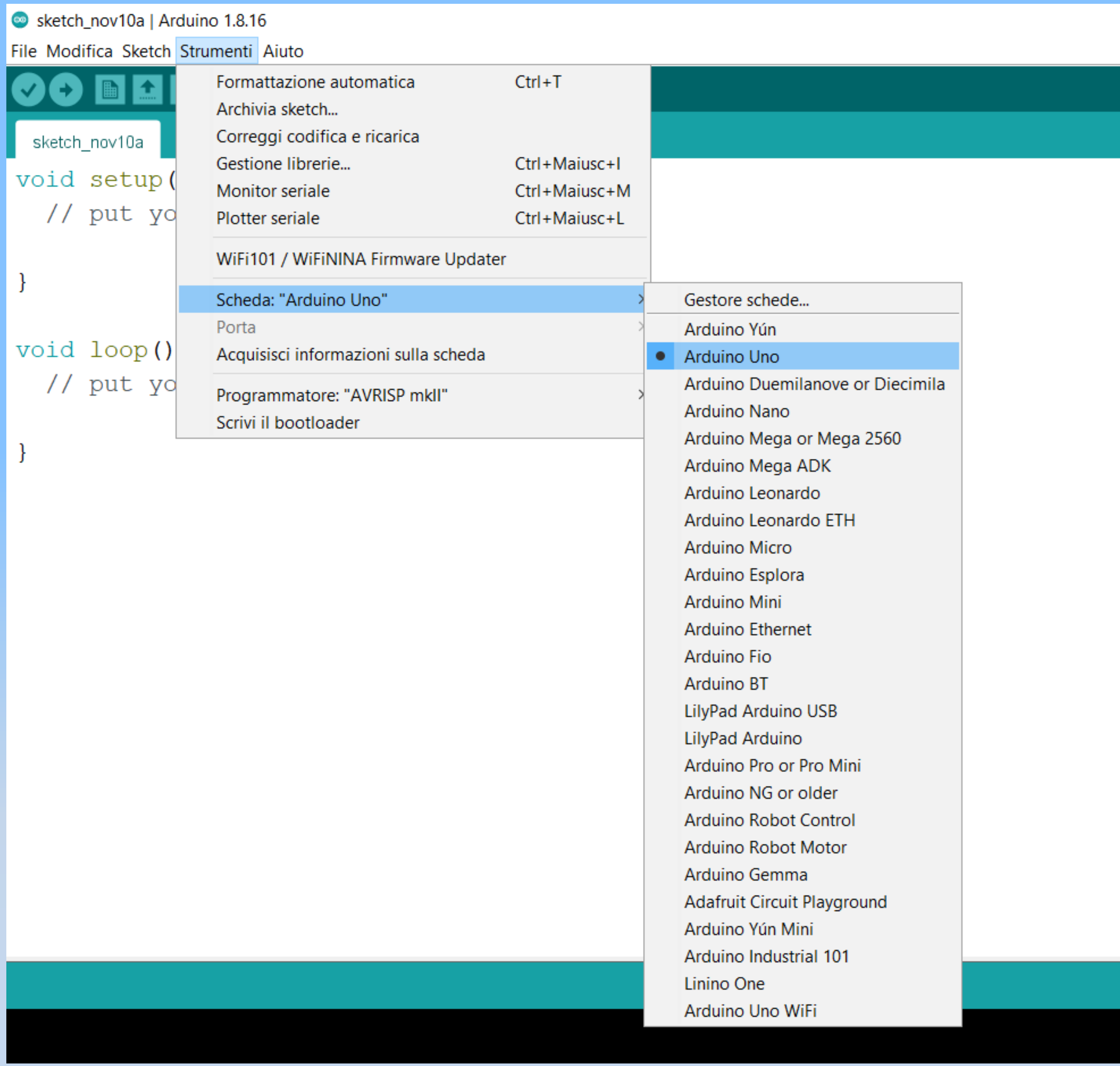
File Modifica Sketch Strumenti Aiuto

Led3

```
long t1,  t2;

void setup() {
// put your setup code here, to run once:
  Serial.begin(9600); // Sets the data rate in bits per second (baud)
                      // for serial data transmission
  pinMode(7, OUTPUT);
  digitalWrite(7,LOW);
}

void loop() {
// put your main code here, to run repeatedly:
  t1 = micros();                 // Number of milliseconds passed since the program started
  digitalWrite(7, HIGH);         // TURN ON THE LED
  delay(11000);                  // WAIT 11 SECONDS
  digitalWrite(7, LOW);          // TURN OFF THE LED
  t2 = micros();                 // Number of milliseconds passed since the program started

  Serial.println(t2-t1);         // CALCULATES AND SHOW t2-t1

  delay(10000);                  // WAIT 10 SECONDS
}
```

Report in the table, for each group, the measurements made (at least 20 times), the errors and the average results obtained.

Discuss the results obtained by comparing them with those of wired logic.


sketch_nov10a | Arduino 1.8.16
File Modifica Sketch Strumenti Aiuto
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void loop()
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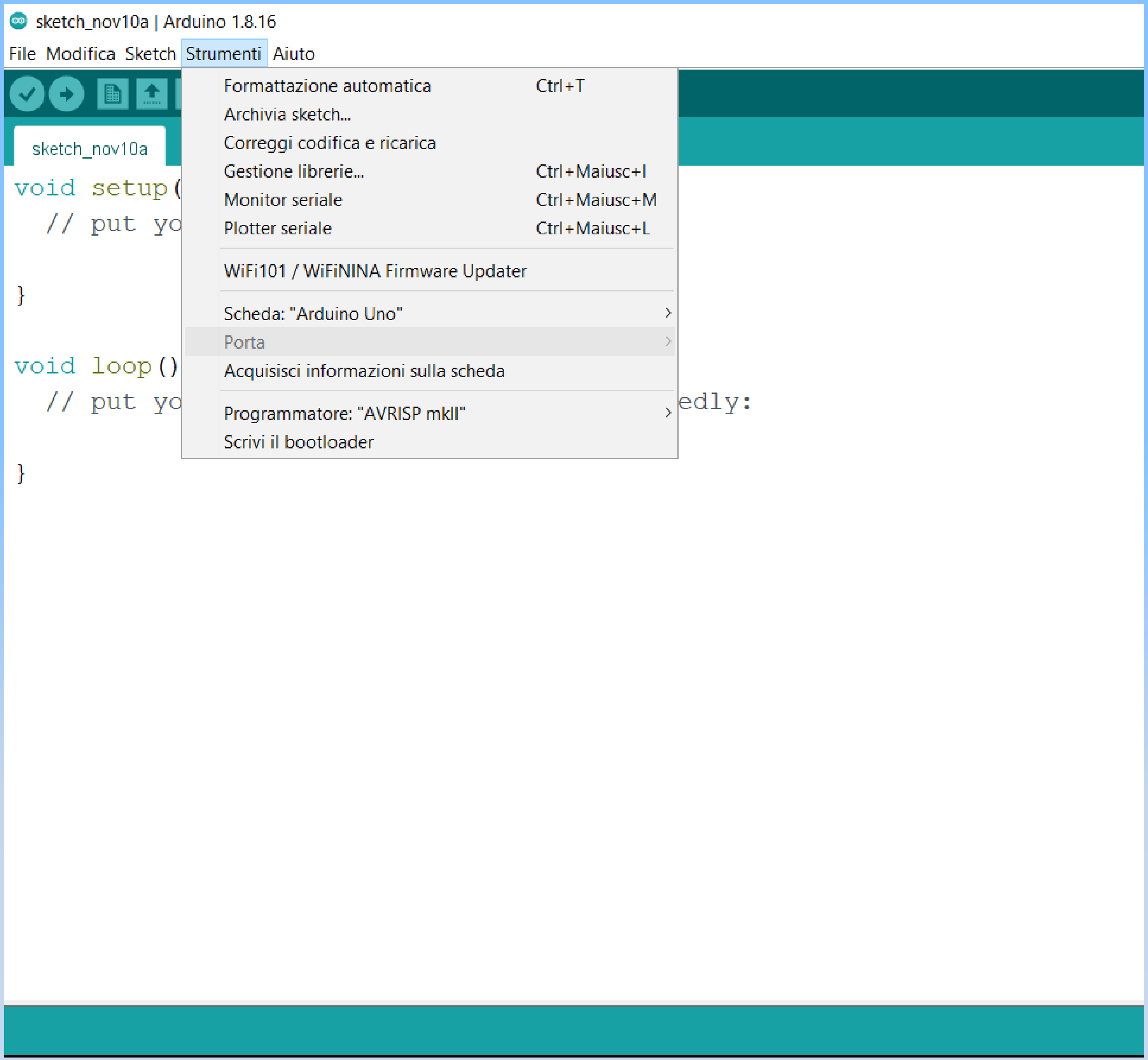
Monitor seriale Ctrl+Maiusc+M
Plotter seriale Ctrl+Maiusc+L
WiFi101 / WiFiNINA Firmware Updater
Scheda: "Arduino Uno"
Porta
Acquisisci informazioni sulla scheda
Programmatore: "AVRISP mkII"
Scrivi il bootloader
Gestore schede...
Arduino Yún
Arduino Uno
Arduino Duemilanove or Diecimila
Arduino Nano
Arduino Mega or Mega 2560
Arduino Mega ADK
Arduino Leonardo
Arduino Leonardo ETH
Arduino Micro
Arduino Esplora
Arduino Mini
Arduino Ethernet
Arduino Fio
Arduino BT
LilyPad Arduino USB
LilyPad Arduino
Arduino Pro or Pro Mini
Arduino NG or older
Arduino Robot Control
Arduino Robot Motor
Arduino Gemma
Adafruit Circuit Playground
Arduino Yún Mini
Arduino Industrial 101
Linino One
Arduino Uno WiFi


sketch_nov10a | Arduino 1.8.16
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Scheda: "Arduino Uno"
Porta
Acquisisci informazioni sulla scheda
Programmatore: "AVRISP mkII"
Scrivi il bootloader

Compile the software and upload it to Arduino

Save the Sketch

Led2 | Arduino 1.8.16

File Modifica Sketch Strumenti Aiuto

Led2

```
long t1,  t2;

void setup() {
// put your setup code here, to run once:
  Serial.begin(9600); // Sets the data rate in bits per second
                      // for serial data transmission
  pinMode(7, OUTPUT);
  digitalWrite(7,LOW);
}

void loop() {
// put your main code here, to run repeatedly:
  t1 = millis();                 // Number of milliseconds pas
  digitalWrite(7, HIGH);         // TURN ON THE LED
  delay(11000);                  // WAIT 11 SECONDS
  digitalWrite(7, LOW);          // TURN OFF THE LED
  t2 = millis();                 // Number of milliseconds pas
```

Experimental experience   n° 3:

Signal and noise

Date 21/11/2022

Dalle ore 10:30 alle ore 13:30

Purpose:

Analyze a signal affected by noise and check if:

- the noise is “white noise”
- the standard deviation of the mean decreases as the square root of the number of acquisitions N

Noise:

- In experimental measurements it is a component joined to a signal.

- It can never be completely eliminated.

- It is distributed on all frequencies in a uniform way.

Setup:

Oscilloscope

Function Generator

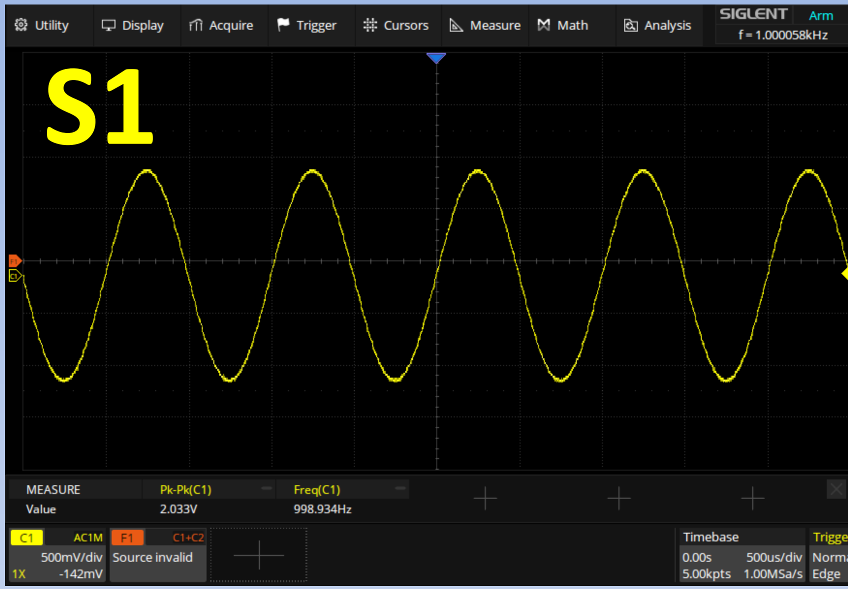
How to proceed :

1. Generate a sinusoidal signal with 1KHz frequency and 2Vpp amplitude (OUTPUT 1).

2. Using the OUTPUT 2 and the "noise" function of the function generator, generate the noise fixing the amplitude at 100mV.

3. Add or subtract the two signals.

4. Acquire the data of the obtained signal.



+

=

S1+S2

Signal to acquire

How to proceed :

- **Perform multiple samplings (e.g. 16) of the values of V (t)** (sum or difference of the two signals).

An acquisition is made up of 5000 points [t, V (t)].

We will see later how to save the data and how to use this new instrumentation.

Data processing 1:

Check if the noise is "white noise"

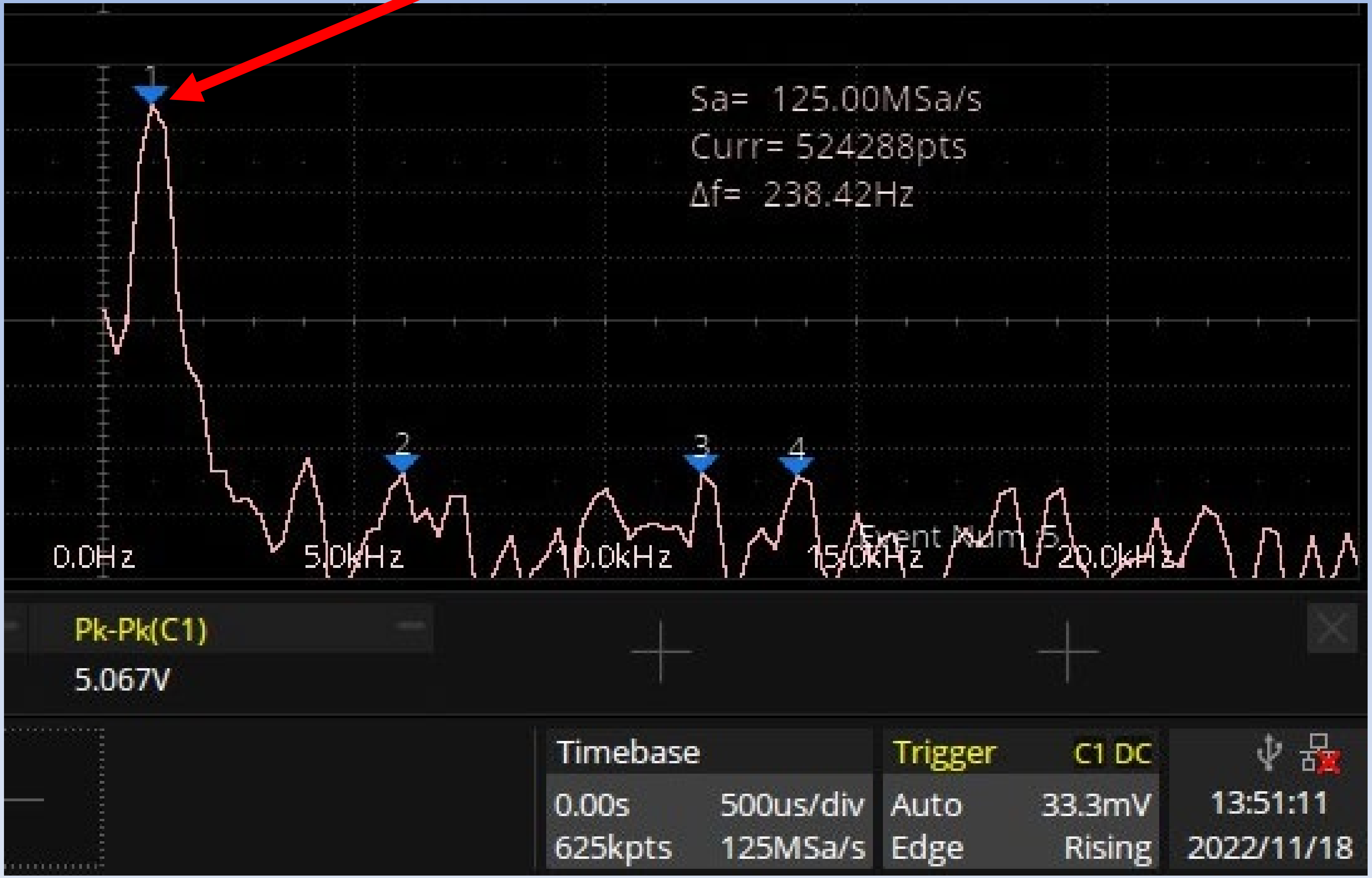
1. Use a suitable software to do the FFT (Fast Fourier Transform) of the data (only the first sampling).

2. Note, in the graph of FFT, the presence of a peak that corresponds to the frequency of the signal used.

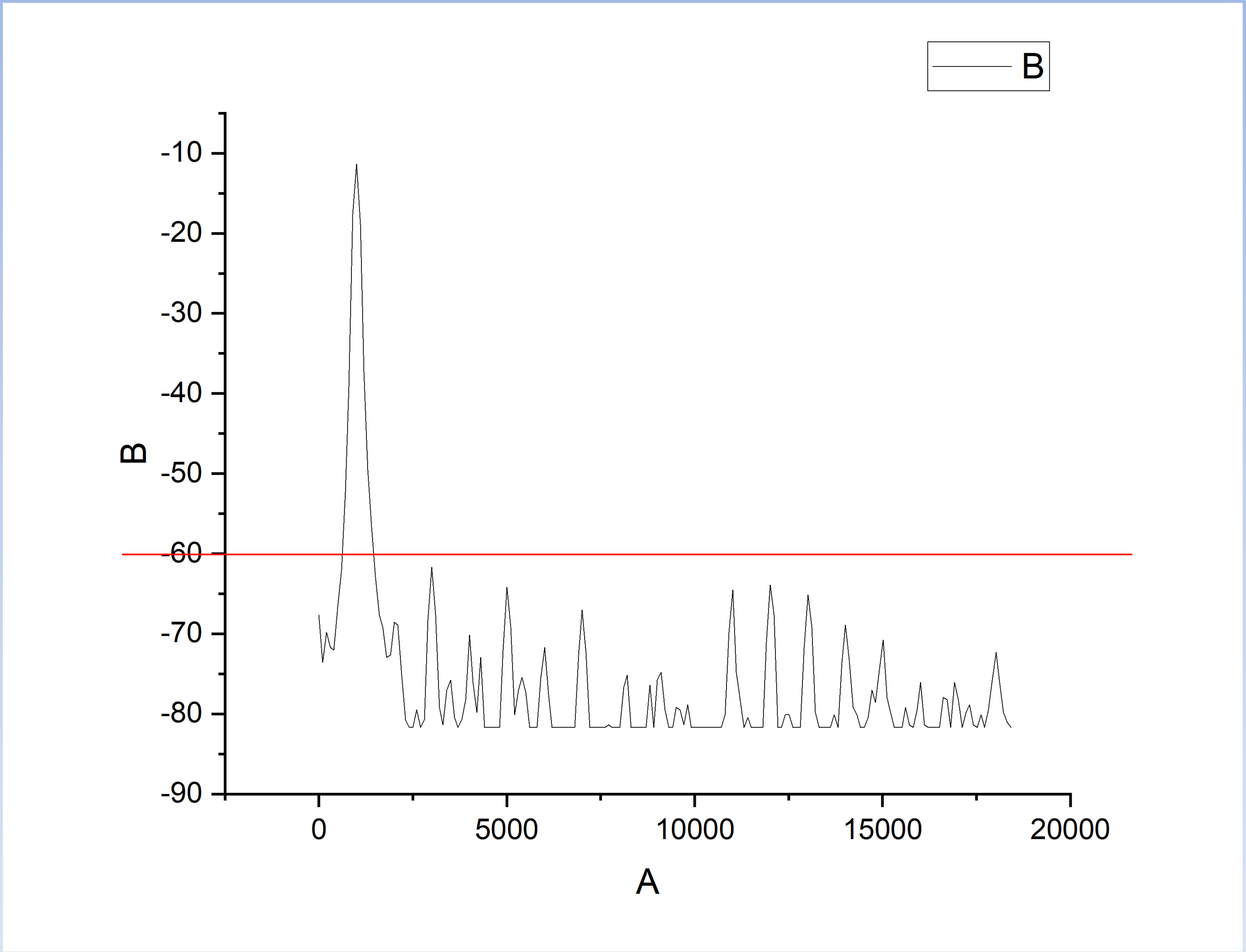
3. Eliminate this peak by substituting zero (expected average value) for those frequency values that generate the peak.

4. Do the IFFT (Inverse Fast Fourier Transform).

Peak to be removed



0	-67,625
100,1001	-73,5625
200,2002	-69,8125
300,3003	-71,6875
400,4004	-72
500,5005	-66,6875
600,6006	-62
700,7007	-52,3125
800,8008	-39,1875
900,9009	-17,625
1001,001	**-11,375**
1101,1011	-18,875
1201,2012	-37,3125
1301,3013	-49,5
1401,4014	-56,6875
1501,5015	-62,9375
1601,6016	-67,625
1701,7017	-69,1875
1801,8018	-72,9375
1901,9019	-72,625
2002,002	-68,5625
2102,1021	-68,875
2202,2022	-75,125
2302,3023	-80,75
2402,4024	-81,6875
2502,5025	-81,6875
2602,6026	-79,5
2702,7027	-81,6875
2802,8028	-80,75
2902,9029	-68,25
3003,003	-61,6875
3103,1031	-67,625



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900,9009	-75,625
1001,001	-70,375
1101,1011	-80,875
1201,2012	-77,3125
1301,3013	-69,5
1401,4014	-66,6875
1501,5015	-62,9375
1601,6016	-67,625
1701,7017	-69,1875
1801,8018	-72,9375
1901,9019	-72,625
2002,002	-68,5625
2102,1021	-68,875
2202,2022	-75,125
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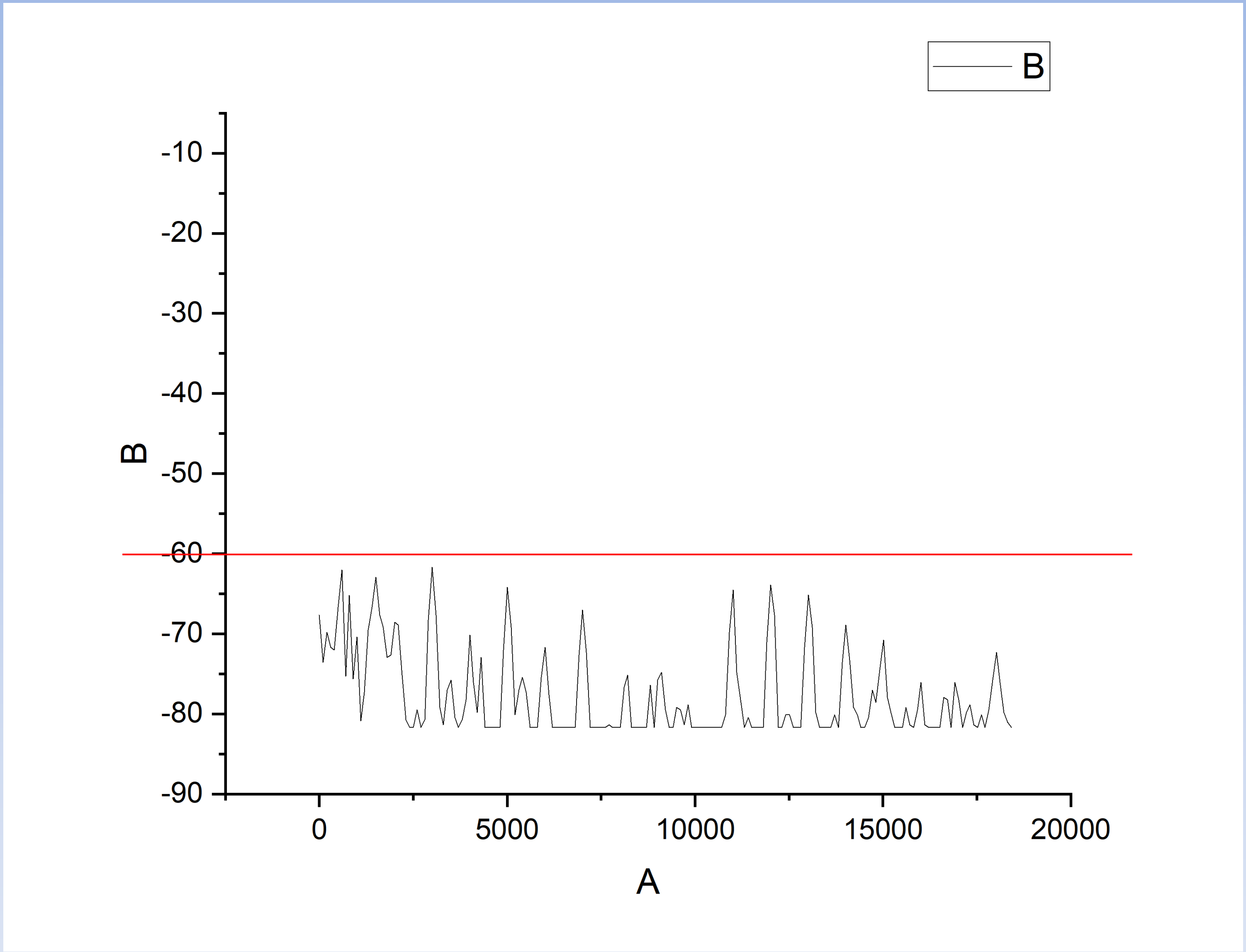
2502,5025	-81,6875
2602,6026	-79,5
2702,7027	-81,6875
2802,8028	-80,75
2902,9029	-68,25
3003,003	-61,6875
3103,1031	-67,625



Data processing 1:

Check if the obtained values (noise) are distributed in a Gaussian way (if it is white noise).

- Make the histogram;
- Calculate the average value <V>;
- Calculate the standard deviation σ_V ;
- Using the Chi-square test, compare the distribution obtained with the theoretical one.

(All this for a single acquisition).

Data processing 2:

In this second part we will see how the random error can be reduced (but not eliminated) by increasing the number of acquisitions.

$$\boxed{\sigma_V = \frac{k}{\sqrt{N}}}$$

Data processing 2:

- Isolate the noise for all other files.

- Add the noise files dot-by-dot in groups of 2, 4, 8, and 16.

(NB: the number of points in a file remains the same)

For each new grouping proceed as above:

- Calculate the average value <V>;

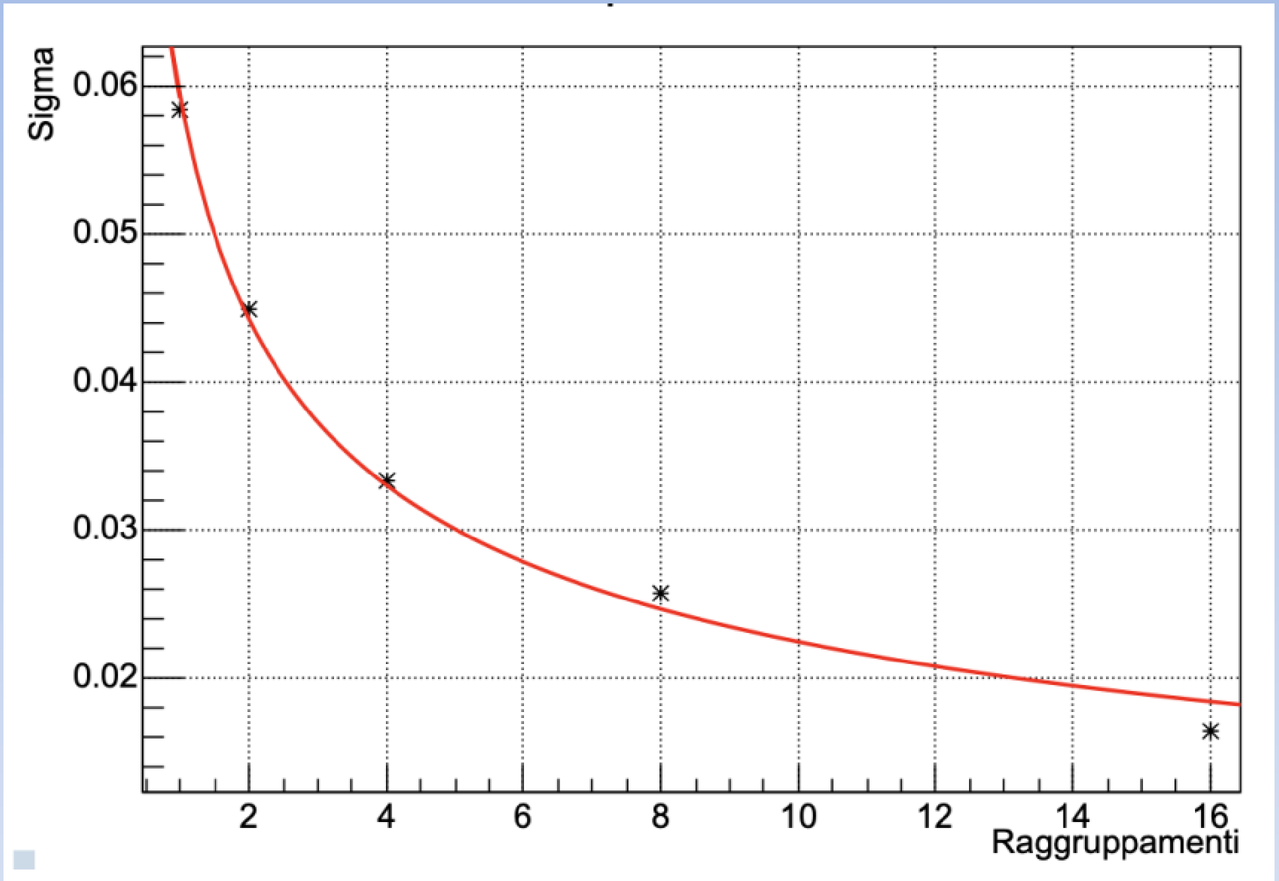
- Calculate the standard deviation σ_v .

Data processing 2:

N Groupings	<V>	$\langle \sigma_v \rangle$
1	V_1	σ_{V1}
2	V_2	σ_{V2}
4	V_3	σ_{V3}
8	V_4	σ_{V4}
16	V_5	σ_{V5}

Report the obtained values of <V> and σ_v in a table (including those of the first sampling).

Data processing 2:



Plot σ_v as a function of N.

Data processing 2:

We combined measures belonging to different acquisitions in groups of 1, 2, 4, 8 and 16 elements.

It can be seen that the $\langle V_i \rangle$ vary little (remain constant) while the $\langle \sigma_{Vi} \rangle$ tend to become smaller.

We must verify that the standard deviation of the mean decreases according to a power law:

$$\boxed{\sigma_V = kN^{\alpha}}$$

Data processing 2:

σ_{Vi} and N are known values, we must find K e α.

Then we proceed with the linearization of the law:

$$\boxed{\sigma_V = \mathrm{k}\mathrm{N}^{\alpha}}$$

As in the previous experience:

$$\boxed{ln\sigma_V = \ln\mathrm{k}\;\alpha ln\mathrm{N}}$$

Least squares method :

The values obtained from the linearization must be reported on a graph ($\ln\sigma_v$ vs $\ln N$) and with the least squares method estimate the value of α.

We expect it to be (within the error) $\alpha = -\frac{1}{2}$ because

the standard deviation of the mean decreases as the square root of the number of acquisitions N.

SDS5034X Oscilloscope

The SDS5034X supports saving setups, reference waveforms, screen shots, and waveform data files to internal storage or external USB storage devices.

BMP
Saves the screen shot to external memory in *.bmp format.

JPG
Saves the screen shot to external memory in *.jpg format.

PNG
Saves the screen shot to external memory in *.png format.

SDS5034X Oscilloscope

Binary Data
Saves the waveform data to external memory in binary (*.bin) format.

CSV Data
Saves the waveform data to the external memory in ".csv" format (**Comma-separated values**).

Matlab Data
Saves the waveform data to external memory in *.dat format which can be imported by Matlab directly.

How to set the number of points to process:

1) ACQUIRE
2) Menu
3) Mem Depth
4) 5 K (or 10K)

How to define a function:

1) MATH
2) Menu
3) Trace: F1 or F2
4) Operation: ON
5) Function: C1+C2

How to save the data:

1) SAVE

2) Save path: external

3) Type: CSV

4) File Manager

Experimental experience n° 4:

The temperature transducer.

Date 28/11/2022

From 10:30 to hours 13:30

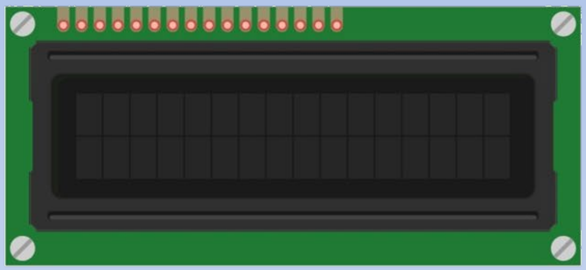
Purpose:

- **Use the LM35 transducer to monitor the temperature**
- **Use an LC display to show the data**
- **Use serial plotter to show graphs**

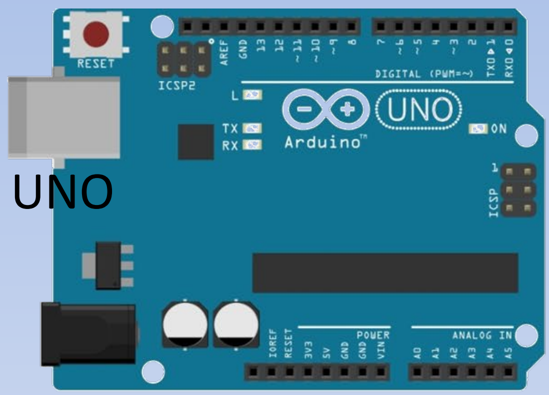
What to use:

RASPBERRY PI

Display LCD

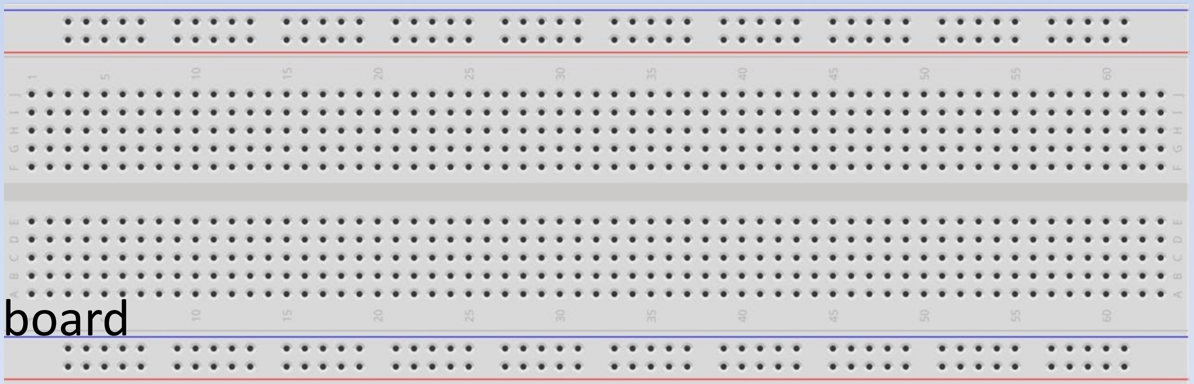


Arduino UNO



Temperature transducer

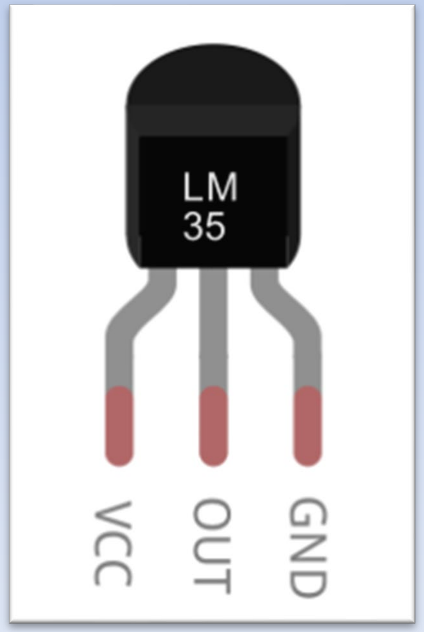
Breadboard



Potentiometer

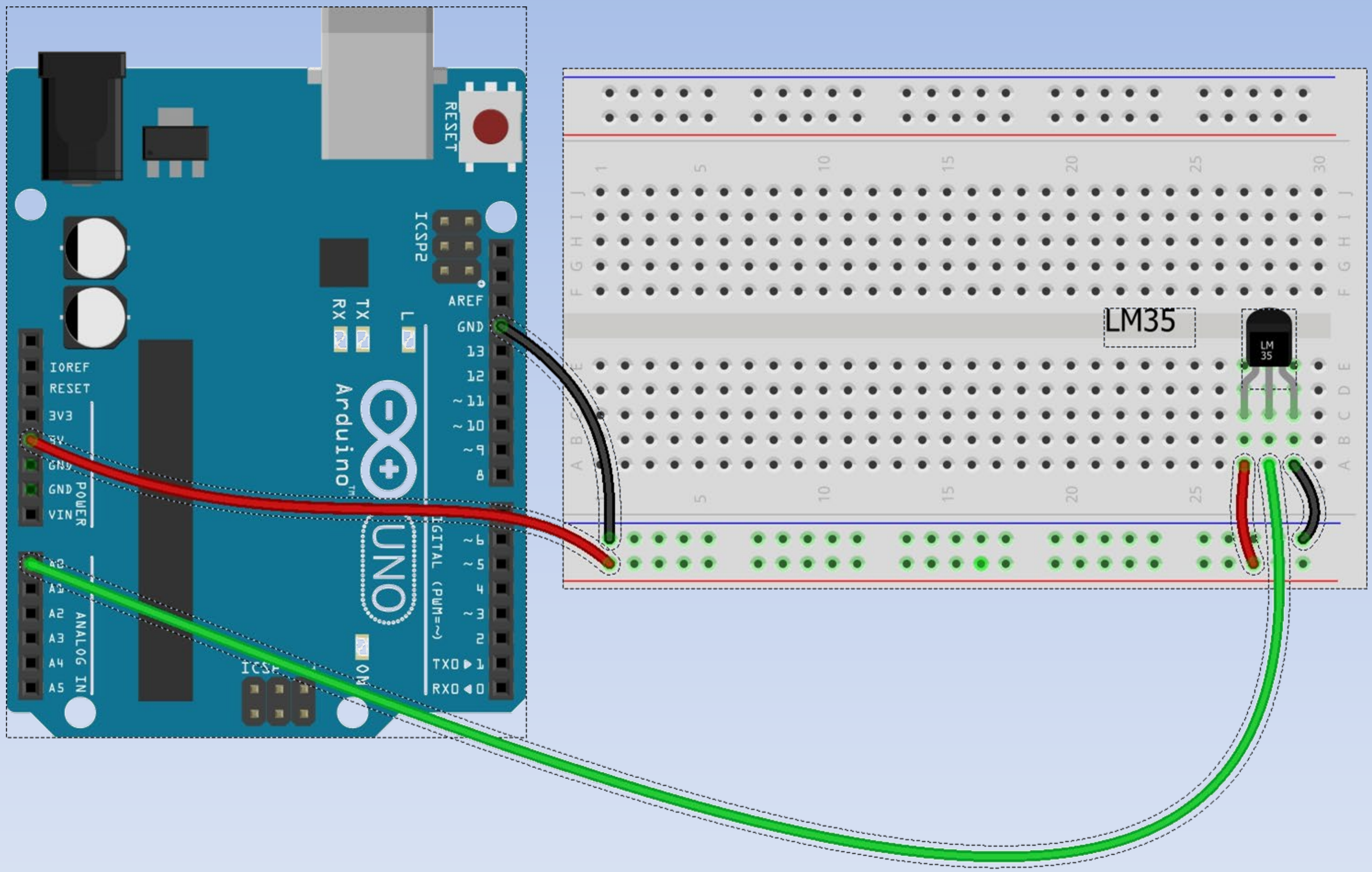
Temperature transducer.

