pmlsc: Large Scale Condensation

The module is written in the ‘pmlsc’ file and called in ‘padmn’, ‘pcumc’, ‘pshcn’, ‘pcldphys’ and ‘pvdfm’ files.

Physical basis for statistical PDF scheme

General Circulation Models (GCMs) typically adopt fractional cloud cover (the volume of cloudy air per total air volume in a grid box) assumption to realistically represent clouds because of their coarse horizontal resolution ($O(100km)$). Statistical cloud schemes assume a subgrid-scale probability density function (PDF) of humidity within the grid. Integration of the specific PDFs will give the cloud fraction and the amount of water condensate consistently.

By means of the “fast condensation” assumption, the cloud water amount in a local area in the grid is

$$q_c = (q_t - q_s)\,\delta\,(q_t - q_s) \tag{hpc.1}$$

where q_s denotes the saturation mixing ratio and q_c does the cloud water ratio. q_t is sum of water vapor and cloud water mixing ratio. $\delta(x)$ denotes the Heviside function of x.

The majority of statistical cloud schemes use the so-called “s-distribution” following Sommeria and Deardorff (1977). A single variable s, which considers the subgrid-scale perturbations of liquid temperature T_l and total water mixing ratio q_t, is employed. s is defined as

$$s = a_L\,(q_t - \alpha_L T_l)$$

where

$$a_L = 1/\left(1 + L\alpha_L/c_p\right), \alpha_L = \partial q_s/\left.\partial T\right|_{T=\bar{T}_l}.$$

For any choice of the PDF of s, denoted as $G(s)$, the grid-mean cloud fraction, C, and cloud water content, q_c, are obtained by integrating $G(s)$ and $(Qc + s)G(s)$,

$$C = \int_{-Q_c}^{\infty} G(s)ds$$

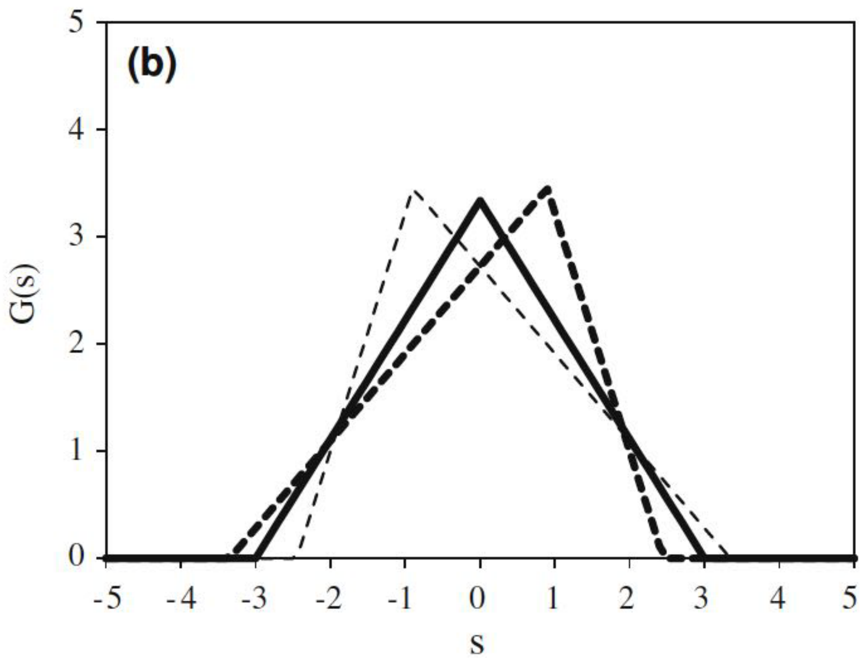
$$\bar{q}_c = \int_{-Q_c}^{\infty} (Q_c + s)\,G(s)ds,$$

where Qc denotes the grid-scale saturation deficit defined as

$$Q_c \equiv a_L\left\{\bar{q}_t - q_s\left(\bar{T}_l, \bar{p}\right)\right\}.$$

Hybrid Prognostic Cloud (HPC) scheme

The statistical scheme implemented in MIROC6 is called Hybrid Prognostic Cloud (HPC) scheme (Watanabe et al. 2009). The HPC scheme proposes two types of shape for the PDF $G(s)$, Double-uniform PDF and Skewed-triangular PDF. Here we focus on Skewed-triangular scheme because MIROC6 adoptes the shape. The physical basics of the scheme are in common with Double-uniform PDF.



