Sci. Data$_0$1.*bib*

Introduction

Fourier transform is the most widely used tools in applied mathematics to analyze signals. The essence of the Fourier transform of a waveform is to decompose or separate the waveform into a sum of sinusoids of different frequencies. If these sinusoids sum to the original waveform, then we have determined the Fourier transform of the waveform. Mathematically speaking, is it possible to write this relation as:

$$\mathcal{F}(f) = \int_{-\infty}^{+\infty} s(t) e^{i2\pi f t} dt,$$

where g(t) is the waveform to be decomposed into a sum of sinusoids and G(f) is the Fourier transform of g(t). As an example, consider the pulse waveform (a) and its Fourier transform (b) shown the following figures.

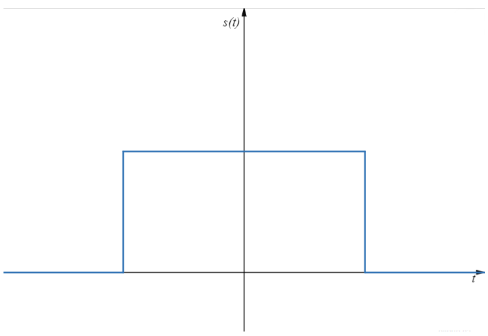

Figure 1: pulse waveform

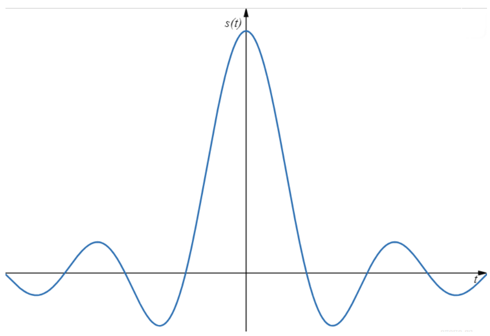

Figure 2: Fourier transform

The purpose of this experiment is to use the Fourier transform to analyze a signal affected by noise and check if the noise is "white noise" and if the standard deviation of the mean decreases as the square root of the number of acquisitions.

1 Materials and Methods

1.1 Equipment And Tools

- Digital storage oscilloscope (Siglent - SDS5034X)
- Waveform generator (Siglent - SDG6022X)

Next, using a function of the digital oscilloscope, we were able to add the two signals. The resulting waveform can be seen in the following figures.

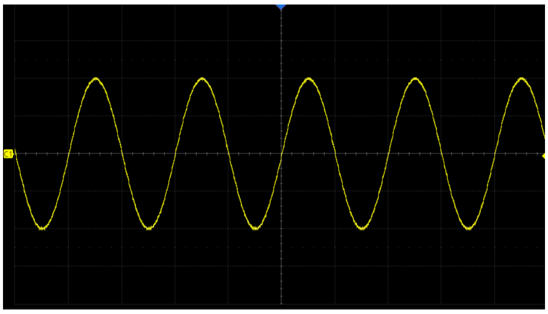

Figure 3: Sinusoidal signal

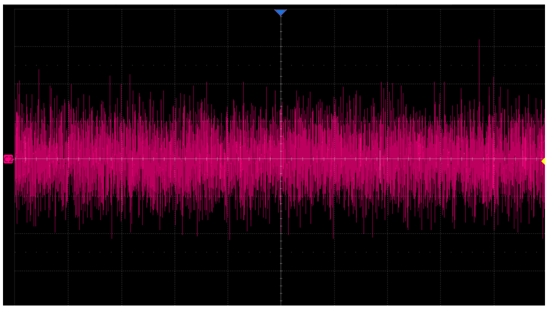

Figure 4: Noise signal

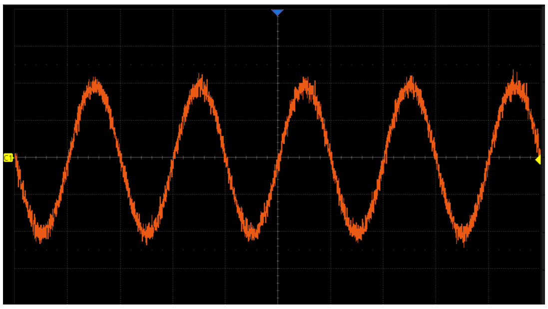

Figure 5: Resulting signal from the addition of the two signals

Finally, we proceeded to acquire the data from the oscilloscope and saved it to a USB flash drive. To acquire the data from the oscilloscope, we followed the steps below:

- press the save/recall button, to open the file acquisitions menu
- select the the file extension, in our case .csv
- select our USB flash drive from the acquisition menu
- press on the touch screen the save as window
- select the file name and press the save button

All the data acquired was analyzed using Julia programming language

2 Results

2.1 Part 1

2.1.1 Original signal and its Fourier transform

Once collected the data, we procede with the visualization of the data in graphical form. The following Fig (6) shows the signal saved from the oscilloscope without any data processing. We can compare this plot with the image acquired directly from the oscilloscope in Fig (??) and check for their similarity.

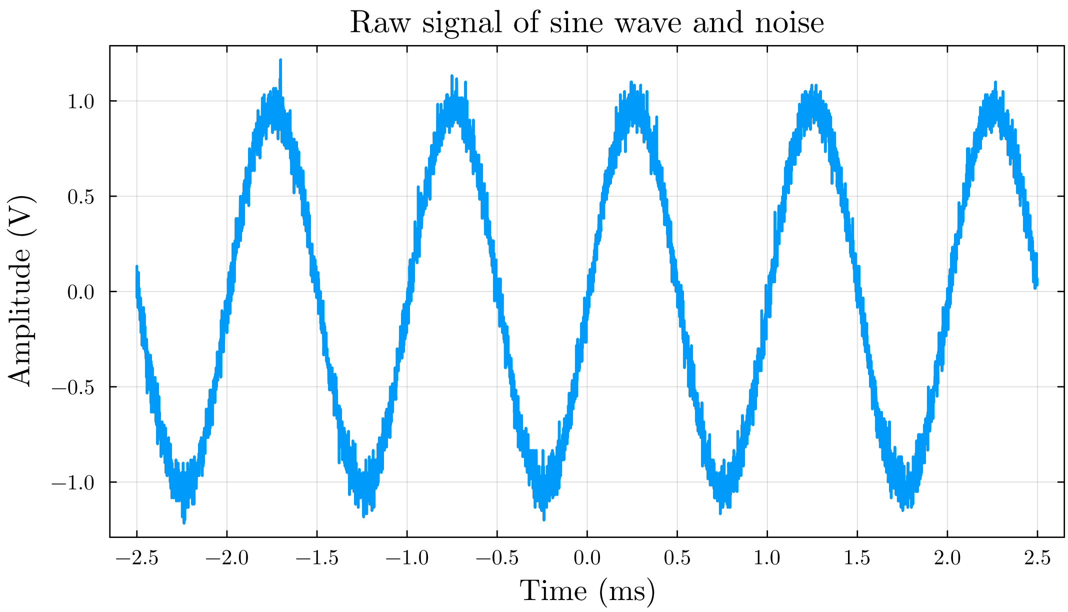

Figure 6: One of our saved signal, composed of one sine wave and some noise.