LM 35	Pins of Arduino
Vcc (left)	5 V (+)
OUT (middle)	A0 (analog in)
GND (right)	GND



Package	TO-92
Function	Temperature transducer
Number of pins	3
Precision	±0.5°C
sensibility	10mV/°C
Maximum temperature	150°C
Minimum temperature	-50°C
Typical operating voltage	4-30V
Output type	Analog

How to connect the transducer with Arduino



Temperature transducer.

Transfer function of LM35

(voltage as a function of T)

To get the temperature in degrees Celsius:

$T(°C) = (V_{OUT} *5/1024) / (10/1000)$

$T(°C) = (V_{OUT} *5/1024)*100$

(V_{OUT} is the value on the analog pin where the transducer is connected)

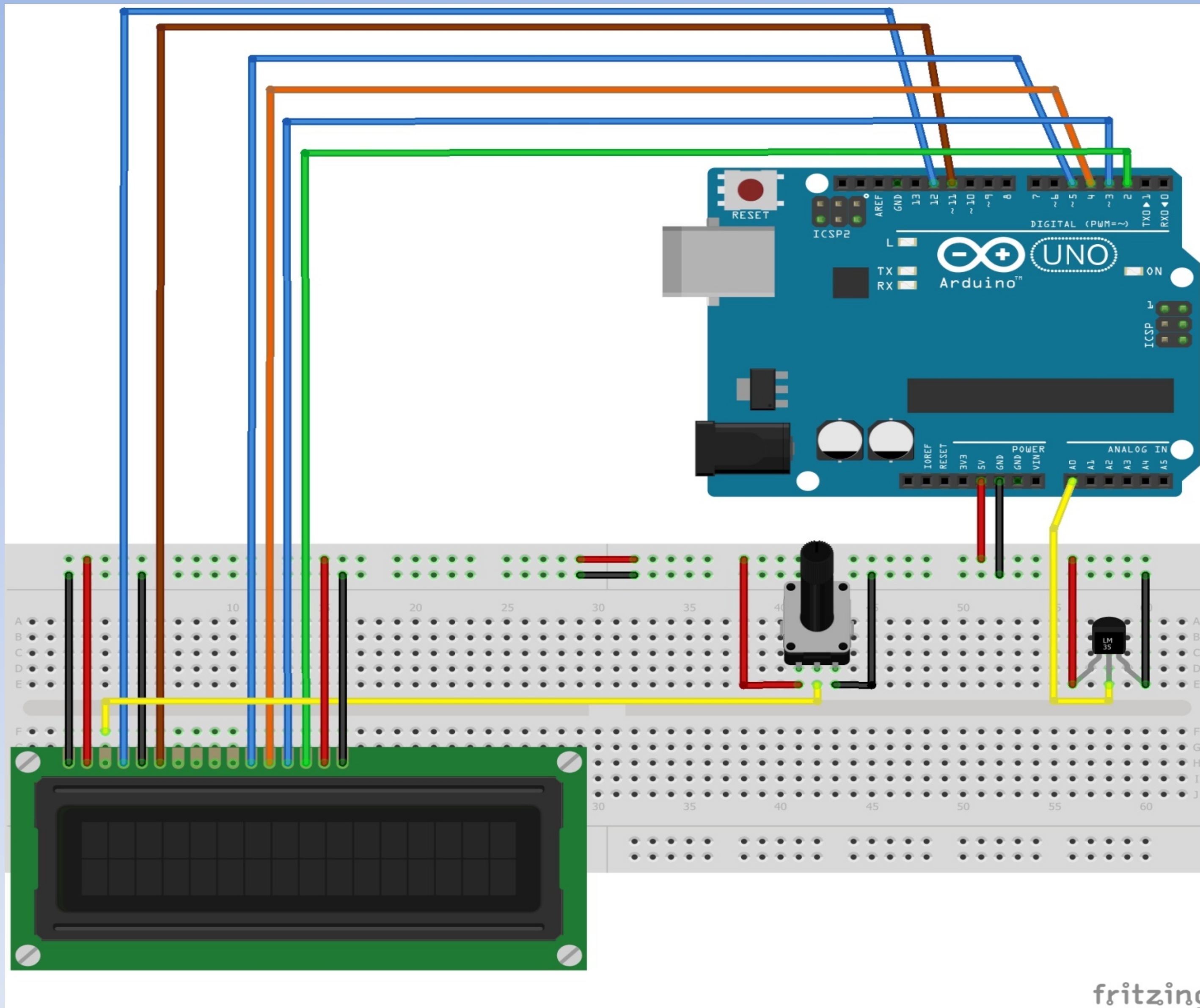
$V_{LM35} = V_{OUT} *5/1024$

How to read the temperature

```
int VOUT;
float  temperature;

void setup() {
  Serial.begin(9600);
                }

void loop() {
          VOUT = analogRead(A0);
         temperature  = ((VOUT * 5.0 / 1024.0)) * 100.0;

         Serial.println(temperature);
         delay(500);
           }
```

Use the Serial Monitor to view the values.

Cable scheme



LCD display connections.

Pins of LCD	Pins of Arduino	Other pins
1	GND	
2	5 V (+)	
3		Central pin of potentiometer
4	12	
5	GND	
6	11	
7-10		
11	5	
12	4	
13	3	
14	2	
15	5 V (+)	
16	GND	

Potentiometer (10KΩ): **Adjust display contrast**

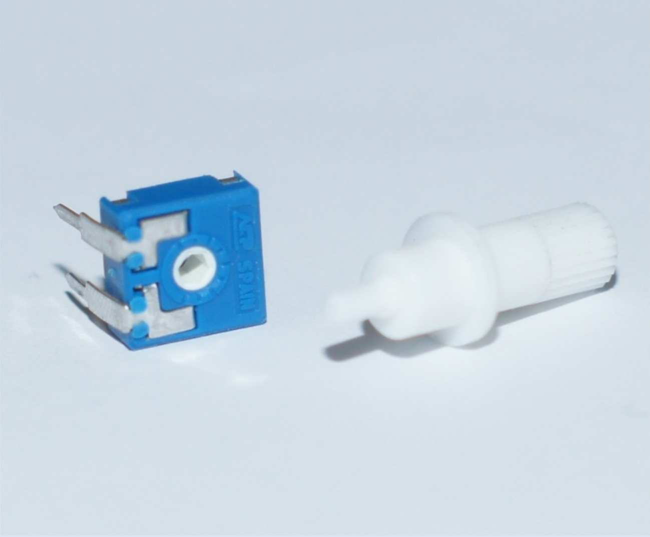
It is a device equivalent to a variable resistive voltage divider.

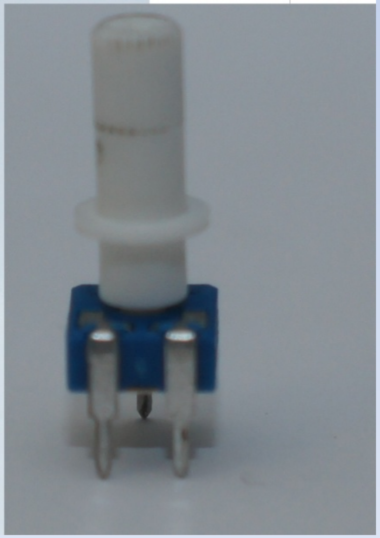
Turning the knob changes the resistance and therefore the voltage.

The middle pin must be connected to pin 3 of the LCD display.

The other two, indifferently, +5 and GND of Arduino

Potentiometer	**Pins of Arduino**
Left	5 V (+)
Right	GND
Central	Pin 3 LCD





Add to the sketch:

- Calculate the average value of T (° C)
  over a number of acquisitions greater than 20
- Calculate the standard deviation of T.
- Max and Min values of T.
- Calculate the temperature in degrees Kelvin T ± ΔT
- Calculate the temperature in Fahrenheit T ± ΔT
- View the numerical values on the LCD display
- Plot all values on the Serial plotter

Anything else you like

Serial Plotter

Serial Plotter receives data from Arduino and visualizes data as waveforms.

It can visualize data for single waveform or data for multiple waveform

in the same graph.

Activate the serial plotter from the Tools menu.

```
void setup() {
Serial.begin(9600);
}

void loop() {
int y1 = analogRead(A0);

Serial.println(y1);

delay(100);
}
```

<= 1 waveform

Serial Plotter

```
void setup() { Serial.begin(9600); }

void loop() {

int y1 = analogRead(A0);
int y2 = analogRead(A1);
int y3 = analogRead(A2);

Serial.print(y1);

Serial.print(" "); //a space ' ' or tab '\t' character is printed between the two values.

Serial.print(y2);

Serial.print(" "); //a space ' ' or tab '\t' character is printed between the two values.

Serial.println(y3); //the last value is followed by a cr and a newline characters.


delay(100);

}
```

<= 3 waveform

To be added to the declarations in order to use the LCD display :

```
#include <LiquidCrystal.h>                  // Include the LCD driver library

LiquidCrystal lcd(12, 11, 5, 4, 3, 2);      // Initialize the library with the numbers
                                            // of the interface pins
```

To be added in setup to use the LCD display :

```
void setup(){

lcd.begin(16, 2);       // Set up the number of columns and rows for the LCD

}
```

Examples:

```
lcd.setCursor(0, 0);              // Set LCD cursor position (column 0, row 0)
lcd.print("Current T is: ");      // Print text to LCD

lcd.setCursor(1, 1);              // Set LCD cursor position (column 1, row 1)
lcd.print("  Celsius   ");        // Print text to LCD

lcd.setCursor(12, 1);             // Set LCD cursor position (column 12, row 1)
lcd.print(Temp);                  // Print current T  to LCD

lcd.clear();                      // Clear LCD
```

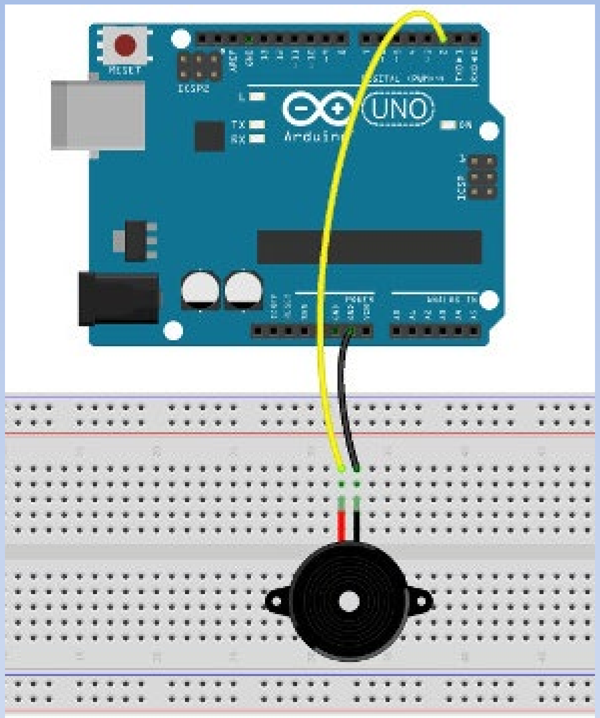
$$\sigma_X = \sqrt{\frac{1}{N}\left(\sum_{i=1}^{N} x_i^2 - N\bar{x}^2\right)} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} x_i^2 - \bar{x}^2}.$$

$$T_{(K)} = T_{(°C)} + 273{,}15$$

$$T_{(F)} = T_{(°C)} \times 1{,}8 + 32$$

Buzzer

```
int buzzerPin = 2;

void setup(){
  pinMode(buzzerPin, OUTPUT);
}

void loop(){
  tone(buzzerPin, 1000, 500);
  delay(1000);
}
```



Experimental experience n° 5:

Windowing

Date 12/12/2022

From 10:30 to hours 13:30

Purpose:

Verify the Windowing effect.

Instrumentation:

Oscilloscope.

Signal generator.

Introduction

An ideal signal, such as a sine wave, is a continuous signal ranging between -∞ and +∞ on the time scale, has a frequency and an amplitude.

If we observe this signal on an oscilloscope, what we see is a signal that has two big differences with the one described above:

1. **It is discrete** (consisting of a finite number of points).

2. **It is truncated** (both right and left, outside the window it is zero).

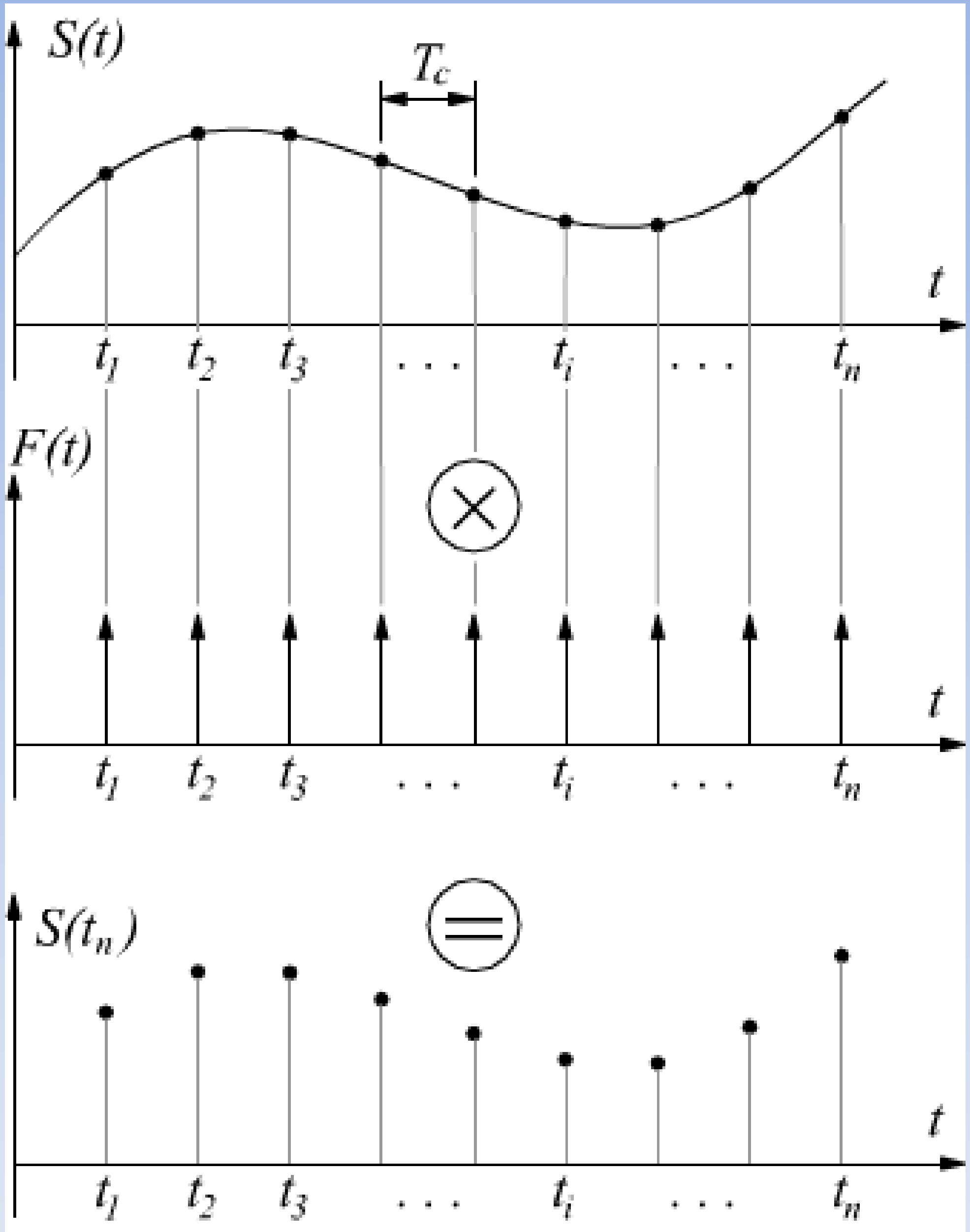
Introduction

Discrete:

In the space of time, discretizing is equivalent to multiplying the original signal by a train of pulses Δt (each equivalent to a Dirac delta) of unit height and distant T from each other.

Introduction

Discrete:



Introduction

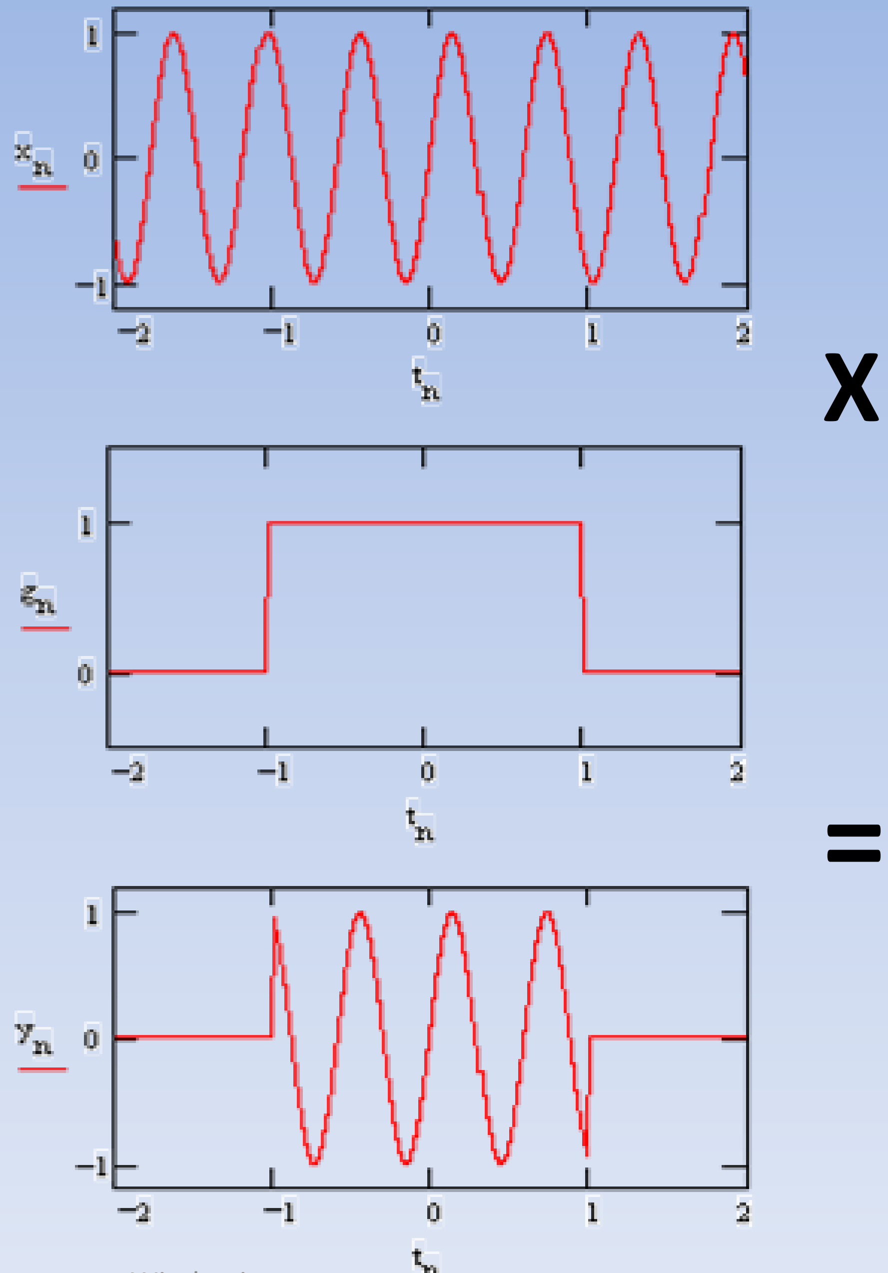
Truncated:

The truncation is due to the observation window.

The original signal is multiplied by a pulse-type signal (or rectangular window) with height equal to 1 and width equal to 2T0 (extends from -T0 to +T0).

Introduction

Truncated:



Introduction

What we observe on the oscilloscope is a multiplication between three signals:

- ideal signal
- Dirac delta train
- pulse signal (or rectangular window)

Introduction

If I do the Fast Fourier Transform

of a signal of this type,

what will the result be?

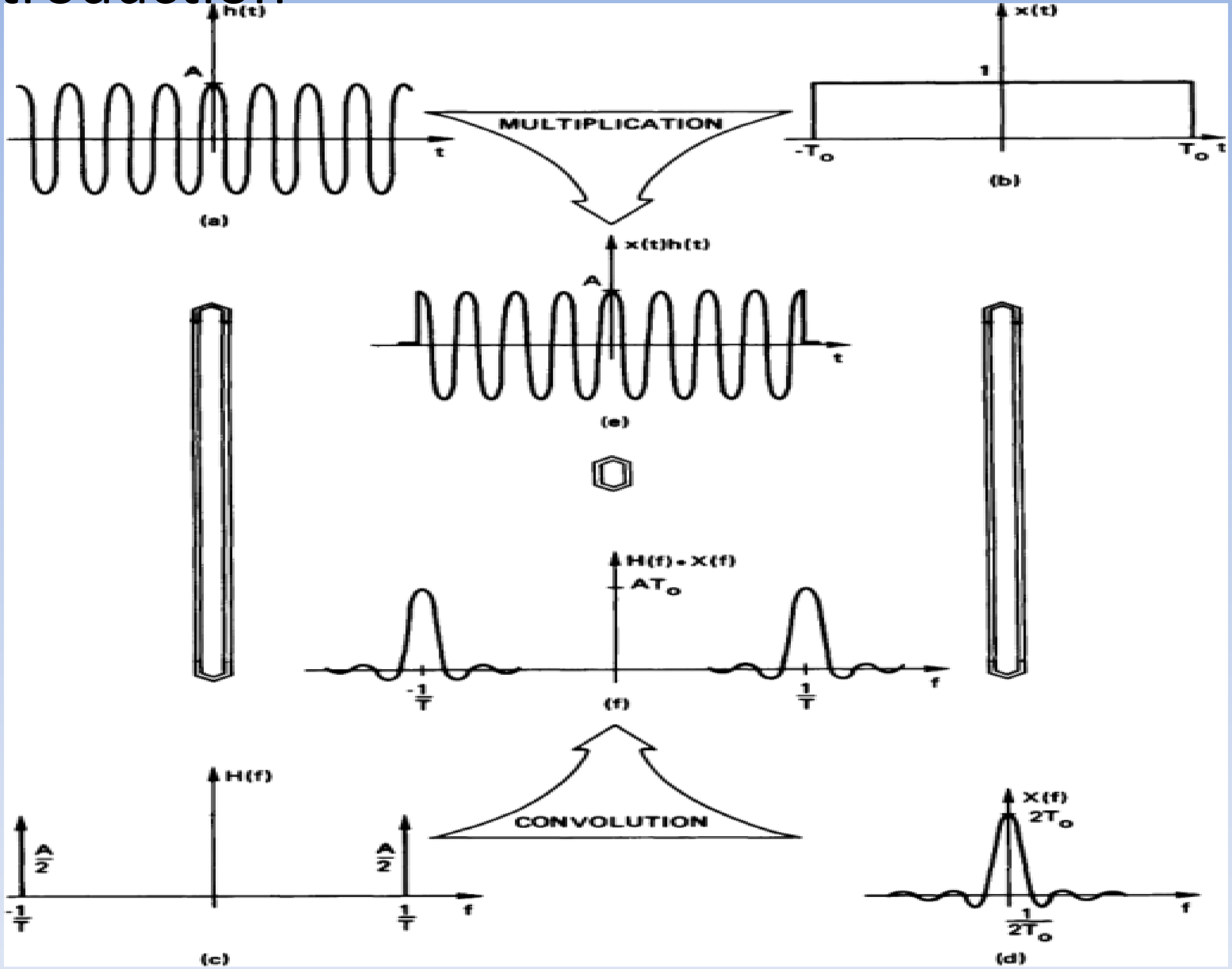
Introduction

What happens in the space of times and phases:

the Fourier transform of an ideal sinusoid is a pair of Dirac delta, while the Fourier transform of a pulse signal (rectangular window) is a function of the type $\frac{\sin(f)}{f}$ (f is a function).

What in the space of time was a multiplication between two signals, in the space of frequencies is a convolution that produces a pair of functions of the type (sin f) / f positioned symmetrically with respect to the y axis.

Introduction

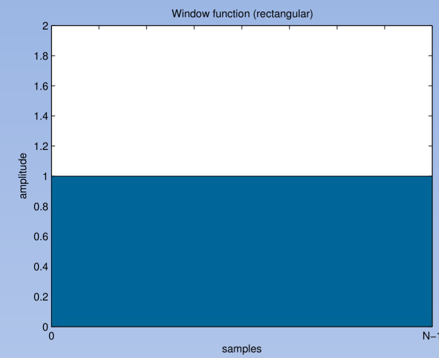


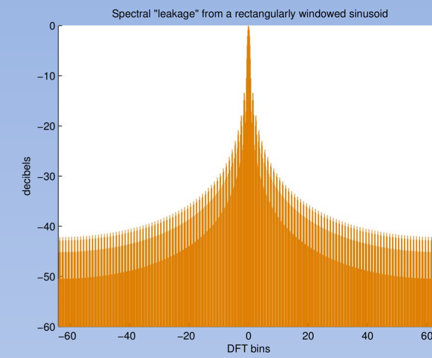
Introduction

In the ideal situation there is no multiplication by a pulse signal (rectangular window) and therefore one would expect peaks that are deltas.

In the real situation, however, we find the peaks that have their own width.

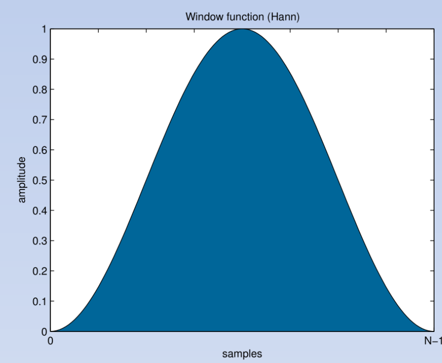
The width of the peaks depends on $1/2T_0$.
It depends on the observation window
(it depends on the full scale used to display the signal on the oscilloscope).

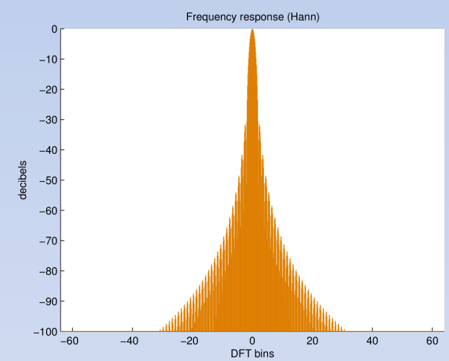




RECTANGLE window:

- The best frequency resolution
- The worst amplitude resolution



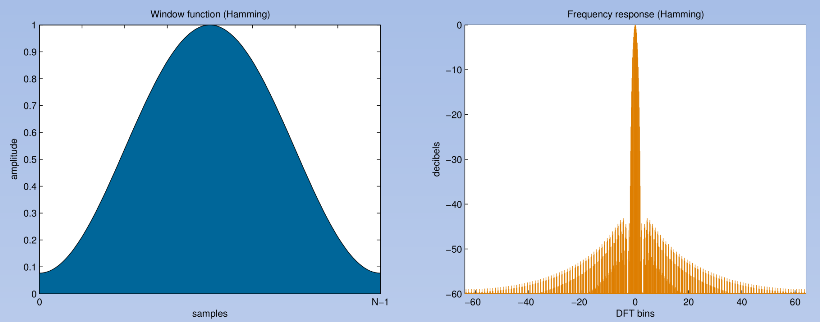


HANNING window:

- Window for making accurate frequency measurements or for resolving two frequencies close to each other
- Poor resolution of amplitude

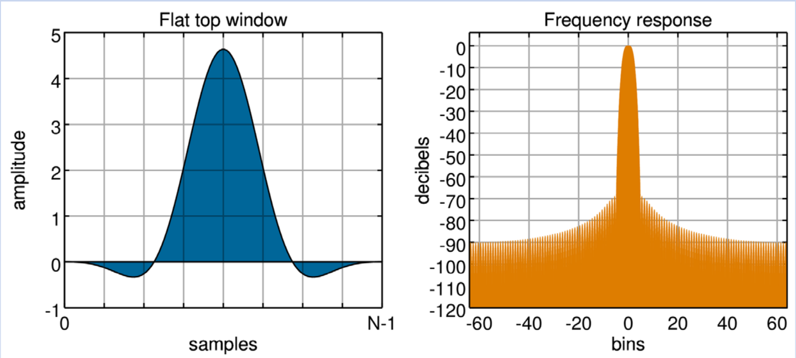
HAMMING window:

- Better frequency resolution
- Poor amplitude resolution

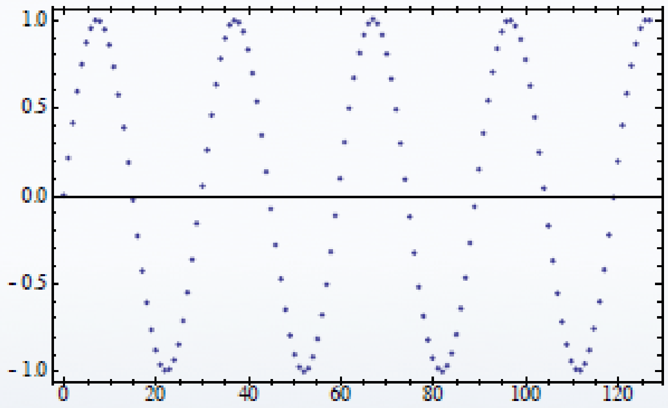


FLAT TOP window:

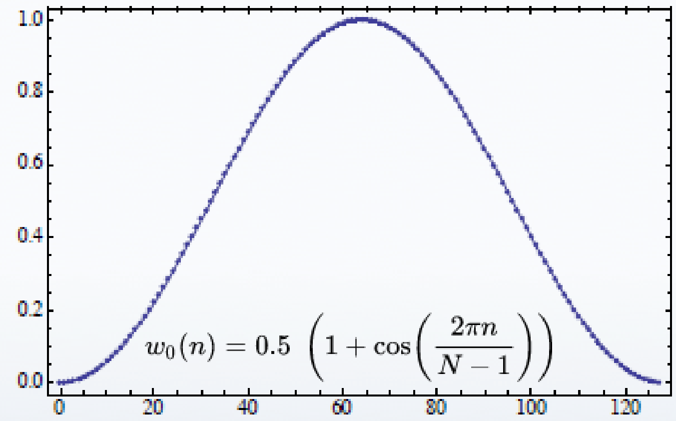
- Poor frequency resolution
- The best amplitude resolution



How a window works (Hanning window example)



X



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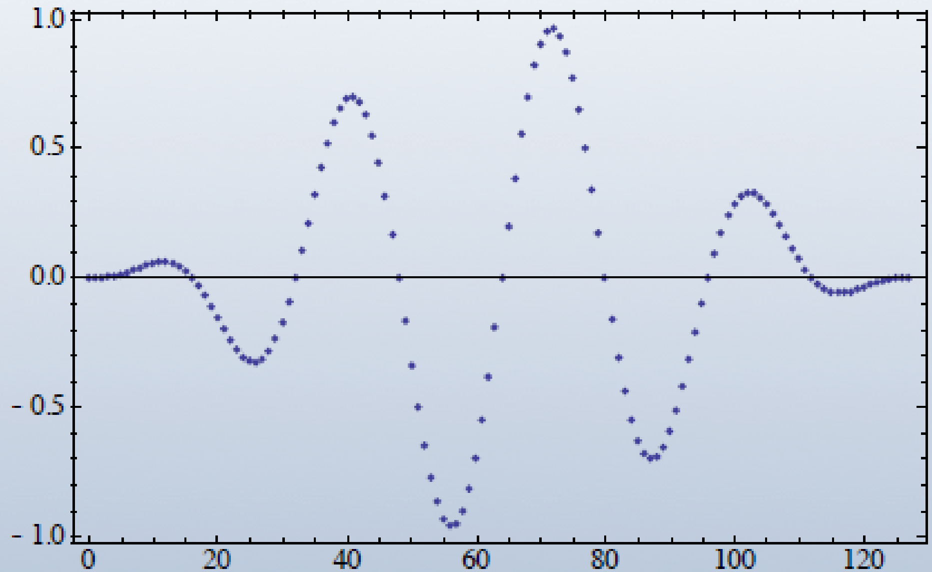
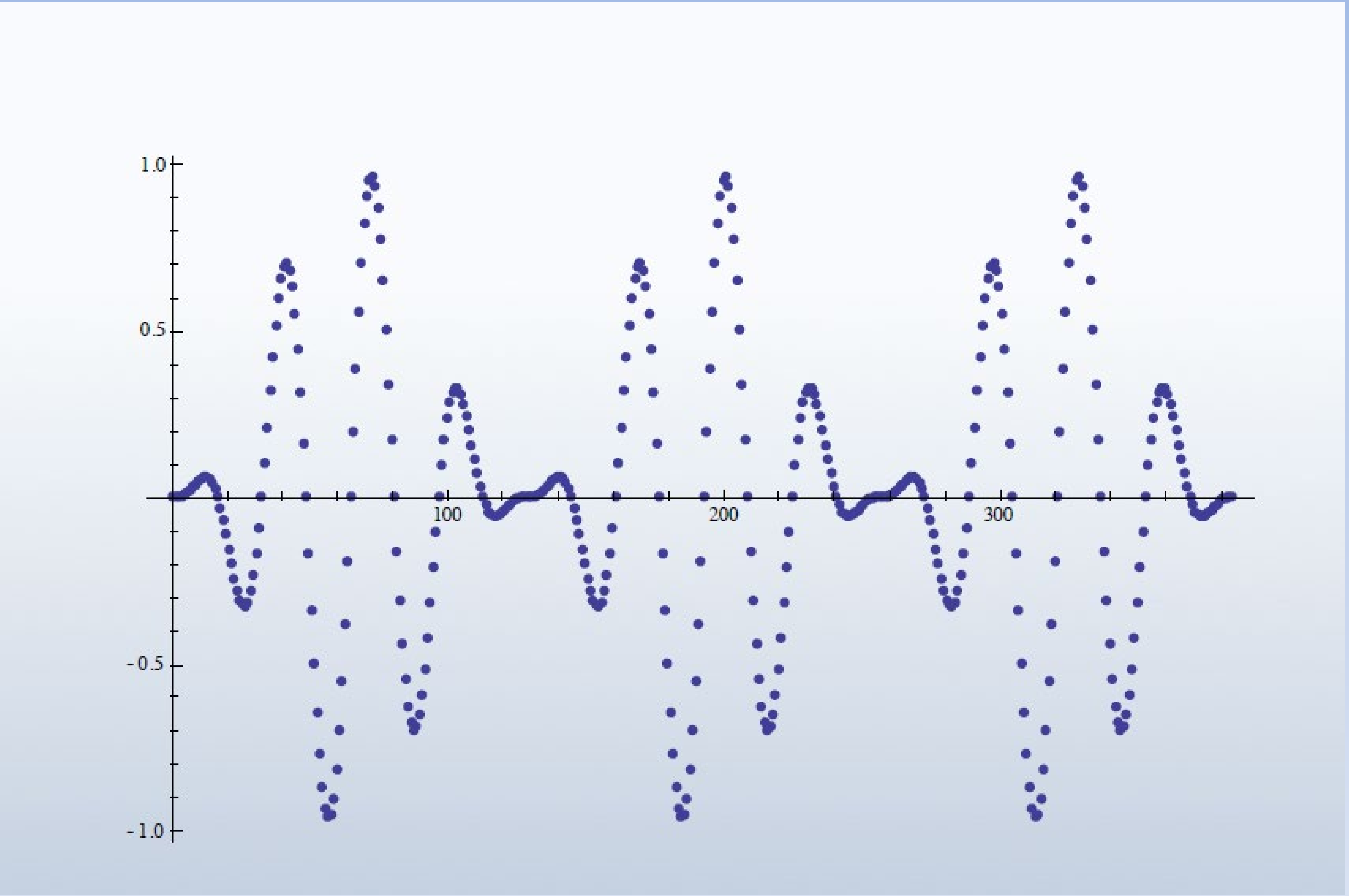
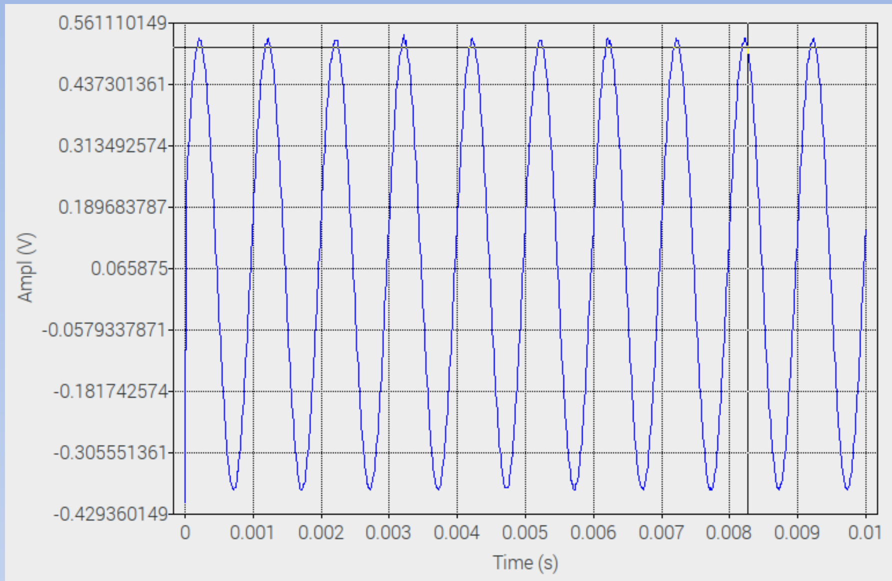


How a window works (Hanning window example)

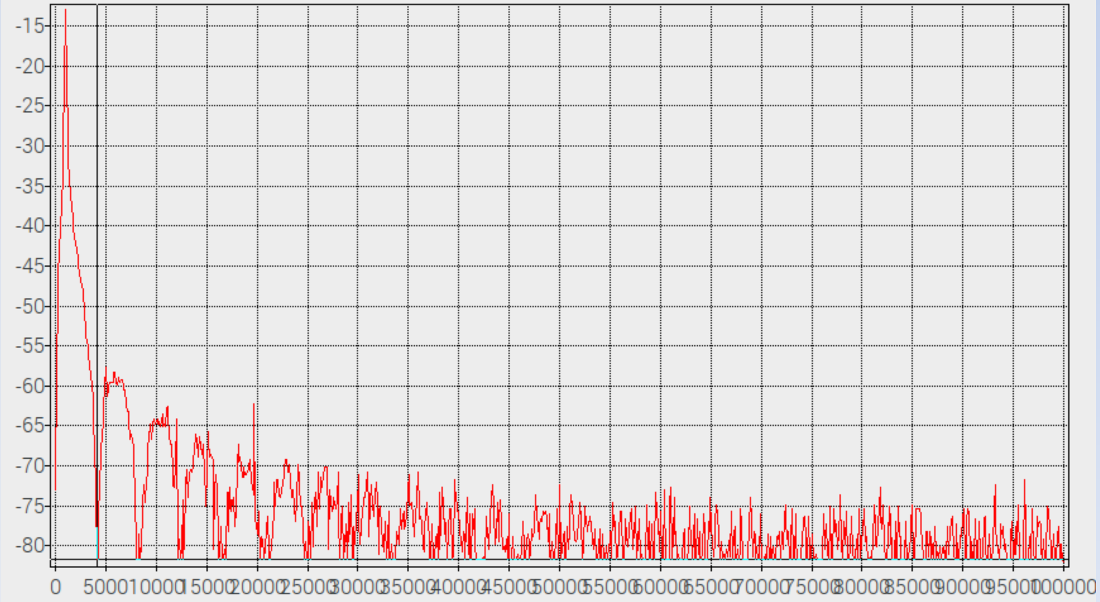


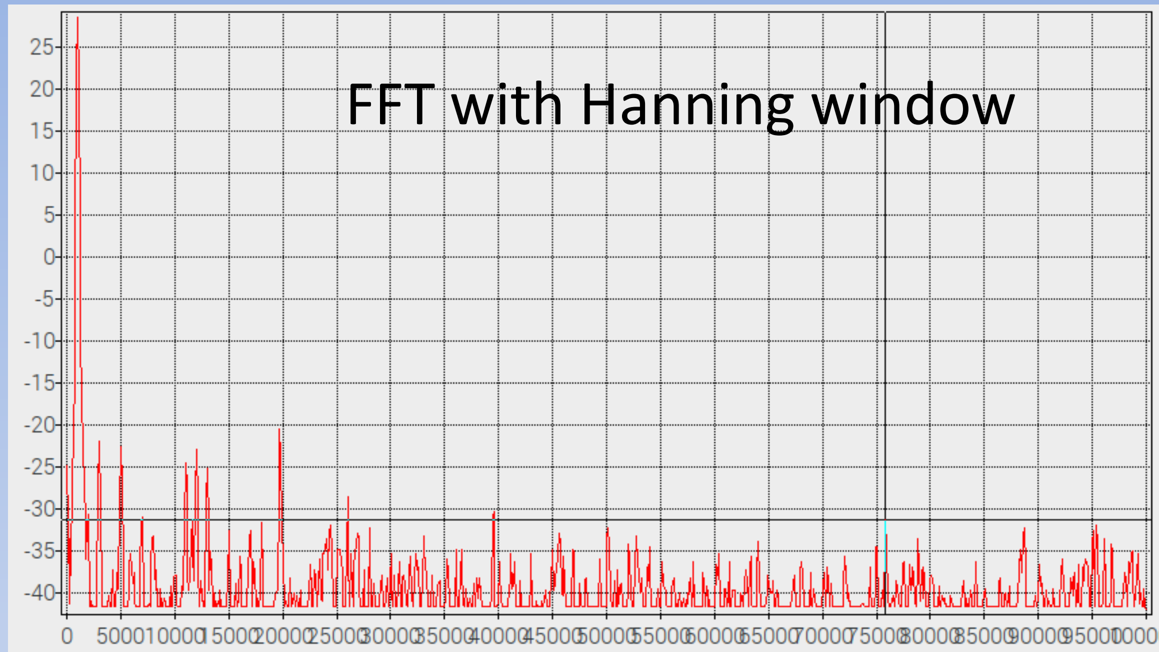
The window restores (approximately) continuity at the edges.