Example of the basis PDF for HPC: skewed-triangular functions. Copied from Fig.1 in Watanabe et al. 2009.

The scheme preicts variance (V) and skewness (S) of the PDF. V, S, the second moment μ_2, and the third moment μ_3 are defined as follows.

$$\mu_2 \equiv V = \int_{-\infty}^{\infty} s^2 G(s) ds$$

$$\mu_3 \equiv \mu_2^{3/2} S = \int_{-\infty}^{\infty} s^3 G(s) ds$$

V and S are affected by cumulus convection, cloud microphysics, turbulent mixing, and advection.

The integrals to obtain C and q_c is symbolically expressed as

$$C = I_C\left(\bar{p}, \bar{T}_l, \bar{q}_t, \mathcal{V}, \mathcal{S}\right) \tag{W09-1}$$

$$\bar{q}_c = I_q\left(\bar{p}, \bar{T}_l, \bar{q}_t, \mathcal{V}, \mathcal{S}\right) \tag{W09-2}$$

where $\bar{p}$ denotes the pressure. The overbars denote the grid-mean quantity.

If the PDF is not too complicated, (1, 2) can be analytically solved for V and S by defining integrand functions $\tilde{I}$ as

$$\mathcal{V} = \tilde{I}_{\mathcal{V}}\left(\bar{p}, \bar{T}_l, \bar{q}_v, \bar{q}_c, C\right) \tag{W09-4}$$

$$\mathcal{S} = \tilde{I}_{\mathcal{S}}\left(\bar{p}, \bar{T}_l, \bar{q}_v, \bar{q}_c, C\right) \tag{W09-5}$$

The relationship between (1, 2) and (4, 5) is quasireversible. The double-uniform function and skewed-triangular function PDFs are selected for $G(s)$ because of their feasibility in analytically nalderiving $\tilde{I}$.

PDF change through processes

The HPC cloud scheme is composed using prognostic equations for four variables determining I, namely, T_l, q_t, V, and S. The prognostic variables can be T_l, q_t, C, and q_c that determine $\tilde{I}$.

Prognostic equations for the PDF variance and skewness are expressed as

$$\frac{D\mathcal{V}}{Dt} = \left.\frac{\Delta\mathcal{V}}{\Delta t}\right|_{\text{conv.}} + \left.\frac{\Delta\mathcal{V}}{\Delta t}\right|_{\text{micro.}} + \left.\frac{\Delta\mathcal{V}}{\Delta t}\right|_{\text{turb.}} + \left.\frac{\Delta\mathcal{V}}{\Delta t}\right|_{\text{others}} - \varepsilon\mathcal{V}$$

$$\frac{D\mathcal{S}}{Dt} = \left.\frac{\Delta\mathcal{S}}{\Delta t}\right|_{\text{conv.}} + \left.\frac{\Delta\mathcal{S}}{\Delta t}\right|_{\text{micro.}} + \left.\frac{\Delta\mathcal{S}}{\Delta t}\right|_{\text{turb.}} + \left.\frac{\Delta\mathcal{S}}{\Delta t}\right|_{\text{others}} - \varepsilon_{\mathcal{S}}$$

where subscripts ‘conv.’, ‘micro.’ and ‘turb.’ indicate cumulus convection, cloud microphysics and turbulent mixing processes, which all affect the PDF shape. The last terms represent dissipation due to subgrid-scale horizontal motions. The specific formulations for each term are described below.

The HPC scheme is referred to as and $G(s)$ is updated every after the process that affects cloud water PDF. $G(s)$ is thus modified several times within a single time step.

