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IE 7374: Machine Learning

Prove that you can write each square matrix A as sum of a symmetric and Anti-symmetric matrix:

$$A = \frac{A + A^T}{2} + \frac{A - A^T}{2} \tag{1}$$

Answer to Question

For formula (1), A can be written as the sum of B and C, where $B = \frac{A+A^T}{2}$, $C = \frac{A-A^T}{2}$.
The idea is to prove that B is symmetric and C is Anti-symmetric.
Prove B is symmetric

$$B^T = (\frac{A + A^T}{2})^T = \frac{A + A^T}{2} = B \tag{2}$$

Prove C is Anti-symmetric

$$C^T = (\frac{A - A^T}{2})^T = -(\frac{A^T - A}{2})^T = -(\frac{A - A^T}{2}) = -C \tag{3}$$

Combining formula(2) and (3), we prove that square matrix A can be as sum of a symmetric B and Anti-symmetric matrix C.