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Portfolio implications of systemic crises ☆

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Abstract

Systemic crises can have grave consequences for investors in international equity markets, because they cause the risk-return trade-off to deteriorate severely for a longer period. We propose a novel approach to include the possibility of systemic crises in asset allocation decisions. By combining regime switching models with Merton [Merton, R.C., 1969. Lifetime portfolio selection under uncertainty: The continuous time case. Review of Economics and Statistics 51, 247–257]-style portfolio construction, our approach captures persistence of crises much better than existing models. Our analysis shows that incorporating systemic crises greatly affects asset allocation decisions, while the costs of ignoring them is substantial. For an expected utility maximizing US investor, who can invest globally these costs range from 1.13% per year of his initial wealth when he has no prior information on the likelihood of a crisis, to over 3% per month if a crisis occurs with almost certainty. If a crisis is taken into account, the investor allocates less to risky assets, and particularly less to the crisis prone emerging markets.

JEL classification: G11; G15; F30; C51

Keywords: Asset allocation; Systemic risk; International finance; Regime switching models

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1. Introduction

Systemic crises can wreak havoc on financial systems, making these crises an important issue for study. De Bandt and Hartmann (2000) and Dow (2000) provide excellent surveys on the characteristics and causes of systemic crises for the different financial markets. In this article we focus on the consequences of systemic crises for investors in international equity markets. International investors suffer from the deterioration of the risk and return characteristics, as systemic crises exhibit a sharp drop in returns, an upswing in volatilities and a rise of the correlations between financial markets, all on a global scale. Evidence of this behavior has been based on the October 1987 stock market crash, and the crises that originated from the emerging markets in the 1990s (e.g. the Mexican crisis of 1994, the Asian crisis of 1997 and the Russian crisis of 1998).[2] Due to their irregular and rare occurrence, standard models that investors use to support their asset allocation decisions typically fail to account for systemic crises, resulting in suboptimal allocations.

Das and Uppal (2004) conclude that the portfolio implications of systemic crises are limited. However, their approach implies that a systemic crises is a short-lived event that is hardly persistent, which is in contrast with recent crises and their aftermaths that lasted several months. If the risk-return trade-off deteriorates for a longer period, the impact of systemic crises will be more severe. To include possible persistence, we investigate their impact by means of a regime switching model in the style of Ang and Bekaert (2002), which we combine with optimal portfolio construction as set out by Merton (1969, 1971). This approach allows us to model the behavior of asset returns on a regime by regime basis, making it both simple and flexible. Formulating and solving the asset allocation problem in continuous time ensures analytical tractability.

We distinguish among two strategies that a utility-maximizing investor can adopt to solve his asset allocation problem: a crisis conscious and a crisis ignorant strategy. The crisis conscious strategy includes a systemic crises as a distinct regime in which all markets encounter a shock, while the crisis ignorant strategy does not. For both strategies, we construct optimal portfolios. By comparing the portfolios we assess the implications and importance of a systemic crisis. For a US-based global investor, who can invest in stock markets in the US, Europe, Japan, Hong Kong, Thailand, Korea and Brazil, and a riskless asset, we find that the crisis conscious strategy leads to a reduction of the investments in risky assets and a shift to countries less prone to a crisis. A small probability of a crisis (of say 5%) already causes these adjustments, and they quickly become more pronounced if the probability increases. Ignoring a crisis is costly, as the investor requires a certainty equivalent return of 1.13% per year as a compensation if he has no information on the ex-ante probability of a crisis. If the investor knows with almost certainty that a crisis occurs, this compensation can easily exceed 3% per month.

This paper extends the analysis of Das and Uppal (2004) in three important aspects. First, our model is better able to capture the persistence of a crisis, because we include a systemic crisis as a distinct regime in a regime switching model, while they incorporate

[2] Roll (1988, 1989), Bertero and Mayer (1990) and King and Wadhwani (1990) studied the October 1987 crash, Calvo and Reinhart (1996) the Mexican crisis, Kaminsky and Schmukler (1999) and Baig and Goldfajn (1999) the Asian crisis, and Kaminsky and Reinhart (2002) the Asian and Russian crises.

it by adding a perfectly correlated jump to a geometric Brownian motion.[3] Second, we analyze the impact of systemic crises in a dynamic setting, which can adapt to changes in the behavior of asset prices. Third, our model without a crisis is more realistic, as the model proposed by Das and Uppal implies a normal distribution with a constant mean and variance. We also extend the work of Liu et al. (2003) by showing the effects of systemic crises on diversification, while their model is limited to a univariate setting with one risky asset only. Our finding that persistence is an important aspect of systemic crises is consistent with their results. Our approach is complementary to Ang and Bekaert (2002, 2004), who consider international asset allocations in a regime-switching framework, as we use a similar framework. Because of the severity of the crisis regime, we find larger effects of regime switches on diversification. As another extension to their work we show how the resulting allocation problem can be solved in continuous time.

In a broader sense our study can be seen as an investigation of the hypothesis that diversification advantages fail to be realized due to increasing correlations during market downturns, such as systemic crises. This claim has been put forward by various authors,[4] but it is not clear how strong this effect is. Ang and Chen (2002) conclude that the costs of ignoring increasing correlations during bear markets are substantial, but Ang and Bekaert (2002) find that diversification advantages remain present. In our approach, an increase in correlations is inherent in a crisis. If the probability with which a crisis hits increases, diversification possibilities erode rapidly and cause large divestments. If the investor faces short sales constraints, he completely withdraws from equity markets.

The outline of the paper is as follows. In Section 2 we discuss how the crisis conscious and crisis ignorant strategies produce optimal portfolios and how the portfolios can be compared. Section 3 presents the actual design of the study, including the data. We discuss the estimation results in Section 4, and derive and compare the allocations produced by the different strategies in Section 5. Section 6 concludes.

2. A crisis conscious and a crisis ignorant strategy

The investor can adopt two strategies to construct an optimal portfolio: a crisis conscious and a crisis ignorant strategy. Both strategies contain a model for the return process and a formulation of the asset allocation problem as their main ingredients. In both cases, a Markov regime switching model describes the return process because of its flexibility to capture heteroskedasticity (see Hamilton and Susmel, 1994) and fat tails (see Timmermann, 2000). The difference between the strategies is the presence of distinct crisis regimes in the model employed by the crisis conscious strategy. The model in the crisis ignorant strategy is as a restricted version of that in the crisis conscious strategy.

We assume that the investor formulates and solves his asset allocation problem in an expected utility, continuous time framework. Because of the continuous time approach,

[3] In a related paper, Das and Uppal (2003) also consider a regime switching model to allow for stronger persistence and conclude that their main conclusions are unaffected. However, the degree of persistence they consider is fairly low compared to our analysis. For higher levels of persistence systemic crises are likely to have more severe consequences, which is also indicated by the result in Das and Uppal (2003) that the effects of systemic crises are increasing and convex for increasing levels of persistence.

[4] See, for instance, Loretan and English (2000), Longin and Solnik (2001), Campbell et al. (2002), Ang and Chen (2002), Ang and Bekaert (2002) and Campbell et al. (2003).

the problem has a closed-form solution, contrary to the numerical approach of Ang and Bekaert (2002). The different strategies lead to different allocations. Because the investor constructs his asset allocation under expected utility, we can determine the economic importance of those differences by calculating the certainty equivalent return needed to compensate the investor for incorrectly using the crisis ignorant strategy.

In the first subsection, we discuss the models for the return process. In the next one, we formulate and solve the investor's asset allocation problem. Section 2.3 shows how the continuous time return process in the model of the asset allocation problem should be constructed to make it consistent with the predictions resulting from the Markov regime switching models. In Section 2.4 we derive the compensation that the investor requires for incorrectly adopting the crisis ignorant strategy. We conclude by showing how the differences between the crisis conscious and crisis ignorant allocations can be explained.

