CCSR/NIES AGCM の解説

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目 次

1 Model Overview. 2

1.1 Characteristics of CCSR/NIES AGCM 2
1.2 Features and structure of the model 3
1.2.1 Basic Features of the Model. 3
1.2.2 Model Execution Flow. 6
1.2.3 Predictive variables. 6
1.2.4 The flow of time evolution of variables 7
1.3 Basic Settings. 14
1.3.1 Coordinate System. 14
1.3.2 Physical Constants. 15

2 Mechanical Processes. 17

2.1 Basic Equations. 17
2.1.1 Basic Equations. 17
2.1.2 Boundary conditions. 19
2.2 Vertical discretization 21
2.2.1 How to take a level. 21
2.2.2 vertical discretization representation. 22
2.3 Horizontal discretization 26
2.3.1 Spectral Expansion. 26
2.3.2 Horizontal Diffusion Term. 28
2.3.3 Spectral representation of the equation 29
2.4 Time integration. 32

2.4.1 Time integration and time filtering by leap frog . . 32
2.4.2 semi-implicit time integration 33
2.4.3 semi-implicit time integration applied 33
2.4.4 Time scheme properties and time step estimates . . 37
2.4.5 Treatment at the beginning of time integration. . . 39
2.5 Summary of the mechanics part 40
2.5.1 Summary of calculations in the mechanics part. . . 40
2.5.2 Conversion of Horizontal Wind to Vorticity and Divergence 41
2.5.3 Calculating Pseudotemperature 41
2.5.4 Calculating the Barometric gradient term 42
2.5.5 Diagnostic calculations of vertical flow. 42
2.5.6 The time-varying term due to advection. 43
2.5.7 Conversion of Predictive Variables to Spectra. . . . 44
2.5.8 Conversion of time-varying terms to spectra. 44
2.5.9 Spectral Value Time Integration 46
2.5.10 Conversion of Prediction Variables to Grid Values . 46
2.5.11 Pseudo etc. p Surface Diffusion Correction 47
2.5.12 Consideration of frictional heat by diffusion. 48
2.5.13 Correction for conservation of mass. 48
2.5.14 Horizontal Diffusion and Rayleigh Friction 50
2.5.15 Time Filter. 50

3 Physical Processes. 52
3.1 Overview of Physical Processes. 52
3.1.1 Fundamental Equations. 52
3.1.2 Time integration of physical processes. 55
3.1.3 Various physical quantities. 56
3.2 cumulus convection 59
3.2.1 Overview of the Cumulus Convection Scheme . . . 59
3.2.2 The Basic Framework of the Arakawa-Schubert Scheme 61
3.2.3 Cloud Model. 62
3.2.4 Cloud Work Function (CWF) 66
3.2.5 Cloud Mass Flux at Cloudbase 66
3.2.6 Cloud Mass Flux, Precipitation 67
3.2.7 Time variation of the average field 67

3.2.8 Evaporation and downdrafting of precipitation . . . 68
3.2.9 cloud water and cloud cover 69
3.3 Massive Coagulation 71
3.3.1 Overview of Large Scale Condensation Schemes. . . 71
3.3.2 Diagnosis of cloud water levels 72
3.3.3 Determination by successive approximation 72
3.3.4 precipitation process. 73
3.3.5 Ice Falling Process. 74
3.3.6 Evaporation process of precipitation. 75
3.3.7 Other Notes. 76
3.4 Radiant Flux. 77
3.4.1 Summary of Radiation Flux Calculations 77
3.4.2 Wavelengths and Subchannels. 78
3.4.3 Calculating the Planck function `MODULE:[PLANKS]` . 79
3.4.4 Calculating the optical thickness by gas absorption `MODULE:[PTFIT]` 80
3.4.5 Optical Thickness by Continuous Absorption and CFC Absorption `MODULE:[CNTCFC]` 80
3.4.6 Scattering optical thickness and scattering moments `MODULE:[SCATMM]` 81
3.4.7 Albedo at Sea Level `MODULE:[SSRFC]` 83
3.4.8 Total Optical Thickness. 83
3.4.9 Planck function expansion `MODULE:[PLKEXP]` 83
3.4.10 Transmission and reflection coefficients of each layer, the source function `MODULE:[TWST]` 84
3.4.11 Combinations of source functions for each layer. . . 87
3.4.12 Radiation flux at each layer boundary `MODULE:[ADDING]` 87
3.4.13 Add in the flux. 89
3.4.14 The temperature derivative of the flux 90
3.4.15 Handling of cloud cover 90
3.4.16 Incidence flux and angle of incidence `MODULE:[SHTINS]` 92
3.4.17 Other Notes. 94
3.5 Vertical Diffusion. 95
3.5.1 Vertical Diffusion Scheme Overview. 95
3.5.2 Basic Formula for Flux Calculations 95

3.5.3 Richardson Number. 96
3.5.4 Diffusion Coefficient. 97
3.5.5 Calculating Flux. 97
3.5.6 Minimum Diffusion Coefficient. 98
3.5.7 Other Notes. 98
3.6 Surface Flux. 99
3.6.1 Overview of the Surface Flux Scheme 99
3.6.2 Basic Formula for Flux Calculations 99
3.6.3 Richardson Number. 100
3.6.4 Bulk factor. 100
3.6.5 Calculating Flux. 101
3.6.6 handling at sea level. 103
3.6.7 Wind Speed Correction 104
3.6.8 Minimum wind speed. 104
3.7 Surface Processes. 105
3.7.1 Overview of Surface Processes. 105
3.7.2 classification of the ground surface. 109
3.7.3 Surface Heat Balance. 110
3.7.4 Set the discrete coordinate system `MODULE:[SETGLV,SETWLV,SETSLV]` 111
3.7.5 Calculating land heat flux and heat capacity `MODULE:[LNDFLX]` 111
3.7.6 Calculating the water flux on land `MODULE:[LNDWFX]` 112
3.7.7 Calculating land surface spill`MODULE:[LNDROF]` . . . 113
3.7.8 Evaluating Albedo on land `MODULE:[LNDALB]` . . . 116
3.7.9 Evaluating roughness on land surface`MODULE:[LNDZO]` 116
3.7.10 Evaluating surface wetness on land`MODULE:[LNDBET]` 117
3.7.11 Calculating heat flux and heat capacity at sea level `MODULE:[SEAFLX]` 118
3.7.12 Evaluating surface wetness at sea level `MODULE:[SEABET]` 118
3.7.13 albedo and roughness at sea level 119
3.7.14 Snow Heat Flux Correction `MODULE:[SNWFLX]` . . . 119
3.7.15 Calculating snow sublimation `MODULE:[SNWSUB]` . . 120
3.7.16 Calculating snowfall `MODULE:[SNWFLP]` 121
3.7.17 Snowmelt calculation `MODULE:[SNWMLP]` 121
3.7.18 Calculating Snow Surface Runoff`MODULE:[SNWROF]` 122
3.7.19 Evaluating albedo on snow-covered surfaces`MODULE:[SNWALB]` 123

3.7.20 Evaluating Surface Roughness on Snow Covered SurfacesMODULE:[SNWZO] 124
3.7.21 Evaluating Surface Wetness on Snow Covered SurfacesMODULE:[SNWBET] 124
3.7.22 Calculating the freezing process MODULE:[LNDFRZ] . 124
3.7.23 oceanic mixed layer model MODULE:[SEAFRZ] 125
3.8 Solving the diffuse balance equation for atmospheric and surface systems 129
3.8.1 Basic Solutions. 129
3.8.2 Fundamental Equations. 129
3.8.3 implicit time difference 130
3.8.4 implicit time difference coupling 132
3.8.5 Solving the Coupling Formula for Time Difference . 135
3.8.6 Combined expression for time difference 139
3.8.7 implicit Treatment of Time Steps in Time Differences 142
3.9 Gravitational wave resistance 144
3.9.1 Gravitational Wave Resistance Scheme Overview . 144
3.9.2 Relationship between local fluid number and momentum flux 144
3.9.3 Momentum fluxes at the surface. 145
3.9.4 Momentum fluxes in the upper levels. 146
3.9.5 The magnitude of the time variation of horizontal wind due to momentum convergence. 146
3.9.6 Other Notes. 147
3.10 Drying convection regulation 148
3.10.1 Overview of Drying Convective Regulation 148
3.10.2 Drying convection regulation procedures. 148

4 Reference list 150

1 Model Overview.

1.1 Characteristics of CCSR/NIES AGCM

AGCM5.4 was developed in collaboration with the Center for Climate System Research (CCSR) at the University of Tokyo. Prepared in collaboration with the National Institute for Environmental Studies (NIES) , The model is a global three-dimensional general circulation model. The features of the model are listed below.

[]@ll@ Header0	Header1	System of equati
System of hydrostatic primitive equations	Area.	Global 3D
	Horizontal Discretization	Spectral Conversion Meth
σ system (Arakawa and Suarez, 1983)	Radiation	2-stream DOM/adding m
(Based on Nakajima and Tanaka, 1986)	A large-scale cloud process	Scheme with the total wa mixing ratio as a forecast
(Based on Le Treut and Li, 1991)	Cumulus Convection	Simplified Arakawa-Schub scheme
Mellor and Yamada(1974) level2		Louis (1979), bulk type
(Considering the convection effect of stomatal resistance, Miller et al. 1992)	Surface Thermal Processes	Multilayer Heat Transfer
Bucket Model		
Scheme based on McFarlane (1987)		The north-south vertical east-west vertical two-dimensional models.

`The vertical one-dimensional model.`

- TAB00000: 18.0
- TAB00000: 18.1
  The mixed-layer coupled model for the ocean

1.2 Features and structure of the model

1.2.1 Basic Features of the Model.

The CCSR/NIES AGCM is a physical description of the global three-dimensional atmosphere, It is a numerical model for computing the time evolution of a system as an initial value problem.

The data to be inputted are as follows.

- Initial data for each forecast variable (horizontal wind speed, temperature, surface pressure, specific humidity, cloud liquid water content, and surface volume)
- Boundary condition data (surface elevation, surface condition, sea surface temperature, etc.)
- Various parameters of the model (atmospheric components, physical process parameters, etc.)

On the other hand, the output looks like the following.

- Data for each forecast parameter and diagnostic parameter, for each time or time average
- Initial data to be used for continuous execution (restart data)
- Progress and various diagnostic messages

Here, the predictor variable is the differential equation of time evolution, which is integrated in time by The data is obtained as a time series, A diagnostic variable is defined as a set of predictor variables, boundary conditions and parameters It is a quantity calculated by some method that does not include time integration.

To be more specific , The model is basically a solution to the following equation (the prediction equation).

\[construct:u-eq-1]

$$\frac{\partial u}{\partial t} = (\mathcal{F}_x)_D + (\mathcal{F}_x)_P \tag{1}$$
$$\frac{\partial v}{\partial t} = (\mathcal{F}_y)_D + (\mathcal{F}_y)_P \tag{2}$$
$$\frac{\partial T}{\partial t} = (Q)_D + (Q)_P \tag{3}$$
$$\frac{\partial p_S}{\partial t} = (M)_D + (M)_P \tag{4}$$
$$\frac{\partial q}{\partial t} = (S)_D + (S)_P \tag{5}$$
$$\frac{\partial T_g}{\partial t} = (Q_g)_D + (Q_g)_P \tag{6}$$

Where, u, v, T, p_S, q, T_g, u, v, T, p_S, q, T_g is, Easterly wind, north-south wind, temperature, surface pressure, specific humidity, etc., respectively It is a forecast variable with a two- or three-dimensional distribution, such as The right-hand side is the term that gives rise to the time variation of each of those forecast variables. This time-varying term $\mathcal{F}_x, \mathcal{F}_y, Q, S, Q_g$ is , It is calculated based on the predictor variables u, v, T, p_S, q, T_g, The terms such as advection due to atmospheric motion represented by u and v (the term of D in the above formula), and The term "cloud and radiation" can be broadly divided into two categories: the term by each process, such as cloud and radiation, and the term by each process (the term in the appendix P). The former is called the dynamic process and the latter is called the physical process.

The main part of the time-varying term of a mechanical process is the advection term, The accurate estimation of the spatial differentiation is important in its calculation. In CCSR/NIES AGCM, the calculation of the horizontal differential term We use spherical harmonic function expansion. The physical processes, on the other hand, are energy conversions due to the phase change of water and radiation absorption and injection, The effects of small-scale atmospheric motion linked to them, The effects of various processes on the surface of the earth, including the effects of , Expressing with parameters in a simple model (parameterization) is important.

The time integration of the predictive equation is , ([construct:u-eq-

1[construct:u-eq-1 (6)

](#struct:u-eq-1)), etc. This is done by approximating the left-hand side by a difference. For example,

$$\frac{\partial q}{\partial t} \rightarrow \frac{q^{t+\Delta t} - q^t}{\Delta t} \tag{7}$$

By making ,

$$q^{t+\Delta t} = q^t + \Delta t \left[(S)_D + (S)_P \right] \tag{8}$$

\brax[struct:sabun]

That would be. Here, S is a function of the forecast variables u, v, T, p_S, q, u, v, T, p_S, q, etc, Depending on which time forecast variables are used in that calculation to evaluate S, There are various possible time difference schemes. In the CCSR/NIES AGCM , Euler method that uses the values from t as they are, The leap frog method using the values from $t + \Delta t/2$, The implicit method using (approximate) values in $t + \Delta t$ is used together.

In the CCSR/NIES AGCM The time integration of the predictors is done separately for mechanical and physical processes. First, the mechanics term is basically a leap frog,

$$\tilde{q}^{t+\Delta t} = q^{t-\Delta t} + 2\Delta t \left(S \right)_D^t \tag{9}$$

However, some terms are treated as implicit. but some terms are treated as implicit. In the physical process , Based on the results of integrating the mechanics terms, Using a combination of the Euler and implicit methods,

$$q^{t+\Delta t} = \tilde{q}^{t+\Delta t} + 2\Delta t \left(S \right)_P \tag{10}$$

I'm looking for. where the Δt of (construct:sabun]) Note that it has been replaced by $2\Delta t$.

1.2.2 Model Execution Flow.

The flow of model execution in brief is as follows. the name of the relevant subroutine.

1. set the parameters of an experiment, coordinates, etc. `MODULE:[SETPAR,PCONST,SETCOR,S`
2. read the initial values of the predictor variable `MODULE:[RDSTRT]`
3. start the time step `MODULE:[TIMSTP]`
4. perform time integration by mechanical processes `MODULE:[DYNMCS]`
5. perform time integration by physical processes `MODULE:[PHYSCS]`
6. advance the time `MODULE:[STPTIM, TFILT]`

Output the data if necessary `MODULE:[HISTOU]`
Output the restart data if necessary `MODULE:[WRRSTR]`

9. 3. Back to

1.2.3 Predictive variables.

The predictor variables are as follows. Parentheses are the coordinate system and $\lambda, \varphi, \sigma, z$ are the coordinates, respectively, Longitude, latitude, dimensionless pressure σ, indicating vertical depth. entries in the list are in units.

[]@lll@ Header0	Header1	Header2	east-west wind sp	
u $(\lambda, \varphi, \sigma)$	"m/score	north-south wind speed	v $(\lambda, \varphi, \sigma)$	
"m/score	atmospheric temperature	T $(\lambda, \varphi, \sigma)$	.L.A.[K.R.I.E.D.]	surfac
p_S (λ, φ)	The "hPa\	specific humidity	q $(\lambda, \varphi, \sigma)$	
[kg/kg\]	Cloud water mixing ratio	l $(\lambda, \varphi, \sigma)$	[kg/kg\]	under tempe
T_g (λ, φ, z)	.L.A.[K.R.I.E.D.]	subterranean moisture	W_g (λ, φ, z)	

"m/m\ W_y (λ, φ) sea-ic
h_I (λ, φ) \0.25\0.25\0.25\0.25\0.25\0.25\0.25\0.25\0.25\0.25\0.25\0.00}.

However, the sea ice thickness is usually only a predictor in the mixed-layer coupled model. In addition, the ground temperature is also higher in the oceans not covered by sea ice Normally, it is not a predictor variable. Also, in the CCSR/NIES AGCM, q and l are not independent variables, In fact, $q + l$ is the forecast variable.

Of these, The surface and subsurface related quantities T_g, W_g, W_y, h_I are At the same time it only stores the amount of one step, but, The atmospheric quantities u, v, T, p_S, q, l are, You need to memorize the amount for two steps at a time. This means that in the time integration of the dynamics of atmospheric quantities This is because the leap forg method is used.

The atmospheric quantities u, v, T, p_S, q, l are , The main routine is a variable administered by the main routine AGCM5. On the other hand, the surface and subsurface quantities of T_g, W_g, W_y, h_I It doesn't appear in the main routine, It is managed by the subroutine `MODULE:[PHYSCS]`, which is a subroutine for physical processes.

1.2.4 The flow of time evolution of variables

We briefly summarize the flow of the model, focusing on the time evolution of the predictor variables.

1. read the initial value `MODULE:[RDSTRT,PRSTRT]`

Initially, the quantities u, v, T, p_S, q, l related to the atmosphere are essentially Two sets of quantities in t and $t - \Delta t$ must be prepared. This can be prepared if you are starting from the output results of the previous model, but the It is not possible to prepare for a departure from normal observations and climate values. In that case, we start from the same value as the value of the two time steps, Launch the calculation using the fine Δt (see below for details).

The initial values for the atmospheric quantities u, v, T, p_S, q, l are read from , This is done with `MODULE:[RDSTRT]`, called by the main routine. On the other hand, the initial values of the surface and underground quantities T_g, W_g, W_y, h_I are read from Conducted by MODULE:[PRSTRT], called by MODULE:[PHYSCS].

2. start the time step MODULE:[TIMSTP]

Forecast variables at time t (and partly in $t-\Delta t$) $u^t, u^{t-\Delta t}, v^t, v^{t-\Delta t}, T^t, T^{t-\Delta t}, p_S^t, p_S^{t-\Delta t}, q^t, q^{t-\Delta}$ shall be complete.

```
$\Delta t$ is essentially an externally given parameter,
```

At regular intervals, the stability of the calculation is evaluated, If there is a risk of calculation instability reduce the size of the Δt MODULE:[TIMSTP].

Set the output of the predictor variable MODULE:[AHSTIN]

In the atmospheric forecast variables, the output is usually The value of time t at this stage $u^t, v^t, T^t, p_S^t, q^t, l^t$ It is. The actual output is performed by the later MODULE:[HISTOU] The timing, which is sent to the buffer here, is

4. mechanical processes MODULE:[DYNMCS]

Solving for the time variation of the predicted variables due to dynamical processes. $u^t, u^{t-\Delta t}, v^t, v^{t-\Delta t}, T^t, T^{t-\Delta t}, p_S^t, p_S^{t-\Delta t}, q^t, q^{t-\Delta t}, l^t, l^{t-\Delta t}$ Based on , Value of the forecast variable in $t+\Delta t$ considering only mechanical processes $\hat{u}^{t+\Delta t}, \hat{v}^{t+\Delta t}, \hat{T}^{t+\Delta t}, \hat{p_S}^{t+\Delta t}, \hat{q}^{t+\Delta t}, \hat{l}^{t+\Delta t}$ Ask for .

```
1. convert to vorticity and divergence ‘MODULE:[UV2VDG, VIRTMD, HGRAD]‘
```

Atmospheric forecast parameters of u, v, T, p_S, q, l In order to estimate the change term due to mechanical processes, we first need to estimate Convert u^t, v^t to the grid values of vorticity and divergence ζ^t, D^t. This is because the equations of mechanics are written in terms of vorticity and divergence. This transformation involves a spatial derivative, This can be done precisely by using the spherical harmonic function expansion MODULE:[UV2VDG].

Furthermore, calculate the pseudotemperature T_v^t, and then set MODULE:[VIRTMD], I still use the spherical harmonic function expansion. Calculates the horizontal differential of surface pressure $\nabla \ln p_S$ MODULE:[HGRAD].

2. calculation of the time-varying term by advection ‘MODULE:[GRDDYN]‘

Using the values in t of u, v, T, p_S, q, l, Due to horizontal and vertical advection, Compute some of the time-varying terms for each atmospheric variable. First, from the continuity equation, vertical velocity $\dot{\sigma}$ and To find the time variation term of p_S diagnostically, Using it, calculate the vertical advection term for u, v, T, q, l. Furthermore, the horizontal advection fluxes of u, v, T, q, l are calculated.

3. convert to a spectrum ‘MODULE:[GD2WD, TENG2W]‘

Value of grid points in $t - \Delta t$ for atmospheric forecast parameters From $u^t, v^t, T^t, p_S^t, q^t, l^t$, Values in Spectral Space in Spherical Harmonic Function Expansion (However, the vorticity is changed to divergence) $\tilde{\zeta}^t, \tilde{D}^t, \tilde{T}^t, \tilde{\pi}^t, \tilde{q}^t, \tilde{l}^t$ (but, $\pi \equiv \ln p_S$) MODULE:[GD2WD].

In addition, the vertical advection of u, v, T, p_S, q, l Expand the time-varying term into a spectrum. Also, by using the derivative in spectral space, Convergence of the horizontal advection flux is calculated, MODULE:[TENG2W]‘ to add to the spectral representation of the time change term.

This allows the ζ, D, T, π, q, l Most terms in the time-varying term are obtained as spectral values. However, among the time-varying terms in ζ, D, T, π, The term that depends linearly on horizontal divergence D is To do time integration by the semi-implicit method, It is not included in the time-varying term at this point.

4. time integration ‘MODULE:[TINTGR]‘

Among the time-varying terms in ζ, D, T, π , We have added a linearly dependent term (the gravitational wave term) to the horizontally diverging D Treat in the semi-implicit method, In addition, the horizontal diffusion of ζ, D, T, q, l By implicitly incorporating the Perform time integration of the mechanical process part. This allows for the Spectral representation of forecast values $\tilde{\zeta}^{t+\Delta t}, \tilde{D}^{t+\Delta t}, \tilde{T}^{t+\Delta t}, \tilde{\pi}^{t+\Delta t}, \tilde{q}^{t+\Delta t}, \tilde{l}^{t+\Delta t}$ is required.

5. `conversion to grid values 'MODULE:[GENGD]'`

From the forecast variables in the spectral representation , $u, v, T, p_S, q, l,$ of $t+\Delta t$ considering only mechanical processes Grid point values for forecast values $\hat{u}^{t+\Delta t}, \hat{v}^{t+\Delta t}, \hat{T}^{t+\Delta t}, \hat{p_S}^{t+\Delta t}, \hat{q}^{t+\Delta t}, \hat{l}^{t+\Delta t}$.

6. `diffusion correction 'MODULE:[CORDIF, CORFRC]'`

Horizontal diffusion is applied on the surface of σ and so on, In large areas of mountain slopes, water vapor is transported uphill, Causing problems such as bringing false precipitation at the top of the mountain. To mitigate that, etc. such that the diffusion of the p surface is close to Insert corrections for T, q, l `MODULE:[CORDIF]`.

Also, heat from friction is added to $\hat{T}$ `MODULE:[CORFRC]`

7. `mass conservation correction 'MODULE:[MASFIX]'`

Saving of the global integral values of q and l is satisfied, and make corrections so that there will be no negative values in q. In addition, the correction is made so that the mass of the dry air is constant.

When I left DYNMCS , The value of the forecast parameter in $t - \Delta t$ has been discarded, Overwritten by the value of the forecast variable in t. The area containing the t forecast variable is , Only the mechanics process is considered. The value of the forecast parameter in $t + \Delta t$ is entered.

5. physical process `MODULE:[PHYSCS]`

Value of the predicted variables in $t + \Delta t$ considering only mechanical processes $\hat{u}^{t+\Delta t}, \hat{v}^{t+\Delta t}, \hat{T}^{t+\Delta t}, \hat{p_S}^{t+\Delta t}, \hat{q}^{t+\Delta t}, \hat{l}^{t+\Delta t}$ and by adding a time-varying term from physical processes to The value of the forecast parameter in $t + \Delta t$ $u^{t+\Delta t}, v^{t+\Delta t}, T^{t+\Delta t}, p_S^{t+\Delta t}, q^{t+\Delta t}, l^{t+\Delta t}$ Ask for .

`Calculation of the basic diagnostic variables 'MODULE:[PSETUP]'`

The basic Find the diagnostic variables.

2. cumulus convection, large-scale condensation ‘MODULE:[CUMLUS, LSCOND]‘

To find the time-varying terms of T, q, l due to cumulus convection, and MODULE:[CUMLUS] Perform time integration with MODULE:[GDINTG] just for that term. In addition, the time-varying terms of T, q, l due to large-scale condensation are found, and MODULE:[LSCOND], Perform time integration with MODULE:[GDINTG] just for that term. Precipitation due to cumulus convection and large scale condensation P_c, P_l, Cloud cover (C_c, C_l, C_c, C_l, etc.) is required. This makes T, q, l Adjusted value for convective condensation process $\hat{T}^{t+\Delta t,a}, \hat{q}^{t+\Delta t,a}, \hat{l}^{t+\Delta t,a}$ That would be.

3. set the surface boundary condition ‘MODULE:[GNDSFC, GNDALB]‘

Set up the surface conditions with given data. The ground state index, sea surface temperature, etc. are set to MODULE:[GNDSFC]. Also, the surface albedo is set to be other than sea level MODULE:[GNDALB]. (The calculation of sea-surface albedo has been incorporated into the radiative flux calculation routine.)

4. calculation of the radiation flux ‘MODULE:[RADCON, RADFLX]‘

Set the atmospheric data for radiation flux calculation MODULE:[RADCON]. Normally, ozone is read from a file. Cloud water and cloud cover are obtained by convection and large-scale condensation, We can also give it to you from the outside here. Using these and $\hat{T}^{t+\Delta t,a}, \hat{q}^{t+\Delta t,a}$ Shortwave and longwave radiation flux F_R, and Calculates the differential coefficient of surface temperature for implicit calculation MODULE:[RADFLX].

5. calculation of the vertical diffuse flux ‘MODULE:[VDFFLX, VFTND1]‘

```
$\hat{u}^{t+\Delta t}, \hat{v}^{t+\Delta t},
```

$\hat{}\{t+\Delta t, a\}, \hat{q}\hat{}\{t + \Delta t, a\}, \hat{l}\hat{}\{t + \Delta t, a\}\$ with, Fluxes in u, v, T, q, l$ by vertical diffusion and Calculate the differential coefficient for implicit calculation `MODULE:[VDFFLX]`. In addition, the implicit solution is computed midway through the LU decomposition, `MODULE:[VFTND1]`.

```
6. calculation of surface processes and time integration of underground variab
```

Calculate the fluxes of \$u, v, T, q \$ between the earth's surface and atmosphere, Considering the conduction of heat in the ground, the energy balance at the surface is Solve with an implicit solution. This allows the surface temperature (T_s) to be diagnostically determined and Value of the ground temperature in the $t + \Delta t$ $T_g^{t+\Delta t}$ is required. In addition, the rate of change of the predicted variables for the first layer of the atmosphere Find $F_{x,1}, F_{y,1}, Q_1, S_1$.

Snow accumulation and snowmelt processes are taken into account, The value of the snowpack in $t + \Delta t$ is determined by $W_y^{t+\Delta t}$, Taking into account the movement of water in the ground Ground moisture $W_g^{t+\Delta t}$ is required.

In the case of the oceanic mixed layer model, the Ocean temperature and sea ice thickness The value in $t + \Delta t$ is found by time integration.

```
7. evaluation of time variation due to radial and vertical diffusion 'MODULE:[
```

Combined radiative flux and vertical diffusion The rate of change of each forecast variable of the atmosphere over time. Seek $\mathcal{F}_x, \mathcal{F}_y, Q, S$ `MODULE:[VFTND2]`. In addition, isolate the contribution from the radiation from `MODULE:[RADTND]`. This is not directly used in the model, but , For the convenience of the data output.

Because we use the implicit method in these calculations, due to changes in surface temperature and atmospheric forecast variables. We are taking into account changes in flux. We'll account for that and the fluxes that break even. Calculate with `MODULE:[FLXCOR]`. This is also for the convenience of data output.

```
8. evaluation of gravitational wave resistance 'MODULE:[GRAVTY]'
```

Calculating the change in atmospheric momentum due to gravitational waves originating from the terrain, Time Dependence of u, v by Vertical Diffusion Add to $\mathcal{F}_x, \mathcal{F}_y$.

`9. evaluation of the atmospheric pressure change term`

Considering the changes in pressure due to precipitation and evaporation, Find the atmospheric pressure change term M.

`10. time integration of physical processes ‘MODULE:[GDINTG]‘`

due to radiation, vertical diffusion, surface processes, gravitational wave resistance, etc. The rate of change of each forecast variable of the atmosphere over time. Using $\mathcal{F}_x, \mathcal{F}_y, Q, M, S$, Find the value of $t + \Delta t$ by time integration.

`11. drying convection adjustment ‘MODULE:[DADJST]‘`

If the calculated T, q, l are unstable for dry convection Drying convection adjustment.

By the above procedure, Value of the forecast parameters in $t + \Delta t$ $u^{t+\Delta t}, v^{t+\Delta t}, T^{t+\Delta t}, p_S^{t+\Delta t}, q^{t+\Delta t}, l^{t+\Delta t}$ is required.

6. time filter `MODULE:[TFILT]`

In order to prevent the leap frog from causing a calculation mode, Apply a time filter. $u^{t-\Delta t}, u^t, u^{t+\Delta t}$ The results of the smoothing operation using the data at the three times of Operate on each variable by replacing it with u^t. (Actually, at the `MODULE:[TFILT]` stage, the Since the information on $u^{t-\Delta t}$ has been erased, This operation is a two-step process. The first stage of operation is done in the mechanical process.)

1.3 Basic Settings.

Here we present the basic setup of the model.

