Sci. Data$_0$1.*bib*

1 Results

1.1 Part 1

1.1.1 Original signal and its Fourier transform

Once collected the data, we procede with the visualization of the data in graphical form. The following Fig (1) shows the signal saved from the oscilloscope without any data processing. We can compare this plot with the image acquired directly from the oscilloscope in Fig (**??**) and check for their similarity.

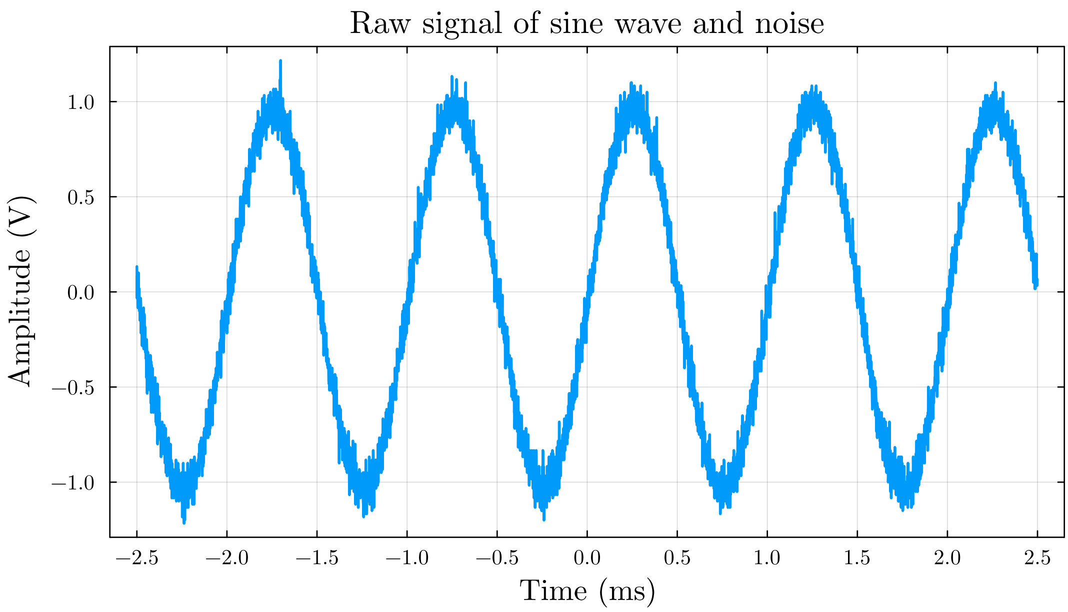
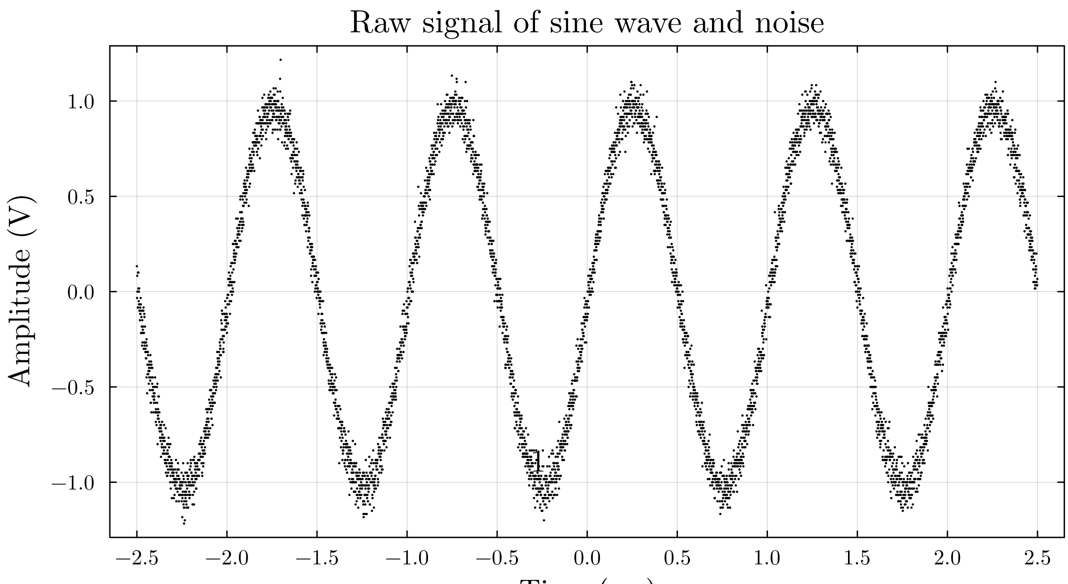


Figure 1: One of our saved signal, composed of one sine wave and some noise.



we must say that the transform has non-zero values both in the positive and in the negative frequency axis. Our signal is of course made from real world data (in other words, real numbers), hence we can say that the final transformation will be an even function and the data on the negative side of the frequency axis will be a reflection of the data on the positive side of the same axis.

