Dynare Tutorial

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Schedule

- Day 1: June 15, 15:30–17:30
  - Introduction to Dynare
  - Standard DSGE Model
  - Log-linearization
- Day 2: June 16, 9:00–12:00
  - Stochastic Simulation with level variables
  - Stochastic Simulation with log-deviation variables
  - Deterministic Simulation

DSGE Models in Modern Macroeconomics

- DSGE model has been emplyed as a workhorse in modern macroeconomic analyses:
  - Real business cycle theory
  - Dynamic labor market analyses
  - Dynamic analyses in public finance
  - New Keynesian macroeconomics with sticky prices
- Solutions to the models take the form of forward looking difference equations with random shocks: $y_t = \Gamma(y_{t+1}, x_t, z_t)$
  eg) Euler equation: $u'(c_t) = \beta E[u'(c_{t+1})(1 + r_{t+1})]$

What is Dynare?

- A Matlab or Octave frontend that solves non-linear dynamic models with forward-looking variables.
- saves considerable amount of economists' labor compared to the time before dynare.
- developed and maintained by Michel Juillard and his team.
- Freely downloadable from `http://www.dynare.org`

What Can Dynare Do?

- Can solve forward looking non-linear models
  - under perfect foresight when the model is deterministic, using a relaxation algorithm which preserves all the non-linearities.
  - under rational expectations when the model is stochastic, using perturbation methods.
- Can estimate parameters of dynamic models using Bayesian techniques.

How Does Dynare Work?

- Economist writes a model to solve in a `.mod` file and parses it to the dynare.
- The Dynare generates a package of Matlab codes to solve/approximate the model.
- Matlab delivers the solution that can be used to compute other properties of the model.
- The solution takes a form, $y = f(x, z)$, where y denotes a vector of endogenous variables, x a vector of state variables, z a vector of shocks, and f is a linear or second-order approximation to the theoretical solution of the model.

Structure of Dynare Code (`.mod` file)

`.mod` file consists of the following five blocks:

- Preamble: declare variables and parameters
- Model: specify the characteristic equations of the model
- Steady State: compute the steady state of the model
- Shocks: specify the nature of shocks
- Computation: compute and simulate the solution

Example: Standard DSGE Model (1)

- Representative worker/consumer:
  - There are a continuum of workers/consumers who live forever and whose mass is normalized to one.
  - all workers/consumers are identical in their preferences and endowment $\Rightarrow$ an analysis to the representative worker is enough to investigate the behavior of all consumers.
  - chooses a series of consumption and labor supply to maximize her life-time utility.
  - consumer's utility maximization:

$$\max_{\{c_t\}_{t=0}^{\infty}} \; E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \qquad (1)$$

$$\text{s.t.} \qquad a_{t+1} = (1 + r_t)a_t + w_t h_t - c_t$$

Example: Standard DSGE Model (2)

- Representative firm:
  - There are a continuum of firms that are identical in their technology $\Rightarrow$ an analysis to the representative firm is enough to investigate the behavior of all firms.
  - produces output using $Y_t = F(K_t, L_t, z_t)$ to maximize profits each period.
  - productivity evolves over time according to a stochastic process.
  - Firm's profit maximization:

$$\max_{K_t, L_t} \ \pi_t = F(K_t, L_t, z_t) - w_t L_t - (r_t + \delta)K_t \quad \text{for all } t \qquad (2)$$

- All markets are perfectly competitive.

Competitive Equilibrium

$\{c_t, h_t, K_t, L_t, w_t, r_t\}_{t=0}^{\infty}$ s.t.

- Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\{c_t, h_t\}_{t=0}^{\infty}$ solves (1)
- Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\{K_t, L_t\}_{t=0}^{\infty}$ solves (2)
- Given $\{c_t, h_t, K_t, L_t\}_{t=0}^{\infty}$, $\{w_t, r_t\}_{t=0}^{\infty}$ clear factor markets.
  - $a_t = K_t$ for all t
  - $h_t = L_t$ for all t
  - $Y(K_t, L_t) = c_t + I_t$ for all t, where $I_t = K_{t+1} - (1-\delta)K_t$

Recursive Formulation of Consumer Problem

$$V(a, K, z) = \max_{c,h} \left\{ u(c, h) + \beta E\left[V(a', K', z'|z) \right] \right\} \tag{3}$$

$$\begin{aligned} \text{s.t. } c + x &= W(K, z)h + R(K, z)a \\ a' &= (1 - \delta)a + x \\ K' &= (1 - \delta)K + X \\ z' &= g(z, \epsilon) \end{aligned}$$

