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0.1 Vertical Discretization

Following Arakawa and Konor (1996) but in the Lorentz grid, the basic equations are discretized vertically by differences. This scheme has the following characteristics.

- Save the total integrated mass
- Save the total integrated energy
- Preserving angular momentum for global integration
- Conservation of total mass-integrated potential temperature
- The hydrostatic pressure equation comes down to local (the altitude of the lower level is independent of the temperature of the upper level)
- For a given temperature distribution, constant in the horizontal direction, the hydrostatic pressure equation becomes accurate and the barometric gradient force becomes zero.
- Isothermal atmosphere stays isothermal forever

0.1.1 Model levels

Model level increases in altitude with the vertical level number k. $k = 1/2$ corresponds with the model bottom ($\eta = 1$), while $k = K + 1/2$ corresponds to the model top ($\eta = 0$). Variables ζ, D, T, q are defined at full levels ($k = 1, 2, \ldots K$), while the vertical velocity $\dot{\eta}$ is defined at half levels ($k = 1/2, 3/2, \ldots K + 1/2$). Using constants $A_{k+1/2}$ and $B_{k+1/2}$ and variable surface pressure p_s, air pressure at half levels are defined as below:

$$p_{k+1/2} = A_{k+1/2} + B_{k+1/2}\, p_s. \tag{1}$$

Thus, the normalized pressure $\sigma \equiv p/p_s$ can be written as below:

$$\sigma_{k+1/2} = \frac{A_{k+1/2}}{p_s} + B_{k+1/2}. \tag{2}$$

Using a reference pressure $p_0 = 1000$ hPa, the hybrid-normalized pressure η is defined as below:

$$\eta_{k+1/2} = \frac{A_{k+1/2}}{p_0} + B_{k+1/2}, \tag{3}$$

which is a constant at all levels and is used as the vertical coordinate by default in MIROC 6.0.

Pressure at full levels are interpolated from half-level pressure by the following formula:

$$p_k = \left\{ \frac{1}{1+\kappa} \left(\frac{p_{k-1/2}^{\kappa+1} - p_{k+1/2}^{\kappa+1}}{p_{k-1/2} - p_{k+1/2}} \right) \right\}^{1/\kappa}. \tag{4}$$

For later use, let us define the following:

$$\Delta\sigma_k \equiv \sigma_{k-1/2} - \sigma_{k+1/2}, \tag{5}$$
$$\Delta B_k \equiv B_{k-1/2} - B_{k+1/2}. \tag{6}$$

0.1.2 Vertical discretization

Basic equations vertically discretized at the η hybrid coordinates are shown below.

1. Continuity equation and diagnosis of the vertical velocity

$$\frac{\partial \pi}{\partial t} = -\sum_{k=1}^{K} \{D_k \Delta\sigma_k + (\mathbf{v}_k \cdot \nabla\pi)\Delta B_k\} \tag{7}$$

In MIRCO 6.0, the discretization is conducted in a manner similar to the σ coordinate, which can be optionally selected and was the default in previous versions, to commonize source codes. Thus, the vertical velocity is represented as $\dot{\sigma} = m\dot{\eta}/p_s$. Furthermore, vertical advection $\dot{\eta}(\partial/\partial\eta)$ is replaced with an equivalent form $m\dot{\eta}/p_s(\partial/\partial\sigma)$.

$$(\dot{\sigma} =)\,\frac{(m\dot{\eta})_{k-1/2}}{p_s} = -B_{k-1/2}\frac{\partial \pi}{\partial t} - \sum_{l=k}^{K} \{D_l \Delta\sigma_l + (\mathbf{v}_l \cdot \nabla\pi)\Delta B_l\} \tag{8}$$

$$\frac{(m\dot{\eta})_{1/2}}{p_s} = \frac{(m\dot{\eta})_{k+1/2}}{p_s} = 0 \tag{9}$$

2. Hydrostatic equation

$$\Phi_1 = \Phi_s + C_p(\sigma_1^{-\kappa} - 1)T_{v,1} \tag{10}$$

$$= \Phi_s + C_p\alpha_1 T_{v,1} \tag{11}$$

$$\Phi_k - \Phi_{k-1} = C_p\left[\left(\frac{p_{k-1/2}}{p_k}\right)^{\kappa} - 1\right]T_{v,k} + C_p\left[1 - \left(\frac{p_{k-1/2}}{p_{k-1}}\right)^{\kappa}\right]T_{v,k-1} \tag{12}$$

$$= C_p\alpha_k T_{v,k} + C_p\beta_{k-1}T_{v,k-1} \tag{13}$$

Here,

$$\alpha_k \equiv \left(\frac{p_{k-1/2}}{p_k}\right)^{\kappa} - 1, \tag{14}$$

$$\beta_k \equiv 1 - \left(\frac{p_{k+1/2}}{p_k}\right)^{\kappa}. \tag{15}$$

3. Equations of motion

$$\frac{\partial \zeta_k}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial (A_v)_k}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(A_u\cos\varphi)_k - \mathcal{D}(\zeta_k) \tag{16}$$

$$\frac{\partial D}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial (A_u)_k}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(A_v\cos\varphi)_k - \nabla_\eta^2(\Phi_k + R\bar{T}\pi + (KE)_k) - \mathcal{D}(D_k) \tag{17}$$

