

Let $A \in \mathbb{R}^{n \times p}$ $(n \geq p)$ be a full-rank matrix. For some $y \in \mathbb{R}^n$, consider the least squares $\min_{x \in \mathbb{R}^p} \|y - Ax\|$. Then write a solution \hat{x} to the least squares by Q and R, where Q and R are the factors of QR decomposition, i.e. A = QR.

$$Ax = y$$

Solve the following linear equation using Gaussian elimination method

$$3x + 5y - 2z = 17$$

$$2x - 2y + 4z = 6$$

$$4x + 4y - z = 19$$

$$\begin{bmatrix}
3 & 5 & -2 & 11 \\
2 & -2 & 4 & 6
\end{bmatrix}$$

$$2 & -2 & 4 & 6
\end{bmatrix}$$

$$2 & -2 & 4 & 6
\end{bmatrix}$$

$$4 & 4 & -1 & 19
\end{bmatrix}$$

$$4 & 4 & -1 & 19
\end{bmatrix}$$

$$0 & -\frac{9}{3} & \frac{5}{3} & \frac{19}{3}
\end{bmatrix}$$

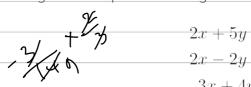
$$0 & 1 & -1 & 1$$

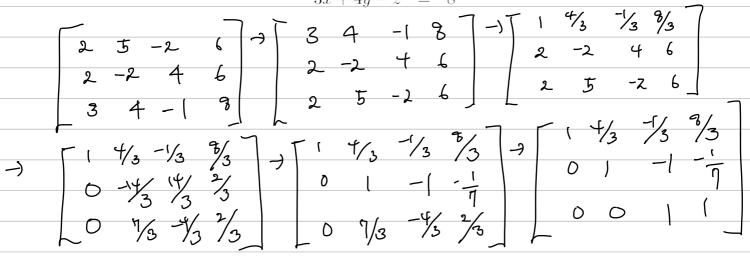
$$0 & -9/3 & \frac{5}{3} & \frac{19}{3}
\end{bmatrix}$$

$$0 & 0 & -1 & -1$$

$$0 & -9/3 & \frac{5}{3} & \frac{19}{3}
\end{bmatrix}$$

Solve the following linear equation using Gaussian elimination method with partial pivoting





$$2=1, y=\frac{6}{7}, x=\frac{13}{7}$$

Let A be some 4 by 4 matrix satisfying

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|-----------------------------------|------|------|-----|------------|---------|-----|----|--|----------------------|-----|-----|----|-----|-------------|-----|-----|----|
| | /h » | | | | M_{2} | | | | | | Az | | | | | | |
| Γ1_ | _0_ | 0 | _0] | Γ 1 | 0 | _0_ | _0 | | <u> [2 </u> | _0_ | _0_ | _0 | | [2] | 1 | _0_ | -3 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | | 1 | 2 | 0 | 0 | A — | 0 | 1 | 4 | 1 |
| 0 | 0 | 1 | 0 | 0 | -0.5 | 1 | 0 | | -1 | 0 | 2 | 0 | _A | 0 | 0 | 3 | -2 |
| 0 | -0- | 0.25 | 1 | | -0.25 | -0- | 1 | | $\lfloor -2 \rfloor$ | -0- | -0- | 2 | | <u>-</u> 0- | -0- | -0- | -2 |

Then, find A, and explain its LU decomposition with each row operation.

$$M \stackrel{-1}{\cdot} M_{2} \stackrel{-1}{\cdot} M_{3} = \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ -1/4 & 1/2 & 0 & 0 \\ 1/4 & 1/4 & 2 & 0 \\ 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix}$$

$$A_{\lambda} = \begin{bmatrix} 2 & 1 & 0 & -3 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 2 \end{bmatrix} = 0$$

$$A = \begin{bmatrix} 1 & 1/2 & 0 & -\frac{3}{2} \\ -\frac{1}{2} & \frac{1}{4} & 2 & \frac{5}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{5}{2} & -\frac{1}{2} \\ 1 & \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

Suppose that for some matrix A, svd(A) in R gives the following result:

```
> svd(A)
$d
[1] 10.9053 6.0228 3.8472
$u
                [,2]
         [,1]
                         [,3]
[1,] -0.48721 -0.18126 0.19771
[2,] -0.16910 -0.76084 -0.61517
[3,] -0.83050 0.10561 0.19620
[4,] 0.21049 -0.61409 0.73755
$v
          [,1]
                   [,2]
                             [,3]
[1,] -0.185133 -0.980175 -0.070586
[2,] -0.982195 0.186891 -0.019103
[3,] -0.031916 -0.065792 0.997323
```

Then what are singular values of A? Can we get a determinant of A?

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AN 4×3 剂智则 叫他们, Set(A)是 空管 与家件.

Suppose that for some matrix A, svd(A) in R gives the following result:

> svd(A)

\$d

[1] 17.1189 9.5683 8.4494

\$u

[,1] [,2] [,3]

[1,] -0.66926 0.68005 0.29938

[2,] -0.65196 -0.34418 -0.67563

[3,] -0.35642 -0.64736 0.67371

\$v

[,1] [,2] [,3]

[1,] -0.66926 -0.68005 0.29938

Then what are singular values of A? Can we get any information on eigenvalues of A or determinant of A?

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