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Mechanical Processes

Basic Equations

Basic Equations

The fundamental equations are a system of primitive equations at the spherical (λ, φ) and η coordinates, given as follows (Arakawa and Konor 1996).

1. Continuity equation

$$\frac{\partial m}{\partial t} + \nabla_\eta \cdot (m \mathbf{v}_H) + \frac{\partial (m \dot{\eta})}{\partial \eta} = 0$$

2. Hydrostatic equation

$$\frac{\partial \Phi}{\partial \eta} = -\frac{R T_v}{p} m$$

3. Equation of motion

$$\frac{\partial \zeta}{\partial t} = \frac{1}{a \cos\varphi} \frac{\partial A_v}{\partial \lambda} - \frac{1}{a \cos\varphi} \frac{\partial}{\partial \varphi} (A_u \cos\varphi) - \mathcal{D}(\zeta)$$

$$\frac{\partial D}{\partial t} = \frac{1}{a \cos\varphi} \frac{\partial A_u}{\partial \lambda} + \frac{1}{a \cos\varphi} \frac{\partial}{\partial \varphi} (A_v \cos\varphi) - \nabla_\eta^2 (\Phi + R\bar{T}\pi + E) - \mathcal{D}(D)$$

4. Thermodynamic equation

$$\begin{gathered}\frac{\partial T}{\partial t} = -\frac{1}{a \cos\varphi} \frac{\partial u T'}{\partial \lambda} - \frac{1}{a} \frac{\partial}{\partial \varphi} (v T' \cos\varphi) + T' D \\ - \dot{\eta} \frac{\partial T}{\partial \eta} + \frac{\kappa T}{\sigma} \left[B \left(\frac{\partial \pi}{\partial t} + \mathbf{v}_H \cdot \nabla_\eta \pi \right) + \frac{m \dot{\eta}}{p_s} \right] + \frac{Q}{C_p} + \frac{Q_{diff}}{C_p} - \mathcal{D}(T)\end{gathered}$$

5. Tracers

ここでは水蒸気の移流方程式を示す。他のトレーサーも同様の方程式に従う。

$$\begin{gathered}\frac{\partial q}{\partial t} = -\frac{1}{a \cos\varphi} \frac{\partial u q}{\partial \lambda} - \frac{1}{a \cos\varphi} \frac{\partial}{\partial \varphi} (v q \cos\varphi) + q D \\ - \dot{\eta} \frac{\partial q}{\partial \eta} + S_q - \mathcal{D}(q)\end{gathered}$$

Here,

$$\begin{gathered}m \equiv \left(\frac{\partial p}{\partial \eta} \right)_{p_s} \\ \theta \equiv T \left(p / p_0 \right)^{-\kappa} \\ \kappa \equiv R / C_p \\ \Phi \equiv g z \\ \pi \equiv \ln p_S \\ \dot{\eta} \equiv \frac{d\eta}{dt} \\ T_v \equiv T (1 + \epsilon_v q) \\ T \equiv \bar{T} + T' \\ \bar{T} \equiv 300 \ \mathrm{K} \\ \zeta \equiv \frac{1}{a \cos\varphi} \frac{\partial v}{\partial \lambda} - \frac{1}{a \cos\varphi} \frac{\partial}{\partial \varphi} (u \cos\varphi) \\ D \equiv \frac{1}{a \cos\varphi} \frac{\partial u}{\partial \lambda} + \frac{1}{a \cos\varphi} \frac{\partial}{\partial \varphi} (v \cos\varphi) \\ A_u \equiv (\zeta + f) v - \dot{\eta} \frac{\partial u}{\partial \eta} - \frac{R T'}{a \cos\varphi} \frac{\partial \pi}{\partial \lambda} + \mathcal{F}_x\end{gathered}$$

$$
\begin{aligned}
A_v &\equiv -(\zeta+f)u - \dot{\eta}\frac{\partial v}{\partial \eta} - \frac{RT'}{a}\frac{\partial \pi}{\partial \varphi} + \mathcal{F}_y \\
E &\equiv \frac{u^2+v^2}{2} \\
\mathbf{v}_H \cdot \nabla &\equiv \frac{u}{a\cos\varphi}\left(\frac{\partial}{\partial \lambda}\right)_\sigma + \frac{v}{a}\left(\frac{\partial}{\partial \varphi}\right)_\sigma \\
\nabla_\eta^2 &\equiv \frac{1}{a^2\cos^2\varphi}\frac{\partial^2}{\partial \lambda^2} + \frac{1}{a^2\cos\varphi}\frac{\partial}{\partial \varphi}\left[\cos\varphi\frac{\partial}{\partial \varphi}\right].
\end{aligned}
$$

$\mathcal{D}(\zeta), \mathcal{D}(D), \mathcal{D}(T), \mathcal{D}(q)$ are horizontal diffusion terms, $\mathcal{F}_\lambda, \mathcal{F}_\varphi$ are forces due to small-scale kinetic processes (treated as 'physical processes'), Q are forces due to radiation, condensation, small-scale kinetic processes, etc. Heating and temperature change due to 'physical processes', and S_q is a water vapor source term due to 'physical processes' such as condensation and small-scale motion. Q_{diff} is the heat of friction and

$$
Q_{diff} = -\mathbf{v}\cdot(\frac{\partial \mathbf{v}}{\partial t})_{diff}.
$$

$(\frac{\partial \mathbf{v}}{\partial t})_{diff}$ is a time-varying term of u, v due to horizontal and vertical diffusion.

Boundary Conditions

鉛直流に関する上下端の境界条件は以下の通りである：

$$
\dot{\eta} = 0 \quad at \;\; \eta = 0, \; 1.
$$

これを用いて連続の式を鉛直積分することで、p_sの予報方程式と、鉛直流の診断方程式が導かれる。