2.1. Regime switching models for the return process

We start with the more general model that is used in the crisis conscious strategy, and consider the restricted version in the crisis ignorant strategy subsequently. The general model for the return process consist of several regimes. The behavior of the return process in a regime corresponding to a normal period is made up of basic components, while its behavior in a regime corresponding with a systemic crisis contains both basic and crisis components. By choosing this setup, a systemic crisis can be clearly interpreted as a simultaneous shock to all assets that comes on top of their normal behavior. We assume that the investor can invest in n assets.

First, consider the set of states that apply to the return process. We assume that each asset i's basic return component can be in a regime Q_i from a set of K regimes. For the crisis component two regimes Q_c are available: presence ($Q_c = 1$) and absence ($Q_c = 0$), and since the crisis is systemic the crisis regime applies to all assets. Consequently, the state that applies to the joint returns, $\tilde{Q}$, is completely defined by the combination of each asset's basic regime and the crisis regime:[5]

$$\tilde{Q} \equiv (Q_1, Q_2, \ldots, Q_n, Q_c). \tag{1}$$

We use Q to denote the combination of basic regimes, $Q \equiv (Q_1, Q_2, \ldots, Q_n)$. The sets $\mathbb{Q}$ and $\tilde{\mathbb{Q}}$ collect all possible state vectors Q and $\tilde{Q}$, respectively. The actually prevailing state will never be known with certainty. Instead, each state prevails with a certain probability, inferred from the data.

The return vector can be split into a basic and a crisis component. For each state $Q \in \mathbb{Q}$ the basic component x is a normally distributed random vector, with a state-specific mean μ_Q and variance matrix Ω_Q. The marginal distribution of asset i's basic component depends only on the regime Q_i that applies. The crisis component consists of a shock, represented by a univariate random variable x_c, to which each asset has a specific sensitivity δ_i. Following Das and Uppal (2004) and Liu et al. (2003) we assume that the shock has a normal distribution, with mean μ_c and variance ω_c. Combining the two components gives the return vector:

$$r = x + Q_c x_c \delta, \quad Q_c \in \{0, 1\}, \tag{2}$$

[5] We use the expression state to refer to a combination of the basic regimes and the crisis regimes.

where δ is the vector of sensitivities. Conditional on the state $\tilde{Q} \in \tilde{\mathbb{Q}}$, the return is normally distributed, being the sum of two (conditionally) normally distributed variables. Under the assumption that the shock and the basic component are independent, the mean vector $\mu_{\tilde{Q}}$ and variance matrix $\Omega_{\tilde{Q}}$ of the return can be written as

$$\mu_{\tilde{Q}} = \mu_Q + Q_c \mu_c \delta, \tag{3}$$

$$\Omega_{\tilde{Q}} = \Omega_Q + Q_c \omega_c \delta \delta'. \tag{4}$$

Because we want the shock to have the same direction for each asset, we require $\delta_i \geqslant 0$ for each i. Consequently, each variance and covariance term will increase if $Q_c = 1$.[6]

The transition probabilities are constant over time, and we impose a specific structure to capture volatility spill-over effects, being the tendency of high volatility in one asset's return to spread to other assets. These effects are widespread and important.[7] Let π^{ab} denote the conditional probability of a switch to state $\tilde{Q}^a$, given that the current state is $\tilde{Q}^b$. First, we impose a structure based on the crisis regime.

- If a crisis occurs neither in the current state ($Q_c^b = 0$) nor in the destination state ($Q_c^a = 0$), we model π^{ab} as the product of the marginal conditional probabilities $\pi_i(Q_i^a|\tilde{Q}^b)$ and the conditional probability that a crisis does not occur, given the current state $\tilde{Q}^b$. Here, π_i^{ab} gives the probability that asset i switches to regime Q_i^a, given that the current state is $\tilde{Q}^b$. The dependence on the state $\tilde{Q}^b$ instead of the asset-specific regime Q_i^b introduces dependence across assets and enables the incorporation of volatility spill-over effects. We use a multinomial logistic model to model this dependence, which we discuss in Appendix A.[8] The alternative of free parameters would lead to an exploding number of free parameters equal to $2K^n \cdot (2K^n - 1)$.
- If the assets enter a crisis regime (so $Q_c^a = 1$ and $Q_c^b = 0$), the basic regime processes switch to the one with the highest volatility. This restriction imposes that global stress triggers local stress.
- For the case that a crisis remains, the same restriction as in the previous case applies. It prohibits illogical switches from high volatility regimes to lower volatility regimes, while a crisis remains present.
- If the assets leave the crisis regime ($Q_c^a = 0$ and $Q_c^b = 1$), the basic regime processes remain in the highest volatility regimes in the next period. After that, they can switch to other regimes. This restriction captures a gradual cooling down of assets after a crisis.

The crisis transition probabilities are independent of the basic regimes. This leads to two parameters for the crisis transition probabilities: π_c^{10}, the probability that a crisis occurs, and π_c^{11}, the probability that a crisis remains. If we assume without loss of generality that the regimes are ordered in ascending order of volatility, we can represent our model as follows:

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[6] The correlation will rise if the relative increase in covariance exceeds the product of the relative increases in volatilities. This condition will generally be satisfied for correlations not too close to 1. Because the change in correlation is completely due to the occurrence of a crisis, our model implies interdependence of assets but not contagion (see Forbes and Rigobon, 2002).

[7] See Hamao et al. (1990), Bekaert and Harvey (1997), Ng (2000), Edwards and Susmel (2001), Lee et al. (2004) and Baele (2005).

[8] Bae et al. (2003) use a related model to investigate contagion.

$$\pi^{ab} = \begin{cases} \pi_1^{ab} \cdot \pi_2^{ab} \ldots \pi_n^{ab} \cdot (1 - \pi_c^{10}) & \text{if } Q_c^a = 0,\ Q_c^b = 0, \\ \pi_c^{10} & \text{if } Q_c^a = 1,\ Q_c^b = 0,\ \forall i\ Q_i^a = K, \\ \pi_c^{11} & \text{if } Q_c^a = 1,\ Q_c^b = 1,\ \forall i\ Q_i^a = K, \\ 1 - \pi_c^{11} & \text{if } Q_c^a = 0,\ Q_c^b = 1,\ \forall i\ Q_i^a = K, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

The crisis ignorant strategy imposes the restriction that transitions to $Q_c^a = 1$ have zero probability, i.e. $\pi_c^{10} = 0$. Consequently, the crisis ignorant strategy leads to different inferences and forecasts about the prevailing regime. Also, if the investor incorrectly follows an ignorant strategy and estimates the parameters of the process, the parameter estimates are likely to be biased.

2.2. The asset allocation problem

The investor is risk averse and maximizes his utility over terminal wealth W_{T}. We assume he has a power utility function:

$$U(W_{\mathrm{T}}) = W_{\mathrm{T}}^{1-\gamma}/(1-\gamma), \quad \gamma > 0,\ \gamma \neq 1, \tag{6}$$

where γ is the investor's coefficient of relative risk aversion.[9] To focus on the effect of a systemic crisis on asset allocation, we do not allow intermediate consumption. As such, our analysis is comparable to Ang and Bekaert (2002), Liu et al. (2003) and Das and Uppal (2004). The investor can trade in continuous time. At each point in time t he chooses to invest proportions of his wealth in the n risky assets, collected in the vector ϕ_t and the remaining part $1 - \phi_t' \iota_n$ in the riskless asset in order to maximize expected utility.

We assume that the investor has an initial endowment W_0. This assumption and a process for the asset prices enables us to derive the investor's self-financing constraint, which describes the dynamics of the wealth process. The returns follow an Itô process $\mathrm{d}r = \mu(r,t)\mathrm{d}t + \Lambda(r,t)\mathrm{d}Z$, where $\mu(r,t)$ is a vector of the instantaneous drift rates, $\Lambda(r,t)$ is a lower triangular $n \times n$ matrix, and $\mathrm{d}Z$ is a vector of n independent Wiener processes. Consequently, the instantaneous variance rate $\Omega(r,t)$ is given by $\Omega(r,t) = \Lambda(r,t)\Lambda(r,t)'$. Below we describe a specific function for the drift and variance rate that makes them consistent with the predictions of the regime switching models of the previous section. For notational convenience we will drop the time and return dependence of μ and Λ. After applying Itô's lemma to find the price processes we end up with the self-financing condition:

$$\frac{\mathrm{d}W}{W} = r_{\mathrm{f}}\,\mathrm{d}t + \phi'\alpha\,\mathrm{d}t + \phi'\Lambda\,\mathrm{d}Z, \tag{7}$$

where $\alpha \equiv \mu + \frac{1}{2}\mathrm{diag}(\Omega) - r_{\mathrm{f}}\iota_n$,[10] r_{f} is the risk-free rate, and $\mathrm{diag}(\Omega)$ denotes a vector containing the diagonal elements of Ω.