If we plot the values on the positive side only, we obtain the graph shown in Fig (3).

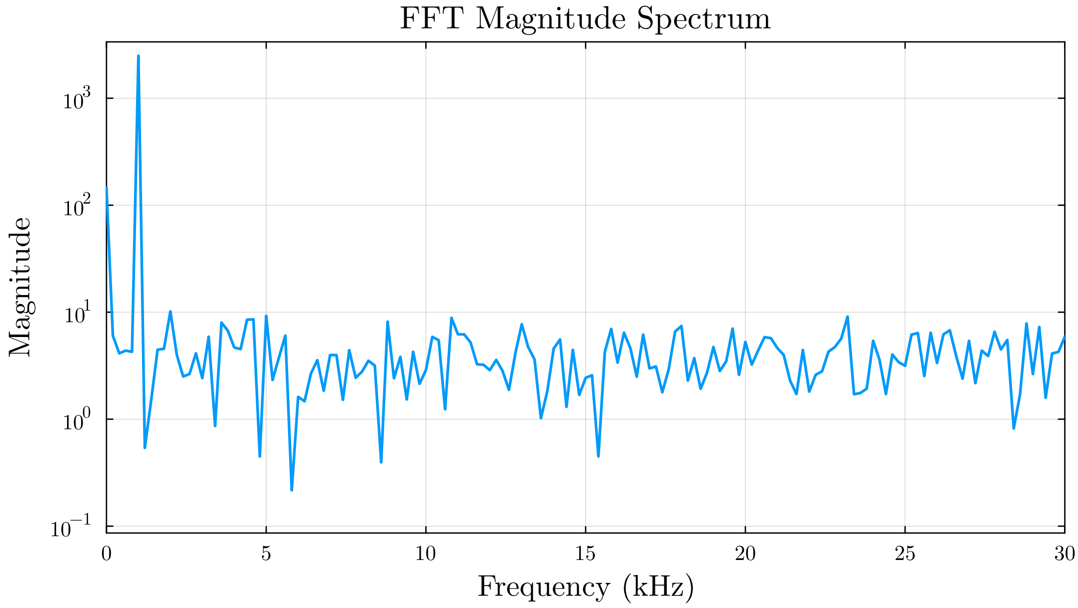


Figure 3: Fast Fourier Transform of the signal

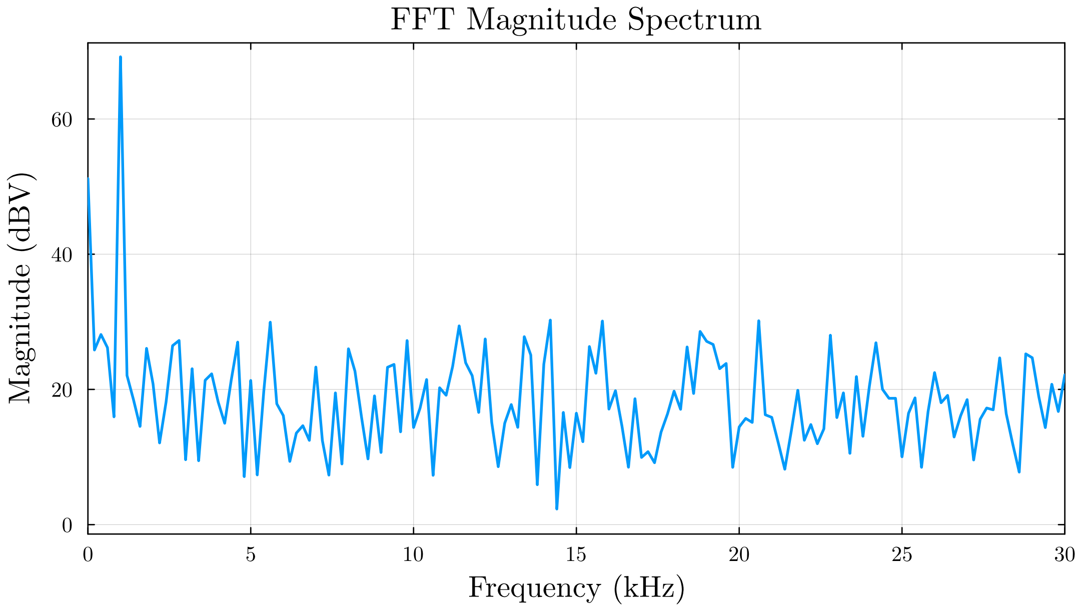


Figure 4: Fast Fourier Transform of the signal in dBV

The plot in Fig (4) shows the Fourier transform in dBV.

1.1.2 Peak cutoff and inverse Fourier transform

We can now proceed in the selection of the frequencies which correspond to the noise. This step is equivalent to a peak removal, since we know that the signal which we are studying is a single sine wave. In practice, we modify the value of the Fourier transform in the position of the peak. We set this value to the mean of the transform in a big enought range (we use the interval I from 1166 to 4500 Hz). We must remember to keep the transformation an even function, so we also modify the value of the transform for the negative frequency to the same value. In formulas, we impose

$$\begin{cases} \mathcal{F}(1000\,Hz) = \langle \mathcal{F}(f) \rangle_I \\ \mathcal{F}(-1000\,Hz) = \mathcal{F}(1000\,Hz) \end{cases} .$$

We can now evaluate the inverse transform in order to reconstruct the original noise. We have a plot for the reconstructed noise in Fig (5).

