# Goldsmith Functions

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### 1 Abstract

## 2 Introduction

text (introduction)

### 3 Proceedings

### 3.1 Inputs

The formulas have several inputs. While some are used as placeholders in definitions, others have actual meaning in the context of the application of the functions.

#### 3.1.1 Definition of Input Variables

Universal input variables of the function set:

- r is a variable which controls the sensitivity of the  $\Lambda_0, \Lambda_{n_s}, \Phi_e, \Phi_0$ , and  $\Phi_{n_s}$  functions.  $r \in \mathbb{Z}$
- $n_s$  is a variable which controls the scaling factor of the  $Rd_{n_s}, Ex_{n_s}, \Phi_{n_s}$ , and  $\Lambda_{n_s}$  functions.  $n_s \in \mathbb{N}_0$
- $n_r$  is a variable which controls the rounding precision of the  $\Gamma_{n_r}$  and  $\mho_{n_r}$  functions. In  $\Gamma_{n_r}$   $n_r$  controls the number of decimal digits, while in  $\mho_{n_r}$  it controls the total number of relevant digits.  $n_r \in \mathbb{N}_0$
- s is the score calculated from c using  $\Phi_0$  or  $\Phi_{n_s}$ .  $s \in [100 \cdot (1-10^{n_s}); 100]$
- q controls whether  $\Gamma_{n_r}$  and  $\mho_{n_r}$  use flooring (truncation) or rounding.  $q \in \{0,1\}$
- c is the main input variable of the  $\Phi$  functions and describes the "purity" or other quality which should be converted into a score.  $c \in [0; 1]$ , except for  $\Phi_0$ , where  $c \in [\Theta(r); 1]$ .
- $\lambda$  is a variable which controls the weight in the averaging function  $\Xi_{\lambda}$ . A higher  $\lambda$  value results in a higher weight for lower inputs, while a  $\lambda$  value close to 0 results in a less adjusted average.  $\lambda \in [0, \infty)$

#### 3.2 Formulas

$$Rd_{n_s}(x) = x \cdot 10^{n_s} + 100 \cdot (1 - 10^{n_s}) \tag{1}$$

$$Ex_{n_s}(x) = Rd_{-n_s}(x) \tag{2}$$

$$\Theta(r) = (1 + 10^{-r})^{-9} \tag{3}$$

$$\beta(r) = 9 \cdot \log_{10}(1 + 10^{-r}) \tag{4}$$

$$\Phi_e(c; r) = 100 \cdot \sqrt{1 + \frac{\log_{10}(c)}{\beta(r)}}$$
(5)

$$\Phi_0(c;r) = \begin{cases} \Phi_e(c;r) & \text{for } c \ge \Theta(r) \\ 0 & \text{for } c < \Theta(r) \end{cases}$$
 (6)

$$\Phi_{n_s}(c;r) = Rd_{n_s}(\Phi_0(c;r)) \tag{7}$$

$$\Lambda_0(s;r) = 10^{\beta(r) \cdot \left(\left(\frac{s}{100}\right)^2 - 1\right)} \tag{8}$$

$$\Lambda_{n_s}(s;r) = \Lambda_0(Ex_{n_s}(s);r) \tag{9}$$

$$\delta(x) = -\log_{10}(1 - x) \tag{10}$$

$$\delta_s(x) = |\delta(x)| \tag{11}$$

$$\chi(x) = \sum_{k=1}^{x} \left(\frac{9}{10^k}\right) = 1 - 10^{-x} \tag{12}$$

$$\Gamma_{n_r}(x;q) = \frac{\lfloor x \cdot 10^{n_r} + 0.5 \cdot q \rfloor}{10^{n_r}}$$
(13)

$$\mathcal{O}_{n_r}(x;q) = \begin{cases}
\Gamma_{n_r - 1 - \lfloor \log_{10}(|x|) \rfloor}(x;q) & \text{for } x \neq 0 \\
0 & \text{for } x = 0
\end{cases}$$
(14)

$$\Xi_{\lambda}(x_1; \dots; x_n) = \frac{\sum_{i=1}^{n} \left( x_i \cdot \left( e^{-\lambda \cdot x_i} \right) \right)}{\sum_{i=1}^{n} \left( e^{-\lambda \cdot x_i} \right)}$$
(15)

$$\varkappa_n(f(x);q) = \frac{\left\lfloor \frac{q}{2} + n \cdot f\left(\frac{\left\lfloor \frac{q}{2} + n \cdot x\right\rfloor}{n}\right)\right\rfloor}{n} \tag{16}$$