1.3.1 Coordinate System.

The coordinate system is basically, Longitude λ, Latitude φ, Normalized Pressure $\sigma = p/p_S$ ($p_S(\lambda, \varphi)$ are surface pressure.) and treat each as orthogonal. However, z is used as the vertical coordinates.

Longitude is discretized at equal intervals `MODULE:[ASETL]`.

$$\lambda_i = 2\pi \frac{i-1}{I} \quad i = 1, \ldots I-1 \tag{11}$$

Latitude is the Gauss latitude φ_j described in Mechanics, and `MODULE:[ASETL]`, Gauss-Legendre derived from the integral formula. This takes $\mu = \sin\varphi$ as its argument J The zero point of the next Legendre polynomial `MODULE:[GAUSS]`.

If J is large, we can approximate

$$\varphi_j = \pi(\frac{1}{2} - \frac{j-1/2}{J}) \quad j = 1, \ldots J-1 \tag{12}$$

Normally, the grid spacing of longitude and latitude is taken as $J = I/2$ almost equally. This is based on the triangular truncation of the spectral method.

Normalized atmospheric pressure (σ) is designed to give a good representation of the vertical structure of the atmosphere, suitably discretized at unequal intervals `MODULE:[ASETS]`. As we will discuss later in Mechanics, the value of the layer boundaries Define the $\sigma_{k-1/2}$ in $k = 1 \ldots K+1$ and then ,

$$\sigma_k = \left\{ \frac{1}{1+\kappa} \left(\frac{\sigma_{k-1/2}^{\kappa+1} - \sigma_{k+1/2}^{\kappa+1}}{\sigma_{k-1/2} - \sigma_{k+1/2}} \right) \right\}^{1/\kappa} \tag{13}$$

Find the σ representing the layer by Figure [a-setup:level]] (#a-setup:level) shows the 20 levels of the standard.

Each forecast variable is all, $(\lambda_i, \varphi_j, \sigma_k)$ or defined on the grid of $(\lambda_i, \varphi_j, z_l)$. (The underground level, z_l, is discussed in the Physical Processes section.)

In the time direction, they are discretized at equally spaced Δt, The time integration of the forecasting equation is performed. However, if the stability of the time integration may be compromised Δt can change.

1.3.2 Physical Constants.

The basic physical constants are shown below `MODULE:[APCON]`.

[]@llll@ Header0 Header1 Header2 Header3 earth radius

a	m	6.37×10^6	acceleration of gravity	g	
ms$^-$2	9.8	atmospheric pressure specific heat	C_p	J kg^{-1} K^{-1}	
1004.6	Atmospheric gas constant	R	J kg^{-1} K^{-1}	287.04	Latent heat water evaporation
L	J kg^{-1}	2.5×10^6	Water vapor constant pressure specific heat	C_v	
J kg^{-1} K^{-1}	1810.	Gas constant of water	R_v	J kg^{-1} K^{-1}	
461.	Density of liquid water	d_{H_2O}	J kg^{-1} K^{-1}	1000.	
e^*(273K)	Pa.	611	Stefan Bolzman	σ_{SB}	
W m^{-2} K^{-4}	5.67	Krman Constant	k		
0.4	Latent heat of ice melting	L_M	J kg^{-1}	3.4×10^5	Water Freez Point
T_M	K	273.15	Constant pressure specific heat of water	C_w	

J kg^{-1}	4,200.	The freezing point of seawater	T_I	K	
271.35	Specific heat ratio of ice at constant pressure	$C_I = C_w - L_M/T_M$		2397.	water vapor molecular weight ratio
$\epsilon = R/R_v$		0.622	coefficient of provisional temperature	$\epsilon_v = \epsilon^{-1} - 1$	
	0.606	Ratio of specific heat to gas constant	$\kappa = R/C_p$		
0.286					

2 Mechanical Processes.

2.1 Basic Equations.

2.1.1 Basic Equations.

The basic equation is , It is a system of primitive equations at the spherical (λ, φ) and σ coordinates, It is given as follows (Haltiner and Williams , 1980).

1. a series of expressions

$$\frac{\partial \pi}{\partial t} + \mathbf{v}_H \cdot \nabla_\sigma \pi = -\nabla_\sigma \cdot \mathbf{v}_H - \frac{\partial \dot{\sigma}}{\partial \sigma} \tag{14}$$

> \\\\blur[mass]

2. hydrostatic pressure formula

$$\frac{\partial \Phi}{\partial \sigma} = -\frac{R T_v}{\sigma} \tag{15}$$

> \blaz

3. equation of motion

$$\frac{\partial \zeta}{\partial t} = \frac{1}{a \cos\varphi} \frac{\partial A_v}{\partial \lambda} - \frac{1}{a \cos\varphi} \frac{\partial}{\partial \varphi}(A_u \cos\varphi) - \mathcal{D}(\zeta) \tag{16}$$

> \\\\.00002}

$$\frac{\partial D}{\partial t} = \frac{1}{a \cos\varphi} \frac{\partial A_u}{\partial \lambda} + \frac{1}{a \cos\varphi} \frac{\partial}{\partial \varphi}(A_v \cos\varphi) - \nabla_\sigma^2 (\Phi + R\bar{T}\pi + E) - \mathcal{D}(D) \tag{17}$$

> \\begin{eqnarray}divergence]

4. thermodynamic equation

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$$\frac{\partial T}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial uT'}{\partial\lambda} - \frac{1}{a}\frac{\partial}{\partial\varphi}(vT'\cos\varphi) + T'D(18)$$

$$-\dot{\sigma}\frac{\partial T}{\partial\sigma} + \kappa T\left(\frac{\partial\pi}{\partial t} + \mathbf{v}_H\cdot\nabla_\sigma\pi + \frac{\dot{\sigma}}{\sigma}\right) + \frac{Q}{C_p} + \frac{Q_{diff}}{C_p} - \mathcal{D}(T)(19)$$

5. water vapor formula

$$\frac{\partial q}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial uq}{\partial\lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(vq\cos\varphi) + qD \tag{20}$$

$$-\dot{\sigma}\frac{\partial q}{\partial\sigma} + S_q - \mathcal{D}(q) \tag{21}$$

Here,

$$\theta \equiv T\left(p/p_0\right)^{-\kappa} \tag{22}$$

$$\kappa \equiv R/C_p \tag{23}$$

$$\Phi \equiv gz \tag{24}$$

$$\pi \equiv \ln p_S \tag{25}$$

$$\dot{\sigma} \equiv \frac{d\sigma}{dt} \tag{26}$$

$$T_v \equiv T(1+\epsilon_v q) \tag{27}$$

$$T \equiv \bar{T}(\sigma) + T' \tag{28}$$

$$\zeta \equiv \frac{1}{a\cos\varphi}\frac{\partial v}{\partial\lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(u\cos\varphi) \tag{29}$$

$$D \equiv \frac{1}{a\cos\varphi}\frac{\partial u}{\partial\lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(v\cos\varphi) \tag{30}$$

$$A_u \equiv (\zeta+f)v - \dot{\sigma}\frac{\partial u}{\partial\sigma} - \frac{RT'}{a\cos\varphi}\frac{\partial\pi}{\partial\lambda} + \mathcal{F}_x \tag{31}$$

$$A_v \equiv -(\zeta+f)u - \dot{\sigma}\frac{\partial v}{\partial\sigma} - \frac{RT'}{a}\frac{\partial\pi}{\partial\varphi} + \mathcal{F}_y \tag{32}$$

$$E \equiv \frac{u^2+v^2}{2} \tag{33}$$

$$\mathbf{v}_H\cdot\nabla \equiv \frac{u}{a\cos\varphi}\left(\frac{\partial}{\partial\lambda}\right)_\sigma + \frac{v}{a}\left(\frac{\partial}{\partial\varphi}\right)_\sigma \tag{34}$$

$$\nabla_\sigma^2 \equiv \frac{1}{a^2\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2} + \frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}\left[\cos\varphi\frac{\partial}{\partial\varphi}\right]. \tag{35}$$

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\[Divergent Definitions]

\brachio[Section B].

\\\.}

\\\.}

$\mathcal{D}(\zeta), \mathcal{D}(D), \mathcal{D}(T), \mathcal{D}(q)$ is the horizontal diffusion term, $\mathcal{F}_\lambda, \mathcal{F}_\varphi$ are forces due to small-scale kinetic processes (treated as 'physical processes'), The Q is a process of 'physical processes' such as radiation, condensation, and small-scale kinetic processes Heating and temperature changes, The S_q is a system of 'physical processes' such as condensation and small-scale kinetic processes The Q_{diff} is a water vapor source term. The Q_{diff} is the frictional heat,

$$Q_{diff} = -\mathbf{v} \cdot (\frac{\partial \mathbf{v}}{\partial t})_{diff}. \tag{36}$$

$($\partial \mathbf{v}_{\overline{\partial t)\_\{diff\}\$ is, The time-varying terms of u,v}}$ due to horizontal and vertical diffusion.

2.1.2 Boundary conditions.

The boundary conditions for lead-direct current are

$$\dot{\sigma} = 0 \quad at \quad \sigma = 0,\ 1. \tag{37}$$

. Thus, from ([\0.25\0.25}(#mass)), The time-varying surface pressure equation and Diagnostic Formula for determining the vertical velocity ($\dot{\sigma}$) in the σ system

$$\frac{\partial \pi}{\partial t} = -\int_0^1 \mathbf{v}_H \cdot \nabla_\sigma \pi d\sigma - \int_0^1 D d\sigma, \tag{38}$$

tendency

$$\dot{\sigma} = -\sigma \frac{\partial \pi}{\partial t} - \int_0^{\sigma} D d\sigma - \int_0^{\sigma} \mathbf{v}_H \cdot \nabla_{\sigma} \pi d\sigma, \tag{39}$$

[vertical speed]

is led.

2.2 Vertical discretization

According to Arakawa and Suarez (1983), The basic equations are discretized vertically by differences. This scheme has the following features.

- Conservation of the total domain-integrated mass
- Save the total integrated energy
- Preserving angular momentum for global integration
- Conservation of total mass-integrated potential temperature
- The hydrostatic pressure equation comes down to local (the altitude of the lower level is independent of the temperature of the upper level)
- Constant in the horizontal direction, for a given temperature distribution, The hydrostatic pressure equation becomes accurate and the barometric gradient force becomes zero.
- The isothermal atmosphere stays at the isothermal level indefinitely

2.2.1 How to take a level.

Number the layers from the bottom to the top. Assume that the physical quantity of ζ, D, T, q is defined in terms of integer levels (layers). On the other hand, $\dot{\sigma}$ is defined by the half-integer level (level). First, let the value of σ at the half-integer level be $\sigma_{k-1/2}, (k = 1, 2, \ldots K)$ is defined. except that level $\frac{1}{2}$ is the lower end ($\sigma = 1$), Level $K + \frac{1}{2}$ should be the uppermost ($\sigma = 0$).

The value of σ for an integer level $\sigma_k, (k = 1, 2, \ldots K)$ is found by the following formula.

$$\sigma_k = \left\{ \frac{1}{1+\kappa} \left(\frac{\sigma_{k-1/2}^{\kappa+1} - \sigma_{k+1/2}^{\kappa+1}}{\sigma_{k-1/2} - \sigma_{k+1/2}} \right) \right\}^{1/\kappa} \tag{40}$$

.0000

Furthermore,

$$\Delta\sigma_k \equiv \sigma_{k-1/2} - \sigma_{k+1/2} \tag{41}$$

Sigma thickness[sigma thickness]

.

2.2.2 vertical discretization representation.

The discretized representation of each equation is as follows.
The equation of continuity, vertical velocity

$$\frac{\partial \pi}{\partial t} = -\sum_{k=1}^{K} (D_k + \mathbf{v}_k \cdot \nabla\pi)\Delta\sigma_k \tag{42}$$

$$\dot{\sigma}_{k-1/2} = -\sigma_{k-1/2}\frac{\partial \pi}{\partial t} - \sum_{l=k}^{K} (D_l + \mathbf{v}_l \cdot \nabla\pi)\Delta\sigma_l \tag{43}$$

$$\dot{\sigma}_{1/2} = \dot{\sigma}_{K+1/2} = 0 \tag{44}$$

2. hydrostatic pressure equation

$$\Phi_1 = \Phi_s + C_p(\sigma_1^{-\kappa} - 1)T_{v,1} \tag{45}$$
$$= \Phi_s + C_p\alpha_1 T_{v,1} \tag{46}$$

$$\Phi_k - \Phi_{k-1} = C_p\left[\left(\frac{\sigma_{k-1/2}}{\sigma_k}\right)^{\kappa} - 1\right]T_{v,k} + C_p\left[1 - \left(\frac{\sigma_{k-1/2}}{\sigma_{k-1}}\right)^{\kappa}\right]T_{v,k-1} \tag{47}$$
$$= C_p\alpha_k T_{v,k} + C_p\beta_{k-1}T_{v,k-1} \tag{48}$$

Here ,

> <span id="Hydrostatic pressure coefficient" label="Hydrostatic pressure coeff

$$\alpha_k = \left(\frac{\sigma_{k-1/2}}{\sigma_k}\right)^{\kappa} - 1 \tag{49}$$

$$\beta_k = 1 - \left(\frac{\sigma_{k+1/2}}{\sigma_k}\right)^{\kappa}. \tag{50}$$

3. equation of motion

$$\frac{\partial \zeta_k}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial (A_v)_k}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(A_u \cos\varphi)_k - \mathcal{D}(\zeta_k) \tag{51}$$

> \\\.com\.}

$$\frac{\partial D}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial (A_u)_k}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(A_v \cos\varphi)_k - \nabla_\sigma^2(\Phi_k + C_p \hat{\kappa}_k \bar{T}_k \pi + (KE)_k) - \mathcal{D}(D_k) \tag{52}$$

Here,

$$(A_u)_k = (\zeta_k + f)v_k - \frac{1}{2\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(u_{k-1} - u_k) + \dot{\sigma}_{k+1/2}(u_k - u_{k+1})] \tag{53}$$

$$-\frac{C_p \hat{\kappa}_k T'_{v,k}}{a\cos\varphi}\frac{\partial \pi}{\partial \lambda} + \mathcal{F}_x \tag{54}$$

$$(A_v)_k = -(\zeta_k + f)u_k - \frac{1}{2\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(v_{k-1} - v_k) + \dot{\sigma}_{k+1/2}(v_k - v_{k+1})] \tag{55}$$

$$-\frac{C_p \hat{\kappa}_k T'_{v,k}}{a}\frac{\partial \pi}{\partial \varphi} + \mathcal{F}_y \tag{56}$$

> <span id="Hatchetkappa" label="Hatchetkappa">\\blade\.com\blade\bladeCoCoCoCo

$$\hat{\kappa}_k = \frac{\sigma_{k-1/2}(\sigma_{k-1/2}^{\kappa} - \sigma_k^{\kappa}) + \sigma_{k+1/2}(\sigma_k^{\kappa} - \sigma_{k+1/2}^{\kappa})}{\sigma_k^{\kappa}(\sigma_{k-1/2} - \sigma_{k+1/2})} \tag{57}$$

$$= \frac{\sigma_{k-1/2}\alpha_k + \sigma_{k+1/2}\beta_k}{\Delta\sigma_k} \tag{58}$$

$$T'_{v,k} = T_{v,k} - \bar{T}_k \tag{59}$$

4. thermodynamic equation

$$\frac{\partial T_k}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial u_k T'_k}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(v_k T'_k \cos\varphi) + H_k \tag{60}$$

$$+\frac{Q_k}{C_p} + \frac{(Q_{diff})_k}{C_p} - \mathcal{D}(T_k) \tag{61}$$

$$\tag{62}$$

```
Where ,
```

$$H_k \equiv T'_k D_k - \frac{1}{\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(\hat{T}_{k-1/2} - T_k) + \dot{\sigma}_{k+1/2}(T_k - \hat{T}_{k+1/2})] \tag{63}$$

$$+\left\{\alpha_k\left[\sigma_{k-1/2}\mathbf{v}_k\cdot\nabla\pi - \sum_{l=k}^{K}(D_l + \mathbf{v}_l\cdot\nabla\pi)\Delta\sigma_l\right]\right. \tag{64}$$

$$\left. +\,\beta_k\left[\sigma_{k+1/2}\mathbf{v}_k\cdot\nabla\pi - \sum_{l=k+1}^{K}(D_l + \mathbf{v}_l\cdot\nabla\pi)\Delta\sigma_l\right]\right\}\frac{1}{\Delta\sigma_k}T_{v,k} \tag{65}$$

$$= T'_k D_k - \frac{1}{\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(\hat{T}_{k-1/2} - T_k) + \dot{\sigma}_{k+1/2}(T_k - \hat{T}_{k+1/2})] \tag{66}$$

$$+\hat{\kappa}_k\mathbf{v}_k\cdot\nabla\pi T_{v,k} \tag{67}$$

$$-\alpha_k\sum_{l=k}^{K}(D_l + \mathbf{v}_l\cdot\nabla\pi)\Delta\sigma_l\frac{T_{v,k}}{\Delta\sigma_k} \tag{68}$$

$$-\beta_k\sum_{l=k+1}^{K}(D_l + \mathbf{v}_l\cdot\nabla\pi)\Delta\sigma_l\frac{T_{v,k}}{\Delta\sigma_k} \tag{69}$$

$$\hat{T}_{k-1/2} = \frac{\left[\left(\frac{\sigma_{k-1/2}}{\sigma_k}\right)^{\kappa} - 1\right]\sigma_{k-1}^{\kappa}T_k + \left[1 - \left(\frac{\sigma_{k-1/2}}{\sigma_{k-1}}\right)^{\kappa}\right]\sigma_k^{\kappa}T_{k-1}}{\sigma_{k-1}^{\kappa} - \sigma_k^{\kappa}} \tag{70}$$

$$= a_k T_k + b_{k-1}T_{k-1} \tag{71}$$

```
> <span id="Temperature Interpolation Factor" label="Temperature Interpolation
```

$$a_k = \alpha_k\left[1 - \left(\frac{\sigma_k}{\sigma_{k-1}}\right)^{\kappa}\right]^{-1} \tag{72}$$

$$b_k = \beta_k\left[\left(\frac{\sigma_k}{\sigma_{k+1}}\right)^{\kappa} - 1\right]^{-1}. \tag{73}$$

5. water vapor formula

$$\frac{\partial q_k}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial u_k q_k}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(v_k q_k \cos\varphi) + R_k + S_{q,k} - \mathcal{D}(q_k) \quad (74)$$

> <span id="q eventually" label="q eventually" label="q eventually">\\blana[q e

$$R_k = q_k D_k - \frac{1}{2\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(q_{k-1} - q_k) + \dot{\sigma}_{k+1/2}(q_k - q_{k+1})] \quad (75)$$

2.3 Horizontal discretization

The horizontal discretization of the The spectral transformation method is used (Bourke, 1988). The differential terms for longitude and latitude are evaluated by orthogonal function expansion, On the other hand, non-linear terms are computed on the grid points.

2.3.1 Spectral Expansion.

As an expansion function system, it is a Laplacian eigenfunction system on a sphere The spherical harmonic function $Y_n^m(\lambda, \mu)$ are used. with $\mu \equiv \sin\varphi$. Y_n^m satisfies the following equation,

$$\nabla_\sigma^2 Y_n^m(\lambda, \mu) = -\frac{n(n+1)}{a^2} Y_n^m(\lambda, \mu) \tag{76}$$

Using the Legendre junction number P_n^m it is written as follows.

$$Y_n^m(\lambda, \mu) = P_n^m(\mu) e^{im\lambda} \tag{77}$$

but \$ n $\geq jmj$\$.

The expansion by the spherical harmonic function is written as ,

$$Y_n^m{}_{ij} \equiv Y_n^m(\lambda_i, \mu_j) \tag{78}$$

If you write ,

$$X_{ij} \equiv X(\lambda_i, \mu_j) = \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{n=|m|}^{N} X_n^m Y_n^m{}_{ij}, \tag{79}$$

\[Spherical Expansion]

The inverse of that is ,

$$X_n^m = \frac{1}{4\pi} \int_{-1}^{1} d\mu \int_0^{\pi} d\lambda X(\lambda, \mu) Y_n^{m*}(\lambda, \mu) \tag{80}$$

$$= \frac{1}{I} \sum_{i=1}^{I} \sum_{j=1}^{J} X_{ij} Y_n^{m*}{}_{ij} w_j \tag{81}$$

\.com[Deployment Factor]

. When evaluating by replacing the integral with the sum, See Gauss's trapezoidal formula for the λ integral, We use the Gauss-Legendre integral formula for the μ integral. μ_j is the Gauss latitude and w_j is the Gauss load. Also, λ_i is a grid of evenly spaced Gauss loads.

Using the spectral expansion, we can obtain a new formula for Gauss-Legendre integration, The grid point values for the terms containing the derivatives are found as follows.

$$\left(\frac{\partial X}{\partial \lambda}\right)_{ij} = \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{n=|m|}^{N} imX_n^m {Y_n^m}_{ij} \tag{82}$$

barometric pressure x].

$$\left(\cos\varphi \frac{\partial X}{\partial \varphi}\right)_{ij} = \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{n=|m|}^{N} X_n^m (1-\mu^2) \frac{\partial}{\partial \mu} {Y_n^m}_{ij} \tag{83}$$

[barometric y].

Furthermore, From the spectral components of ζ and D, The grid point values for u, v are obtained as follows.

$$u_{ij} = \frac{1}{\cos\varphi} \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{\substack{n=|m| \\ n\neq 0}}^{N} \left\{ \frac{a}{n(n+1)} \zeta_n^m (1-\mu^2) \frac{\partial}{\partial \mu} {Y_n^m}_{ij} - \frac{ima}{n(n+1)} D_n^m {Y_n^m}_{ij} \right\} \tag{84}$$

\[Seeking U].

$$v_{ij} = \frac{1}{\cos\varphi} \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{\substack{n=|m| \\ n\neq 0}}^{N} \left\{ -\frac{ima}{n(n+1)} \zeta_n^m {Y_n^m}_{ij} - \frac{a}{n(n+1)} D_n^m (1-\mu^2) \frac{\partial}{\partial \mu} {Y_n^m}_{ij} \right\} \tag{85}$$

\[V seeking]

The derivative that appears in the advection term of the equation is, It is required as follows.

[A integral]

$$\left(\frac{1}{a\cos\varphi}\frac{\partial A}{\partial\lambda}\right)_n^m = \frac{1}{4\pi}\int_{-1}^{1} d\mu \int_0^{\pi} d\lambda \frac{1}{a\cos\varphi}\frac{\partial A}{\partial\lambda} Y_n^{m*} \tag{86}$$

$$= \frac{1}{4\pi}\int_{-1}^{1} d\mu \int_0^{\pi} d\lambda\, imA\cos\varphi \frac{1}{a(1-\mu^2)} Y_n^{m*} \tag{87}$$

$$= \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} imA_{ij}\cos\varphi_j {Y_n^{m*}}_{ij} \frac{w_j}{a(1-\mu_j^2)} \tag{88}$$

B integral].

$$\left(\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(A\cos\varphi)\right)_n^m = \frac{1}{4\pi a}\int_{-1}^{1} d\mu \int_0^{\pi} d\lambda \frac{\partial}{\partial\mu}(A\cos\varphi) Y_n^{m*} \tag{89}$$

$$= -\frac{1}{4\pi a}\int_{-1}^{1} d\mu \int_0^{\pi} d\lambda A\cos\varphi \frac{\partial}{\partial\mu} Y_n^{m*} \tag{90}$$

$$= -\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} A_{ij}\cos\varphi_j (1-\mu_j^2)\frac{\partial}{\partial\mu} {Y_n^{m*}}_{ij} \frac{w_j}{a(1-\mu_j^2)} \tag{91}$$

Further ,

$$\left(\nabla_\sigma^2 X\right)_n^m = -\frac{n(n+1)}{a^2} X_n^m \tag{92}$$

to evaluate the term ∇^2.

2.3.2 Horizontal Diffusion Term.

The horizontal diffusion term is entered in the form ∇^{N_D} as follows.

$$\mathcal{D}(\zeta) = K_{MH}\left[(-1)^{N_D/2}\nabla^{N_D} - \left(\frac{2}{a^2}\right)^{N_D/2}\right]\zeta, \tag{93}$$

Regular[Horizontal Diffusion].

$$\mathcal{D}(D) = K_{MH} \left[(-1)^{N_D/2} \nabla^{N_D} - \left(\frac{2}{a^2} \right)^{N_D/2} \right] D, \tag{94}$$

$$\mathcal{D}(T) = (-1)^{N_D/2} K_{HH} \nabla^{N_D} T, \tag{95}$$

$$\mathcal{D}(q) = (-1)^{N_D/2} K_{EH} \nabla^{N_D} q. \tag{96}$$

This horizontal diffusion term has strong implications for computational stability. In order to represent selective horizontal diffusion on small scales, For N_D, $4 \sim 16$ is used. The extra terms on the diffusion of vorticity and divergence are It represents that the term for rigid body rotation in $n = 1$ does not decay.

2.3.3 Spectral representation of the equation

1. a series of equations

$$\frac{\partial \pi_m^m}{\partial t} = - \sum_{k=1}^{K} (D_n^m)_k \Delta \sigma_k \tag{97}$$

$$+ \frac{1}{I} \sum_{i=1}^{I} \sum_{j=1}^{J} Z_{ij} Y_n^{m*}{}_{ij} w_j, \tag{98}$$

Here,

$$Z \equiv - \sum_{k=1}^{K} \mathbf{v}_k \cdot \nabla \pi. \tag{99}$$

2. equation of motion

$$\frac{\partial \zeta_n^m}{\partial t} = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} im(A_v)_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{100}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(A_u)_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{101}$$

$$-(\mathcal{D}_M)_n^m\zeta_n^m\ , \tag{102}$$

$$\frac{\partial \tilde{D}_n^m}{\partial t} = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} im(A_u)_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{103}$$

$$-\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(A_v)_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{104}$$

$$-\frac{n(n+1)}{a^2}\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} E_{ij}{Y_n^{m*}}_{ij}w_j \tag{105}$$

$$+\frac{n(n+1)}{a^2}(\Phi_n^m + C_p\hat{\kappa}_k\bar{T}_k\pi_n^m) - (\mathcal{D}_M)_n^m D_n^m, \tag{106}$$

However,

$$(\mathcal{D}_M)_n^m = K_{MH}\left[\left(\frac{n(n+1)}{a^2}\right)^{N_D/2} - \left(\frac{2}{a^2}\right)^{N_D/2}\right]. \tag{107}$$

3. thermodynamic equation

$$\frac{\partial T_n^m}{\partial t} = -\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} imu_{ij}T'_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{108}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} v_{ij}T'_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{109}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}\left(H_{ij} + \frac{Q_{ij}+Q_{diff}}{C_p}\right){Y_n^{m*}}_{ij}w_j \tag{110}$$

$$-(\tilde{\mathcal{D}}_H)_n^m T_n^m\ , \tag{111}$$

However,

$$
(\mathcal{D}_H)_n^m = K_{HH} \left(\frac{n(n+1)}{a^2} \right)^{N_D/2}. \tag{112}
$$

4. water vapor formula

$$
\frac{\partial q_n^m}{\partial t} = -\frac{1}{I} \sum_{i=1}^{I} \sum_{j=1}^{J} imu_{ij} q_{ij} \cos \varphi_j Y_n^{m*}{}_{ij} \frac{w_j}{a(1-\mu_j^2)} \tag{113}
$$

$$
+\frac{1}{I} \sum_{i=1}^{I} \sum_{j=1}^{J} v_{ij} q_{ij} \cos \varphi_j (1-\mu_j^2) \frac{\partial}{\partial \mu} Y_n^{m*}{}_{ij} \frac{w_j}{a(1-\mu_j^2)} \tag{114}
$$

$$
+\frac{1}{I} \sum_{i=1}^{I} \sum_{j=1}^{J} \left(\hat{R}_{ij} + S_{q,ij} \right) Y_n^{m*}{}_{ij} w_j \tag{115}
$$

$$
+(\mathcal{D}_H)_n^m q_n^m \tag{116}
$$

However,

$$
(\mathcal{D}_E)_n^m = K_{EH} \left(\frac{n(n+1)}{a^2} \right)^{N_D/2}. \tag{117}
$$

2.4 Time integration.

The time difference scheme is essentially a leap frog. However, the diffusion terms and physical process terms are backward or forward differences. A time filter (Asselin, 1972) is used to suppress the computational mode. In order to increase the value of Δt, we use a time filter (Asselin, 1972), Applying the semi-implicit method to the gravitational wave term (Bourke, 1988).

2.4.1 Time integration and time filtering by leap frog

We use leap frog as a time integration scheme for advection terms and so on. The backward difference of $2\Delta t$ is used for the horizontal diffusion term. The pseudo p surface correction of the diffusion term and the frictional heat by horizontal diffusion term are combined with treated as a correction and becomes a forward difference in $2\Delta t$. The physical process terms $(\mathcal{F}_\lambda, \mathcal{F}_\varphi, Q, S_q)$ are treated as We still use the forward difference of $2\Delta t$. (However, for the calculation of the time-varying term of vertical diffusion, we treat it as a backward difference. Please refer to the chapter on physical processes for details.)

Representing each of the forecast variables as X, we obtain

$$\hat{X}^{t+\Delta t} = \bar{X}^{t-\Delta t} + 2\Delta t \dot{X}_{adv}\left(X^t\right) + 2\Delta t \dot{X}_{dif}\left(\hat{X}^{t+\Delta t}\right) \tag{118}$$

\$ {*adv*} *\$ is an advection term etc, \$* {dif} \$ is a horizontal diffusion term.