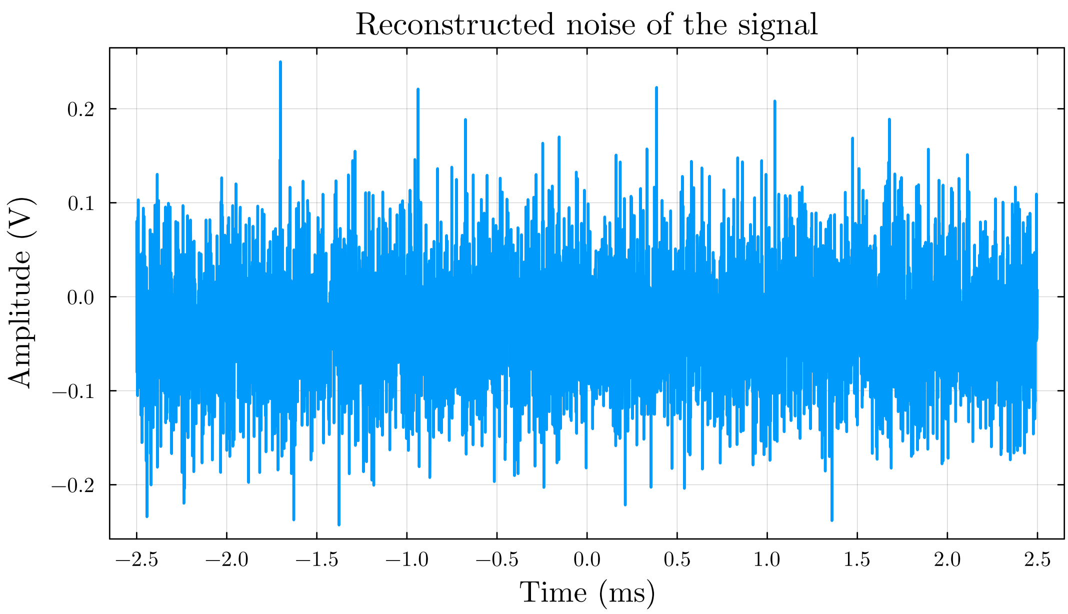


Figure 5: Reconstructed noise obtained from the inverse Fourier transform

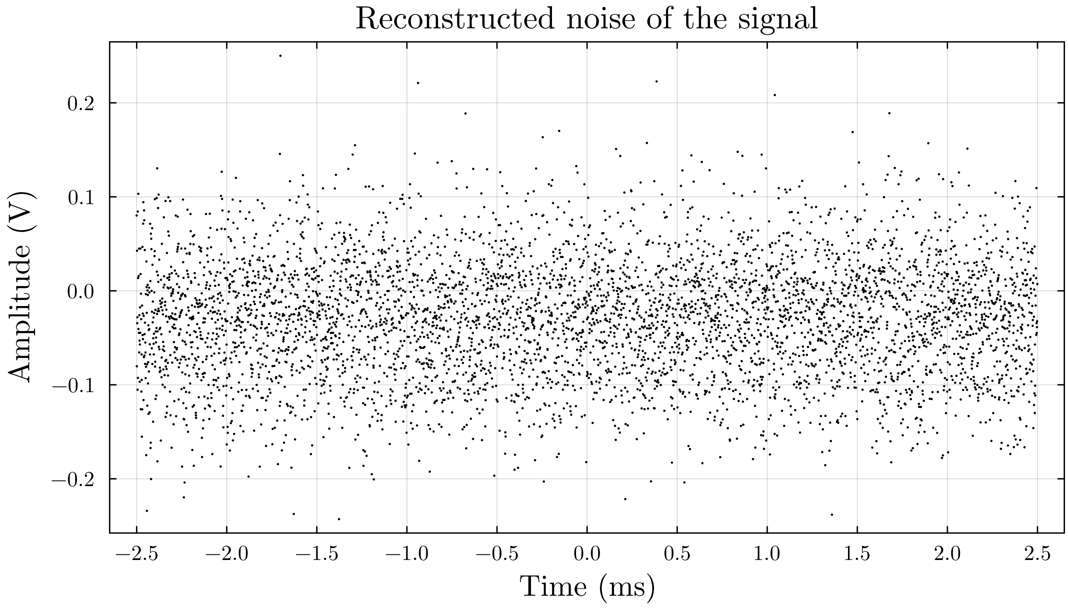


Figure 6: Reconstructed noise obtained from the inverse Fourier transform

1.1.3 Statistical analysis of the reconstructed noise

From the data obtained from the previos section, we can conduct a brief statistical analysis. The main purpose of this section is to check whether the noise obtained can be classified as white noise, that is noise that follows a normal distribution (also called Gaussian distribution).