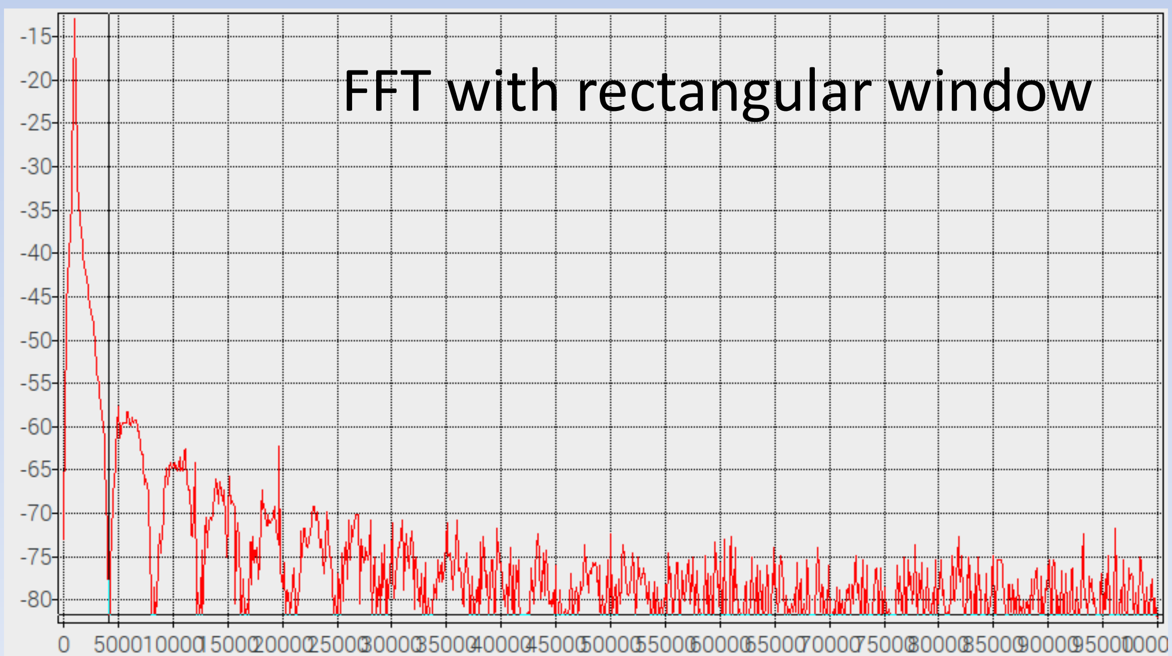
Sinusoidal signal 1 KHz - 1 Vpp - time window 10 ms



FFT with rectangular window




FFT with Hanning window
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
0


FFT with rectangular window
-15
-20
-25
-30
-35
-40
-45
-50
-55
-60
-65
-70
-75
-80
0

Data acquisition:

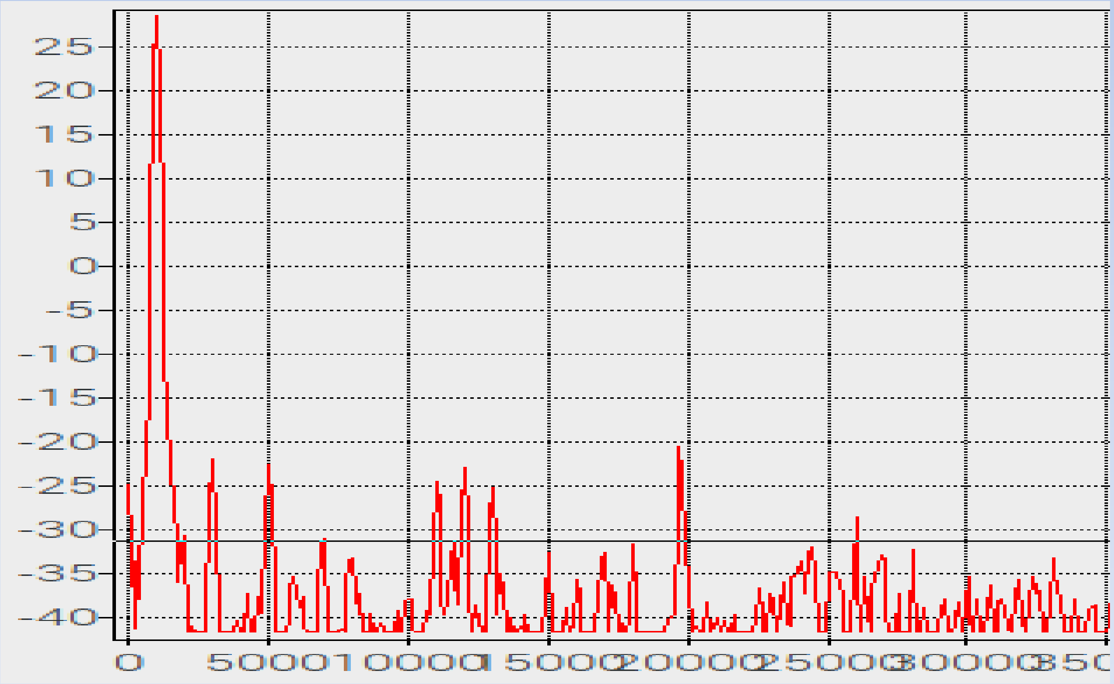
1. Generate a sinusoidal signal with frequency 1KHz and amplitude 1 Vpp.
2. Select the Hanning window on the oscilloscope
3. Set the length of the time window to 2 ms.
4. Save the data relating to the displayed signal.
5. Save the FFT of the signal.
6. Repeat from step 2 for the following time window values: 5 ms, 10 ms, 20 ms, 50 ms.

Data processing (oscilloscope FFT data series):

1. Evaluate the width Δf of the peaks.
2. Plot on a graph T_0 as a function of t.

$T_0 = 1/\Delta f$ (reciprocal of the peak width)

t = (2,5,10,20,50)ms

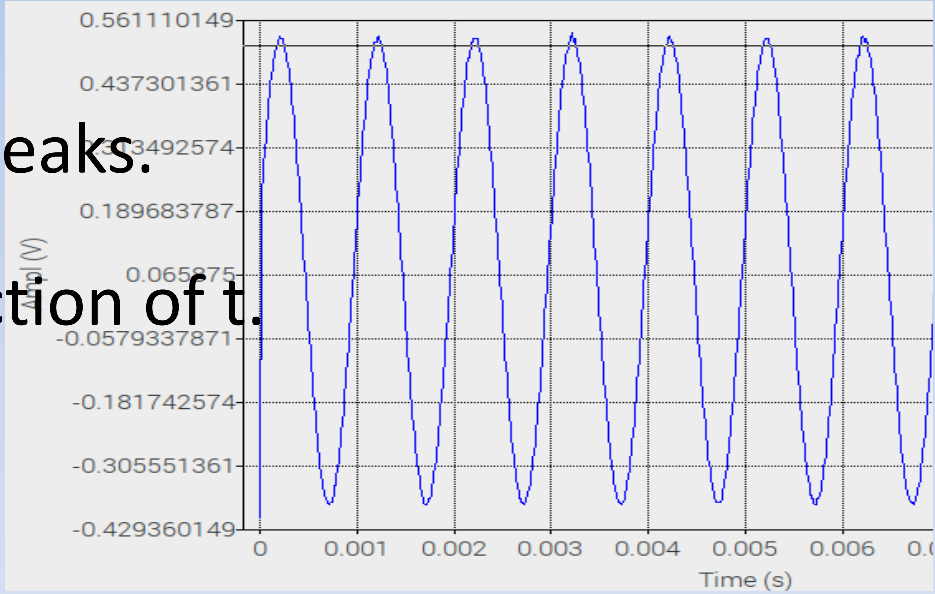


Data processing (oscilloscope signal data series):

1. Calculate, with special software using a Hanning window, the FFT of the data relating to the acquired signals.

2. Evaluate the width of the peaks.

3. Plot on a graph T_0 as a function of t.



$T_0 = 1/\Delta f$ (reciprocal of the peak width) $t = (2,5,10,20,50)$ms

Data processing:

For each graphic:

- Check if the points obtained lie on a line or not. Perform the FIT of the data (for both cases) and calculate the intercept, slope and correlation coefficient.
- **Compare the width of the peaks obtained in the two series of measurements and draw the appropriate conclusions.**

How to set the number of points to process:

1) ACQUIRE
2) Menu
3) Mem Depth
4) 10 K

How to define a function:

1) MATH
2) Menu
3) Trace: F1
4) Operation: ON
5) Function: Math
6) FFT
7) C1

How to save the data:

1) SAVE
2) Save path: external
3) Type: CSV
4) File Manager

Experimental experience n° 6:

Control of a DC motor

Date 13/01/2023

From 10:30 to hours 13:30

Purpose:

- Adjust the speed of a DC motor according to the amplitude of the signal generated by a waveform generator.
- Explain / interpret the waveforms displayed with the oscilloscope.

Instrumentation:

Waveform generator

Oscilloscope

Variable power supply

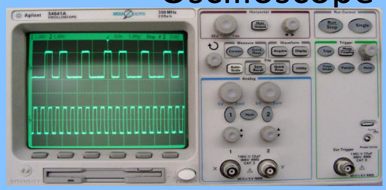
Arduino UNO board

Raspberry

What to use :

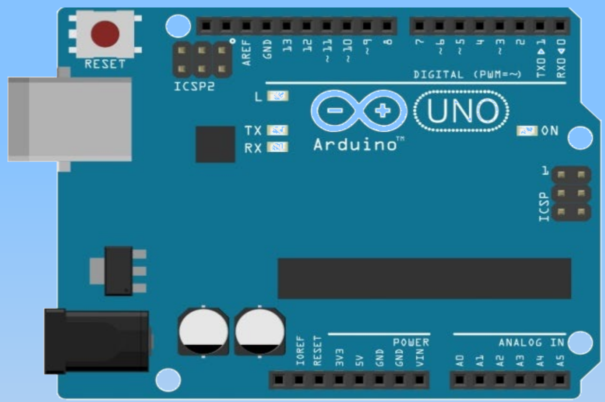
Waveform generator

Oscilloscope

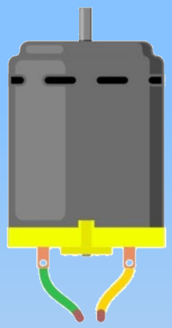


Variable power supply

Arduino

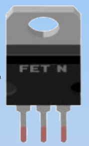


DC Motor



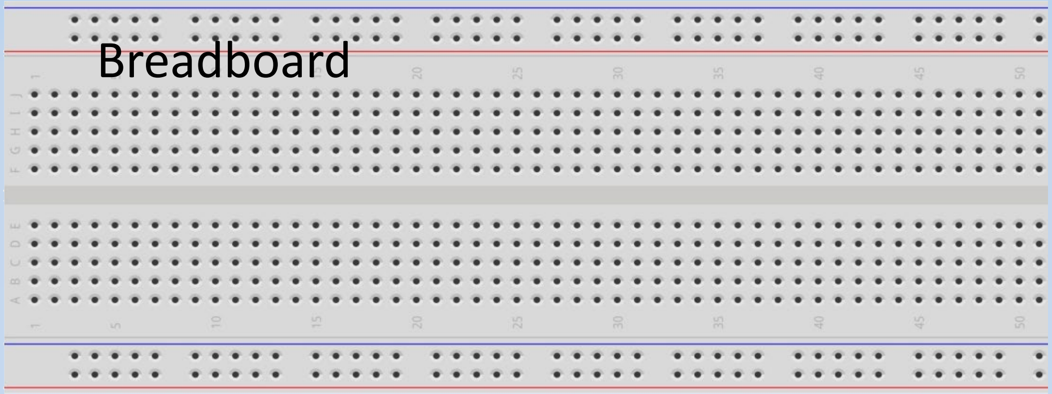
Diode

1N4007



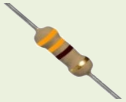
Transistor

IRF 520



Breadboard

Green LED

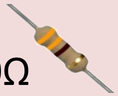


260Ω

Fhoto-resistor

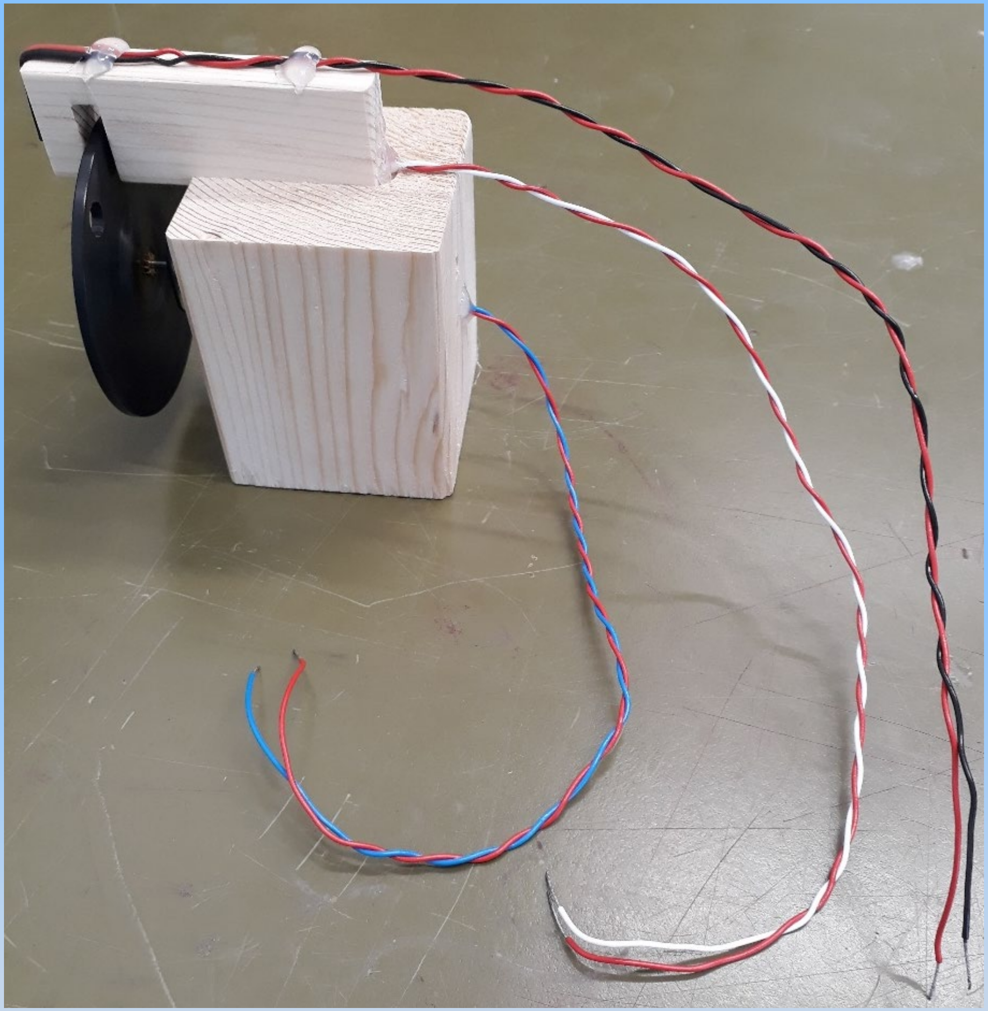
20KΩ

Red LED



340Ω

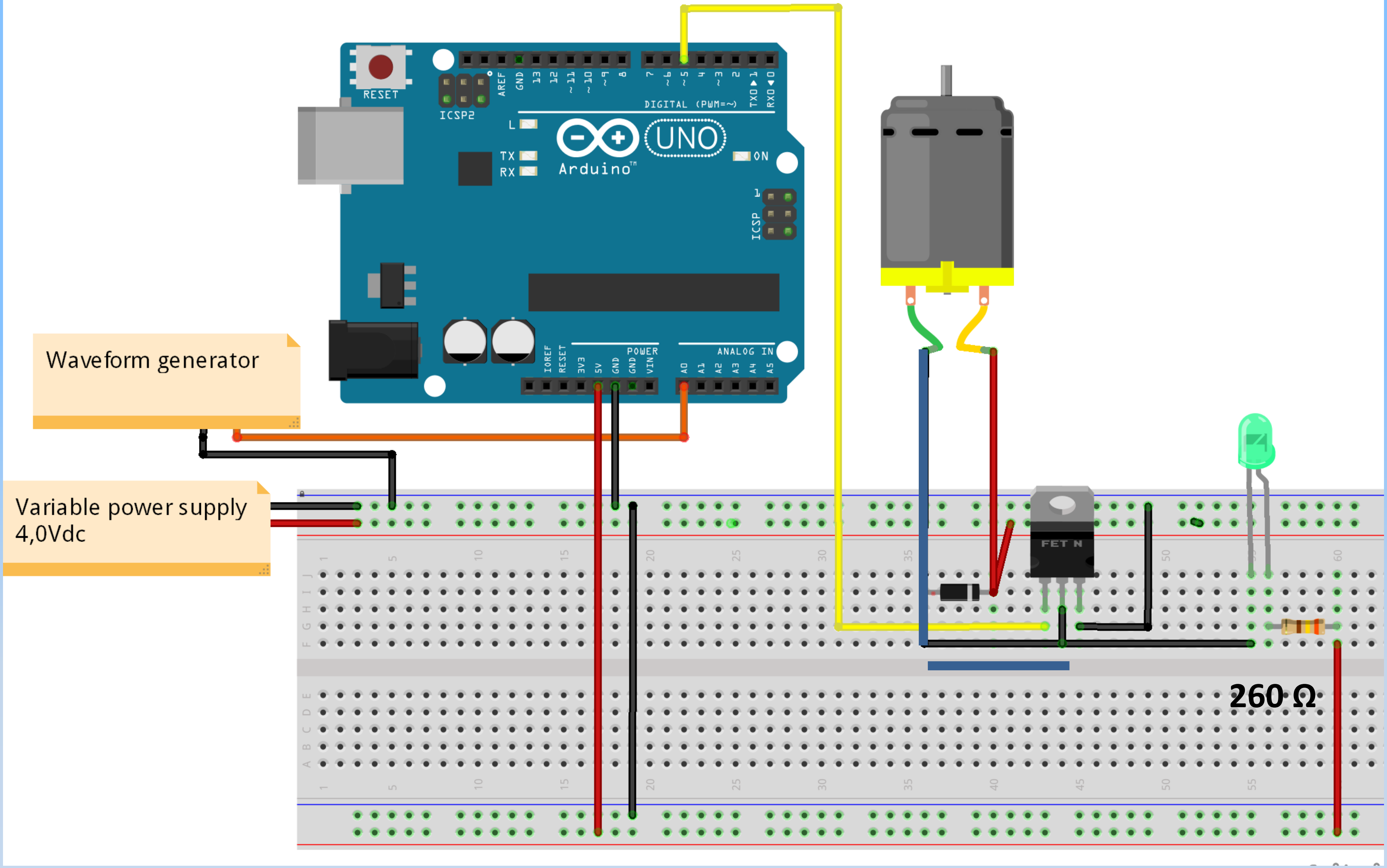
Motor, **LED** and **Photoresistor** are already assembled in the setup



Color of the wires:

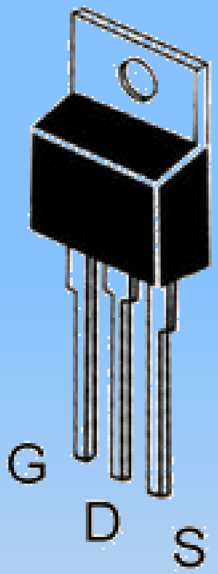
- Motor: blue, red
- Photoresistor: white, red
- LED: black, red $^{+}$ (pay attention to polarity)

Circuit assembly diagram.



Transistor IRF 520

Power MOSFET, fast switching, robust device, low thermal resistance and low cost.



Gate: pin 5 of Arduino

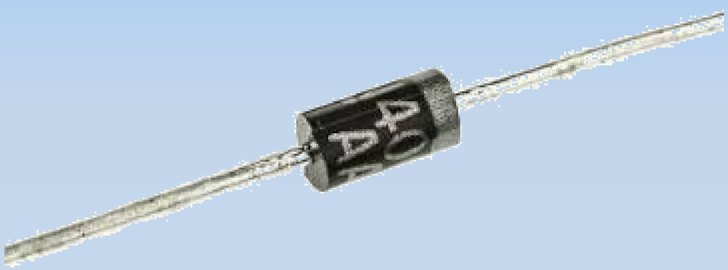
Drain: blue wire of motor
and negative side of green LED

Source: GND of Arduino

Diode 1N4007

Connect the diode between the blue wire and the red wire of the motor to avoid eddy currents.

(Reverse polarized)



Variable power supply

Positive: Red column of the breadboard

Negative : Black column of the breadboard

Set the values : 4,0V 0,3A

Signal generator

Positive: pin A0 of Arduino

Negative: Black column of the breadboard

Signal generator

Initial wave form: sine wave

- Frequency: 0,1 Hz
- Amplitude: 5,0 V
- Offset: 2,5 V
- Phase: 0,0

Step number 1:

- Upload the sketch.
- Activate the serial plotter.

- Explain what the graphs shown represent.
- Does the speed of the disk follow the shape of the used signal?
- Observe the green LED and the rotation of the disk: are they synchronized?

  (max light = max speed, low light = low speed)

Step number 1:

- Change the waveform: **square wave** and **sawtooth** (using the same frequency, amplitude, offset and phase values).

- Try to vary the frequency just a bit.

Describe everything in the report with data and screenshots

Step number 2:

- Connect the pin n.5 on the oscilloscope and explain what the shown signal represents.

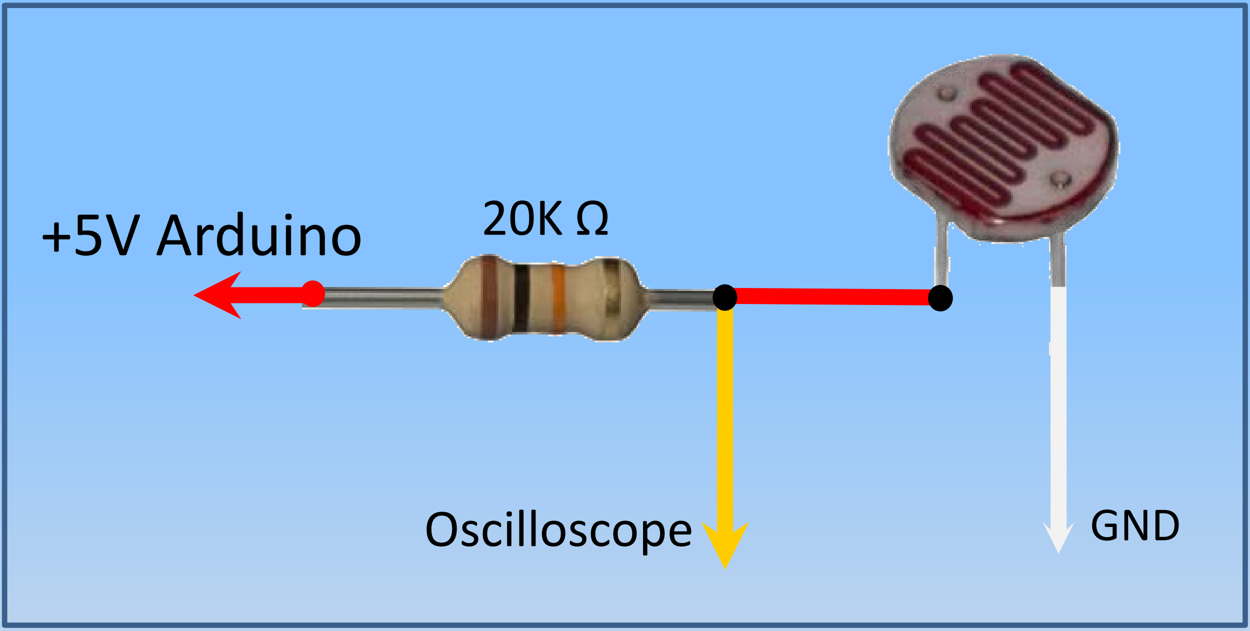
- Measure frequency, period and amplitude at 2 or 3 different times.

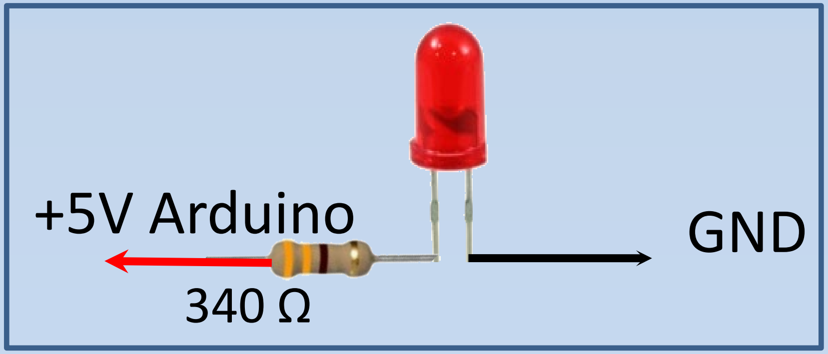
- Capture at least one screenshot.

Describe everything in the report with data and screenshots.

Step number 3:

Connect the photoresistor and the LED as shown in the pictures.





Step number 3:

Edit the sketch adding this statement in void setup()

```
analogWrite(pinMotor,speedmotor);
```

Edit void loop() deactivating the statement

```
// speedMotor = ((signal_input*255)/1023);
```

Step number 3:

- View on the oscilloscope the signal taken between photoresistor and GND and explain what it represents.

- Measure frequency and pulse duration in 2 or 3 different moments and capture at least one screenshot.

- Vary the power supply voltage between 1.5V and 4V max

Describe everything in the report with data and screenshots.

Sketch

```
const int pinSignal = A0;
const int pinMotor = 5;

int MaxValue = 1023;

long signal_input = 0;
long speedMotor = 255;

void setup()
{
Serial.begin(9600);
pinMode(pinMotor,OUTPUT);
}
```

```
void loop() {

signal_input = analogRead(pinSignal);          // values 0 ÷ 1023

speedMotor = ((signal_input*255)/1023);        // values 0 ÷ 255

analogWrite(pinMotor,speedMotor);

Serial.print(MaxValue);
Serial.print("\t");
Serial.print(signal_input);
Serial.print("\t");
Serial.println(speedMotor);
delay(100);

}
```

Oscilloscope: How to save screen shot :

Run / Stop button

Utility Menu

SAVE / Recall

Type: jpg, png ….

File Manager …..

Rasberry: How to save screen shot :

Run the LXTerminal

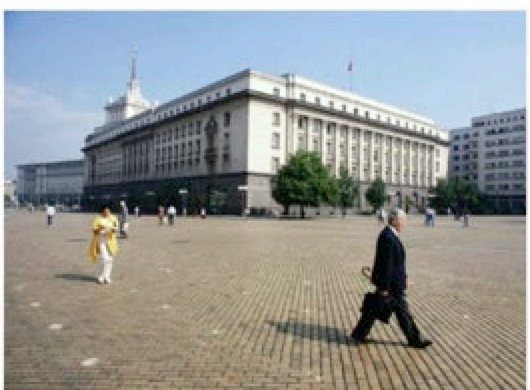
Type the command: scrot -s

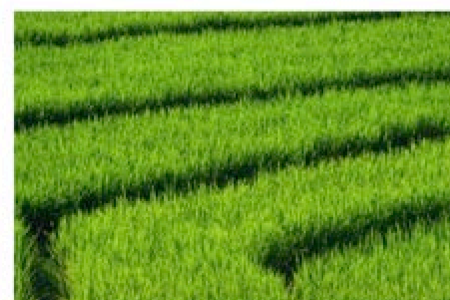
Press: Enter

You will find the file in the folder:

/home/pi

Analisi di Fourier





TRASFORMATA DI FOURIER (FT)
funzioni non periodiche

•Ogni funzione continua *f(x)* anche se non periodica (purché abbia area finita) può essere espressa come *integrale* di sinusoidi **complesse** opportunamente pesate. (*u*=frequenza)

antitrasformata di Fourier

$$f(x)=\int_{-\infty}^{+\infty}F(u)e^{j2\pi ux}du$$

Trasformata di Fourier

$$F(u)=\int_{-\infty}^{+\infty}f(x)e^{-j2\pi ux}dx$$

Dominio spaziale (o temporale) o dominio diretto

Dominio delle frequenze o dominio trasformato

TRASFORMATA DI FOURIER 1D
di una funzione continua

La trasformata di Fourier di una funzione continua e integrabile è una funzione complessa nel dominio delle frequenze (*u* = frequenza). In <u>coordinate polari</u> si ha :

$$\boxed{F(u) = \mathcal{F}\left[f(x)\right] = \Re(u) + j\Im(u) = \left|F(u)\right| e^{j\phi(u)}}$$

Spettro $\quad \left|F(u)\right| = \left[\Re(u)^2 + \Im(u)^2\right]^{1/2}$

Fase $\quad \phi(u) = \tan^{-1}\left[\frac{\Im(u)}{\Re(u)}\right]$

Potenza spettrale (densità spettrale) $\quad \left|F(u)\right|^2 = \Re(u)^2 + \Im(u)^2$

TRASFORMATA DI FOURIER 2D
di una funzione continua

Trasformata
$$F(u,v)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

Antitrasformata
$$f(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)e^{j2\pi(ux+vy)}dudv$$

Spettro
$$|F(u,v)|=\left[\Re(u,v)^2+\Im(u,v)^2\right]^{1/2}$$

Fase
$$\phi(u,v)=\tan^{-1}\left[\frac{\Im(u,v)}{\Re(u,v)}\right]$$

Potenza spettrale
(densità spettrale)
$$|F(u,v)|^2=\Re(u,v)^2+\Im(u,v)^2$$

TRASFORMATA DI FOURIER DISCRETA 2D DFT

$$f(x,y) = f(x_0 + x\Delta x, y_0 + y\Delta y) \qquad x=0,\dots,M\text{-}1;\ y=0,\dots,N\text{-}1$$

$$F(u,v) = F(u_0 + u\Delta u, v_0 + v\Delta v) \qquad u=0,\dots,M\text{-}1;\ v=0,\dots,N\text{-}1$$

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)} \qquad \textbf{DFT}$$

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)} \qquad \textbf{DFT}^{-1}$$

f* (*x,y*) campionata: *Δx* , *Δy

***F* (*u,v*) campionata: *Δu=1/MΔx* , *Δv=1/NΔy*.**

***f*(*x,y*)=*f*(*x+M,y+N*)**

***F* (*u,v*)=*F* (*u+M,v+N*)**

PERIODICITA'

TRASFORMATA DI FOURIER
(funzione complessa)

$$\boxed{F(u,v) = \Re(u,v) + j\Im(u,v) = \left|F(u,v)\right| e^{j\phi(u,v)}}$$

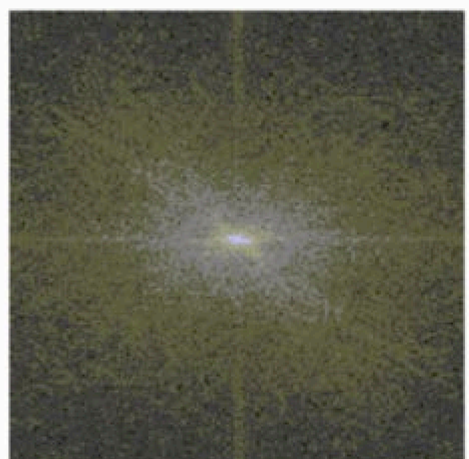
Spettro

$$\left|F(u,v)\right| = \left[\Re(u,v)^2 + \Im(u,v)^2\right]^{1/2}$$

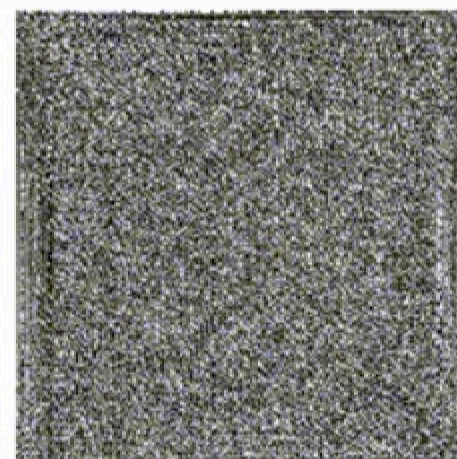
Fase

$$\phi(u,v) = \tan^{-1}\left[\frac{\Im(u,v)}{\Re(u,v)}\right]$$

$f(x,y)$



$|F(u,v)|$

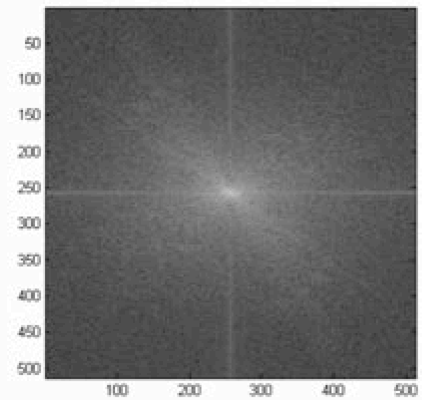


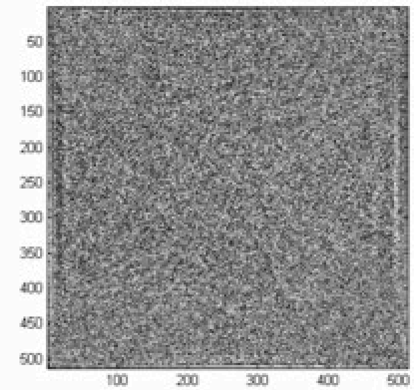
$\Phi\,(u,v)$

ANALISI-SINTESI

$$|F(u,v)| \qquad \phi(u,v)$$

DFT



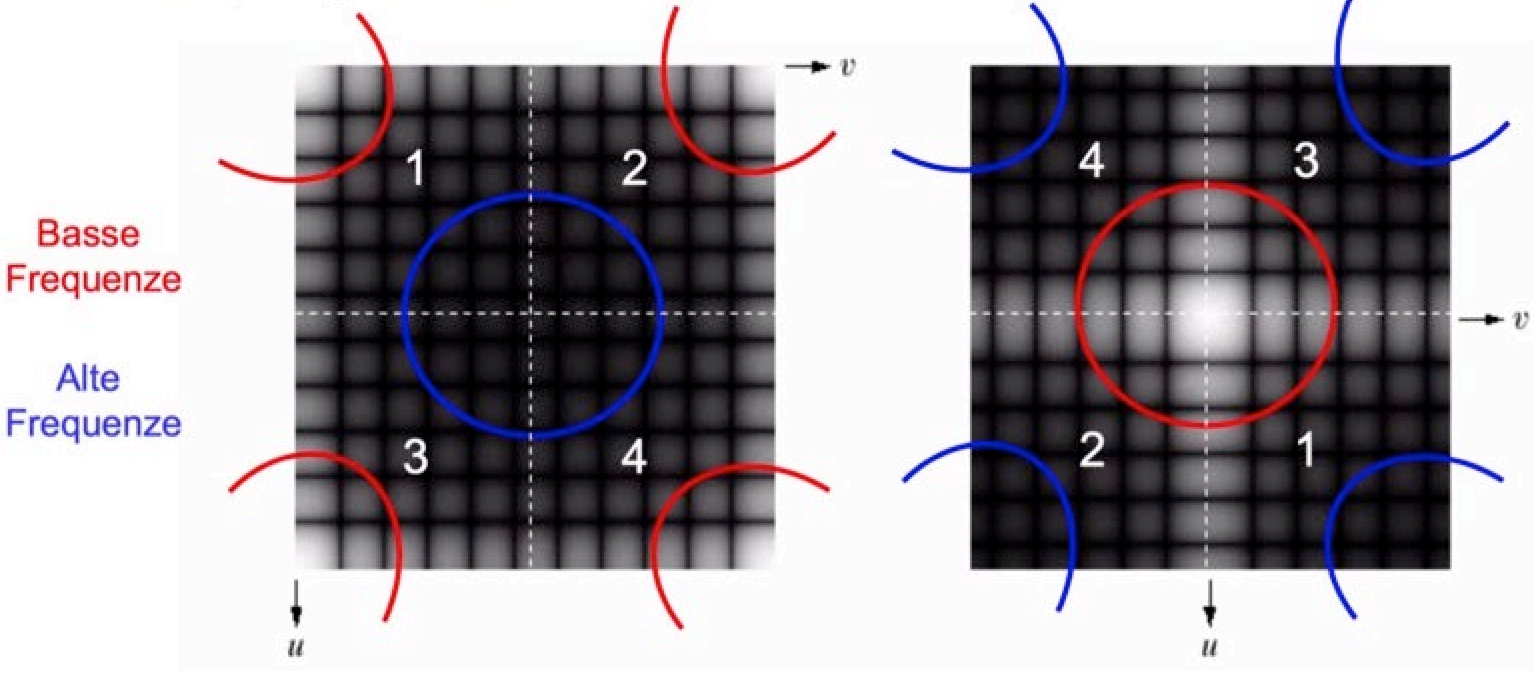


$$\boxed{F(u,v) = |F(u,v)|e^{j\phi(u,v)}}$$

DFT^{-1}

Trasformata di Fourier: Immagini

- La trasformata di una immagine è periodica e simmetrica: il valore (0,0) è in alto a sinistra