In \$ ˆ{t+Δ*t*}\$*, thereisahorizontaldiffusionterm, Pseudo, etc.p* Correction of frictional heat (\$ {*dis*} *)bysurfaceandhorizontaldiffusionandphysicalprocesses(* {phy} \$) have been added, \$ {X}ˆ{t+Δ*t*}\$.

$$X^{t+\Delta t} = \hat{X}^{t+\Delta t} + 2\Delta t \dot{X}_{dis}\left(\hat{X}^{t+\Delta t}\right) + 2\Delta t \dot{X}_{phy}\left(\hat{X}^{t+\Delta t}\right) \tag{119}$$

To remove the calculation mode in leap frog The time filter of Asselin (1972) is applied at every step. I.e. ,

$$\bar{X}^t = (1 - 2\epsilon_f)X^t + \epsilon_f\left(\bar{X}^{t-\Delta t} + X^{t+\Delta t}\right) \tag{120}$$

and $\bar{X}$. Normally 0.05 is used as the ϵ_f.

2.4.2 semi-implicit time integration

For mechanics calculations, the leap frog is basically used, We treat some terms as implicit. Here, the implicit considers the trapezoidal implicit. For the vector quantity $\mathbf{q}$, The value in t is converted to $\mathbf{q}$, The value in $t + \Delta t$ was converted to $\mathbf{q}^+$, If you write the value of $t - \Delta t$ as $\mathbf{q}^-$, What is trapezoidal implicit? $(\mathbf{q}^+ + \mathbf{q}^-)/2$. We use the time-varying terms evaluated by using Now, as a time-varying term in q, The term A is treated in the leap forg method and the term B is treated in the trapezoidal implicit method. Assume that A is nonlinear with respect to q, while B is linear. In other words,

$$\mathbf{q}^+ = \mathbf{q}^- + 2\Delta t \mathcal{A}(\mathbf{q}) + 2\Delta t B(\mathbf{q}^+ + \mathbf{q}^-)/2 \tag{121}$$

where B is a square matrix. Then, $\Delta\mathbf{q} \equiv \mathbf{q}^+ - \mathbf{q}$ And then you can write,

$$(I - \Delta t B)\Delta\mathbf{q} = 2\Delta t \left(\mathcal{A}(\mathbf{q}) + B\mathbf{q}\right) \tag{122}$$

This can be easily solved by matrix operations.

2.4.3 semi-implicit time integration applied

Then, we apply this method and treat the term of linear gravity waves as implicit. This allows us to reduce the time step (Δt).

In the system of equations, the basic field is such that $T = \bar{T}_k$ Separation of the linear gravity wave term and the other terms (with the index NG). Vertical Vector Representation Using $\mathbf{D} = \{D_k\}$, $\mathbf{T} = \{T_k\}$,

$$\frac{\partial \pi}{\partial t} = \left(\frac{\partial \pi}{\partial t}\right)_{NG} - \mathbf{C} \cdot \mathbf{D}, \tag{123}$$

$$\frac{\partial \mathbf{D}}{\partial t} = \left(\frac{\partial \mathbf{D}}{\partial t}\right)_{NG} - \nabla^2_\sigma(\mathbf{\Phi}_S + \underline{W}\mathbf{T} + \mathbf{G}\pi) - \mathcal{D}_M\mathbf{D}, \tag{124}$$

$$\frac{\partial \mathbf{T}}{\partial t} = \left(\frac{\partial \mathbf{T}}{\partial t}\right)_{NG} - \underline{h}\mathbf{D} - \mathcal{D}_H \mathbf{T}, \tag{125}$$

where the non-gravitational wave term is,

$$\left(\frac{\partial \pi}{\partial t}\right)^{NG} = -\sum_{k=1}^{K} \mathbf{v}_k \cdot \nabla \pi \Delta\sigma_k \tag{126}$$

$$= Z_k \tag{127}$$

<span id="Section Z" label="Section Z" label="Section Z" label=">\\.}

$$\dot{\sigma}^{NG}_{k-1/2} = -\sigma_{k-1/2}\left(\frac{\partial \pi}{\partial t}\right)^{NG} - \sum_{l=k}^{K} \mathbf{v}_l \cdot \nabla \pi \Delta\sigma_l \tag{128}$$

$$\left(\frac{\partial D}{\partial t}\right)^{NG} = \frac{1}{a\cos\varphi}\frac{\partial (A_u)_k}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(A_v \cos\varphi)_k - \nabla^2_\sigma \hat{E}_k - \mathcal{D}(D_k) \tag{129}$$

$$\left(\frac{\partial T_k}{\partial t}\right)^{NG} = -\frac{1}{a\cos\varphi}\frac{\partial u_k T'_k}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(v_k T'_k \cos\varphi) + \hat{H}_k - \mathcal{D}(T_k) \tag{130}$$

$$\hat{H}_k = T'_k D_k \tag{131}$$

$$-\frac{1}{\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(\hat{T'}_{k-1/2} - T'_k) + \dot{\sigma}_{k+1/2}(T'_k - \hat{T'}_{k+1/2})] \tag{132}$$

$$-\frac{1}{\Delta\sigma_k}[\dot{\sigma}^{NG}_{k-1/2}(\hat{\bar{T}}_{k-1/2} - \bar{T}_k) + \dot{\sigma}^{NG}_{k+1/2}(\bar{T}_k - \hat{\bar{T}}_{k+1/2})] \tag{133}$$

$$+\hat{\kappa}_k T_{v,k} \mathbf{v}_k \cdot \nabla \pi \tag{134}$$

$$-\frac{\alpha_k}{\Delta\sigma_k} T_{v,k} \sum_{l=k}^{K} \mathbf{v}_l \cdot \nabla \pi \Delta\sigma_l - \frac{\beta_k}{\Delta\sigma_k} T_{v,k} \sum_{l=k+1}^{K} \mathbf{v}_l \cdot \nabla \pi \Delta\sigma_l \tag{135}$$

$$-\frac{\alpha_k}{\Delta\sigma_k} T'_{v,k} \sum_{l=k}^{K} D_l \Delta\sigma_l - \frac{\beta_k}{\Delta\sigma_k} T'_{v,k} \sum_{l=k+1}^{K} D_l \Delta\sigma_l \tag{136}$$

$$+\frac{Q_k + (Q_{diff})_k}{C_p} \tag{137}$$

$$\hat{E}_k = E_k + \sum_{k=1}^{K} W_{kl}(T_{v,l} - T_l) \tag{138}$$

where the vector and matrix of the gravitational wave terms (underlined) are,

$$C_k = \Delta\sigma_k \tag{139}$$

\c[Coefficient C]

$$W_{kl} = C_p \alpha_l \delta_{k \geq l} + C_p \beta_l \delta_{k-1 \geq l} \tag{140}$$

$$G_k = \hat{\kappa}_k C_p \bar{T}_k \tag{141}$$

$$\underline{h} = \underline{\underline{Q}}\,\underline{S} - \underline{R} \tag{142}$$

$$Q_{kl} = \frac{1}{\Delta\sigma_k}(\hat{\bar{T}}_{k-1/2} - \bar{T}_k)\delta_{k=l} + \frac{1}{\Delta\sigma_k}(\bar{T}_k - \hat{\bar{T}}_{k+1/2})\delta_{k+1=l} \tag{143}$$

$$S_{kl} = \sigma_{k-1/2}\Delta\sigma_l - \Delta\sigma_l \delta_{k \leq l} \tag{144}$$

$$R_{kl} = -\left(\frac{\alpha_k}{\Delta\sigma_k}\Delta\sigma_l \delta_{k \leq l} + \frac{\beta_k}{\Delta\sigma_k}\Delta\sigma_l \delta_{k+1 \leq l}\right)\bar{T}_k. \tag{145}$$

[Coefficient R].

Here, for example, $\delta_{k\leq l}$ is the same as It is a function that is 1 if \$ k $\leq l$\$$istrueand0otherwise$.

Using the following expression,

$$\delta_t X \equiv \frac{1}{2\Delta t}\left(X^{t+\Delta t} - X^{t-\Delta t}\right) \tag{146}$$

\centric=“Shemiinp”> .

$$\overline{X}^t \equiv \frac{1}{2}\left(X^{t+\Delta t} + X^{t-\Delta t}\right) \tag{147}$$

$$= X^{t-\Delta t} + \delta_t X \Delta t, \tag{148}$$

Applying the semi-implicit method to a system of equations,

$$\delta_t \pi = \left(\frac{\partial \pi}{\partial t}\right)_{NG} - \mathbf{C}\cdot\overline{\mathbf{D}}^t \tag{149}$$

[semi-imp pi]

$$\delta_t \mathbf{D} = \left(\frac{\partial \mathbf{D}}{\partial t}\right)_{NG} - \nabla_\sigma^2(\mathbf{\Phi}_S + \underline{W}\overline{\mathbf{T}}^t + \mathbf{G}\overline{\pi}^t) - \mathcal{D}_M(\mathbf{D}^{t-\Delta t} + 2\Delta t \delta_t \mathbf{D}) \tag{150}$$

[semi-imp D\[semi-imp D]

$$\delta_t \mathbf{T} = \left(\frac{\partial \mathbf{T}}{\partial t}\right)_{NG} - \underline{h}\overline{\mathbf{D}}^t - \mathcal{D}_H(\mathbf{T}^{t-\Delta t} + 2\Delta t \delta_t \mathbf{T}) \tag{151}$$

[semi-imp T]

So..,

\brachos[semi-imp barD]

$$
\begin{aligned}
\left\{(1+2\Delta t\mathcal{D}_H)(1+2\Delta t\mathcal{D}_M)\underline{I} - (\Delta t)^2(\underline{W}\ \underline{h} + (1+2\Delta t\mathcal{D}_M)\mathbf{G}\mathbf{C}^T)\nabla_\sigma^2\right\}\overline{\mathbf{D}}^t & \quad (152)\\
= (1+2\Delta t\mathcal{D}_H)(1+\Delta t\mathcal{D}_M)\mathbf{D}^{t-\Delta t} + \Delta t\left(\frac{\partial \mathbf{D}}{\partial t}\right)_{NG} & \quad (153)\\
-\Delta t\nabla_\sigma^2\left\{(1+2\Delta t\mathcal{D}_H)\mathbf{\Phi}_S + \underline{W}\left[(1-2\Delta t\mathcal{D}_H)\mathbf{T}^{t-\Delta t} + \Delta t\left(\frac{\partial \mathbf{T}}{\partial t}\right)_{NG}\right]\right. & \quad (154)\\
\left.+(1+2\Delta t\mathcal{D}_H)\mathbf{G}\left[\pi^{t-\Delta t} + \Delta t\left(\frac{\partial \pi}{\partial t}\right)_{NG}\right]\right\} & \quad (155)
\end{aligned}
$$

Since we use the spherical harmonic expansion, we can use it, and the above formula can be solved for $\overline{\mathbf{D}_n^m}^t$. After that,

$$
D^{t+\Delta t} = 2\overline{\mathbf{D}}^t - D^{t-\Delta t} \tag{156}
$$

and (semi-imp pi], (semi-imp T\) The value in $t+\Delta t$ according to $\hat{X}^{t+\Delta t}$ is required .

2.4.4 Time scheme properties and time step estimates

advectional equation

$$
\frac{\partial X}{\partial t} = c\frac{\partial X}{\partial x} \tag{157}
$$

Consider the stability of the leap frog discretization in Now, If we place the difference between

$$
X^{n+1} = X^{n-1} + 2ik\Delta t X^n \tag{158}
$$

where Here , \lambda = X{n+1}/Xn = Xn/X{n-1}}\\bars}] So,

$$
\lambda^2 = 1 + 2ikc\Delta t\lambda\ . \tag{159}
$$

The solution is labeled $kc\Delta t = p$,

$$\lambda = -ip \pm \sqrt{1-p^2} \tag{160}$$

This absolute value is

$$|\lambda| = \begin{cases} 1|p| \leq 1 \\ p \pm \sqrt{p^2-1} \ \ |p| > 1 \end{cases} \tag{161}$$

In the case of $|p| > 1$, it would be $|\lambda| > 1$, The solution becomes exponentially larger in absolute value with time. This indicates that the computation is unstable.

On the other hand, in the case of $|p| \leq 1$, the value is $|\lambda| = 1$, The calculation is neutral. However, there are two solutions for λ, One of them, when $\Delta t \to 1$ is set to This is a $\lambda \to 1$, but.., The other is $\lambda \to -1$. This indicates a time-varying solution. This mode is called "calculation mode", One of the problems with the leap frog method. This mode can be applied by applying a time filter to the It can be attenuated.

The terms of the $|p| = kc\Delta t \leq 1$ are, Given the horizontal discretization grid spacing Δx, the This will cause the maximum value of k to be than one person can be in a position to do so. From becoming ,

$$\Delta t \leq \frac{\Delta x}{\pi c} \tag{162}$$

In the case of the spectral model, the maximum wavenumber of For the spectral model, the maximum wavenumber is determined by N, Earth radius is set to a,

$$\Delta t \leq \frac{a}{Nc} \tag{163}$$

This is the stability condition.

To guarantee the stability of the integral, As for c, it has the fastest advection and propagation speed, You may use a time step smaller than Δt which is determined by the semi-implicit method. If semi-implicit is not used, the propagation speed of the gravitational wave ($c \sim 300m/s$)

is the criterion for stability, When semi-implicit is used, advection by the east-west wind is usually This is a limiting factor. Therefore, the Δt sets U_{max} as the maximum value of the east-west wind,

$$\Delta t \leq \frac{a}{NU_{max}} \tag{164}$$

In practice, this is multiplied by a safety factor. In practice, this should be multiplied by a safety factor.

2.4.5 Treatment at the beginning of time integration.

Not calculated by AGCM, If you start with an appropriate initial value, you can use a model-consistent You cannot give the physical quantities for two times of t and $t - \Delta t$. However, if you give an inconsistent value for $t - \Delta t$, then you should not give an inconsistent value for $t - \Delta t$, A large calculation mode is generated.

So, first, as $X^{\Delta t/4} = X^0$, in the time step of 1/4 {X^{{X}{DΔt/2} = X^0+X^0+{{X}^{DΔt/2} = X^0+t/2nDentro{X}^0n0}]andfurthermore,inthetimestepof \\lopen}t = X^{X}{X}^{X}{Δt/2}}}t = X^0+}t}t}t/2}tAnd,intheoriginaltimestep,nnltraLab X^{2Δt} = X^0+2ttt}^{X}{t}]andthenperformthecalculationwithleapfrogasusual,Theoccurr

2.5 Summary of the mechanics part

Here, we duplicate the previous description, Enumerate the calculations performed in the mechanical part.

2.5.1 Summary of calculations in the mechanics part.

The mechanical processes are calculated in the following order.

1. the transformation of horizontal wind into vorticity and divergence `MODULE:[UV2VDG(dvect)]`
2. calculation of pseudotemperature `MODULE:[VIRTMD(dvtmp)]`
3. calculation of the barometric gradient term `MODULE:[HGRAD(dvect)]`
4. diagnostic calculation of vertical flow `MODULE:[GRDDYN/PSDOT(dgdyn)]`
5. time change term due to advection `MODULE:[GRDDYN(dgdyn)]`
6. convert the predictive variable to a spectrum `MODULE:[GD2WD(dg2wd)]`
7. convert the time-varying term into a spectrum `MODULE:[TENG2W(dg2wd)]`
8. time integration of spectral values `MODULE:[TINTGR(dintg)]`
9. convert the predictive variables to grid values `MODULE:[GENGD(dgeng)]`
10. pseudo etc. p plane spreading correction `MODULE:[CORDIF(ddifc)]`
11. consideration of frictional heat by diffusion `MODULE:[CORFRC(ddifc)]`
12. correction for conservation of mass `MODULE:[MASFIX(dmfix)]`
13. (physical process) `MODULE:[PHYSCS(padmn)]`
14. (time filter) `MODULE:[TFILT(aadvn)]`

2.5.2 Conversion of Horizontal Wind to Vorticity and Divergence

Grid point values for horizontal wind u_{ij}, v_{ij} from the grid point values of vorticity and divergence ζ_{ij}, D_{ij}. First, the spectra of vorticity and divergence Ask for ζ_n^m, D_n^m,

$$\zeta_n^m = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} imv_{ij}\cos\varphi_j Y_n^{m*}{}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{165}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} u_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}Y_n^{m*}{}_{ij}\frac{w_j}{a(1-\mu_j^2)} \ , \tag{166}$$

$$D_n^m = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} imu_{ij}\cos\varphi_j Y_n^{m*}{}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{167}$$

$$-\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} v_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}Y_n^{m*}{}_{ij}\frac{w_j}{a(1-\mu_j^2)};. \tag{168}$$

[d-summ:uv-D\[d-summ:uv-D].

And more,

$$\zeta_{ij} = \mathcal{R}\mathbf{e}\sum_{m=-N}^{N}\sum_{n=|m|}^{N}\zeta_n^m Y_n^m{}_{ij} \ , \tag{169}$$

and so on.

2.5.3 Calculating Pseudotemperature

Provisional Temperature T_v is ,

$$T_v = T(1+\epsilon_v q - l) \ , \tag{170}$$

However, it is $\epsilon_v = R_v/R - 1$, R_v is a gas constant for water vapor (461 Jkg^{-1}K^{-1}) R is a gas constant of air (287.04 Jkg^{-1}K^{-1}) .

2.5.4 Calculating the Barometric gradient term

The barometric gradient term $\nabla\pi = \frac{1}{p_S}\nabla p_S$ is , First, we need to get the π_n^m

$$\pi_n^m = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(\ln p_S)_{ij}Y_n^{m*}{}_{ij}w_j \ , \tag{171}$$

to a spectral representation and then ,

$$\frac{1}{a\cos\varphi}\left(\frac{\partial\pi}{\partial\lambda}\right)_{ij} = \frac{1}{a\cos\varphi}\mathcal{R}\mathbf{e}\sum_{m=-N}^{N}\sum_{n=|m|}^{N} im\tilde{X}_n^m Y_n^m{}_{ij} \ , \tag{172}$$

$$\frac{1}{a}\left(\frac{\partial\pi}{\partial\varphi}\right)_{ij} = \frac{1}{a\cos\varphi}\mathcal{R}\mathbf{e}\sum_{m=-N}^{N}\sum_{n=|m|}^{N}\pi_n^m(1-\mu^2)\frac{\partial}{\partial\mu}Y_n^m{}_{ij} \ . \tag{173}$$

2.5.5 Diagnostic calculations of vertical flow.

Barometric pressure change term, and lead DC,

$$\frac{\partial\pi}{\partial t} = -\sum_{k=1}^{K}(D_k + \mathbf{v}_k\cdot\nabla\pi)\Delta\sigma_k \tag{174}$$

$$\dot\sigma_{k-1/2} = -\sigma_{k-1/2}\frac{\partial\pi}{\partial t} - \sum_{l=k}^{K}(D_l + \mathbf{v}_l\cdot\nabla\pi)\Delta\sigma_l \tag{175}$$

and its non-gravity component.

$$\left(\frac{\partial\pi}{\partial t}\right)^{NG} = -\sum_{k=1}^{K}\mathbf{v}_k\cdot\nabla\pi\Delta\sigma_k \tag{176}$$

$$\tag{177}$$

$$\dot\sigma_{k-1/2}^{NG} = -\sigma_{k-1/2}\left(\frac{\partial\pi}{\partial t}\right)^{NG} - \sum_{l=k}^{K}\mathbf{v}_l\cdot\nabla\pi\Delta\sigma_l \tag{178}$$

2.5.6 The time-varying term due to advection.

Momentum advection term:

$$(A_u)_k = (\zeta_k + f)v_k - \frac{1}{2\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(u_{k-1} - u_k) + \dot{\sigma}_{k+1/2}(u_k - u_{k+1})] \tag{179}$$

$$-\frac{C_p\hat{\kappa}_k T'_{v,k}}{a\cos\varphi}\frac{\partial\pi}{\partial\lambda} \tag{180}$$

$$(A_v)_k = -(\zeta_k + f)u_k - \frac{1}{2\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(v_{k-1} - v_k) + \dot{\sigma}_{k+1/2}(v_k - v_{k+1})] \tag{181}$$

$$-\frac{C_p\hat{\kappa}_k T'_{v,k}}{a}\frac{\partial\pi}{\partial\varphi} \tag{182}$$

$$\hat{E}_k = \frac{1}{2}(u^2 + v^2) + \sum_{k'=1}^{k}[C_p\alpha_k(T_v - T)_{k'} + C_p\beta_k(T_v - T)_{k'-1}] \tag{183}$$

Temperature Advection Term:

$$(uT')_k = u_k(T_k - \bar{T}) \tag{184}$$

$$(vT')_k = v_k(T_k - \bar{T}) \tag{185}$$

$$\hat{H}_k = T'_k D_k \tag{186}$$

$$-\frac{1}{\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(\hat{T}'_{k-1/2} - T'_k) + \dot{\sigma}_{k+1/2}(T'_k - \hat{T}'_{k+1/2})] \tag{187}$$

$$-\frac{1}{\Delta\sigma_k}[\dot{\sigma}^{NG}_{k-1/2}(\hat{\bar{T}}_{k-1/2} - \bar{T}_k) + \dot{\sigma}^{NG}_{k+1/2}(\bar{T}_k - \hat{\bar{T}}_{k+1/2})] \tag{188}$$

$$+\hat{\kappa}_k T_{v,k}\mathbf{v}_k\cdot\nabla\pi \tag{189}$$

$$-\frac{\alpha_k}{\Delta\sigma_k}T_{v,k}\sum_{l=k}^{K}\mathbf{v}_l\cdot\nabla\pi\Delta\sigma_l - \frac{\beta_k}{\Delta\sigma_k}T_{v,k}\sum_{l=k+1}^{K}\mathbf{v}_l\cdot\nabla\pi\Delta\sigma_l \tag{190}$$

$$-\frac{\alpha_k}{\Delta\sigma_k}T'_{v,k}\sum_{l=k}^{K}D_l\Delta\sigma_l - \frac{\beta_k}{\Delta\sigma_k}T'_{v,k}\sum_{l=k+1}^{K}D_l\Delta\sigma_l \tag{191}$$

Water Vapor Advection Term:

$$(uq)_k = u_k q_k \tag{192}$$

$$(vq)_k = v_k q_k \tag{193}$$

$$R_k = q_k D_k - \frac{1}{2\Delta\sigma_k}[\dot{\sigma}_{k-1/2}(q_{k-1} - q_k) + \dot{\sigma}_{k+1/2}(q_k - q_{k+1})] \tag{194}$$

2.5.7 Conversion of Predictive Variables to Spectra.

(\eqnarrayd-summ:uv-D (194)

) to
$u_{ij}^{t-\Delta t}, v_{ij}^{t-\Delta t}$. Spectral representation of vorticity and divergence Convert to ζ_n^m, D_n^m. Furthermore , Temperature $T^{t-\Delta t}$, Specific Humidity $q^{t-\Delta t}$, $\pi = \ln p_S^{t-\Delta t}$.

$$X_n^m = \frac{1}{I} \sum_{i=1}^{I} \sum_{j=1}^{J} X_{ij} {Y_n^{m*}}_{ij} w_j \ , \tag{195}$$

to a spectral representation.

2.5.8 Conversion of time-varying terms to spectra.

Time Variation Term of Vorticity

$$\frac{\partial \zeta_n^m}{\partial t} = \frac{1}{I} \sum_{i=1}^{I} \sum_{j=1}^{J} im(A_v)_{ij} \cos\varphi_j {Y_n^{m*}}_{ij} \frac{w_j}{a(1-\mu_j^2)} \tag{196}$$

$$+\frac{1}{I} \sum_{i=1}^{I} \sum_{j=1}^{J} (A_u)_{ij} \cos\varphi_j (1-\mu_j^2) \frac{\partial}{\partial \mu} {Y_n^{m*}}_{ij} \frac{w_j}{a(1-\mu_j^2)} \tag{197}$$

$$\tag{198}$$

The non-gravity wave component of the time-varying term of the divergence

$$\left(\frac{\partial D_n^m}{\partial t}\right)^{NG} = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} im(A_u)_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{199}$$

$$-\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(A_v)_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{200}$$

$$-\frac{n(n+1)}{a^2}\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}\hat{E}_{ij}{Y_n^{m*}}_{ij}w_j \tag{201}$$

$$\tag{202}$$

The non-gravitational component of the time-varying term of temperature

$$\left(\frac{\partial T_n^m}{\partial t}\right)^{NG} = -\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} im(uT')_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{203}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(vT')_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{204}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}\hat{H}_{ij}{Y_n^{m*}}_{ij}w_j \tag{205}$$

Time-varying terms for water vapor

$$\frac{\partial q_n^m}{\partial t} = -\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J} im(uq)_{ij}\cos\varphi_j {Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{206}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}(vq)_{ij}\cos\varphi_j(1-\mu_j^2)\frac{\partial}{\partial\mu}{Y_n^{m*}}_{ij}\frac{w_j}{a(1-\mu_j^2)} \tag{207}$$

$$+\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{J}R_{ij}{Y_n^{m*}}_{ij}w_j \tag{208}$$

2.5.9 Spectral Value Time Integration

Equations in matrix form

$$\left\{(1+2\Delta t\mathcal{D}_H)(1+2\Delta t\mathcal{D}_M)\underline{I}-(\Delta t)^2(\underline{W}\ \underline{h}+(1+2\Delta t\mathcal{D}_M)\mathbf{G}\mathbf{C}^T)\nabla_\sigma^2\right\}\overline{\mathbf{D}}^t \tag{209}$$

$$=(1+2\Delta t\mathcal{D}_H)(1-\Delta t\mathcal{D}_M)\mathbf{D}^{t-\Delta t}+\Delta t\left(\frac{\partial \mathbf{D}}{\partial t}\right)_{NG} \tag{210}$$

$$-\Delta t\nabla_\sigma^2\left\{(1+2\Delta t\mathcal{D}_H)\mathbf{\Phi}_S+\underline{W}\left[(1-2\Delta t\mathcal{D}_H)\mathbf{T}^{t-\Delta t}+\Delta t\left(\frac{\partial \mathbf{T}}{\partial t}\right)_{NG}\right]\right. \tag{211}$$

$$\left.+(1+2\Delta t\mathcal{D}_H)\mathbf{G}\left[\pi^{t-\Delta t}+\Delta t\left(\frac{\partial \pi}{\partial t}\right)_{NG}\right]\right\} \tag{212}$$

By using LU decomposition to solve for Ask for $\bar{D}$,

$$\frac{\partial \mathbf{T}}{\partial t}=\left(\frac{\partial \mathbf{T}}{\partial t}\right)_{NG}-\underline{h}\mathbf{D} \tag{213}$$

$$\frac{\partial \pi}{\partial t}=\left(\frac{\partial \pi}{\partial t}\right)_{NG}-\mathbf{C}\cdot\mathbf{D} \tag{214}$$

by. $\partial\mathbf{T}/\partial t$, $\partial\pi/\partial t$ and calculate the value of the spectrum in $t+\Delta t$ by finding

$$\zeta^{t+\Delta t}=\left(\zeta^{t-\Delta t}+2\Delta t\frac{\partial \zeta}{\partial t}\right)(1+2\Delta t\mathcal{D}_M)^{-1} \tag{215}$$

$$D^{t+\Delta t}=2\bar{D}-D^{t-\Delta t} \tag{216}$$

$$T^{t+\Delta t}=\left(T^{t-\Delta t}+2\Delta t\frac{\partial T}{\partial t}\right)(1+2\Delta t\mathcal{D}_H)^{-1} \tag{217}$$

$$q^{t+\Delta t}=\left(q^{t-\Delta t}+2\Delta t\frac{\partial q}{\partial t}\right)(1+2\Delta t\mathcal{D}_E)^{-1} \tag{218}$$

$$\pi^{t+\Delta t}=\pi^{t-\Delta t}+2\Delta t\frac{\partial \pi}{\partial t} \tag{219}$$

2.5.10 Conversion of Prediction Variables to Grid Values

Spectral values of vorticity and divergence from ζ_n^m, D_n^m Find the grid values for the horizontal wind speed u_{ij}, v_{ij}.

$$u_{ij} = \frac{1}{\cos\varphi_j} \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{\substack{n=|m| \\ n\neq 0}}^{N} \left\{ \frac{a}{n(n+1)} \zeta_n^m (1-\mu^2) \frac{\partial}{\partial\mu} Y_{n\ ij}^m - \frac{ima}{n(n+1)} D_n^m Y_{n\ ij}^m \right\} \tag{220}$$

$$v_{ij} = \frac{1}{\cos\varphi_j} \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{\substack{n=|m| \\ n\neq 0}}^{N} \left\{ -\frac{ima}{n(n+1)} \zeta_n^m Y_{n\ ij}^m - \frac{a}{n(n+1)} \tilde{D}_n^m (1-\mu^2) \frac{\partial}{\partial\mu} Y_{n\ ij}^m \right\} \tag{221}$$

Furthermore ,

$$T_{ij} = \mathcal{R}\mathbf{e} \sum_{m=-N}^{N} \sum_{n=|m|}^{N} T_n^m Y_{n\ ij}^m \ , \tag{222}$$

T_{ij}, π_{ij}, q_{ij} are obtained by such methods as

$$p_{S_{ij}} = \exp \pi_{ij} \tag{223}$$

Calculate the .