We begin by calculating the mean (μ) and the standard deviation (σ) of the data, obtaining $\mu = -0.02925\, mV$ and $\sigma = 0.06231 mV$. As expected, the noise has a mean value very close to zero and the error is bigger than the mean, which means that the zero is inside the confidence range of 68%.

We will now use the chi squared test to check how well the normal distribution fits our data. The Fig (7) lets us see the expected values for this distribution in overlay with the normalized histogram of the data from the noise.

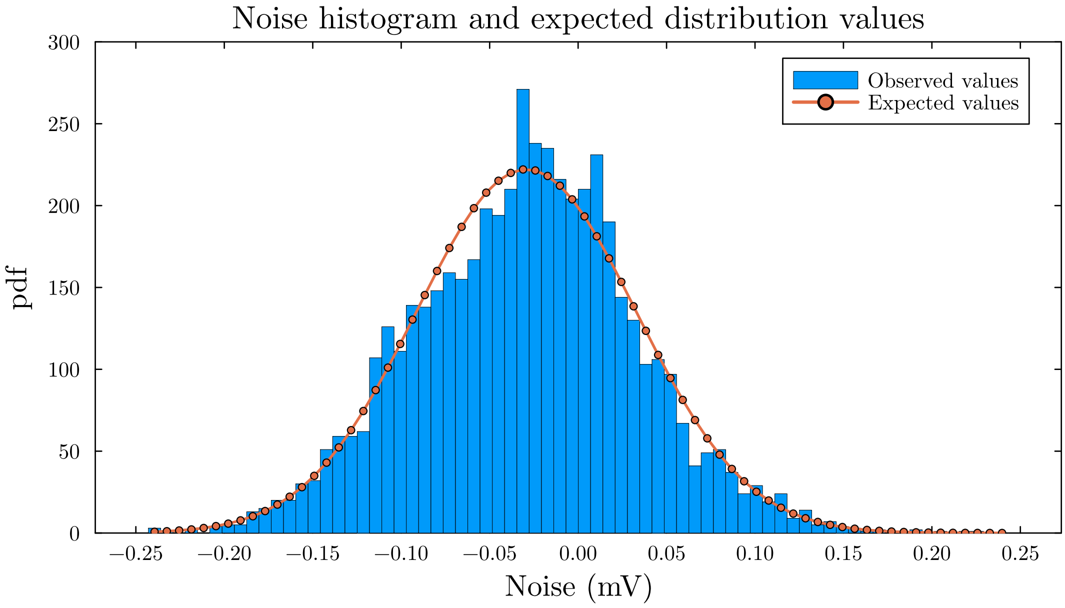


Figure 7: Histogram of the noise in overlay with the expected values given by the Gaussian distribution.