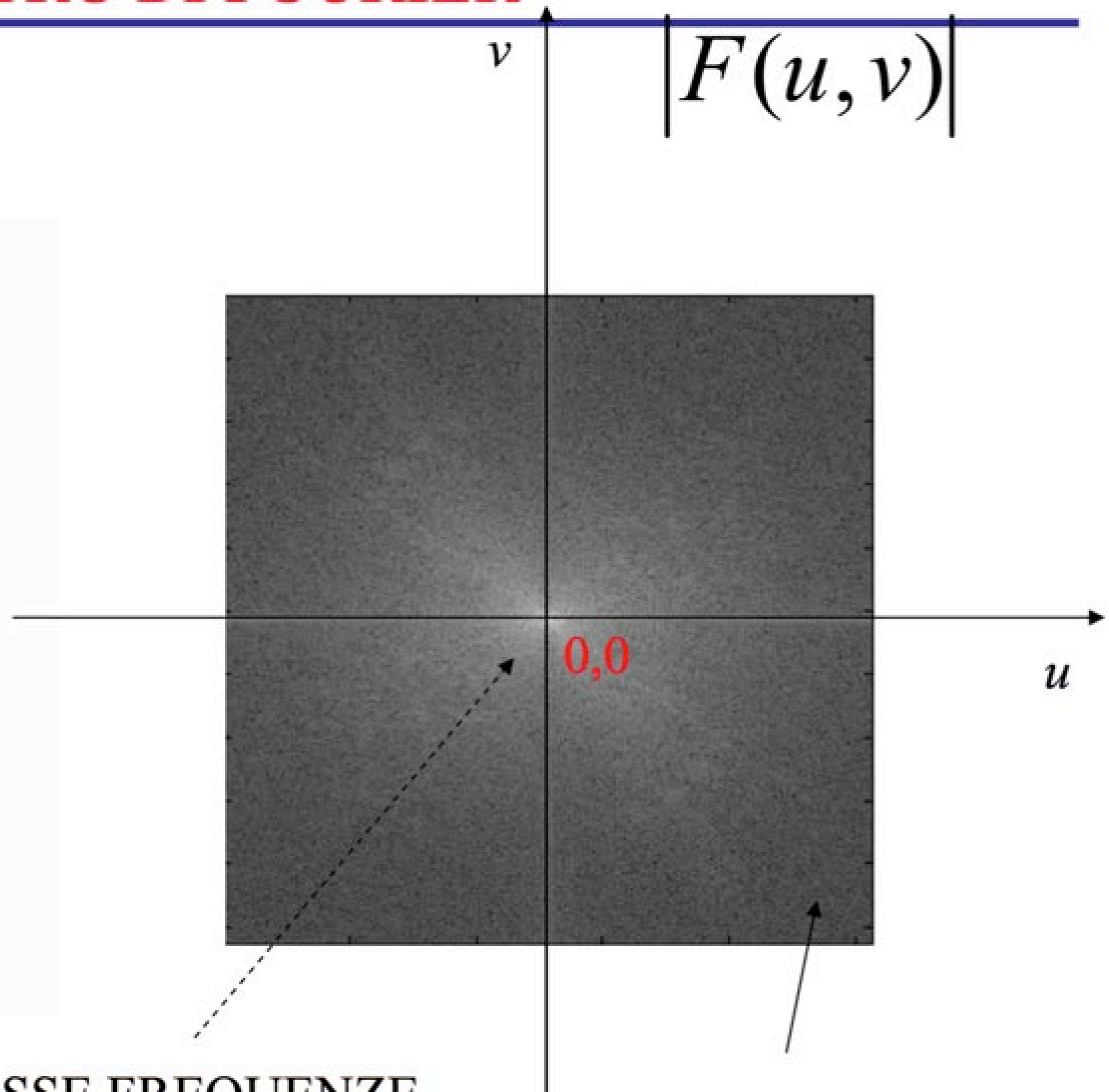


- Per leggibilità (e utilità) si preferisce traslare lo spettro per avere (0,0) al centro

RAPPRESENTAZIONE SPETTRO DI FOURIER

$$f(x,y)$$

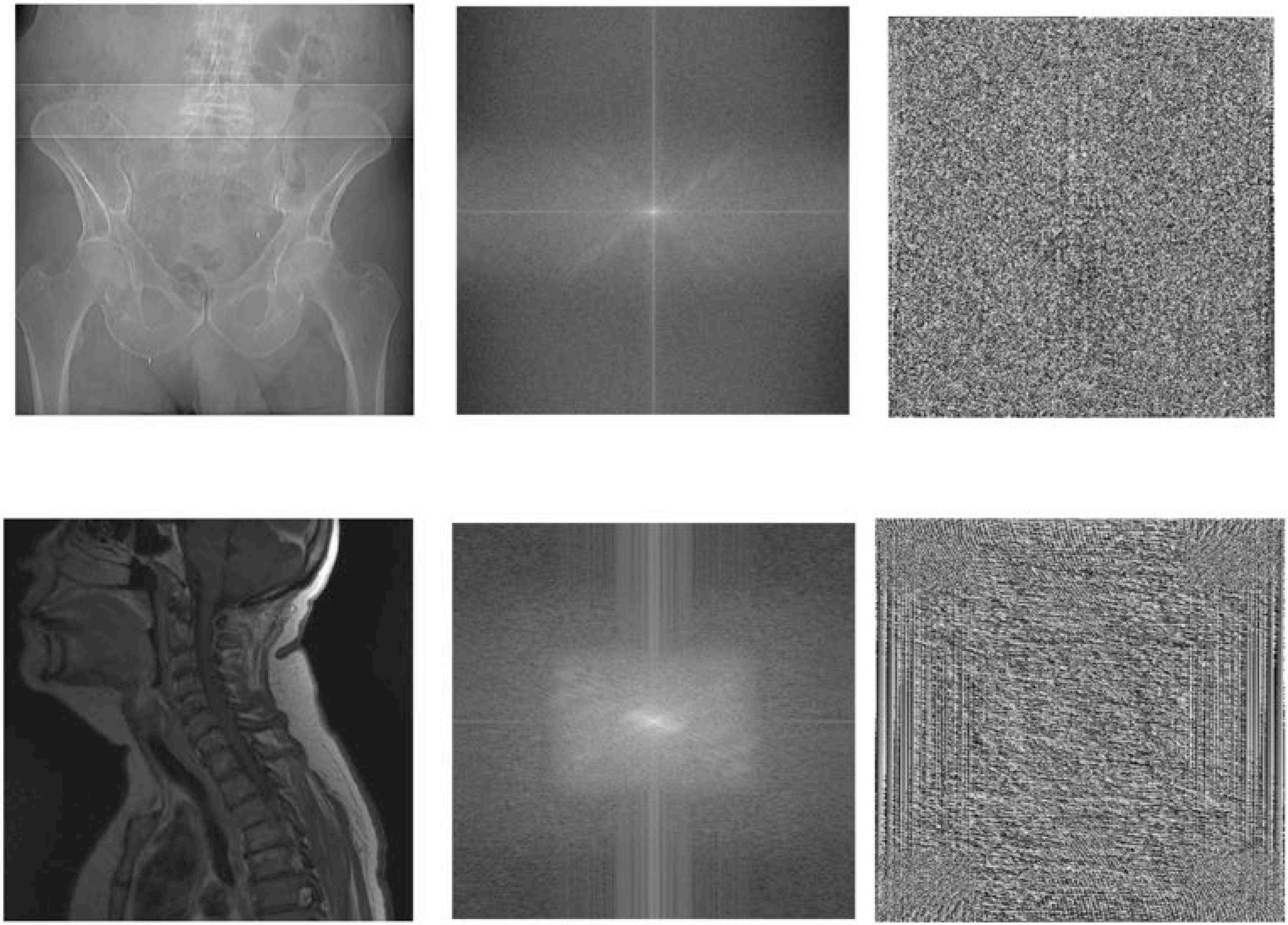
$$\left|F(u,v)\right|$$



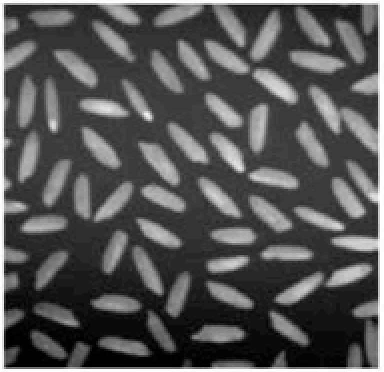
BASSE FREQUENZE

ALTE FREQUENZE

DISCRETE FOURIER TRANSFORM DFT

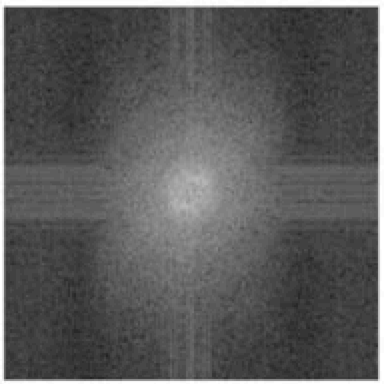


MODULO E FASE

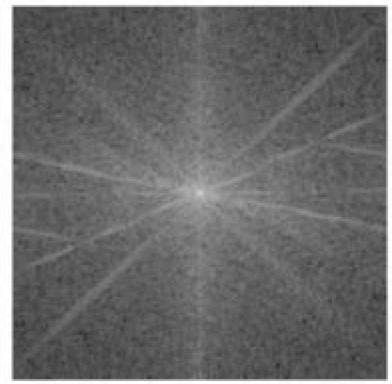


$f(x,y)$

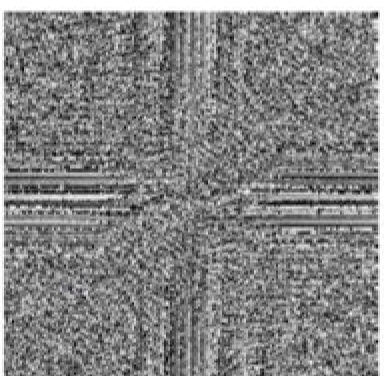
$g(x,y)$



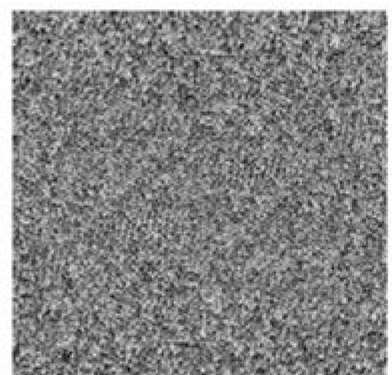
$|F(u,v)|$



$|G(u,v)|$

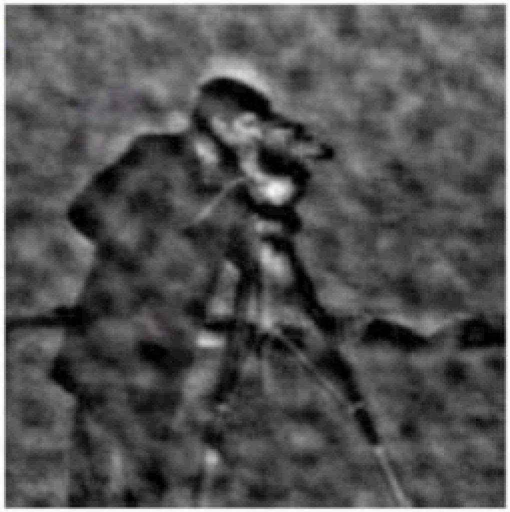


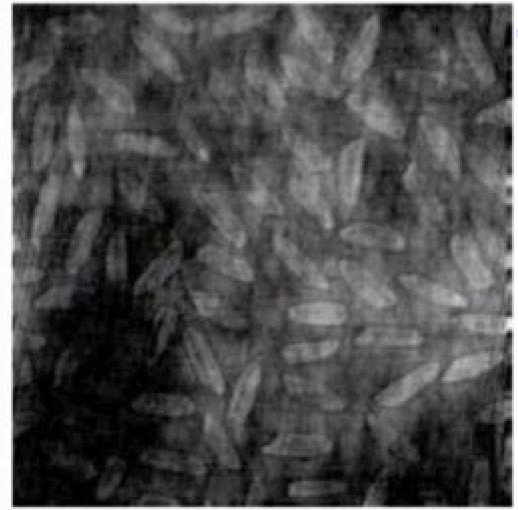
$\angle F(u,v)$



$\angle G(u,v)$

MODULO E FASE





$$F^{-1}\left[|F(u,v)|e^{(j\angle G(u,v))}\right] \qquad F^{-1}\left[|G(u,v)|e^{(j\angle F(u,v))}\right]$$

•La fase contiene le informazione essenziali sulla struttura dell'immagine e le informazioni sulla posizione.

•L'ampiezza contiene solo l'informazione relativa alla presenza o meno delle strutture nell'immagine.

PROPRIETÀ DELLA DFT

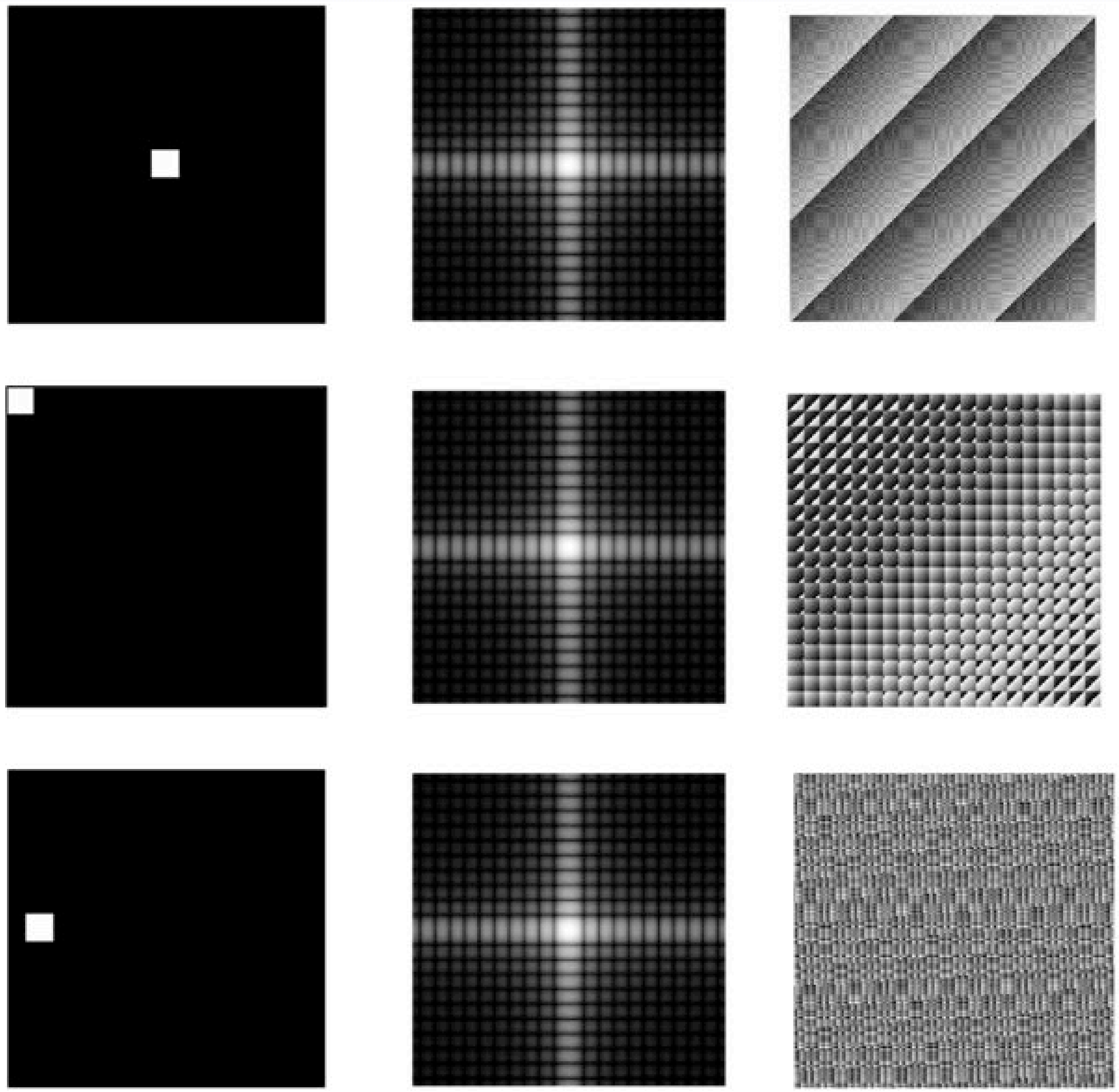
Separabilità

$$
\begin{tikzcd}[column sep=huge]
\boxed{f(x,y)} \arrow[r, Rightarrow, "\textit{DFT 1D sulle righe}"'] & \boxed{F(x,v)} \arrow[r, Rightarrow, "\textit{DFT 1D sulle colonne}"'] & \boxed{F(u,v)}
\end{tikzcd}
$$

$$
\begin{aligned}
F(u,v) &= \frac{1}{NM}\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{N}+\frac{vy}{M}\right)} \\
&= \frac{1}{N}\sum_{x=0}^{N-1}\left(\underbrace{\boxed{\frac{1}{M}\sum_{y=0}^{M-1} f(x,y)\, e^{-j2\pi\left(\frac{vy}{M}\right)}}}_{\mathbf{F(x,v)}}\right) e^{-j2\pi\left(\frac{ux}{N}\right)}
\end{aligned}
$$

PROPRIETÀ DELLA DFT

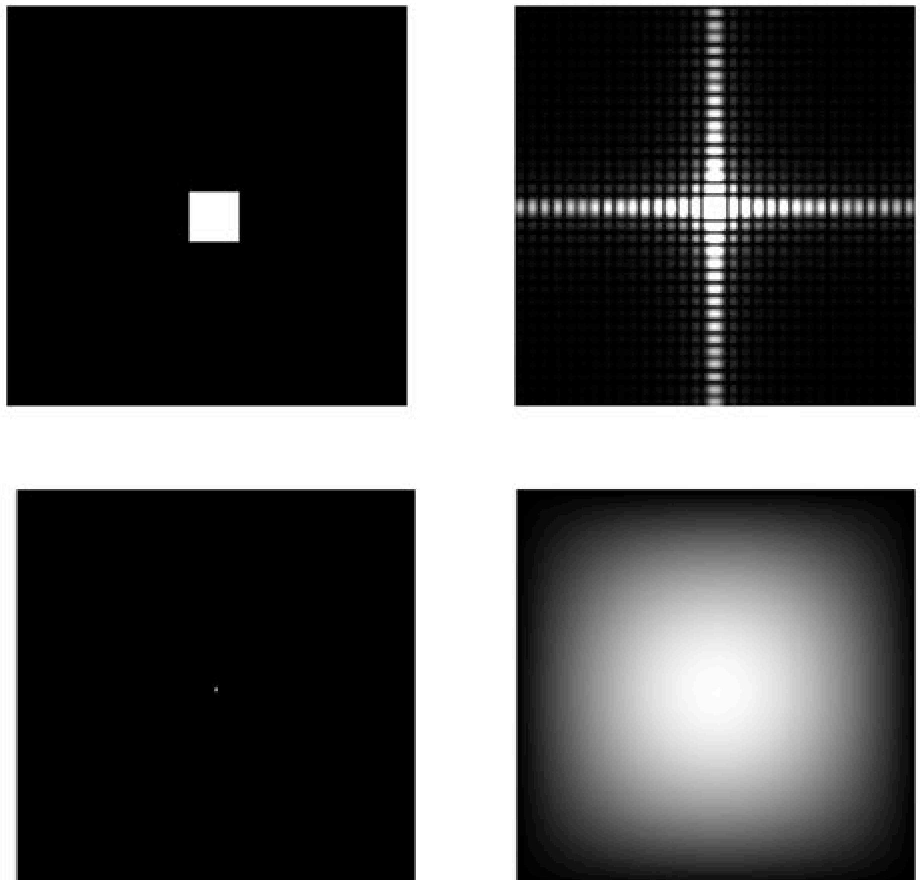
• Spettro invariante per traslazione



PROPRIETÀ DELLA DFT

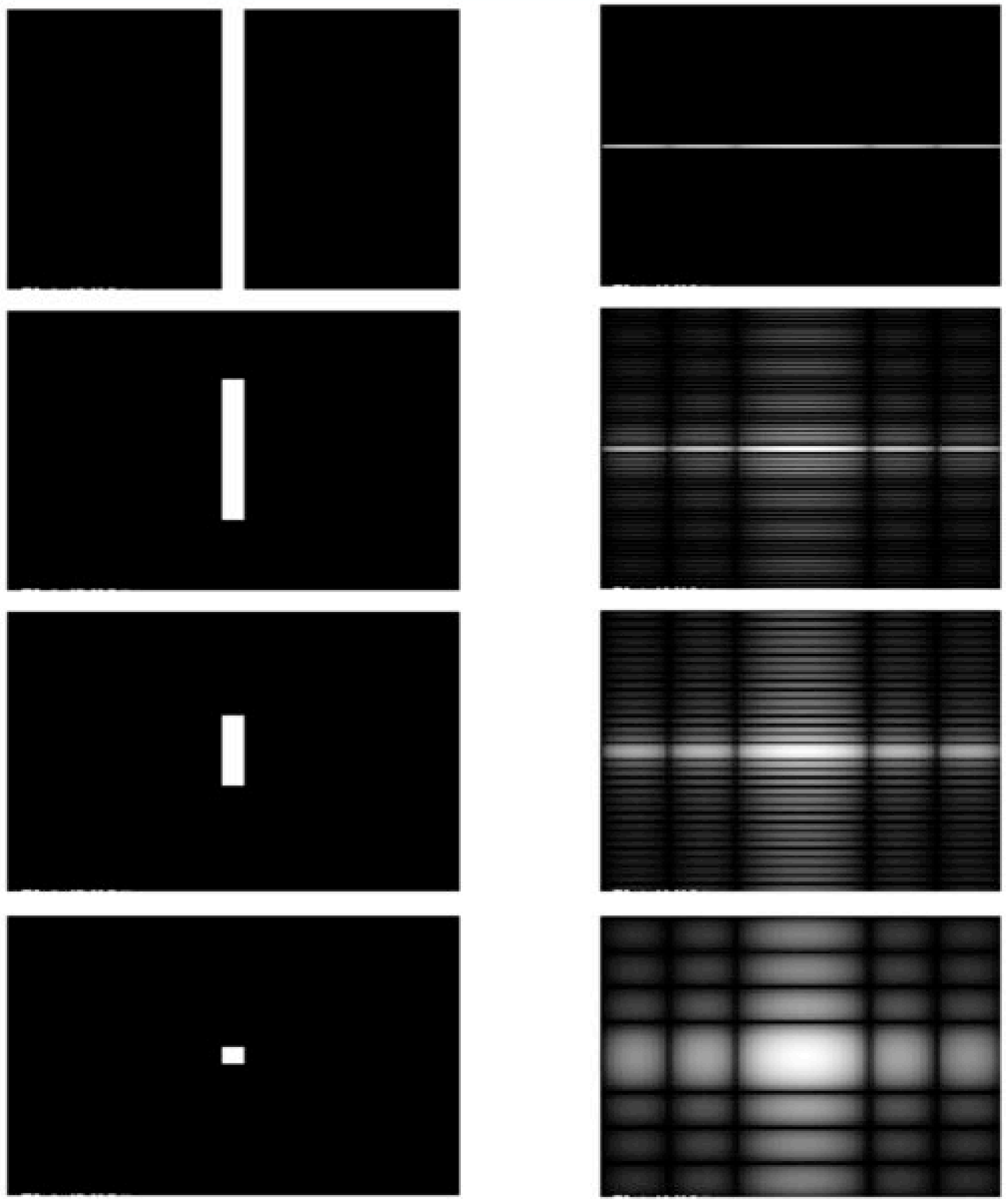
Scaling

$$f(\alpha x, \beta y) \Leftrightarrow \frac{1}{|\alpha\beta|} F\left(\frac{u}{\alpha}, \frac{v}{\beta}\right)$$



PROPRIETÀ DELLA DFT

- Scaling



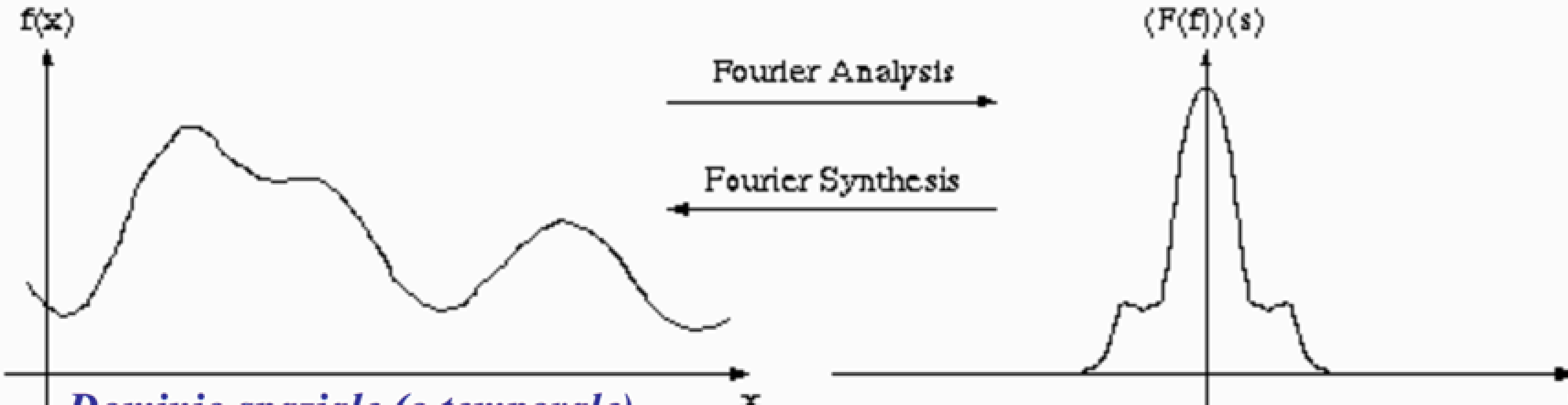
PROPRIETÀ DELLA DFT

Trasformazione reversibile: ANALISI-SINTESI

$$F^{-1}F[f(x,y)] = f(x,y)$$

Antitrasformata (SINTESI) — *Trasformata (ANALISI)*

$f(x) = F^{-1}[F(u)]$ — $F(u) = F[f(x)]$



Dominio spaziale (o temporale) o dominio diretto

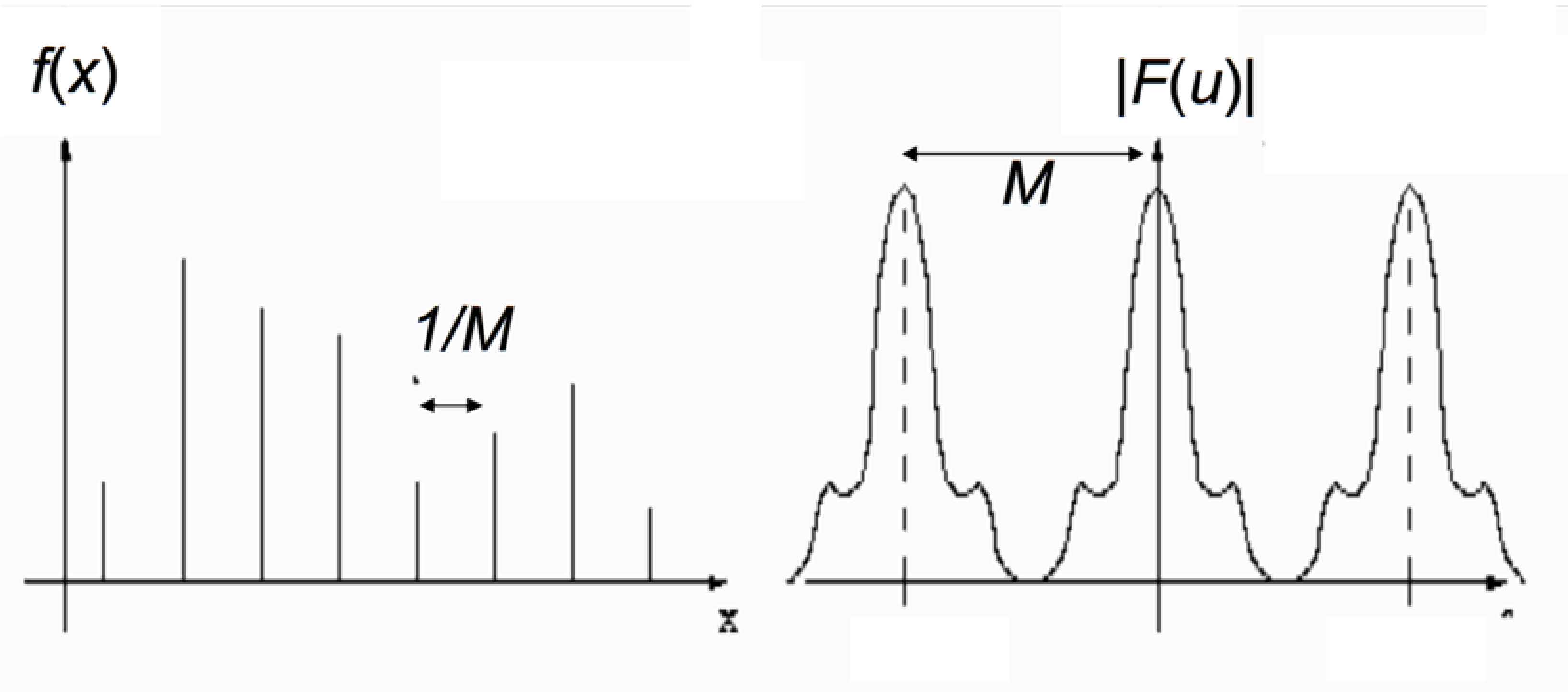
Dominio delle frequenze o dominio trasormato

PROPRIETÀ DELLA DFT

Periodicità della DFT: f(x) campionata (vale anche per DTFT)

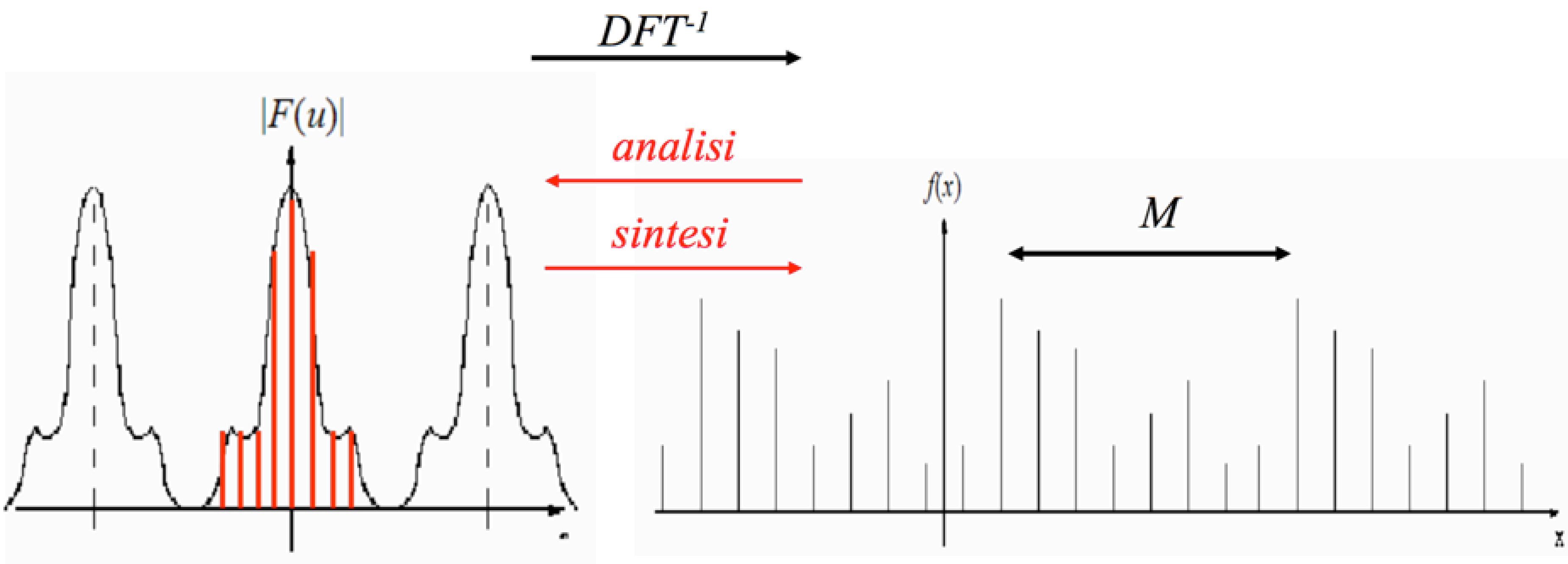
$$F(u) = F(u+M)$$

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x) e^{-j\frac{2\pi}{M}ux} \quad u = 0,\ldots,M-1$$



PROPRIETÀ DELLA DFT

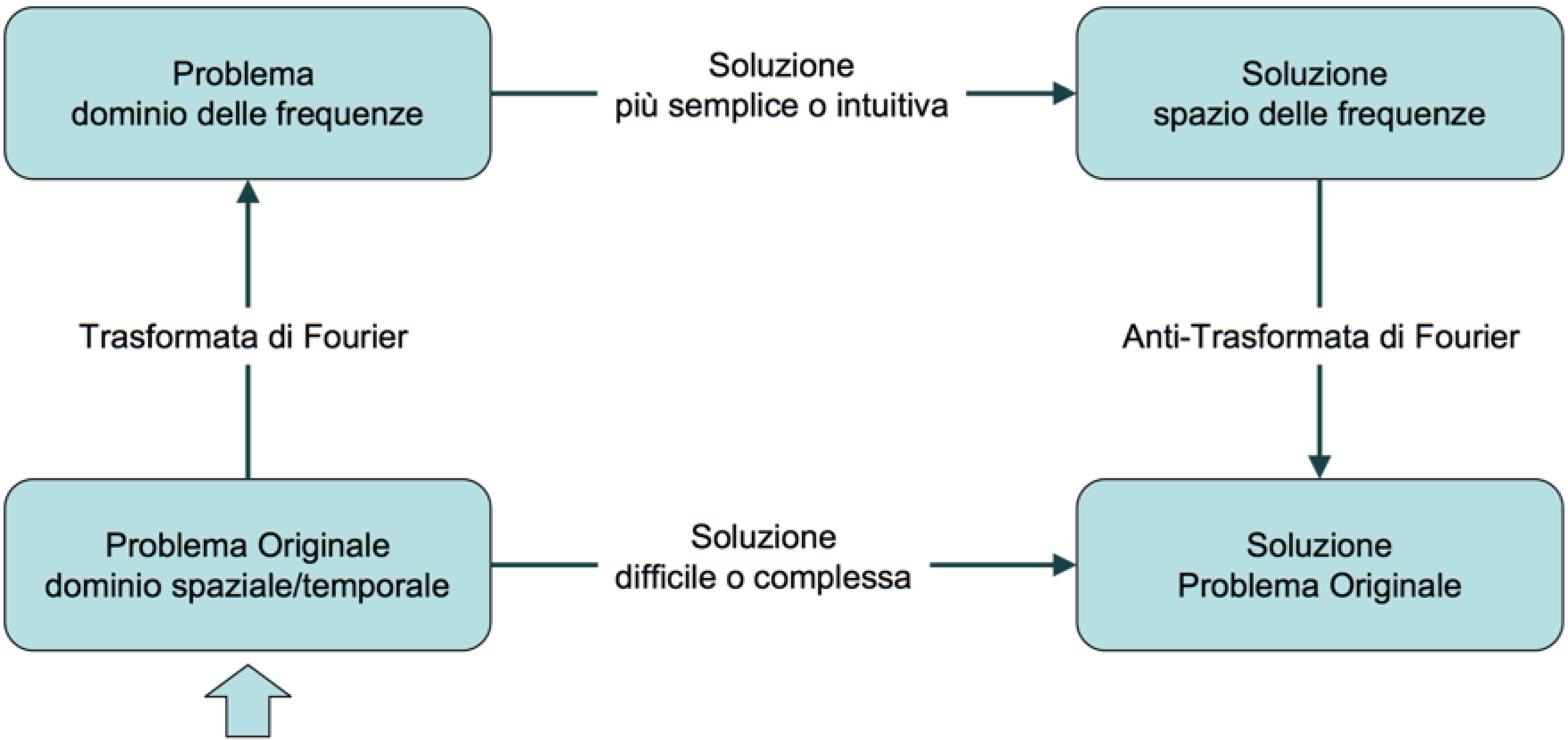
Periodicità di antitrasformata



$$f(x) = \sum_{u=0}^{M-1} F(u) e^{j\frac{2\pi}{M}ux} \qquad x = 0, \ldots, M-1$$

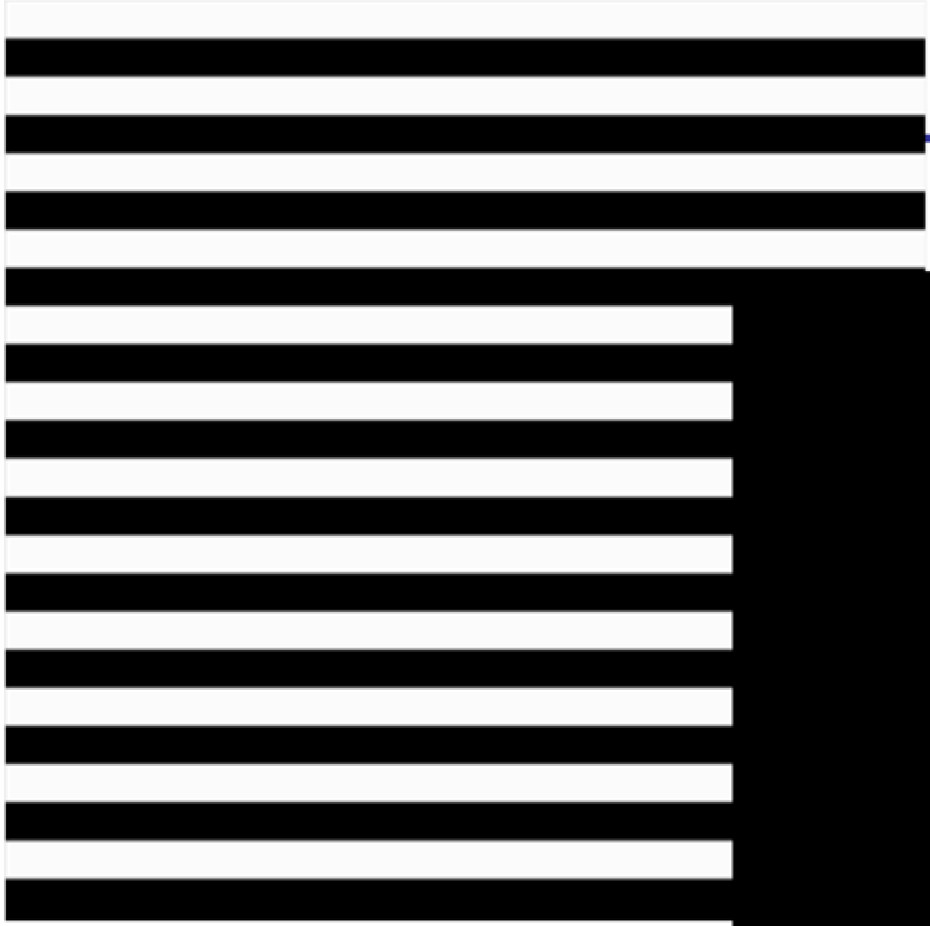
Trasformata di Fourier

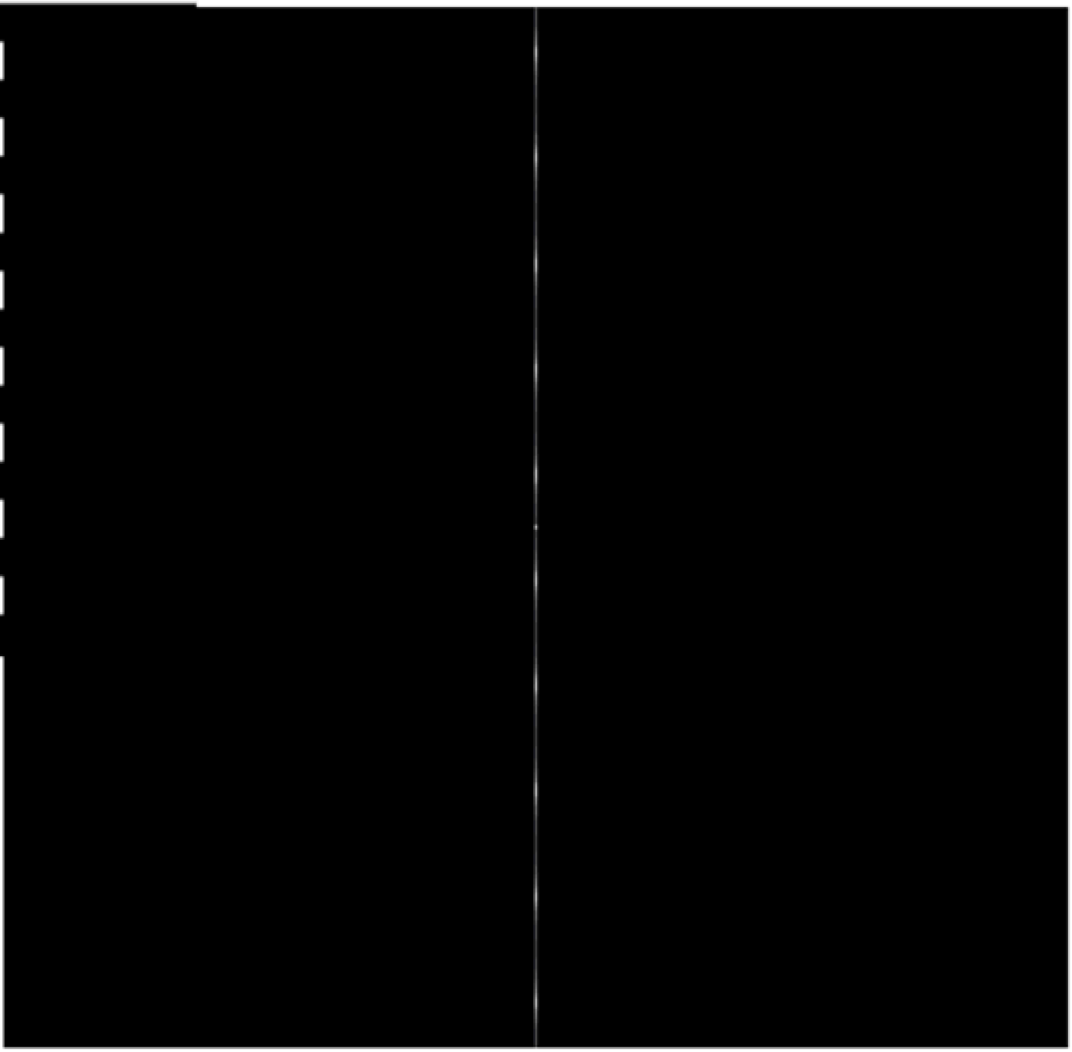
- Dominio Spaziale/Temporale vs. Frequenze