2.5.11 Pseudo etc. p Surface Diffusion Correction

Horizontal diffusion is applied on the surface of σ and so on, In large areas of mountain slopes, water vapor is transported uphill, causing problems such as bringing false precipitation at the top of the mountain. To mitigate it, such as close to the diffusion of the p surface Insert a correction for T, q, l.

$$\mathcal{D}_p(T) = (-1)^{N_D/2} K \nabla_p^{N_D} T \simeq (-1)^{N_D/2} K \nabla_\sigma^{N_D} T - \frac{\partial\sigma}{\partial p} (-1)^{N_D/2} K \nabla_\sigma^{N_D} p \cdot \frac{\partial T}{\partial\sigma} \tag{224}$$

$$= (-1)^{N_D/2} K \nabla_\sigma^{N_D} T - (-1)^{N_D/2} K \nabla_\sigma^{N_D} \pi \cdot \sigma \frac{\partial T}{\partial\sigma} \tag{225}$$

$$= \mathcal{D}(T) - \mathcal{D}(\pi) \sigma \frac{\partial T}{\partial\sigma} \tag{226}$$

so,

$$T_k \leftarrow T_k - 2\Delta t \sigma_k \frac{T_{k+1} - T_{k-1}}{\sigma_{k+1} - \sigma_{k-1}} \mathcal{D}(\pi) \tag{227}$$

And so on. $\mathcal{D}(\pi)$ is equal to the spectral value of pi in π_n^m The spectral representation of the diffusion coefficient multiplied by Converted into a grid value.

2.5.12 Consideration of frictional heat by diffusion.

Frictional heat from diffusion is ,

$$Q_{DIF} = -\left(u_{ij}\mathcal{D}(u)_{ij} + v_{ij}\mathcal{D}(v)_{ij}\right) \tag{228}$$

It is estimated that Therefore,

$$T_k \leftarrow T_k - \frac{2\Delta t}{C_p}\left(u_{ij}\mathcal{D}(u)_{ij} + v_{ij}\mathcal{D}(v)_{ij}\right) \tag{229}$$

2.5.13 Correction for conservation of mass.

The spectral method is not used, The global integration of the $\pi = \ln p_S$ is preserved except for rounding errors, The conservation of the mass, i.e., the global integration of p_S, is not guaranteed. Also, as the spectral wavenumber expires, it is not possible to preserve the global integration of p_S, Negative values of the grid points of water vapor are sometimes observed. For these reasons, Let the mass of dry air and water vapor, the mass of cloud water be preserved, Furthermore, corrections are made to remove the negative water vapor content in the region.

First, at the beginning of the dynamics calculation, `MODULE:[FIXMAS]` is added, Global integrals of water vapor and cloud water are calculated for M_q, M_l.

$$M_q^0 = \sum_{ijk} q p_S \Delta\lambda_i w_j \Delta\sigma_k \tag{230}$$

$$M_l^0 = \sum_{ijk} l p_S \Delta\lambda_i w_j \Delta\sigma_k \tag{231}$$

In the first step of the calculation, the global integrals of Calculate and memorize dry mass M_d.

$$M_d^0 = \sum_{ijk} (1 - q - l) p_S \Delta\lambda_i w_j \Delta\sigma_k \tag{232}$$

At the end of the dynamics calculation, `MODULE:[MASFIX]`, The correction is performed as follows.

First, for the grid points with negative water vapor content, The water vapor is distributed from the grid points directly below, Remove negative water vapor. If this is \$q_k < 0 \$,

$$q_k' = 0 \tag{233}$$

$$q_{k-1}' = q_{k-1} + \frac{\Delta p_k}{\Delta p_{k-1}} q_k \tag{234}$$

```
However, this is only done for $q_{k-1}' \ge 0 $.
```

The value is then set to zero for the grid points not removed by the above procedure.

3. calculate the global integration value M_q and Make sure this matches the M_q^0, Multiply the global water vapor content by a constant percentage.

$$q'' = \frac{M_q^0}{M_q} q' \tag{235}$$

4. correcting the dry air mass. Similarly, calculate M_d and

$$p_S'' = \frac{M_d^0}{M_d} p_S \tag{236}$$

2.5.14 Horizontal Diffusion and Rayleigh Friction

The coefficients of horizontal diffusion can be expressed spectrally,

$$\mathcal{D}_{M\,n}^{\;m} = K_M \left[\left(\frac{n(n+1)}{a^2} \right)^{N_D/2} - \left(\frac{2}{a^2} \right)^{N_D/2} \right] + K_R \tag{237}$$

$$\mathcal{D}_{H\,n}^{\;m} = K_M \left(\frac{n(n+1)}{a^2} \right)^{N_D/2} \tag{238}$$

$$\mathcal{D}_{E\,n}^{\;m} = K_E \left(\frac{n(n+1)}{a^2} \right)^{N_D/2} \tag{239}$$

The K_R is the Rayleigh coefficient of friction. The Rayleigh coefficient of friction is

$$K_R = K_R^0 \left[1 + \tanh \left(\frac{z - z_R}{H_R} \right) \right] \tag{240}$$

The profile is given as However,

$$z = -H \ln \sigma \tag{241}$$

It is approximated as follows. Standard values are, $K_R^0 = (30day)^{-1}$, $z_R = -H \ln \sigma_{top}$ (σ_{top} : the top level of the model), $H = 8000$ m, $H_R = 7000$ m. .

2.5.15 Time Filter.

To remove the computation mode in leap frog Applying Asselin's (1972) time filter at every step.

$$\bar{T}^t = (1 - 2\epsilon_f) T^t + \epsilon_f \left(\bar{T}^{t-\Delta t} + T^{t+\Delta t} \right) \tag{242}$$

and $\bar{T}$ are obtained. $T^{t-\Delta t}$, which is used in the next step of the mechanical process, is Use this $\bar{T}^t$. For a ϵ_f it is standard to use 0.05.

In practice, you should use First, in the `MODULE:[GENGD]` conversion of the predictor to a grid of values, the following variables are used,

$$\bar{T}^{t*} = (1 - \epsilon_f)^{-1} \left[(1 - 2\epsilon_f) T^t + \epsilon_f \bar{T}^{t-\Delta t} \right] \tag{243}$$

and when the physical process is complete, the After determining the value of $T^{t+\Delta t}$, you can use `MODULE:[TFILT]` to determine the value of the $T^{t+\Delta t}$,

$$\bar{T}^t = (1 - \epsilon_f) \bar{T}^{t*} + \epsilon_f \bar{T}^{t+\Delta t} \tag{244}$$

3 Physical Processes.

3.1 Overview of Physical Processes.

As a physical process, we can consider the following

- cumulus convection process
- large-scale condensation process
- radiation process
- vertical diffusion process
- surface flux
- Surface and underground processes
- gravitational wave resistance

The time-varying terms of the forecast variables due to these processes Calculate $F_x, F_y, Q, M, S,\ F_x, F_y, Q, M, S$ and do time integration. In addition, in order to evaluate the atmospheric and surface fluxes Using the surface sub-model. In the surface sub-model, Ground Temperature T_g, Ground Moisture W_g, Snowpack W_y, etc. It is used as a predictor variable.

3.1.1 Fundamental Equations.

Equation of motion of the atmosphere in the σ coordinate system, thermodynamic equation, Consider the equation for a sequence of substances such as water vapor. The vertical fluxes of momentum, heat, water vapor, etc. are considered, Find the time variation due to its convergence. All vertical fluxes are positive upward.

1. equation of motion

$$\rho \frac{du}{dt} = \frac{\partial Fu}{\partial \sigma} \tag{245}$$

> \\centric

$$\rho\frac{dv}{dt} = \frac{\partial Fv}{\partial\sigma} \tag{246}$$

u, v: East-West, North-South Wind;
Fu, Fv: Their vertical flux.

2. thermodynamic equation

$$\rho\frac{dc_pT}{dt} = \frac{T}{\theta}\frac{\partial F\theta}{\partial\sigma} + \frac{\partial FR}{\partial\sigma} \tag{247}$$

T: Temperature ;
c_p: Constant Pressure Specific Heat;
$\theta=T(p/p_0)^{-R/c_p}=T(p/p_0)^{-\kappa}$: Hot Position;
$F\theta$: Vertical Sensible Heat Flux;
FR: Vertical Radiation Flux.

If we write $\theta' = T(p/p_s)^{-\kappa} = T\sigma^{-\kappa}$, then this is

$$\rho\frac{dc_pT}{dt} = \sigma^{\kappa}\frac{\partial F\theta'}{\partial\sigma} + \frac{\partial FR}{\partial\sigma} \tag{248}$$

As far as vertical 1D processes are concerned, Instead of the θ, consider the θ'. For the sake of simplicity, unless there is a risk of confusion, Write θ' as θ.

3. water vapor continuity formula

$$\rho\frac{dq}{dt} = \frac{\partial Fq}{\partial\sigma} \tag{249}$$

q: Specific Humidity;
$F{q}$: Vertical Steam Flux.

Fundamental Equations in the Ground

Considered in terms of z coordinates with the downward direction positive. After all, all vertical fluxes are positive upward.

4. thermal formula

$$\frac{\partial C_g G}{\partial t} = \frac{\partial Fg}{\partial z} + Sg \tag{250}$$

```
$G$: Ground Temperature; $C_g$: Constant Pressure Specific Heat;
$F{g}$: Vertical Heat Flux;
$Sg$; Heating term (due to phase change, etc.).
```

5. formula for ground moisture

$$C_w \frac{\partial w}{\partial t} = \frac{\partial Fw}{\partial z} + Sw \tag{251}$$

```
$w$: Ground Moisture;
$F{w}$: Lead Water Flux;
$Sw$; Sources of water (spills, etc.).
```

6. energy balance equation

At the surface, an energy balance is established.

$$F\theta + LFq + FR - Fg = \Delta s \ \ (\sigma = 1, z = 0) \tag{252}$$

```
$L$: Latent Heat of Evaporation;
$\Delta s$: Surface energy balance (due to phase change, etc.).
```

7. surface water balance

$$Pg + Fw - Rg = 0 \tag{253}$$

```
$Pg$: Precipitation;
$Rg$: Surface Runoff.
```

8. the snow balance

$$\frac{\partial Wy}{\partial t} = Py - Fy - My \tag{254}$$

```
$Wy$: Snow cover(kg/m$^2$);
$Py$: Snowfall;
$Fy$: Sublimation;
$My$: Snowmelt.
```

3.1.2 Time integration of physical processes.

Classifying physical processes in terms of the time integration of predictor variables, The order of execution can be divided into the following three categories.

1. cumulus convection and large-scale condensation
2. radiation, vertical diffusion, ground boundary layer and surface processes
3. gravitational wave resistance, mass regulation, dry convection regulation

Cumulus convection and large-scale condensation,

$$\hat{T}^{t+\Delta t,(1)} = \hat{T}^{t+\Delta t} + 2\Delta t Q_{CUM}(\hat{T}^{t+\Delta t}) \tag{255}$$

$$\hat{T}^{t+\Delta t,(2)} = \hat{T}^{t+\Delta t,(1)} + 2\Delta t Q_{LSC}(\hat{T}^{t+\Delta t,(1)}) \tag{256}$$

by the usual Euler difference. Large-scale condensation schemes include , Note that the updated values are passed on by the cumulus convection scheme. In practice, the output of the heating rate and so on are used in the routines for cumulus convection and large-scale condensation, Time integration is done by the immediately following `MODULE:[GDINTG]`.

Radiation in the following groups, vertical diffusion, ground boundary layer and surface processes calculations are essentially all of these updated values ($\hat{T}^{t+\Delta t,(1)}$, $\hat{q}^{t+\Delta t,(2)}$, $\hat{T}^{t+\Delta t,(1)}$, $\hat{q}^{t+\Delta t,(2)}$, etc.) This is done by using However, in order to calculate some of the terms as implicit, the Calculate all of these terms together and calculate the heating rate, etc, Finally, we do time integration. In other words, symbolically,

$$\hat{T}^{t+\Delta t,(3)} = \hat{T}^{t+\Delta t,(2)} + 2\Delta t Q_{RAD,DIF,SFC}(\hat{T}^{t+\Delta t,(2)}, \hat{T}^{t+\Delta t,(3)}) \quad (257)$$

That would be.

As for gravitational wave resistance, mass regulation and dry convection regulation, It is similar to cumulus convection and large-scale condensation.

$$\hat{T}^{t+\Delta t,(4)} = \hat{T}^{t+\Delta t,(3)} + 2\Delta t Q_{ADJ}(\hat{T}^{t+\Delta t,(3)}) \quad (258)$$

3.1.3 Various physical quantities.

A simple calculation from the predictive variables can be used to find Definitions of various physical quantities. Some of these are , Calculated with `MODULE:[PSETUP]`.

1. temporary temperature

Provisional Temperature T_v is ,

$$T_v = T(1 + \epsilon_v q - l) \quad (259)$$

2. air density

The atmospheric density, ρ, is calculated as follows

$$\rho = \frac{p}{RT_v} \quad (260)$$

3. high degree

The high degree z is a mechanical process The same method is used to calculate the geopotential.

$$z = \frac{\Phi}{g} \tag{261}$$

$$\Phi_1 = \Phi_s + C_p(\sigma_1^{-\kappa} - 1)T_{v,1} \tag{262}$$

$$\Phi_k - \Phi_{k-1} = C_p \left[\left(\frac{\sigma_{k-1/2}}{\sigma_k} \right)^{\kappa} - 1 \right] T_{v,k} + C_p \left[1 - \left(\frac{\sigma_{k-1/2}}{\sigma_{k-1}} \right)^{\kappa} \right] T_{v,k-1} \tag{263}$$

4. layer boundary temperature

The temperature of the boundary of the layer is determined by the temperature of the $\ln p$, i.e., the temperature of the boundary relative to the $\ln \sigma$ Perform a linear interpolation and calculate.

$$T_{k-1/2} = \frac{\ln \sigma_{k-1} - \ln \sigma_{k-1/2}}{\ln \sigma_{k-1} - \ln \sigma_k} T_k + \frac{\ln \sigma_{k-1/2} - \ln \sigma_k}{\ln \sigma_{k-1} - \ln \sigma_k} T_{k-1} \tag{264}$$

5. saturated specific humidity

Saturation Specific Humidity $q^*(T,p)$ is approximated using the saturation vapor pressure $e^*(T)$,

$$q^*(T,p) = \frac{\epsilon e^*(T)}{p}. \tag{265}$$

Here, it is $\epsilon = 0.622$,

$$\frac{1}{e_v^*}\frac{\partial e_v^*}{\partial T} = \frac{L}{R_v T^2} \tag{266}$$

`> <span id="e-sat" label="e-sat">\\blade[e-sat]< /span>`

Therefore, if the latent heat of evaporation (L) and the gas constant (R_v) of the water vapor are held constant, then the number of vapor particles will be reduced,

$$e^*(T) = e^*(T = 273K) \exp\left[\frac{L}{R_v}\left(\frac{1}{273} - \frac{1}{T}\right)\right] , \tag{267}$$

```
$e^*(T=273{K}) = 611$ is a \\blank\blank\blank\blank\.com.
```

```
(From [\\\begin{eqnarray}e-sat\end{eqnarray}] (#e-sat)),
```

$$\frac{\partial q^*}{\partial T} = \frac{L}{R_v T^2} q^*(T, p). \tag{268}$$

Here, if the temperature is lower than the freezing point 273.15 K Use the sublimation latent heat $L + L_M$ as the latent heat L.

6. dry static energy, wet static energy

Dry static energy s is

$$s = C_p T + gz \ , \tag{269}$$

Wet Static Energy h is

$$h = C_p T + gz + Lq \ , \tag{270}$$

. defined by .

3.2 cumulus convection

3.2.1 Overview of the Cumulus Convection Scheme

The cumulus convection scheme is , This figure represents the condensation, precipitation and convection processes involved in cumulus convection, Due to the latent heat release and associated convective motion Calculate precipitation with temperature and with changes in water vapor content. We also calculate the cloud water content and cloud coverage involved in the radiation. The main input data are temperature T and specific humidity q, The output data is the time rate of change of temperature and specific humidity, $\partial T/\partial t, \partial q/\partial t, \partial l/\partial t$, The cloud water content of the cumulus clouds used for radiation is l^{cR} Cloud volume C^c.

The framework of the cumulus convection scheme is Basically based on Arakawa and Schubert (1974). Vertical air columns in one horizontal grid. Considered as the basic unit of parameterization. Clouds are determined by the temperature, specific humidity, cloud water content and Characterized by a vertical upward mass flux, Considering multiple clouds with different cloud tops within a single vertical air column. Clouds occupy part of the horizontal lattice, and the rest of the surrounding region is There is a downward flow equal to the cloud mass flux (compensating downward flow). This compensatory downward flow and outflow of air into the surrounding region in the clouds (detraining) The temperature and the specific humidity field in the surrounding region are changed by The area of the upwelling of the cumulus convection is assumed to be small, The lattice-averaged temperature and specific humidity fields and Since we treat the temperature and specific humidity fields in the surrounding area as the same, we are able to This gives the changes in the lattice mean temperature and specific humidity.

It is the cloud model that determines the temperature, specific humidity and cloud water content in clouds. Here, we use an entrained-purume model, As with Moorthi and Suarez (1992) , We assume a linear mass flux increase with respect to height. The cloud base is used as the lifted condensation height of the surface atmosphere, of the percentage of air uptake (entrainment) in the surrounding area. Consider multiple cloud top altitudes depending on the difference. However, if a cloud with a cloud base cannot exist, then Consider the possibility of clouds with higher cloud bases.

The mass flux of each cloud is diagnostically determined using the cloud work function. The cloud work function is defined as It is defined as the vertical integral of the work done by buoyancy. This cloud-work function is driven by the compensating downward motion of cumulus clouds, etc. It gives a mass flux that approaches zero at a certain relaxation time.

In addition, the evaporation of precipitation and The effect of the downdrafting that goes with it. Consider in a very simple way .

The outline of the calculation procedure is as follows. Parentheses are the names of the corresponding subroutines.

1. cloud-bottom height as the lifted condensation height of the surface atmosphere Evaluate .

2. using a cloud model, Corresponding to each cloud top altitude of cloud temperature, specific humidity, cloud water content, and mass flux (relative value) Calculates the vertical distribution `MODULE:[UPDRF]`.

3. calculate the cloud work function `MODULE:[CWF]`.

4. due to a cloud of unit mass fluxes. Calculates the hypothetical change of temperature and specific humidity in the surrounding area `MODULE:[CLDTST]`.

5. for a hypothetical change in temperature and specific humidity Calculate the cloud work function `MODULE:[CWF]`.

6. using the cloud work function before and after the virtual change Calculates the cloud mass flux at the cloud base `MODULE:[CBFLX]`.

7. the cloud mass flux detrainment. Calculate the vertical distribution and precipitation `MODULE:[CMFLX]`.

8. evaluate cloud water and cloud cover due to cumulus clouds `MODULE:[CMCLD]`.

9. by detainment. Calculate the change of temperature and specific humidity `MODULE:[CLDDET]`.

10. by compensatory downstream flow. Calculate the change of temperature and specific humidity `MODULE:[CLDSBH]`.
11. evaporation of precipitation and The downdraft. of cloud temperature, specific humidity and mass flux Calculates the vertical distribution `MODULE:[DWNEVP]`.
12. by downdraft detrainment. Calculate the change of temperature and specific humidity `MODULE:[CLDDDE]`.
13. by the compensatory upward flow of downdrafts. Calculate the change of temperature and specific humidity `MODULE:[CLDSBH]`.

3.2.2 The Basic Framework of the Arakawa-Schubert Scheme

Cloud Mass Flux M, Detrainment D is,

$$M(z) = M_B \eta(z) \ , \tag{271}$$

$$D(z) = M_B \eta(z_T) \delta(z - z_T) \ . \tag{272}$$

represented as . The mass flux at the cloud base (M_B) is the mass flux at z_B, η is a dimensionless mass flux in it.

From this, the time variation of the mean field is calculated as

$$\frac{\partial \bar{h}}{\partial t} = M \frac{\partial \bar{h}}{\partial z} + D(h^t - \bar{h}) \ , \tag{273}$$

$$\frac{\partial \bar{q}}{\partial t} = M \frac{\partial \bar{q}}{\partial z} + D(q^t + l^t - \bar{q}) \ . \tag{274}$$

However, $\bar{h}, \bar{q}$ are based on the wet static energy of the mean field and the specific humidity, h^t, q^t, l^t are the air in the detrainment It is the wet static energy, specific humidity, and cloud water content.

η, h^t, q^t, l^t are required by the cloud model. M_B is obtained by the closure assumption using the cloud work function.

3.2.3 Cloud Model.

The cloud model is essentially an entrained-purume model. Each type of cloud is characterized by an entrainment rate, It will have various cloud top heights accordingly. However, for the sake of later calculations, Here, you can specify the cloud top altitude, By finding the corresponding entrainment rate Find the vertical structure of clouds. By assuming a linear mass flux increase with respect to height. This calculation is simplified to a form that does not include a sequential approximation.

Let's set the cloudbase altitude at z_T, The lifted condensation altitude of the surface atmosphere, i.e., the height of condensation,

$$\bar{q}(0) \geq \bar{q}^*(z) + \frac{\gamma}{L(1+\gamma)} \left(\bar{h}(0) - \bar{h}(z) \right) \ , \tag{275}$$

Define it as the minimum z that meets the following criteria

The dimensionless mass flux η is, The entrainment rate is set to λ,

$$\frac{\partial \eta}{\partial z} = \lambda \ , \tag{276}$$

Namely,

$$\eta(z) = 1 + \lambda(z - z_B) \tag{277}$$

$$\equiv 1 + \lambda \hat{\eta}(z) \ . \tag{278}$$

The balance on wet static energy h^c and total water content w^c in the clouds is,

$$\frac{\partial}{\partial z}(\eta h^c) = \lambda \bar{h} \ , \tag{279}$$

$$\frac{\partial}{\partial z}(\eta w^c) = \lambda \bar{q} - \pi \ . \tag{280}$$

Here, $\bar{h}, \bar{q}, \pi$ are respectively, h and q, in mean field, are precipitation generation.

Integrating,

$$\eta(z)h^c(z) = h^c(z_B) + \lambda \int_{z_B}^{z} \bar{h}(\xi)d\xi \tag{281}$$

$$\equiv h^c(z_B) + \lambda \hat{h}^c(z) \ , \tag{282}$$

$$\eta(z)w^c(z) = w^c(z_B) + \lambda \int_{z_B}^{z} \bar{q}(\xi)d\xi - R(z) \tag{283}$$

$$\equiv w^c(z_B) + \lambda \hat{w}^c(z) - R(z) \tag{284}$$

$$\equiv \eta(z)w^a(z) - R(z) \ . \tag{285}$$

The mass flux is assumed to be zero at the surface, It is assumed to increase linearly below the cloud base,

$$\eta(z) = \frac{z}{z_B} \quad (z < z_B) \ . \tag{286}$$

By calculating the entrainment below this cloud base, h^c, w^c are required at cloudbase. That is, ,

$$h^c(z_B) = \frac{1}{z_B} \int_0^{z_B} \bar{h}(z)dz \ , \tag{287}$$

$$w^c(z_B) = \frac{1}{z_B} \int_0^{z_B} \bar{q}(z)dz \ . \tag{288}$$

The buoyancy per unit mass flux due to clouds is ,

$$B = \frac{g}{\bar{T}}(T_v^c - \bar{T}_v) \tag{289}$$

$$= \frac{g}{\bar{T}}\left[T^c(1 + \epsilon q^c - l^c) - \bar{T}(1 + \epsilon \bar{q})\right] \tag{290}$$

$$\simeq \frac{g}{\bar{T}}\left[(T^c - \bar{T}) - \bar{T}\left(\epsilon(q^c - \bar{q}) - l^c\right)\right] \tag{291}$$

$$\simeq \frac{g}{\bar{T}}\left[\frac{1}{C_p(1+\gamma)}(h^c - \bar{h}^*) + \bar{T}\left(\epsilon\frac{\gamma}{L(1+\gamma)}(h^c - \bar{h}^*) + \epsilon(\bar{q}^* - \bar{q}) - l^c\right)\right] \tag{292}$$

where T_v is the provisional temperature and q^* is the saturation specific humidity, $\epsilon = R_{H_2O}/R_{air} - 1$, It is $\gamma = L/C_p \partial q^*/\partial T$, $\bar{q}^*, \bar{h}^*$ indicate the values at mean-field saturation, respectively. q^c, l^c are the amounts of cloud water vapor and cloud water,

$$q^c = q^*(T^c) \simeq \bar{q}^* + \frac{1}{L(1+\gamma)}(h^c - \bar{h}^*) \, , \tag{293}$$

$$l^c = w^c - q^c \, . \tag{294}$$

For the cloud top z_T, the buoyancy B is assumed to be zero. Thus, solving the $B(z_T) = 0$ corresponds to the given cloud top height of z_T λ can be obtained. Here, for precipitation rate $R(z)$ integrated from the ground upward, we have a problem, Using the known function $r(z)$ Assume that it is represented.

$$R(z) = \eta(z) r(z) \left[w^a(z) - q^c(z) \right] \, . \tag{295}$$

So..,

$$\frac{\bar{T}}{g} B \simeq \frac{1}{1+\gamma} \left[\frac{1}{C_p} + \bar{T}(\epsilon + 1 - r) \frac{\gamma}{L} \right] (h^c - \bar{h}^*) + (\epsilon + 1 - r)\bar{T}\bar{q}^* - \epsilon \bar{T} \bar{q} - \bar{T}(1-r) w^a \tag{296}$$

\$B(z_T) =0 \$ is easy to solve and,

$$\lambda = \frac{a \left[h^c(z_B) - \bar{h}^*(z_T) \right] + \bar{T}(z_T) \left[b - (1 - r(z_T)) q^c(z_B) \right]}{a \left[\hat{\eta}(z_T) \bar{h}^*(z_T) - \hat{h}^c(z_T) \right] - \bar{T}(z_T) \left[b \hat{\eta}(z_T) - (1 - r(z_T)) \hat{q}^c_t(z_T) \right]} \tag{297}$$

However,

$$a \equiv \frac{1}{1+\gamma} \left[\frac{1}{C_p} + \bar{T}(z_T) \left(\epsilon + 1 - r(z_T) \right) \frac{\gamma}{L} \right] \, , \tag{298}$$

$$b \equiv \left(\epsilon + 1 - r(z_T) \right) \bar{q}^*(z_T) - \epsilon \bar{q}(z_T) \, . \tag{299}$$

As mentioned above, you should specify λ to obtain z_T, A physically meaningful λ for a given z_T There is no guarantee that we will seek it. That scrutiny is necessary, but here it is, The smaller the λ is, the more the z_T is Take into account that it should be lower.

$$\frac{\partial \lambda}{\partial z_T} < 0 \tag{300}$$

We will examine whether or not the If the value is not satisfied, assume that the cloud with cloud top z_T does not exist. Also, a minimum value has been set for λ, We assume that there are no smaller λ clouds. This means that the entrainment rate can be reduced by Given the inverse proportions , The equivalent of having a maximum in the size of the plume.

Cloud water content $l^c(z)$ is ,

$$l^c(z) = w^a(z) - q^c(z) - R(z)/\eta(z) \tag{301}$$

$$= (1 - r(z)) \left[w^a(z) - q^c(z) \right] \ . \tag{302}$$

However, in the case of $w^a(z) < q^c(z)$, it is $l^c(z) = 0$. Furthermore, it is unlikely that a precipitation event will be followed by cloudy water, $R(z)$ must be an increasing function of z. This will limit the $r(z)$.

The characteristic value of the detrainment air is ,

$$h^t = h^c(z_T) \ , \tag{303}$$

$$q^t = q^c(z_T) \ , \tag{304}$$

$$l^t = l^c(z_T) \ . \tag{305}$$

In the case of \$ h^c(z_B) < h}^* (z_T) \$, Suppose that clouds do not exist. In this case,

$$\bar{h}(z'_B) > \bar{h}^*(z_T) \ , \quad z_B < z < z_T \tag{306}$$

If there is a z'_B that satisfies , The area directly above it has been renamed z_B,

$$h^c(z_B) = \bar{h}(z'_B) \ , \tag{307}$$

$$w^c(z_B) = \bar{q}(z'_B) \tag{308}$$

Seek as .

3.2.4 Cloud Work Function (CWF)

The cloud work function (CWF), A is,

$$A \equiv \int_{z_B}^{z_T} B\eta dz \tag{309}$$

It is,

$$A = \int_{z_B}^{z_T} \frac{g}{\bar{T}} \left[(T^c - \bar{T}) + \bar{T} \left\{ \epsilon(q^c - \bar{q}) - l^c \right\} \right] \eta dz \; . \tag{310}$$

[p-cum:cwf]

Essentially, the work associated with the downdraft, discussed below, should be It should be accounted for, but we'll ignore it here for simplicity's sake.

In this calculation, we start at the bottom and Once a positively buoyant cloud is negatively buoyant, if , Since there should be cloud tops where they are supposed to be negative, Assume that the cloud with the cloud top we are considering does not exist.

3.2.5 Cloud Mass Flux at Cloudbase

The cloud mass flux at the cloud base is , On some time scale τ_a, Cloud action determines the cloud work function to be close to zero I make the assumption that.

In order to estimate it, we firstly estimated the unit cloud-bottom mass flux of M_0 Find the time variation of the mean field.

$$\frac{\partial \bar{h}'}{\partial t} = M_0 \eta \frac{\partial \bar{h}}{\partial z} + \eta(z_T) \delta(z - z_T)(h^t - \bar{h}) \; , \tag{311}$$

$$\frac{\partial \bar{q}'}{\partial t} = M_0 \eta \frac{\partial \bar{q}}{\partial z} + \eta(z_T) \delta(z - z_T)(q^t + l^t - \bar{q}) \; . \tag{312}$$

With this,

$$\bar{h}' = \bar{h} + \frac{\partial \bar{h}'}{\partial t} \delta t , \tag{313}$$

$$\bar{q}' = \bar{q} + \frac{\partial \bar{q}'}{\partial t} \delta t \tag{314}$$

and using $\bar{h}', \bar{q}'$ Let A' be the cloud work function calculated from (p-cum:cwf]).

So..,

$$M_B = \frac{A}{A - A'} \frac{\delta t}{\tau_a} M_0 \tag{315}$$

That would be. Here, when obtaining A', the original $\bar{h}', \bar{q}'$ were used I should recalculate the vertical structure of the clouds, Now we are using the same cloud structure.