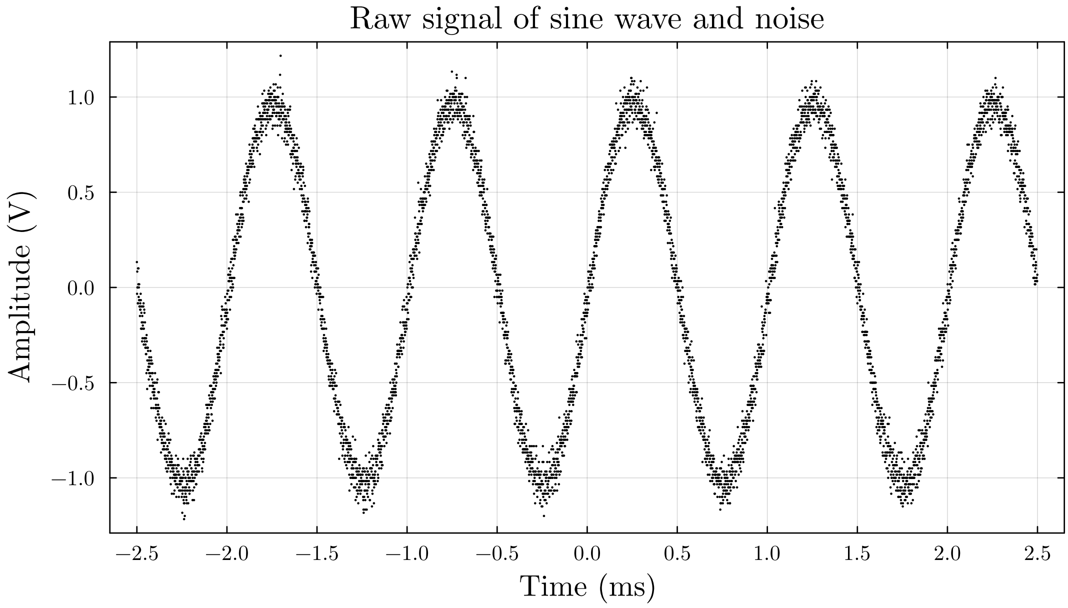

Figure 7: One of our saved signal, composed of one sine wave and some noise.

As our second step, we compute the Fourier transform of the previous signal, without using any windowing function. To do so, we take advantage of the library ”FFTW” of the julia programming language, this library implements both the complex and real fourier transform, among with the inverse fourier transform and many other methods. For this experiment we will use the complex fourier transform.

After the transformation, we obtain an array of complex values. We will consider the modulus of these numbers (which is the amplitude of the given frequency). Furthermore, we must say that the transform has non-zero values both in the positive and in the negative frequency axis. Our signal is of course made from real world data (in other words, real numbers), hence we can say that the final transformation will be an even function and the data on the negative side of the frequency axis will be a reflection of the data on the positive side of the same axis.

If we plot the values on the positive side only, we obtain the graph shown in Fig (8).

