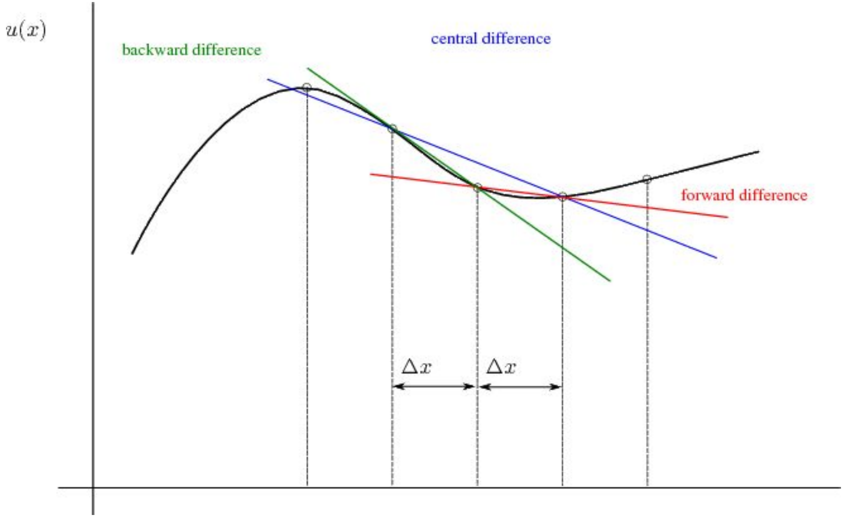
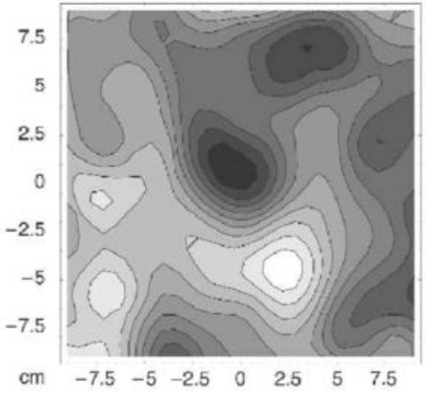
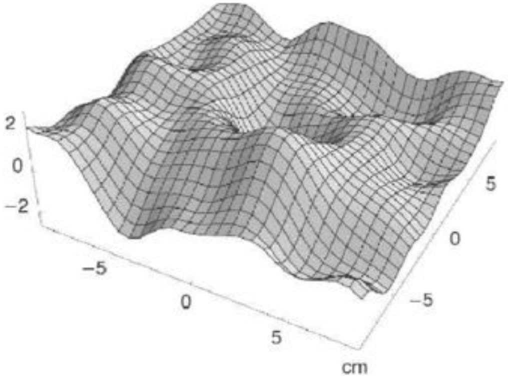
Сверточные нейронные сети (CNN)

Почему не перцептрон?