3.2.6 Cloud Mass Flux, Precipitation

The sum of the clouds at each cloud top altitude, Cloud Mass Flux M

$$M(z) = \int^{i} M_B^i \eta^i(z) . \tag{316}$$

Also, precipitation flux $P(z)$ is

$$P(z) = \int_i M_B^i \left[R^i(z_T) - R^i(z) \right] . \tag{317}$$

3.2.7 Time variation of the average field

by compensated downstream flow and detraining. The time variation of the mean field is calculated as follows

$$\frac{\partial \bar{h}}{\partial t} = M \frac{\partial \bar{h}}{\partial z} + \int_i D^i ((h^t)^i - \bar{h}) , \tag{318}$$

$$\frac{\partial \bar{q}}{\partial t} = M \frac{\partial \bar{q}}{\partial z} + \int_i D^i ((q^t)^i + (l^t)^i - \bar{q}(z_T^i)) . \tag{319}$$

However, it is $D^i = M_B^i \eta^i(z_T^i)$.

3.2.8 Evaporation and downdrafting of precipitation

Precipitation falls through the unsaturated atmosphere, while some of it evaporates. In addition, some of them form a downdraft.

Evaporation Rate E is ,

$$E = \rho a_e {\rho_p}^{b_e} \left(\bar{q}_w - \bar{q}\right) \ , \tag{320}$$

Note that $\bar{q}_w$ is the saturation specific humidity corresponding to the wet bulb temperature,

$$\bar{q}_w = \bar{q} + \frac{\bar{q}^* - \bar{q}}{1 + \frac{L}{C_P}\frac{\partial q^*}{\partial T}} \ . \tag{321}$$

a_e, b_e is a parameter of the microphysics. ρ_p is the density of precipitation particles and V_T is the terminal velocity of precipitation,

$$\rho_p = \frac{P}{V_T} \ . \tag{322}$$

The current standard values are $a_e = 0.25$, $b_e = 1$ and $V_T = 10$ m/s.

For downdrafting, we make the following assumptions.

- $\bar{h}$ decreases monotonically with altitude above cloudbase If the upper end of the region is set to z_d, the downdraft is It occurs in the region of $z < z_d$.
- A certain percentage of the precipitation evaporation that occurs at each altitude It is used to form downdrafts. Evaporation of precipitation has just saturated it. The air in the surrounding area. Taken into the downdraft (Entrainment).
- In $z < z_B$, detraining occurs, The mass flux decreases linearly.

That is, in $z_B < z < z_d$, the mass flux $M^d(z)$, The downdraft air masses $h^d(z), q^d(z)$ follow the following equation. Upon evaporation of precipitation, the wet static energy should be conserved, and the specific humidity when saturated by evaporation. Note that this is $\bar{q}_w$.

$$\frac{\partial M^d}{\partial z} = -f_d \frac{E}{\bar{q}_w - \bar{q}} \ , \tag{323}$$

$$\frac{\partial}{\partial z}(M^d h^d) = \bar{h}\frac{\partial M^d}{\partial z} \ , \tag{324}$$
$$\frac{\partial}{\partial z}(M^d q^d) = \bar{q}_w \frac{\partial M^d}{\partial z} \ . \tag{325}$$

In the above equation, f_d is the portion of the evaporation that is taken up by the downdraft, $(1 - f_d)$ evaporates directly into the mean field. However, the downdraft mass flux M^d is The total mass flux of cloud base shall not exceed the f_m of M. The current standard value is $f_d = 0.5, f_m = 1.0$.

3.2.9 cloud water and cloud cover

The lattice-averaged cloud water content used for radiation, l^{cR}, is Strong upwelling areas of cumulus clouds, including cloud water l^c If the ratio of the ratio to the δ^c ,

$$l^{cR} = \delta^c l^c \ . \tag{326}$$

The mass flux M^c is the same as this δ^c Using the vertical velocity of the upstream stream, v^c

$$M = \delta^c \rho v^c \tag{327}$$

So, in the end,

$$l^{cR} = \frac{M^c}{\rho v^c} l^c = \alpha M^c l^c \ . \tag{328}$$

The cloud cover used to estimate radiation, C^c, is , that there is actually a horizontal spread in the distribution of upwelling and cloud water. It is reasonable to take a larger value than this δ^c. Here, in brief,

$$C^c = \beta M_B \tag{329}$$

. The current standard values are $\alpha = 0.3$ and $\beta = 10$.

3.3 Massive Coagulation

3.3.1 Overview of Large Scale Condensation Schemes.

Large-scale condensation schemes are , This is a representation of the condensation processes involved in clouds other than cumulus convection, Calculating latent heat release and water vapor reduction, precipitation. We also calculate the cloud water content and cloud coverage involved in the radiation. The main input data are temperature T, specific humidity q, and cloud cover l, The output data is the time rate of change of temperature, specific humidity and cloud water content, $\partial T/\partial t, \partial q/\partial t, \partial l/\partial t$, The cloud cover is C.

In the CCSR/NIES AGCM, in addition to the water-vapor mixture ratio (specific humidity q), the Cloud water content (l) is also a forecast variable in the model. In fact, in this large-scale condensation routine First, calculate the sum of the two, the total amount of water ($q^t = q+l$), We are dividing it again into cloud water and water vapor, In effect, the forecast variable is a single total water volume (q^t). By assuming the distribution of the variation of q^t in the grid, Diagnosis of the cloud cover and cloud water content in each grid. The conversion of cloud water into precipitation and the evaporation of precipitation during its fall are also considered.

The outline of the calculation procedure is as follows.

1. add the amount of water vapor (q) and the amount of cloud water (l) Total water volume q^t The temperature has evaporated the cloud water, Set the liquid water temperature T_l.
2. assuming the distribution of the variation in q^t, Find the cloud cover and separate it again into cloud water and water vapor.
3. considering the temperature change due to condensation, By successive approximation Determine the cloud cover, cloud water content, and water vapor distribution.
4. evaluate the conversion of cloud water to precipitation.
5. evaluate the ice fall.
6. evaluate precipitation and evaporation of falling ice.

3.3.2 Diagnosis of cloud water levels

When the grid-averaged total water volume $\overline{q}^t = \overline{q} + \overline{l}$ is given, Distribution of the total water volume q^t in the grid, Between $(1-b)\overline{q}^t$ and $(1+b)\overline{q}^t$ It is assumed to be uniformly distributed. That is, the probability density function is ,

$$F(q^t) = \begin{cases} (2b\overline{q}^t)^{-1} & (1-b)\overline{q}^t < q^t < (1+b)\overline{q}^t \\ 0 \text{その他} \end{cases} . \tag{330}$$

We consider this distribution to be a horizontal distribution. On the other hand, the saturation specific humidity is based on the grid average of $\overline{q}^*$.

In the grid, Consider the presence of a cloud in a region in $q^t > q^*$ (Figure [lsclsc:fig-cloud]).

Then, as shown by the shading in the figure, the The horizontal ratio of the portion of the total water volume exceeding saturation C is ,

$$C = \begin{cases} 0 & (1+b)\overline{q}^t \leq \overline{q}^* \\ \dfrac{(1+b)\overline{q}^t - \overline{q}^*}{2b\overline{q}^t} & (1-b)\overline{q}^t < \overline{q}^* < (1+b)\overline{q}^t \\ 1 & (1-b)\overline{q}^t \leq \overline{q}^* \end{cases} \tag{331}$$

and this is the cloud cover (horizontal cloud coverage).

In addition, the cloud cover of l is in the region of $q^t > q^*$ This is an integral of $q^t - q^*$,

$$l = \begin{cases} 0 & (1+b)\overline{q}^t \leq \overline{q}^* \\ \dfrac{\left[(1+b)\overline{q}^t - \overline{q}^*\right]^2}{4b\overline{q}^t} & (1-b)\overline{q}^t \leq \overline{q}^* \leq (1+b)\overline{q}^t \\ \overline{q}^t - \overline{q}^* & (1-b)\overline{q}^t \geq \overline{q}^* \end{cases} \tag{332}$$

[p-lsc:l]

3.3.3 Determination by successive approximation

First, from the Water Vapor q and Cloud Water l and the Temperature T, Find the total water volume q^t and liquid water temperature T_l.

$$q^t = q + l \; , \tag{333}$$

$$T_l = T - \frac{L}{C_P} l \; . \tag{334}$$

The T_l corresponds to the temperature at which all cloud water is evaporated. $T^{(0)} = T_l$, $l^{(0)} = 0$

By saturation specific humidity relative to temperature T_l, Assuming that the cloud water content evaluated by the aforementioned method is $l^{(1)}$, It changes the temperature,

$$T^{(1)} = T_l + \frac{L}{C_P} l^{(1)} \; . \tag{335}$$

The cloud water content evaluated using the saturation specific humidity versus temperature was estimated from $l^{(2)}$, The resulting temperature change is solved by successive approximation as $T^{(2)} \ldots$ In order to speed up this sequential convergence, we use the Newton method. I.e., instead of (ie, ([p-lsc:itereate1])

$$T^{(1)} = T_l + \frac{L}{C_P} l^{(1)} \left(1 - \frac{L}{C_P} \frac{dl}{dT} \right)^{-1} \tag{336}$$

. dl/dT can be obtained analytically using ([p-lsc:l]).

3.3.4 precipitation process.

Precipitation occurs dependent on the amount of cloud water diagnosed. If the precipitation rate (in 1/s) is set to P,

$$P = l/\tau_P \; . \tag{337}$$

τ_P is the time scale of precipitation,

$$\tau_P = \tau_0 \left\{ 1 - \exp\left[-\left(\frac{l}{l_C}\right)^2 \right] \right\}^{-1} . \tag{338}$$

where l_C is the critical cloud water content, In view of the Bergeron-Findeisen effect ,

$$l_C = \begin{cases} l_C^0 \;\; T \geq T_0 \\ l_C^0 \left\{ 1 + \alpha \exp\left[-\beta (T - Tc)^2 \right] \right\}^{-1} \;\; T_0 > T > T_c \\ l_C^0 (1+\alpha)^{-1} T \leq T_c \end{cases} . \tag{339}$$

$l_C^0 = 10^{-4}$, $\alpha = 50$, $\beta = 0.03$, $T_0 = 273.15$ K, $T_c = 258.15$ K
Precipitation results in a decrease in l.

$$P = l/\tau_P , \tag{340}$$
$$\frac{\partial l}{\partial t} = -P . \tag{341}$$

Integrating this during Δt,

$$P\Delta t = \left\{ 1 - \exp(-\Delta t/\tau_P) \right\} l . \tag{342}$$

Precipitation flux at a certain height, p If (unit kg m^{-2} s^{-1}) is set to F_P

$$F_P(p) = \int_0^p P \frac{dp}{g} . \tag{343}$$

3.3.5 Ice Falling Process.

Cloud water is divided into ice and water clouds depending on the temperature. The ice cloud ratio is

$$f_I = \frac{T_0 - T}{T_0 - T_1} \tag{344}$$

(but with a maximum value of 1 and a minimum value of 0). Also, $T_0 = 273.15K, T_1 = 258.15K$. The ice cloud will fall at a slow speed, Consider the effect. Rate of descent V_S is,

$$V_S = V_S^0(\rho_a f_I l)^\gamma \ . \tag{345}$$

However, $V_S^0 = 3$ m/s, $\gamma = 0.17$. So..,

$$\tau_S = \frac{\Delta p}{\rho g V_S} \tag{346}$$

as well as precipitation.

3.3.6 Evaporation process of precipitation.

Evaporation of precipitation The evaporation of precipitation, E, is estimated as follows.

$$E = k_E(q^w - q)\frac{F_P}{V_T} \ . \tag{347}$$

However, if $q^w < q$ is set, this should be zero. The q_w is the saturation specific humidity corresponding to the wet bulb temperature,

$$q^w = q + \frac{q^* - q}{1 + \frac{L}{C_P}\frac{\partial q^*}{\partial T}} \ . \tag{348}$$

This means that precipitation is

$$F_P(p) = \int_0^p (P - E)\frac{dp}{g} \tag{349}$$

The temperature drop due to evaporation is estimated to be We also estimate the temperature drop due to evaporation.

$$\frac{\partial T}{\partial t} = -\frac{L}{C_P}E \ . \tag{350}$$

3.3.7 Other Notes.

Calculations are made from the topmost layer down. For convenience, the calculation is based on the precipitation from the upper layers of the We start by evaluating evaporation in that layer.

2. fallen ice in the layer just below. It will be treated the same as the cloud water that already exists in that layer, incorporated into the total water volume.

3.4 Radiant Flux.

3.4.1 Summary of Radiation Flux Calculations

The CCSR/NIES AGCM radiation calculation scheme is , Discrete Ordinate Method and It was created based on the k-Distribution Method. by gases and clouds/aerosols Considering the absorption, emission, and scattering processes of solar and terrestrial radiation, Calculate the value of the radiation flux at each level. The main input data are temperature T, specific humidity q, cloud cover l, and cloud cover C, The output data are upward and downward radiation fluxes, F^-, F^+, and the differential coefficient of upward radiation flux with respect to surface temperature This is a dF^-/dT_g.

The calculation is done for several wavelengths. Each wavelength range is based on the k-distribution method, It is further divided into several sub-channels. As for gaseous absorption, Band Absorption in H_2O, CO_2, O_3, N_2O, CH_4 and , Continuous absorption of H_2O, O_2, O_3 and CFC absorption is incorporated. As for the scattering, we can use Rayleigh scattering of gases and Scattering by cloud and aerosol particles is incorporated.

The outline of the calculation procedure is as follows (subroutine names in parentheses).

1. calculate the Planck function from atmospheric temperature `MODULE:[PLANKS]`.

2. in each subchannel, Calculates the optical thickness due to gas absorption `MODULE:[PTFIT]`.

3. by continuous absorption and CFC absorption calculate the optical thickness `MODULE:[CNTCFC]`.

4. of Rayleigh scattering and particle scattering Calculates the optical thickness and scattering moment `MODULE:[SCATMM]`.

5. from the optical thickness and solar zenith angle of the scattering, Seeking sea-level albedo `MODULE:[SSRFC]`.

6. for each sub-channel, Expand the Planck function by optical thickness `MODULE:[PLKEXP]`.

7. for each sub-channel, Calculates the transmission coefficient, reflection coefficient, and source function for each layer `MODULE:[TWST]`

8. by adding method, the calculate the radiation flux `MODULE:[ADDING]`

To account for the partial coverage of clouds, The transmission coefficients, reflection coefficients and source functions for each layer are Calculated separately for cloud cover and cloud-free conditions, Multiply the cloud cover by the weight of the cloud cover and take the average. We also consider the cloud cover of the cumulus clouds. In addition, we also perform several additions and calculate the clear-sky radiation flux.

3.4.2 Wavelengths and Subchannels.

The basics of radiative flux calculations are , Beer-Lambert's Law

$$F^\lambda(z) = F^\lambda(0) exp(-k^\lambda z) \tag{351}$$

. where F^λ is the radiant flux density at the wavelength of λ. k^λ is the absorption coefficient. In order to calculate the radiative fluxes related to the heating rate, we need to calculate the An integration operation with respect to the wavelength is required.

$$F(z) = \int F^\lambda(z) d\lambda = \int F^\lambda(0) exp(-k^\lambda z) d\lambda \tag{352}$$

[p-rad:beer]

However, the absorption and emission of radiation by gas molecules is not Due to the complicated wavelength dependence of the absorption line structure of the molecule, It is not easy to evaluate this integration precisely. The method designed to make the relatively precise calculation easier is It is a k-distribution method. Within a certain wavelength range, the absorption coefficient of k Consider the density function $F(k)$ for λ, (p-rad:beer])

$$\int F^\lambda(0) exp(-k^\lambda z) d\lambda \simeq \int \bar{F}^k(0) exp(-kz) F(k) dk \tag{353}$$

which is approximated by where $barF^k(0)$ is In the $z = 0$, the absorption coefficient in this wavelength range has a value of k It is a flux averaged over a wavelength. This expression shows that $\bar{F}_k, F(k)$ are the same as the k If you have a relatively smooth function,

$$\int F^\lambda(0)exp(-k^\lambda z)d\lambda \simeq \sum \bar{F}^i(0)exp(-k^i z)F^i \tag{354}$$

with the addition of a finite number of exponential terms (subchannels), such that It is possible to calculate relatively precisely. This method is furthermore , It has the advantage that it is easy to consider absorption and scattering at the same time.

In the CCSR/NIES AGCM , By changing the radiation parameter data, the Calculations can be performed at various wavelengths. Not currently used as a standard, The wavelength range is divided into 18 parts. In addition, each wavelength range is divided into one to six sub-channels (corresponding to the i in the above formula), There will be 37 channels in total. The wavelength range is a wavenumber (cm^{-1}) 50, 250, 400, 550, 770, 990, 1100, 1400, 2000, 2500, 4000, 14500, 31500, 33000, 34500, 36000, 43000, 46000, 50000 Divided by.

3.4.3 Calculating the Planck function `MODULE:[PLANKS]`

The Planck function $\overline{B}^w(T)$ integrated in each wavelength range is, Evaluate by the following formula.

$$\overline{B}^w(T) = \lambda^{-2} T \exp\left\{ \sum_{n=0} 4B_n^w (\bar{\lambda}^w T)^{-n} \right\} \tag{355}$$

$\bar{\lambda}^w$ is the average wavelength of the wavelength range, B_n^w is a parameter defined by function fitting. This is the atmospheric temperature of each layer, T_l, and the boundary atmospheric temperature of each layer, $T_{l+1/2}$ and surface temperature T_g.

In the following, the index w is basically abbreviated for the wavelength range.

3.4.4 Calculating the optical thickness by gas absorption `MODULE:[PTFIT]`

The optical thickness of the gas absorption is determined by using the index m as the type of molecule, It looks like the following.

$$\tau^g = \sum_{m=1} N_m k^{(m)} C^{(m)} \tag{356}$$

where $k^{(m)}$ is the absorption coefficient of the molecule m, which is different for each subchannel.

$$k^{(m)} = \exp \left\{ \sum_{i=0} N_i \sum_{j=0} N_j A_{ij}^{(m)} (\ln p)^i (T - T_{STD})^j \right\} \tag{357}$$

as a function of temperature T(K) and atmospheric pressure p(hPa). $C^{(m)}$ is the amount of gas in the layer represented by mol cm^{-2}, Volume Mixing Ratio r (in ppmv) to ,

$$C = 1 \times 10^{-5} \frac{p}{R_u T} \Delta z \cdot r \tag{358}$$

And it can be calculated that . Note that R_u is the gas constant per mole (8.31 J mol$_{-1}$ K^{-1}), The unit of air layer thickness Δz is in km. The volume mixing ratio r at ppmv is Mass Mixture Ratio q to ,

$$r = 10^6 R^{(m)} / R^{(air)} q = 10^6 M^{(air)} / M^{(m)} \tag{359}$$

This can be converted by . $R^{(m)}, R^{(air)}$ are Gas constant per target molecule and atmospheric mass, respectively, $M^{(m)}, M^{(air)}$ is It is the average molecular weight of the target molecule and the atmosphere, respectively.

This calculation is done for each sub-channel and each layer.

3.4.5 Optical Thickness by Continuous Absorption and CFC Absorption `MODULE:[CNTCFC]`

The optical thickness of the H_2O continuous absorption τ^{H_2O} is , Think of it as a dimer, Basically, it is evaluated in proportion to the square of the volume mixing ratio of water vapor.

$$\tau^{H_2O} = (A^{H_2O} + f(T)\hat{A}^{H_2O})(r^{H_2O})^2 \rho \Delta z \tag{360}$$

The $f(T)$ for the $\hat{A}$ section is , The temperature dependence of the absorption of the dimer. Furthermore, in the wavelength range where normal gas absorption is ignored, the Incorporate a contribution proportional to the square of the volume mixing ratio of water vapor.

The continuous absorption of O_2 is assumed to be constant in the mixing ratio,

$$\tau^{O_2} = A^{O_2} \rho \Delta z \tag{361}$$

.

The continuous absorption of O_3 is based on the mixing ratio r^{O_3} and incorporates a temperature dependence,

$$\tau^{O_3} = \sum_{n=0} 2A_n^{O_3} r^{O_3} {\frac{T}{T_{STD}}}^{n} \rho \Delta z \tag{362}$$

Absorption of CFCs is considered for N_m types of CFCs,

$$\tau^{CFC} = \sum_{m=1} N_m A_m^{CFC} r^{(m)} \rho \Delta z \tag{363}$$

The sum of these optical thicknesses is τ^{CON}.

$$\tau^{CON} = \tau^{H_2O} + \tau^{O_2} + \tau^{O_3} + \tau^{CFC} \tag{364}$$

This calculation is performed for each wavelength range and each layer.

3.4.6 Scattering optical thickness and scattering moments `MODULE:[SCATMM]`

The optical thicknesses of Rayleigh scattering and particle dissipation (including scattering and absorption) are

$$\tau^s = \left(e^R + \sum_{p=1} N_p e_m^{(p)} r^{(p)} \right) \rho \Delta z \tag{365}$$

where e^R is the dissipation coefficient of Rayleigh scattering, The $e^{(p)}$ is the dissipation factor of the particle p, $r^{(p)}$ converted to standard conditions It is the volume mixing ratio of the particle p.

Here, the mass mixing ratio of cloud water from l The conversion of cloud grains to standard state-conversion volume mixing ratios (ppmv) is as follows.

$$r = 10^6 \frac{p_{STD}}{RT_{STD}} / \rho_w \tag{366}$$

However, ρ_w is the density of cloud particles.

On the other hand, the scattering-induced part of the optical thickness, τ_s^s, is

$$\tau_s^s = \left(s^R + \sum_{p=1} N_p s_m^{(p)} r^{(p)} \right) \rho \Delta z \tag{367}$$

where s^R is the scattering coefficient of Rayleigh scattering, $s^{(p)}$ is the scattering coefficient for the particle p.

Also, the standardized scattering moments g (asymmetry factor) and f (forward scattering factor) were not

$$g = \frac{1}{\tau_s} \left[\left(g^R + \sum_{p=1} N_p g_m^{(p)} r^{(p)} \right) \rho \Delta z \right] \tag{368}$$

$$f = \frac{1}{\tau_s} \left[\left(f^R + \sum_{p=1} N_p f_m^{(p)} r^{(p)} \right) \rho \Delta z \right] \tag{369}$$

Here, g^R, f^R are the scattering moments of Rayleigh scattering, $g^{(p)}$, $f^{(p)}$ is the scattering moment of the particle p.

This calculation is performed for each wavelength range and each layer.

3.4.7 Albedo at Sea Level `MODULE:[SSRFC]`

Albedo α_s at sea level is the vertical addition of the optical thickness of the scattering Using $< \tau^s >$ and the solar incidence angle factor μ_0,

$$\alpha_s = \exp\left\{\sum_{i,j} C_{ij} \mathcal{T}^j {\mu_0}^j\right\} \tag{370}$$

expressed as follows. However,

$$\mathcal{T} = (4 < \tau^s > /\mu)^{-1} \tag{371}$$

It is.

This calculation is done for each wavelength.

3.4.8 Total Optical Thickness.

Gaseous band absorption, continuous absorption, Rayleigh scattering, particle scattering and absorption All things considered, the optical thickness is ,

$$\tau = \tau^g + \tau^{CON} + \tau^s \tag{372}$$

where τ^g is different for each subchannel. Here, since τ^g is different for each subchannel, The calculation is done for each sub-channel and each layer.

3.4.9 Planck function expansion `MODULE:[PLKEXP]`

In each layer, the Planck function B is

$$B(\tau') = b_0 + b_1\tau' + b_2 (\tau')^2 \tag{373}$$

and obtain the expansion coefficients $b_0, b_1, b, 2$. Here, as $B(0)$ B at the top of each layer (bordering the top layer), As $B(\tau)$, B at the bottom edge of each layer (the boundary with the layer below), As $B(\tau/2)$, use the B at the representative level of each layer.

$$b_0 = B(0) \tag{374}$$

$$b_1 = (4B(\tau/2) - B(\tau) - 3B(0))/\tau \tag{375}$$

$$b_2 = 2(B(\tau) + B(0) - 2B(\tau/2))/\tau^2 \tag{376}$$

This calculation is done for each sub-channel and each layer.

3.4.10 Transmission and reflection coefficients of each layer, the source function MODULE:[TWST]

So far obtained, optical thickness τ, optical thickness of scattering τ^s, Scattering Moments g, f, Expansion Coefficient for Planck Function b_0, b_1, b_2, Using the solar incidence angle factor μ_0, Assuming a uniform layer, and using the two-stream approximation Transmission Coefficient R, Reflection Coefficient T, Downward Radiation Source Function ϵ^+, Find the upward radiation source function ϵ^-.

The single-scattering albedo ω is,

$$\omega = \tau_s^s/\tau \tag{377}$$

The contribution from the forward scattering factor f is Corrected Optical Thickness τ^*, The single-scattering albedo ω^*, asymmetric factor g^* is,

$$\tau^* = \frac{\tau}{1 - \omega f} \tag{378}$$

$$\omega^* = \frac{(1-f)\omega}{1 - \omega f} \tag{379}$$

$$g^* = \frac{g - f}{1 - f} \tag{380}$$

From now on, as a phase function of the normalized scattering,

$$\hat{P}^{\pm} = \omega^* W^{-2} \left(1 \pm 3g^* \mu\right)/2 \tag{381}$$

$$\hat{S}_s^{\pm} = \omega^* W^{-} \left(1 \pm 3g^* \mu\mu_0\right)/2 \tag{382}$$

However, μ is a two-stream directional cosine, and

$$\mu \equiv \begin{cases} 1/\sqrt{3} & \text{可視・近赤外域} \\ 1/1.66 & \text{赤外域} \end{cases} \tag{383}$$

$$W^{-} \equiv \mu^{-1/2} \tag{384}$$

Furthermore,

$$X = \mu^{-1} - (\hat{P}^{+} - \hat{P}^{-}) \tag{385}$$

$$Y = \mu^{-1} - (\hat{P}^{+} + \hat{P}^{-}) \tag{386}$$

$$\hat{\sigma}_s^{\pm} = \hat{S}_s^{+} \pm \hat{S}_s^{-} \tag{387}$$

$$\lambda = \sqrt{XY} \tag{388}$$

the reflectance R and transmission T become

$$\frac{A^{+}\tau^*}{A^{-}\tau^*} = \frac{X(1 + e^{-\lambda\tau^*}) - \lambda(1 - e^{-\lambda\tau^*})}{X(1 + e^{-\lambda\tau^*}) + \lambda(1 - e^{-\lambda\tau^*})} \tag{389}$$

$$\frac{B^{+}\tau^*}{B^{-}\tau^*} = \frac{X(1 - e^{-\lambda\tau^*}) - \lambda(1 + e^{-\lambda\tau^*})}{X(1 - e^{-\lambda\tau^*}) + \lambda(1 + e^{-\lambda\tau^*})} \tag{390}$$

$$R = \frac{1}{2}\left(\frac{A^{+}\tau^*}{A^{-}\tau^*} + \frac{B^{+}\tau^*}{B^{-}\tau^*}\right) \tag{391}$$

$$T = \frac{1}{2}\left(\frac{A^{+}\tau^*}{A^{-}\tau^*} - \frac{B^{+}\tau^*}{B^{-}\tau^*}\right) \tag{392}$$

Next, we first find the source function from which the Planck function is derived.

$$\hat{b}_n = 2\pi(1-\omega^*)W^- b_n \quad n = 0, 1, 2 \tag{393}$$

The expansion coefficients of the radiant function can be found from

$$D_2^{\pm} = \frac{\hat{b}_2}{Y} \tag{394}$$

$$D_1^{\pm} = \frac{\hat{b}_1}{Y} \mp \frac{2\hat{b}_2}{XY} \tag{395}$$

$$D_0^{\pm} = \frac{\hat{b}_0}{Y} + \frac{2\hat{b}_2}{XY^2} \mp \frac{\hat{b}_1}{XY} \tag{396}$$

$$\tag{397}$$

$$D^{\pm}(0) = D_0^p m \tag{398}$$

$$D^{\pm}(\tau^*) = D_0^p m + D_1^p m\tau^* + D_2^p m\tau^{*2} \tag{399}$$

The source function $\hat{\epsilon}_A^{\pm}$, which is derived from the Planck function, is

$$\hat{\epsilon}_A^- = D^-(0) - RD^+(0) - TD^-(\tau^*) \tag{400}$$

$$\hat{\epsilon}_A^+ = D^+(0) - TD^+(0) - RD^-(\tau^*) \tag{401}$$

On the other hand, the source function of the solar-induced radiation is

$$Q\gamma = \frac{X\hat{\sigma}_s^+ + \mu_0^{-1}\hat{\sigma}_s^-}{\lambda^2 - \mu_0^{-2}} \tag{402}$$

than ,

$$V_s^{\pm} = \frac{1}{2}\left[Q\gamma \pm \left(\frac{Q\gamma}{\mu X} + \frac{\hat{\sigma}_s^-}{X}\right)\right] \tag{403}$$

we obtain the following by using

$$\hat{\epsilon}_S^- = V_s^- - RV_s^+ - TV_s^- e^{-\tau^*/\mu_0} \tag{404}$$

$$\hat{\epsilon}_S^+ = V_s^+ - TV_s^+ - RV_s^- e^{-\tau^*/\mu_0} \tag{405}$$

This calculation is done for each sub-channel and each layer.