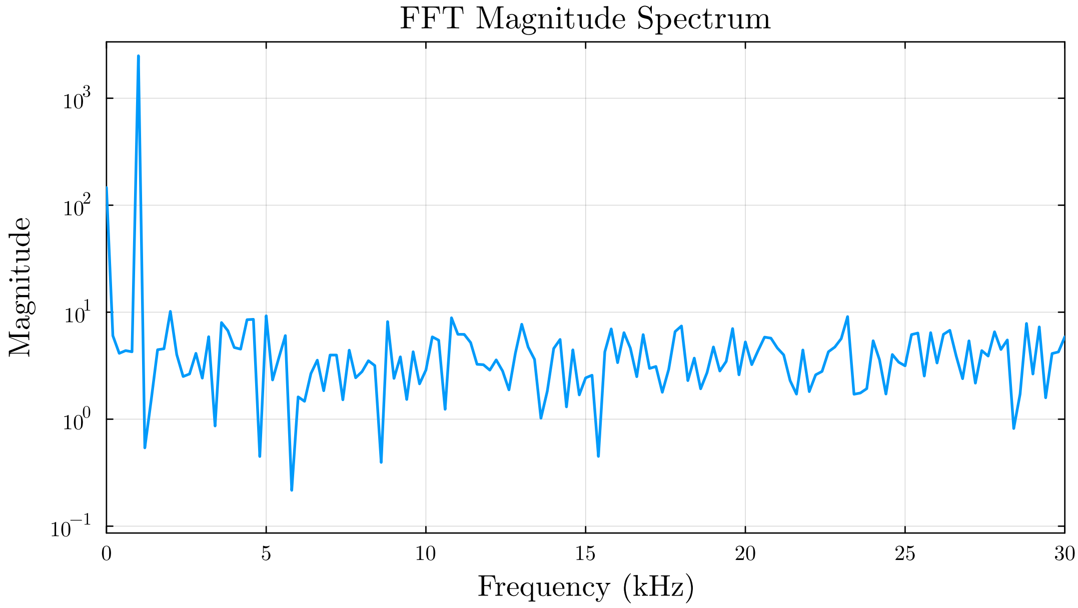

Figure 8: Fast Fourier Transform of the signal

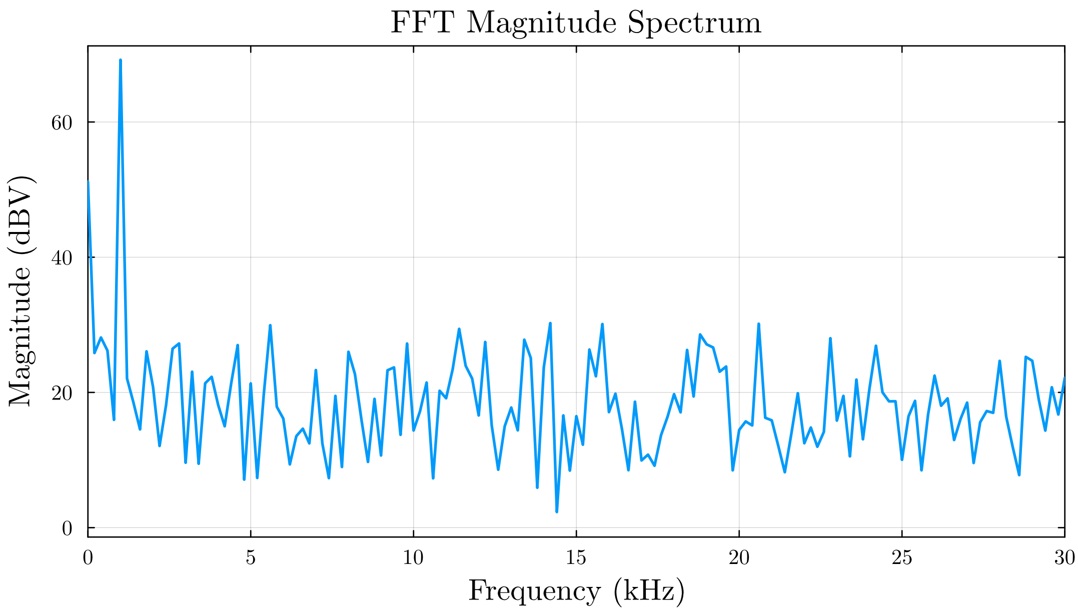

Figure 9: Fast Fourier Transform of the signal in dBV

The plot in Fig (9) shows the Fourier transform in dBV.

2.1.2 Peak cutoff and inverse Fourier transform

We can now proceed in the selection of the frequencies which correspond to the noise. This step is equivalent to a peak removal, since we know that the signal which we are studying is a single sine wave. In practice, we modify the value of the Fourier transform in the position of the peak. We set this value to the mean of the transform in a big enought range (we use the interval I from 1166 to 4500 Hz). We must remember to keep the transformation an even function, so we also modify the value of the transform for the negative frequency to the same value. In formulas, we impose

$$\begin{cases} \mathcal{F}(1000\,Hz) = \mathcal{F}(f)_I \\ \mathcal{F}(-1000\,Hz) = \mathcal{F}(1000\,Hz) \end{cases}.$$

We can now evaluate the inverse transform in order to reconstruct the original noise. We have a plot for the reconstructed noise in Fig (10).

