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Summary of the dynamics component

In this section, we enumerate the calculations performed in the Mechanical Engineering Department, although they overlap with the previous descriptions.

Summary of Calculations for the Dynamicscs Component

The mechanical processes are calculated in the following order.

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2. calculation of pseudotemperature `MODULE:[VIRTMD(dvtmp)]`
3. calculation of the barometric gradient term `MODULE:[HGRAD(dvect)]`
4. diagnostic calculation of vertical flow `MODULE:[GRDDYN/PSDOT(dgdyn)]`
5. time change term due to advection `MODULE:[GRDDYN(dgdyn)]`
6. convert the predictive variable to a spectrum `MODULE:[GD2WD(dg2wd)]`
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11. consideration of frictional heat by diffusion `MODULE:[CORFRC(ddifc)]`
12. correction for conservation of mass `MODULE:[MASFIX(dmfix)]`
13. (physical process) `MODULE:[PHYSCS(padmn)]`
14. (time filter) `MODULE:[TFILT(aadvn)]`

Conversion of Horizontal Wind to Vorticity and Divergence

Obtain grid values of vorticity and divergence from the grid values of u_{ij}, v_{ij} for horizontal wind. First, we obtain the spectra of vorticity and divergence from ζ_n^m, D_n^m,

$$\zeta_n^m = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} imv_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} + \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} u_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \ ,$$

$$D_n^m = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} imu_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} - \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} v_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)}; .$$

I'll take that further,

$$\zeta_{ij} = \mathcal{R}\mathbf{e}\sum_{m=-N}^{N}\sum_{n=|m|}^{N}\zeta_n^m {Y_n^m}_{ij} \ ,$$

and so on.

Calculating Virtual Temperature

Virtual Temperature T_v is ,

$$T_v = T(1+\epsilon_v q - l) \ ,$$

However, it is $\epsilon_v = R_v/R - 1$ and R_v is the gas constant for water vapor (461 Jkg^{-1}K^{-1}) and R is the gas constant for air (287.04 Jkg^{-1}K^{-1}).

Calculating the Barometric gradient term

The barometric gradient term $\nabla\pi = \frac{1}{p_S}\nabla p_S$ is first used to define the π_n^m

$$\pi_n^m = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(\ln p_S)_{ij}{Y_n^{m*}}_{ij}w_j \ ,$$

to a spectral representation and then ,

$$\frac{1}{a\cos\varphi}\left(\frac{\partial\pi}{\partial\lambda}\right)_{ij} = \frac{1}{a\cos\varphi}\mathcal{R}\mathbf{e}\sum_{m=-N}^{N}\sum_{n=|m|}^{N} im\tilde{X}_n^m Y_n^m{}_{ij} \ ,$$

$$\frac{1}{a}\left(\frac{\partial\pi}{\partial\varphi}\right)_{ij} = \frac{1}{a\cos\varphi}\mathcal{R}\mathbf{e}\sum_{m=-N}^{N}\sum_{n=|m|}^{N} \pi_n^m (1-\mu^2)\frac{\partial}{\partial\mu}Y_n^m{}_{ij} \ .$$

Diagnostic calculations of vertical flow

Barometric pressure change term, and lead DC,

$$\frac{\partial\pi}{\partial t} = -\sum_{k=1}^{K}\{D_k\Delta\sigma_k + (\mathbf{v}_k\cdot\nabla\pi)\Delta B_k\}$$

$$\frac{(m\dot{\eta})_{k-1/2}}{p_s} = -B_{k-1/2}\frac{\partial\pi}{\partial t} - \sum_{l=k}^{K}\{D_l\Delta\sigma_l + (\mathbf{v}_l\cdot\nabla\pi)\Delta B_l\}$$

and its non-gravity components.

$$\left(\frac{\partial\pi}{\partial t}\right)^{NG} = -\sum_{k=1}^{K}\mathbf{v}_k\cdot\nabla\pi\Delta B_k$$

$$\frac{(m\dot{\eta})_{k-1/2}^{NG}}{p_s} = -B_{k-1/2}\left(\frac{\partial\pi}{\partial t}\right)^{NG} - \sum_{l=k}^{K}\mathbf{v}_l\cdot\nabla\pi\Delta B_l$$