3.4.11 Combinations of source functions for each layer.

The Planck function origin and solar-induced origin The combined source function is

$$\epsilon^{\pm} = \epsilon_A^{\pm} + \hat{\epsilon}_S^{\pm} e^{-<\tau^*>/\mu_0} F_0 \tag{406}$$
$$\tag{407}$$

However, the $< \tau^* >$ is not a good match for the upper atmosphere. However, $< \tau^* >$ has a value of to the top of the layer we're considering now. It is the total optical thickness of the τ^*, It is the incident flux in the wavelength range under consideration in F_0. In other words, $e^{-<\tau^*>/\mu_0} F_0$ is It is the incident flux at the top of the layer under consideration. This calculation is actually ,

$$e^{-<\tau^*>/\mu_0} = \Pi' e^{-\tau^*/\mu_0} \tag{408}$$

The procedure is as follows. Π' will be taken from the uppermost layer of the atmosphere by Represents the product up to one layer above the layer we're considering now.

This calculation is done for each sub-channel and each layer.

3.4.12 Radiation flux at each layer boundary `MODULE:[ADDING]`

Transmission coefficient of each layer R_l, Reflection coefficient T_l, Radiation source function $\epsilon_l^{\pm}$ is required in all layers of l, The radiation fluxes at each layer boundary can be obtained by using the adding method. This means that the two layers of R, T, ϵ are known, The R, T, ϵ of the whole combined layer of the two layers can be easily calculated by It is an exploitation of what is required . In a homogeneous layer, the reflectance of the incident light from above, the transmission coefficient and the It is the same as the reflectance and transmittance of the incident light from below, Because it is different in heterogeneous layers composed of multiple layers, The reflectance, transmittance and transmittance of the incident light from above $(R^+, T^+,\ R^+, T^+)$ Distinguish between the reflectance and the transmittance of the incident light from below (R^-, T^-) and the reflectance of the incident light from below (R^-, T^-). Now, in layer 1 above and layer 2 below, these If $R_1^{\pm}, T\pm_1, \epsilon_1^{\pm}, R_2^{\pm}, T\pm_2, \epsilon_2^{\pm}$ are known, Value in the composite layer $R_{1,2}^{\pm}, T\pm_{1,2}, \epsilon_{1,2}^{\pm}$ is It looks like the following.

$$R_{1,2}^+ = R_1^+ + T_1^-(1 - R_2^+ R_1^-)^{-1} R_2^+ T_1^+ \tag{409}$$

$$R_{1,2}^- = R_2^- + T_2^+(1 - R_1^+ R_2^-)^{-1} R_1^- T_2^- \tag{410}$$

$$T_{1,2}^+ = T_2^+(1 - R_1^+ R_2^-)^{-1} T_1^+ \tag{411}$$

$$T_{1,2}^- = T_1^-(1 - R_1^+ R_2^-)^{-1} T_2^- \tag{412}$$

$$\epsilon_{1,2}^+ = \epsilon_2^+ + T_2^+(1 - R_2^+ R_1^-)^{-1}(R_1^- \epsilon_2^- + \epsilon_1^+) \tag{413}$$

$$\epsilon_{1,2}^- = \epsilon_1^- + T_1^-(1 - R_2^+ R_1^-)^{-1}(R_2^+ \epsilon_1^+ + \epsilon_2^-) \tag{414}$$

Let's say there are layers 1, 2, $\ldots N$ from the top. However, the surface is considered to be a single layer and is the N layer. Reflectance and source function of the layers from the n to the N layer as a single layer Given the $R_{n,N}^+, \epsilon_{n,N}^-$, $R_{n,N}^+, \epsilon_{n,N}^-$,

$$R_{n,N}^+ = R_n^+ + T_n^-(1 - R_{n+1,N}^+ R_n^-)^{-1} R_{n+1,N}^+ T_n^+ \tag{415}$$

$$\epsilon_{n,N}^- = \epsilon_n^- + T_n^-(1 - R_{n,N}^+ R_n^-)^{-1}(R_{n,N}^+ \epsilon_n^+ + \epsilon_{n,N}^-) \tag{416}$$

This is the value at the surface

$$R_{N,N}^+ = R_N^+ = 2W^{+2} \alpha_s \tag{417}$$

$$\epsilon_{N,N}^- = \epsilon_N^- = W^+ \left(2\alpha_s \mu_0 e^{-<\tau^*>/\mu_0} F_0 + 2\pi(1-\alpha_s) B_N\right) \tag{418}$$

It can be solved by $n = N-1, \ldots 1$ in sequence, starting from However,

$$W^+ \equiv \mu^{1/2} \tag{419}$$

In the next section, we consider the reflectance and source function of the layers from the first to the n as a single layer Given the $R_{1,n}^-, \epsilon_{1,n}^+$, $R_{1,n}^-, \epsilon_{1,n}^+$,

$$R^-_{1,n} = R^-_n + T^+_n(1 - R^+_{1,n-1}R^-_n)^{-1}R^-_{1,n-1}T^-_n \tag{420}$$
$$\epsilon^+_{1,n} = \epsilon^+_n + T^+_n(1 - R^+_{1,n-1}R^-_n)^{-1}(R^-_{1,n-1}\epsilon^-_n + \epsilon^+_{1,n-1}) \tag{421}$$

and this is also $R^-_{1,1} = R^-_1, \epsilon^+_{1,1} = \epsilon^+_1$ It can be solved by $n = 2, \dots N$, starting from

With these , Downward flux at the boundary between layers n and $n+1$ $u^+_{n,n+1}$ and upward flux $u^-_{n,n+1}$ is , $1 \sim n$ The combination of layers and $n+1 \sim N$ Reduced to a matter between two layers of combined layers,

$$u^+_{n+1/2} = (1 - R^-_{1,n}R^+_{n+1,N})^{-1}(\epsilon^+_{1,n} + R^-_{1,n}\epsilon^-_{n+1,N}) \tag{422}$$
$$u^-_{n+1/2} = R^+_{n+1,N}u^+_{n,n+1} + \epsilon^-_{n+1,N} \tag{423}$$

It can be written as. However, the flux at the top of the atmosphere is not

$$u^+_{1/2} = 0 \tag{424}$$
$$u^-_{1/2} = \epsilon^-_{1,N} \tag{425}$$

Finally, since this flux is scaled , We rescaled and added direct solar incidence to the Find the radiation flux.

$$F^+_{n+1/2} = \frac{W^+}{\bar{W}}u^+_{n+1/2} + \mu_0 e^{-<\tau^*>_{1,n}/\mu_0}F_0 \tag{426}$$
$$\tag{427}$$
$$F^-_{n+1/2} = \frac{W^+}{\bar{W}}u^-_{n+1/2} \tag{428}$$
$$\tag{429}$$

This calculation is done for each sub-channel.

3.4.13 Add in the flux.

If the radiation flux $F^{\pm}_c$ is found for each subchannel in each layer, the It corresponds to a wavelength representative of the subchannel By applying a weight (w_c) and adding them together, The wavelength-integrated flux is found.

$$\bar{F}^{\pm} = \sum_c w_c F^{\pm} \tag{430}$$

In practice, the short wavelength range (solar region), Divided into long wavelengths (earth's radiation region) and added together. In addition, a part of the short wavelength region (shorter than the wavelength of 0.7μ) The downward flux at the surface is obtained as PAR (photosynthetically active radiation).

3.4.14 The temperature derivative of the flux

To solve for surface temperature by implicit, Differential term of upward flux with respect to surface temperature Calculating dF^-/dT_g. Therefore, the value for temperatures 1K higher than T_g We also obtained $\overline{B}^w(T_g + 1)$ and used it to Redo the flux calculation using the addition method, The difference from the original value is set to dF^-/dT_g. This is a meaningful value only in the long-wavelength region (Earth's radiation region).

3.4.15 Handling of cloud cover

In the CCSR/NIES AGCM , Considering the horizontal coverage of clouds in a single grid. There are two types of clouds

1. stratus cloud. Diagnosed by the large scale condensation scheme `MODULE:[LSCOND]`. For each layer (n), the lattice-averaged cloud water content of l^l_n and The horizontal coverage factor (cloud cover) C^l_n is defined.

2. cumulus clouds. Diagnosed by the cumulus convection scheme `MODULE:[CUMLUS]`. For each layer (n) the lattice-averaged cloud water content l^c_n is defined, but Horizontal coverage (cloud cover) C^c shall be constant in the vertical direction.

In these treatments, we assume that the stratocumulus clouds overlap randomly in a vertical direction, Assuming that the cumulus cloud always occupies the same area in the upper and lower layers (Assume that the cloud cover is 0 or 1 if it is confined to that region). To do so, we perform the following calculations.

1. optical thickness of Rayleigh and particle scattering/absorption, etc. τ^s, τ_s^s, g, f,
    1. when cloud water of the l_n^l/C_n^l exists (stratocumulus)
    2. when there are no clouds at all
    3. when cloud water in the cloud cover of l_n^c/C^c is present (cumulus clouds)

Calculate for.

2. reflection and transmission coefficients for each layer, The radiant source function (Planck function origin, insolation origin) is Calculate for each of the three cases above. The values for no clouds. R°, in the case of stratus clouds R^l, in the case of cumulus clouds R^c and so on.

3. reflection and transmission coefficients for each layer, The source function is averaged with the weight of the cloud cover of the stratocumulus, C^l. The averages are represented by $\bar{}$,

$$\bar{R} = (1 - C^l)R^\circ + C^l R^l \tag{431}$$

$$\bar{T} = (1 - C^l)T^\circ + C^l T^l \tag{432}$$

$$\bar{\epsilon} = (1 - C^l)\epsilon_A^\circ + C^l \epsilon_A^l \tag{433}$$

$$+ \left[(1 - C^l)\epsilon_S^\circ + C^l \epsilon_S^l\right] e^{-\overline{<\tau^*>}/\mu_0} F_0 \tag{434}$$

However, the However ,

$$e^{-\overline{<\tau^*>}/\mu_0} = \Pi' \left[(1 - C^l)e^{-\tau^{*\circ}/\mu_0} + C_l e^{-\tau^{*l}/\mu_0}\right] \tag{435}$$

It is. Also,

$$\epsilon^\circ = \epsilon_A^\circ + \epsilon_S^\circ e^{-<\tau^{*\circ}>/\mu_0} F_0 \tag{436}$$
$$\epsilon^c = \epsilon_A^c + \epsilon_S^c e^{-<\tau^{*c}>/\mu_0} F_0 \tag{437}$$

Seek also.

4. when the characteristic values of the average (e.g., $\bar{R}$) are used, When using a characteristic value without clouds (e.g., R°), When the characteristic values of cumulus clouds (e.g., R^c) are used, fluxes by adding, respectively. Find $\bar{F}, F^\circ, F^c$.

5. the final flux we seek is

$$F = (1 - C^c)\bar{F} + C^c F^c \tag{438}$$

`($F^\circ$ is used to estimate cloud radiative forcing`

I'm doing the math.)

3.4.16 Incidence flux and angle of incidence `MODULE:[SHTINS]`

Incident Flux F_0 is , Solar constant, F_{00}, The distance between the sun and the earth, The ratio of the ratio to the time average is r_s.

$$F_0 = F_0 0 r_s^- 2 \tag{439}$$

Here, r_s asks for the following.

$$M \equiv 2\pi(d - d_0) \tag{440}$$

As ,

$$r_s = a_0 - a_1 \cos M - a_2 \cos 2M - a_3 \cos 3M \tag{441}$$

Note that d is the time in days since the beginning of the year.

The angle of incidence is obtained as follows. Solar angle position ω_s

$$\omega_s = M + b_1 \sin M + b_2 \sin 2M + b_3 \sin 3M \tag{442}$$

As the solar declination δ_s is

$$\sin \delta_s = \sin \epsilon \sin(\omega_s - \omega_0) \tag{443}$$

Then the angle of incidence factor $\mu = \cos \zeta$ (where ζ is the zenith angle) is

$$\mu = \cos \zeta = \cos \varphi \cos \delta_s \cos h + \sin \varphi \sin \delta_s \tag{444}$$

φ is a latitude, h is the time angle (local time minus π).

Assuming that the eccentricity of the Earth's orbit is e (Katayama, 1974),

$$a_0 = 1 + e^2 \tag{445}$$
$$a_1 = e - 3/8e^3 - 5/32e^5 \tag{446}$$
$$a_2 = 1/2e^2 - 1/3e^4 \tag{447}$$
$$a_3 = 3/8e^3 - 135/64^5 \tag{448}$$
$$b_1 = 2e - 1/4e^3 + 5/96e^5 \tag{449}$$
$$b_2 = 5/4e^2 - 11/24e^4 \tag{450}$$
$$b_3 = 13/12e^3 - 645/940e^5 \tag{451}$$
$$\tag{452}$$

It is also possible to give average annual insolation. In this case, the annual mean incidence and the annual mean angle of incidence are It approximates to be as follows.

$$\overline{F} = F_{00}/\pi \tag{453}$$

$$\overline{\mu} \simeq 0.410 + 0.590 \cos^2 \varphi. \tag{454}$$

3.4.17 Other Notes.

The calculation of the radiation is usually not done at every step. To do so, we have to save the radiation flux, If the time is not used for radiation calculation, it is used. As for the shortwave radiation, Percentage of time (time that is $\mu_0 > 0$) between next calculation time (f) and Using the solar incidence angle factor ($\bar{\mu_0}$) averaged over the daylight hours Seeking Flux $\bar{F}$,

$$F = f \frac{\mu_0}{\bar{\mu_0}} \bar{F} \tag{455}$$

.

2. cloud water depends on the temperature, Treated as water and ice cloud particles. Percentage treated as ice clouds f_I is ,

$$f_I = \frac{T_0 - T}{T_0 - T_1} \tag{456}$$

```
(but with a maximum value of 1 and a minimum value of 0). Also,
$T_0 = 273.15{K}, T_1 = 258.15{K}$.
```

3.5 Vertical Diffusion.

3.5.1 Vertical Diffusion Scheme Overview.

The vertical diffusion scheme, due to sub-grid scale turbulent diffusion. Evaluating the vertical flux of physical quantities. The main input data are wind speed, u, v, u, v, temperature T, specific humidity q, and cloud cover l, The output data are the vertical fluxes of momentum, heat, water vapor, cloud water and It is the differential value for obtaining an implicit solution.

To estimate the vertical diffusion coefficient, the Mellor and Yamada (1974, 1982). The turbulent closure model. Using level 2 parameterization.

The outline of the calculation procedure is as follows.

1. as the stability of the atmosphere. Richardson numbers.
2. calculate the diffusion coefficient from Richardson number `MODULE:[VDFCOF]`.
3. calculate the flux and its derivative from the diffusion coefficient.

3.5.2 Basic Formula for Flux Calculations

The vertical diffuse flux in the atmosphere is , Using the diffusion coefficient K, it is evaluated as follows.

$$Fu = K_M \frac{\partial u}{\partial \sigma} \tag{457}$$

$$Fv = K_M \frac{\partial v}{\partial \sigma} \tag{458}$$

$$F\theta = K_H \frac{\partial \theta}{\partial \sigma} \tag{459}$$

$$Fq = K_q \frac{\partial q}{\partial \sigma} \tag{460}$$

3.5.3 Richardson Number.

The standard for atmospheric stratospheric stability, Bulk Richardson number R_{iB} is

$$R_{iB} = \frac{\frac{g}{\theta_s}\frac{\Delta\theta}{\Delta z}}{\left(\frac{\Delta u}{\Delta z}\right)^2 + \left(\frac{\Delta v}{\Delta z}\right)^2} \tag{461}$$

. defined by . Here, $(\Delta A)_{k-1/2}$ represents $A_k - A_{k-1}$. The $(\Delta z)_{k-1/2}$ is based on the hydrostatic pressure equation,

$$(\Delta z)_{k-1/2} = \frac{RTv_k}{g}\frac{(\Delta\sigma)_{k-1/2}}{\sigma_{k-1/2}} \tag{462}$$

The flux Ricahrdson number R_{if} is ,

$$R_{if} = \frac{1}{2\beta_2}\left[\beta_1 + \beta_4 R_{iB} - \sqrt{(\beta_1 + \beta_4 R_{iB})^2 - 4\beta_2\beta_3 R_{iB}}\right], \tag{463}$$

However,

$$\alpha_1 = 3A_2\gamma_1 \tag{464}$$

$$\alpha_2 = 3A_2(\gamma_1 + \gamma_2) \tag{465}$$

$$\beta_1 = A_1 B_1(\gamma_1 - C_1) \tag{466}$$

$$\beta_2 = A_1[B_1(\gamma_1 - C_1) + 6A_1 + 3A_2] \tag{467}$$

$$\beta_3 = A_2 B_1 \gamma_1 \tag{468}$$

$$\beta_4 = A_2[B_1(\gamma_1 + \gamma_2) - 3A_1], \tag{469}$$

$$(A_1, B_1, A_2, B_2, C_1) = (0.92, 16.6, 0.74, 10.1, 0.08), \tag{470}$$

$$\gamma_1 = \frac{1}{3} - \frac{2A_1}{B_1}, \;\; \gamma_2 = \frac{B_2}{B_1} + 6\frac{A_1}{B_1}. \tag{471}$$

The relationship between the R_{iB} and the R_{if} is illustrated in this figure, Figure [p-dif:rib-rif]] (#p-dif:rib-rif).

3.5.4 Diffusion Coefficient.

The diffusion coefficient is , For each layer boundary ($k-1/2$ level) , It is given as follows.

$$K_M = l^2 \frac{\Delta|\mathbf{v}|}{\Delta z} S_M \tag{472}$$

$$K_H = K_q = l^2 \frac{\Delta|\mathbf{v}|}{\Delta z} S_H \tag{473}$$

Here, S_M, S_H are,

$$\widetilde{S_H} = \frac{\alpha_1 - \alpha_2 R_{if}}{1 - R_{if}} \tag{474}$$

$$\widetilde{S_M} = \frac{\beta_1 - \beta_2 R_{if}}{\beta_3 - \beta_4 R_{if}} \widetilde{S_H}, \tag{475}$$

with ,

$$S_M = B_1^{1/2} (1 - R_{if})^{1/2} \widetilde{S_M}^{3/2} \tag{476}$$

$$S_H = B_1^{1/2} (1 - R_{if})^{1/2} \widetilde{S_M}^{1/2} \widetilde{S_H}. \tag{477}$$

l is a mixing distance, according to Blakadar (1962),

$$l = \frac{kz}{1 + kz/l_0} \tag{478}$$

Take. k is a Krman constant. The current standard value is $l_0 = 200$ m.

If S_H, S_M are shown as functions of R_{if}, Figure [p-dif:smsh-rif]] (#p-dif:smsh-rif).

3.5.5 Calculating Flux.

Using the above, we calculate the fluxes and flux derivatives.

$$Fu_{k-1/2} = K_{M,k-1/2}(u_{k-1} - u_k)/(\sigma_{k-1} - \sigma_k) \tag{479}$$

$$Fv_{k-1/2} = K_{M,k-1/2}(v_{k-1} - v_k)/(\sigma_{k-1} - \sigma_k) \tag{480}$$

$$F\theta_{k-1/2} = K_{H,k-1/2}(\theta_{k-1} - \theta_k)/(\sigma_{k-1} - \sigma_k) \tag{481}$$

$$Fq_{k-1/2} = K_{q,k-1/2}(q_{k-1} - q_k)/(\sigma_{k-1} - \sigma_k) \tag{482}$$

$$\frac{\partial Fu_{k-1/2}}{\partial u_{k-1}} = \frac{\partial Fv_{k-1/2}}{\partial v_{k-1}} = -\frac{\partial Fu_{k-1/2}}{\partial u_k} = -\frac{\partial Fv_{k-1/2}}{\partial v_k} = K_{M,k-1/2}/(\sigma_{k-1} - \sigma_k) \tag{483}$$

$$\frac{\partial F\theta_{k-1/2}}{\partial T_{k-1}} = \sigma_{k-1}^{-\kappa} K_{H,k-1/2}/(\sigma_{k-1} - \sigma_k) \tag{484}$$

$$\frac{\partial F\theta_{k-1/2}}{\partial T_k} = \sigma_k^{-\kappa} K_{H,k-1/2}/(\sigma_{k-1} - \sigma_k) \tag{485}$$

$$\frac{\partial Fq_{k-1/2}}{\partial u_{k-1}} = -\frac{\partial Fq_{k-1/2}}{\partial u_k} = K_{q,k-1/2}/(\sigma_{k-1} - \sigma_k) \tag{486}$$

3.5.6 Minimum Diffusion Coefficient.

In the very stable case, the above estimate gives zero as the diffusion coefficient. As it is, the model's behavior can be modified in various ways Set a suitable minimum value as it will have a negative effect. The current standard values are the same for all fluxes and $K_{min} = 0.15$ m^2/s

3.5.7 Other Notes.

I'm calling the shallow cumulus convection `MODULE:[SHLCOF]`, By default, this is a dummy.

3.6 Surface Flux.

3.6.1 Overview of the Surface Flux Scheme

The surface flux scheme is , due to turbulent transport in the ground boundary layer. Assessing the flux of physical quantities between atmospheric surfaces. The main input data are wind speed u, v, u, v, temperature T, and specific humidity q, The output data are the vertical fluxes of momentum, heat, water vapor and It is the differential value for obtaining an implicit solution.

Bulk coefficients are obtained according to Louis(1979), Louis *et al.*(1982). However, we take into account the difference between the momentum and heat roughness in the correction.

The outline of the calculation procedure is as follows.

1. as the stability of the atmosphere. Richardson numbers.
2. calculate the bulk coefficient from Richardson number `MODULE:[PSFCL]`.
3. calculate the flux and its derivative from the bulk coefficient.
4. if necessary, using the required flux After taking into account the roughness effect of sea level, the effect of free convection, and wind speed correction, Do the math again.

3.6.2 Basic Formula for Flux Calculations

Surface Flux \$ Fu, Fv, Fθ, $Fq\$areUsingthebulkcoefficientsC_M, C_H, C_E$ It is expressed as follows.

$$Fu = -\rho C_M |\mathbf{v}| u \tag{487}$$

$$Fv = -\rho C_M |\mathbf{v}| v \tag{488}$$

$$F\theta = \rho c_p C_H |\mathbf{v}| (\theta_g - \theta) \tag{489}$$

$$Fq^P = \rho C_E|\mathbf{v}|(q_g - q) \tag{490}$$

However, the Fq^P is the amount of possible evaporation. The calculation of actual evaporation is a combination of “surface processes” and We will discuss this in the section “Solving Diffusion-Based Budget Equations for Atmospheric Surface Systems”.

3.6.3 Richardson Number.

The standard of stability between atmospheric surfaces, Bulk Richardson number R_{iB} is

$$R_{iB} = \frac{\frac{g}{\theta_s}(\theta_1 - \theta(z_0))/z_1}{(u_1/z_1)^2} = \frac{g}{\theta_s}\frac{T_1(p_s/p_1)^\kappa - T_0}{u_1^2/z_1} f_T. \tag{491}$$

Here,

$$f_T = (\theta_1 - \theta(z_0))/(\theta_1 - \theta_0) \tag{492}$$

is a correction factor and is approximated from the uncorrected bulk Richardson number, while Here, the calculation method is abbreviated.

3.6.4 Bulk factor.

The bulk coefficients C_M, C_H, C_E are Louis(1979), Louis *et al.*(1982). However, we take into account the difference between the momentum and heat roughness in the correction. i.e., the roughness to momentum, heat, and water vapor. z_{0M}, z_{0H}, z_{0E} respectively In general, it is $z_{0M} > z_{0H}, z_{0E}$, but heat and water vapor are also Bulk factor for flux from the height of the z_{0M} First find the $\widetilde{C_H}$ and $\widetilde{C_E}$ and then correct them.

$$C_M = \begin{cases} C_{0M}[1 + (b_M/e_M)R_{iB}]^{-e_M} R_{iB} \geq 0 \\ C_{0M}\left[1 - b_M R_{iB}\left(1 + d_M b_M C_{0M}\sqrt{\frac{z_1}{z_{0M}}|R_{iB}|}\right)^{-1}\right] R_{iB} < 0 \end{cases} \tag{493}$$

$$\widetilde{C_H} = \begin{cases} \widetilde{C_{0H}}[1+(b_H/e_H)R_{iB}]^{-e_H} R_{iB} \geq 0 \\ \widetilde{C_{0H}} \left[1 - b_H R_{iB} \left(1 + d_H b_H \widetilde{C_{0H}} \sqrt{\frac{z_1}{z_{0M}}|R_{iB}|}\right)^{-1}\right] R_{iB} < 0 \end{cases} \tag{494}$$

$$C_H = \widetilde{C_H} f_T \tag{495}$$

$$\widetilde{C_E} = \begin{cases} \widetilde{C_{0E}}[1+(b_E/e_E)R_{iB}]^{-e_E} R_{iB} \geq 0 \\ \widetilde{C_{0E}} \left[1 - b_E R_{iB} \left(1 + d_E b_E \widetilde{C_{0E}} \sqrt{\frac{z_1}{z_{0M}}|R_{iB}|}\right)^{-1}\right] R_{iB} < 0 \end{cases} \tag{496}$$

$$C_E = \widetilde{C_E} f_q \tag{497}$$

The $C_{0M}, \widetilde{C_{0H}}, \widetilde{C_{0E}}$ are The bulk coefficient (for fluxes from z_{0M}) at neutral,

$$C_{0M} = \widetilde{C_{0H}} = \widetilde{C_{0E}} = \frac{k^2}{\left[\ln\left(\frac{z_1}{z_{0M}}\right)\right]^2}. \tag{498}$$

Correction Factor f_q is ,

$$f_q = (q_1 - q(z_0))/(q_1 - q^*(\theta_0)) \tag{499}$$

but the method of calculation is abbreviated. The coefficient is $(b_M, d_M, e_M) = (9.4, 7.4, 2.0)$, $(b_H, d_H, e_H) = (b_E, d_E, e_E) = (9.4, 5.3, 2.0)$.

The dependence of the bulk coefficients on Ri_B is illustrated in Fig, Figure [p-sflx:cm] (#p-sflx:cm), Figure [p-sflx:ch] (#p-sflx:ch).

3.6.5 Calculating Flux.

This will calculate the flux.

$$\hat{F}u_{1/2} = -\rho_{1/2} C_M |\mathbf{v}_1| u_1 \tag{500}$$

$$\hat{F}v_{1/2} = -\rho_{1/2} C_M |\mathbf{v}_1| v_1 \tag{501}$$

$$\hat{F}\theta_{1/2} = \rho_{1/2} c_p C_H |\mathbf{v}_1| \left(T_0 - \sigma_1^{-\kappa} T_1 \right) \tag{502}$$

$$\hat{F}q^P_{1/2} = \rho_{1/2} C_E |\mathbf{v}_1| \left(q^*(T_0) - q_1 \right) \tag{503}$$

The differential term is as follows

$$\frac{\partial F u_{1/2}}{\partial u_1} = \frac{\partial F v_{1/2}}{\partial v_1} = -\rho_{1/2} C_M |\mathbf{v}_1| \tag{504}$$

$$\frac{\partial F \theta_{1/2}}{\partial T_1} = -\rho_{1/2} c_p C_H |\mathbf{v}_1| \sigma_1^{-\kappa} \tag{505}$$

$$\frac{\partial F \theta_{1/2}}{\partial T_0} = \rho_{1/2} c_p C_H |\mathbf{v}_1| \tag{506}$$

$$\frac{\partial F q_{1/2}}{\partial q_1} = -\beta \rho_{1/2} C_E |\mathbf{v}_1| \tag{507}$$

$$\frac{\partial F q^P_{1/2}}{\partial T_0} = \beta \rho_{1/2} C_E |\mathbf{v}_1| \left(\frac{dq^*}{dT} \right)_{1/2} \tag{508}$$

Here, it's important to note, T_0 is a quantity that is not required at this time. Epidermal temperature is , Conditions for surface heat balance

$$F\theta(T_0, T_1) + L\beta F q^P(T_0, q_1) + FR(T_0) - Fg(T_0, G_1) = 0 \tag{509}$$

Determined to meet. At this point, for T_0, we use the one from the previous time step for evaluation. The true flux value that meets the surface balance is , It is determined by solving this equation in conjunction with surface processes. In that sense, I have added $\hat{}$ to the flux above.

3.6.6 handling at sea level.

At sea level, we follow Miller et al. (1992) and consider the following two effects.

- Free convection is preeminent when the wind speed is low
- The roughness of the sea surface varies with the wind speed.

The effect of free convection is calculated using the buoyancy flux F_B,

$$F_B = F\theta/c_p + \epsilon T_0 F_q^P \tag{510}$$

When I was in $F_B > 0$,

$$w^* = (H_B F_B)^{1/3} \tag{511}$$

$$|\mathbf{v}_1| = \left(u_1^2 + v_1^2 + (w^*)^2\right)^{1/2} \tag{512}$$

to be considered by making H_B corresponds to the thickness scale of the mixing layer. The current standard value is $H_B = 2000$ m.

The roughness variation of the sea surface is represented by the friction velocity u^*

$$u^* = \left(\sqrt{Fu^2 + Fv^2}/\rho\right)^{1/2} \tag{513}$$

with ,

$$Z_{0M} = A_M + B_M(u^*)^2/g + C_M\nu/u^* \tag{514}$$
$$Z_{0H} = A_H + B_H(u^*)^2/g + C_H\nu/u^* \tag{515}$$
$$Z_{0E} = A_E + B_E(u^*)^2/g + C_E\nu/u^* \tag{516}$$

Evaluate as follows. $\nu = 1.5 \times 10^{-5}$ m^2 s^{-1} is It is the kinematic viscosity of the atmosphere, The standard values for the other coefficients are \$(A_M, B_M, C_M) = (0, 0.018, 0.11) \$, \$(A_H, B_H, C_H) = $(1.4\times10\hat{}\{-5\}, 0, 0.4)$\$, \$$(A_E, B_E, C_E) = (1.3 \times 10\hat{}\{-4\}, 0, 0.62)$\$.