ESEMPIO DI DFT 2D

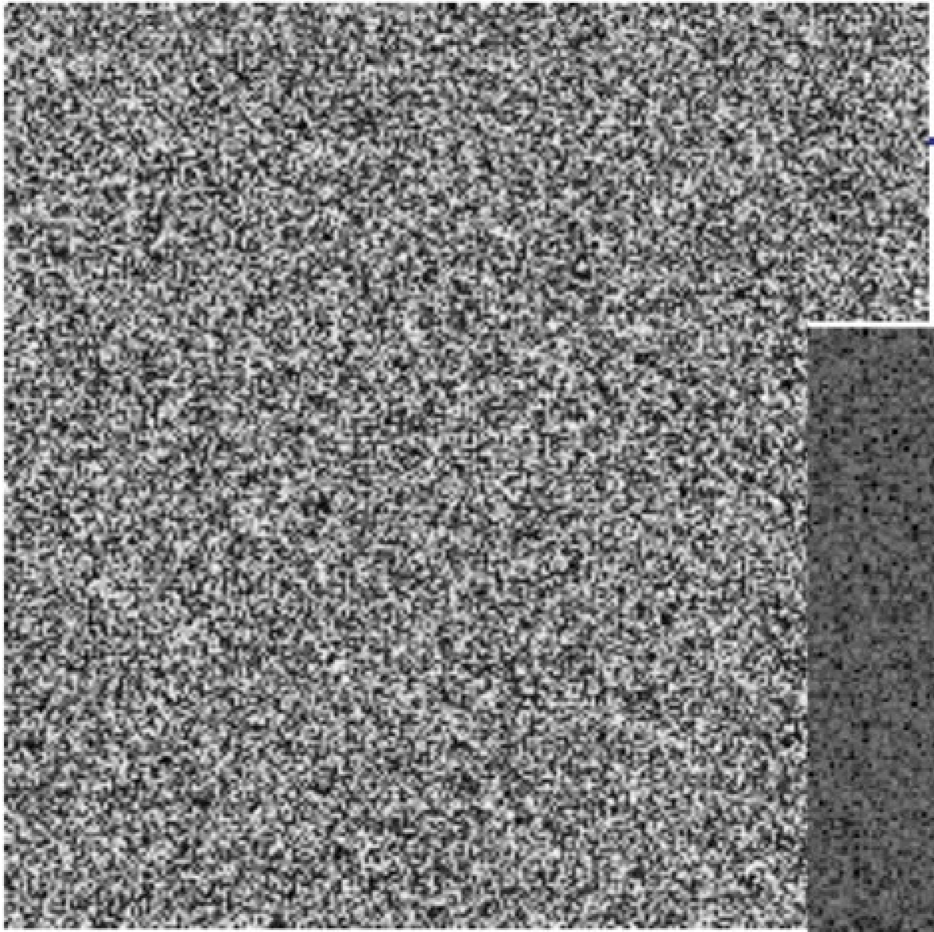
senza rumore (noise)

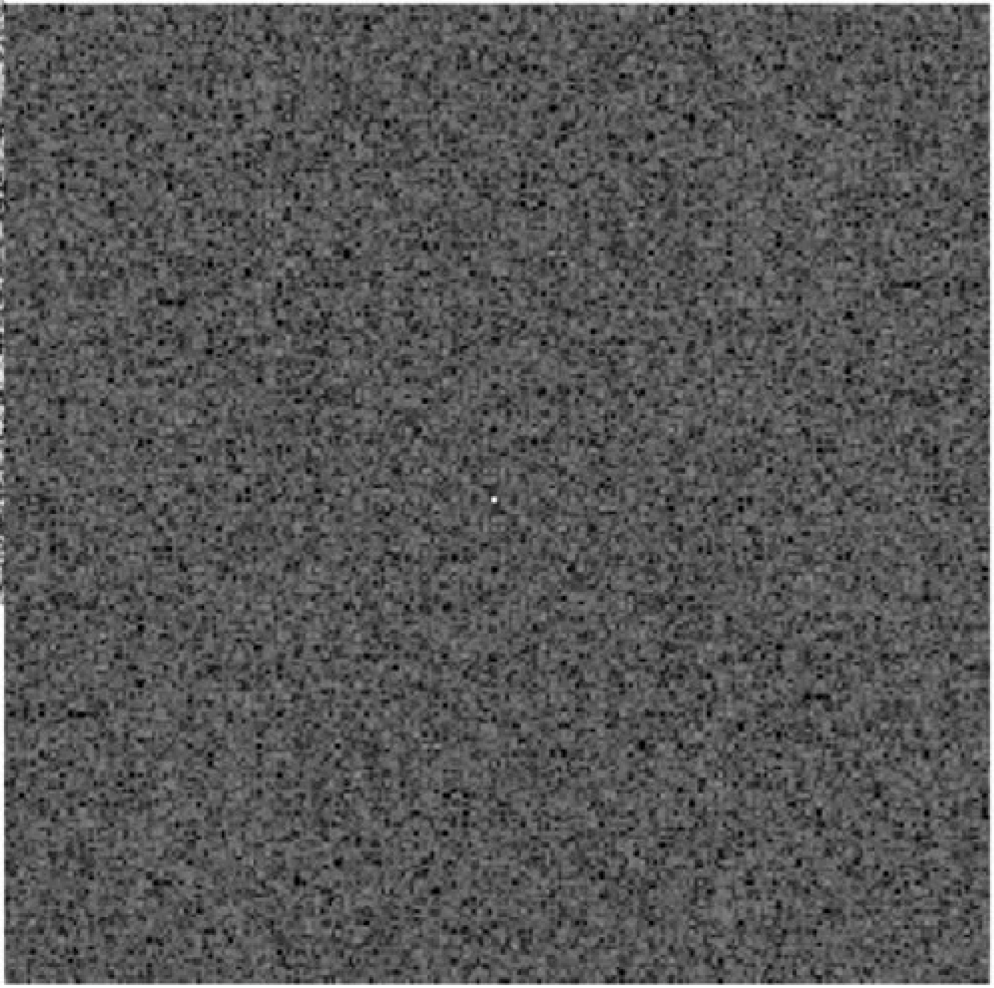




ESEMPIO DI DFT 2D

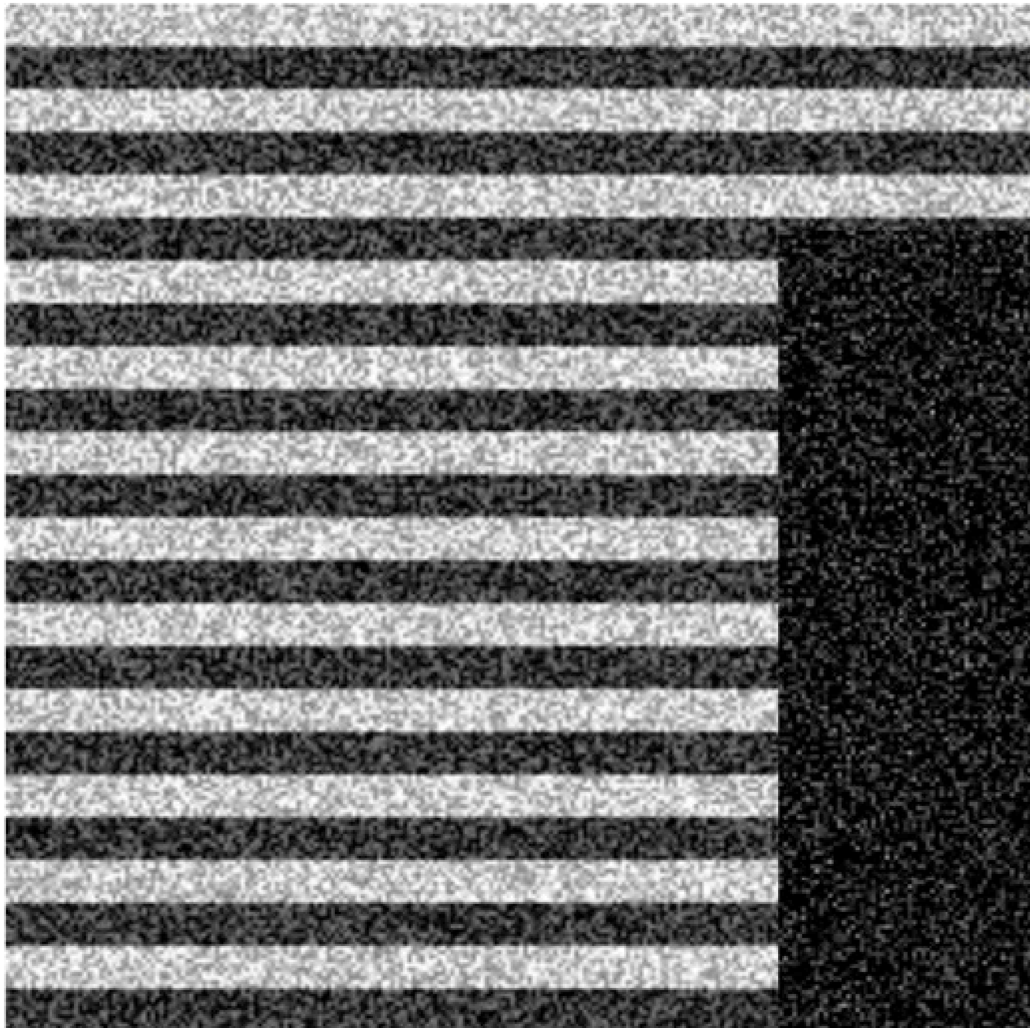
rumore (noise)

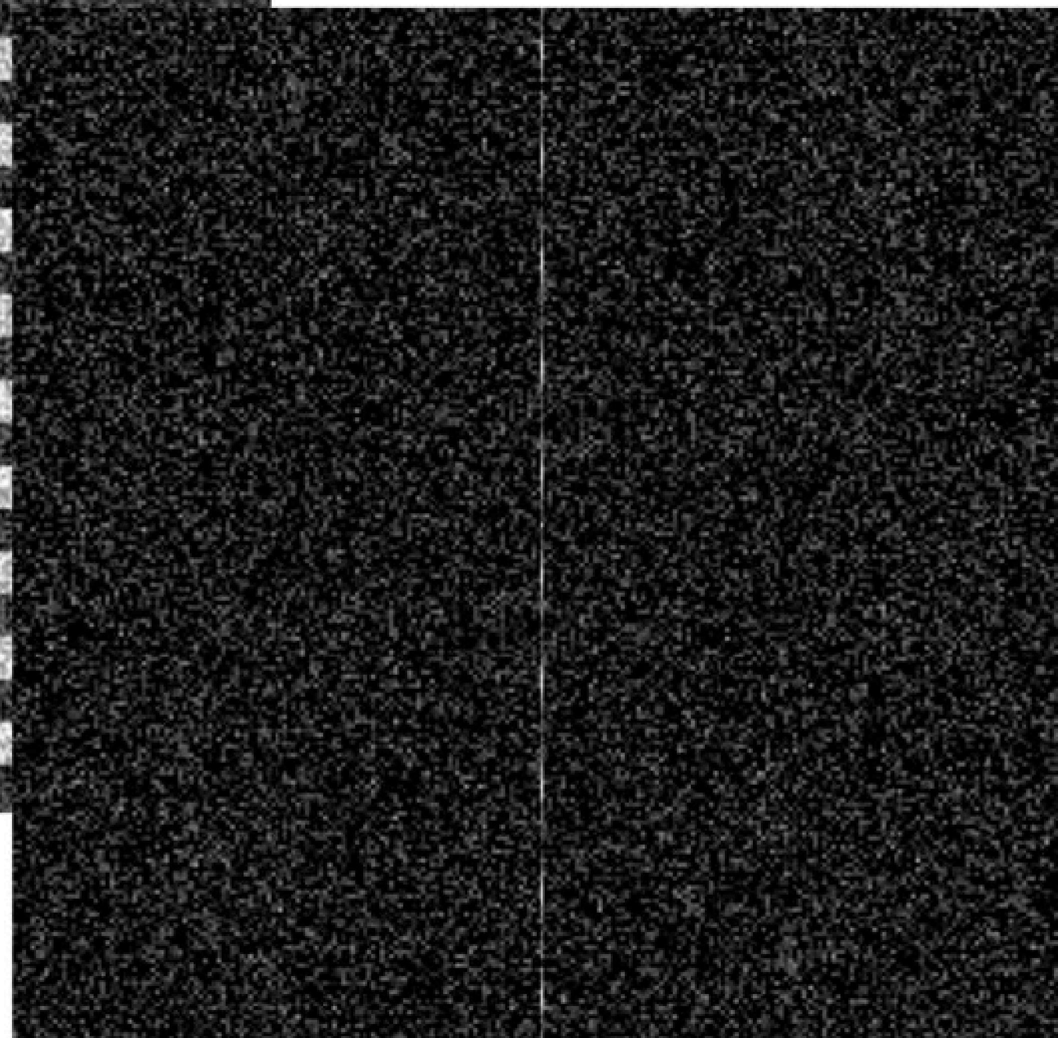




ESEMPIO DI DFT 2D

con rumore (noise)





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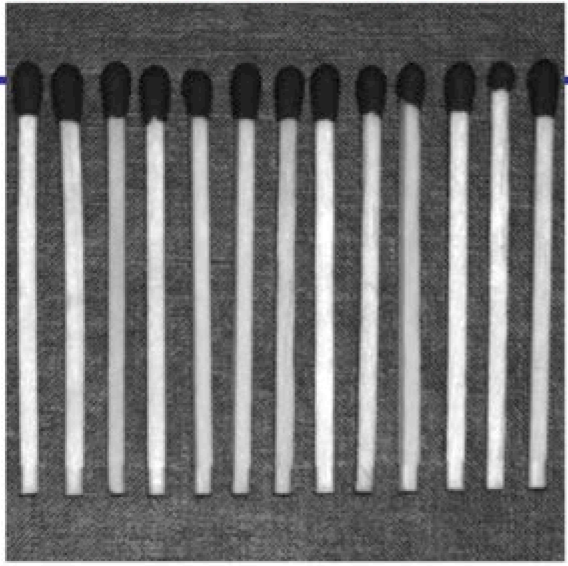
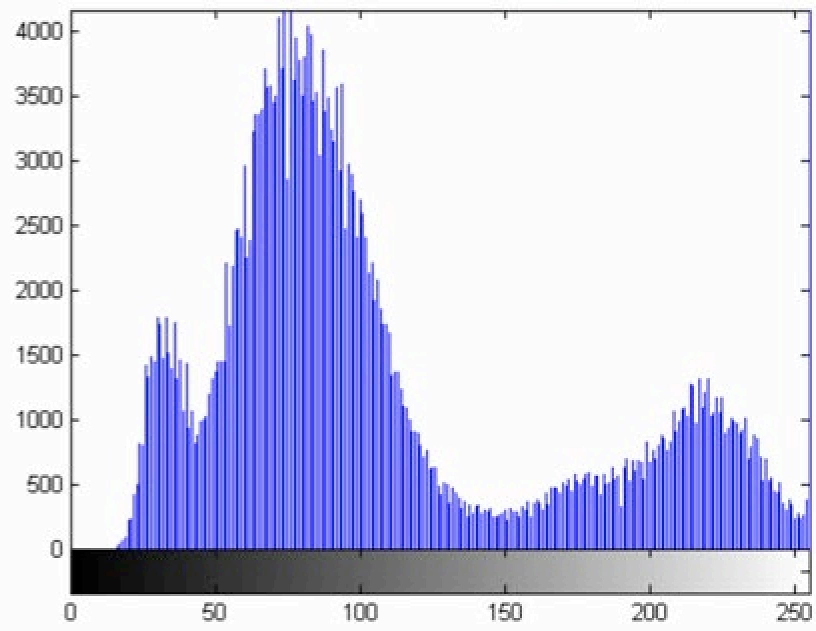
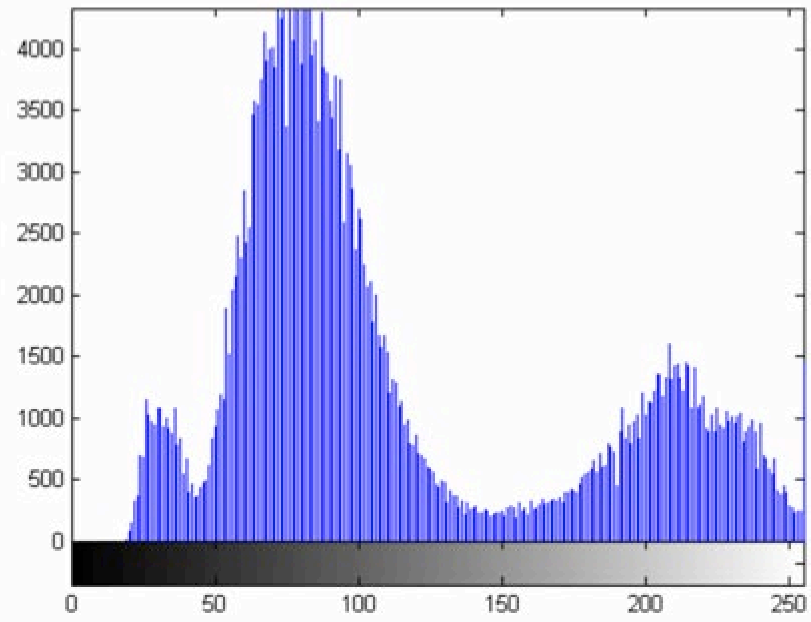


ANALISI ISTOGRAMMA





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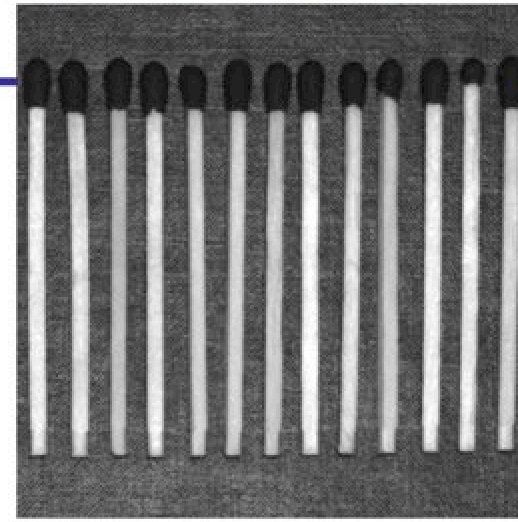

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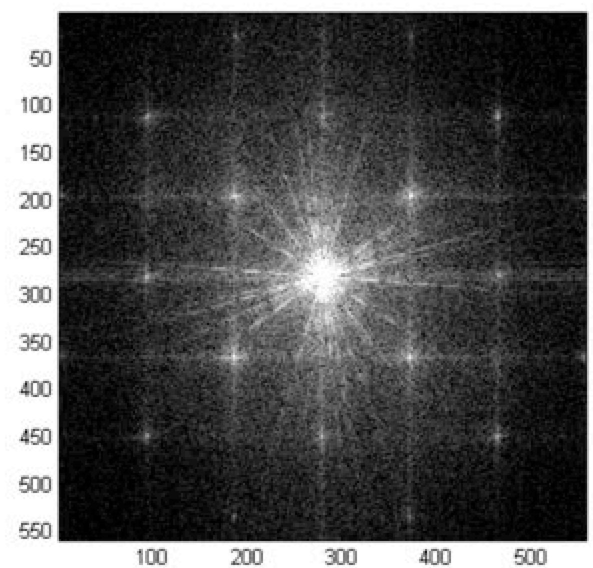

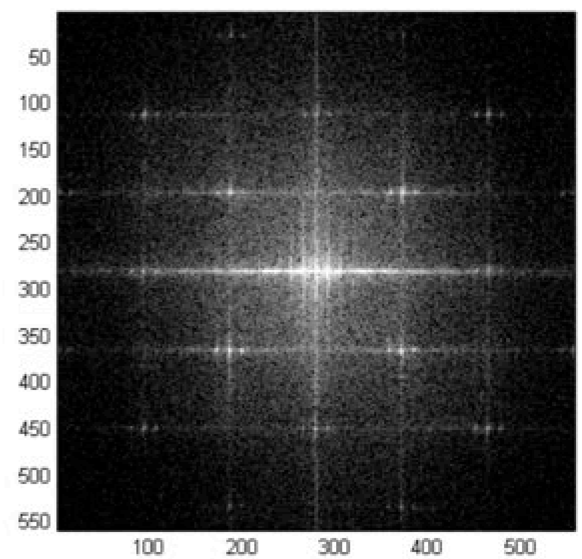

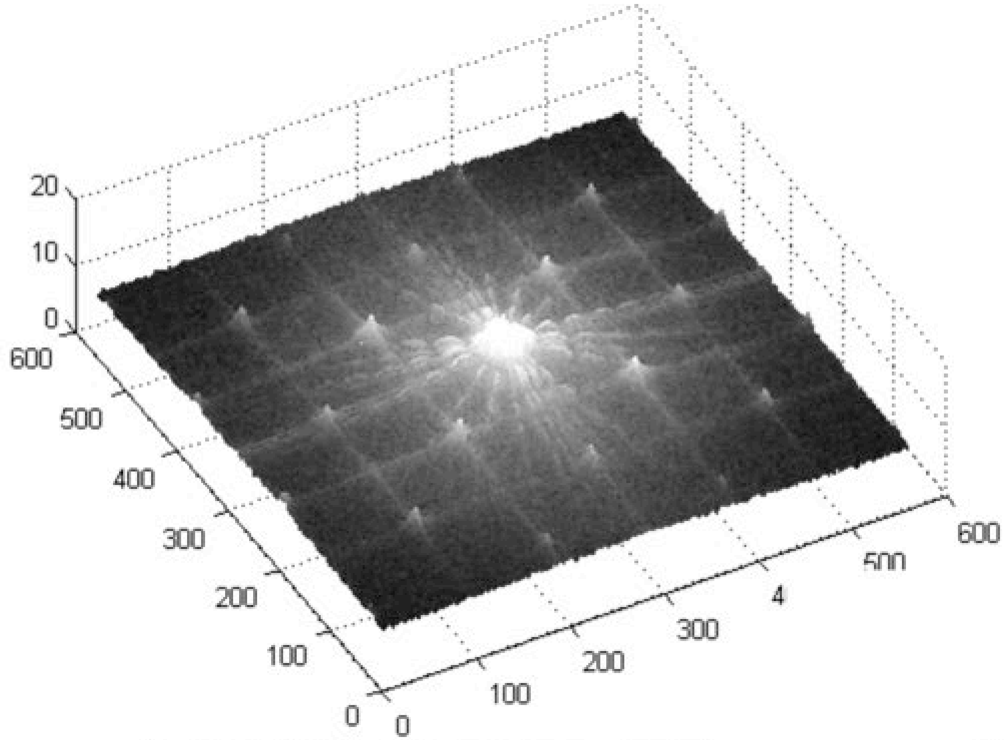
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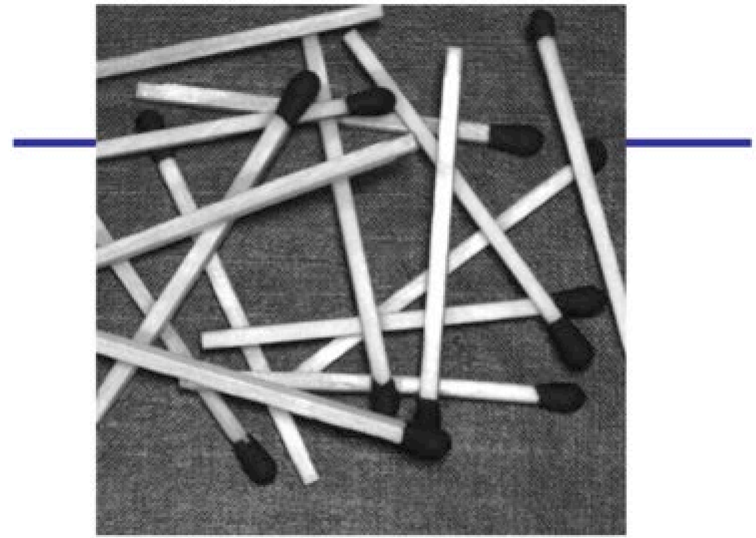

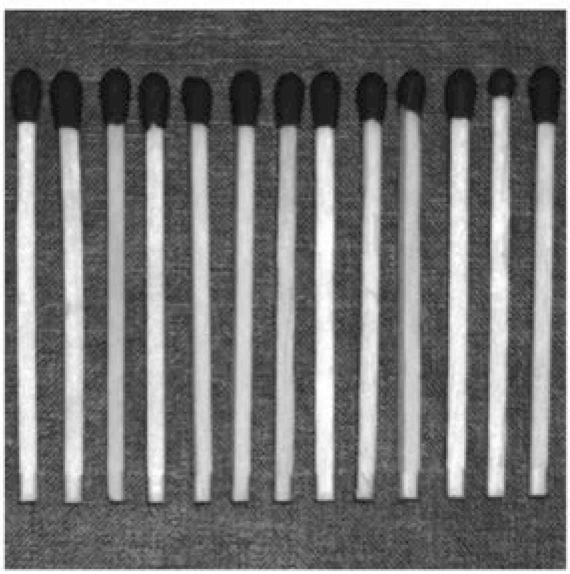

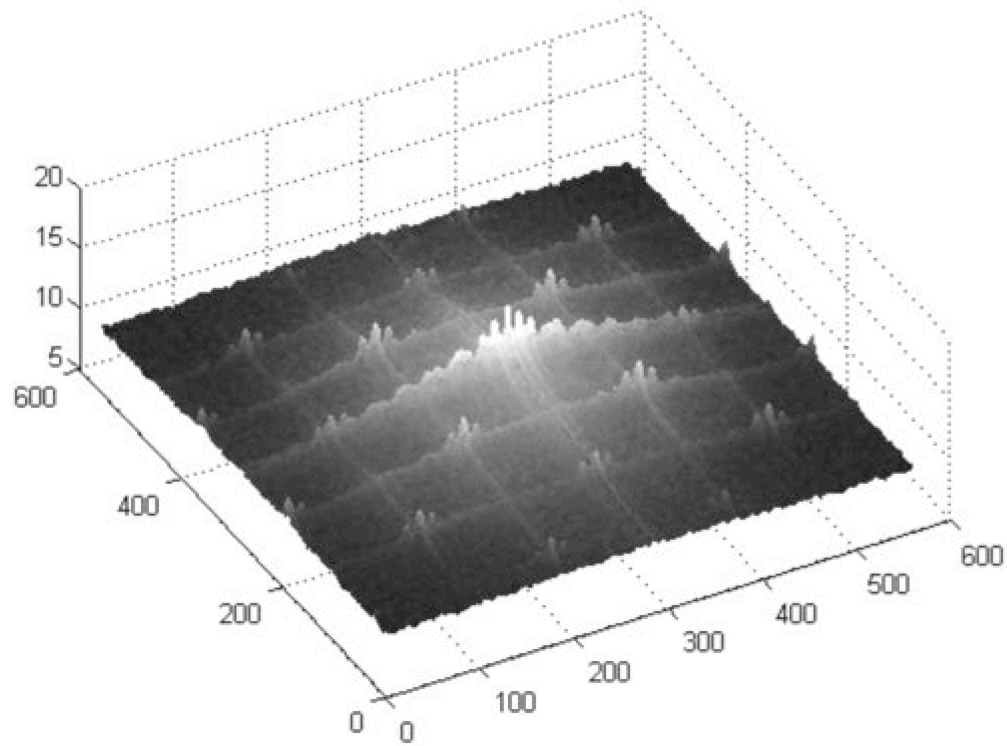
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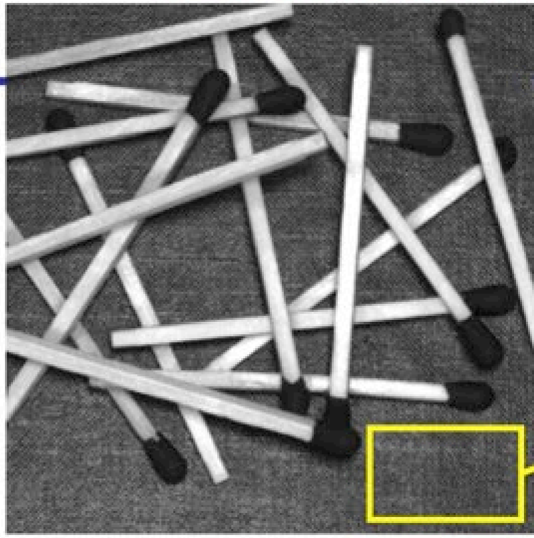

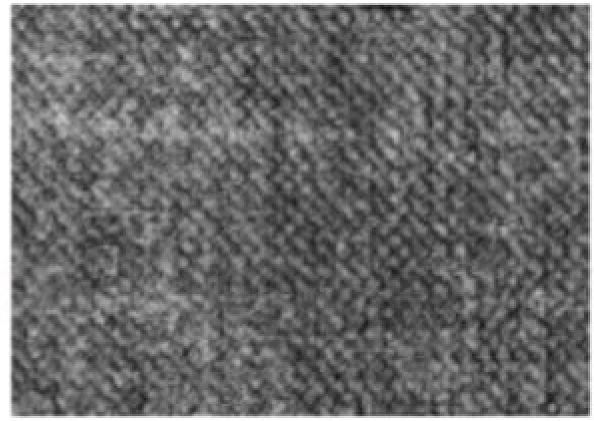

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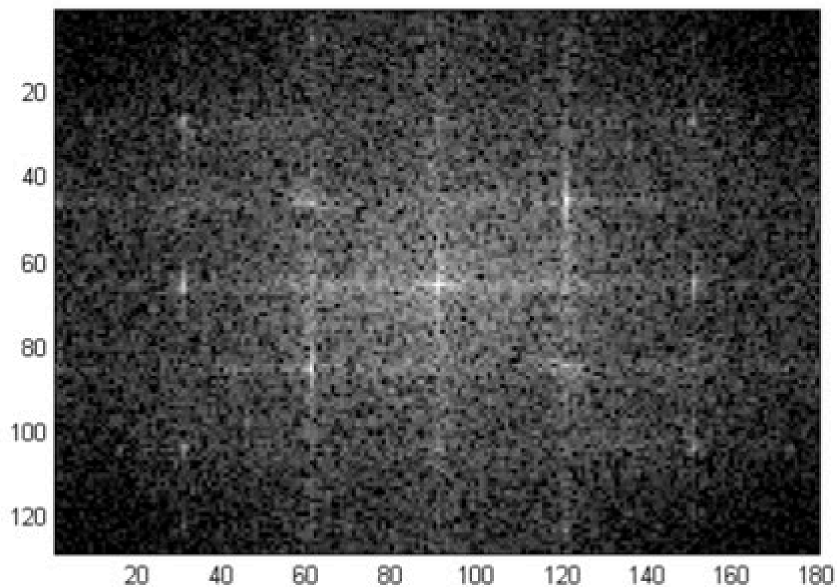

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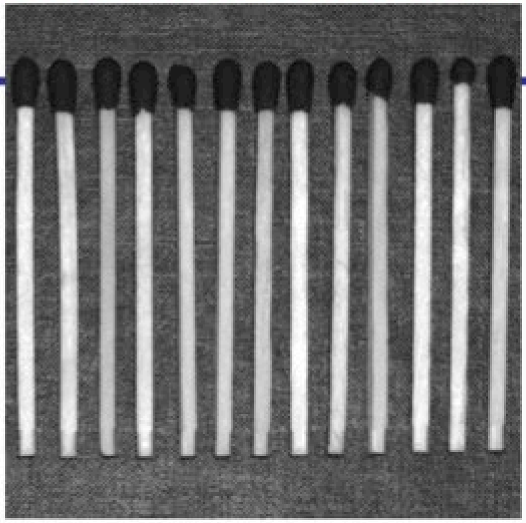

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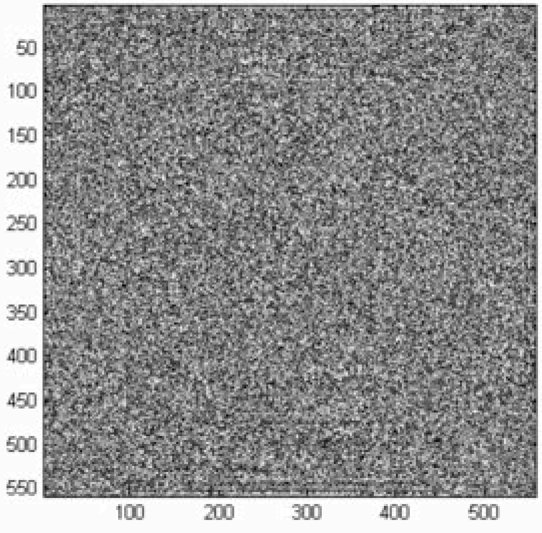
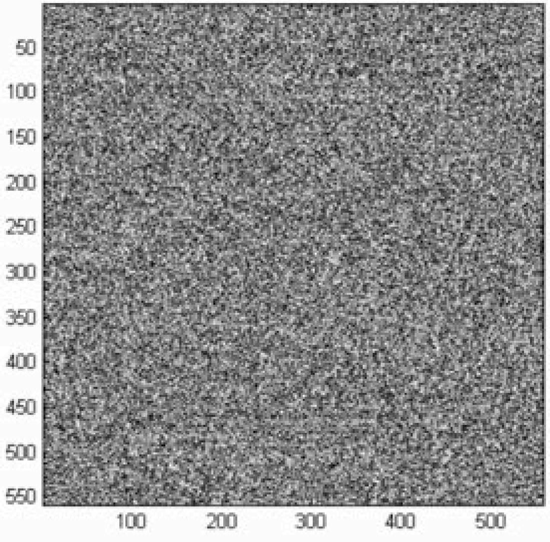
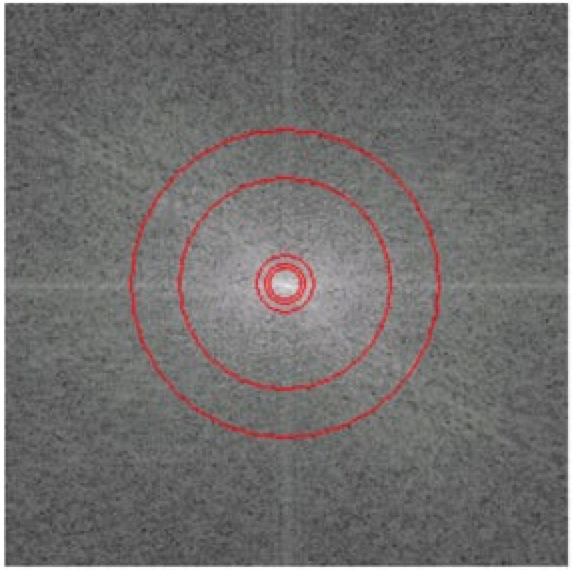




Trasformata di Fourier: Immagini

- Manipolazione dello spettro

$$P(u) = \left|F(u)\right|^2 = \Re(u)^2 + \Im(u)^2$$ Potenza spettrale

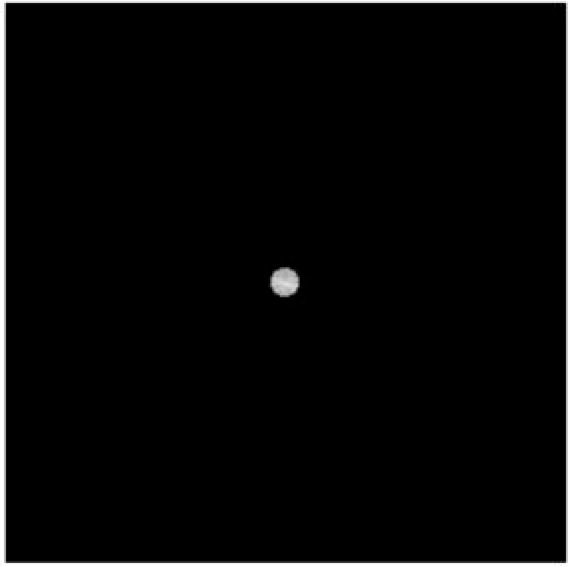


- Le circonferenze (dal centro) racchiudono il 90, 93, 95, 99, 99.5% della potenza spettrale totale

