# Benchmark Functions for Bayesian Optimization

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## 0 Benchmark Functions for Bayesian Optimization

- All functions are implemented in https://github.com/jungtaekkim/bayeso-benchmarks.
- We refer to [Surjanovic and Bingham, 2013] for specific forms of many benchmark functions, which are described in https://www.sfu.ca/~ssurjano.
- The details of bayeso [Kim and Choi, 2017] and bayeso-benchmarks are introduced in http://bayeso.org.
- We will introduce the following benchmark functions in this article.
  1. Ackley
  2. Beale
  3. Bohachevsky
  4. Branin
  5. Cosines
  6. Eggholder
  7. Goldstein-Price
  8. Gramacy & Lee (2012)
  9. Hartmann 6D
  10. Holder Table
  11. Rosenbrock
  12. Six-Hump Camel
  13. Sphere

## 1 Ackley Function

Given a $d$-dimensional input, $\mathbf{x} \coloneqq [x_1, \ldots, x_d] \in [-32.768, 32.768]^d$,

$$y = -20 \exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(2\pi x_i)\right) + 20 + \exp(1). \tag{1}$$

A global optimum is 0, at $\mathbf{x}^\star = [0, \ldots, 0] \in \mathbb{R}^d$.

## 2 Beale Function

Given a two-dimensional input, $\mathbf{x} \coloneqq [x_1, x_2] \in [-4.5, 4.5]^2$,

$$y = (1.5 - x_1 + x_1 x_2)^2 + (2.25 - x_1 + x_1 x_2^2)^2 + (2.625 - x_1 + x_1 x_2^3)^2. \tag{2}$$

A global optimum is 0, at $\mathbf{x}^\star = [3.0, 0.5]$.

## 3 Bohachevsky Function

Given a two-dimensional input, $\mathbf{x} \coloneqq [x_1, x_2] \in [-100, 100]^2$,

$$y = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1) - 0.4\cos(4\pi x_2) + 0.7. \tag{3}$$

A global optimum is 0, at $\mathbf{x}^\star = [0, 0]$.

## 4 Branin Function

Given a two-dimensional input, $-5 \leq x_1 \leq 10$, $0 \leq x_2 \leq 15$,

$$y = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right) + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10. \tag{4}$$

Global optima are 0, at $\mathbf{x}^\star = [-\pi, 12.275]$, $[\pi, 2.275]$, and $[9.42478, 2.475]$.

## 5 Cosines Function

Given a $d$-dimensional input, $\mathbf{x} \coloneqq [x_1, \ldots, x_d] \in [-2\pi, 2\pi]^d$,

$$y = \sum_{i=1}^{d} \cos(x_i)\left(\frac{0.1}{2\pi}|x_i| - 1\right). \tag{5}$$

A global optimum is $-d$, at $\mathbf{x}^\star = [0, \ldots, 0] \in \mathbb{R}^d$.

## 6 Eggholder Function

Given a two-dimensional input, $\mathbf{x} \coloneqq [x_1, x_2] \in [-512, 512]^2$,

$$y = -(x_2 + 47)\sin\left(\sqrt{|x_2 + 0.5x_1 + 47|}\right) - x_1\sin\left(\sqrt{|x_1 - (x_2 + 47)|}\right). \tag{6}$$

A global optimum is $-959.6407$, at $\mathbf{x}^\star = [512.0, 404.2319]$.

## 7 Goldstein-Price Function

Given a two-dimensional input, $\mathbf{x} \coloneqq [x_1, x_2] \in [-2, 2]^2$,

$$y = \left[1 + A(B - 14x_2 + 6x_1 x_2 + 3x_2^2)\right]\left[30 + C(D + 48x_2 - 36x_1 x_2 + 27x_2^2)\right], \tag{7}$$

where

$$A = (x_1 + x_2 + 1)^2, \tag{8}$$

$$B = 19 - 14x_1 + 3x_1^2, \tag{9}$$

$$C = (2x_1 - 3x_2)^2, \tag{10}$$

$$D = 18 - 32x_1 + 12x_1^2. \tag{11}$$

A global optimum is 3, at $\mathbf{x}^\star = [0, -1]$.