Recursive Competitive Equilibrium

RCE consists of a value function $V(a, K, z)$, individual decision rules $c(a, K, z)$, $h(a, K, z)$, $x(a, K, z)$ aggregate decision rules $C(K, z)$, $L(K, z)$, $X(K, z)$, and the factor prices $W(K, z)$, $R(K, z)$ that satisfy, for all (K, z),

1. conumer's utility maximization in (3).
2. firm's profit maixmization in (2).
3. individthe consistency of individual and aggregate decisions:
    - $c(K, K, z) = C(K, z)$
    - $h(K, K, z) = L(K, z)$
    - $x(K, K, z) = X(K, z)$
4. the aggregate resource constraint:
   $C(K, z) + X(K, z) = Y(K, z)$

Charateristic Equations (1)

Consumer's Utility Maximization Requires:

- Optimal consumption (Euler equation):

$$u_c(c,h) = \beta E\left[u_c(c',h')(1+r')\right]$$

- Optimal labor supply:

$$\frac{u_h(c,h)}{u_c(c,h)} = w$$

- Budget constraint:

$$a' = (1+r)a + wh - c$$

Charateristic Equations (2)

Firm's Profit Maximization Requires:

- Optimal capital demand:

$$F_K(z, K, L) = r + \delta$$

- Optimal labor demand:

$$F_L(z, K, L) = w$$

- Stochastic process for the productivity shock:

$$z' = g(z, \epsilon')$$

Charateristic Equations (3)

Market Clearing Requires:

- Capital market:

$$a = K$$

- Labor market:

$$h = L$$

Computation of RCE

Difficult to solve a system of non-linear difference equations ⇒ numercial approximation to solution

1. Calibration
2. Computational Algorithms
   - Log-linearization around steady state
   - Second or higher order Taylor approximation
3. Deterministic evolution of equilibrium:
   - Temporary/Permanent changes in productivity
   - Policy changes
4. Stochastic fluctuations of equilibrium
   - Cyclical properties
   - Impulse responses

Economist takes care of (1) while Dynare does the rest

Calibration: Prescott (1986)

- Preferences: $u(c) = \log c + A \log(1 - h)$
- Production technology: $F(K, L, z) = zK^{\alpha} L^{1-\alpha}$
- Stochastic process for productivity:

$$\log z_{t+1} = \rho \log z_t + \epsilon_{t+1}, \quad \text{where } \epsilon_{t+1} \sim N(0, \sigma_\epsilon^2)$$

- Parameter values for the quarterly frequency:
  - Capital share: $\alpha = 0.36$
  - Capital depreciation: $\delta = 0.025$
  - Interest rate: $r = 0.01$
  - Discount factor: $\beta = 1/(1 + r) = 0.99$
  - Persistence of shock: $\rho = 0.95$
  - Standard deviation of innovation to shock: $\sigma_\epsilon = 0.007$
  - Utility from leisure: A to be chosen so that $h = 1/3$ in steady state

Calibrated Charateristic Equations

$$\frac{1}{c} = \beta E\left[\frac{1}{c'}\left(1 + r'\right)\right]$$

$$\frac{A}{1-h} = \frac{w}{c}$$

$$a' = (1 + r)a + wh - c$$

$$r = \alpha z\left(\frac{K}{L}\right)^{\alpha - 1} - \delta$$

$$w = (1 - \alpha)z\left(\frac{K}{L}\right)^{\alpha}$$

$$\log z' = \rho \log z + \epsilon', \quad \epsilon \sim N(0, \sigma_\epsilon^2)$$

$$a = K, \quad h = L$$

Log-Linearization of the Model

- Impossible to solve the model analytically since the characteristic equations are highly nonlinear.
- Interested in the behavior of the model economy at the business cycle frequency assuming that the economy is fluctuating around its long-run equilibrum (steady state).
- Examine the local dynamics aound the steady state.
- Useful to linear-aprroximate the model in log-deviation form of variables $\Rightarrow$ percentage deviations from the steady state.

- There are several ways to derive log-deviation form of a variable, each has its own advantage.
- The common idea of all methods is to apply the first-order Taylor expansion to a nonlinear function to get a locally linear approximation
- In this lecture, a simple and general method that directly apply the first-order Taylor expansion to the original nonllinear function to approximate with a linear function in log-diviation from.
- Let $\hat{x}_t$ be the log deviagtion of x_t from its steady state value $\bar{x}$.

$$\hat{x}_t = \ln x_t - \ln \bar{x} \approx \frac{x_t - \bar{x}}{\bar{x}}$$

Univariate Case: Log-linearize $y_t = f(x_t)$.

