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Assignment 7

IE 7374: Machine Learning

Derive the θ_{MAP} for a generative model where y is from a $N(\theta^T x, \sigma^2)$ and pior θ is from a $[Laplace(\lambda)\ or\ Laplace(0, \frac{1}{\lambda})]$ distribution.

Answer to Question

The generative formula when $\theta \sim Laplace(0, \frac{1}{\lambda})$,

$$\begin{aligned}
\theta_{MAP} &= argmax \prod_{i=1}^{N} P(y_i|\theta^T x_i, \sigma^2)P(\theta) \\
&= argmax \prod_{i=1}^{N} [\frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{-(y_i - \theta^T x_i)^2}{2\sigma^2} \cdot \frac{\lambda}{2} \exp^{-\lambda|\theta|}] \\
&\Longrightarrow argmax \sum_{i=1}^{N} [\frac{-(y_i - \theta^T x_i)^2}{2\sigma^2} - \lambda|\theta|]
\end{aligned}$$

Similarly, we can get the form for $\theta \sim Laplace(\lambda)$. These forms are related to decrease the errors of lasso regression. In this way, a relationship between probability/MAP and Cost/MLE will be established, among which the type of MLE will be confirmed by the prior distribution and Gaussian's.