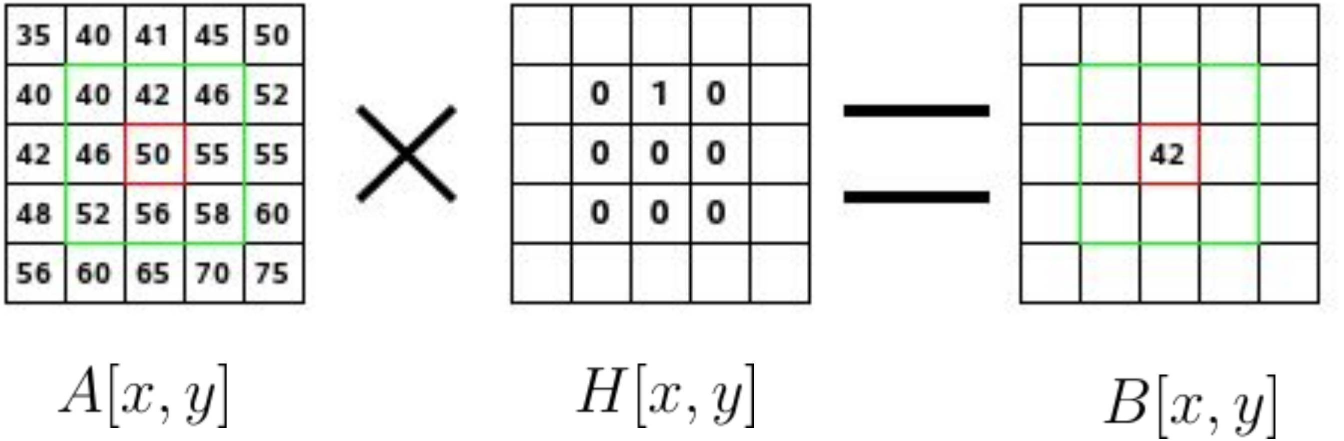
- Двумерность данных
- Непрерывность







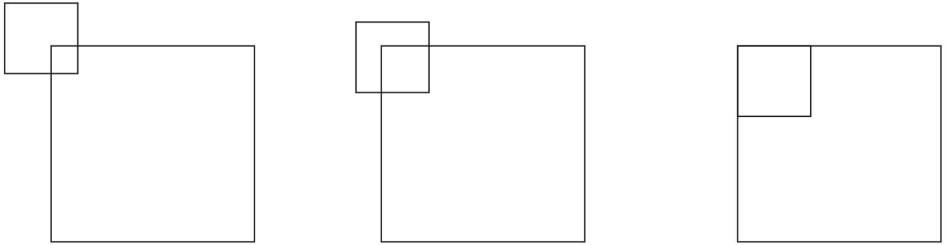
Свертка



$$B[x,y] = \sum_{u=-n}^{+n} \sum_{v=-n}^{+n} A[x+u, y+v] H[u,v]$$

Свертка

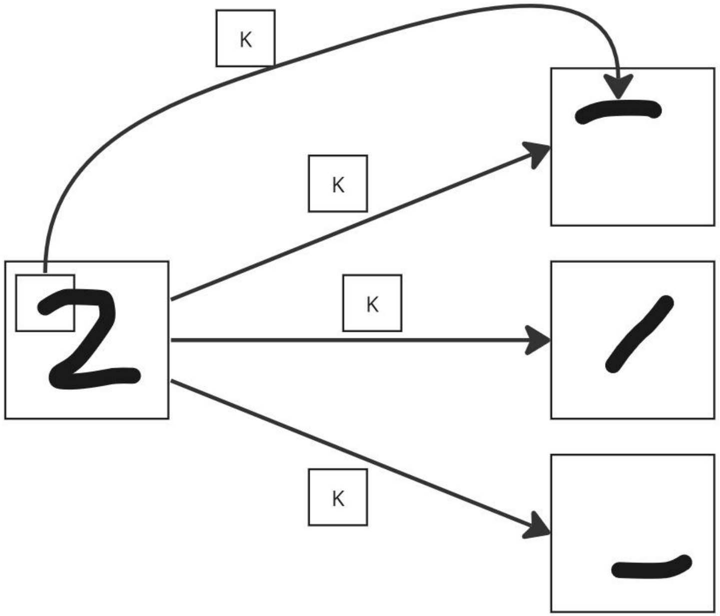
Свертки и границы

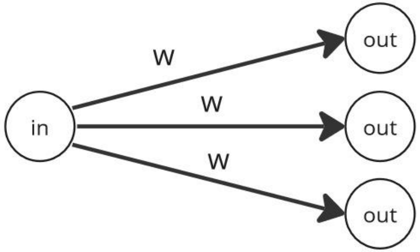


full same valid

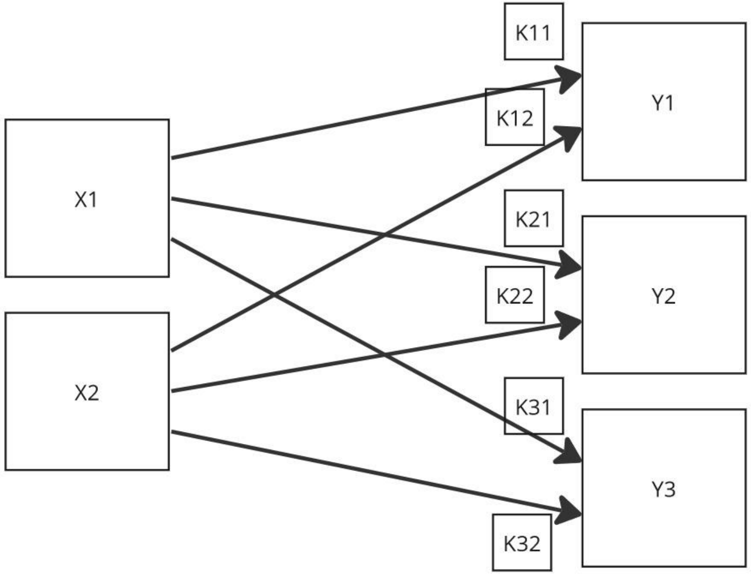