Evaluating the chi squared gives us a result of $\chi^2 = 149.77$, with $d = 70 - 3 = 67$ degrees of freedom. We can now obtain the reduced chi squared $\tilde{\chi}^2 = \chi^2/d = 2.2354$. In this case, the probability to obtain a χ^2 ... is less than 0.05%, hence the distridution does not represent at all the collected data. The Table (2) sums up all the statistical data collected in this section.

Quantity	Value
Mean (μ)	$-0.02925\,mV$
Standard error (σ)	$0.06231mV$
Degrees of freedom (d)	67
Chi squared (χ^2)	149.77
Reduced chi squared ($\tilde{\chi}^2$)	2.2354

Table 1: Summary table for the statistical quantities

1.2 Part 2

1.2.1 Evaluation of the mean of multiple signals

After the first part on the statistical analysis of the noise of a single signal, we proceed with the study of the treatment of the noise using multiple signals. In particular, we will make use of all 32 different datasets of the same signal to verify whether the error on the signal will fall off as the square root of the number of measures.

We begin from an intuitive graph, Fig (8), where we show the signal obtained averaging different numbers of datasets. We can see how the error is reduced from one average to the other.

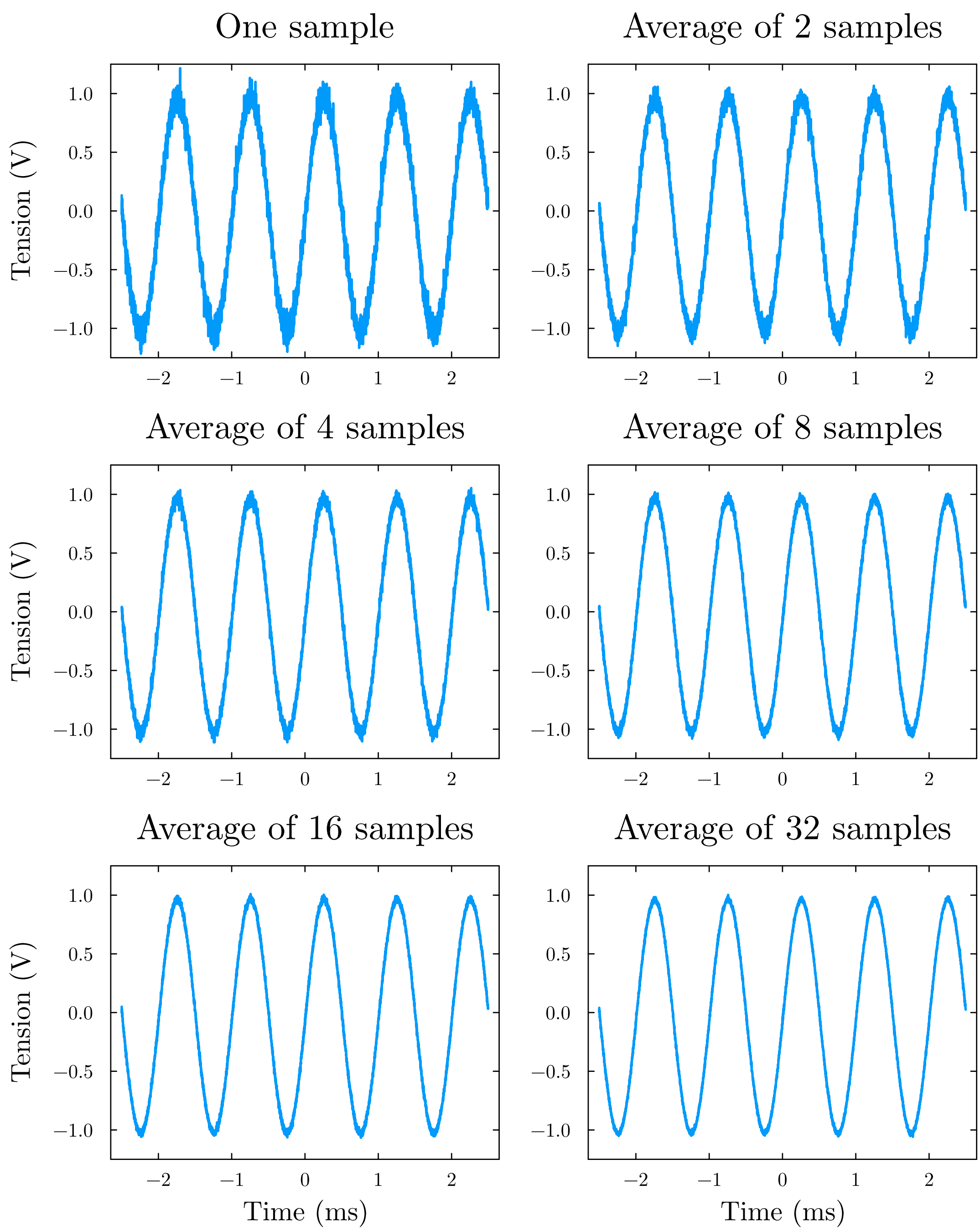


Figure 8: Signal obtained from the average of different numbers of signal datasets.

In Fig (9), we can see a similar plot as the one before, but in this graph we only consider the reconstructed noise. Note that we first reconstruct the noise for every dataset, then we apply the average over these results. Here the decay of the noise is much more clear to see.