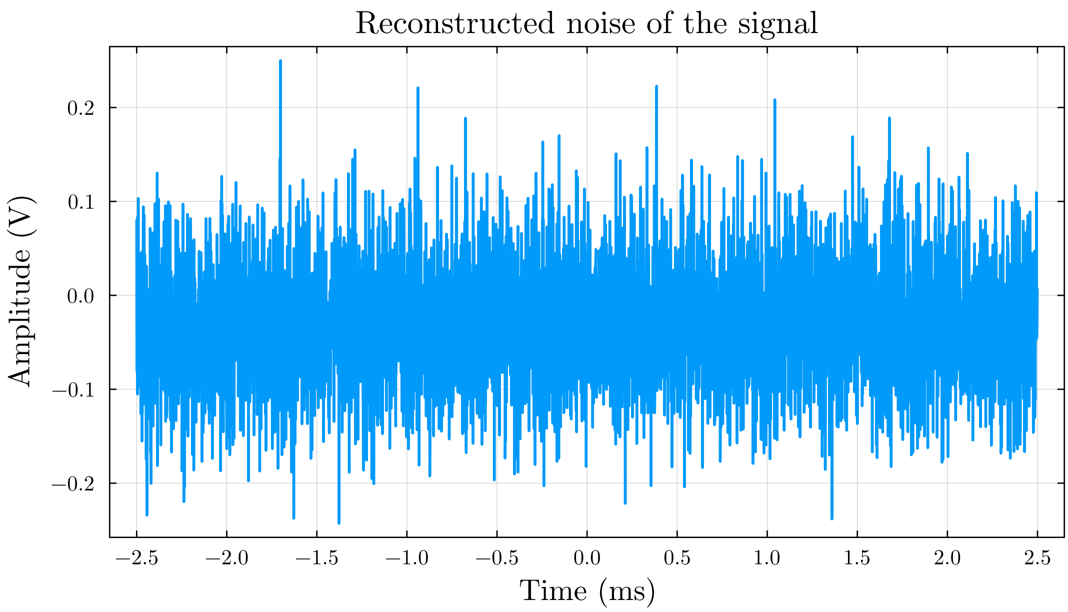

Figure 10: Reconstructed noise obtained from the inverse Fourier transform

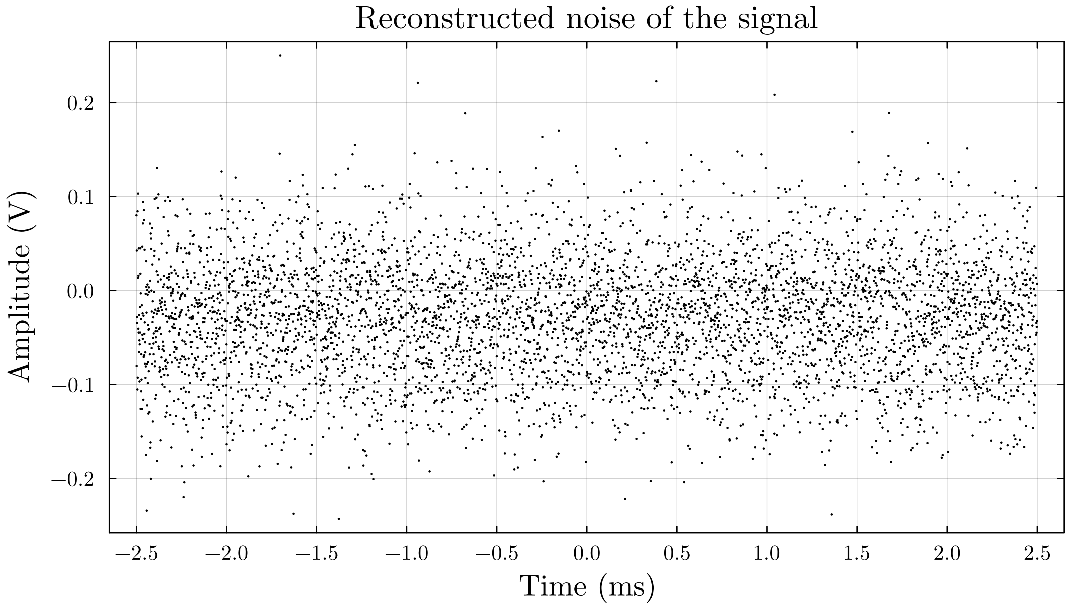

Figure 11: Reconstructed noise obtained from the inverse Fourier transform

2.1.3 Statistical analysis of the reconstructed noise

From the data obtained from the previos section, we can conduct a brief statistical analysis. The main purpose of this section is to check whether the noise obtained can be classified as white noise, that is noise that follows a normal distribution (also called Gaussian distribution).

