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Computational Physics

Numerical Integration Techniques for Sinusoidal Functions

1. Introduction

Numerical integration, or quadrature, is a fundamental technique used to estimate definite integrals when analytical methods are intractable. This report investigates the application of three prominent numerical integration methods:

- **Trapezoidal Rule**
- **Simpson's Rule**
- **Gaussian Quadrature (4-point)**

Each method is applied to compute the definite integral of f(x) = sin(x) over a user-specified interval [a,b][a, b]. We then compare their accuracy against the known exact integral of

$\int_0^\pi \sin(x)\,dx = 2.$

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2. Program Overview

The C++ program consists of the following components:

2.1 Function Definition

The function being integrated is:

```
double f(double x) {
   return sin(x);
}
```

2.2 Trapezoidal Rule

The trapezoidal rule approximates the area under the curve by summing trapezoids:

Integral≈ $$\approx \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right]$$

Implemented in:

```
double trapezoidal(double a, double b, int n) { ... }
```

2.3 Simpson’s Rule

Simpson’s rule uses parabolic segments to approximate the function:

$$\text{Integral} \approx \frac{h}{3}\left[f(a) + 4\sum f(\text{odd } x_i) + 2\sum f(\text{even } x_i) + f(b)\right]$$

Implemented in:

```
double simpson(double a, double b, int n) { ... }
```

2.4 Gaussian Quadrature (4-point)

Gaussian Quadrature computes the weighted sum of function values at strategically chosen points:

$$\text{Integral} \approx \sum_{i=1}^{n} c_i f\left(\frac{b-a}{2}x_i + \frac{b+a}{2}\right)$$

Implemented in:

```
double gaussianQuadrature(double a, double b) { ... }
```

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3. Input and Execution

The user is prompted to enter:

- Lower limit a
- Upper limit b
- Number of subintervals n

Sample output for inputs a = 0, b = π, n = 4:

```
Trapezoidal rule result: 1.8961
Simpson rule result: 2.0046
Gaussian Quadrature result: 2
Exact result: 2.0
Difference for Trap.: 0.1039
Difference for Simpson's.: -0.0046
```

4. Analysis and Discussion

4.1 Accuracy

Method	Approximate Value	Error (vs exact = 2)
Trapezoidal Rule	~1.8961	~0.1039
Simpson’s Rule	~2.0046	~0.0046
Gaussian Quadrature	2.0000	0.0000

- **Trapezoidal Rule** underestimates the integral due to linear approximation.
- **Simpson’s Rule** yields much higher accuracy, due to its quadratic interpolation.
- **Gaussian Quadrature** is extremely accurate with just 4 points because it is optimized for polynomials and well-behaved functions like sine.

4.2 Convergence

- **Trapezoidal** converges linearly: increasing `n` improves accuracy, but slowly.
- **Simpson** converges faster (order O(h^4)), requiring fewer subintervals for good accuracy.
- **Gaussian Quadrature** doesn't rely on subintervals and offers high precision with few evaluations, particularly effective for smooth integrands.

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5. Conclusion

The study effectively demonstrates the performance of different numerical integration methods:

- For rough estimates or where computational simplicity is needed, **Trapezoidal Rule** suffices.
- For higher accuracy with moderate complexity, **Simpson's Rule** is ideal.
- For precision and efficiency, especially with smooth functions, **Gaussian Quadrature** is superior.

This program is useful for educational purposes to compare and understand the strengths and limitations of numerical integration methods in a hands-on way.

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6. Recommendations for Improvement

- Extend the code to handle user-defined functions using function pointers or lambda functions.
- Allow dynamic Gaussian point selection (2, 3, 5, etc.) for flexible precision.
- Include graphical output for visual comparison using external libraries.

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