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Mechanical Processes

Basic Equations

Basic Equations

The fundamental equations are a system of primitive equations at the spherical (λ, φ) and η coordinates, given as follows (Arakawa and Konor 1996).

1. Continuity equation

$$\frac{\partial m}{\partial t} + \nabla_\eta \cdot (m\mathbf{v}_H) + \frac{\partial (m\dot{\eta})}{\partial \eta} = 0$$

2. Hydrostatic equation

$$\frac{\partial \Phi}{\partial \eta} = -\frac{RT_v}{p} m$$

3. Equation of motion

$$\frac{\partial \zeta}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial A_v}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(A_u \cos\varphi) - \mathcal{D}(\zeta)$$

$$\frac{\partial D}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial A_u}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(A_v \cos\varphi) - \nabla_\eta^2(\Phi + R\bar{T}\pi + E) - \mathcal{D}(D)$$

4. Thermodynamic equation

$$\begin{aligned} \frac{\partial T}{\partial t} &= -\frac{1}{a\cos\varphi}\frac{\partial uT'}{\partial \lambda} - \frac{1}{a}\frac{\partial}{\partial \varphi}(vT'\cos\varphi) + T'D \\ -\dot{\eta}\frac{\partial T}{\partial \eta} &+ \frac{\kappa T}{\sigma}\left[B\left(\frac{\partial \pi}{\partial t} + \mathbf{v}_H \cdot \nabla_\eta \pi\right) + \frac{m\dot{\eta}}{p_s}\right] + \frac{Q}{C_p} + \frac{Q_{diff}}{C_p} - \mathcal{D}(T) \end{aligned}$$

5. Tracers

ここでは水蒸気の移流方程式を示す。他のトレーサーも同様の方程式に従う。

$$\begin{aligned} \frac{\partial q}{\partial t} &= -\frac{1}{a\cos\varphi}\frac{\partial uq}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(vq\cos\varphi) + qD \\ &- \dot{\eta}\frac{\partial q}{\partial \eta} + S_q - \mathcal{D}(q) \end{aligned}$$

Here,

$$\begin{aligned} m &\equiv \left(\frac{\partial p}{\partial \eta}\right)_{p_s} \\ \theta &\equiv T\,(p/p_0)^{-\kappa} \\ \kappa &\equiv R/C_p \\ \Phi &\equiv gz \\ \pi &\equiv \ln p_S \\ \dot{\eta} &\equiv \frac{d\eta}{dt} \\ T_v &\equiv T(1+\epsilon_v q) \\ T &\equiv \bar{T} + T' \\ \zeta &\equiv \frac{1}{a\cos\varphi}\frac{\partial v}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(u\cos\varphi) \\ D &\equiv \frac{1}{a\cos\varphi}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(v\cos\varphi) \\ A_u &\equiv (\zeta+f)v - \dot{\eta}\frac{\partial u}{\partial \eta} - \frac{RT'}{a\cos\varphi}\frac{\partial \pi}{\partial \lambda} + \mathcal{F}_x \\ A_v &\equiv -(\zeta+f)u - \dot{\eta}\frac{\partial v}{\partial \eta} - \frac{RT'}{a}\frac{\partial \pi}{\partial \varphi} + \mathcal{F}_y \end{aligned}$$

$$
E \equiv \frac{u^2+v^2}{2}
$$

$$
\mathbf{v}_H \cdot \nabla \equiv \frac{u}{a\cos\varphi}\left(\frac{\partial}{\partial\lambda}\right)_\sigma + \frac{v}{a}\left(\frac{\partial}{\partial\varphi}\right)_\sigma
$$

$$
\nabla_\eta^2 \equiv \frac{1}{a^2\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2} + \frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}\left[\cos\varphi\frac{\partial}{\partial\varphi}\right].
$$

$\mathcal{D}(\zeta), \mathcal{D}(D), \mathcal{D}(T), \mathcal{D}(q)$ are horizontal diffusion terms, $\mathcal{F}_\lambda, \mathcal{F}_\varphi$ are forces due to small-scale kinetic processes (treated as 'physical processes'), Q are forces due to radiation, condensation, small-scale kinetic processes, etc. Heating and temperature change due to 'physical processes', and S_q is a water vapor source term due to 'physical processes' such as condensation and small-scale motion. Q_{diff} is the heat of friction and

$$
Q_{diff} = -\mathbf{v} \cdot (\frac{\partial \mathbf{v}}{\partial t})_{diff}.
$$

$(\frac{\partial \mathbf{v}}{\partial t})_{diff}$ is a time-varying term of u, v due to horizontal and vertical diffusion.

Boundary Conditions

鉛直流に関する上下端の境界条件は以下の通りである：

$$
\dot{\eta} = 0 \quad at \;\; \eta = 0,\ 1.
$$

これを用いて連続の式を鉛直積分することで、p_sの予報方程式と、$\dot{\eta}$の診断方程式が導かれる。

なお、実際の離散化においてはηを陽に用いない表式を用いている。例えば、鉛直移流項$\dot{\eta}(\partial/\partial\eta)$は以下の等式を利用して、$(m\dot{\eta}/p_s)(\partial/\partial\sigma)$に置き換えて離散化している。

$$
\dot{\eta}\frac{\partial}{\partial\eta} = \dot{\eta}\frac{\partial p}{\partial\eta}\frac{\partial}{\partial p} = m\dot{\eta}\frac{\partial}{\partial p} = m\dot{\eta}\frac{\partial}{\partial(p_s\sigma)} = \frac{m\dot{\eta}}{p_s}\frac{\partial}{\partial\sigma}
$$