The asset allocation problem can be solved using standard stochastic control techniques.[11] We show in Appendix A that the optimal portfolio is given by

[9] For $\gamma = 1$ the utility function is defined as log utility $U(W_{\mathrm{T}}) = \ln W_{\mathrm{T}}$.

[10] The expression can be interpreted as an excess, arithmetic mean return.

[11] See for example Léonard and Van Long (1992).

$$\phi^* = \gamma^{-1}\Omega^{-1}\alpha = \gamma^{-1}\Omega^{-1}\left(\mu + \frac{1}{2}\mathrm{diag}(\Omega) - r_{\mathrm{f}}\iota_n\right). \tag{8}$$

Though this expression has the same structure as the solution to a standard mean–variance optimization problem, ϕ^* depends on time and the observed returns via μ and Ω. This expression also applies to the log-utility investor.

2.3. The Itô process for returns

Brigo (2002) describes a way to derive continuous time processes whose corresponding density at a certain point in time is a mixture of densities from the same family.[12] This approach has two advantages. First, it facilitates the use of the powerful techniques developed for continuous time finance. Second, the drift rates and variance rates that characterize the Itô processes incorporate the different regimes directly, so no extra state variables for the different regimes are introduced. Consequently, the regimes are implicitly present in the portfolio optimization (i.e. in the parameters of the Itô process), and do not lead to regime-specific Itô processes.

Since the distribution of $r_{\tau+1}$ conditional on its filtration is a mixture of normal distributions (see Hamilton, 1994, Chapter 22), we can apply a multivariate extension of Theorem 2 in Brigo (2002). Consequently, the Itô process starting at $t_0 = \tau$ with $r_{t_0} = 0$ has a mixture density at time $\tau + 1$ which corresponds with the mixture model implied by the regime switching model, if the instantaneous drift rate $\mu(r, t)$ and instantaneous variance rate $\Omega(r, t)$ are given by

$$\mu(r,t) = \sum_{\tilde{Q}\in\tilde{\mathbb{Q}}} \pi(\tilde{Q}, r, t)\mu_{\tilde{Q}}, \tag{9}$$

$$\Omega(r,t) = \sum_{\tilde{Q}\in\tilde{\mathbb{Q}}} \pi(\tilde{Q}, r, t)\Omega_{\tilde{Q}} \tag{10}$$

with

$$\pi(\tilde{Q}, r, t) = \frac{\xi_{\tau+1|\tau}(\tilde{Q}) \cdot f(r_t; \mu_{\tilde{Q}}(t-\tau), \Omega_{\tilde{Q}}(t-\tau))}{\sum_{\hat{Q}\in\tilde{\mathbb{Q}}} \xi_{\tau+1|\tau}(\hat{Q}) \cdot f(r_t; \mu_{\hat{Q}}(t-\tau), \Omega_{\hat{Q}}(t-\tau))}, \tag{11}$$

where $\tau < t \leqslant \tau + 1$, and $\xi_{\tau+1|\tau}(\tilde{Q}) = Pr(\tilde{Q}_{\tau+1}|\mathscr{F}_\tau)$ gives the forecast probability that state $\tilde{Q}$ is prevailing at time $\tau + 1$. $\Lambda(r, t)$ can then be found by applying a Cholesky decomposition to $\Omega(r, t)$. For $t = \tau$, Eq. (11) reduces to $\pi(\tilde{Q}, 0, \tau) = \xi_{\tau+1|\tau}(\tilde{Q})$.

The drift and variance rate constructed by Eqs. (9)–(11) have an appealing interpretation. The drift and variance are a probability weighted average of the mean and variance parameters for the different states. These probabilities have a clear interpretation as inference probabilities (see Hamilton, 1994, Eq. (22.4.5). that are updated with a Bayesian rule, with $\xi_{\tau+1|\tau}(\tilde{Q}) = Pr(Q_{\tau+1}|\mathscr{F}_\tau)$ as prior probability for the prevailing regime and $\pi(\tilde{Q}, r, t) = Pr(Q_{\tau+1}|r_t, \mathscr{F}_\tau)$ as its posterior probability.

[12] Applications of this technique can be found in Alexander and Narayanan (2001), Alexander (2004) and Brigo and Mercurio (2002).

2.4. Comparing portfolios

Though the expression for the optimal portfolio Eq. (8) is the same for both the crisis conscious and the crisis ignorant strategy, the resulting portfolios (ϕ^c and ϕ^i, respectively), will differ because of differences in $\mu_{\tilde{Q}}, \Omega_{\tilde{Q}}$ and $\pi(\tilde{Q}, r, t)$. To assess the economic impact of the differences in portfolios we calculate the certainty equivalent return needed to compensate the investor for using the crisis ignorant strategy, when he should have used the crisis conscious one. Since the first does not take a crisis into account, the resulting portfolio will be suboptimal and yield lower utility. The certainty equivalent return shows by how much the initial wealth of the investor should be raised to compensate him for this utility loss and hence the cost of ignoring a crisis. We derive in Appendix A that the certainty equivalent return $\bar{r}$ needed for compensation equals

$$\bar{r} = [h(\phi^c) - h(\phi^i)](T - t), \tag{12}$$

where ϕ^c denotes the crisis conscious portfolio, ϕ^i denotes the crisis ignorant portfolio and $h(\phi) = \phi'\alpha - \frac{1}{2}\gamma\phi'\Omega\phi$. This expression depends only on the coefficient of risk aversion γ via the function h and the portfolio ϕ. It is easy to show that $h(\gamma^{-1}\phi) = \gamma^{-1}h(\phi)$. Consequently, the certainty equivalent return needed to compensate a power utility investor can be derived from the certainty equivalent return for the log-utility investor. Moreover, the certainty equivalent return is a linear function of the investor's horizon T.

The portfolio differences can stem from differences in the estimates for the basic regimes, the estimation effect, and the absence of crisis regimes, the crisis effect. An analysis of these differences provides insights into the importance of both sources. Suppose that the differences in parameter estimates explain just a small part of the changes in the optimal allocations. In that case, the crisis regime is the main driver of the portfolio adjustments. Alternatively, if the differences in parameter estimates explain most of the changes in optimal portfolios, the influence of the crisis itself is limited. The observations that belong most likely to the crisis regime cause outlier problems in the crisis ignorant case.

In order to disentangle the differences between the optimal allocations produced by the crisis conscious and ignorant strategies we introduce a myopic strategy. This strategy uses the same estimates as the crisis conscious strategy, but excludes a crisis regime in the forecasts it makes, as it constructs only forecasts for the basic states:

$$\xi^m_{\tau+1|\tau}(Q) = \xi_{\tau+1|\tau}(Q, Q^c = 0) + \xi_{\tau+1|\tau}(Q, Q^c = 1), \quad Q \in \mathbb{Q}. \tag{13}$$

The myopic strategy produces an allocation ϕ^m. We interpret the differences between the myopic and the crisis ignorant strategy $\phi^e \equiv \phi^m - \phi^i$ as the estimation effect, and the differences between the crisis conscious and myopic strategy as the crisis effect, $\phi^s \equiv \phi^c - \phi^m$. We also calculate $h(\phi^m)$, and derive the economic importance of the estimation effect as $\bar{r}^e = [h(\phi^m) - h(\phi^i)](T - t)$ and the importance of the crisis effect as $\bar{r}^s = [h(\phi^c) - h(\phi^m)](T - t)$.