Пространство за границами заполняется в зависимости от задачи

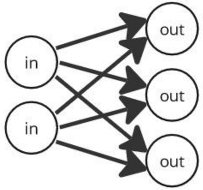
Сверточный слой





Сверточный слой

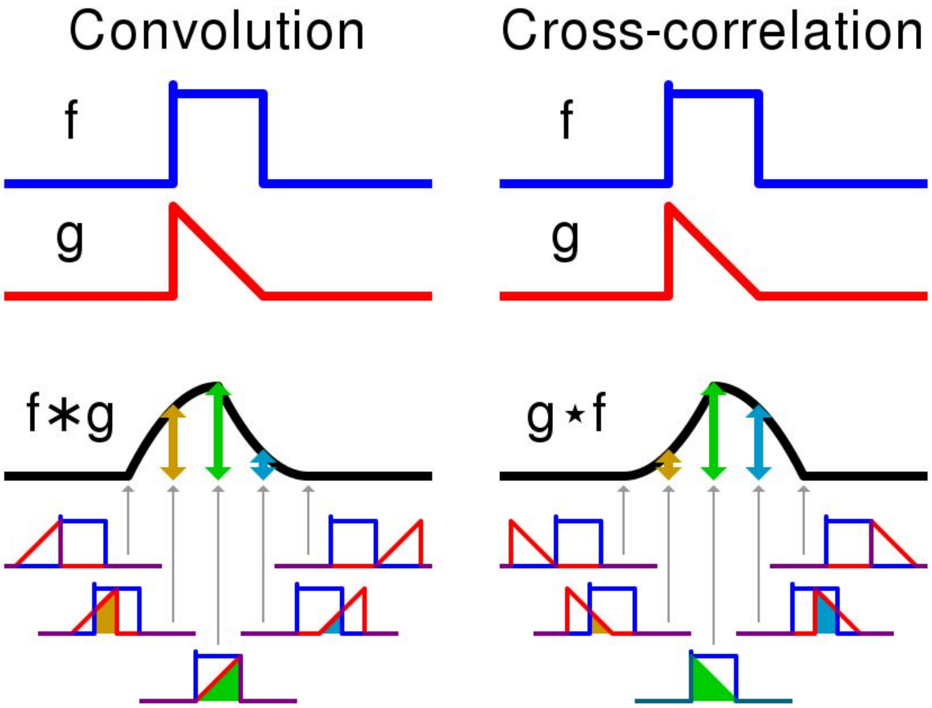




$$y = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} \cdot x$$

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \\ K_{31} & K_{32} \end{pmatrix} \star \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix}$$