For the above calculations, $Fu, Fv, F\theta, Fq$ are required, Perform successive approximation calculations.

3.6.7 Wind Speed Correction

In general, the roughness of the ground surface is greater on large surfaces than on small surfaces. The downward transport of momentum is so efficient that the wind just above it is weak, The difference in wind speed cancels out the difference in roughness in C_D.

In the model, the wind speed passed to the surface flux calculation is It is a value calculated by time integration of the mechanical processes, The values are smoothed by spectral expansion. This is the reason why the surface of the ground with widely different roughnesses, such as sea level and land level, is In an area that is mixed on a small scale , I can't describe this compensation effect well. Therefore, once the momentum flux is calculated and The wind speed in the lowest layer of the atmosphere is corrected for by it, and then Recalculate the momentum, heat, and water fluxes again.

3.6.8 Minimum wind speed.

Consider the effects of small-scale exercise, Surface wind speed in the calculation of surface flux Set the minimum value of $|\mathbf{v}_1|$. The current standard values are the same for all fluxes and It is 3 m/s.

3.7 Surface Processes.

3.7.1 Overview of Surface Processes.

The surface processes are the processes of momentum, heat, and water between the atmosphere and the earth's surface Giving boundary conditions at the lower end of the atmosphere through flux exchange. Surface processes are described in the Ground Temperature T_g, Ground Moisture W_g, Handle your own forecast variables, such as snow cover W_y, Thermal inertia of the ground surface, water accumulation, snow accumulation, Evaluate the accumulation of sea ice and other factors. The main input data are the diffusion of geophysical quantities between the atmosphere and the surface and the fluxes due to radiation and precipitation. The output data are the surface temperature (T_s) and Various boundary condition parameters such as albedo and roughness.

Surface processes are classified as follows.

1. geothermal diffusion processes - Determining the geothermal temperature structure
2. geological and hydrological processes - Determining the structure of underground water, runoff, etc.
3. snow accumulation process - snow accumulation, snowmelt, etc., expression of snow-related processes
4. oceanic mixed layer processes - determining ocean temperature and sea ice thickness (optional)

We briefly list the characteristics of the CCSR/NIES AGCM surface processes:

Evaluate the thermal conductivity and water diffusion (optional) of multiple layers in the ground.

2. assessing the surface heat balance using the surface temperature.
3. diffusional conduction of heat and water is solved by the implicit method.

Snow is not treated as a separate layer, but is evaluated together with the first surface layer.

5. assessing oceanic mixed layers and sea ice in multiple layers (optional)

The scheme is outlined in detail along the calculation flow. the part in the square brackets is the name of the corresponding subroutine and the part in ( ) is the file name. The entries enclosed in parentheses refer to the descriptions in other sections.

1. (Evaluate surface fluxes `MODULE:[SFCFLX(psfcm)]`) The heat, water (evaporation), and momentum fluxes between the atmosphere and the surface are Estimate in bulk. However, the evaporation efficiency (β) is set to 1.

2. evaluate the surface roughness `MODULE:[GNDZ0(pgsfc)]` Basically, it depends on the file and the surface type. I can get it from the outside, Change according to snowfall and other factors.

3. evaluate the heat flux and heat capacity within the ground surface . `MODULE:[LNDFLX(pglnd), SEAFLX(pgsea), SNWFLX(pgsnw)]` Estimate the heat capacity of each layer of land and sea, The heat flux at each layer boundary is estimated from the heat transfer equation. If there is snowfall, change the heat capacity and flux.

4. evaluate the water flux and capacity of the land surface `MODULE:[LNDWFX(pglnd)]` Estimate the capacity of each layer of water on land, Estimate the water flux at each layer boundary from the water diffusion equation

5. evaluate the evaporation efficiency `MODULE:[GNDBET(pgsfc)]` For the land surface, the calculation depends on soil moisture and stomatal resistance.

6. implicit solution of geo-thermal conduction up to the middle `MODULE:[GNDHT1(pggnd)]` Evaluate the temperature change due to heat conduction in the ground. However, taking into account the surface temperature changes Since it is done implicitly, only the first step is done here.

7. solving the surface heat balance `MODULE:[SLVSFC(pgslv)]` Solving the equation for the heat balance between the surface temperature and Time variation of temperature and specific humidity in the first layer of the atmosphere. Using this, the heat/water (evaporation) flux between the atmosphere and the ground surface and compensate for the heat transfer flux at the surface. If there is snow or ice and the surface temperature is above the freezing point, With the surface temperature as the freezing point, Evaluate the residual flux as the flux used for snowmelt.

8. implicit solution for geothermal conduction `MODULE:[GNDHT2(pggnd)]` Since the change in surface temperature was obtained, we used it to determine Solving for changes in ground temperature due to heat conduction in the ground.

9. evaluate snow reduction by snow sublimation `MODULE:[SNWSUB(pgsnw)]` For snow cover conditions, the calculated evaporation (sublimation) fluxes allow Decrease the snowpack.

10. assessing the increase in snow cover due to snowfall `MODULE:[SNWFLP(pgsnw)]` It discriminates between snowfall and rainfall and increases the snowpack when it falls.

11. assessing snowfall reduction due to snowmelt `MODULE:[SNWMLP(pgsnw)]` If the surface temperature or first layer temperature exceeds the freezing point during snowfall , If the snow melts, it will keep the temperature below the freezing point, Decrease the snowpack.

12. implicit solution for groundwater diffusion `MODULE:[GNDWTR(pggnd)]` Solving changes in subsurface moisture due to subsurface water fluxes.

13. evaluate precipitation interception by snowpack `MODULE:[SNWROF(pgsnw)]` Snowfall prevents precipitation infiltration into the soil, Rainfall and snowmelt water (part of it) will be runoff.

14. assessing surface runoff `MODULE:[LNDROF(pglnd)]` Calculating surface runoff of rainfall and snowmelt water. Bucket Model, New

Bucket Model, There are 3 evaluation methods to choose from, including runoff evaluation using permeability.

15. evaluate the freezing process MODULE:[LNDFRZ(pglnd)] Freezing and thawing of groundwater and Calculate the temperature change due to the latent heat release associated with it. However, this routine is optional, Usually skipped.
16. assessing the growth of sea ice MODULE:[SEAICE(pgsea)] If you specify the marine mixed layer option , Calculate the increase or decrease in the thickness of the sea ice due to heat conduction.
17. evaluate the melting of sea ice surface MODULE:[SEAMLT(pgsea)] If the surface temperature or first layer temperature of the sea ice exceeds the freezing point, The temperature is kept below the freezing point, Decrease the thickness of the sea ice.
18. nudge the ocean temperature MODULE:[SEANDG(pgsea)] With the oceanic mixed layer option, the given Nudging to get closer to the temperature. It can be added to the temperature of the sea surface.
19. evaluate the wind speed on the ground MODULE:[SLVWND(pggnd)] Solving for changes in wind speed in the first layer of the atmosphere.

Also, some of the routines mentioned above are In addition, the following subroutines for the evaluation of land, sea and snow surfaces are used There's a split.

1. setting the boundary conditions MODULE:[GNDSFC(pgbnd)]
    1. read the surface conditions MODULE:[GETIDX(pgbnd)]
    2. read the sea level condition MODULE:[GETSEA(pgbnd)]
    3. set the condition of sea level MODULE:[SEATMP(pgsea)]
2. evaluate the surface albedo MODULE:[GNDALB(pgsfc)]
    1. load the albedo MODULE:[GETALB(pgbnd)]

    2. change the land surface albedo `MODULE:[LNDALB(pglnd)]`
    3. change the sea-surface albedo `MODULE:[SEAALB(pgsea)]`
    4. change the snow albedo `MODULE:[SNWALB(pgsnw)]`

3. evaluate the surface roughness `MODULE:[GNDZ0(pgsfc)]`

    1. read the roughness `MODULE:[GETZ0(pgbnd)]`
    2. change the land surface roughness `MODULE:[LNDZ0(pglnd)]`
    3. change the sea surface roughness `MODULE:[SEAZ0(pgsea)]`
    4. change the snow surface roughness `MODULE:[SNWZ0(pgsnw)]`

4. evaluate the surface wetness `MODULE:[GNDBET(pgsfc)]`

    1. read the degree of wetness `MODULE:[GETBET(pgbnd)]`
    2. change the surface wetness `MODULE:[LNDBET(pglnd)]`
    3. change the degree of sea surface wetness `MODULE:[SEABET(pgsea)]`
    4. change the snow surface wetness `MODULE:[SNWBET(pgsnw)]`

3.7.2 classification of the ground surface.

The ground surface is a condition given by the outside world, According to the surface type m, they are classified as follows.

[]@ll@ Header0	Header1	m
requirement	-I can't even begin to tell you what to do.	mixed-layered ocean
Sea Ice (given from outside)	0	Sea level (providing temp from outside)
land ice	≥ 2	land surface

Furthermore, depending on the possible internal changes in the ice conditions, Take the following ground surface conditions n.

[]@ll@ Header0 Header1n state0 Sea surface without ice1 Sea Ice and Land Ice≥ 2 land surface

These are defined in `MODULE:[GNDSFC(pgsfc)]`.

3.7.3 Surface Heat Balance.

The conditions of the surface heat balance are ,

$$F\theta(T_0, T_1) + L\beta Fq^P(T_0, q_1) + FR(T_0) - Fg(T_0, G_1) = 0 \tag{517}$$

[p-sfc:sfc-balance].

It is. $F\theta, Fq^P, FR, Fg$, $F\theta, Fq^P, FR, Fg$ is , Atmospheric and subsurface forecast variables before evaluating surface processes and , The evaluation is performed using the In the T_0 used at that time, this balance is generally not met.

There are several ways to solve this problem.

1. how to consider only T_0 as an unknown
2. how to consider T_0, T_1, q_1, G_1 as unknown numbers

The CCSR/NIES AGCM uses the latter method. In doing so, it is necessary to combine and solve for all the variables of the atmospheric and ground layers. The details are described in the section “Solving the Diffusion-Based Budget Equations for Atmospheric Surface Systems”.

There are two ways to evaluate the evaporation terms $\beta Fq^P(T_0, q_1)$.

1. as a $\beta = 1$ Solved for ([\p-sfc:sfc-balance](#p-sfc:sfc-balance)) Fq^P (possible evaporation amount) multiplied by β.
2. using β. (p-sfc:sfc-balance]) directly solve.

The temperatures used in the calculations in βFq^P are different between the former and the latter. In the former case, the temperature in the case of $\beta = 1$, In the latter case, the actual temperature is used.

The CCSR/NIES AGCM uses the former method as standard. The result of solving (with a }(#p-sfc:sfc-balance) on a snow or ice surface If the T_0 exceeds the freezing point, Or, when T_0 divides the freezing temperature of seawater at the sea surface (in the case of oceanic mixed-layer model) by fixing the temperature of the T_0 at the freezing point and calculating each flux, (p-sfc:sfc-balance]) and the residuals (energy residuals) of the formula Suppose it is used for freezing and thawing snow and ice.

3.7.4 Set the discrete coordinate system MODULE: [SETGLV,SETWLV,SETSLV]

The ground is discretized with the z coordinate system. Land temperature is layer zg_l, water content is layer zw_l, Ocean temperature is defined in layer zs_l. The l increases from the top to the bottom. The flux is defined by the layer boundary $zg_{k-1/2}, zw_{k-1/2}$.

Also, consider a layer of zero thickness on the $z = 0$, Define skin temperature T_s. For convenience, it is represented by $l = 0$ and $zg_0 = zg_{1/2} = zg_{-1/2} = 0$.

3.7.5 Calculating land heat flux and heat capacity MODULE: [LNDFLX]

Physical quantities, such as heat and moisture fluxes in the ground, and wetness The evaluation of surface characteristics is based on whether the surface is sea or land, and in the case of land surface This is done separately if there is snowfall or not. In the following, we will first evaluate the evaluation method for the land surface case without snow. We shall describe in brief. We will describe the difference between the case of sea level and snow surface in detail later.

The heat capacity of the land surface is ,

$$Cg_l = \tilde{C}g_l(zg_{l+1/2} - zg_{l-1/2}) = \tilde{C}g_l \Delta zg_l \ . \tag{518}$$

where $\tilde{C}g_l$ is the volume specific heat.

The land heat flux is treated as a constant heat transfer coefficient (which may depend on l).

$$Fg_{l-1/2} = Kg_{l-1/2}(G_l - G_{l-1})(zg_l - zg_{l-1}) \ , \tag{519}$$

$$\frac{\partial Fg_{l-1/2}}{\partial G_l} = -\frac{\partial Fg_{l-1/2}}{\partial G_{l-1}} = Kg_{l-1/2}/(zg_l - zg_{l-1}) \ . \tag{520}$$

3.7.6 Calculating the water flux on land `MODULE:[LNDWFX]`

The capacity of water in each layer C_w is ,

$$Cw_l = \rho_w (zw_{l+1/2} - zw_{l-1/2}) = \rho_w \Delta zw_l \ . \tag{521}$$

However, in practice, it is not possible to store this much water. The maximum storage capacity, i.e., the saturation capacity, is defined as the saturation water content of ws,

$$Cws_l = ws_l \rho_w (zw_{l+1/2} - zw_{l-1/2}) = ws_l \rho_w \Delta zw_l \ . \tag{522}$$

The basic formula for the groundwater flux can be written as follows.

$$Fw = -K_w \left(\frac{\partial w}{\partial z} + g_w \right) \tag{523}$$

\[basic-Fw]

Here, g_w represents the effect of gravity.
There are two ways to evaluate the groundwater flux on land.

1. fixed diffusion coefficient method
2. moisture content dependent diffusion coefficient method `MODULE:[HYDFLX]`

In the method of fixed diffusion coefficients, we simply express it as follows. K_w is the diffusion coefficient and ρ_w is the density of liquid water. where the gravitational potential term g_w in (basic-Fw]) is Ignore it.

$$Fw_{l-1/2} = \rho_w Kw_{l-1/2} (w_l - w_{l-1}) / (zw_l - zw_{l-1}) \ , \tag{524}$$

$$\frac{\partial Fw_{l-1/2}}{\partial w_l} = -\frac{\partial Fw_{l-1/2}}{\partial w_{l-1}} = \rho_w Kw_{l-1/2} / (zw_l - zw_{l-1}) \ . \tag{525}$$

On the other hand, the method with the moisture content dependent diffusion coefficient is not applicable, Using the hydraulic potential, we obtain the following.

$$Fw_{l-1/2} = \rho_w Kw_{l-1/2}(W_{l-1/2})^{2B+3}((\psi_l - \psi_{l-1})/(zw_l - zw_{l-1}) - 1) , \tag{526}$$

$$\frac{\partial Fw_{l-1/2}}{\partial w_l} = \rho_w Kw_{l-1/2}(W_{l-1/2})^{2B+3}\frac{\partial \psi_l}{\partial w_l}/(zw_l - zw_{l-1}) , \tag{527}$$

$$-\frac{\partial Fw_{l-1/2}}{\partial w_{l-1}} = \rho_w Kw_{l-1/2}(W_{l-1/2})^{2B+3}\frac{\partial \psi_{l-1}}{\partial w_{l-1}}/(zw_l - zw_{l-1}) . \tag{528}$$

where Kw is the saturated hydraulic conductivity, W is the saturation degree, and ψ is the pressure potential, It is given as follows.

$$W_l = w_l/ws_l \ , \quad W_{l-1/2} = (W_{l-1} + W_l)/(Cw_{l-1} + Cw_l) , \tag{529}$$

$$\psi_l = \psi s_l (W_l)^{-B} \ , \quad \frac{\partial \psi_l}{\partial w_l} = -B\psi_l W_l/ws . \tag{530}$$

Kw, B, and ψs are constants and may depend on the ground surface types m and l.

3.7.7 Calculating land surface spill`MODULE:[LNDROF]`

The following three methods can be used to evaluate runoff.

1. bucket model
2. new bucket model
3. surface runoff with consideration of infiltration capacity

In the bucket model ,

$$w_1^{m+1,*} = w_1^{m+1} + \frac{P-E}{Cw_1}\Delta t \tag{531}$$

and this is

$$w_1^{m+1,*} > w_s \tag{532}$$

with the outflow as R_B,

$$w_1^{m+1} = w_s \tag{533}$$
$$R_B = Cw_1(w_1^{m+1,*} - w_s)/\delta t \tag{534}$$

. Other than that ,

$$w_1^{m+1} = w_1^{m+1,*} \tag{535}$$
$$R_B = 0 \ . \tag{536}$$

The new bucket model (Kondo, 1993) is based on the idea that surface soil types and depths are spatially It is a model for estimating the average groundwater infiltration rate for non-uniform cases. It was originally developed to estimate the daily average outflow of , Here, we changed it to be used at each time step. In the new bucket model, precipitation infiltration and post-runoff soil moisture are estimated as follows.

$$w_1^{m+1} = w_1^m + (ws_1 - w_1^m)\tanh\left(\frac{(P-E)\tau_B}{Cw_1(ws_1 - w_1^m)}\right)\Delta t/\tau_B \ . \tag{537}$$

Here, τ_B is a time constant (standard value 3600s). In this case, the runoff volume (R_N) is calculated from the surface water balance

$$R_N = P - E - Cw_1(w_1^{m+1} - w_1^m)/\Delta t \tag{538}$$
$$= (P-E)\left(1 - \frac{\tanh x}{x}\right) \tag{539}$$

It is estimated that . However,

$$x = \frac{(P-E)\tau_B}{Cw_1(ws_1 - w_1^m)} \ . \tag{540}$$

The evaluation of the surface runoff R_I considering soil infiltration capacity is based on the evaluation of the infiltration capacity of the C_I, Assuming that the intensity of stratiform rainfall is P_l and that of convective rainfall is P_c, Given below.

$$
R_I = \left\{ \begin{array}{l} P_c \exp[-(C_I - P_l)/(P_c/\kappa)] \ \ (P_l \leq C_I) \\ P_l + P_c - C_I \ \ (P_l > C_I) \end{array} \right. . \tag{541}
$$

[inf-exs]

The amount of precipitation input percolating to the ground surface is modified as follows.

$$
\widetilde{P_c} = \left\{ \begin{array}{l} P_c - R_I \ \ (P_l \leq C_I) \\ 0 \ \ (P_l > C_I) \end{array} \right. , \tag{542}
$$

$$
\widetilde{P_l} = \left\{ \begin{array}{l} P_l \ \ (P_l \leq C_I) \\ C_I \ \ (P_l > C_I) \end{array} \right. . \tag{543}
$$

On the formula (index[inf-exs]), convective rainfall intensity probability $f(P_c)$ It is derived from the following equation, which assumes an exponential distribution.

$$
f(P_c) = \frac{\kappa}{\overline{\overline{P_c}}} \exp\left[-\frac{\kappa}{\overline{\overline{P_c}}} P_c \right] , \tag{544}
$$

$$
\frac{R_I}{\kappa} = \int_{\widetilde{C_I}}^{\infty} (P_c - \widetilde{C_I}) f(P_c) d(P_c) . \tag{545}
$$

However, assuming that stratocumulus rainfall is uniformly infiltrating, the effective infiltration capacity of $\widetilde{C_I} = C_I - P_l$. κ is the percentage of convective rainfall area in the total grid area, It is a constant (0.6 by default).

When considering multi-layered soil moisture transfer, We can also consider the drainage from each layer proportional to the permeability coefficient.

3.7.8 Evaluating Albedo on land `MODULE:[LNDALB]`

The evaluation of albedo is basically based on a constant value given by an external source. There are two ways to give it.

1. give a distribution in a file
2. specify a value for each surface type m

For each wavelength band, we can give two components in the visible and near-infrared (The same values are used in the standard).

We can also consider the effects of surface wetness and solar radiation zenith angle as follows (Not considered in the standard).

$$\alpha = \alpha - f_w \alpha w_1 \ , \tag{546}$$

$$\alpha = \alpha + \Delta f_\zeta (1-\alpha)(1-\cos^2\zeta) \ . \tag{547}$$

Here, the wetness factor (f_w) and the zenith angle factor (f_ζ) are constants.

3.7.9 Evaluating roughness on land surface`MODULE:[LNDZO]`

The evaluation of roughness is basically based on a constant value given by an external source. There are two ways to give it.

1. give a distribution in a file
2. specify a value for each surface type m

The roughness z_{0H} for heat and the roughness z_{0E} for water vapor are If not given, the roughness to momentum is a constant multiple of z_{0M}. ($z_{0H} = z_{0E} = 0.1 z_{0M}$ by default)

3.7.10 Evaluating surface wetness on land`MODULE:[LNDBET]`

On land ice, β has a constant value of 1. For non-icy land surfaces, we can use several evaluation methods as follows.

1. using an externally given constant value. As a way of giving ,

    1. give a distribution in a file
    2. specify a value for each surface type m

There are two possibilities.

2. soil moisture calculated as a function of w.

Define the saturation degree $W \equiv w/w_s$, We give it as a function.

```
Function type 1.
```

The critical saturation is 1 if it exceeds the critical saturation value of W_c, and is linearly dependent below it.

$$\beta = \min\left(W/W_c, 1\right) \tag{548}$$

```
2. that depend nonlinearly on function type 2. $W$.
```

$$\beta = 1 - \exp\left[-3(W/W_c)^a\right] \tag{549}$$

In the following, we describe the different treatment of the sea surface from that of the land surface.

3.7.11 Calculating heat flux and heat capacity at sea level MODULE:[SEAFLX]

At the sea surface, the heat capacity varies depending on the presence of sea ice. Volumetric Specific Heat of Seawater $\tilde{C}_s$ and Using the volume specific heat of sea ice with $\tilde{C}_i$, h_i is used as the thickness of the sea ice,

$$
Cg_l = \begin{cases} \tilde{C}_s(zg_{l+1/2} - zg_{l-1/2})(h_i \leq zg_{l-1/2}) \\ \tilde{C}_s(zg_{l+1/2} - h_i) + \tilde{C}_i(h_i - zg_{l-1/2})(zg_{l-1/2} < h_i < zg_{l+1/2}) \\ \tilde{C}_i(zg_{l+1/2} - zg_{l-1/2})(h_i \geq zg_{l+1/2}) \end{cases} \quad (550)
$$

Even at sea level, the thermal conductivity is kept constant (depending on l).

$$
Fg_{l-1/2} = Ks_{l-1/2}(G_l - G_{l-1})/(zs_l - zs_{l-1}) \ , \quad (551)
$$

$$
\frac{\partial Fg_{l-1/2}}{\partial G_l} = -\frac{\partial Fg_{l-1/2}}{\partial G_l - 1} = Ks_{l-1/2}/(zs_l - zs_{l-1}) \ . \quad (552)
$$

However, in areas where there is sea ice, The temperature of the boundary between sea ice and seawater is set to T_i (= 271.15K), Let the thermal conductivity be the value of the sea ice.

$$
Fg_{l-1/2} = \begin{cases} Ks_{l-1/2}(G_l - G_{l-1})/(zs_l - zs_{l-1}) \ \ (h_i < zg_{l-3/2}) \\ Ks_{l-1/2}(G_l - T_i)/(zs_l - h_i) \ \ (zg_{l-3/2} \leq h_i < zg_{l-1/2}) \\ K_i(T_i - G_{l-1})/(h_i - zs_{l-1}) \ \ (zg_{l-1/2} \leq h_i < zg_{l+1/2}) \\ K_i(G_l - G_{l-1})(zs_l - zs_{l-1}) \ \ (h_i \geq zg_{l+1/2}) \end{cases} \quad (553)
$$

Heat fluxes in the oceans outside the sea ice area are significant because This is only the case with the oceanic mixed layer model.

3.7.12 Evaluating surface wetness at sea level MODULE:[SEABET]

The surface moisture content of the β used to evaluate evaporation is , The constant value of 1 for the sea surface and sea ice.

3.7.13 albedo and roughness at sea level

The albedo at the sea surface, which is not covered by sea ice, is in the radiation routine, Calculated at each wavelength as a function of atmospheric optical thickness and solar incidence angle `MODULE:[SSRFC]` .

The roughness of the ocean surface that is not covered by the ocean is determined in the surface flux routine, Calculated as a function of momentum flux `MODULE:[SEAZOF]` .

The albedo and roughness of the sea surface covered with sea ice are Give a constant value. `MODULE:[SEAALB, SEAZO]`. The current standard value is 0.7, albedo, The roughness is 1 $\times 10^{-3}$ m.

In the following, we describe the different treatment of the snow surface from that of the land surface.

3.7.14 Snow Heat Flux Correction `MODULE:[SNWFLX]`

The snow is treated thermally as the same layer as the first layer of the ground surface. That is, the heat capacity and thermal diffusivity of the first layer are The shape will be changed by the presence of snow.

The heat capacity is expressed as a simple sum of Let C_y be the specific heat per mass of snow and W_y be the mass per unit area of snow,

$$Cg_l = Cg'_l + C_y W_y \ . \tag{554}$$

However, Cg'_l is the heat capacity in the absence of snow.

The heat flux is defined as the hypothetical temperature at the snow-soil interface, which is set to G_I,

$$Fg_{1/2} = K_y(G_I - T_0)/h_y = Kg_{1/2}(G_1 - G_I)(zg_1 - zg_0) \ . \tag{555}$$

However, the depth of the snow cover in h_y is Let ρ_y be the density of snow, and $h_y = W_y/\rho_y$ be the density of snow. Eliminating G_I from the equation above,

$$Fg_{1/2} = \left[(K_y/h_y)^{-1} + (K_g/(zg_1 - zg_0))^{-1} \right]^{-1} (G_1 - T_0) \tag{556}$$

$$= \left[(K_y(G_1 - T_0)/h_y)^{-1} + (Fg_{1/2})^{-1} \right]^{-1} . \tag{557}$$

However, the $Fg'_{1/2}$ is the flux when there is no snow. Therefore, if this has already been calculated, By taking the harmonic mean of that and the snow only flux, Fluxes are required in the presence of snow. Also, the temperature differential coefficient of the fluxes $\frac{\partial Fg_{1/2}}{\partial G_1}$ and $\frac{\partial Fg_{1/2}}{\partial T_0}$ is similarly obtained by the harmonic mean of the temperature differential coefficients.

If there is more than a certain amount of snowfall , The temperature G_1 should be regarded as the temperature of the snowpack rather than the temperature of the soil. To account for such cases, in fact, the In the above formula, instead of h_y, $h_y/2$ is used, Furthermore, not only $F_{1/2}$, but also $F_{3/2}$ is changed by snow Handling.

$$Fg_{1/2} = \left[(K_y(G_1 - T_0)/(h_y/2))^{-1} + (Fg'_{1/2})^{-1} \right]^{-1} , \tag{558}$$

$$Fg_{3/2} = \left[(K_y(G_2 - G_1)/(h_y/2))^{-1} + (Fg'_{3/2})^{-1} \right]^{-1} . \tag{559}$$

3.7.15 Calculating snow sublimation `MODULE:[SNWSUB]`

Decrease the snowpack by the amount of sublimation flux.

$$\tilde{Wy} = Wy - Fq_1 \Delta t . \tag{560}$$

When the snowfall is fully sublimated, the missing water flux is evaporated from the soil. The energy balance of the earth's surface is assumed to be the same as if the surface moisture flux were all sublimated We need to correct for the soil temperature because it has been calculated.

$$\tilde{G}_1 = G_1 + L_M(Fq_1 \Delta t - Wy)/Cg_1 . \tag{561}$$

3.7.16 Calculating snowfall `MODULE:[SNWFLP]`

When precipitation arrives at the ground surface, it is judged whether it is solid (snow) or liquid (rain).

Atmosphere First Layer Wet Bulb Temperature Tw_1

$$Tw_1 = T_1 - L/Cp(q^* - q_1)/(1 + L/Cp\frac{\partial q^*}{\partial T}) \tag{562}$$

If the freezing point (Tw_1) is lower than the freezing point (Tm), it is assumed to be snow, and if it is higher than the freezing point (Tm), it is assumed to be rain. The reason why the wet-ball temperature is used is that the temperature of precipitation reaching the surface is This is to incorporate effects that depend on the likelihood of evaporation during the fall of precipitation.

In the case of snowfall, the snowpack is increased by the amount of snowfall.

$$\tilde{Wy} = Wy + Py\Delta t \ . \tag{563}$$

Py is a snowfall flux.

3.7.17 Snowmelt calculation `MODULE:[SNWMLP]`

If the surface energy balance (Δs) is positive, as a result of the calculation of the surface energy balance and/or Soil first layer in areas with snow cover (including snowpack) When the temperature of the soil is higher than the freezing point, the amount of snowmelt is calculated and the latent heat of melting is calculated. Make corrections.

If the soil temperature before correction is set to $\hat{G}_1$, If the snowmelt occurred to the extent that it would resolve the energy balance Snowmelt $\tilde{My}\Delta t$ and soil temperature $\tilde{G}_1$ are ,

When the $\hat{G}_1 \geq Tm$,

$$\tilde{My}\Delta t = (Cg_1(\hat{G}_1 - Tm) + \Delta s\Delta t)/L_M \ , \tag{564}$$

$$\tilde{G_1} = Tm\,. \tag{565}$$

When the $\hat{G}_1 < Tm$,

$$\tilde{My}\Delta t = \Delta s \Delta t/(L_M + Cp_I(Tm - \hat{G}_1))\ , \tag{566}$$

$$\tilde{G_1} = \hat{G}_1\ . \tag{567}$$

In the case of $\hat{G}_1 < Tm$, the temperature of the part of the snow that melts in the energy balance except for the snow is I'm assuming it doesn't change. L_M is the latent heat of melting, Tm is the freezing point, and Cp_I is the specific heat of ice.

