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Assignment 8

IE 7374: Machine Learning

Prove that if negative-log likehood for Logistic regression is Convex.

Answer to Question

$X = \{x_1, \ldots, x_i, \ldots, x_n\}^T_{n\times d}$, $x_i \in \mathbb{R}^d$
$p(x_i|\theta) = \frac{1}{1+exp(-\theta x_i)}$

$$\begin{aligned}
P &= -log[p(X|\theta)] \\
&= -log[\sum_{i=1}^{n} p(x_i|\theta)] \\
&= \sum_{i=1}^{n} log[1+exp(-\theta x_i)]
\end{aligned}$$

Then, we calculate the second derivative of P with respect to θ, to see if it is greater or equal than 0. If so, we prove it is a convex function.

$$\begin{aligned}
\frac{\partial P}{\partial \theta} &= \sum_{i=1}^{n} \frac{1}{1+exp(-\theta x_i)} \cdot exp(-\theta x_i) \cdot (-x_i) \\
\frac{\partial^2 P}{\partial \theta^2} &= \sum_{i=1}^{n} [-(1+exp(-\theta x_i))^{-2} \cdot exp(-\theta x_i) \cdot (-x_i) \cdot exp(-\theta x_i) \cdot (-x_i) + \\
&\quad (1+exp(-\theta x_i))^{-1} \cdot exp(-\theta x_i) \cdot (-x_i) \cdot (-x_i)] \\
&= \sum_{i=1}^{n} [(1+exp(-\theta x_i))^{-2} \cdot exp(-2\theta x_i) \cdot x_i^2][(1+exp(-\theta x_i)) \cdot exp(\theta x_i) - 1] \\
&= \sum_{i=1}^{n} [(1+exp(-\theta x_i))^{-2} \cdot exp(-2\theta x_i) \cdot x_i^2] \cdot exp(\theta x_i) \\
&\geq 0
\end{aligned}$$

Negative-log likehood for logistic regression is a Convex.