Cumulus convection

The total effect of cumulus convection to the PDF moments is written as

$$\left.\frac{\Delta \mathcal{V}}{\Delta t}\right|_{\text{conv.}} = M_c \frac{\partial \mathcal{V}}{\partial z} + \frac{\Delta \tilde{I}_{\mathcal{V}}}{\Delta t} \tag{W09-14}$$

$$\left.\frac{\Delta \mathcal{S}}{\Delta t}\right|_{\text{conv.}} = M_c \frac{\partial \mathcal{S}}{\partial z} + \frac{\Delta \tilde{I}_{\mathcal{S}}}{\Delta t} \tag{W09-15}$$

M_c is the cumulus mass-flux including updraft in the convection tower and downdraft in the environment. The vertical transport of the PDF moments is represented by the first terms on the right side hand of (14, 15).

Cumulus convections modify the grid-mean T_l, q_t, and q_c by upward transportation of grid-mean moist static energy, q_v, and q_c. Detrainment also affects these variables. The detrainment of the cloudy air mass is considered, as in Bushell et al. (2003),

$$\left.\frac{\partial C}{\partial t}\right|_{\text{conv.}} = D(1-C)$$

The second terms on the right hand side of (14, 15) indicates that the changes in the PDF moments is calculated consistent with the changes in the grid-scale temperature, humidity, cloud water, and cloud fraction.

$$\Delta \tilde{I}_{\mathcal{X}} = \tilde{I}_{\mathcal{X}}\left(\bar{p}, \bar{T}_l + \Delta \bar{T}_l, \bar{q}_v + \Delta \bar{q}_v, \bar{q}_c + \Delta \bar{q}_c, C + \Delta C\right) - \tilde{I}_{\mathcal{X}}\left(\bar{p}, \bar{T}_{l,} \bar{q}_v, \bar{q}_c, C\right) \tag{W09-16}$$

where $\mathcal{X}$ is either $\mathcal{V}$ or $\mathcal{S}$.

Cloud Microphysics

The tendency due to microphysical processes can be written in a similar manner to the cumulus convection effect.

$$\left.\frac{\Delta \mathcal{V}}{\Delta t}\right|_{\text{micro.}} = \frac{\Delta \tilde{I}_{\mathcal{V}}}{\Delta t}$$

$$\left.\frac{\Delta \mathcal{S}}{\Delta t}\right|_{\text{micro.}} = \frac{\Delta \tilde{I}_{\mathcal{S}}}{\Delta t}$$

Changes in $\bar{T}_l, \bar{q}_v, \text{and} \bar{q}_c$ are derived from microphysical tendency terms including precipitation, evaporation, melting/freezing.

Turbulent mixing

From the definition of s, the PDF variance $\mathcal{V}$ becomes

$$\mathcal{V} = a_L^2 \left(\overline{q_t'^2} + \alpha_L^2 \Pi \overline{\theta_l'^2} - 2\alpha_L \Pi \overline{q_t' \theta_l'} \right),$$

where Π is the Exner function. Assuming the level-2 closure in Nakanishi and Niino (2004), the time evolution of V can be derived as

$$\begin{aligned} \left.\frac{\Delta \mathcal{V}}{\Delta t}\right|_{\mathrm{turb.}} =& 2a_L^2 \left[(\alpha_L \Pi)^2 K_H \left(\frac{\partial \bar{\theta}_l}{\partial z} \right)^2 + K_q \left(\frac{\partial \bar{q}_t}{\partial z} \right)^2 \right. \\ & \left. -\alpha_L \Pi \left(K_H + K_q \right) \frac{\partial \bar{\theta}_l}{\partial z} \frac{\partial \bar{q}_t}{\partial z} \right] - \frac{2q}{\Lambda_2} \mathcal{V}, \end{aligned} \tag{W09-28}$$

where K_H and K_q are the mixing coefficients for sensible heat and moisture, respectively. $q^2 = \overline{u'^2 + v'^2 + w'^2}$ denotes the turbulent kinetic energy. The other symbols follow the original notation.

Since the turbulence production does not affect the PDF shape parameter defined by the third moment (cf. Tompkins 2002), the skewness change $\Delta \mathcal{S} / \left.\Delta t\right|_{\mathrm{turb.}}$ is simply calculated due to the variance change in (28).