$$(A_u)_k = (\zeta_k + f)v_k - \left[\frac{(m\dot{\eta})_{k-1/2}}{p_s}\frac{u_{k-1} - u_k}{\Delta\sigma_{k-1} + \Delta\sigma_k} + \frac{(m\dot{\eta})_{k+1/2}}{p_s}\frac{u_k - u_{k+1}}{\Delta\sigma_k + \Delta\sigma_{k+1}}\right] \tag{18}$$

$$- \frac{1}{a\cos\varphi}\frac{\partial \pi}{\partial \lambda}(C_p T_{v,k}\hat{\kappa} - R\bar{T}) + \mathcal{F}_x \tag{19}$$

$$(A_v)_k = -(\zeta_k + f)u_k - \left[\frac{(m\dot{\eta})_{k-1/2}}{p_s}\frac{v_{k-1} - v_k}{\Delta\sigma_{k-1} + \Delta\sigma_k} + \frac{(m\dot{\eta})_{k+1/2}}{p_s}\frac{v_k - v_{k+1}}{\Delta\sigma_k + \Delta\sigma_{k+1}}\right] \tag{20}$$

$$- \frac{1}{a}\frac{\partial \pi}{\partial \varphi}(C_p T_{v,k}\hat{\kappa} - R\bar{T}) + \mathcal{F}_y \tag{21}$$

$$\hat{\kappa}_k = \frac{B_{k-1/2}\alpha_k + B_{k+1/2}\beta_k}{\Delta\sigma_k} \tag{22}$$

4. Thermodynamic equation

$$\frac{\partial T_k}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial u_k T'_k}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(v_k T'_k \cos\varphi) + H_k + \frac{Q_k}{C_p} + \frac{(Q_{diff})_k}{C_p} - \mathcal{D}(T_k) \tag{23}$$

Here,

$$\begin{aligned}
H_k &\equiv T'_k D_k - \left[\frac{(m\dot{\eta})_{k-1/2}}{p_s}\frac{\hat{T}_{k-1/2} - T_k}{\Delta\sigma_k} + \frac{(m\dot{\eta})_{k+1/2}}{p_s}\frac{T_k - \hat{T}_{k+1/2}}{\Delta\sigma_k}\right] && (24)\\
&+ \left\{\alpha_k \left[B_{k-1/2}\mathbf{v}_k \cdot \nabla\pi - \sum_{l=k}^{K}(D_l\Delta\sigma_l + (\mathbf{v}_l \cdot \nabla\pi)\Delta B_l)\right]\right. && (25)\\
&+ \left.\beta_k \left[B_{k+1/2}\mathbf{v}_k \cdot \nabla\pi - \sum_{l=k+1}^{K}(D_l\Delta\sigma_l + (\mathbf{v}_l \cdot \nabla\pi)\Delta B_l)\right]\right\} \frac{1}{\Delta\sigma_k}T_{v,k} && (26)\\
&= T'_k D_k - \left[\frac{(m\dot{\eta})_{k-1/2}}{p_s}\frac{\hat{T}_{k-1/2} - T_k}{\Delta\sigma_l} + \frac{(m\dot{\eta})_{k+1/2}}{p_s}\frac{T_k - \hat{T}_{k+1/2}}{\Delta\sigma_l}\right] && (27)\\
&+ \hat{\kappa}_k(\mathbf{v}_k \cdot \nabla\pi)T_{v,k} && (28)\\
&- \alpha_k \sum_{l=k}^{K}(D_l\Delta\sigma_l + (\mathbf{v}_l \cdot \nabla\pi)\Delta B_l)\frac{T_{v,k}}{\Delta\sigma_k} && (29)\\
&- \beta_k \sum_{l=k+1}^{K}(D_l\Delta\sigma_l + (\mathbf{v}_l \cdot \nabla\pi)\Delta B_l)\frac{T_{v,k}}{\Delta\sigma_k}, && (30)\\
\hat{T}_{k-1/2} &= a_k T_k + b_{k-1}T_{k-1}, && (31)\\
a_k &= \alpha_k \left[1 - \left(\frac{p_k}{p_{k-1}}\right)^{\kappa}\right]^{-1}, && (32)\\
b_k &= \beta_k \left[\left(\frac{p_k}{p_{k+1}}\right)^{\kappa} - 1\right]^{-1}. && (33)
\end{aligned}$$

5. Tracers

$$\begin{aligned}
\frac{\partial q_k}{\partial t} &= -\frac{1}{a\cos\varphi}\frac{\partial u_k q_k}{\partial\lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(v_k q_k \cos\varphi) + R_k + S_{q,k} - \mathcal{D}(q_k) && (34)\\
R_k &= q_k D_k - \frac{1}{2}\left[\frac{(m\dot{\eta})_{k-1/2}}{p_s}\frac{q_{k-1} - q_k}{\Delta\sigma_k} + \frac{(m\dot{\eta})_{k+1/2}}{p_s}\frac{q_k - q_{k+1}}{\Delta\sigma_k}\right] && (35)
\end{aligned}$$

0.1.3 Differences from σ-coordinate

In MIROC 6.0, the discretization is conducted in a similar form to the σ coordinate. Thus, differences of discretized equations between η and σ coordinates are relatively small, which are listed below:

- In the σ coordinate, $A_{k+1/2}$ is equal to zero at all levels.
- While ΔB_k and $\Delta\sigma_k$ are different in the η coordinates, those are equivalent to each other in the σ coordinate.