Cross-Correlation and Convolution

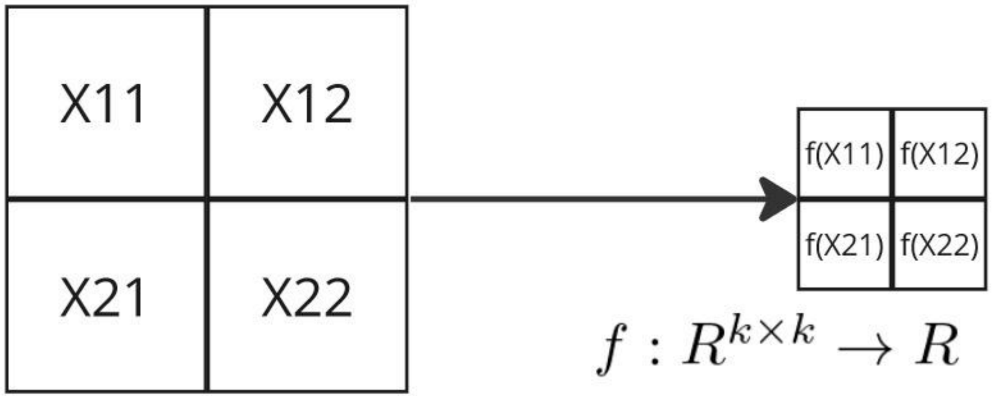


Свертка в нашем понимании - это кросс-корреляция

Свертка - похожая операция, но одна из функций изменяется в обратном направлении

Sub-sampling layer (pooling)

- Average Pooling
- Max Pooling



$$f : R^{k \times k} \rightarrow R$$

LeNet-5

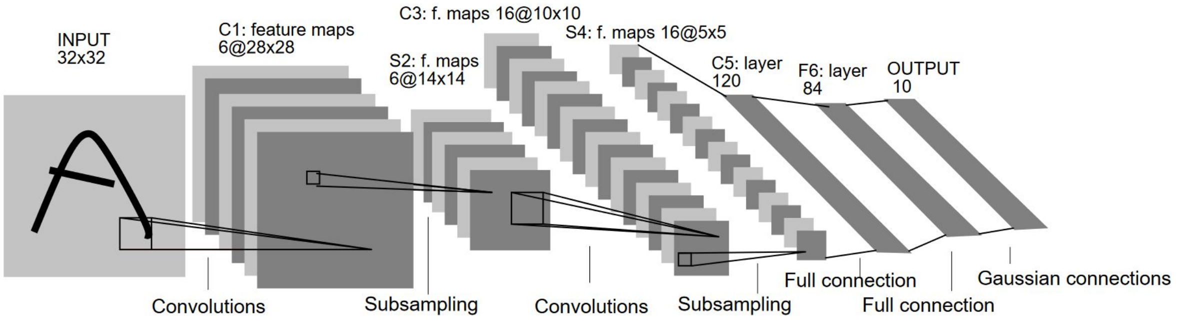


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

Layer		Feature Map	Size	Kernel Size
Input	Image	1	32x32	-
1	Convolution	6	28x28	5x5
2	Average Pooling	6	14x14	2x2
3	Convolution	16	10x10	5x5
4	Average Pooling	16	5x5	2x2
5	Convolution	120	1x1	5x5
6	FC	-	84	-
Output	FC	-	10	-

Последний сверточный слой - обычная “valid” свертка с ядром размера 5x5 над картами признаков размером 5x5, на выходе получается набор карт признаков размера 1x1 или просто вектор скаляров.

Обучение сверточного слоя

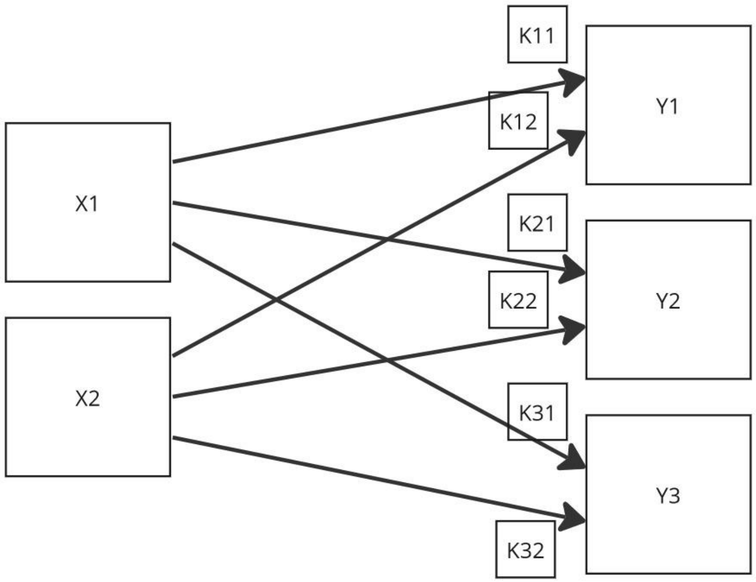
Обучаемые параметры: $\vec{\mathcal{B}}, \mathcal{K}$