Subgrid-scale horizontal eddy

In the planetary boundary layer, the subgrid-scale inhomogeneity is dissipated due to the turbulent mixing. In free atmosphere, the grid box will be homogenized mainly due to mesoscale motions, which are expressed by the Newtonian damping as in (Tompkins 2002): $\varepsilon_{\mathcal{V}} = \frac{\mathcal{V}}{\tau_h}, \varepsilon_{\mathcal{S}} = \frac{\mathcal{S}}{\tau_h}$, where the relaxation timescale is parameterized by the horizontal wind shear as

$$\tau_h^{-1} = C_s^2 \left\{ \left(\frac{\partial \bar{u}}{\partial x} \right)^2 + \left(\frac{\partial \bar{v}}{\partial y} \right)^2 \right\}^{1/2}$$

The coefficient C_s is set to 0.23 following Tompkins 2002.

Other processes

Dynamics, shallow convection, radiation, mass source, and dissipation heating processes change the grid-mean temperature and humidity. Such effects on the shape of PDF are included following (16).

Solving procedures

The shape of the Skewed-triangular PDF is represented as follows. The widths defined by positions of the left and right edges on the s-coordinate are denoted as a and b, respectively. The position of the top, denoted as q, is constrained by $a+b+q=0$. By definition, $q \leq b$ and $a \leq q$ must be satisfied. The PDF is then expressed as

$$G(s) = \begin{cases} -\frac{2(s-b)}{(b-q)(b-a)} & \text{for } q < s \leq b \\ \frac{2(s-a)}{(q-a)(b-a)} & \text{for } a < s \leq q \end{cases}$$

The pmlsc module includes two main subroutines, PDF2CLD and CLD2PDF. The subroutine PDF2CLD calculates C and $\bar{q}_c$ given $\bar{p}, T_l, \bar{q}_t, \mathcal{V}, \mathcal{S}$. The subroutine CLD2PDF calculates $\mathcal{V}$ and $\mathcal{S}$ given $\bar{p}, T_l, \bar{q}_t, \bar{q}_c, C$. We will derive the concrete calculation processes in this subsection.

PDF2CLD

From μ_1, μ_2, μ_3 **To** a, b, q The first, second, and third moments of the PDF is calculated as follows.

$$\mu_1 = \int_{q-a}^{q+b} sG(s)ds = q + \frac{b-a}{3} \tag{E08-7}$$

$$\mu_2 = \int_{q-a}^{q+b} (s-\mu_1)^2 G(s)ds = \frac{a^2+ab+b^2}{18} \tag{E08-8}$$

$$\mu_3 = \int_{q-a}^{q+b} (s-\mu_1)^3 G(s)ds = \frac{(b-a)\left(2a^2+5ab+2b^2\right)}{270} \tag{E08-9}$$

From (7,8,9), we will derive the solution for a, b, q given μ_1, μ_2, μ_3.

We define $\delta \equiv b-a, \beta \equiv ab$. (8,9) are

$$\delta^2 + 3\beta = 18\mu_2$$

$$\delta\left(\beta + 12\mu_2\right) = 90\mu_3$$

Eliminate β or δ from these equations, you will get the equations.

$$\delta^3 - 54\mu_2\delta + 270\mu_3 = 0 \tag{E08-10}$$

$$\beta = 6\mu_2 - \frac{1}{3}\delta^2 \tag{E08-11}$$

We apply the formula for the solution of a cubic equation to (10) to obtain δ.

$$\delta = 2\sqrt{18\mu_2}\cos\left(\frac{1}{3}\cos^{-1}\left(\frac{-135\mu_3}{\sqrt{(18\mu_2)^3}}\right) + \frac{4}{3}\pi\right)$$

β is obtained from (11). We define $\alpha \equiv \sqrt{\delta^2 + 4\beta}$ for simplicity. Finally, a, b, q is calculated as follows.

$$a = (\alpha - \delta)/2$$

$$b = (\alpha + \delta)/2$$

$$q = \mu_1 - \delta/3$$

From PDF to C and qc Once the PDF $G(s)$ is determined by the parameters a, b, q, the cloud fraction C and grid-mean cloud water mixing ratio $\bar{q_c}$ are derived as follows.