Time change term due to advection

Momentum advection term:

$$\begin{aligned}(A_u)_k = (\zeta_k + f)v_k &- \left[\frac{(m\dot{\eta})_{k-1/2}}{p_s}\frac{u_{k-1}-u_k}{\Delta\sigma_{k-1}+\Delta\sigma_k} + \frac{(m\dot{\eta})_{k+1/2}}{p_s}\frac{u_k-u_{k+1}}{\Delta\sigma_k+\Delta\sigma_{k+1}}\right] \\ &- \frac{1}{a\cos\varphi}\frac{\partial\pi}{\partial\lambda}(C_pT_{v,k}\hat{\kappa} - R\bar{T}) + \mathcal{F}_x\end{aligned}$$

$$\begin{aligned}(A_v)_k = -(\zeta_k + f)u_k &- \left[\frac{(m\dot{\eta})_{k-1/2}}{p_s}\frac{v_{k-1}-v_k}{\Delta\sigma_{k-1}+\Delta\sigma_k} + \frac{(m\dot{\eta})_{k+1/2}}{p_s}\frac{v_k-v_{k+1}}{\Delta\sigma_k+\Delta\sigma_{k+1}}\right] \\ &- \frac{1}{a}\frac{\partial\pi}{\partial\varphi}(C_pT_{v,k}\hat{\kappa} - R\bar{T}) + \mathcal{F}_y\end{aligned}$$

Temperature advection term:

$$(uT')_k = u_k(T_k - \bar{T})$$

$$(vT')_k = v_k(T_k - \bar{T})$$

$$\begin{aligned} H_k = T'_k D_k - &\left[\frac{(m\dot{\eta})_{k-1/2}}{p_s} \frac{\hat{T}_{k-1/2} - T_k}{\Delta\sigma_l} + \frac{(m\dot{\eta})_{k+1/2}}{p_s} \frac{T_k - \hat{T}_{k+1/2}}{\Delta\sigma_l} \right] \\ &+ \hat{\kappa}_k \mathbf{v}_k \cdot \nabla\pi T_{v,k} \\ &- \alpha_k \sum_{l=k}^{K} (D_l \Delta\sigma_l + (\mathbf{v}_l \cdot \nabla\pi) \Delta B_l) \frac{T_{v,k}}{\Delta\sigma_k} \\ &- \beta_k \sum_{l=k+1}^{K} (D_l \Delta\sigma_l + (\mathbf{v}_l \cdot \nabla\pi) \Delta B_l) \frac{T_{v,k}}{\Delta\sigma_k} \end{aligned}$$

Water vapor advection term:

$$(uq)_k = u_k q_k$$

$$(vq)_k = v_k q_k$$

$$R_k = q_k D_k - \frac{1}{2} \left[\frac{(m\dot{\eta})_{k-1/2}}{p_s} \frac{q_{k-1} - q_k}{\Delta\sigma_k} + \frac{(m\dot{\eta})_{k+1/2}}{p_s} \frac{q_k - q_{k+1}}{\Delta\sigma_k} \right]$$

Spectral Representation of Prognostic Variables

Convert $u_{ij}^{t-\Delta t}, v_{ij}^{t-\Delta t}$ to a spectral representation of vorticity and divergence ζ_n^m, D_n^m. Furthermore, converting the temperature $T^{t-\Delta t}$, specific humidity $q^{t-\Delta t}$, and $\pi = \ln p_S^{t-\Delta t}$ to

$$X_n^m = \frac{1}{I} \sum_{i=1}^{I} \sum_{j=1}^{J} X_{ij} {Y_n^{m*}}_{ij} w_j \ ,$$

to a spectral representation.

Spectral Representation of Tendencies

Time Variation Term of Vorticity

$$\frac{\partial \zeta_n^m}{\partial t} = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} im(A_v)_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(A_u)_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)}$$

The non-gravity wave component of the time-varying term of the divergence

$$\left(\frac{\partial D_n^m}{\partial t}\right)^{NG} = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} im(A_u)_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)}$$

$$-\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(A_v)_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)}$$

$$-\frac{n(n+1)}{a^2}\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}\hat{E}_{ij}{Y_n^{m*}}_{ij}w_j$$

The non-gravity wave component of the time-varying term of temperature

$$\left(\frac{\partial T_n^m}{\partial t}\right)^{NG} = -\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} im(uT')_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(vT')_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}\hat{H}_{ij}{Y_n^{m*}}_{ij}w_j$$

Time-varying term of water vapor

$$\frac{\partial q_n^m}{\partial t} = -\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} im(uq)_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(vq)_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}R_{ij}{Y_n^{m*}}_{ij}w_j$$

Spectral value time integration

Equations in matrix form

$$\{(1+2\Delta t\mathcal{D}_H)(1+2\Delta t\mathcal{D}_M)\underline{I} - (\Delta t)^2(\underline{W}\ \underline{h} + (1+2\Delta t\mathcal{D}_M)\mathbf{G}\mathbf{C}^T)\nabla_\sigma^2\}\ \overline{\mathbf{D}}^t$$
$$= (1+2\Delta t\mathcal{D}_H)(1-\Delta t\mathcal{D}_M)\mathbf{D}^{t-\Delta t} + \Delta t\left(\frac{\partial \mathbf{D}}{\partial t}\right)_{NG}$$
$$-\Delta t\nabla_\sigma^2\left\{(1+2\Delta t\mathcal{D}_H)_S + \underline{W}\left[(1-2\Delta t\mathcal{D}_H)\mathbf{T}^{t-\Delta t} + \Delta t\left(\frac{\partial \mathbf{T}}{\partial t}\right)_{NG}\right]\right.$$
$$\left. +(1+2\Delta t\mathcal{D}_H)\mathbf{G}\left[\pi^{t-\Delta t} + \Delta t\left(\frac{\partial \pi}{\partial t}\right)_{NG}\right]\right\}.$$