#### 3.3 Explanation

#### 3.3.1 Explanation of Functions

- $Rd_{n_s}(x)$ :  $Rd_{n_s}$  describes a scaling function. It is mainly used to convert the output of  $\Phi_0$  from a 0 to 100 range to an  $100 \cdot (1-10^n)$  to 100 range.  $Rd_{n_s}(Rd_{n_s}(x)) = Rd_{2 \cdot n_s}(x)$ ;  $Rd_{n_s}(100) = 100$ ;  $Rd_0(x) = x$
- $Ex_{n_s}(x)$ :  $Ex_{n_s}$  is the inverse scaling function of  $Rd_{n_s}$ . Its mainly used to convert the input of  $\Lambda_{n_s}$  from a  $100 \cdot (1-10^n)$  to 100 range to an 0 to 100 range.  $Ex_{n_s}(Ex_{n_s}(x)) = Ex_{2\cdot n_s}(x)$ ;  $Ex_{n_s}(100) = 100$ ;  $Ex_0(x) = x$
- $\Theta(r)$ :  $\Theta$  is a function used to determine the minimum c value required to get a  $\Phi_0$  score above 0 and it is equal to  $\Lambda_0(0;r)$ . It also defines the lower bound of the domain of the  $\Phi_e$

function.

- $\beta(r)$ :  $\beta$  is an intermediate function for r, and it forms the implementation of the sensitivity control in  $\Phi_e$ ;  $\Phi_0$ ;  $\Phi_{ns}$ ;  $\Lambda_0$  and  $\Lambda_{ns}$ . It is not intended to be used as a stand-alone function.
- $\Phi_e(c;r)$ :  $\Phi_e$  is the main conversion function of the Goldsmith functions and converts a c between  $\Theta(r)$  and 1 to an s value between 0 and 100.
- $\Phi_0(c;r)$ :  $\Phi_0$  is used to extend the domain of  $\Phi_e$  to 0 to 1.
- $\bullet \Phi_{n_s}(c;r)$ :
- $\Lambda_0(s;r)$ :
- $\Lambda_{n_s}(s;r)$ :
- $\bullet \ \delta(x)$ :
- $\bullet$   $\delta_s(x)$ :
- $\bullet \chi(x)$ :
- $\Gamma_{n_r}(x)$ :
- $\mho_{n_x}(x)$ :
- $\Xi_{\lambda}(x_1;\ldots;x_n)$ :
- $\bullet \varkappa_n(f(x);q)$ :

#### 3.4 Outputs

#### 3.4.1 Interpretation of outputs

#### 3.5 Examples

There are several possible areas of application for the Goldsmith functions. Some of these applications include the purity of precious metals, reliability, accuracy of AI algorithms, data fidelity, purity of silicon in the semiconductor industry and many more. Below are some example values.

- 99.999998% gold = 88.19r8 0 gold
- 99.9999999954% silicon = 69.92r12 0 silicon
- 10.0% gold content in ore = 79.43r0 0 gold content in ore
- 58.3% gold content in 14k gold = 60.91r1 0 gold content in 14k gold
- 93.0% process yield = 43.55r2 0 process yield
- 0.6% copper content in ore = 42.42r0 0 copper content in ore
- 99.5% uptime = 66.54r4 0 uptime

The general notation is given by the following standard.

•  $[100 \cdot c]\%$   $[quantity] = [\Phi_{n_s}(c;r)]r[r] - [n_s]$  [quantity]

The notation standard is intended for the use in various fields listed in the abstract and is deliberately kept concise.

## 4 Conclusions

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## 5 References

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# 6 Appendices

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