# 8 Gramacy & Lee (2012) Function

Given a one-dimensional input, $x \in [0.5, 2.5]$,

$$y = \frac{\sin(10\pi x)}{2x} + (x-1)^4. \tag{12}$$

A global optimum is 0.54856405, at $x^\star = -0.86901113$.

# 9 Hartmann 6D Function

Given a six-dimensional input, $\mathbf{x} \coloneqq [x_1, \ldots, x_6] \in [0, 1]^6$,

$$y = -\sum_{i=1}^{4} \alpha_i \exp\left(-\sum_{j=1}^{6} A_{ij}(x_j - P_{ij})^2\right), \tag{13}$$

where $\boldsymbol{\alpha} = [1.0, 1.2, 3.0, 3.2]^\top$,

$$\boldsymbol{A} = \begin{pmatrix} 10 & 3 & 17 & 3.5 & 1.7 & 8 \\ 0.05 & 10 & 17 & 0.1 & 8 & 14 \\ 3 & 3.5 & 1.7 & 10 & 17 & 8 \\ 17 & 8 & 0.05 & 10 & 0.1 & 14 \end{pmatrix}, \tag{14}$$

$$\boldsymbol{P} = 10^{-4} \begin{pmatrix} 1312 & 1696 & 5569 & 124 & 8283 & 5886 \\ 2329 & 4135 & 8307 & 3736 & 1004 & 9991 \\ 2348 & 1451 & 3522 & 2883 & 3047 & 6650 \\ 4047 & 8828 & 8732 & 5743 & 1091 & 381 \end{pmatrix}. \tag{15}$$

A global optimum is $-3.32237$, at $\mathbf{x}^\star = [0.20169, 0.150011, 0.476874, 0.275332, 0.311652, 0.6573]$.

# 10 Holder Table Function

Given a two-dimensional input, $\mathbf{x} \coloneqq [x_1, x_2] \in [-10, 10]^2$,

$$y = -\left|\sin(x_1)\cos(x_2)\exp\left(\left|1 - \frac{\sqrt{x_1^2 + x_2^2}}{\pi}\right|\right)\right|. \tag{16}$$

Global optima are $-19.2085$, at $\mathbf{x}^\star = [8.05502, 9.66459]$, $[8.05502, -9.66459]$, $[-8.05502, 9.66459]$, and $[-8.05502, -9.66459]$.

## 11 Rosenbrock Function

Given a $d$-dimensional input, $\mathbf{x} \coloneqq [x_1, \ldots, x_d] \in [-2.048, 2.048]^d$,

$$y = \sum_{i=1}^{d-1} \left[ 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]. \tag{17}$$

A global optimum is 0, at $\mathbf{x}^\star = [1, \ldots, 1] \in \mathbb{R}^d$.

## 12 Six-Hump Camel Function

Given a two-dimensional input, $-3 \leq x_1 \leq 3$, $-2 \leq x_2 \leq 2$,

$$y = \left( 4 - 2.1x_1^2 + \frac{x_1^4}{3} \right) x_1^2 + x_1 x_2 + (-4 + 4x_2^2)x_2^2. \tag{18}$$

Global optima are $-1.0316$, at $\mathbf{x}^\star = [0.0898, -0.7126]$ and $[-0.0898, 0.7126]$.

## 13 Sphere Function

Given a $d$-dimensional input, $\mathbf{x} \coloneqq [x_1, \ldots, x_d] \in [-5.12, 5.12]^d$,

$$y = \sum_{i=1}^{d} x_i^2. \tag{19}$$

A global optimum is 0, at $\mathbf{x}^\star = [0, \ldots, 0] \in \mathbb{R}^d$.

## References


J. Kim and S. Choi. bayeso: A Bayesian optimization framework in Python. `http://bayeso.org`, 2017.

S. Surjanovic and D. Bingham. Virtual library of simulation experiments: Test functions and datasets. `http://www.sfu.ca/~ssurjano`, 2013.