3. Design of the analysis

Central in our analysis of the impact of systemic crises is a US investor who wants to diversify his portfolio internationally. He can invest worldwide and does not only consider the developed markets US, Europe, Japan and Hong Kong, but also the emerging markets

Table 1
Descriptive statistics

	US	Europe	Japan	Hong Kong	Thailand	Korea	Brazil
(a) Univariate statistics							
Mean	0.50	0.53	0.31	0.69	0.31	0.41	0.32
Volatility	4.38	4.75	6.48	9.31	10.22	10.54	15.42
Skewness	−0.76	−0.72	0.07	−1.08	−0.44	0.36	−0.49
Kurtosis	5.98	4.78	3.48	8.43	6.01	5.81	6.01
Minimum	−24.45	−21.65	−22.18	−57.58	−41.88	−41.37	−84.79
Maximum	12.05	12.69	21.04	28.37	38.14	53.17	44.84
(b) Correlation matrix							
US	1	0.64	0.30	0.42	0.32	0.28	0.21
Europe	0.64	1	0.48	0.50	0.31	0.25	0.25
Japan	0.30	0.48	1	0.30	0.25	0.37	0.15
Hong Kong	0.42	0.50	0.30	1	0.39	0.21	0.21
Thailand	0.32	0.31	0.25	0.39	1	0.39	0.13
Korea	0.28	0.25	0.37	0.21	0.39	1	0.12
Brazil	0.21	0.25	0.15	0.21	0.13	0.12	1

This table provides descriptive statistics for the data set consisting of the monthly excess gross returns (in %) for the MSCI US, MSCI Europe, MSCI Japan, MSCI Hong Kong, IFC Thailand, IFC Korea and IFC Brazil indexes, running from January 1976 to December 2004. Panel (a) presents univariate statistics, panel (b) shows the correlation matrix.

Thailand, Korea and Brazil, which can extend diversification opportunities.[13] We represent each market by an index. We assume that for each country 2 regimes can be distinguished. The estimation results are used to construct allocations. Based on the differences between the crisis conscious and crisis ignorant allocations we determine the impact of systemic crises.

We base the analysis on monthly returns, mainly because monthly data are available with the longest history. A systemic crisis is a rare event, necessitating a long history to get an accurate estimate of its probability. A longer horizon also improves the estimate for the mean returns in the different regimes. Each developed market is approximated by its corresponding gross return index from Morgan Stanley Capital International (MSCI). For the emerging markets we use the gross return indexes provided by Standard & Poors/International Finance Corporation (IFC), both provided by DataStream. We use the start of the IFC indexes, December 31, 1975 as a starting point for our analysis and collect the index values in dollars till December 31, 2004, resulting in 348 returns. We construct excess returns by subtracting the 1-month T-bill return from Ibbotson Associates, Inc.

The summary statistics in Table 1 show the familiar picture of small, positive means, non-zero skewness and fat tails. Generally, the minimum exceeds the maximum in absolute value. The correlation matrix shows low levels of correlation, particularly for the emerging markets, implying the presence of diversification possibilities. However, Hong Kong, Thailand, Korea and Brazil may be less attractive due to their relatively high levels of volatility.

[13] Early studies (see e.g. Harvey, 1995) find significant diversification opportunities, but more recent studies show these may be less when transaction costs and investment constraints are taken into account (see Bekaert and Harvey, 2003, for a discussion).

The regime switching models we propose in Section 2 belong to the standard regime switching models as discussed in Hamilton (1994). We use the expectation maximization algorithm (see Hamilton, 1990) to estimate the parameters in the models. To ensure that the estimated covariance matrix is positive definite, we assume that the correlation matrix is independent of the basic state vectors.

The expression for the optimal portfolio in Eq. (8) defines an asset allocation strategy in continuous time. This means that we can derive the evolution of portfolios over time for a given price path. We construct paths of portfolios based on daily prices. The data we use are the daily prices of the mentioned gross return indexes in dollar terms, also gathered from DataStream. To keep the daily and monthly data sets consistent, we use the 1-month T-bill rate that was prevailing at the beginning of the month to compute the daily excess returns.

The portfolio at a given day of the month reflects two sources of information: a prior probability based on the information at the beginning of the month, and the information present in the returns observed until that day. The second source of information is used to update the prior probability to a posterior probability as in Eq. (11). We interpret the prior probability as the outcome of an investor's thorough analysis of the likelihood of a state. Because of its thoroughness such an analysis is conducted at a limited frequency, i.e. once per month. We represent the outcomes of the analysis by a Markov chain. The prior probability can be regarded as an informative prior, because it is based on all available information at the moment it is determined. We will also consider the allocation that results if the investor does not use the observed stock price path to determine the prior probabilities. Instead, the investor uses unconditional probabilities for each regime, which are only based on the transition matrices. The results from this analysis can serve as a benchmark and can be compared to the results in Das and Uppal (2004).

4. Estimation results

We start the discussion of the estimation results with the estimates for the parameters of the marginal normal distributions presented in Table 2. The crisis regime that distinguishes the two strategies contains a shock with an estimated mean of −0.63% and a volatility of 1.54%. The shock has been normalized such that the US has a sensitivity of 1. The other countries (except Europe) are more sensitive to the shock. In particular the emerging countries Korea and Thailand are more than 10 times as sensitive as the US. The considerable difference in log likelihood values[14] of 19.0, provides evidence in favor of the addition of a crisis regime. However standard statistical tests cannot be used, because several parameters are not identified under the null hypothesis.[15] In both models, the two basic regimes for each asset can be distinguished by their volatility levels, as reported by Ramchand and Susmel (1998), Ang and Bekaert (2002) and Graflund and Nilsson (2003). In the remainder we will therefore use the terms low volatility regime and high volatility regime to distinguish between the regimes.

Combining the estimates for the shock and the basic high volatility regimes yields the crisis regime, which exhibits a sharp drop in expected returns and an increase in volatili-

[14] The log likelihood values for the models without and with a crisis equal −8029.5 and −8010.5, respectively.
[15] Hansen (1992)'s method to formally test whether the addition of a regime is a significant improvement is less attractive, as its complexity grows exponentially in the number of parameters.

Table 2
Univariate parameter estimates

		Crisis ignorant		Crisis conscious		
		Low	High	Low	High	Crisis
US	μ	0.90	0.35	0.95	0.47	−0.16
	$\sqrt{\omega}$	2.47	4.99	2.21	4.73	4.97
						1.00
EU	μ	1.10	0.23	1.17	0.26	−0.30
	$\sqrt{\omega}$	3.09	5.46	3.07	5.35	5.52
						0.88
Japan	μ	0.44	0.03	0.20	0.74	−1.46
	$\sqrt{\omega}$	5.14	8.29	4.92	8.26	9.83
						3.47
Hong Kong	μ	0.93	0.57	0.92	0.74	−2.15
	$\sqrt{\omega}$	6.00	13.34	5.77	12.98	14.75
						4.55
Thailand	μ	0.80	−0.53	−0.22	1.51	−7.03
	$\sqrt{\omega}$	6.29	15.28	5.24	11.44	23.62
						13.46
Korea	μ	0.34	1.28	0.19	1.94	−6.55
	$\sqrt{\omega}$	7.26	15.69	6.56	12.44	24.04
						13.40
Brazil	μ	0.96	−0.73	0.86	−0.01	−2.43
	$\sqrt{\omega}$	9.83	20.91	8.49	19.49	20.35
						3.81
	μ_c					−0.63
	$\sqrt{\omega_c}$					1.54

This tables reports the estimates for the mean parameters (μ) and volatility parameters ($\sqrt{\omega}$) of the marginal distributions of the excess monthly equity return (in %) for the US, Europe, Japan, Hong Kong, Thailand, Korea and Brazil under the low and high volatility regimes. The first two columns present the estimates for the crisis ignorant strategy; the second two for the crisis conscious strategy. The parameters for the crisis conscious strategy also contain estimates for the shock: a mean (μ_c) and variance ($\sqrt{\omega_c}$). The last column contains the estimates for the sensitivity to a systemic crisis (δ) and the resulting mean and volatility. The sensitivity of the US market has been normalized to 1.

ties. Because of their large sensitivity to a crisis, these effects are most pronounced for emerging markets, possibly explaining the fat tails reported by Susmel (2001). As in Ang and Chen (2002) and Forbes and Rigobon (2002), correlations in the crisis regime are also higher (see Table 3). Obviously, the risk-return trade-off for each asset deteriorates, while diversification possibilities become less. Consequently, risky assets become less attractive on a global scale. The exact consequences become clear in the next section.