$$C = \begin{cases} 0 & \text{if } b < -Q_c \\ \frac{(Q_c+b)^2}{(b-q)(b-a)} & \text{if } q \le -Q_c \le b \\ \frac{(Q_c+a)^2}{(q-a)(b-a)} & \text{if } a \le -Q_c \le q \\ 1 & \text{if } -Q_c < a \end{cases} \tag{E08-15}$$

$$\bar{q_c} = \begin{cases} 0 & \text{if } b < -Q_c \\ \frac{1}{3}C\left(Q_c + b\right) & \text{if } q \le -Q_c \le b \\ Q_c - \frac{1}{3}(1-C)\left(Q_c + a\right) & \text{if } a \le -Q_c \le q \\ Q_c & \text{if } -Q_c < a \end{cases} \tag{E08-16}$$

CLD2PDF

From $\bar{q_c}, C$ To a, b, q We can not determine the position of Q_c in the triangle at the beginning of the calculation. Thus we calculate a, b assuming that $a \le -Q_c \le q$ at first. If the calculated parameters are physically consistent with the PDF ($a + b \ge 0$), a, b, q are determined. Otherwise, we regard $q \le -Q_c \le b$ and then a, b, q are derived.

1. $a \le -Q_c \le q$

From (16), a is derived as follows.

$$a = \frac{3\left(Q_c - q_c\right)}{1 - C} - Q_c$$

We eliminate q from (15) using $q = -a - b$. The quadratic equation for b is obtained.

$$b^2 + ab - 2a^2 + \left(Q_c + a\right)^2 / (1 - C) = 0 \tag{E08-17}$$

The physically meaningful solution for b is

$$b = \left(-a\sqrt{9a^2 - 4\left(Q_c + a\right)^2 / (1 - C)}\right)/2 \tag{E08-18}$$

2. $q \leq -Q_c \leq b$

From (16), b is

$$b = \frac{3q_c}{C} - Q_c$$

We eliminate q from (15) using $q = -a - b$. The quadratic equation of a is obtained.

$$a^2 + ab - 2b^2 + \left(Q_c + b\right)^2 / C = 0 \tag{E08-17}$$

The physically meaningful solution for a is

$$a = \left(-b - \sqrt{9b^2 - 4\left(Q_c + b\right)^2 / C}\right)/2 \tag{E08-18}$$

Adjustment of Cloud Fraction When there is no physically meaningful solution for (18), C is adjusted so that a reasonable solution is obtained. The critical conditions for the existence of real solutions for (18) are as follows.

$$\begin{array}{ll} 9a^2 - 4\left(Q_c + a\right)^2 / (1 - C) = 0 & (a \leq -Q_c \leq q) \\ 9b^2 - 4\left(Q_c + b\right)^2 / C = 0 & (q \leq -Q_c \leq b) \end{array}$$

Eliminate a and b uging (17), we get the relationship between C and q_c,

$$\begin{array}{ll} 9\left(\frac{3(Q_c - q_c)}{1-C} - Q_c\right)^2 = \frac{4}{1-C}\left(\frac{3(Q_c - q_c)}{1-C}\right)^2 & (a \leq -Q_c \leq q) \\ 9\left(\frac{3q_c}{C} - Q_c\right)^2 = \frac{4}{C}\left(\frac{3q_c}{C}\right)^2 & (q \leq -Q_c \leq b) \end{array}$$

We take the square root of the both sides of the equations and define $\gamma_1 \equiv \sqrt{1 - C}$ and $\gamma_2 \equiv \sqrt{C}$. The cubic equations for γ is obtained.

$$\begin{array}{ll} \gamma_1^3 - 3\left(1-\frac{q_c}{Q_c}\right)\gamma_1 \pm 2\left(1-\frac{q_c}{Q_c}\right) = 0 & (a \le -Q_c \le q) \\ \gamma_2^3 - 3\frac{q_c}{Q_c}\gamma_2 \pm 2\frac{q_c}{Q_c} = 0 & (q \le -Q_c \le b) \end{array}$$

We define $R_1 = 1 - \frac{q_c}{Q_c}, R_2 = \frac{q_c}{Q_c}$.

$$\gamma^2 = \begin{cases} -4R\sinh^2\left(\frac{1}{3}\sinh^{-1}\left(\frac{1}{\sqrt{-R}}\right)\right) & (R < 0) \\ 4R\cos^2\left(\frac{1}{3}\cos^{-1}\left(\frac{1}{\sqrt{R}}\right) + \frac{4}{3}\pi\right) & (R > 1) \end{cases} \tag{E08-26}$$

Note that, $\gamma = \gamma_1, R = R_1$ $(a \le -Q_c \le q)$ or $\gamma = \gamma_2, R = R_2$ $(q \le -Q_c \le b)$.