Trasformata di Fourier: Immagini

- Manipolazione dello spettro

Originale



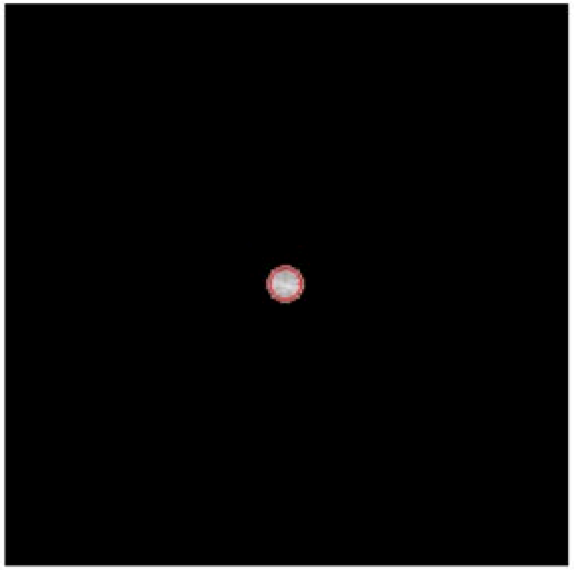
Spettro (90%)

Ricostruita

Trasformata di Fourier: Immagini

- Manipolazione dello spettro

Originale



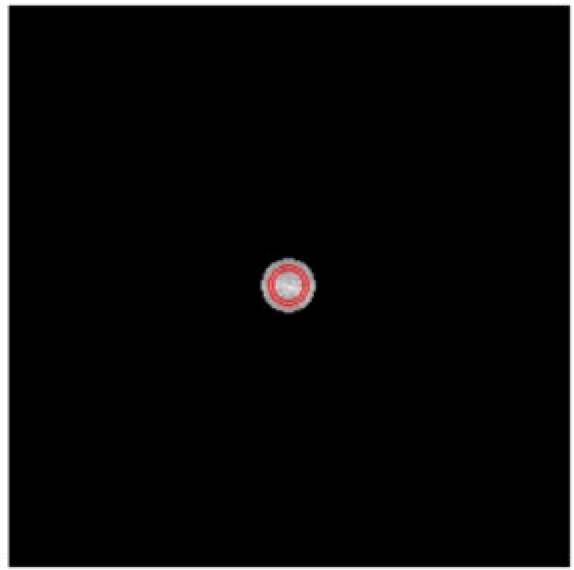
Spettro (93%)

Ricostruita

Trasformata di Fourier: Immagini

- Manipolazione dello spettro

Originale



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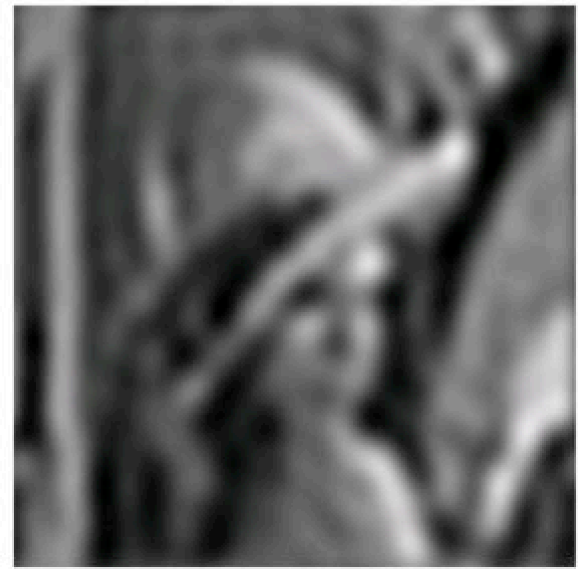
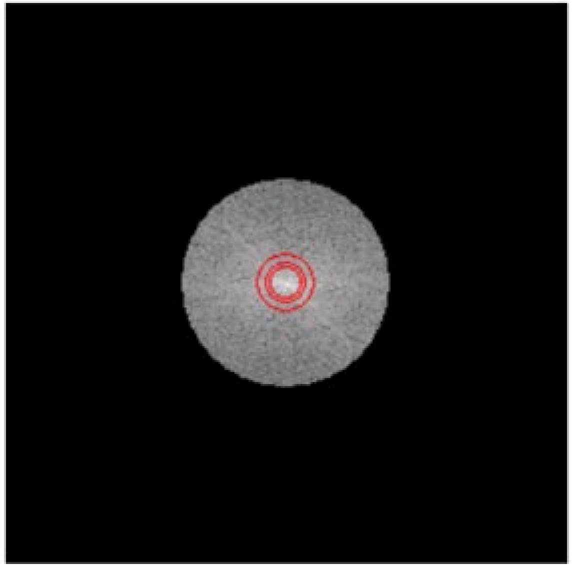


Ricostruita

Trasformata di Fourier: Immagini

- Manipolazione dello spettro

Originale

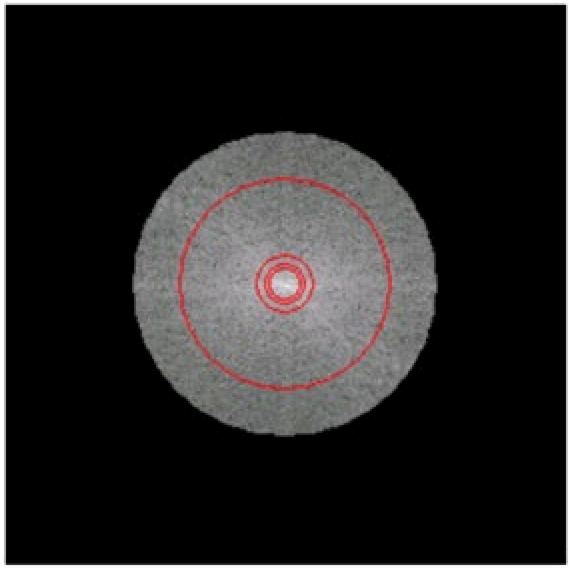


Spettro (99%)

Ricostruita

Trasformata di Fourier: Immagini

- Manipolazione dello spettro



Originale | Spettro (99.5%) | Ricostruita

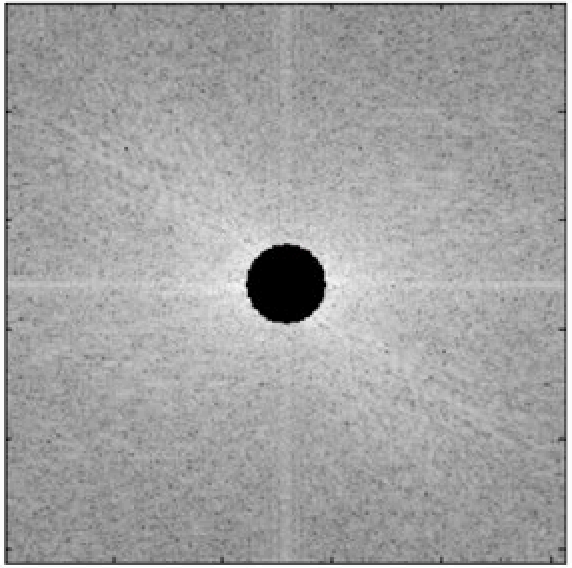
Trasformata di Fourier: Immagini

- Ringing

Trasformata di Fourier: Immagini

- Manipolazione dello spettro

Originale



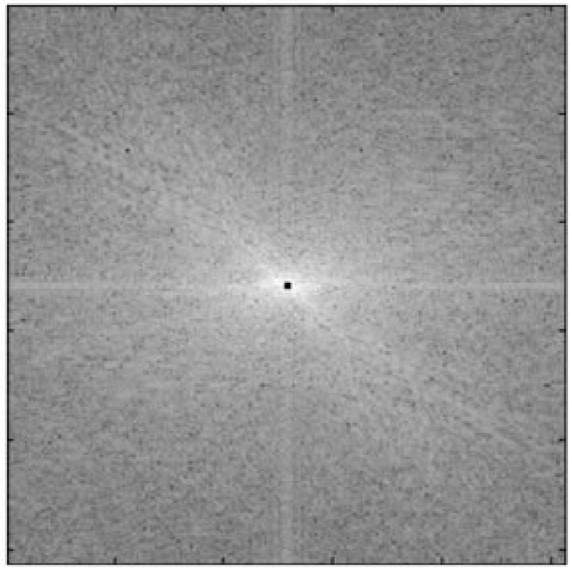
Spettro

Ricostruita

Trasformata di Fourier: Immagini

- Manipolazione dello spettro

Originale



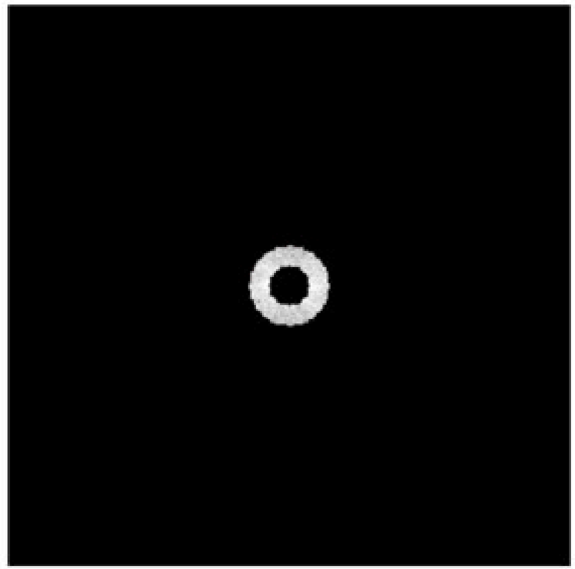
Spettro

Ricostruita

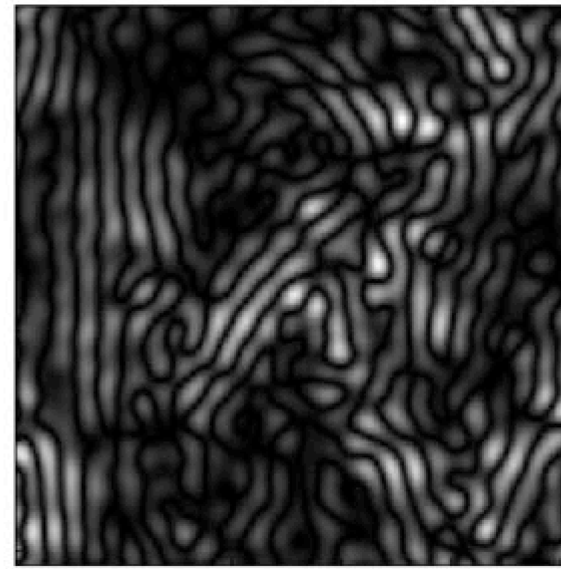
Trasformata di Fourier: Immagini

- Manipolazione dello spettro

Originale



Spettro

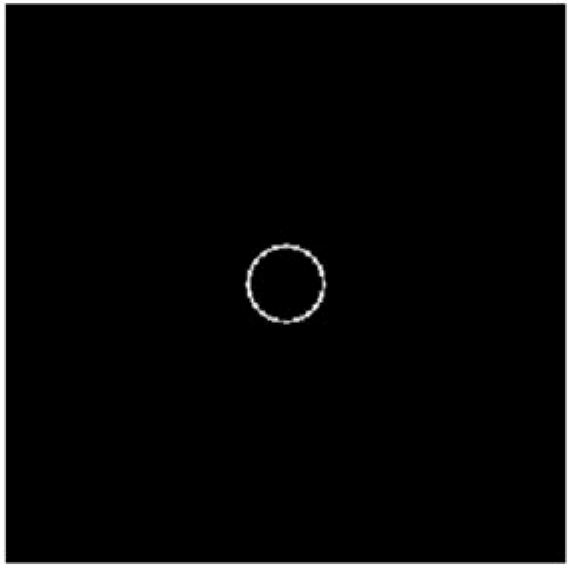


Ricostruita

Trasformata di Fourier: Immagini

- Manipolazione dello spettro

Originale



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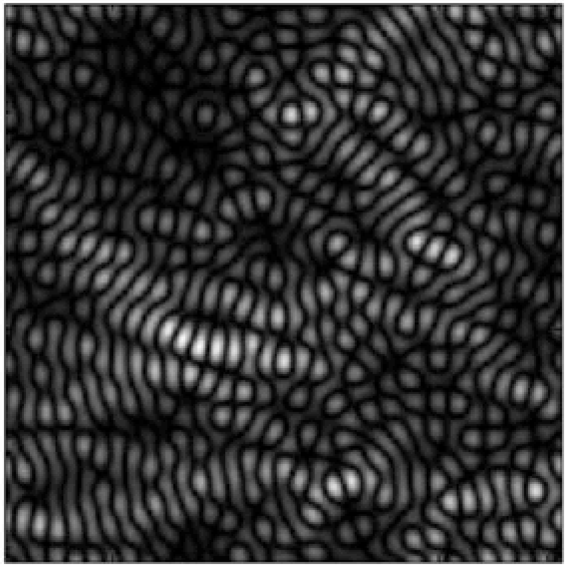


Ricostruita

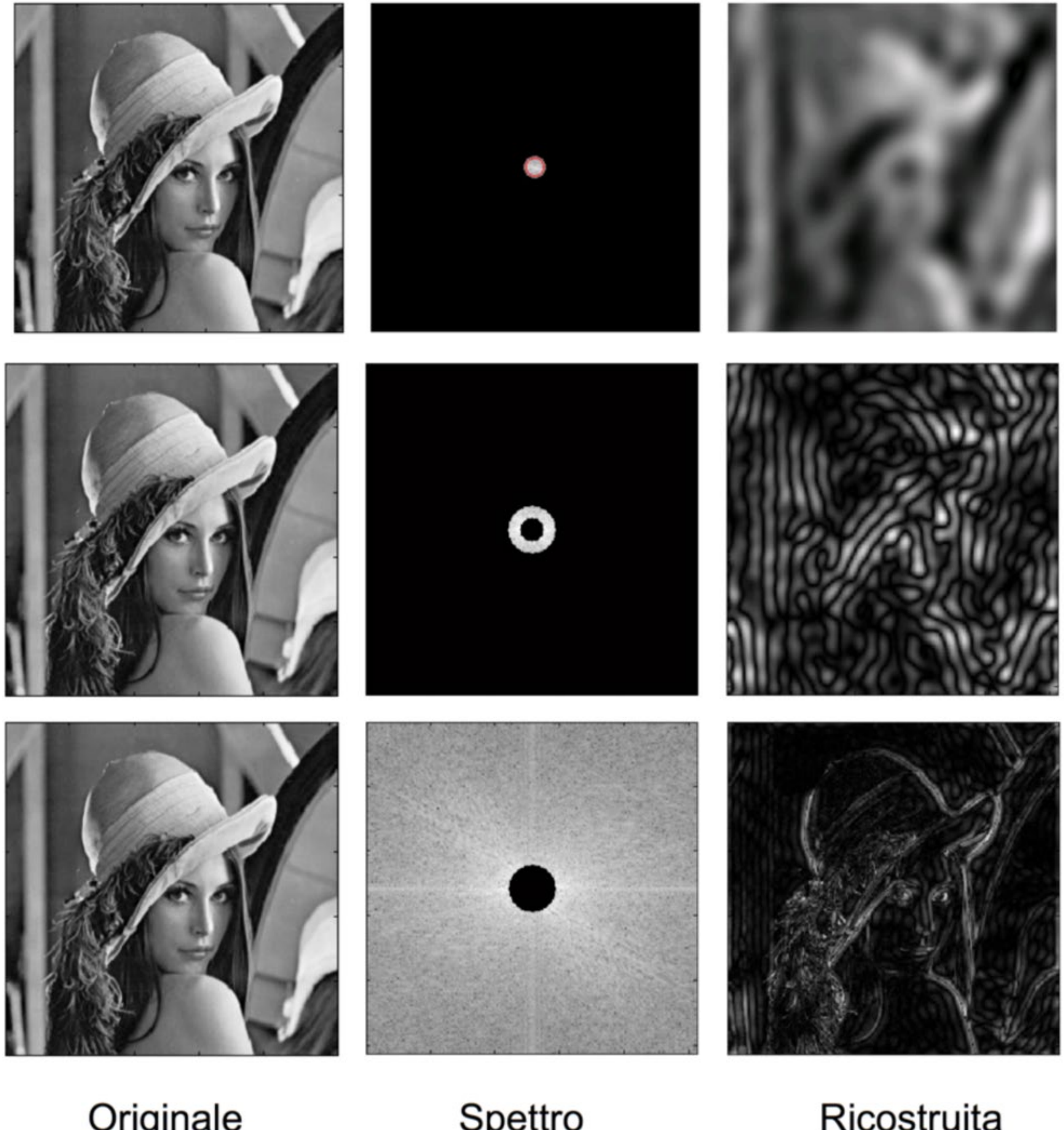
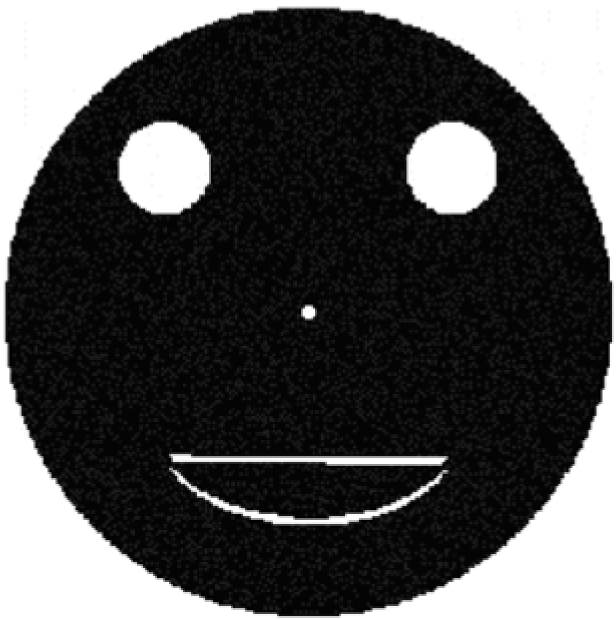
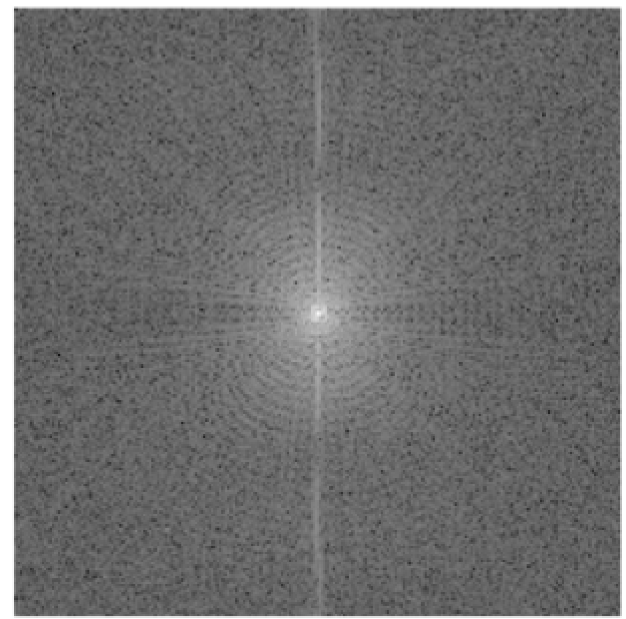
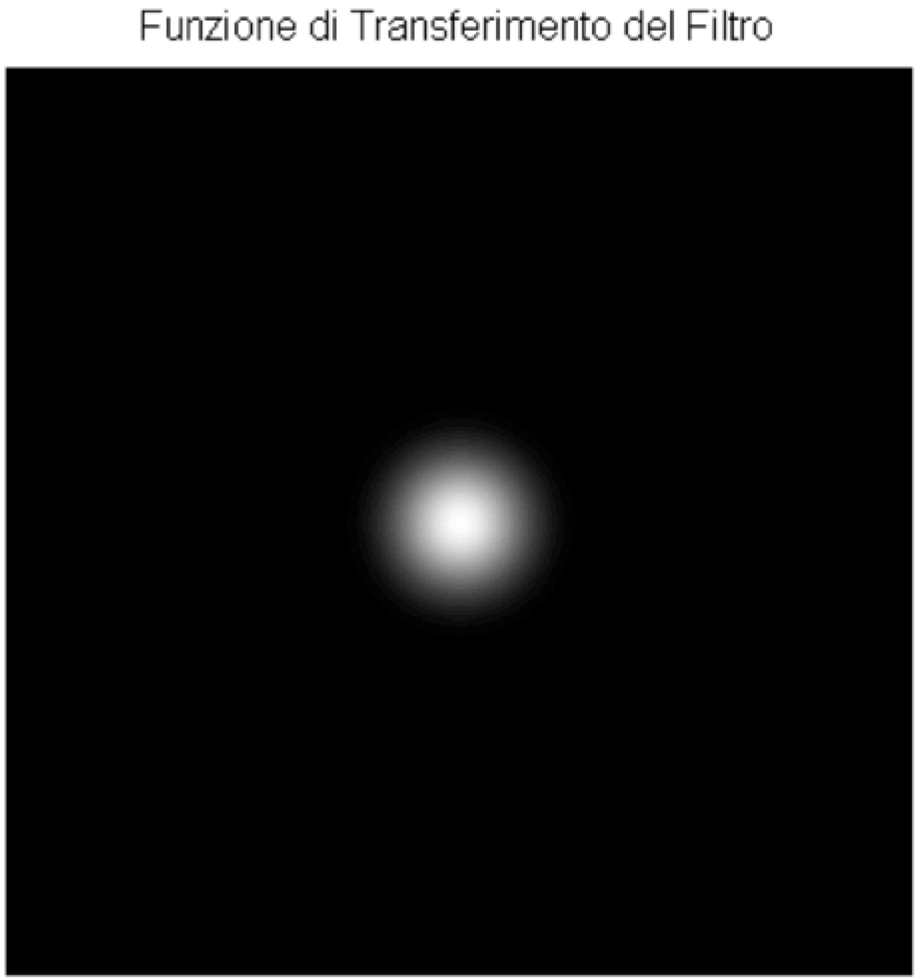

Originale
Spettro
Ricostruita

Immagine semplice



Spettro di Fourier





Il filtro si applica con un prodotto di convoluzione nel dominio della trasformata dell'immagine

Spettro di Fourier dell'immagine filtrata

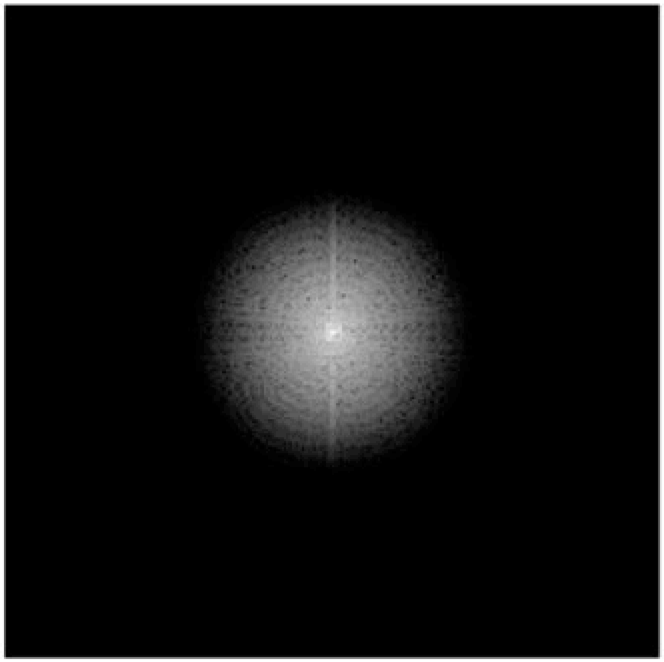


Immagine Filtrata

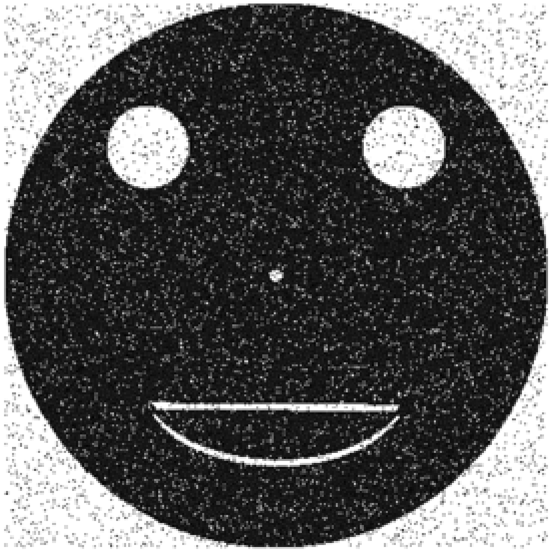
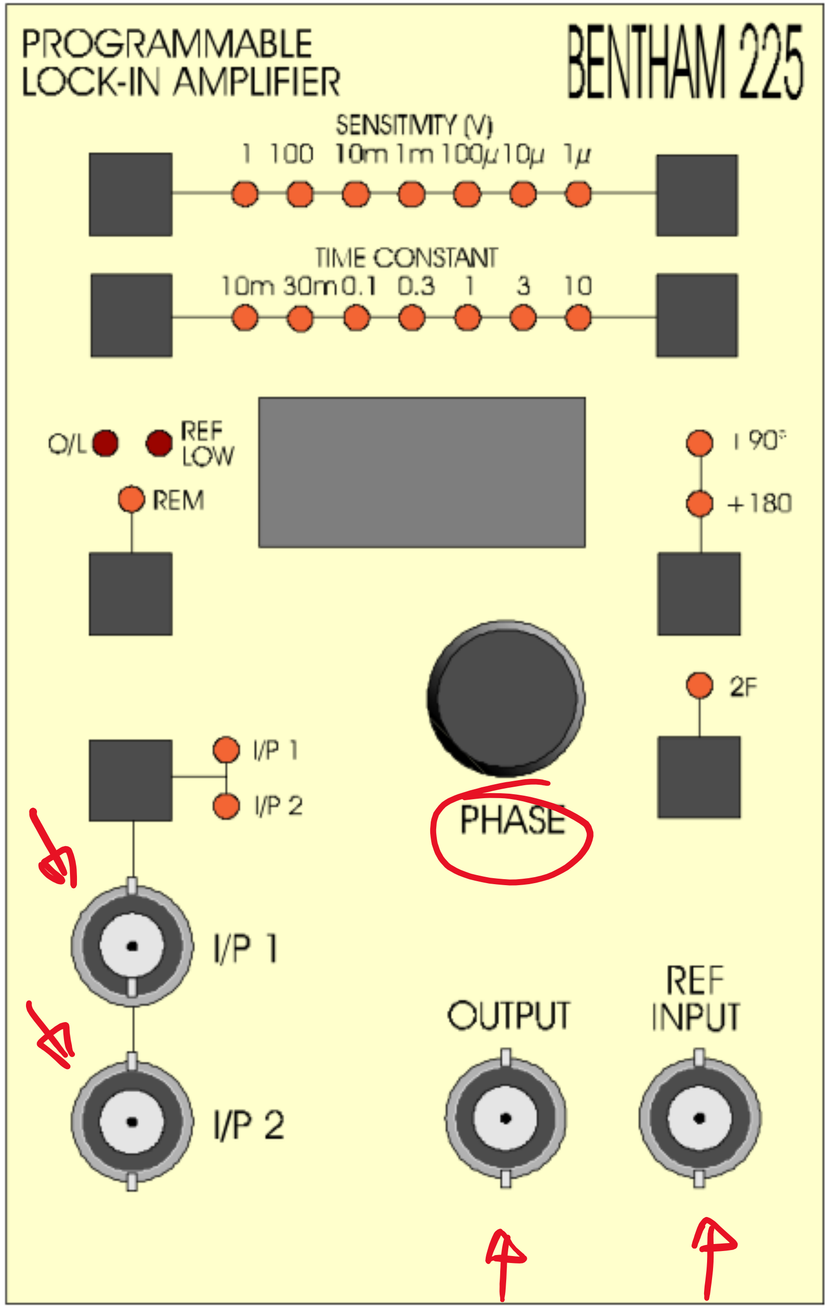
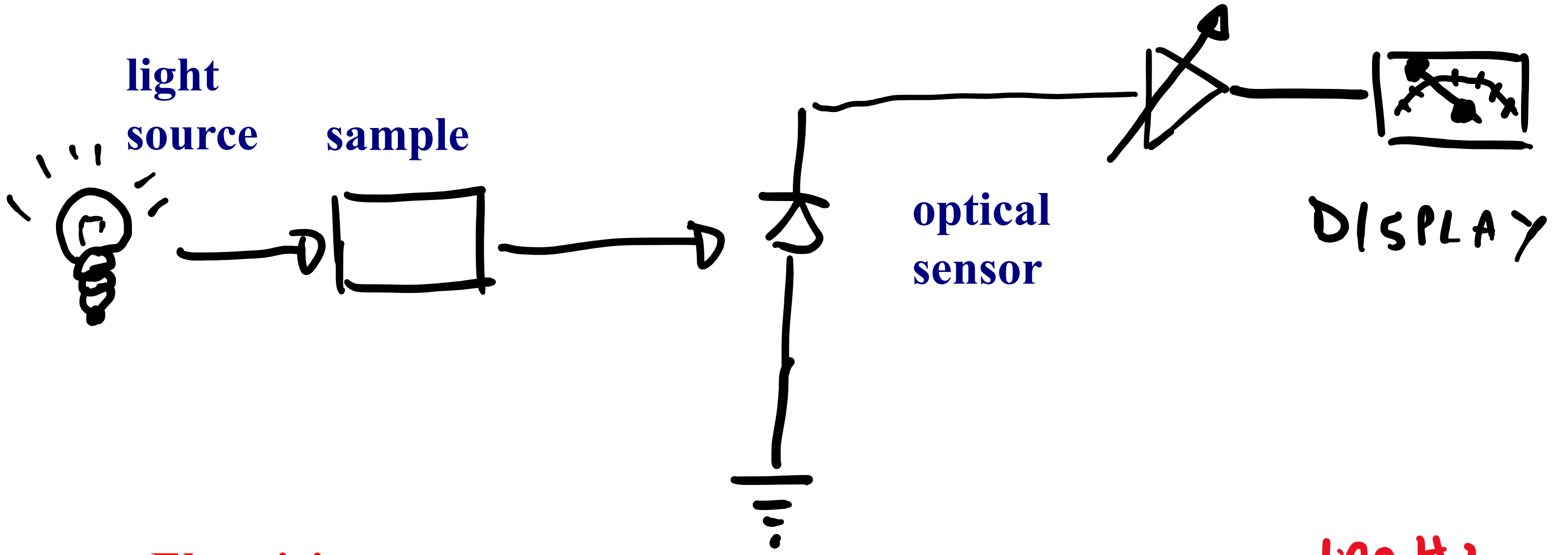


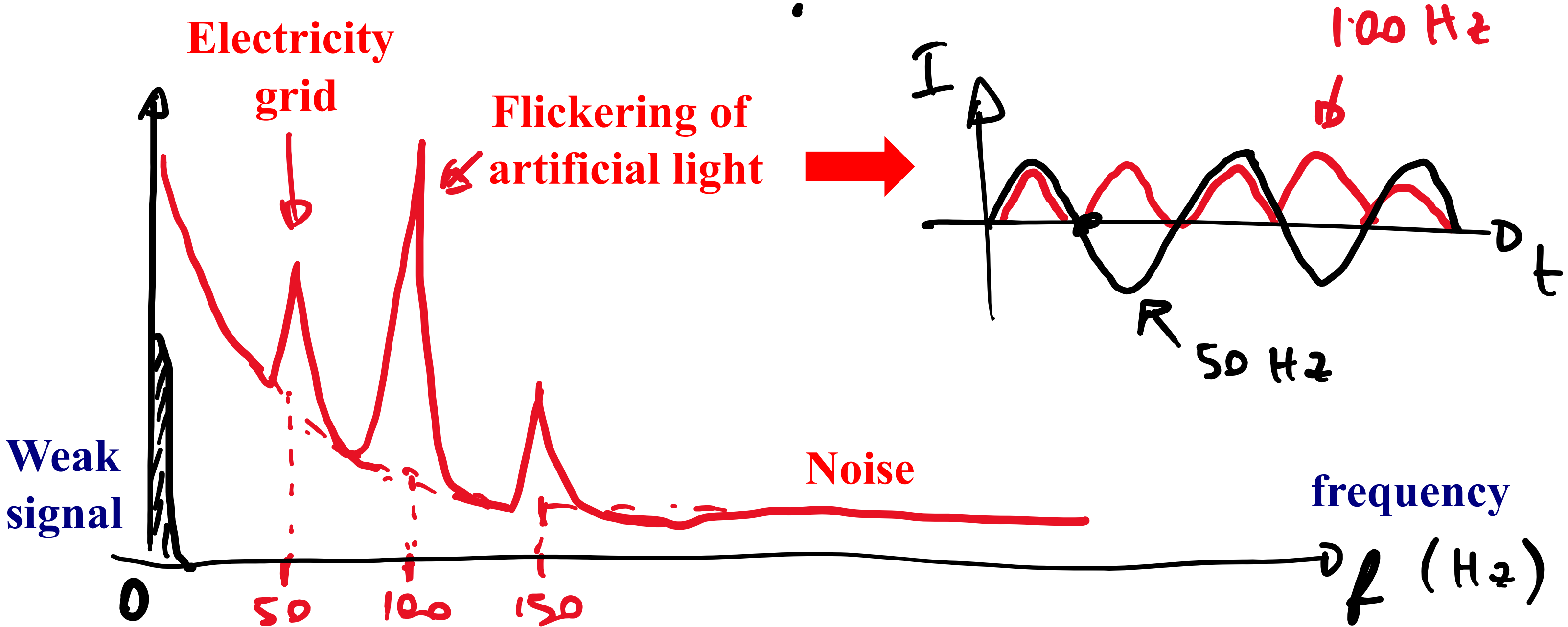
Immagine Filtrata



Measurement of the absorption of light in a material

Didactic scheme:





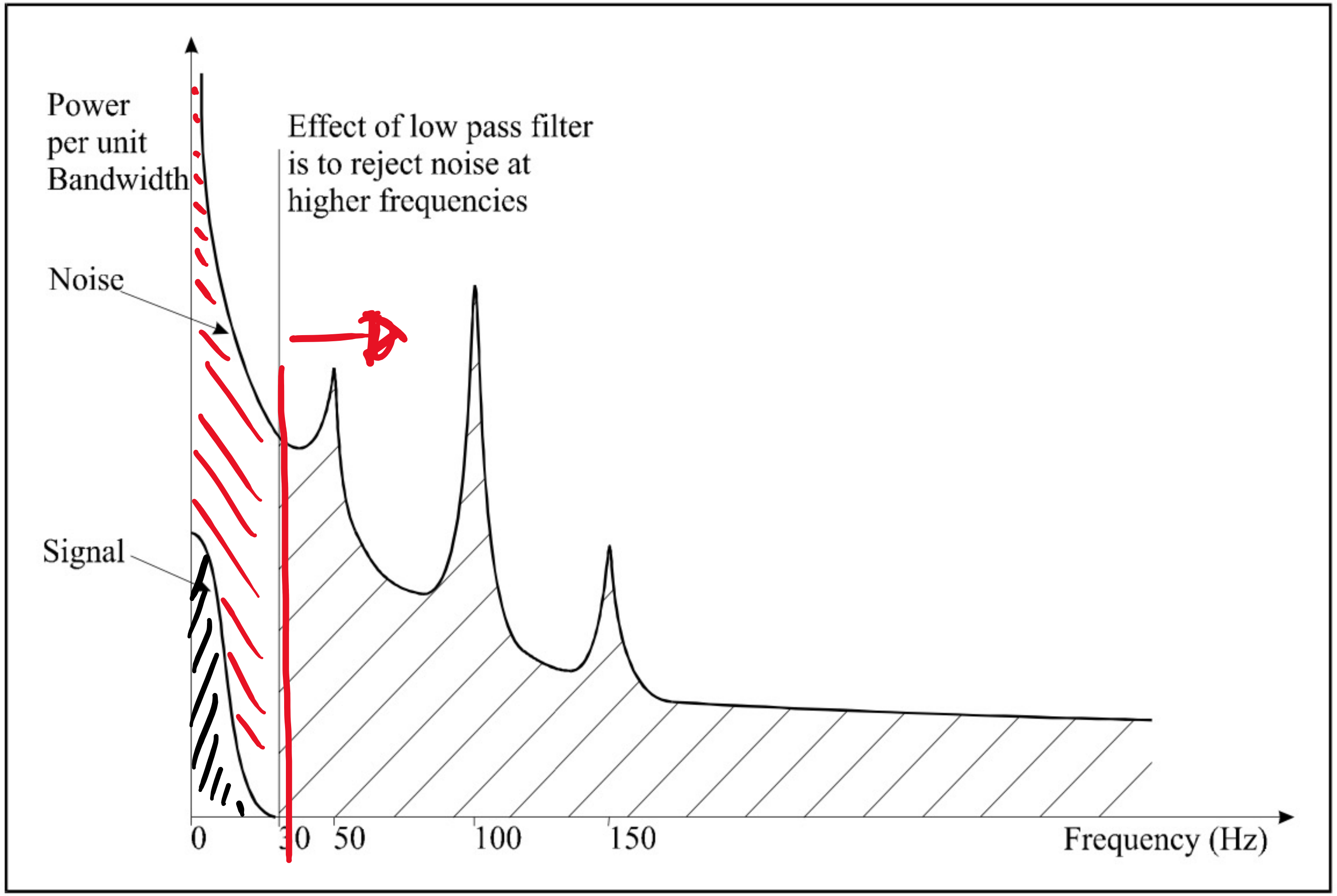


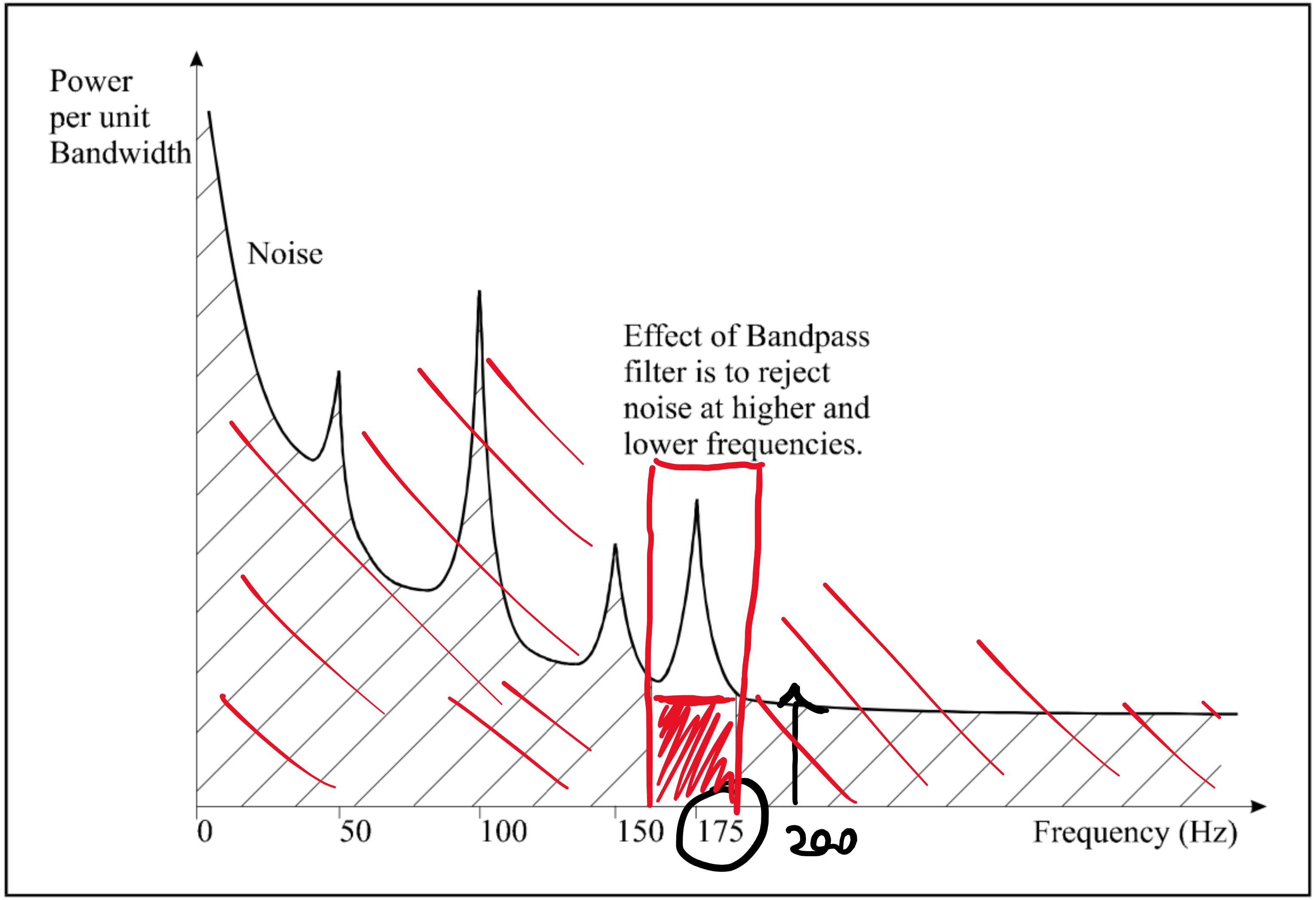
Fig 3 Effect of Low Pass Filter

The low pass filter is not a sufficient solution if the signal is too weak compared to the noise

Possible solutions:

1) Move the signal from f=0 to f=175 Hz (for instance)

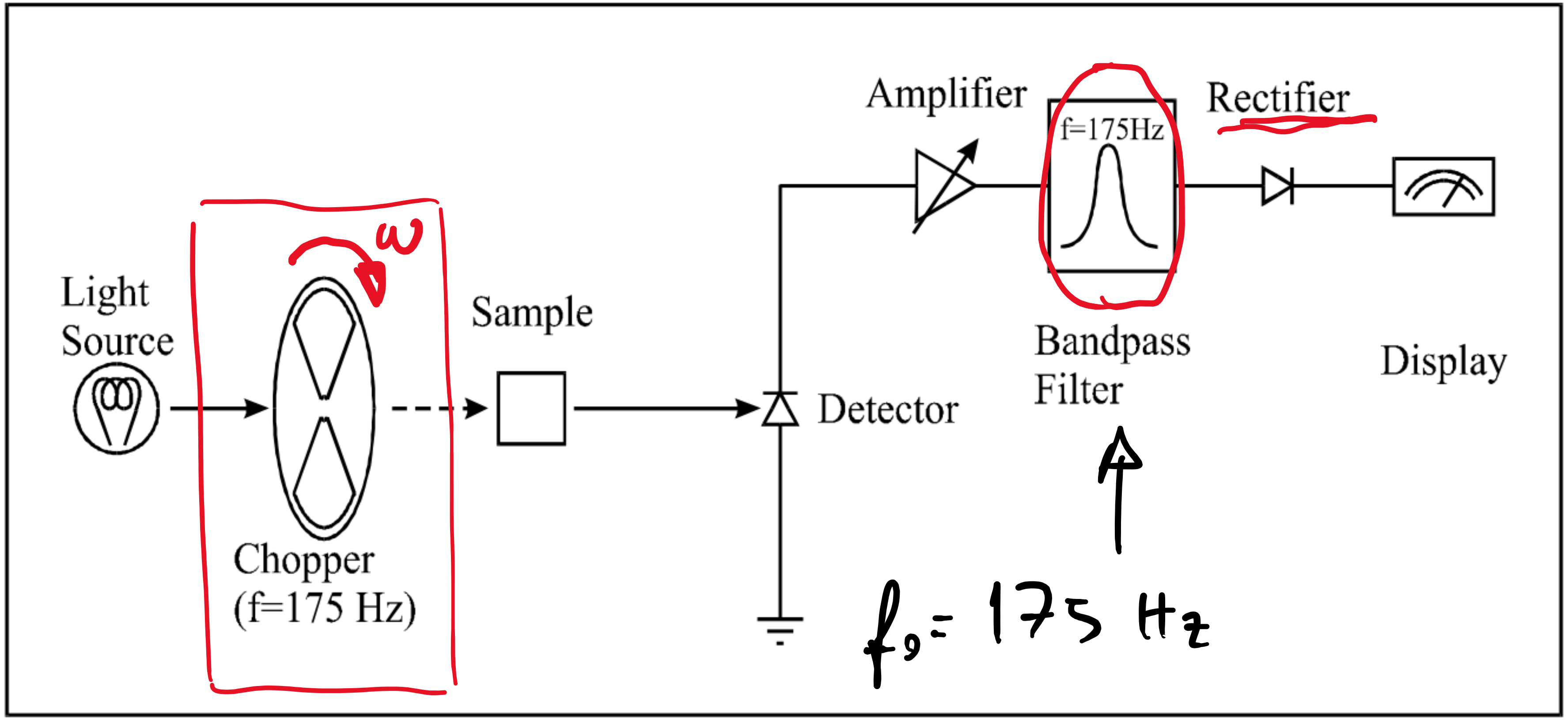
2) Use a bandpass filter



we choose f = 175 Hz because it is not a multiple of 50 Hz, so it is not a harmonic of the frequency of the electrical grid

Fig 4 Effect of Bandpass Filter at 175 Hz.