Using LU decomposition, $\bar{D}$ is obtained by solving for

$$\frac{\partial \mathbf{T}}{\partial t} = \left(\frac{\partial \mathbf{T}}{\partial t}\right)_{NG} - \underline{h}\mathbf{D}$$

$$\frac{\partial \pi}{\partial t} = \left(\frac{\partial \pi}{\partial t}\right)_{NG} - \mathbf{C}\cdot\mathbf{D}$$

Calculate the value of the spectrum in $\partial\mathbf{T}/\partial t$, $\partial\pi/\partial t$ and then calculate the value of the spectrum in $t+\Delta t$ using

$$\zeta^{t+\Delta t} = \left(\zeta^{t-\Delta t} + 2\Delta t\frac{\partial \zeta}{\partial t}\right)(1+2\Delta t\mathcal{D}_M)^{-1}$$
$$D^{t+\Delta t} = 2\bar{D} - D^{t-\Delta t}$$
$$T^{t+\Delta t} = \left(T^{t-\Delta t} + 2\Delta t\frac{\partial T}{\partial t}\right)(1+2\Delta t\mathcal{D}_H)^{-1}$$
$$q^{t+\Delta t} = \left(q^{t-\Delta t} + 2\Delta t\frac{\partial q}{\partial t}\right)(1+2\Delta t\mathcal{D}_E)^{-1}$$
$$\pi^{t+\Delta t} = \pi^{t-\Delta t} + 2\Delta t\frac{\partial \pi}{\partial t}$$

Conversion of Prognostic Variables to Grid Values

Obtain grid values of horizontal wind speed from the spectral values of vorticity and divergence (ζ_n^m, D_n^m) u_{ij}, v_{ij}.

$$u_{ij} = \frac{1}{\cos\varphi_j}\mathcal{R}\mathbf{e}\sum_{m=-N}^{N}\sum_{\substack{n=|m|\\ n\neq 0}}^{N}\left\{\frac{a}{n(n+1)}\zeta_n^m(1-\mu^2)\frac{\partial}{\partial\mu}{Y_n^m}_{ij} - \frac{ima}{n(n+1)}D_n^m{Y_n^m}_{ij}\right\}$$

$$v_{ij} = \frac{1}{\cos\varphi_j} \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{\substack{n=|m| \\ n\neq 0}}^{N} \left\{ -\frac{ima}{n(n+1)} \zeta_n^m Y_n^m{}_{ij} - \frac{a}{n(n+1)} \tilde{D}_n^m (1-\mu^2) \frac{\partial}{\partial\mu} Y_n^m{}_{ij} \right\}$$

Furthermore,

$$T_{ij} = \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{n=|m|}^{N} T_n^m Y_n^m{}_{ij} \ ,$$

T_{ij}, π_{ij}, q_{ij}, and so on,

$$p_{S_{ij}} = \exp \pi_{ij}$$

to calculate.

Pseudo-p Surface Diffusion Correction

The horizontal diffusion is applied on the $\eta-$plane, but it can cause problems in large slopes, such as transporting water vapor uphill and causing false precipitation at the top of a mountain. To mitigate this problem, corrections have been made for T, q, l to make the diffusion closer to that of the p surface, e.g., for T, q, l.

$$\begin{aligned}
\mathcal{D}_p(T) = (-1)^{N_D/2} K \nabla_p^{N_D} T \simeq (-1)^{N_D/2} K \nabla_\eta^{N_D} T - \frac{\partial\sigma}{\partial p} (-1)^{N_D/2} K \nabla_\eta^{N_D} p \cdot \frac{\partial T}{\partial\sigma} \\
= (-1)^{N_D/2} K \nabla_\eta^{N_D} T - (-1)^{N_D/2} K \nabla_\eta^{N_D} \pi \cdot \sigma \frac{\partial T}{\partial\sigma} \\
= \mathcal{D}(T) - \mathcal{D}(\pi) \sigma \frac{\partial T}{\partial\sigma}
\end{aligned}$$

So,

$$T_k \leftarrow T_k - 2\Delta t \sigma_k \frac{T_{k+1} - T_{k-1}}{\sigma_{k+1} - \sigma_{k-1}} \mathcal{D}(\pi)$$

and so on. In $\mathcal{D}(\pi)$, the spectral value of pi is converted to a grid by multiplying the spectral value of π_n^m by the spectral representation of the diffusion coefficient.