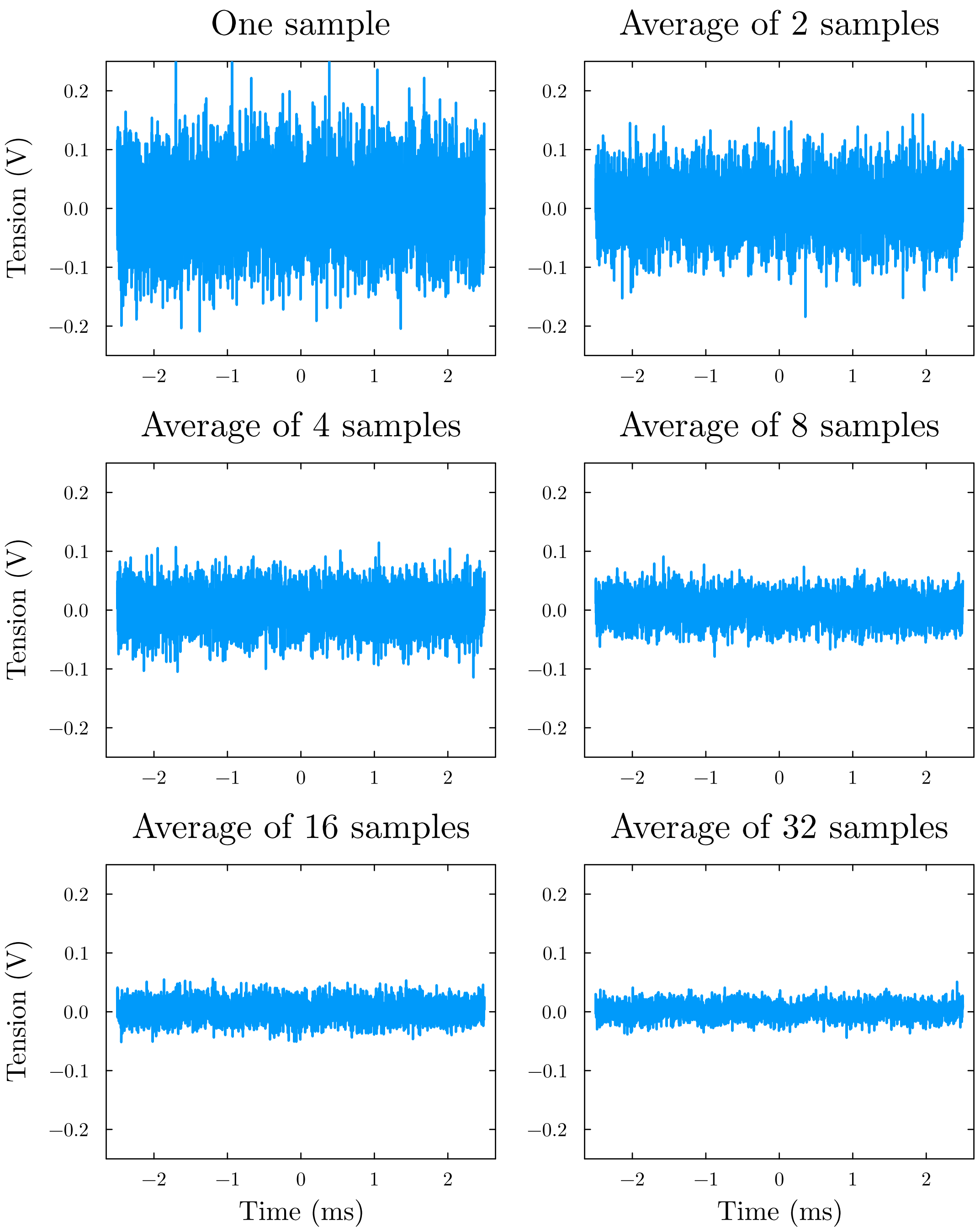


Figure 9: Reconstructed noise obtained from the average of different numbers of noise datasets.

We then conduct some statistical analysis over the data in the last plot of Fig (9), that is the evaluation of the standard error. The results are presented in the Table (**??**).

Average of # datasets	Standard error
1	0.06237
2	0.04435
4	0.03121
8	0.02233
16	0.01598
32	0.01180

Table 2: Variation of the standard error of the reconstructed noise in function of the number of datasets condidered for the mean.