- $LHS \approx \bar{y} + (y_t - \bar{y}) = \bar{y} + \bar{y}\hat{y}_t$
- $RHS \approx f(\bar{x}) + f'(\bar{x})(x_t - \bar{x}) = f(\bar{x}) + f'(\bar{x})\bar{x}\hat{x}_t$
- Since $\bar{y} = f(\bar{x})$, equating both sides yields

$$\bar{y}\hat{y}_t = f'(\bar{x})\bar{x}\hat{x}_t \tag{4}$$

Multivariate Case: Log-linearize $z_t = g(x_t, y_t)$.

- $LHS \approx \bar{z} + \bar{z}\hat{z}_t$
- $RHS \approx g(\bar{x}, \bar{y}) + g_x(\bar{x}, \bar{y})\bar{x}\hat{x}_t + g_y(\bar{x}, \bar{y})\bar{y}\hat{y}_t$
- Since $\bar{z} = g(\bar{x}, \bar{y})$, equating both sides yields

$$\bar{z}\hat{z}_t = g_x(\bar{x}, \bar{y})\bar{x}\hat{x}_t + g_y(\bar{x}, \bar{y})\bar{y}\hat{y}_t \tag{5}$$

Example of Univariate Case: $k_{t+1} = sk_t^{\alpha} + (1-\delta)k_t$.

- Use eq. (4)

$$\bar{k}\hat{k}_{t+1} = \left(s\alpha\bar{k}^{\alpha-1} + 1 - \delta\right)\bar{k}\hat{k}_t$$
$$\hat{k}_{t+1} = \left(s\alpha\bar{k}^{\alpha-1} + 1 - \delta\right)\hat{k}_t$$

- Since $\bar{k} = s\bar{k}^{\alpha} + (1-\delta)\bar{k}$ in the steady state, $s\bar{k}^{\alpha-1} = \delta$.

$$\hat{k}_{t+1} = \big(1 - (1-\alpha)\delta\big)\hat{k}_t$$

Example of Multivariate Case: $k_{t+1} = sz_t k_t^{\alpha} + (1-\delta)k_t$.

- Use eq. (5)

$$\bar{k}\hat{k}_{t+1} = \left(s\alpha\bar{z}\bar{k}^{\alpha-1} + 1 - \delta\right)\bar{k}\hat{k}_t + s\bar{k}^{\alpha}\bar{z}\hat{z}_t$$
$$\hat{k}_{t+1} = \left(s\alpha\bar{z}\bar{k}^{\alpha-1} + 1 - \delta\right)\hat{k}_t + \left(s\bar{z}\bar{k}^{\alpha-1}\right)\hat{z}_t$$

- Since $\bar{k} = s\bar{z}\bar{k}^{\alpha} + (1-\delta)\bar{k}$ in the steady state, $s\bar{z}\bar{k}^{\alpha-1} = \delta$.

$$\hat{k}_{t+1} = \left(1 - (1-\alpha)\delta\right)\hat{k}_t + \delta\hat{z}_t$$

Log-Linearized Characteristic Equations

$$\frac{1}{c} = \beta E\left[\frac{1}{c'}\Big(1+r'\Big)\right] \Rightarrow 0 = E\left[\hat{c}_t - \hat{c}_{t+1} + \frac{\bar{r}}{1+\bar{r}}\hat{r}_{t+1}\right]$$

$$\frac{A}{1-h} = \frac{w}{c} \Rightarrow \hat{h}_t = \frac{1-\bar{h}}{\bar{h}}\big(\hat{w}_t - \hat{c}_t\big)$$

$$a' = (1+r)a + wh - c \Rightarrow \hat{c}_t = \phi_a\big(\hat{r}_t + \hat{a}_t\big) + (1-\phi_a)\big(\hat{w}_t + \hat{h}_t\big) + \left(\frac{1}{\bar{r}}\right)\phi_a\big(\hat{a}_t - \hat{a}_{t+1}\big)$$

$$r = \alpha z\left(\frac{K}{L}\right)^{\alpha-1} - \delta \Rightarrow \hat{r}_t = \left(\frac{\bar{r}+\delta}{\bar{r}}\right)\Big(\hat{z}_t + (\alpha-1)\big(\hat{K}_t - \hat{L}_t\big)\Big)$$

$$w = (1-\alpha)z\left(\frac{K}{L}\right)^{\alpha} \Rightarrow \hat{w}_t = \hat{z}_t + \alpha\big(\hat{K}_t - \hat{L}_t\big)$$

$$\log z' = \rho\log z + \epsilon' \Rightarrow \hat{z}_{t+1} = \rho\hat{z}_t + \epsilon_{t+1}$$

$$a = K, \quad h = L \Rightarrow \hat{a}_t = \hat{K}_t, \quad \hat{h}_t = \hat{L}_t$$