Linear Algebra Questions (Make sure to attend the Online session)

1. Given the matrices:

$$A = \begin{bmatrix} -1 & 23 & 10 \\ 0 & -2 & -11 \end{bmatrix}, \quad B = \begin{bmatrix} -6 & 2 & 10 \\ 3 & -3 & 4 \\ -5 & -11 & 9 \\ 1 & -1 & 9 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 9 - 5 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix}, \quad E = \begin{bmatrix} 3 \\ 5 \\ -11 \\ 7 \end{bmatrix}, \quad G = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}$$

- a) What is the dimension of each matrix?
- b) Which matrices are square?
- c) Which matrices are symmetric?
- d) Which matrix has the entry at row 3 and column 2 equal to -11?
- e) Which matrices has the entry at row 1 and column 3 equal to 10?
- f) Which are column matrices?
- g) Which are row matrices?
- h) Find AT,CT,ET,GT. (T -> Transpose)

2. A, B, C, D and E are matrices given by:

$$A = \begin{bmatrix} -1 & 1 & -2 \ 0 & -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 0 \ 0 & -3 & 4 \ -1 & -2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 9 & -5 & 7 \ 0 & 5 & 2 \end{bmatrix},$$
 $D = \begin{bmatrix} -2 & 6 \ -5 & 2 \end{bmatrix}, \quad E = \begin{bmatrix} 3 \ 5 \ -11 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 0 & 2 \ -2 & -3 & 4 \ 1 & 4 & -3 \end{bmatrix}$

Find if possible:

- a) AB
- b) BC
- c) AD
- d) EF
- e) FE

3. Find the determinant of the matrix M:

$$M = \begin{pmatrix} 15 & 10 \\ 3 & 2 \end{pmatrix} \qquad M = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 3 & 2 & -1 \end{pmatrix}$$

4. Find the inverse matrix A-1 to the matrix A:

$$A = \begin{pmatrix} -3 & -2 \\ 3 & 3 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

- 5. What does it mean if three equations are linearly independent?
 - a. Two of the equations can be combined to come up with the third equation.
 - b. There is no way to combine any two equations to come up with the third equation.
 - c. The graphical representations of the equations are lines that do not intersect.
 - d. The graphical representations of the equations are lines that do intersect.
- 6. Let

$$f(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{x}^{\top} \boldsymbol{A} \boldsymbol{y} + \boldsymbol{x}^{\top} \boldsymbol{B} \boldsymbol{x} - \boldsymbol{C} \boldsymbol{y} + \boldsymbol{D}$$

with $\boldsymbol{x} \in \mathbb{R}^M$, $\boldsymbol{y} \in \mathbb{R}^N$, function $f : \mathbb{R}^M \times \mathbb{R}^N \to \mathbb{R}$.

Compute the dimensions of the matrices A, B, C, D for the function so that the mathematical expression is valid.