Предыдущий слой: $\vec{\mathcal{X}}$

Выход слоя: $\vec{\mathcal{Y}}$

Известно: $\dfrac{\delta\vec{\mathcal{E}}}{\delta\mathcal{Y}} = \begin{pmatrix} \dfrac{\delta\mathcal{E}}{\delta Y_1} \\ \dfrac{\delta\mathcal{E}}{\delta Y_2} \\ \dfrac{\delta\mathcal{E}}{\delta Y_3} \end{pmatrix}$

Нужно найти: $\dfrac{\delta\vec{\mathcal{E}}}{\delta\mathcal{B}}, \dfrac{\delta\mathcal{E}}{\delta\mathcal{K}}, \dfrac{\delta\vec{\mathcal{E}}}{\delta\mathcal{X}}$



$$\vec{\mathcal{Y}} = \mathcal{K} \star \vec{\mathcal{X}} + \vec{\mathcal{B}}$$

Аналогии

$$\frac{\delta \mathcal{E}}{\delta \mathcal{K}} = \begin{pmatrix} \frac{\delta \mathcal{E}}{\delta K_{11}} & \frac{\delta \mathcal{E}}{\delta K_{12}} \\ \frac{\delta \mathcal{E}}{\delta K_{21}} & \frac{\delta \mathcal{E}}{\delta K_{22}} \\ \frac{\delta \mathcal{E}}{\delta K_{31}} & \frac{\delta \mathcal{E}}{\delta K_{32}} \end{pmatrix} \qquad \frac{\delta \mathcal{E}}{\delta W} = \begin{pmatrix} \frac{\delta \mathcal{E}}{\delta w_{11}} & \frac{\delta \mathcal{E}}{\delta w_{12}} \\ \frac{\delta \mathcal{E}}{\delta w_{21}} & \frac{\delta \mathcal{E}}{\delta w_{22}} \\ \frac{\delta \mathcal{E}}{\delta w_{31}} & \frac{\delta \mathcal{E}}{\delta w_{32}} \end{pmatrix}$$

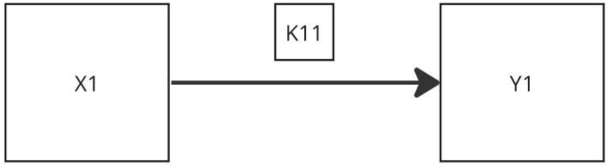
$$\frac{\delta \mathcal{E}}{\delta K_{32}} \in R^{k \times k} \qquad \frac{\delta \mathcal{E}}{\delta w_{32}} \in R$$

Bias error

$$Y_1 = B_1 + X_1 \star K_{11} + X_2 \star K_{12}$$
$$\mathrm{Y}_2 = B_2 + X_1 \star K_{21} + X_2 \star K_{22}$$
$$\mathrm{Y}_3 = B_3 + X_1 \star K_{31} + X_2 \star K_{32}$$

$$\frac{\delta \vec{\mathcal{E}}}{\delta B_1} = \frac{\delta \vec{\mathcal{E}}}{\delta Y_1} \circ \frac{\delta \vec{Y_1}}{\delta B_1}$$