Chopper: a device that makes the light pulsed instead of continuous



Q = quality factor of a measurement

$$Q = \frac{f_0}{\Delta f}$$

f_0 is the central frequency of the bandpass filter

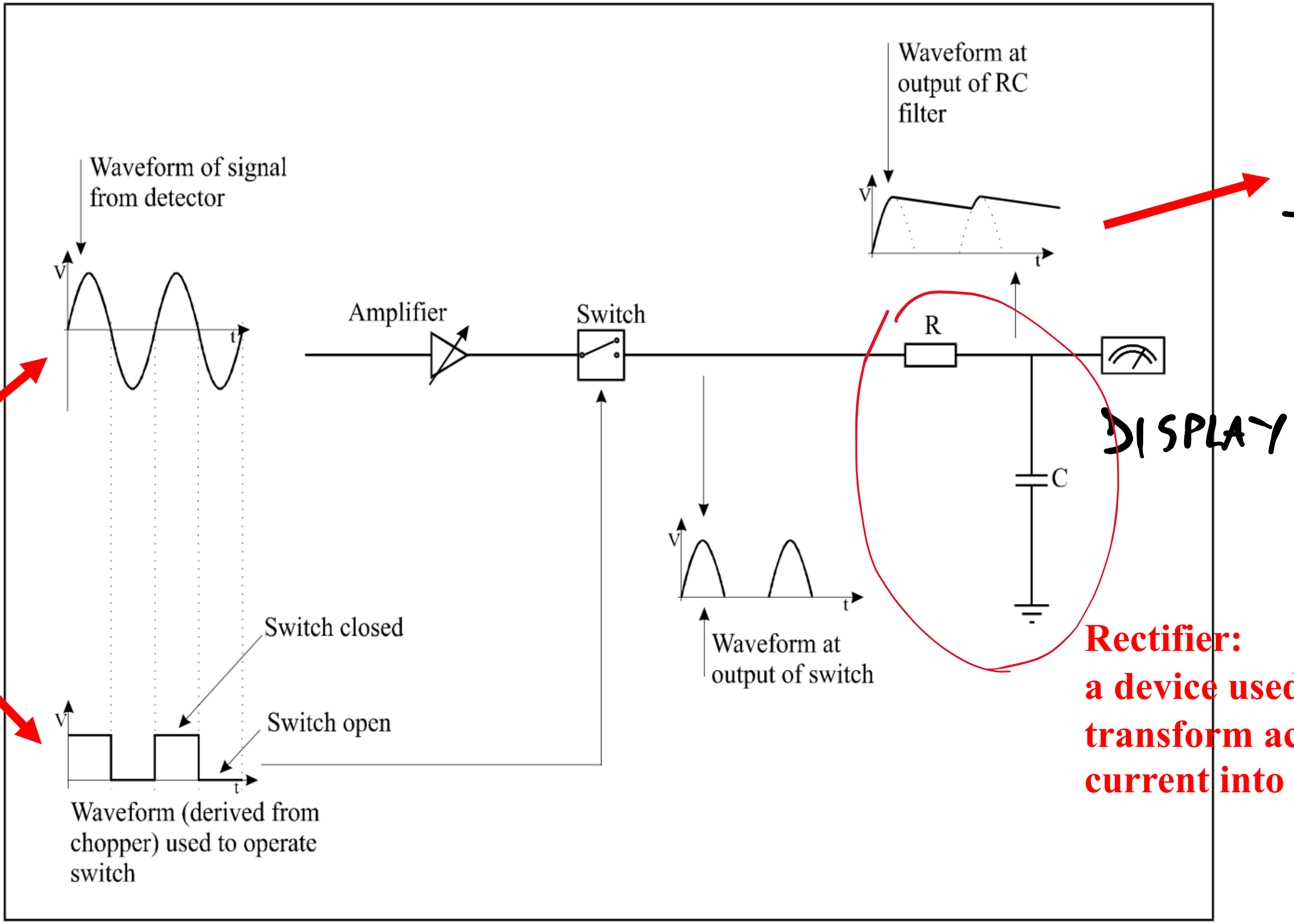
Δf is the width in frequencies of the bandpass filter, bandwidth

For an optical filter $Q \simeq 100 \Rightarrow \begin{cases} f_0 \simeq 175 \text{ Hz} \\ \Delta f \simeq 1{,}7 \text{ Hz} \end{cases}$

High precision measurements require Q> 1000 !!!

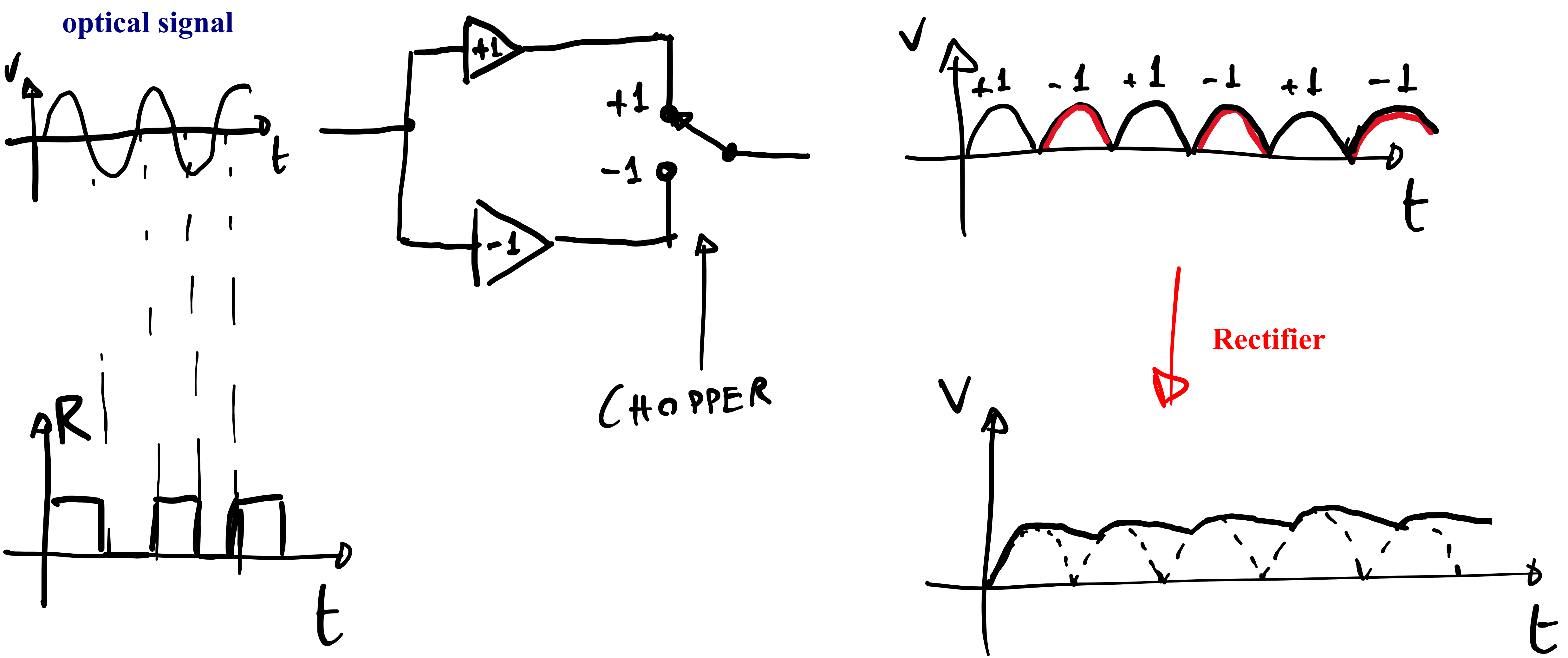
All these problems are overcome by the phase sensitive detector.

Two signals locked in frequency



Rectifier:
a device used to transform ac electric current into dc current

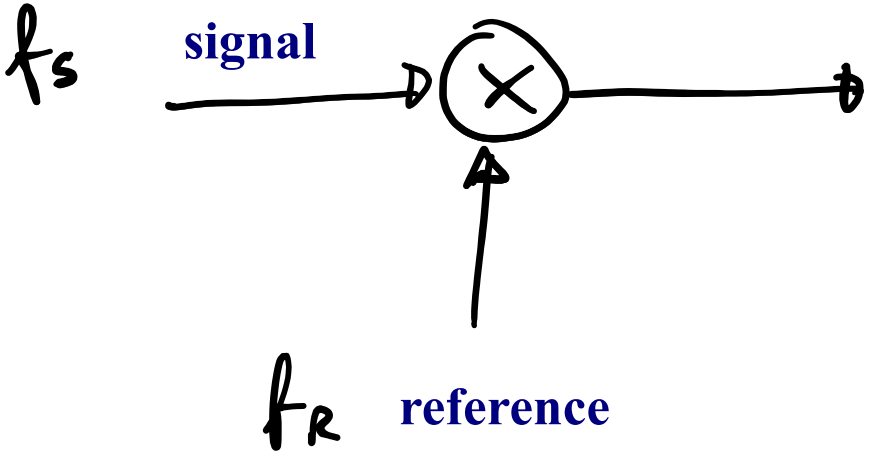
Fig 6 Synchronous Filter



The switch shows well the phase lock operation, but in practice it cannot be used because it generates noise. We need another practical solution …

A real lock-in amplifier uses a multiplier

Multiply those two signals which are periodic



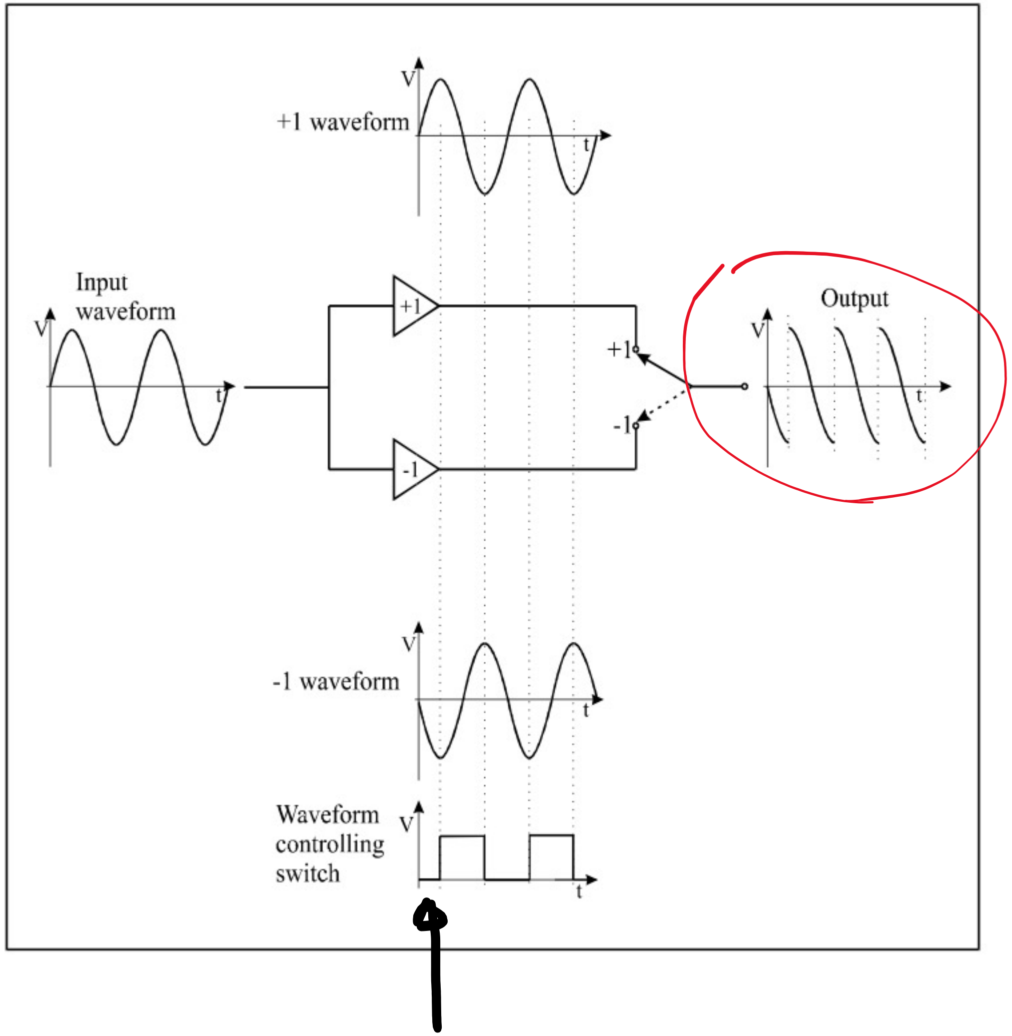
Signal with components

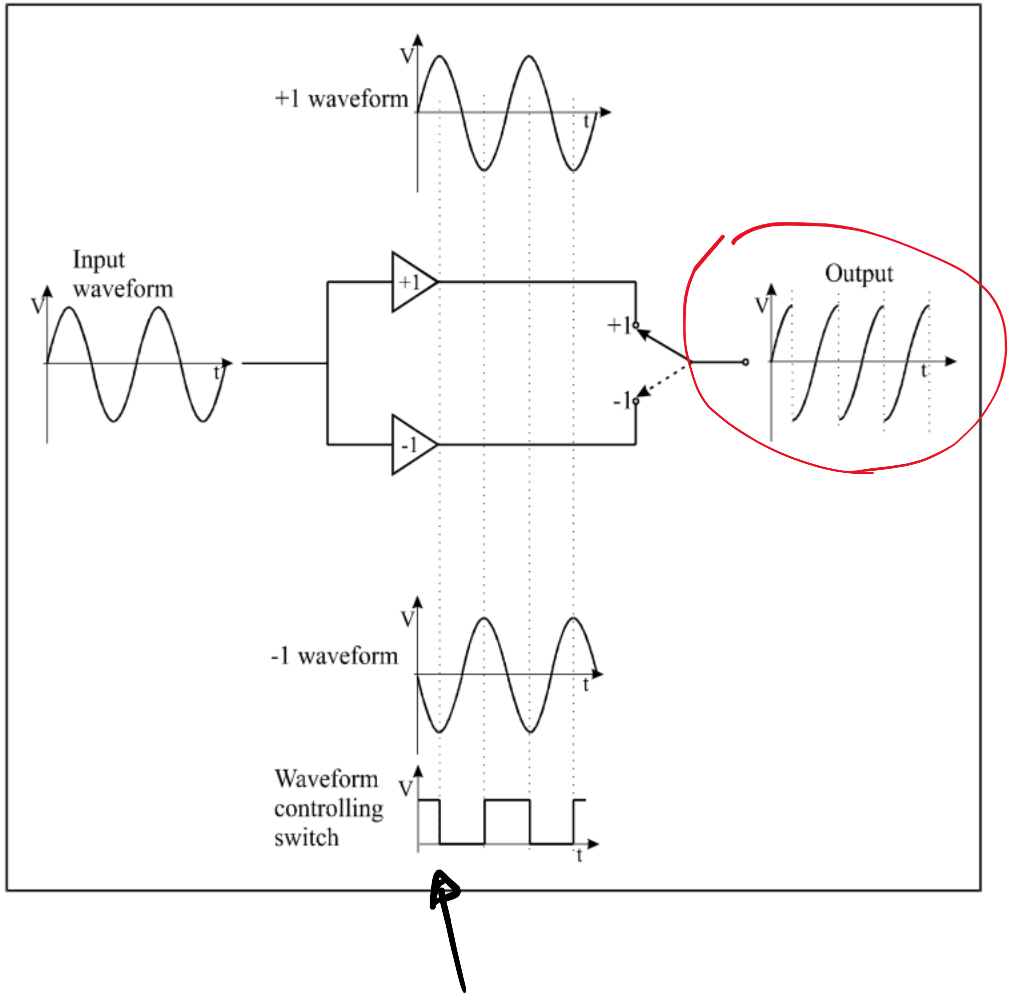
$(f_s + f_R)$ **and** $(f_s - f_R)$

If $f_s \simeq f_R \Rightarrow f_s - f_R \simeq 0$

Werner formulas

With a low pass filter one can erase the component (f_s+f_r)
and one can save the component (f_s-f_r) which practically is a DC signal





Delay between the reference signal and optical response

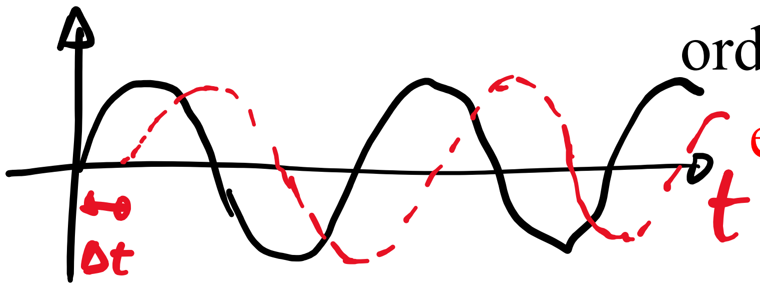
Birefringent material

light travels with two different speeds because the material has two refractive indices (ordinary and extraordinary)

The lock-in amplifier allows to measure the delay between the ordinary and the extraordinary beam, that is the birefringence [1]

signal

ordinary



extraordinary

reference

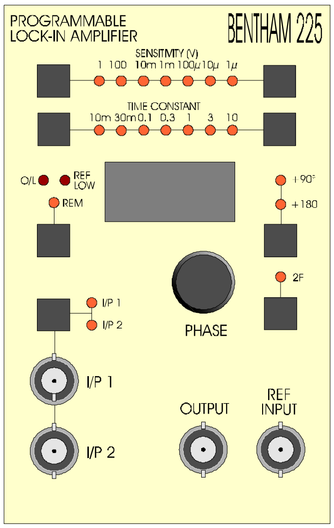
phase difference

1) H. A. Van Sprang (1991) Combined Tilt and Thickness Measurements on Nematic Liquid Crystal Samples, Molecular Crystals and Liquid Crystals, 199:1, 19-26, DOI: 10.1080/00268949108030913
http://dx.doi.org/10.1080/00268949108030913

BENTHAM

225 Lock-in Amplifier

1. WHAT IS A LOCK-IN?



There are a number of ways of visualising the operation and significance of a lock-in amplifier. As an introduction to the subject there follows a simple intuitive account biased towards light measurement applications.

All lock-in amplifiers, whether analogue or digital, rely on the concept of phase sensitive detection for their operation.

Stated simply, phase sensitive detection refers to the demodulation or rectification of an ac signal by a circuit which is controlled by a reference waveform derived from the device which caused the signal to be modulated. The phase sensitive detector effectively responds to signals which are coherent (same frequency and phase) with the reference waveform and rejects all others.

In a light measurement system the device which causes the signal to be modulated is usually a chopper, the reference waveform is an output coherent with the chopping action provided by the chopper and the ac signal is the signal from the photodetector.

As the lock-in is a solution to a measurement problem we can usefully describe its action and composition by looking at the sort of problems that occur when light measurements are pushed to the limit.

Consider a simple light measurement system being used to measure transmission. Light from a stable light source is passed through a sample and reaches a detector. The resulting electrical signal from the detector is amplified and displayed on a meter. The meter reading gives an indication of the amount of light transmitted by the sample.

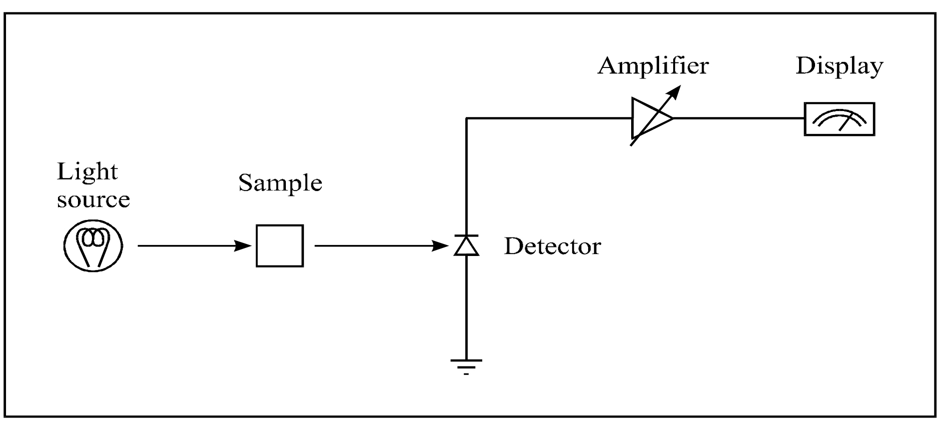


Fig 1 Schematic of Lock-in Device

With medium to high transmission samples this system would be expected to give precise and reproducible results.

Samples of increasing optical density could be accommodated by increasing the gain of the amplifier. What is always noticeable in such systems, however, is that as the signal level falls and the amplifier gain is increased so the precision with which the results can be recorded also falls. This is due to noise.

Noise in this sense is anything which contributes to the meter reading but which is not due to the parameter being measured. It is generated in all parts of the electrical circuitry but in light measurement systems it is dominated by noise from the detector or noise associated with the optical signal.

The following diagram shows the distribution of noise and signal power from the optical detector in terms of power per unit bandwidth as a function of frequency. This can be used for a situation where the density of the sample is so high that the signal is smaller than the noise with the result is that the instrument becomes unusable.

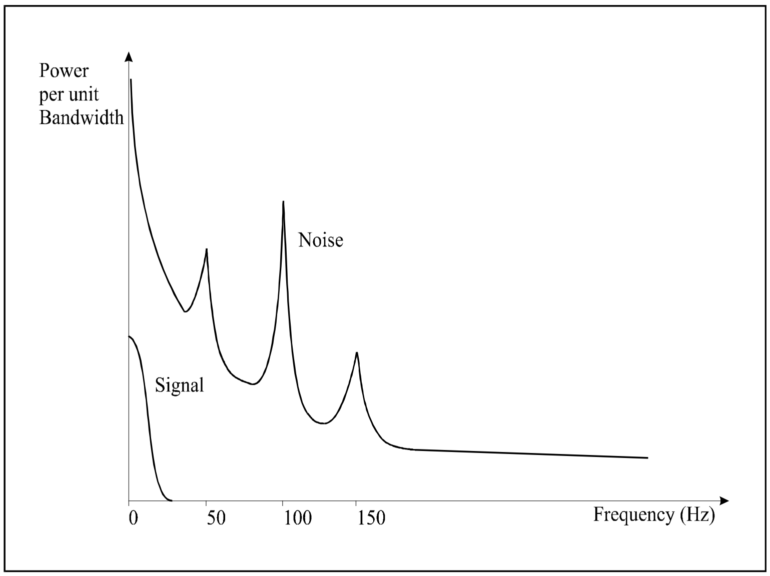


Fig. 2

The most noticeable feature of this curve is the steady increase in noise power which occurs as zero Hz is approached.

In our transmission measuring instrument this low frequency noise has several sources including flicker noise associated with semiconductor devices, variations in dark current (especially in photomultipliers) and variations in ambient light leaking into the instrument and reaching the detector.

At higher frequencies the spectrum flattens out to give a reasonably constant shot noise background which is associated with the quantum nature of light. The small peaks of 50Hz and 150Hz are due to electrical interference from the power system. The larger peak at 100Hz is due to light from room lighting leaking into the instrument and reaching the detector.

It is important to note that the y axis in this diagram is in units of power per unit bandwidth so the total noise and signal powers are represented by the area under the corresponding curves. Clearly therefor we can immediately improve the signal to noise ratio in this system by using an electronic filter to reject the higher frequency components which do not contain any signal information. Unfortunately the information relating to the transmission of the sample is also near zero hertz, so using a low pass filter to reject all noise components above say 30Hz will give only a small improvement in signal to noise ratio.

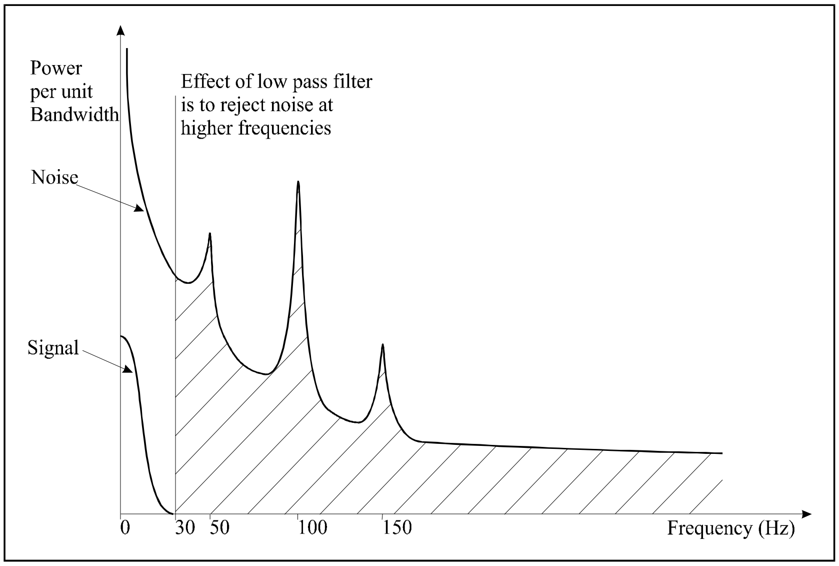


Fig 3 Effect of Low Pass Filter

What we really need to do if we want to measure high optical density with this system is to move the signal information away from the high noise zero hertz region.

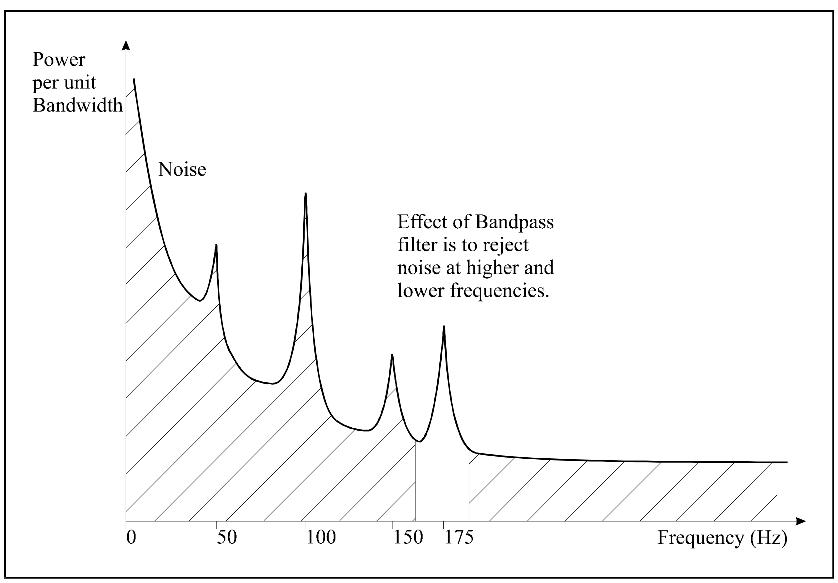


Fig 4 Effect of Bandpass Filter at 175 Hz.

We can do this by placing an optical chopper, which will periodically interrupt the light, between the light source and the detector. The diagram below shows the detector output spectrum in this new situation with the chopper running at 175Hz.

From the diagram this looks like a good move. We have moved the signal away from a region where the background noise is high to a region where it is low. We can now pass the signal through an electronic bandpass filter which will reject both the noise at higher and lower frequencies (including zero hertz) and hence significantly improve the signal to noise ratio.

The problem now is that the signal is ac, i.e. its average value is zero so to record a value from it we must first rectify it. We might end up with an arrangement as shown in Fig 5 which includes an amplifier, a tuned filter whose centre frequency is at 175Hz, a rectifying circuit and a display.

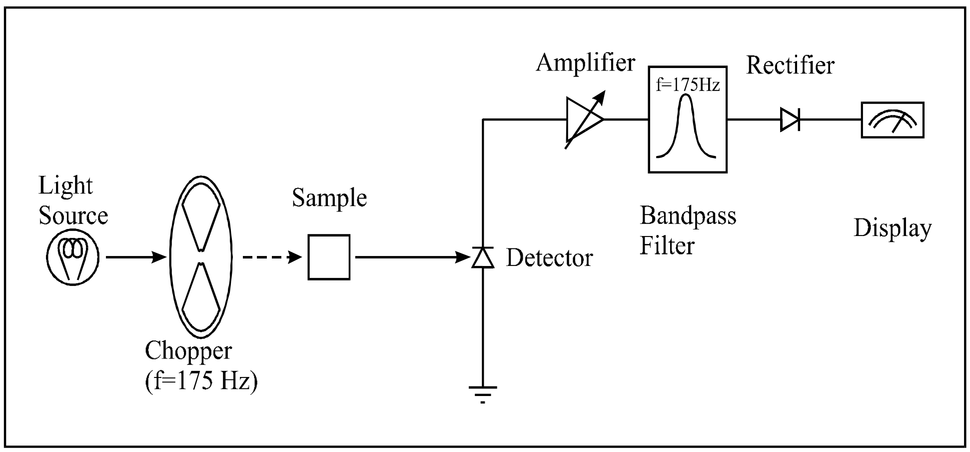


Fig 5

Tuned amplifiers, as these devices are called, are used in some simple systems but they suffer from 3 major disadvantages.

The first concerns Q.

Q is defined as the centre frequency of a filter divided by its bandwidth.

In this application the narrower the bandwidth of the filter, the greater is the noise rejection.

The maximum Q typically achievable for a tuned amplifier is in the region of 100, but in a demanding measurement situation we might need a Q of 1000 to achieve acceptable signal to noise ratio.

Secondly, if such a filter could be produced any small shift in chopping frequency would result in large changes in output due to misalignment between signal frequency and filter centre frequency.

The third problem lies in the rectifying device and the way it responds to noise which passes through the filter. Using a normal rectifying device such as a semiconductor diode the noise itself will be rectified and will thus give rise to a dc level at the meter which will be indistinguishable from that derived from the signal.

All these problems are overcome by the phase sensitive detector.

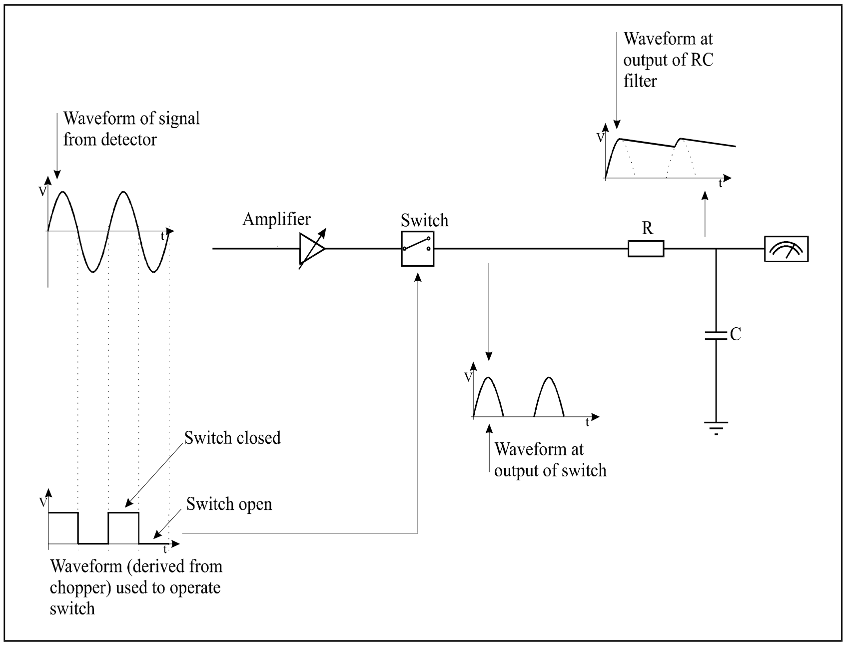


Fig 6 Synchronous Filter

As first glance this device looks very simple.

As before there is circuitry to amplify low level signals, but now there is no tuned filter or rectifying diode. Instead the amplifier is followed by a switch which is operated by a waveform derived from the chopper. When the level from the chopper is high the switch is closed and the output of the amplifier is connected directly to a low pass filter consisting of a resistor (R) and a capacitor (C). When the output of the chopper is low the switch is open and no connection is made.

Rectification of the signal occurs when the waveform controlling the switch is exactly in phase with the ac signal at the input to the switch, hence the sometimes used description, synchronous rectifier. More importantly when the switch is closed the noise associated with the signal passes through un-rectified to the low pass RC filter beyond where it is smoothed or averaged to its mean value of zero.

The device behaves as a bandpass filter and performs the same function as a tuned amplifier followed by a rectifier but with the following advantages.

1) The effective bandwidth and hence noise rejecting capability of the device is determined only by the values of the components used in the low pass filter. In fact the bandwidth is given by 1/4T where T is the time constant[1] of the RC filter. A time constant of 2.5 seconds will thus give a bandwidth of 0.1Hz which at centre frequency of 175Hz corresponds to a Q of 1750.

2) The centre frequency of the filter is locked (hence lock-in) onto the chopper's frequency. The signal can never drift outside the pass band of the filter.

3) Any noise which is present at the output is equally distributed about the mean value resulting from demodulation of the signal. It does not give rise to a dc level as with the diode rectifier and increasing the time constant will reduce its' magnitude.

An alternative approach is to visualising the phase sensitive detector is to consider the switch as a multiplier.

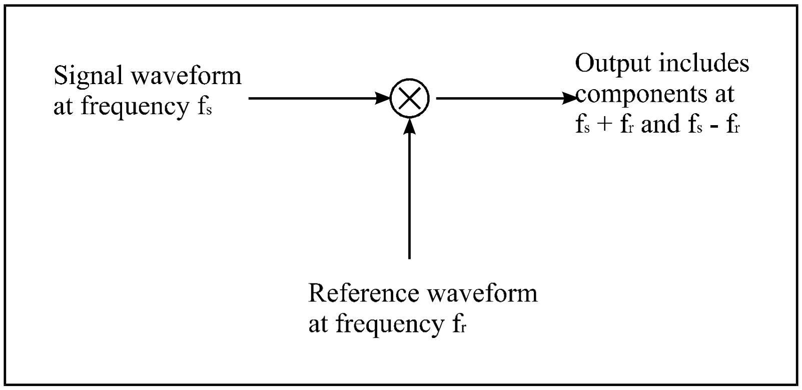


Fig 7

Assuming that both the signal and reference waveforms are sinusoidal then the output of the multiplier will contain components at frequencies of f_s+f_r and f_s-f_r where f_s and f_r are the frequencies of the signal and reference waveforms respectively.