The addition of a crisis regime has important effects on the other estimates. Because of decreasing volatility estimates, the risk within each regime becomes less, particularly in the high volatility regime. Moreover, the means in the high volatility regimes increase considerably, indicating that the few crisis observations differ substantially from normal high volatility periods.

Table 4 presents the estimates for the parameters of the logistic functions that we use to construct the regime transition probabilities (see Appendix A for more details). In total,

Table 3
Correlation estimates

	US	EU	JP	HK	TH	KO	BR
(a) Crisis ignorant							
US	1	0.65	0.29	0.46	0.28	0.27	0.23
EU	0.65	1	0.48	0.53	0.30	0.27	0.27
JP	0.29	0.48	1	0.32	0.18	0.38	0.15
HK	0.46	0.53	0.32	1	0.35	0.22	0.25
TH	0.28	0.30	0.18	0.35	1	0.24	0.11
KO	0.27	0.27	0.38	0.22	0.24	1	0.13
BR	0.23	0.27	0.15	0.25	0.11	0.13	1
(b) Crisis conscious							
US	1	0.64	0.30	0.47	0.33	0.26	0.24
EU	0.64	1	0.49	0.54	0.32	0.26	0.27
JP	0.30	0.49	1	0.32	0.15	0.35	0.16
HK	0.47	0.54	0.32	1	0.37	0.21	0.26
TH	0.33	0.32	0.15	0.37	1	0.21	0.10
KO	0.26	0.26	0.35	0.21	0.21	1	0.14
BR	0.24	0.27	0.16	0.26	0.10	0.14	1
(c) Crisis regime							
US	1	0.67	0.41	0.54	0.42	0.39	0.31
EU	0.67	1	0.53	0.57	0.37	0.34	0.32
JP	0.41	0.53	1	0.49	0.53	0.62	0.28
HK	0.54	0.57	0.49	1	0.57	0.50	0.36
TH	0.42	0.37	0.53	0.57	1	0.80	0.30
KO	0.39	0.34	0.62	0.50	0.80	1	0.32
BR	0.31	0.32	0.28	0.36	0.30	0.32	1

This table reports estimates for the correlations between the different countries for the crisis ignorant (panel a) and crisis conscious strategy (panel b) and the resulting correlations for the crisis regime (panel c). The correlations are assumed to be independent of the basic regimes.

we have $2 \cdot 7^2 = 98$ parameters from which we construct the 128×128 ($=2^7$) basic transition matrix. The diagonal elements give the estimates that correspond with no regime switch for a country, given that the other countries are currently in their low volatility regimes. The off-diagonal elements give the volatility spill-over estimates, which are restricted to be negative (positive) for switches from the low (high) volatility regimes. Because of these restrictions, volatility spill-over effects increase the probability that countries are in their high volatility regimes. As an example, consider the Hong Kong market, which has a probability of remaining in its low volatility regime, given that the other markets are also in their low volatility regimes equal to $e^{3.75}/(1 + e^{3.75}) = 0.98$. However, if the US is in its high volatility regime, this probability decreases to $e^{3.75-0.48}/(1 + e^{3.75-0.48}) = 0.96$.

The high values for the positive diagonal elements (exceptions are the high volatility regimes for Japan and Korea) in Table 4 indicate that each regime is in itself strongly persistent. Volatility spill-over effects mainly affect the probability of a switch from the low volatility to the high volatility regimes. Third, volatility spill-over effects are mainly present among the developed markets, and from emerging markets to developed markets. Other studies finds volatility spill-over effects from the US and Japan to other Asian markets (see Bekaert and Harvey, 1997; Ng, 2000; Lee et al., 2004), but this may be due to differences in the applied methods.

Table 4
Estimates for the multinomial model for the regime transition probabilities for the crisis ignorant (panels a and b) and the crisis conscious strategy (panels c and d)

	US	EU	JP	HK	TH	KO	BR
(a) Crisis ignorant: low volatility regimes							
US	39.90	−87.42	−37.81	−0.48	0.00	0.00	−15.53
EU	−37.40	106.71	0.00	0.00	0.00	−16.33	0.00
JP	0.00	0.00	120.41	0.00	0.00	0.00	−0.16
HK	0.00	−36.40	−56.48	3.75	0.00	0.00	−1.15
TH	−38.54	0.00	−81.05	0.00	3.33	−0.08	0.00
KO	−609.63	−18.79	−0.18	0.00	0.00	18.74	0.00
BR	−37.85	−36.55	−47.41	−0.05	0.00	0.00	19.43
(b) Crisis ignorant: high volatility regimes							
US	1.53	0.00	16.52	0.00	0.06	0.00	0.00
EU	0.00	1.67	28.34	0.00	0.00	12.40	0.00
JP	0.00	0.00	−46.09	0.00	0.00	0.00	0.00
HK	0.00	0.00	0.00	2.22	0.00	19.54	0.00
TH	14.49	0.00	0.00	0.00	1.42	20.98	0.00
KO	0.00	0.00	0.00	1.34	13.64	−19.11	0.00
BR	0.00	0.00	31.20	0.00	0.00	0.00	3.45
(c) Crisis conscious: low volatility regimes							
US	3.97	−95.63	0.00	0.00	0.00	0.00	0.00
EU	0.00	137.55	0.00	0.00	0.00	−14.32	0.00
JP	0.00	0.00	32.26	0.00	0.00	0.00	0.00
HK	0.00	−27.92	−27.99	3.74	0.00	0.00	−2.60
TH	0.00	−26.31	−30.46	0.00	2.66	−2.56	−0.37
KO	0.00	−14.86	0.00	0.00	0.00	17.86	0.00
BR	0.00	−56.18	−2.18	−0.64	0.00	0.00	4.62
(d) Crisis conscious: low volatility regimes							
US	4.80	2.04	22.83	1.73	0.00	14.46	1.65
EU	0.00	−0.10	0.00	0.00	0.00	0.00	0.00
JP	0.00	0.00	−59.11	0.00	0.00	0.00	0.00
HK	0.00	0.00	11.75	1.14	0.00	12.33	0.00
TH	0.00	0.00	35.48	0.00	1.69	11.25	0.83
KO	0.00	0.00	0.00	0.00	13.72	−23.39	0.00
BR	0.00	0.00	25.46	0.00	0.00	0.00	1.51

The multinomial model is specified in Appendix A.2. Each column in the panels (a) and (c) (panels (b) and (d)) corresponds with remaining in the low (high) volatility regime. The diagonal elements corresponds with the case that all countries are in their low volatility regimes. The off-diagonal elements in a column give the volatility spillover effects that occur from the country corresponding with the row to the country corresponding with the column. The off-diagonal elements in the panels (a) and (c) (panels (b) and (d)) are restricted to be negative (positive).

A crisis has a probability of 0.0031 to occur, and is highly persistent, as indicated by the probability of 0.93 of remaining in the crisis regime. Unconditionally, if no prior information on the prevailing regimes is available, the crisis regime occurs with a probability of 0.045. Das and Uppal (2004) estimate a probability on a systemic jump of 0.0501 for the developed markets and of 0.0138 for the US with emerging markets, which is comparable to the unconditional probability we find. However, in their model, a systemic jump at this instant does not affect the probability of a jump in the next instant, which remains relatively low as a consequence.