Consideration of frictional heat from diffusion.

Frictional heat from diffusion is ,

$$Q_{DIF} = -\left(u_{ij}\mathcal{D}(u)_{ij} + v_{ij}\mathcal{D}(v)_{ij}\right)$$

It is estimated that Therefore,

$$T_k \leftarrow T_k - \frac{2\Delta t}{C_p}\left(u_{ij}\mathcal{D}(u)_{ij} + v_{ij}\mathcal{D}(v)_{ij}\right)$$

Correction for conservation of mass

In the spectral method, the global integral of $\pi = \ln p_S$ is preserved with rounding errors removed, but the preservation of the mass, i.e. the global integral of p_S is not guaranteed. Moreover, a wavenumber break in the spectra sometimes results in negative values of the water vapor grid points. For this reason, we perform a correction to preserve the masses of dry air, water vapor, and cloud water, and to remove the regions with negative water vapor content.

At the beginning of the dynamics calculations, the global integrals of `MODULE:[FIXMAS]`, water vapor, and cloud water are calculated for M_q, M_l.

$$M_q^0 = \sum_{ijk} q p_S \Delta\lambda_i w_j \Delta\sigma_k M_l^0 = \sum_{ijk} l p_S \Delta\lambda_i w_j \Delta\sigma_k$$

In the first step of the calculation, the dry mass M_d is calculated and stored.

$$M_d^0 = \sum_{ijk}(1-q-l) p_S \Delta\lambda_i w_j \Delta\sigma_k$$

At the end of the calculation, `MODULE:[MASFIX]`, the following procedure is followed.

First, negative water vapor is removed by dividing the water vapor from the grid points immediately below the grid points. Suppose that $q_k < 0 $ is used,

$$q'_k = 0 q'_{k-1} = q_{k-1} + \frac{\Delta p_k}{\Delta p_{k-1}} q_k$$

However, this should only be done if it is $q_{k-1}' 睚 0 $.

Next, set the value to zero for the grid points not removed by the above procedure.

3. calculate the global integral value of M_q and multiply the global water vapor content by a fixed percentage so that it is the same as that of M_q^0.

$$q'' = \frac{M_q^0}{M_q} q'$$

4. correct for dry air mass Likewise calculate M_d,

$$p_S'' = \frac{M_d^0}{M_d} p_S$$

Horizontal Diffusion and Rayleigh Friction

The coefficients of horizontal diffusion can be expressed spectrally,

$$\mathcal{D}_{M_n}^{\ m} = K_M \left[\left(\frac{n(n+1)}{a^2} \right)^{N_D/2} - \left(\frac{2}{a^2} \right)^{N_D/2} \right] + K_R$$

$$\mathcal{D}_{H_n}^{\ m} = K_M \left(\frac{n(n+1)}{a^2} \right)^{N_D/2}$$

$$\mathcal{D}_{E_n}^{\ m} = K_E \left(\frac{n(n+1)}{a^2} \right)^{N_D/2}$$

K_R is the Rayleigh coefficient of friction. The Rayleigh coefficient of friction is

$$K_R = K_R^0 \left[1 + \tanh \left(\frac{z - z_R}{H_R} \right) \right]$$

However, the profile is given in the same way as However,

$$z = -H \ln \sigma$$

The results are approximate to those of $K_R^0 = (30day)^{-1}$ and $z_R = -H \ln \sigma_{top}$. The standard values are $K_R^0 = (30day)^{-1}$, $z_R = -H \ln \sigma_{top}$ (σ_{top}: top level of the model), $H = 8000$ m, and $H_R = 7000$ m.

Time Filter.

Apply the time filter of Asselin (1972) to remove computational modes in leap frog at every step.

$$\bar{T}^t = (1 - 2\epsilon_f) T^t + \epsilon_f \left(\bar{T}^{t-\Delta t} + T^{t+\Delta t} \right)$$

and $\bar{T}$ are obtained. This $\bar{T}^t$ is used as the $T^{t-\Delta t}$ for the next step of the mechanical process. As a rule, 0.05 is used for ϵ_f.

In fact, the first step is to convert the predictor to a grid value at the `MODULE:[GENGD]`,

$$\bar{T}^{t*} = (1-\epsilon_f)^{-1}\left[(1-2\epsilon_f)T^t + \epsilon_f \bar{T}^{t-\Delta t}\right]$$

and after the physical process is finished and the value of $T^{t+\Delta t}$ is fixed, the `MODULE:[TFILT]` can be used to determine

$$\bar{T}^t = (1-\epsilon_f)\bar{T}^{t*} + \epsilon_f \bar{T}^{t+\Delta t}$$

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