1 kernel error



$$y_{i,j} = \underbrace{\sum_l \sum_m x_{i+l,j+m} k_{l,m}}_{\text{Свертка}}$$

$$y_{11} = x_{11}\underline{k_{11}} + x_{12}k_{12} + x_{21}k_{21} + x_{22}k_{22}$$
$$y_{12} = x_{12}\underline{k_{11}} + x_{13}k_{12} + x_{22}k_{21} + x_{23}k_{22}$$
$$y_{21} = x_{21}\underline{k_{11}} + x_{22}k_{12} + x_{31}k_{21} + x_{32}k_{22}$$
$$y_{22} = x_{22}\underline{k_{11}} + x_{23}k_{12} + x_{32}k_{21} + x_{33}k_{22}$$

$$\frac{\delta \mathcal{E}}{\delta k_{l,m}} = \sum_i \sum_j \frac{\delta \mathcal{E}}{\delta y_{i,j}} \frac{\delta y_{i,j}}{\delta k_{l,m}} = \sum_i \sum_j \frac{\delta \mathcal{E}}{\delta y_{i,j}} x_{i+l,j+m}$$

$$\frac{\delta \mathcal{E}}{\delta K} = X \star \frac{\delta \mathcal{E}}{\delta Y}$$

Kernels error

$$Y_1 = B_1 + X_1 \star K_{11} + X_2 \star K_{12}$$

$$\mathrm{Y}_2 = B_2 + X_1 \star K_{21} + X_2 \star K_{22}$$

$$\mathrm{Y}_3 = B_3 + X_1 \star K_{31} + X_2 \star K_{32}$$

$$\frac{\delta \mathcal{E}}{\delta \mathcal{K}} = \begin{pmatrix} \frac{\delta \mathcal{E}}{\delta K_{11}} & \frac{\delta \mathcal{E}}{\delta K_{12}} \\ \frac{\delta \mathcal{E}}{\delta K_{21}} & \frac{\delta \mathcal{E}}{\delta K_{22}} \\ \frac{\delta \mathcal{E}}{\delta K_{31}} & \frac{\delta \mathcal{E}}{\delta K_{32}} \end{pmatrix} = \begin{pmatrix} X_1 \star \frac{\delta \mathcal{E}}{\delta Y_1} & X_2 \star \frac{\delta \mathcal{E}}{\delta Y_1} \\ X_1 \star \frac{\delta \mathcal{E}}{\delta Y_2} & X_2 \star \frac{\delta \mathcal{E}}{\delta Y_2} \\ X_1 \star \frac{\delta \mathcal{E}}{\delta Y_3} & X_2 \star \frac{\delta \mathcal{E}}{\delta Y_3} \end{pmatrix}$$

Input error

$$\frac{\delta \mathcal{E}}{\delta X} = \frac{\delta \mathcal{E}}{\delta Y} *_{full} K$$

$$Y_1 = B_1 + \underline{X_1} \star K_{11} + X_2 \star K_{12}$$
$$Y_2 = B_2 + \underline{X_1} \star K_{21} + X_2 \star K_{22}$$
$$Y_3 = B_3 + \underline{X_1} \star K_{31} + X_2 \star K_{32}$$

$$\frac{\delta \vec{\mathcal{E}}}{\delta \mathcal{X}} = \begin{pmatrix} \frac{\delta \mathcal{E}}{\delta Y_1} *_{full} K_{11} + \frac{\delta \mathcal{E}}{\delta Y_2} *_{full} K_{21} + \frac{\delta \mathcal{E}}{\delta Y_3} *_{full} K_{31} \\ \frac{\delta \mathcal{E}}{\delta Y_1} *_{full} K_{12} + \frac{\delta \mathcal{E}}{\delta Y_2} *_{full} K_{22} + \frac{\delta \mathcal{E}}{\delta Y_3} *_{full} K_{32} \end{pmatrix}$$

$$\frac{\delta \vec{\mathcal{E}}}{\delta \mathcal{X}} = \mathcal{K}^T *_{full} \frac{\delta \vec{\mathcal{E}}}{\delta \mathcal{Y}}$$

Источники

https://www.youtube.com/watch?v=Lakz2MoHy6o

https://youtube.com/playlist?list=PLZHQObOWTQDNU6R1_67000Dx_ZCJB-3pi&si=v-cUtG-ELLVHjlRa