If $f_s=f_r$ as is the case where the reference waveform is derived from the device which is modulating the signal then there will be an output at 0Hz i.e. dc. Any other component in the signal e.g. a noise component at a frequency of f_n will give rise to an ac output at frequencies of f_n-f_r and f_n+f_r which will be smoothed or averaged to the mean value of zero by the low pass filter.

As the time constant (RC) of the filter is increased so the attenuation of the higher frequency components from the multiplier will increase thus effectively reducing the bandwidth of the overall device.

Some Practical Points

So far we have considered the phase sensitive demodulator as a single pole switch which is capable of synchronous rectification of only half of the signal. For a signal recovery device such a waste of signal information would be unacceptable. Commercial lock-in amplifiers therefore include a full wave synchronous demodulator which usually works by using inverting and non-inverting amplifiers to produce anti-phase versions of the signal (i.e. 180° out of phase).

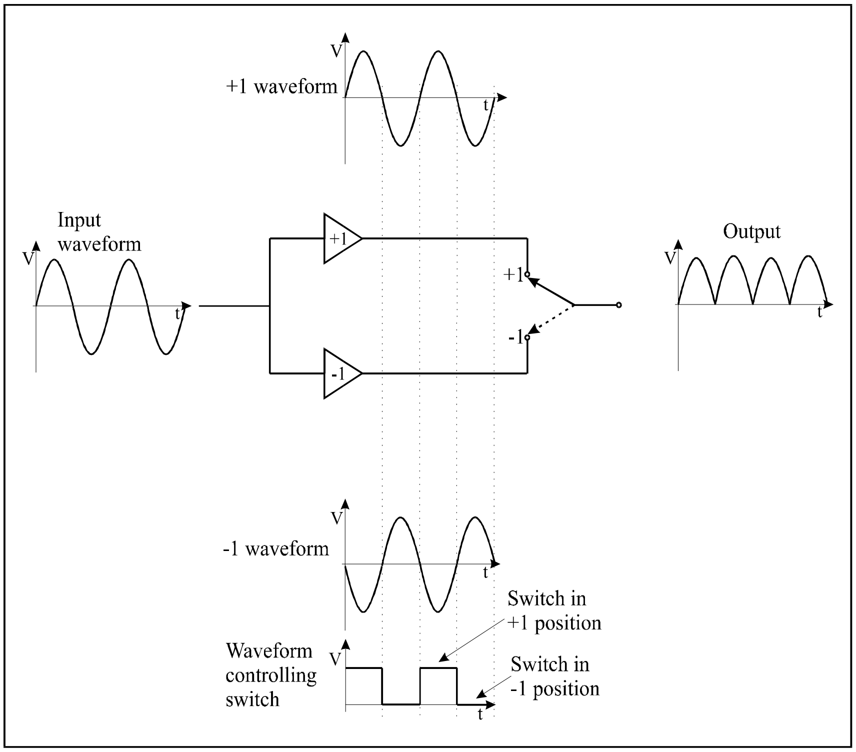


Fig 8

In all the diagrams shown so far the reference waveform available at the phase sensitive demodulator has conveniently been in phase with the signal - the condition for maximum output and hence best signal to noise ratio.

In any real experimental set-up we can not always be sure that the relationship between the signal from the detector and the reference waveform from the chopper will be in phase. The phase relationship will be dependant on the position of the chopped light beam relative to the reference pick-up on the chopper and also on any phase modification of the signal introduced by the detector.

To allow perfect phase matching at the demodulator, lock-in amplifiers include comprehensive and flexible phase shifting circuitry which allows a phase shift of over 360 degrees to be introduced. The phase controls on the lock-in usually take the form of a continuously variable of 0-95 degrees and 3 fixed increments of 90 degrees giving 365 degrees in total.

The following diagrams show how the output of the psd varies with the reference to signal phase relationship. In figs 9a, 9b and 9c the reference waveform has been shifted by 90°, 180° and 270° respectively from that in Fig 8.

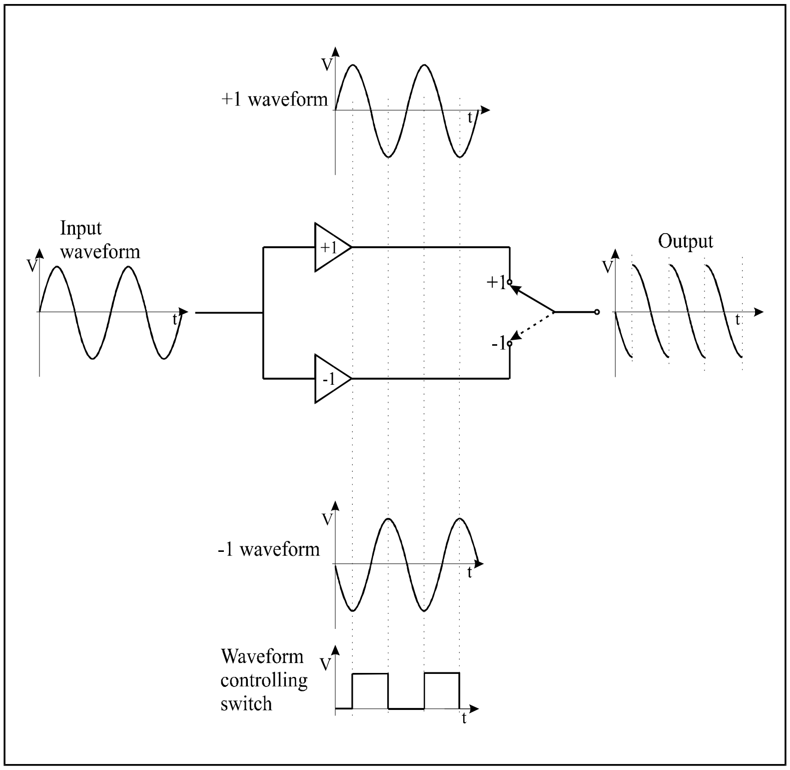

+1 waveform
Input
waveform
Output
+1
-1
-1 waveform
Waveform
controlling
switch
V
t

Fig 9a

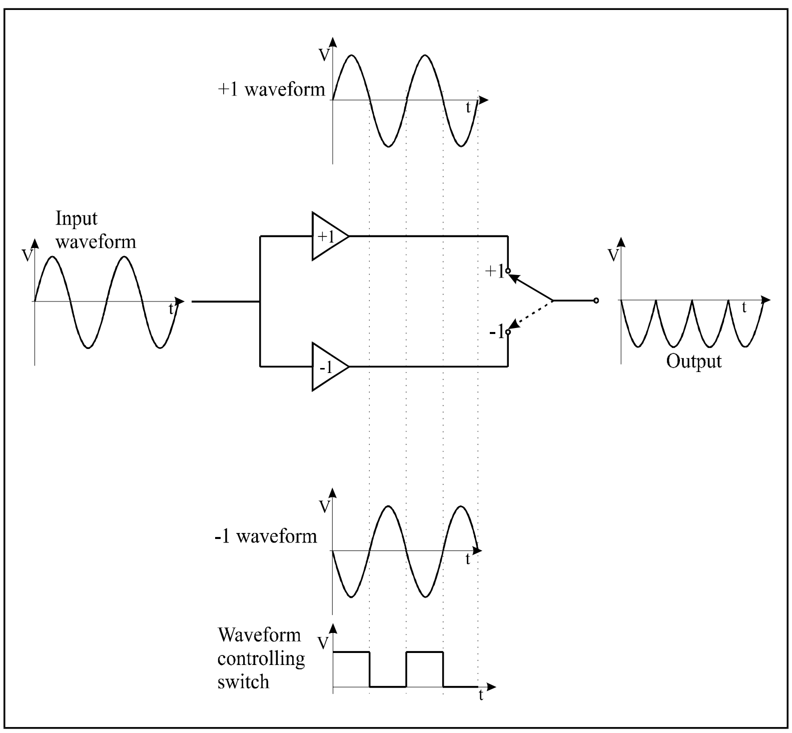

+1 waveform
Input
waveform
+1
-1
Output
-1 waveform
Waveform
controlling
switch
V
t

Fig 9b

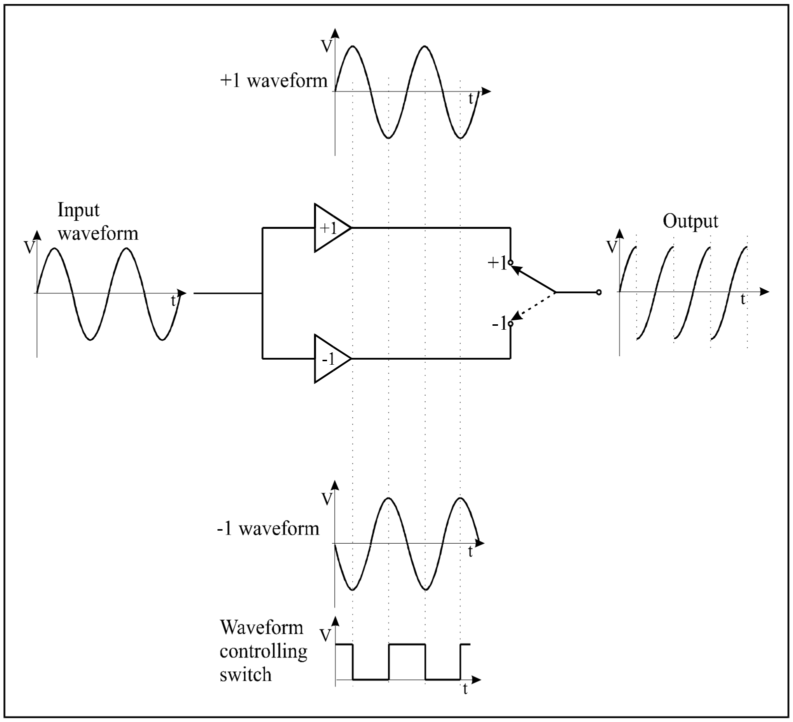


Fig 9c

Harmonic Response

In the multiplier model of our psd we made the assumption that the two waveforms were sinusoidal, but in practice, when using a simple switch we are effectively multiplying the signal not by a sine wave but by a square wave (the switch is either closed or open). Now a square wave is equivalent to a sine wave of the fundamental frequency plus all the odd harmonics of the fundamental[2]. A psd using a simple switch therefore behaves as a filter centred at the fundamental but with windows at each odd harmonic.

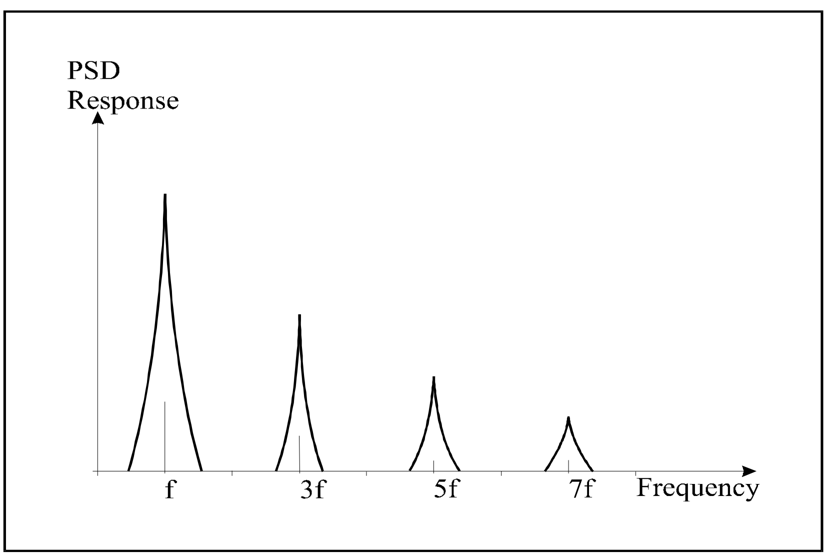


Fig 10

At first glance this looks to be a problem in that the noise present in these harmonic windows will be passed through to the low pass filter and degrade the signal to noise ratio at the output of the lock-in.

However, this problem is not as bad as it looks, especially if the modulating device is a chopper.

First of all the harmonic windows do not have the same “sensitivity” as the fundamental but reduce as 1/harmonic reflecting the Fourier composition of the square wave switching the demodulator[2].

Secondly, if the modulating device is a chopper, then the signal itself will not be sinusoidal but more typically triangular or trapezoidal both of which have significant odd harmonic content so the harmonic windows will also transfer additional signal information to the output of the lock-in.

The most dangerous situation is one in which a harmonic window coincides with a point in the spectrum where a large discrete interference is present but this should never occur in a properly designed light measurement system with the correct choice of chopping frequency.

It is unfortunately the case that lock-ins are sometimes used to compensate for incorrect optical design.

The most commonly encountered misconception regarding the use of lock-in amplifiers in light measurement concerns their ability to disregard a constant dc light level most commonly caused by ambient light leaking into the system and reaching the detector. This after all is the reason for including the chopper so that the signal information is shifted away from the dc region.

The next intuitive step taken by many is to assume that there is no longer any need to make the system light tight - as long as ambient light reaching the detector does not cause saturation everything will be OK.

This is not correct.

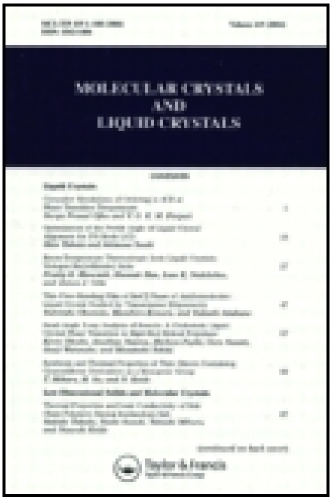
Referring back to the earlier paragraphs we pointed out that the shot noise background is caused by the light itself and is associated with its quantum nature.

Ambient light leaking into chopped light systems will always degrade signal to noise performance even though it does not give rise to a dc output from the lock-in.

You can often improve signal to noise ratio just by making things light tight - if you cut down light leaks you may be able to reduce the time constant and make your measurements faster.

1 The time constant of the RC circuit in seconds is given by the product of the resistance in ohms and the capacitance in farrads.

BENTHAM INSTRUMENTS LTD
2 Boulton Road, Reading, Berkshire, RG2 0NH
Tel: +44 (0)118 975 1355
Fax: +44 (0)118 931 2971
Email: sales@bentham.co.uk



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Combined Tilt and Thickness Measurements on Nematic Liquid Crystal Samples

H. A. Van Sprang [a]

[a] Philips Research Laboratories , P.O. Box 80, 000 5600, JA Eindhoven, The Netherlands

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Combined Tilt and Thickness Measurements on Nematic Liquid Crystal Samples

H. A. van SPRANG
Philips Research Laboratories, P.O. Box 80.000 5600 JA Eindhoven, The Netherlands

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A new technique is presented which enables simultaneous measurement of sample thickness and director tilt of non-twisted nematic liquid crystal cells up to about 20 μm thick. It is shown that only n_o and n_e, the ordinary and extra ordinary refractive indices of the liquid crystal, are required with no more than three digit accuracy.

Keywords: Nematic liquid crystals, birefringence

It is common knowledge that on many surfaces non-zero director tilt angles can be obtained for the director of a nematic liquid crystal. In the course of time a number of experimental techniques has been developed which enable measurement of this tilt angle. Unfortunately, no such technique can be generally applied to arbitrary samples, as all of them have their own restrictions. A summary of those methods applicable to normal, flat substrate covered, test cells is presented in Table I. The only method applicable over the whole range of tilt angles is the magnetic null method.[1] For use with "classical" (non superconducting) magnets of about 2 T one requires samples of more than 10 μm thickness. For studying the tilt in samples of thickness below 10 μm we are left with either a capacitive[1] or an interferometric method.[2,3] In both methods a strong dependency is observed of the tilt angle obtained on the sample thickness.

We wanted to find a method which allows for accurate determination of tilt angles without a need to measure (in a preparatory measurement) cell thickness or capacitance of an empty cell because the sample thickness might easily change during introduction of liquid crystal.

The method we propose is based on the crystal rotation method[4,5] but it is more powerful due to the fact that we measure the optical phase shift instead of the transmission as a function of the rotation angle. In Figure 1 a block diagram is shown of the experimental set up. The liquid crystal sample is manufactured in such a way that no twist, splay or bend are present. It can be rotated around an axis parallel to the substrates. The director is in a plane perpendicular to the rotation axis of the sample.

The incident linear polarized light is in general elliptically polarized upon exit

TABLE I

Comparison of methods to measure tilt angles

Method	Accessible Range θ (°)	Accuracy θ (°)	Required Quantities	Required d (μm)	References	Remarks
Magneto optic null method	0–90	0.1	—	>10	1	Only for non-twisted samples
Capacitive	0–90	0.1	C_0, $\varepsilon_\parallel$, $\varepsilon_\perp$	—	1	Requires C_0 meas. on empty cell; suited for twisted samples
Interferometry	15–75	0.5	d, n_e, n_0	—	2, 3	Requires d to be constant before and after filling
Crystal rotation	0–13 77–90	0.2	n_e, n_0	>10	4, 5	$\theta < 9°$ in twisted structures
Conoscopy	0–17 73–90	0.2	n_e, n_0	>20	6	

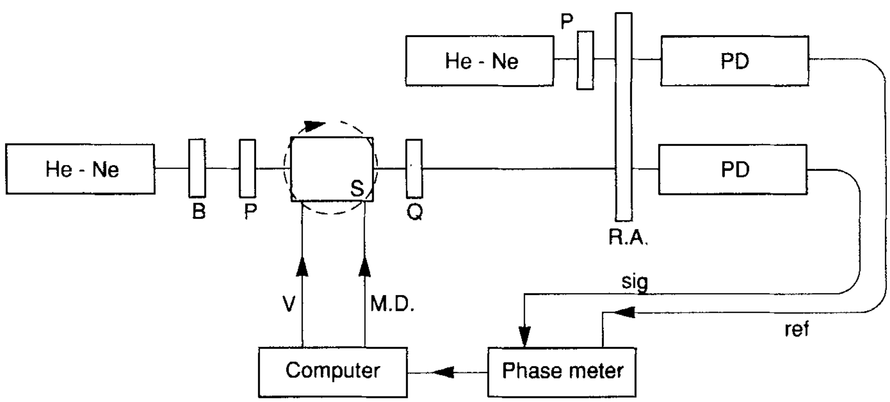

FIGURE 1 Block diagram of the experimental set-up. He–Ne indicates a He–Ne laser, B a Beam expander, P a fixed polarizer, Q a quarter waveplate, RA a rotating analyser and PD a photodetector. The rotation of the sample (Motor Drive M.D.) and the applied voltage V are controlled by a Micro computer.

from the sample. The quarter wave plate transforms it into linear polarized light with a rotation of the plane of polarization equal to δ. This value is then measured using a rotating analyser as described before in Reference 7.

The expression for the phase difference in terms of the angle of incidence ϕ with the substrate normal and the director tilt θ with respect to the substrate is.[1]

$$\delta = \frac{2\pi d}{\lambda}\left[\frac{n_0^2 - n_e^2}{n^2}\sin\theta\cos\theta\sin\phi + \frac{n_0 n_e}{n^2}\sqrt{n^2 - \sin^2\phi} - \sqrt{n_0^2 - \sin^2\phi}\right]$$

where

$$n^2 = n_0^2\cos^2\theta + n_e^2\sin^2\theta$$

It is this equation which is used directly in our procedure. First, the phase difference δ is measured for a scan of ϕ from $-30°$ to $30°$. The second step is a fit of these data to the above equation using d and θ as unknown variables. This leads to an almost independent determination of d and θ due to the fact that both variables occur in a different way in the expression for δ. Obviously d is directly proportional to δ and determines the position of the δ–ϕ curve on an absolute scale while θ determines the shape of the curve. As can be learnt from the expression for δ only n_e and n_0 are left as "known" parameters. We measured the relevant indices in a slightly modified Pulfrich Refractometer which has an accuracy of better than 1.10^{-4}. In Table II we present the refractive indices for ROTN 3010 around room temperature. This liquid crystal has been used in all samples presented here.

The fit of the experimental data to the theoretical formula is performed on an IBM mainframe computer using standard IMSL Fortran software. A typical example of the results is shown in Figure 2 for a 1.91 μm thick cell with a Polyimid orienting layer.

Obviously there is a scattering of the experimental data around the fitted results, expressed as a standard deviation of 0.98. In Figure 3 we show the results for an

TABLE II

Refractive indices for ROTN 3010 at λ = 632 nm

t (°C)	n_e	n_0
20.0	1.6391	1.4984
20.5	1.6387	1.4983
21.0	1.6384	1.4981
21.5	1.6380	1.4980
22.0	1.6376	1.4979
22.5	1.6372	1.4978
23.0	1.6369	1.4977
23.5	1.6365	1.4976
24.0	1.6361	1.4975
24.5	1.6357	1.4974
25.0	1.6354	1.4973

FIGURE 2 Phase Retardation versus rotation angle for a 1.91 μm thick PI cell with θ = 0.27°. Measured at 22.2°C.

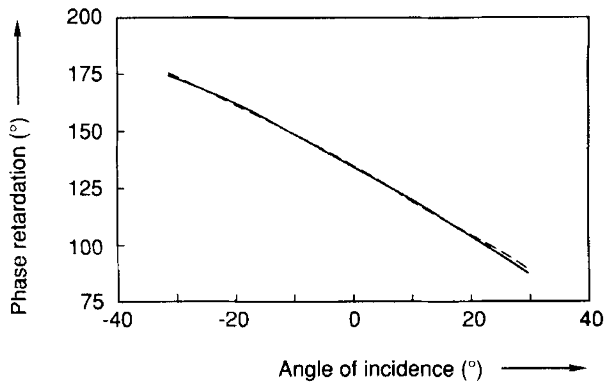

FIGURE 3 Phase Retardation v.s. rotation angle for a 2.16 μm thick SiO_2 sample with θ = 25.13°. Measured at 25.3°C.

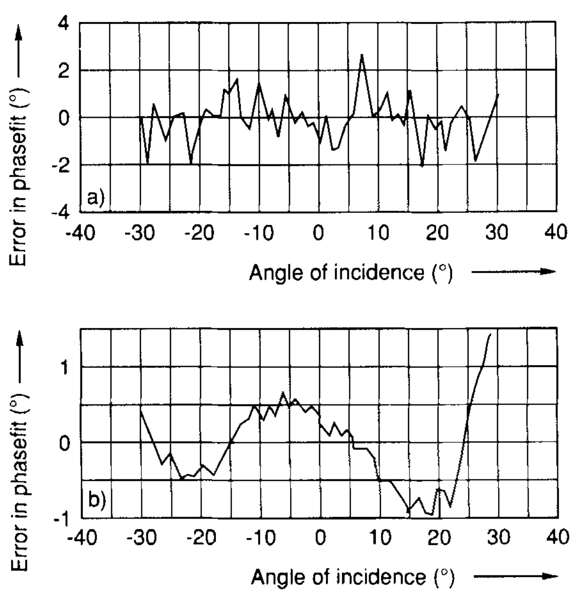

FIGURE 4 Comparison of the deviation of the fitted data from the experimental data as a function of the rotation angle: a) data for cell in Figure 2, b) data for cell in Figure 3.

evaporated SiO_2 layer which had been rubbed subsequently. As can be seen the shape of the curve is different as is the observed deviation of the fit from the experimental data which amounts to a standard deviation of 0.54° only in Figure 3. In Figure 4 we compare the SiO_2 and PI cell. Apparently in the PI cell the deviation consists of abrupt humps as a function of ϕ while a gradual change is observed in Figure 4b for the SiO_2 cell. To understand this difference the experimental technique must be considered in more detail. During the experiment the sample rotates over 60° and the laser beam (diameter $\simeq$ 1 mm) passes through slightly different positions at the liquid crystal-orienting layer interface. From the photographs in Figure 5 it is immediately clear that PI is much less homogeneous at a microscopic scale than is SiO_2. Thus inhomogeneity is supposed to be the reason for the substantial noise in PI samples. Although individual differences occur, it is generally true that SiO_2 cells exhibit a more gradually changing noise

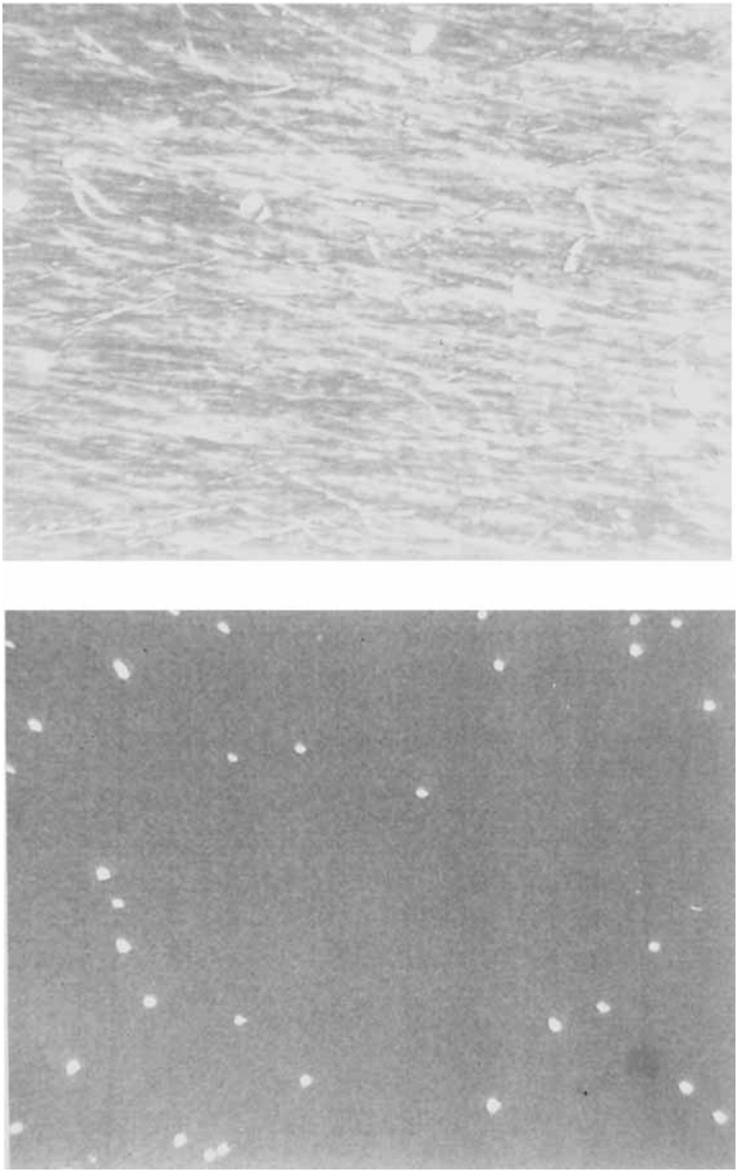

FIGURE 5 Micro photographs showing the structural difference between: a) PI oriented and b) rubbed SiO_2 oriented samples. The photographs were taken under dark-field illumination and using crossed polarizers. (Magnification 100×). See Color Plate I.

as a function of ϕ than PI samples. Figures 6–8 contain further illustrations of the method.

Although the method is powerful, it also has its limitations. We can summarize them as follows:

1. For cells with δ (632 nm) > 360°, or equivalently $d\delta n$ > 1.58 μm, it is necessary to know the absolute value of δ and not the value modulo (360°) as determined in the setup of Figure 1. For this reason we use samples supplied with ITO electrodes to which an increasing voltage up to 10V is applied. The number of times a 360° phase jump is encountered is registered by the computer and used to determine δ at $\phi = 0$. For samples much thicker than 20 μ the phase jumps might occur within a too small voltage range and hence the δ value could be wrong by 360° or more.
2. As discussed before a certain inhomogeneity might lead to a higher noise level. This does not seem to greatly influence the accuracy of the fitted results.
3. The accuracy of the refractive index data is very important to obtain accurate

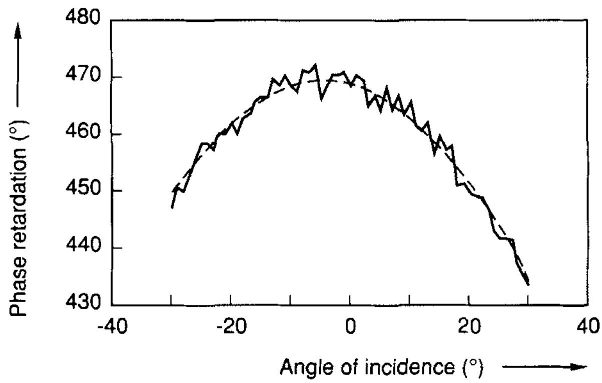

FIGURE 6 Phase retardation vs rotation angle for a 5.89 μm thick cell with a PI orienting layer and with θ = 1.33°. Measured at 21.4°C.

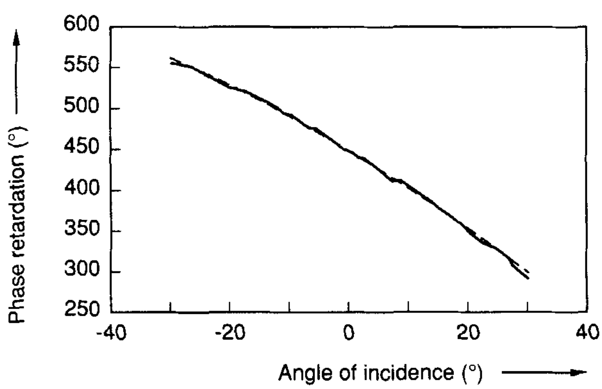

FIGURE 7 Phase retardation vs rotation angle for a 61.8 μm SiO_2 cell with θ = 22.85°. Measured at 25.0°C.

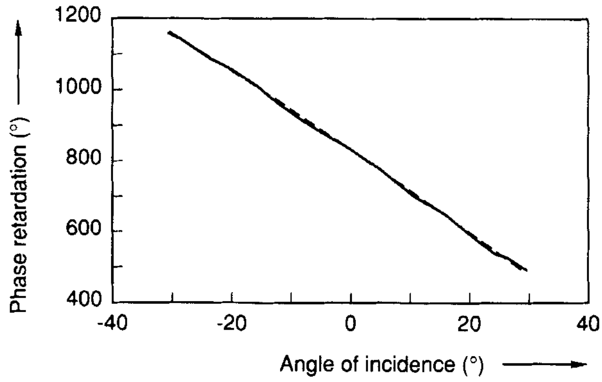

FIGURE 8 Phase Retardation vs rotation angle for a 14.59 μm SiO_2 cell with θ = 30.47°. Measured at 21.4°C.

TABLE III

Influence of the variation of n_e and n_0 on the obtained θ and d values

n_e	n_0	θ (°)	d (μm)
1.6353	1.4973	22.85	6.82
1.635	1.498	22.87	6.87
1.635	1.497	22.85	6.82
1.635	1.496	22.83	6.77
1.636	1.497	22.84	6.77
1.634	1.497	22.85	6.87
1.634	1.498	22.87	6.92
1.636	1.496	22.82	6.72
1.655	1.517	23.14	6.84

TABLE IV

Variation of n_e and n_0 for a thin sample

n_e	n_0	θ (°)	d (μm)
1.6351	1.4972	25.13	2.16
1.635	1.497	25.13	2.16
1.636	1.498	25.15	2.16
1.636	1.496	25.10	2.13
1.634	1.498	25.16	2.19

θ and d values from the fit. In Table III we show how n_e and n_0 influence the results for the data of Figure 7. In general this allows for the conclusion that for values of n_e and n_0 within 1 10^{-3} from their exact value an accuracy in θ is still obtained of better than 0.1 deg while the thickness is more accurate than 0.1 μm. From Tables IV and V similar results can be extracted for the data displayed previously in Figures 4 and 6 respectively. Thus in general the same conclusions hold independent of tilt or thickness.

Concluding we can say that the method is apparently accurate enough for appli-

TABLE V

Variation of θ and d with n_e and n_0

n_e	n_0	θ (°)	d (μm)
1.6380	1.4980	1.33	5.89
1.637	1.499	1.33	5.98
1.639	1.497	1.32	5.81
1.64	1.50	1.33	5.89

cation to experimental samples without twist having a tilt between 0–90° degrees and a thickness up to about 20 μm.

Inhomogeneity of the orienting layer leads to noise in the experimental data which does not significantly influence the results of the fit. The method described above yields a reliable way to determine both tilt angle and thickness in cells which are made under normal TN processing conditions but without a twisted director pattern.

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This lock-in amplifier is a simple instrument. Usually you have some kind information on the specifics of the device and as the minimum setup two inputs, one output and one so called reference input.
There is only one thing that we can regulate in this instrument: the phase of the signal.
In practice you acquire the signal from I/P 1 or 2 and you compare the signal to the reference that must be build in a suitable way in the setup. Then we can change the phase between the input signal and the reference signal. This is important because it allows the so-called "phase-locking-acquisition", relevant to avoid noise and useful to acquire very weak signals.
As of today most instruments include inside themselves the phase lock-in device, but usually it's not explicitly stated and it's dangerous since we don't know what the instrument is doing.
The lock-in amplifier most important feature is the lock-in. The phase gives, adds or reduces the delay between the main signal that you want to acquire and the reference one. If we want to measure the absorption of light in a material, we shine the sample and part of the light transmitted will be sent to the optical sensor (photodiode). Then from the latter we obtain the intensity of the signal by a suitable amplifier and reader we can output the signal.
If the material is very opaque or the light source is very weak the signal we obtain is very bad. Supposing that the light source is continuous we should obtain a signal very close to zero. Now in the environment during which we are performing this experiment we'll observe this weak signal.
In a dark room we could expect the noise represented by the red line. In every environment we have e-m field everywhere due to the electric sources around us.
Usually this kind of noise is called pink noise which is typical of electronic objects. If the signal is very weak probably the main noise coming from the environment will produce this kind of noise which will have an amplitude higher than the signal one.
Moreover there are typical noise peaks of pink noise. The reason behind these peaks is due to signals coming from electric systems in our laboratory and the power supply usually works at 50 hz (frequency of the laboratory/home). In US 60 hz. The other peaks are harmonics of the first fundamental oscillations which is 50 hz. The second harmonics is higher than the fundamental which is strange; usually the harmonics have lower amplitude compared to the first one.
It's designed like this because it's a very peculiar harmonic: it's due to the artificial light that can enter in a dark room. We would receive light noise from artificial light sources in experiments involving this kind of setup. Now the artificial light oscillates usually at 100 Hz and this is due to the alternating power which we use to power our instruments. In the graph DX the black line represents the AC power supply at 50 Hz but our lamps don't care about positive or negative parts of the current you are providing. All artificial light sources have an oscillation two times larger than the basic frequency of the power source.
When using LED this effect is due to the rectification, since it works only in one current direction. However all our light sources oscillates two times much compared to the power source energy (50 Hz). This oscillations produce a signal with twice the frequency of the power source one. This is true for any light source however.
The weak signal (close to 0 frequency) is the one we want to observe with the lock in. To obtain a clean signal we can use a low pass filter. Doing so we can erase the high frequency and just consider the low frequencies. If we do so we then erased the effects of before but the noise can be very high as amplitude with respect to the noise. It's a first step but it won't be enough.
Everything to the left is saved applying the filter.
Then as we observed increasing the frequencies the peak reduced their amplitude, so the next solution will be to shift the signal where the peaks are in some way. In this way we would compare our signal to a lower level of the noise. We transform our signal from a 0 frequency to an oscillating frequency: we select 175 because it's not a harmonics of our signal, we have to move the signal in a place where it's zero. Using one of the harmonics frequencies will give us the previous problem.

Then we apply a band-pass filter: we select a range of frequencies where we know our signal is. Using a band pass filter means selecting a restricted bandwidth between 150 and 175. We compare our signal to the lower level of the noise in this way.
Now we have to select a quality of this procedure. Quality factor of our acquisition is the ratio between the frequency you are working and the width of the observation windows you are using to look for your frequency using the band-pass filter. 1000 is a good quality factor when dealing with this sort of experiments. A QF of typical optical experiments is less than 100. So it's too low.
So a standard electronic optical filter is not enough to obtain a good quality factor sufficient to say you are observing a weak signal and not the noise. However the price of a filter grows exponentially with the quality factor.
We have to make sure that our signals maintains stable around the same frequency we are using with the window. The stabilization in frequency is very difficult because even changing the temperature will cause oscillating properties in frequency for the circuits. There are two reasons to not buy the very good filter: -very high cost, -it's difficult to stabilize the frequency in an oscillating window which is narrow. You observe you signal with a window using a band pass filter.
Now to move the signal from 0 to 175 Hz we have to use an instrument to create a pulsating light from a static continuous one. This is the chopper, you select a certain frequency for it. The chopper is rotating at a certain angular frequency.
It's just a disk rotating with some holes on the surface. After producing the oscillating light it travels in the sample and it's then detected. You put an amplifier in case the signal is very weak and then we have a band-pass filter centered on the frequency of the pulsating light. then we put the rectifier because we want to measure the intensity. The rectifier is used to measure something not constant by making it that way. We don't want to have an oscillating signal but one as much as possible constant. Then we have a display that shows the interested intensity.
Now one solution to problem cost vs. good QF is the schematic phase sensitive detector. This uses the phase lock-in: if two signals are phase locked it means they are oscillating at the same frequency.
The only requirement used for this approach is that the signal is periodic.
Th chopper controls the periodicity of the signal and then as the output we have the signal after the procedure. In principle these two signals should have the same frequency
The same signal we are using to control the frequency of the chopper will be used for the control signal.
With the reference signal we can operate with a switch: electrical open= no electrical signal is transmitted, closed=it will transmit the signal. For the high level the switch will be closed while for the low one it will be open. It means that our signal won't be acquired in a complete way but only a portion; only when the switch will be closed i.e. we have transmission of electric current.
So with this setup we can select only the positive part of the oscillating current flowing trough the circuit. And then by using the rectifier circuit which is just a resistance and a capacitor in the simplest case we can obtain a waveform with a flat band.
This scheme is neat because has no band pass filter so the cost is zero. Secondly the switch is a simple device. We are selecting the signal with a very high precision.
In this case the quality factor we should get is related to the electrical components involved in the circuit. Easily a QF of 1000.
The main problem in this scheme is that the switch is a very bad device since it produces a lot of noise and also the frequency at which the switch is working will be very low. We are also losing half of the initial information since we are selecting only the positive side of the signal.
We have to do the same scheme without a mechanical switch: by using two switches we can do something better. We could save all the signal information. For instance we could have an amplifier for just changing the sign of the signal and another amplifier which does nothing. Then you have a switch between +1 and -1.

The switch is connected to -1 when is collecting the negative end of our signal and +1 and the positive part. But using an amplifier which changes the sign we obtain for the negative part a positive part (change of sign of signal using operational amplifier).

Now we solved one part of the problem but the switches (mechanical switches) are producing some noise. To solve this problem: consider two periodic signals (f_s and f_r) we can multiply them with a multiplier. We obtain a signal with components with frequencies equal to the sum of the initial signals (higher frequencies) and components with difference frequencies of them (lower frequencies). This is done trough the usage of Werner formula. If the frequencies are practically the same since we are using a chopper this is a constant signal after applying this procedure.

Multiplier is a cheap circuit to produce a high frequency and a low frequency component. We can cut the high frequencies with a low pass filter and save just the lower ones. We save just the low frequency component which is just a DC signal.

Lock-in amplifier is then a good device with an adequate multiplier to change the phase, to add some retard to one of your two signals.

As the lockin amplifier has a phase control we are not obliged to acquire the signal at 0 frequency; we can change that by adding a delay. Adding the phase means adding a delay between the signals. In practice having the multiplier you can select a part of the signal that gives information about your physical experiments.

This is important. If we have some retard coming from our experiment we can measure this retard.