5. Portfolio construction

Based on the estimates of the previous section we construct optimal portfolios for the crisis conscious and crisis ignorant strategies. The portfolios vary over time and depend on the filtration of the return processes, limiting the relevancy of an analysis of static portfolios. Instead, we concentrate on two situations. First, we consider the influence of a crisis when the investor uses uninformative forecast probabilities for the likelihood of the different regimes, or in other words has no prior knowledge on the state of the economy. In the second situation we analyze the effects of a crisis when the investor uses informative forecasts during the Asian crisis. We concentrate on October 1997, the month in which the Hong Kong market crashed. The second situation is the more interesting one, since we can observe how both strategies perform in a real-life situation. The first situation will be useful as a benchmark and enables a comparison with the results of Das and Uppal (2004) and Liu et al. (2003).

For both cases we conduct an analysis consisting of the same steps. We present and motivate the steps here, together with their main outcomes. We start by deriving and comparing the optimal allocations for the log-utility investor. Though the assumption of log utility is unrealistic due to its low degree of risk aversion, the log utility portfolio is popular in asset allocation studies. The optimal portfolio for a power utility investor is the log utility portfolio scaled by the inverse of his coefficient of relative risk aversion and an investment in the riskless asset to meet the budget constraint. We find that the crisis conscious strategy invests considerably less in risky assets. An inspection of only the risky part shows that the crisis conscious strategy shifts investment to countries less prone to a crisis.

Next, we determine the economic importance of the differences between the two strategies. We calculate the certainty equivalent return that the investor requires as a compensation for adopting the crisis ignorant strategy, when the crisis conscious strategy is appropriate. For the uninformative case, the costs of ignoring the possibility of a systemic crisis are limited, but large enough not to neglect them, particularly for longer horizons. When a crisis takes place with almost certainty, the certainty equivalent return rises substantially, also for more risk averse investors.

We conclude the analysis by investigating what can explain the differences between the crisis conscious and crisis ignorant portfolios: the differences in the parameters estimates for the basic component, or the hedging demand due to the possibility of a crisis. To accomplish this we use the myopic strategy introduced in Section 2.4. This strategy uses the same estimates for the basic part as the crisis conscious model, but excludes the crisis regime from the forecasts it makes. Consequently, the differences between the portfolios produced by the crisis ignorant and the myopic strategies are due to different parameter estimates for the basic component. On the other hand, the differences between the myopic strategy and the crisis conscious strategy stem solely from the crisis regime. These latter differences have the clear interpretation of a hedging demand. We find that the investor hedges against a crisis by taking a long position in the US, Europe, and the riskless asset and a short position in the stock markets of the other countries.

In Kole et al. (2005) we examine the crisis conscious and the crisis ignorant strategy for different months of the Asian crisis, investigate how imposing short sales constraints influence our results and consider portfolio turnover. Other months of the Asian crisis lead to similar conclusions as October 1997. If the investor faces short sales constraints, he completely withdraws from equity markets when the inference probability of the crisis regime rises. Finally, the portfolio turnover indicates that the crisis conscious strategy implies more trading.

5.1. Static analysis

Table 5 presents the portfolios that a log-utility investor would construct, if he has no information on the price path of the assets so far. Most importantly, the crisis conscious strategy results in a less aggressive allocation than the crisis ignorant strategy. Overall, the position is less leveraged: the investor lends 3.79 times his initial wealth opposed to 4.71 under the crisis ignorant strategy. Das and Uppal (2004) report similar, though less pronounced results. Leverage is large because we report the log utility portfolio. An investor with coefficient of relative risk aversion equal to 5 would invest 4% in the risk free asset, adopting the crisis conscious, or lend 14% of his wealth, if he adopts the crisis ignorant strategy. The risky asset portfolio itself does not change much.

It is costly to ignore the possibility of a crisis. A log-utility investor who incorrectly adopts the crisis ignorant strategy requires a certainty equivalent return of 0.09% per month (or 1.13% per year) as compensation. For more risk averse investors, the required compensation becomes less, as the certainty equivalent return should be divided by their coefficient of relative risk aversion. A comparison of this result with findings of Das and Uppal (2004) highlights the importance of persistence. Das and Uppal (2004) report that an investor with coefficient of relative risk aversion of 3 and a horizon of 1 year requires a return of 0.1% per annum as compensation for incorrectly ignoring the systemic jumps in their model. A similar investor in our approach would require a return of 0.38% for incorrectly following the crisis ignorant strategy. Since both systemic events have a comparable probability of occurrence, we conclude that persistence increases the impact of systemic crises. Das and Uppal also show that the certainty equivalent return is an increasing and convex function of the probability of a systemic crisis (see Das and Uppal, 2003, Section 5.2.5, Appendix A.2 and Fig. 4), but the degree of persistence they consider is fairly low.

Table 5
Static optimal portfolios

	Log utility portfolio				Risky assets portfolio			
	Crisis ignorant	Crisis conscious	ϕ^e	ϕ^s	Crisis ignorant	Crisis conscious	ϕ^e	ϕ^s
US	1.61	1.15	−0.68	0.22	0.28	0.24	−0.11	0.06
Europe	3.59	2.98	−1.06	0.44	0.63	0.62	−0.15	0.14
Japan	−0.70	−0.39	0.45	−0.13	−0.12	−0.08	0.07	−0.03
Hong Kong	0.26	0.27	0.08	−0.07	0.05	0.06	0.02	−0.01
Thailand	0.02	0.07	0.55	−0.50	0.00	0.01	0.10	−0.09
Korea	0.60	0.44	0.30	−0.46	0.11	0.09	0.06	−0.08
Brazil	0.33	0.27	−0.03	−0.03	0.06	0.06	0.00	0.00
Risk free	−4.71	−3.79	0.40	0.52	0	0	0	0

This table reports the optimal portfolios for the crisis ignorant and crisis conscious strategies for different situations: log utility and an investment in risky assets only. The portfolios are the initial portfolios ($t = 0$) based on the unconditional inference probabilities. The portfolio weights for the different countries and the risk-free asset are reported in the first two columns. The differences between the allocations are decomposed in an estimation effect ϕ^e and a crisis effect ϕ^s.

Overall, the decomposition of the differences shows that the estimation effect causes a more prudent allocation: leverage is decreased by 0.40. However, within the risky asset part of the portfolio, investments shift from the US and Europe to Asia, mainly because the estimation effect makes the high volatility regimes for the Asian markets more attractive. Of course, this implies that the crisis regime entails substantial risk for investments in Asian equity. The crisis effect cause large divestments in Asia, which are partly directed towards the US and European markets and partly to the riskless asset. For the Asian markets the estimation effect and crisis effect cancel out more or less, but for the developed market the estimation effect dominates. Also in an economic sense the crisis effect is the more important. A log-utility investor requires a compensation of 0.31% per month for ignoring the crisis effect. If he also ignores the estimation effect, the required compensation reduces to 0.1% per month.

5.2. October 1997: The Asian crisis

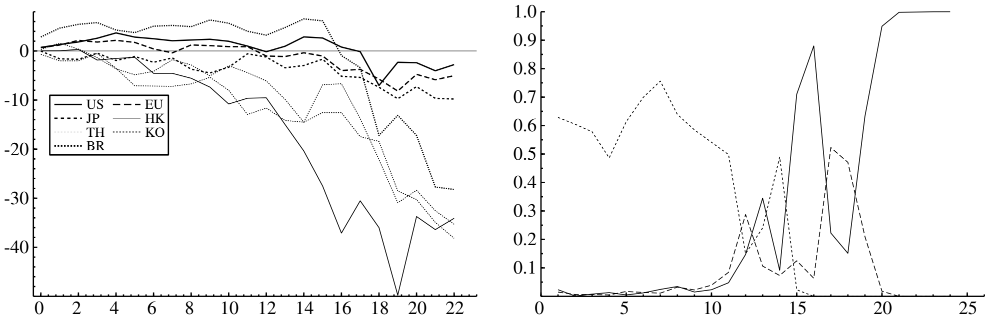
To study an informative setting, we investigate the implications for asset allocation in October 1997, the month during which the Hong Kong market crashed. We take the estimates presented in the previous sections as given. The inference probabilities that the investor uses are constructed by applying the filtering technique described in Hamilton (1994) on the data up to September 1997. This setup enables us to observe how the dive of the Hong Kong market influenced the inference probabilities and consequently the investor's asset allocation. The calculation of certainty equivalent returns and decompositions can help us to understand the changes in optimal asset allocations over time, caused by the continuous updating of the inference probabilities.