The actual calculation procedure is as follows. If the solution for (18) is not a real number, C is adjusted using (26). Then we solve (18) again.

From a, b, q To μ_2, μ_3 By definition, the PDF moments are expressed in terms of a and b.

$$\mu_2 = \frac{a^2 + ab + b^2}{6}$$

$$\mu_3 = \frac{-(a+b)ab}{10}$$

Treatment of cloud ice and in-cloud water vapor

Because the original HPC scheme by Watanabe et al. (2009) does not consider the cloud ice, it is modified when coupled with the Wilson and Ballard (1999) ice microphysics. Since the statistical PDF scheme employs a 'fast condensation' assumption that is no more valid for ice, the ice mixing ratio is assumed to be conserved in the large scale condensation process.

Here we assume that - the water vapor mixing ratio within the cloudy area in a grid is constant - cloud ice preferentially exists in areas with large total water content

Based on these assumptions, the cloud fraction and each condensate mixing ratios are diagnosed. The notations for the mixing ratios (q_l, q_i, q_v, q_{vi}) of liquid water (subscript l), ice (subscript i), vapor (subscript v), in-cloud vapor (subscript vi) are employed.

At first the total condensate mixing ratio $q_c = q_l + q_i$ is diagnosed from q_t and T_l assuming that ice does not exist in the grid. The saturation mixing ratio is set for liquid (q_{satl}).

Mixed-phase cloud is generated when the condensate amount is more than the ice content $(q_c > q_i)$, whereas the cloud fraction and vapor amount are adjusted in the case of a pure ice cloud when the condensate amount is less than the ice content ($q_c < q_i$). Specifically, q_c, C and q_{vi} are calculated as follows.

1. $q_c > q_i$

Liquid-phase clouds and ice clouds coexist.

$$q_l = q_c - q_i$$

$$q_{vi} = q_{\mathrm{satl}}$$

2. $q_c < q_i$

Only ice clouds exist ($q_l = 0$). In this case, C and q_{vi} are rediagnosed. We eliminate Q_c in (15,16) assuming that $q_c = q_i$. Equations for C are given as

$$C^3 = \frac{9q_i^2}{(b-q)(b-q)} \; (q \leq -Q_c \leq b)$$

$$C^3 + 3C^2 = 4 - \frac{9\left(q_i + a\right)^2}{(q-a)(b-a)} \; (a \leq -Q_c < q)$$

From these equations, C is obtained as follows.

$$C = \begin{cases} \sqrt[3]{\frac{9q_i^2}{(b-q)(b-a)}} & \left(0 \leq q_i \leq \frac{(b-q)^2}{3(b-a)}\right) \\ 2\cos\left(\frac{1}{3}\cos^{-1}\left(1 - \frac{9(q_i+a)^2}{2(q-a)(b-a)}\right)\right) - 1 & \left(\frac{(b-q)^2}{3(b-a)} < q_i \leq -a\right) \\ 1 & (-a < q_i) \end{cases}$$

,where

$$Q_c = \frac{3q_i}{C} - b = \sqrt[3]{3q_i(b-q)(b-a)} - b.$$

Given Q_c, $q_{vi} = q_t - Q_c$ is calculated as follows.

$$q_{vi} = \begin{cases} q_t - \frac{3q_i}{C} + b & \left(0 \leq q_i \leq \frac{(b-q)^2}{3(b-a)}\right) \\ q_t - \frac{3(q_i+a)}{2+C} + a & \left(\frac{(b-q)^2}{3(b-a)} < q_i \leq -a\right) \\ q_t - q_i & (-a < q_i) \end{cases}$$