We begin by calculating the mean (μ) and the standard deviation (σ) of the data, obtaining $\mu = -0.02925\, mV$ and $\sigma = 0.06231 mV$. As expected, the noise has a mean value very close to zero and the error is bigger than the mean, which means that the zero is inside the confidence range of 68%.

We will now use the chi squared test to check how well the normal distribution fits our data. The Fig (12) lets us see the expected values for this distribution in overlay with the normalized histogram of the data from the noise.

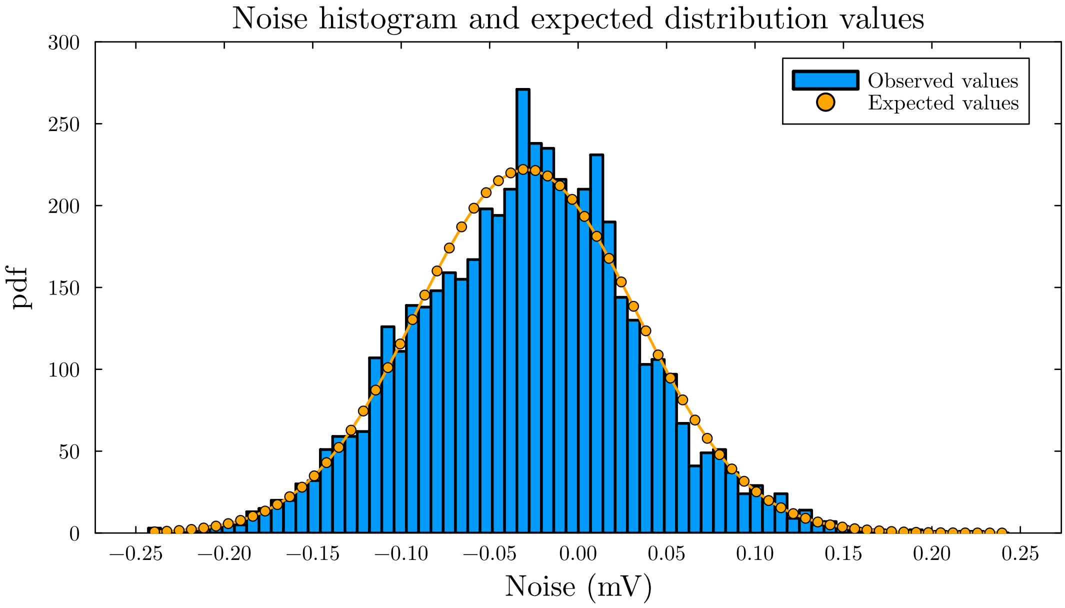

Figure 12: Histogram of the noise in overlay with the expected values given by the Gaussian distribution.

Evaluating the chi squared gives us a result of $\chi^2 = 149.77$, with $d = 70 - 3 = 67$ degrees of freedom. We can now obtain the reduced chi squared $\tilde{\chi}^2 = \chi^2/d = 2.2354$. In this case, the probability to obtain a χ^2 ... is less than 0.05%, hence the distridution does not represent at all the collected data. The Table (1) sums up all the statistical data collected in this section.

Quantity	Value
Mean (μ)	$-0.02925\,mV$
Standard error (σ)	$0.06231mV$
Degrees of freedom (d)	67
Chi squared (χ^2)	149.77
Reduced chi squared ($\tilde{\chi}^2$)	2.2354

Table 1: Summary table for the statistical quantities