The Asian crisis hit financial markets during the second half of 1997.[16] In August 1997 the Thai market crashed. Only after the crash of the Hong Kong market, the shocks in Asia were considered as a global crisis. An inspection of the inference probabilities of our model produces a similar picture, as they were below 0.05 by the end of August and September, but almost 1 by the end of October.

Fig. 1(a) shows that the cumulative returns in the Asian markets are already negative during the beginning of the month, but the US, Europe and Brazil realize small, positive returns. However, after October 17, 1997 (the 13th trading day of that month) the Hong Kong market starts to dive: from −9.5% to −50% in 7 days. The Thai and Korean market move in lock step, but the other markets also suffer large price drops, in particular the Brazilian market. On October 20, the inference probability for the crisis regime in the crisis conscious model climbs to 0.71 and remains high for the rest of the month. So the conscious strategy deems the crisis regime the most likely one for the second half of the month.

The allocations in Fig. 2 show that adopting the crisis conscious strategy leads to less risky allocations. Though leverage in Fig. 2(h) starts at similar levels, the crisis conscious strategy reduces it more strongly as the probability of a crisis increases. This reduction already starts before the dive of the Hong Kong market. It even advises no leverage but a long position in the riskless asset by the end of the month, contrary to the crisis

[16] See Kamin (1999) for a broad discussion of the symptoms of the Asian crisis. Kaminsky and Schmukler (1999) investigate the causes of daily market fluctuations during the Asian crisis.



(a) return path (b) conscious

Fig. 1. The return path (excess returns, in %) of the indexes for the US, Europe (EU), Japan (JP), Hong Kong (HK), Thailand (TH), Korea (KO) and Brazil (BR) (panel a) and the resulting inferences for the crisis conscious strategy (panel b) and the crisis ignorant strategy (panel c) for each trading day in October 1997 (numbered consecutively). The inference probabilities are constructed by updating the forecast probabilities based on the returns to September 1997 in a Bayesian fashion as given in Eq. (11). We only plot the inferences for a state vector, if the inferences have exceeded 0.4 at least once: for the crisis conscious strategy that is US and Thailand high volatility, others low (dashed line), US, Hong Kong and Thailand high volatility, others low (long dashed line), and the crisis state (solid line).

ignorant strategy. Foreign markets quickly become less attractive in the crisis conscious strategy, as it advocates short positions in Japan, Hong Kong, Thailand and Korea. Applying the crisis ignorant strategy leads to reduced exposure in Japan and Hong Kong (which is again favored by the end of the month), but a stable long position in Korea. Moreover, the crisis conscious strategy moves out of the Hong Kong market much faster.

The differences between the allocations in Fig. 2 are not only pronounced but also economically important. Both before and after the dive of the Hong Kong market, the certainty equivalent in Fig. 3 is higher than 0.3% per month, clearly exceeding the 0.09% per month of the uninformative case. Moreover, it rises dramatically (to at most 4.0%) after the crash of the Hong Kong market. Of course, these returns are lower for more risk averse investors, but we stress that they correspond with a 1-month horizon.

A rise in the inference probabilities causes a rapid deterioration of diversification opportunities. Already before the crisis hits, investments are reduced and in some countries short position are taken. The deterioration we report is worse than Ang and Bekaert (2002) find for the prevailing of a bearish regime, but the crisis regime in our model surpasses the severity of a bearish, high volatility regime.

We conclude the analysis by considering the estimation and crisis effects. The crisis effect in Fig. 2 causes a hedging demand for US and European equity and the riskless asset. Over time the hedging demand for the riskless asset increases strongly. Positions in other markets are more and more reduced, particularly in crisis prone Korea and Thailand. The estimation effect presents a less clear picture. In the uninformative case, the estimation effect causes investments to shift from the riskless asset to the risky assets. Now we observe a preference for Japan, Korea and Thailand. Within the risky asset part we observe a tendency to more aggressive allocations, but leverage is not increased much. The certainty equivalent returns associated with missing the crisis effect are also considerable, being always positive and rising rapidly. Of course, missing the crisis effect becomes

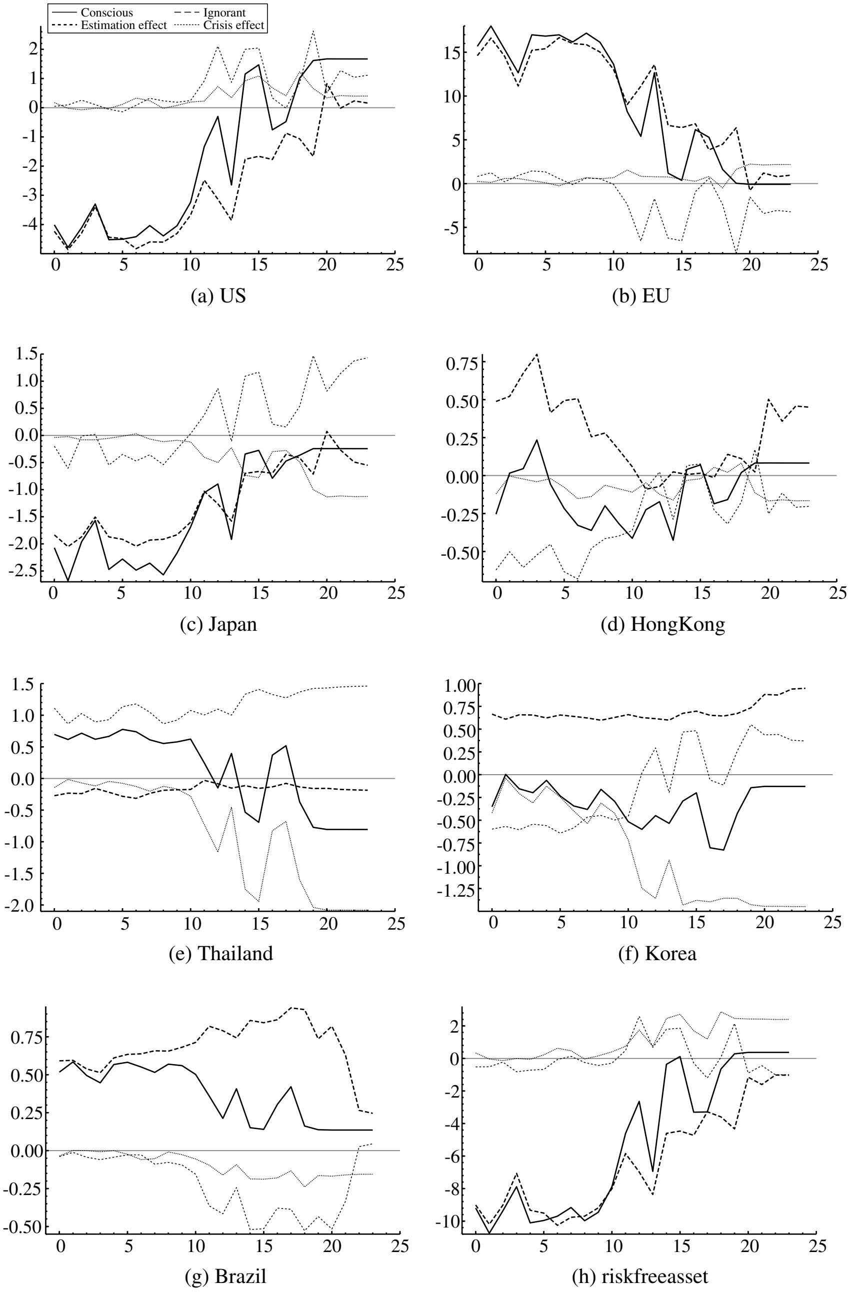


Fig. 2. The proportion invested in the different countries for the crisis conscious and crisis ignorant strategy and a decomposition of the differences for each trading day in October 1997 (numbered consecutively). We assume the investor has a log utility function. The portfolios are based on the estimates in Tables 2 and 3 and the inference probabilities that are constructed by updating the forecast probabilities based on the returns to September 1997 in a Bayesian fashion as given in Eq. (11). The portfolio differences between the two portfolios are decomposed in an estimation and a crisis effect.

