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Mechanical Processes

Basic Equations

Basic Equations

The fundamental equations are a system of primitive equations at the spherical (λ, φ) and η coordinates, given as follows (Arakawa and Konor 1996).

1. Continuity equation

$$\frac{\partial m}{\partial t} + \nabla_\eta \cdot (m\mathbf{v}_H) + \frac{\partial (m\dot{\eta})}{\partial \eta} = 0$$

2. Hydrostatic equation

$$\frac{\partial \Phi}{\partial \eta} = -\frac{RT_v}{p} m$$

3. Equation of motion

$$\frac{\partial \zeta}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial A_v}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(A_u \cos\varphi) - \mathcal{D}(\zeta)$$

$$\frac{\partial D}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial A_u}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(A_v \cos\varphi) - \nabla_\eta^2(\Phi + R\bar{T}\pi + E) - \mathcal{D}(D)$$

4. Thermodynamic equation

$$\frac{\partial T}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial uT'}{\partial \lambda} - \frac{1}{a}\frac{\partial}{\partial \varphi}(vT'\cos\varphi) + T'D$$
$$-\dot{\eta}\frac{\partial T}{\partial \eta} + \frac{\kappa T}{\sigma}\left[B\left(\frac{\partial \pi}{\partial t} + \mathbf{v}_H \cdot \nabla_\eta \pi\right) + \frac{m\dot{\eta}}{p_s}\right] + \frac{Q}{C_p} + \frac{Q_{diff}}{C_p} - \mathcal{D}(T)$$

5. Tracers

ここでは水蒸気の移流方程式を示す。他のトレーサーも同様の方程式に従う。

$$\frac{\partial q}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial uq}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(vq\cos\varphi) + qD$$

$$-\dot{\eta}\frac{\partial q}{\partial \eta} + S_q - \mathcal{D}(q)$$

Here,

$$m \equiv \left(\frac{\partial p}{\partial \eta}\right)_{p_s}$$

$$\theta \equiv T(p/p_0)^{-\kappa}$$

$$\kappa \equiv R/C_p$$

$$\Phi \equiv gz$$

$$\pi \equiv \ln p_S$$

$$\dot{\eta} \equiv \frac{d\eta}{dt}$$

$$T_v \equiv T(1+\epsilon_v q)$$

$$T \equiv \bar{T} + T'$$

$$\bar{T} \equiv 300 \text{ K}$$

$$\zeta \equiv \frac{1}{a\cos\varphi}\frac{\partial v}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(u\cos\varphi)$$

$$D \equiv \frac{1}{a\cos\varphi}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(v\cos\varphi)$$

$$A_u \equiv (\zeta+f)v - \dot{\eta}\frac{\partial u}{\partial \eta} - \frac{RT'}{a\cos\varphi}\frac{\partial \pi}{\partial \lambda} + \mathcal{F}_x$$

$$A_v \equiv -(\zeta+f)u - \dot{\eta}\frac{\partial v}{\partial \eta} - \frac{RT'}{a}\frac{\partial \pi}{\partial \varphi} + \mathcal{F}_y$$

$$E \equiv \frac{u^2+v^2}{2}$$

$$\mathbf{v}_H \cdot \nabla \equiv \frac{u}{a\cos\varphi}\left(\frac{\partial}{\partial \lambda}\right)_\sigma + \frac{v}{a}\left(\frac{\partial}{\partial \varphi}\right)_\sigma$$

$$\nabla_\eta^2 \equiv \frac{1}{a^2\cos^2\varphi}\frac{\partial^2}{\partial \lambda^2} + \frac{1}{a^2\cos\varphi}\frac{\partial}{\partial \varphi}\left[\cos\varphi\frac{\partial}{\partial \varphi}\right].$$

$\mathcal{D}(\zeta), \mathcal{D}(D), \mathcal{D}(T), \mathcal{D}(q)$ are horizontal diffusion terms, $\mathcal{F}_\lambda, \mathcal{F}_\varphi$ are forces due to small-scale kinetic processes (treated as 'physical processes'), Q are forces due to radiation, condensation, small-scale kinetic processes, etc. Heating and temperature change due to 'physical processes', and S_q is a water vapor source term due to 'physical processes' such as condensation and small-scale motion. Q_{diff} is the heat of friction and

$$Q_{diff} = -\mathbf{v} \cdot (\frac{\partial \mathbf{v}}{\partial t})_{diff}.$$

$(\frac{\partial \mathbf{v}}{\partial t})_{diff}$ is a time-varying term of u, v due to horizontal and vertical diffusion.

Boundary Conditions
鉛直流に関する上下端の境界条件は以下の通りである：

$$\dot{\eta} = 0 \quad at \quad \eta = 0,\ 1.$$

これを用いて連続の式を鉛直積分することで、p_s の予報方程式と、鉛直流の診断方程式が導かれる。