We will use this last data to verify our assumption, so we will assume a power law in these points. We will use the least square method applied to the power law case in order to find the exponent, which we expect to be approximately $-\frac{1}{2}$. The Fig (10) lets us have a look at our fit.

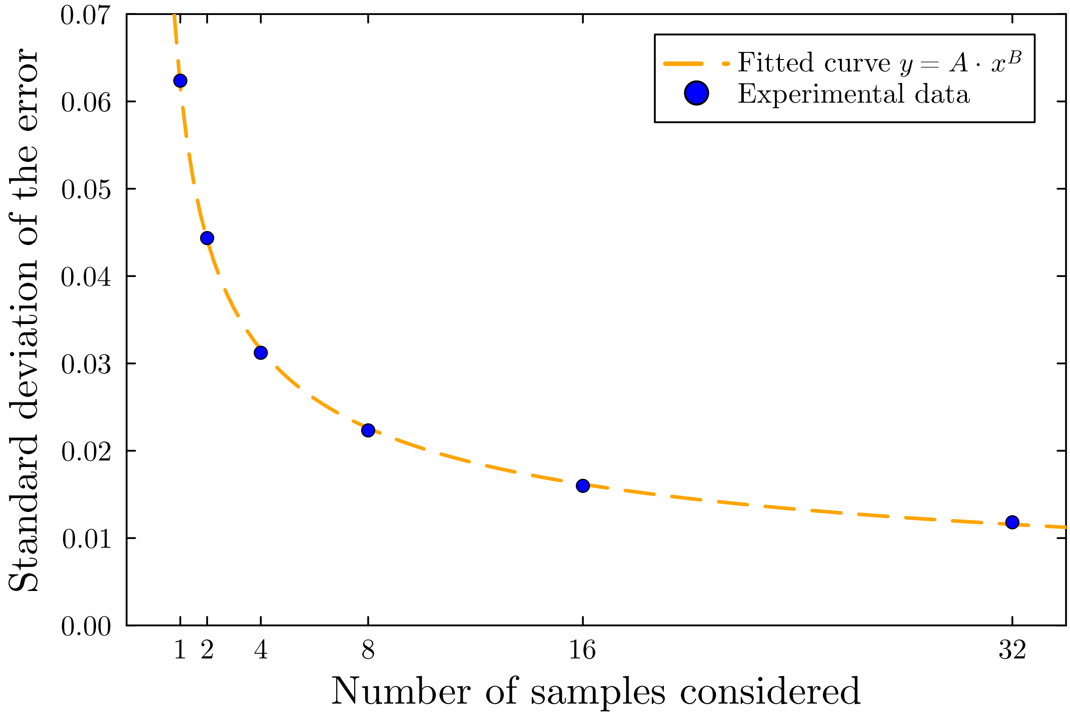


Figure 10: The standard errors of the noise and the best power law fit.

The parameters and the error are reported in Table (3).

Parameter	Value	Error (if necessary)
A	-2.785	0.011
B	-0.4831	0.0054
R^2	0.9995	/

Table 3: All the parameter for the power law fit $y = A \cdot x^B$.