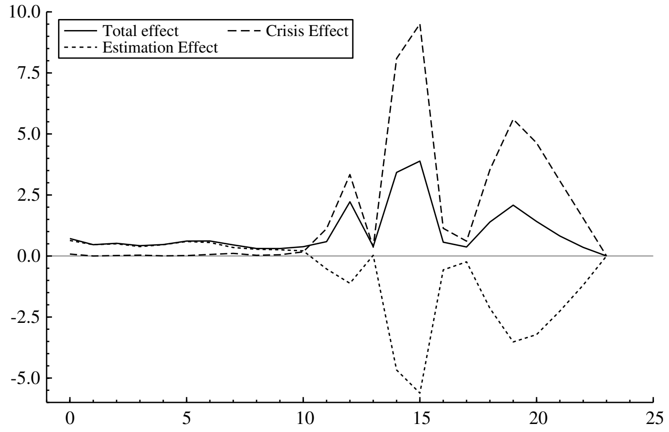


Fig. 3. Certainty equivalent return (in %) needed to compensate the investor for adopting the suboptimal ignorant strategy and a decomposition in an estimation and crisis effect for each trading day in October 1997 (numbered consecutively). We assume the investor has a log utility function and a horizon of 1 month. The portfolios during October 1997 and the corresponding decompositions are given in Fig. 2.

extremely expensive after a crisis has occurred (with a maximum certainty equivalent return at 10%). The estimation effect does not always harm the investor's utility. But influences utility less (in absolute sense) than the crisis effect.

6. Conclusions

A systemic crisis in international equity markets can put investors in dire straits, because of the simultaneous decrease in expected returns and increase in volatilities and correlations. Standard models supporting asset allocation decisions typically fail to fully capture systemic crises due to their irregular and relatively rare occurrences. In this paper, we have proposed a framework to determine the impact of systemic crises on asset allocations, which combines regime switching models with optimal portfolio construction in continuous time. An investor can adopt a crisis conscious strategy that includes systemic crises, and, as an alternative, a crisis ignorant strategy which excludes it. We have studied the allocations of a US-based, global investor, who maximizes his expected utility by investing globally in equity markets and a riskless asset in a general uninformative case and the informative case of the Asian crisis. In the crisis conscious strategy, the investor decreases leverage and shifts investments to countries that are less prone to a crisis. The certainty equivalent returns indicates that these differences are economically important and quickly rise during October 1997.

The pronounced portfolio differences and their economic importance indicate that persistence is an important characteristic of systemic crises. We have estimated that the probability of remaining in the crisis regime for another month equals 0.90. Because of this persistence, we find stronger evidence advocating the incorporation of systemic crises in asset allocation decisions than reported by Das and Uppal (2004, 2003). Crisis persistence also explains the large differences between the crisis conscious and ignorant strategies and the large required compensation if a crisis occurs with almost certainty.

Systemic crises seriously diminish diversification possibilities. If a crisis has a small probability of occurrence, this effect is present but limited. As the probability increases, the impact becomes larger and leads to short positions in several markets. When the investors faces short sales constraints, he withdraws from equity markets completely. These findings are complementary to Ang and Bekaert (2002) who conclude that the presence of a bear regime in a regime switching model does not extinguish diversification possibilities. However, because of its severity the crisis regime in our model causes a much stronger deterioration in the risk-return trade-off than the bear regime in their model.

Appendix A

A.1. Solving the asset allocation problem

In this appendix we derive the optimal portfolio and the certainty equivalent return to compensate for suboptimal portfolios. The optimal portfolio solves

$$\max_{\phi_s, t \leqslant s \leqslant T} E_t[U(W_{\mathrm{T}})] \quad \text{subject to } \mathrm{d}W/W = r_{\mathrm{f}}\,\mathrm{d}t + \phi'\alpha\,\mathrm{d}t + \phi'\Lambda\,\mathrm{d}Z. \tag{14}$$

Via the indirect utility function $V(W,t) = \max_{\phi_s, t\leqslant s\leqslant T} E_t[U(W_{\mathrm{T}})]$, we derive the Hamilton–Jacobi–Bellman equation:

$$\max_{\phi}\left(\frac{\partial V}{\partial t} + (r_{\mathrm{f}} + \alpha\phi')W\frac{\partial V}{\partial W} + \frac{1}{2}\phi'\Omega\phi W^2\frac{\partial^2 V}{\partial W^2}\right) = 0. \tag{15}$$

We conjecture (and verify) that the indirect utility function is of the form $V(W,t) = C(t)\cdot W^{1-\gamma}/(1-\gamma)$ and derive expressions for the derivatives in Eq. (15).[17] Differentiating Eq. (15) with respect to ϕ, and solving the first-order conditions yield the optimal portfolio in Eq. (8).

The certainty equivalent return $\bar{r}$ to compensate the investor for selecting the inappropriate crisis ignorant portfolio ϕ^i instead of the optimal crisis conscious portfolio ϕ^c solves:

$$V(e^{\bar{r}}W_t, t; \phi^i) = V(W_t, t; \phi^c). \tag{16}$$

Using the functional form of the value function, we find after some rearrangements that $\bar{r} = (\ln C(t;\phi^c) - \ln C(t;\phi^i))/(1-\gamma)$, which is independent of wealth. To identify $C(t)$, consider the Hamiltonian Eq. (15) at the presumed optimal solution ϕ^*. This equation implies an ordinary differential equation for $C(t)$:

$$\mathrm{d}C = -(1-\gamma)\left[r_{\mathrm{f}} + \phi^{*\prime}\alpha - \frac{1}{2}\gamma\phi^{*\prime}\Omega\phi^*\right]C(t)$$

that can be solved straightforwardly, yielding:

$$C(t;\phi^*) = \exp[(1-\gamma)(r_{\mathrm{f}} + h(\phi^*))(T-t)] \tag{17}$$

with $h(\phi^*) = \phi^{*\prime}\alpha - \frac{1}{2}\gamma\phi^{*\prime}\Omega\phi^*$. We used the boundary condition $V(W,T;\phi^*) = U(W_{\mathrm{T}})$, to solve for the integration constant. Substituting this into the expression for $\bar{r}$ yields Eq. (12).

[17] For $\gamma = 1$ we use $V = \ln[C(t)W]$.

A.2. A multinomial model for regime transition probabilities

In this appendix we discuss the specification of the multinomial logistic model for the regime transition probabilities in more detail. The probability π_i^{ab} gives the marginal probability that asset i will switch to regime Q_i^a, given that the current regime is $\tilde{Q}^b$, and that a crisis is absent in both the current and the destination regime, and has functional form:

$$\pi_i^{ab} = \frac{\exp f(Q_i^a, Q^b)}{1 + \sum_{k=1}^{K-1} \exp f(Q_i^a = k, Q^b)}. \tag{18}$$

The summation in the denominator excludes the Kth basic regime to ensure that the probabilities add up to 1. We specify the function $f(Q_i^a, Q^b)$ as

$$f(Q_i^a = k, Q^b) = \psi_{i,k,Q_i^b} + \sum_{j=1, j\neq i}^{n} \sum_{k'=2}^{K} \psi_{i,j,k,k'} I(Q_j^b = k'), \tag{19}$$

where ψ_{i,k,Q_i^b} and $\psi_{i,j,k,k'}$ are constants, and $I()$ denotes the indicator function. If all assets $j \neq i$ are in their low volatility regimes ($Q_j^b = 1$), the function returns the constant ψ_{i,k,Q_j^b}. For the assets that are in a higher volatility level k', constants $\psi_{i,j,k'}$ are added to it. We require $\psi_{i,j,k,k'} < 0$ for $k < k', \psi_{i,j,k,k'} > 0$ for $k \geqslant k'$, and $\psi_{i,j,k,k'} < \psi_{i,j,k,k''}$ for $k' < k''$ to ensure that the volatility spill-over effects increase the probability of higher volatility regimes.

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