The actual snowmelt and soil temperature are based on the current amount of snow and soil temperature in the case of full melting of the Wy,

$$My\Delta t = \begin{cases} \tilde{My}\Delta t(\tilde{My}\Delta t \leq Wy) \\ Wy(\tilde{My}\Delta t > Wy) \end{cases}\ , \tag{568}$$

$$G_1 = \begin{cases} \tilde{G_1}(\tilde{My}\Delta t \leq Wy) \\ \hat{G}_1 + (\Delta s \Delta t - L_M Wy - Cp_I Wy(Tm - \hat{G}_1))/(Cg_1 - Cp_I Wy)(\tilde{My}\Delta t > Wy) \end{cases} \tag{569}$$

3.7.18 Calculating Snow Surface RunoffMODULE:[SNWROF]

If there is a snow Wy, prior to calculating the land surface runoff The runoff due to snow accumulation was evaluated as follows, Exclude moisture from the surface input. Snowmelt water (My) is added to the surface water input here.

$$Is = \begin{cases} 1 - Wy/Wy_{Ci}\ \ (Wy < Wy_{Ci}) \\ 0\ \ (Wy \geq Wy_{Ci}) \end{cases}\ , \tag{570}$$

$$\widetilde{P_c} = IsP_c \ , \tag{571}$$

$$\widetilde{P_l} = Is(P_c + My) \ , \tag{572}$$

$$R_S = (1 - Is)(P_c + P_l + My) \ . \tag{573}$$

where Is is the surface infiltration rate due to snow cover. The standard value of critical snowpack for infiltration, Wy_{Ci}, is 200 kg/m^2.

3.7.19 Evaluating albedo on snow-covered surfaces`MODULE:[SNWALB]`

If you have a snow Wy , The ratio of snow cover is considered to be proportional to the square root of the snowpack, Albedo approaches the snow value αs in proportion to the square root of the snowpack (The critical value of Wy_C is 200 kg/m^2 in standard).

$$\alpha = \begin{cases} \alpha' + (\alpha s - \alpha')\sqrt{Wy/Wy_C} \ \ (Wy < Wy_C) \\ \alpha s (Wy \geq Wy_C) \end{cases} . \tag{574}$$

Also, when melting occurs and the snowpack is wet, the snow albedo The smaller effect is considered as follows.

$$\alpha s = \begin{cases} \alpha s_d \ \ (T_0 \leq Td) \\ \alpha s_d - (\alpha s_m - \alpha s_d)(T_0 - Td)/(Td - Tm) \ \ (Td < T_0 \leq Tm) \\ \alpha s_m \ \ (T_0 > Tm) \end{cases} \tag{575}$$

where T_0 is the surface temperature. Dry Snow Albedo αs_d, Wet Snow Albedo αs_m The standard values for The critical temperatures (Td and Tm) are 258.15 and 273.15, respectively.

Furthermore, as in the absence of snow, we can take into account the effect of the zenith angle dependence of solar radiation (Not considered in the standard).

3.7.20 Evaluating Surface Roughness on Snow Covered SurfacesMODULE:[SNWZ0]

If you have a snow Wy , The ratio of snow cover is considered to be proportional to the square root of the snowpack, Surface roughness approaches snow roughness in proportion to the square root of the snowpack. (The critical value, Wy_C, is 200 kg/m^2 in standard).

$$z_0 = \begin{cases} z_0' + (z_0 s - z_0')\sqrt{Wy/Wy_C} & (Wy < Wy_C) \\ z_0 s(Wy \geq Wy_C) \end{cases} . \tag{576}$$

The standard values for snow roughness are for momentum, temperature and water vapor, respectively. 10^{-2}, 10^{-3}, 10^{-3}.

3.7.21 Evaluating Surface Wetness on Snow Covered SurfacesMODULE:[SNWBET]

If you have a snow Wy , The ratio of snow cover is considered to be proportional to the square root of the snowpack, Surface wetness approaches snow wetness 1 in proportion to the square root of the snowpack (The critical value of Wy_C is 200 kg/m^2 in standard).

$$\beta = \begin{cases} \beta' + (1 - \beta')\sqrt{Wy/Wy_C} & (Wy < Wy_C) \\ 1(Wy \geq Wy_C) \end{cases} . \tag{577}$$

In the following section, we describe the optional surface processes.

3.7.22 Calculating the freezing process MODULE:[LNDFRZ]

To use this option, you must use the The number of vertical layers in the ground and the level of each layer must be equal.

After calculating the ground temperature by thermal diffusion,

- If the ground temperature is lower than the freezing point and liquid moisture is present, the freezing of moisture will be
- If the ground temperature is higher than the freezing point and solid moisture is present, the water will melt.

Calculate.

Assuming that the ice content of the l layer is w_{Fl}, the freezing water (Δw_{Fl}) is

$$\Delta w_{Fl} = \begin{cases} -w_{Fl} \;\; (\Delta_0 w_{Fl} \leq -w_{Fl}) \\ \Delta_0 w_{Fl} \;\; (-w_{Fl} < \Delta_0 w_{Fl} \leq w_l - w_{Fl}) \\ w_l - w_{Fl} \;\; (\Delta_0 w_{Fl} > w_{Fl}) \end{cases} . \tag{578}$$

where the negative Δw_{Fl} represents the water to be melted. The $\Delta_0 w_{Fl}$ will freeze/thaw until the soil temperature reaches the freezing point. This is the value of Δw_{Fl} if it were to occur, given by

$$\Delta_0 w_{Fl} = Cg_l (Tm - G_l)/(L_M Cw_l) . \tag{579}$$

Tm has an ice point of 273.16K.

The change in soil temperature due to the latent heat of the soil moisture phase change is given by

$$\tilde{G}_l = G_l + L_M Cw_l \Delta w_{Fl}/Cg_l . \tag{580}$$

3.7.23 oceanic mixed layer model `MODULE:[SEAFRZ]`

In the oceanic mixed layer model , By solving for the heat balance of the oceans, the temperature of the oceans and Determine the time variation of sea ice thickness.

Multi-layered models are possible, but, Here we will take a single layer model of thickness D as an example. The predictor variables are temperature (G) and sea ice thickness (h_I).

First, determine the heat capacity and surface flux of the ocean. `MODULE:[SEAFLX]` The heat capacity of the oceans is , The specific heat of water C_w, the specific heat of ice C_I, and the density of water and ice as ρ_w,

$$C_s = C_I \rho_w h_I + C_w \rho_w (D - h_I) \tag{581}$$

In the absence of sea ice, the heat transfer flux is

$$Fs_{1/2} = K_s \frac{G - T_0}{d/2} \tag{582}$$

On the other hand, if there is sea ice,

$$Fs_{1/2} = K_I \frac{T_I - T_0}{h_I} \tag{583}$$

where T_I is the freezing temperature of sea ice at 271.35 K. where T_I is the freezing temperature of sea ice at 271.35 K.

Heat flux in the $z = D$ is usually zero while the $Fs_{1+1/2}$ is usually zero, It can be given from the outside. It is used in the case of flux correction considering oceanic heat transport.

2. using this heat flux and heat capacity As with the land surface, determine the change in temperature (G).

The melting of the sea ice surface is treated in the same way as snow.
`MODULE:[SEAFLX]`
First, I'll set the melting value, $\tilde{M}_I$, to When the $G \geq T_I$,

$$M_I = \frac{C_s(G - T_I) + \Delta s \Delta t)}{\rho_w (C_w - C_I) T_I} \tag{584}$$

When the $G < T_I$,

$$M_I = \frac{\Delta s \Delta t)}{\rho_w (C_w - C_I) G} \tag{585}$$

Estimate in , However, if it has melted completely, set the value to $M_I = h_I$ and compensate for the heat.

$$G \leftarrow G + \frac{\Delta s \Delta t - \rho_w (C_w - C_I) h_I G}{C_w + \rho_w (C_w - C_I) h_I} \tag{586}$$

Varying the thickness of the ice,

$$h_I \leftarrow h_I - M_I \tag{587}$$

Then, vary the heat capacity correspondingly.

$$C_s = C_s + \rho_w (C_w - C_I) h_I \tag{588}$$

The next step is to consider the growth process from the bottom of the sea ice.

```
1. when there is no sea ice ($h_I=0$)
```

When the $G < T_I$,

$$\tilde{f}_I = \frac{C_s(T_I - G) - \Delta s \Delta t)}{\rho_w (C_w - C_I) T_I} \tag{589}$$

When the $G \geq T_I$,

$$\tilde{f}_I = \frac{-\Delta s \Delta t)}{\rho_w (C_w - C_I) G} \tag{590}$$

Estimate in . When it is positive, sea ice is produced. Here, Δs is T_0 is $T_I = 271.35$ K or less Note that it is positive when the

$$h_I \leftarrow f_I \tag{591}$$

$$C_s \leftarrow C_s - \rho_w (C_w - C_I) f_I \tag{592}$$

$$G \leftarrow \max(G, T_I) \tag{593}$$

```
2. when the sea ice is already present ($h_I>0$)
```

Heat fluxes from the seawater beneath the sea ice to the bottom of the sea ice.

$$F_b = K_s \frac{G - T_I}{D/2 - h_I} \tag{594}$$

Estimate in . The difference between F_b and the heat flux from the ocean to the top of $Fs_{1/2}$ Used for the growth or melting of sea ice.

$$f_I = \frac{Fs_{1/2} - F_b}{\rho_w (C_w - C_I) G} \Delta t \tag{595}$$

So..,

$$h_I \leftarrow h_I + f_I \tag{596}$$
$$G \leftarrow G \frac{C_s}{C_s - \rho_w (C_w - C_I) f_I} \tag{597}$$
$$C_s \leftarrow C_s - \rho_w (C_w - C_I) f_I \tag{598}$$

5. give an external reference temperature of G_{ref} You can nudging it.

$$G \leftarrow G + \frac{G_{ref} - G}{\tau} \Delta t \tag{599}$$

This is a heat flux

$$F_n = C_s \frac{G_{ref} - G}{\tau} \tag{600}$$

The equivalent of giving a .

To do flux correction, Provide the appropriate τ and perform nudging, Remember the F_n, You can give it to me as $Fs_{1+1/2}$.

3.8 Solving the diffuse balance equation for atmospheric and surface systems

3.8.1 Basic Solutions.

Radiative, vertical diffusion, ground boundary layer and surface processes are Some terms are treated as implicit in the Compute the time-varying term and do time integration at the end. As a time-varying term for the vector quantity q, The term $\mathcal{A}$ for the Euler method and the term $\mathcal{B}$ for the implicit method are considered separately.

$$\mathbf{q}^{+} = \mathbf{q} + 2\Delta t \mathcal{A}(\mathbf{q}) + 2\Delta t \mathcal{B}(\mathbf{q}^{+}) \ . \tag{601}$$

It is difficult to solve this in the general case, but, It can be solved by linearizing B in an approximate manner.

$$\mathcal{B}(\mathbf{q}^{+}) \simeq \mathcal{B}(\mathbf{q}) + B(\mathbf{q}^{+} - \mathbf{q}) \tag{602}$$

using the matrix B as Here,

$$B_{ij} = \frac{\partial \mathcal{B}_i}{\partial q_j} \tag{603}$$

It is. So.., $\Delta \mathbf{q} \equiv \mathbf{q}^{+} - \mathbf{q}$ And then you can write,

$$(I - 2\Delta t B)\Delta \mathbf{q} = 2\Delta t \left(\mathcal{A}(\mathbf{q}) + \mathcal{B}(\mathbf{q})\right) \ . \tag{604}$$

It is easy to solve in principle by matrix operations.

3.8.2 Fundamental Equations.

Radiation, vertical diffusion, ground boundary layer and surface processes The equations are basically expressed as follows.

$$\frac{\partial u}{\partial t} = -g\frac{\partial}{\partial p}F_u \ , \tag{605}$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial}{\partial p}F_v \ , \tag{606}$$

$$c_p\frac{\partial T}{\partial t} = -g\frac{\partial}{\partial p}(F_T + F_R) \ , \tag{607}$$

$$\frac{\partial q}{\partial t} = -g\frac{\partial}{\partial p}F_q \ , \tag{608}$$

$$C_g\frac{\partial G}{\partial t} = -\frac{\partial}{\partial z}F_g \ . \tag{609}$$

where F_u, F_v, F_T, F_q are By vertical diffusion, u, v, c_pT, q, respectively is the vertical upward flux density of Also, the F_R is a radiant It is a vertical upward energy flux density.

The atmosphere is discretized in the $\sigma = p/p_S$ coordinate system. Wind speed, temperature, etc. are defined in layer σ_k. The flux is defined by the layer boundary $\sigma_{k-1/2}$. k increases from lower to higher levels. Also, $\sigma_{1/2} = 1$, This is the $\sigma_k \simeq (\sigma_{k-1/2}+\sigma_{k+1/2})/2$. σ The coordinates are only available when we consider a one-dimensional vertical process, It can be considered to be the same as p coordinates except for the difference in the constant (p_S) times. Here,

$$\Delta\sigma_k = \sigma_{k-1/2} - \sigma_{k+1/2} \ , \tag{610}$$

$$\Delta m_k = \frac{1}{g}\Delta p_k = \frac{p_S}{g}(\sigma_{k-1/2} - \sigma_{k+1/2}) \tag{611}$$

And write .

3.8.3 implicit time difference

For terms that can be linearized, such as the vertical diffusion term, we use the implicit method. Diffusion coefficients and other factors also depend on the forecast variables, The coefficients are only calculated first, not iteratively. However, the treatment of the time step is devised to improve the stability (see below).

For example, the discretized equation of u ([[u-eq.orig]](#u-eq.orig)) is

$$(u_k^{m+1} - u_k^m)/\Delta t = (Fu_{k-1/2}^{m+1} - Fu_{k+1/2}^{m+1})/\Delta m_k \tag{612}$$

Here, m is a time step. Since $Fu_{k-1/2}$ etc. is a function of u_k, its dependency is linearized and the

$$Fu_{k-1/2}^{m+1} = Fu_{k-1/2}^m + \sum_{k'=1}^{K} \frac{\partial Fu_{k-1/2}^m}{\partial u_{k'}} (u_{k'}^{m+1} - u_{k'}^m) \tag{613}$$

\[u-flux.next]

Thus, if you put $\delta u_k \equiv (u_k^{m+1} - u_k^m)/\Delta t$,

$$\Delta m_k \delta u_k = \left(Fu_{k-1/2}^m + \sum_{k'=1}^{K} \frac{\partial Fu_{k-1/2}^m}{\partial u_{k'}} \Delta t \delta u_{k'} - Fu_{k+1/2}^m - \sum_{k'=1}^{K} \frac{\partial Fu_{k+1/2}^m}{\partial u_{k'}} \Delta t \delta u_{k'} \right) \tag{614}$$

Namely,

$$\Delta m_k \delta u_k - \sum_{k'=1}^{K} \left(\frac{\partial Fu_{k-1/2}^m}{\partial u_{k'}} - \frac{\partial Fu_{k+1/2}^m}{\partial u_{k'}} \right) \Delta t \delta u_{k'} = Fu_{k-1/2}^m - Fu_{k+1/2}^m \tag{615}$$

It can be written in the following matrix form

$$\sum_{k'=1}^{K} M_{k,k'}^u \delta u_k' = Fu_{k-1/2}^m - Fu_{k+1/2}^m \tag{616}$$

$$M_{k,k'}^u \equiv \Delta m_k \delta_{k,k'} - \left(\frac{\partial Fu_{k-1/2}^m}{\partial u_{k'}} - \frac{\partial Fu_{k+1/2}^m}{\partial u_{k'}} \right) \Delta t \tag{617}$$

[u-matrix]

This can be solved by LU decomposition or some other method. Normally, $M_{k,k'}^u$ is easy to solve since it is a triple diagonal. After solving it, ([u-flux.next]), you can use We calculate the consistent flux to this method. The same is true for v.

3.8.4 implicit time difference coupling

Temperature, specific humidity, and ground temperature are not as simple as those in the previous section.

$$c_p \Delta m_k \delta T_k - \sum_{k'=0}^{K} \left(\frac{\partial F\theta^m_{k-1/2}}{\partial T_{k'}} - \frac{\partial F\theta^m_{k+1/2}}{\partial T_{k'}} \right) \Delta t \delta T_{k'} - \sum_{k'=0}^{K} \left(\frac{\partial FR^m_{k-1/2}}{\partial T_{k'}} - \frac{\partial FR^m_{k+1/2}}{\partial T_{k'}} \right) \Delta t \delta T_{k'} \tag{618}$$

$$= (F\theta^m_{k-1/2} - F\theta^m_{k+1/2}) + (FR^m_{k-1/2} - FR^m_{k+1/2}) \tag{619}$$

[deq-theta]

$$\Delta m_k \delta q_k - \sum_{k'=0}^{K} \left(\frac{\partial Fq^m_{k-1/2}}{\partial q_{k'}} - \frac{\partial Fq^m_{k+1/2}}{\partial q_{k'}} \right) \Delta t \delta q_{k'} = (Fq^m_{k-1/2} - Fq^m_{k+1/2}) \tag{620}$$

[deq-q]

$$Cg_l \Delta z_l \delta G_l + \sum_{l'=0}^{L} \left(\frac{\partial Fg^m_{l-1/2}}{\partial G_{l'}} - \frac{\partial Fg^m_{l+1/2}}{\partial G_{l'}} \right) \Delta t \delta T_{k'} = -(Fg^m_{l-1/2} - Fg^m_{l+1/2}) \tag{621}$$

[deq-g]

Here, $\sum_{k'}$ and $\sum_{l'}$ in the above equations are Note that I took this from $k'=0$, $l'=0$. because, This is because the flux at the surface is as follows

$$F\theta_{1/2} = c_p C_H |\mathbf{v}_{1/2}| (\theta_0 - \theta_1) \tag{622}$$

$$Fq_{1/2} = \beta C_E |\mathbf{v}_{1/2}| (q_0 - q_1) \tag{623}$$

$$Fg_{1/2} = K_g (G_1 - G_0)/z_1 \tag{624}$$

Where the surface skin temperature is set to T_0, $\theta_0 = T_0$, $q_0 = q^*(T_0)$ (saturated specific humidity), $G_0 = T_0$. They all depend on the T_0. Also, the value of the FR_k depends on the T_0 for all k values.

(as with [[u-matrix]](#u-matrix)), using the matrices M^θ, M^q, M^g ([deq-theta] (#deq-theta)), ([deq-q] (#deq-q)), ([deq-g\] (#deq-g)), and ([deq-g\] (#deq-g)) are rewritten, For $k \geq 2$ (for θ, q) or $l \geq 1$ (for G),

$$\sum_{k'=1}^{K} M^\theta_{k,k'} \delta T_{k'} = (F\theta^m_{k-1/2} - F\theta^m_{k+1/2}) + (FR^m_{k-1/2} - FR^m_{k+1/2}) \tag{625}$$

$$+ \left(\frac{\partial FR^m_{k-1/2}}{\partial T_0} - \frac{\partial FR^m_{k+1/2}}{\partial T_0} \right) \Delta t \delta T_0 \ , \tag{626}$$

\[combo-theta2\blazer]

$$\sum_{k'=1}^{K} M^q_{k,k'} \delta q_{k'} = (Fq^m_{k-1/2} - Fq^m_{k+1/2}) \ , \tag{627}$$

[combo-q2]

$$\sum_{l'=0}^{L} M^g_{l,l'} \delta G_{l'} = -(Fg^m_{l-1/2} - Fg^m_{l+1/2}) \ . \tag{628}$$

[combo-g2]

However,

$$M^\theta_{k,k'} \equiv c_p \Delta m_k \delta_{k,k'} - \left(\frac{\partial F\theta^m_{k-1/2}}{\partial T_{k'}} - \frac{\partial F\theta^m_{k+1/2}}{\partial T_{k'}} \right) \Delta t - \left(\frac{\partial FR^m_{k-1/2}}{\partial T_{k'}} - \frac{\partial FR^m_{k+1/2}}{\partial T_{k'}} \right) \Delta t, \tag{629}$$

$$M^q_{k,k'} \equiv \Delta m_k \delta_{k,k'} - \left(\frac{\partial Fq^m_{k-1/2}}{\partial q_{k'}} - \frac{\partial Fq^m_{k+1/2}}{\partial q_{k'}} \right) \Delta t \ , \tag{630}$$

$$M^g_{l,l'} \equiv Cg_l \Delta z_l \delta_{l,l'} - \left(\frac{\partial Fg^m_{l-1/2}}{\partial G_{l'}} - \frac{\partial Fg^m_{l+1/2}}{\partial G_{l'}} \right) \Delta t \ . \tag{631}$$

In case of $k = 1$ (for θ, q) or $l = 0$ (for G),

$$\sum_{k'=1}^{K} M^\theta_{1,k'} \delta T_{k'} + \frac{\partial F\theta^m_{1/2}}{\partial T_1} \Delta t \delta T_1 = (F\theta^m_{1/2} - F\theta^m_{3/2}) + (FR^m_{1/2} - FR^m_{3/2}) \tag{632}$$

$$+ \frac{\partial F\theta^m_{1/2}}{\partial T_0} \Delta t \delta T_0 \tag{633}$$

$$+ \left(\frac{\partial FR^m_{1/2}}{\partial T_0} - \frac{\partial FR^m_{3/2}}{\partial T_0} \right) \Delta t \delta T_0 \ , \tag{634}$$

[comb-theta]

$$\sum_{k'=1}^{K} M^q_{1,k'} \delta q_{k'} - \frac{\partial Fq^m_{1/2}}{\partial q_1} \Delta t \delta q_1 = (Fq^m_{1/2} - Fq^m_{3/2}) + \frac{\partial Fq^m_{1/2}}{\partial T_0} \Delta t \delta T_0 \ , \tag{635}$$

\centric.

$$\sum_{l'=1}^{L} M^g_{0,l'} \delta G_{l'} + \left(\frac{\partial F\theta^m_{1/2}}{\partial T_0} + L \frac{\partial Fq^m_{1/2}}{\partial T_0} + \frac{\partial FR^m_{1/2}}{\partial T_0} - \frac{\partial Fg^m_{1/2}}{\partial T_0} \right) \Delta t \delta T_0 \tag{636}$$

$$= -F\theta^m - LFq^m - FR^m + Fg^m_{1/2} \tag{637}$$

$$-\frac{\partial F\theta^m_{1/2}}{\partial T_1} \Delta t \delta T_1 - L \frac{\partial Fq^m_{1/2}}{\partial q_1} \Delta t \delta q_1 - \frac{\partial FR^m_{1/2}}{\partial T_1} \Delta t \delta T_1 + \frac{\partial Fg^m_{1/2}}{\partial G_1} \Delta t \delta G_1 \tag{638}$$

[combo-g]

However,

$$M^\theta_{1,k'} \equiv c_p \Delta m_1 \delta_{1,k'} - \left(-\frac{\partial F\theta^m_{3/2}}{\partial T_{k'}} \right) \Delta t - \left(\frac{\partial FR^m_{1/2}}{\partial T_{k'}} - \frac{\partial FR^m_{3/2}}{\partial T_{k'}} \right) \Delta t \ , \tag{639}$$

$$
M^q_{1,k'} \equiv \Delta m_1 \delta_{1,k'} - \left(- \frac{\partial F q^m_{3/2}}{\partial q_{k'}} \right) \Delta t \ , \tag{640}
$$

$$
M^g_{0,l'} \equiv \left(- \frac{\partial F g^m_{1/2}}{\partial G_{l'}} \right) \Delta t \ . \tag{641}
$$

However, ([comb-g]](#comb-g)), the balance condition of the ground surface

$$
F\theta^{m+1} + LFq^{m+1} + FR^{m+1} - Fg^{m+1} = 0 \tag{642}
$$

as the case of $l = 0$ in the soil temperature equation, (Note that it is not included in the formula of deq-g].

These, ([comb-theta2] (#comb-theta2)), ([comb-q2] (#comb-q2)), ([comb-g2] (#comb-g2)), ([comb-theta](#comb-theta)), (comb-q), ([comb-g\lopencomb-g]) for the $2K + L + 1$ unknowns, There are equations of equality that can be solved. In practice, the LU decomposition can be used to solve the problem.

Once you're untied, ([u-flux.next]](#u-flux.next)) as well as , Consistent flux should be sought.

3.8.5 Solving the Coupling Formula for Time Difference

([comb-theta]](#comb-theta)), etc., can be written as follows.

$$
\sum_{k'=1}^{K} (M_{k,k'} + \delta_{1,k}\delta_{1,k'}\alpha) = F_k + \delta_{1,k}(F_s + \gamma T_0) \tag{643}
$$

Where, F_s, α, γ The term in Section 3.1 is a term associated with surface flux, The others are terms associated with vertical diffusion. Here, if we reverse the top and bottom and represent it as a matrix, we get the following.

$$\begin{pmatrix} M_{KK} \cdots M_{K1} \\ \vdots\vdots \\ M_{1K} \cdots M_{11} + \alpha \end{pmatrix} \begin{pmatrix} T_K \\ \vdots \\ T_1 \end{pmatrix} = \begin{pmatrix} F_K \\ \vdots \\ F_1 + F_s + \gamma T_0 \end{pmatrix} \tag{644}$$

For the sake of brevity, we shall now refer to this document as $K = 3$. For the sake of notation simplicity, we will now refer to it as $K = 3$. You can't lose the.

$$\begin{pmatrix} M_{33}M_{32}M_{31} \\ M_{23}M_{22}M_{21} \\ M_{13}M_{12}M_{11} + \alpha \end{pmatrix} \begin{pmatrix} T_3 \\ T_2 \\ T_1 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_2 \\ F_1 + F_s + \gamma T_0 \end{pmatrix} \tag{645}$$

Here, The expression for $F_s = 0, \alpha = 0, \gamma = 0$, (This corresponds to the case where flux replacement at the surface is not considered.) by LU decomposition.

$$\begin{pmatrix} M_{33}M_{32}M_{31} \\ M_{23}M_{22}M_{21} \\ M_{13}M_{12}M_{11} \end{pmatrix} \begin{pmatrix} T'_3 \\ T'_2 \\ T'_1 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_2 \\ F_1 \end{pmatrix} \tag{646}$$

[summe-0]

LU. Take it apart,

$$\begin{pmatrix} 100 \\ L_{23}10 \\ L_{13}L_{12}1 \end{pmatrix} \begin{pmatrix} U_{33}U_{32}U_{31} \\ 0U_{22}U_{21} \\ 00U_{11} \end{pmatrix} \begin{pmatrix} T'_3 \\ T'_2 \\ T'_1 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_2 \\ F_1 \end{pmatrix} \tag{647}$$

Now..,

$$\begin{pmatrix} 100 \\ L_{23}10 \\ L_{13}L_{12}1 \end{pmatrix} \begin{pmatrix} f'_3 \\ f'_2 \\ f'_1 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_2 \\ F_1 \end{pmatrix} \tag{648}$$

[solve-z]

for f' (which can be easily solved by starting from $f'_3 = F_3$), And then..,

$$\begin{pmatrix} U_{33}U_{32}U_{31} \\ 0U_{22}U_{21} \\ 00U_{11} \end{pmatrix} \begin{pmatrix} T'_3 \\ T'_2 \\ T'_1 \end{pmatrix} = \begin{pmatrix} f'_3 \\ f'_2 \\ f'_1 \end{pmatrix} \tag{649}$$

for f', starting from $x'_1 = z'_1/U_{11}$, and solving in sequence.
For $\alpha \neq 0, \gamma \neq 0$, the LU decomposition is

$$\begin{pmatrix} 100 \\ L_{23}10 \\ L_{13}L_{12}1 \end{pmatrix} \begin{pmatrix} U_{33}U_{32}U_{31} \\ 0U_{22}U_{21} \\ 00U_{11} + \alpha \end{pmatrix} \begin{pmatrix} T_3 \\ T_2 \\ T_1 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_2 \\ F_1 + F_s + \gamma T_0 \end{pmatrix} \tag{650}$$

Now..,

$$\begin{pmatrix} 100 \\ L_{23}10 \\ L_{13}L_{12}1 \end{pmatrix} \begin{pmatrix} f_3 \\ f_2 \\ f_1 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_2 \\ F_1 + F_s + \gamma T_0 \end{pmatrix} \tag{651}$$

However, comparing this with ([solve-z]), we see the following relationship.

$$\begin{pmatrix} f_3 \\ f_2 \\ f_1 \end{pmatrix} = \begin{pmatrix} f'_3 \\ f'_2 \\ f'_1 + F_s + \gamma T_0 \end{pmatrix} \tag{652}$$

With this,

$$\begin{pmatrix} U_{33}U_{32}U_{31} \\ 0U_{22}U_{21} \\ 00U_{11} + \alpha \end{pmatrix} \begin{pmatrix} T_3 \\ T_2 \\ T_1 \end{pmatrix} = \begin{pmatrix} f'_3 \\ f'_2 \\ f'_1 + F_s + \gamma x_0 \end{pmatrix} \tag{653}$$

[solve-x]

The result is That is, ,

$$(U_{11} + \alpha)T_1 = f'_1 + F_s + \gamma T_0 \tag{654}$$

[solve-1]

where, $U_{k,k'}$ and, f'_k are,

In other words, without considering the surface flux term Note that this can be obtained by performing LU decomposition. The physical meaning of these terms is , During the flux exchange process with the ground surface, The entire atmosphere has a heat capacity of U_{11}, Flux (f'_1) from the top Indicates that it can be regarded as one layer to be supplied.

([comb-theta2]](#comb-theta2)) and (comb-theta]), ([combo-q2](#comb-q2)) and (combo-q\\clean}), ([comb-g2](#comb-g2)) and (comb-g) (the formula corresponding to [[solve-1 (654)

](#solve-1)) is obtained and is as follows

$$(U_{11}^T + \alpha^T)\delta T_1 - \gamma^T \delta T_0 = f_1^T + F\theta_{1/2} \tag{655}$$

$$(U_{11}^q + \alpha^q)\delta q_1 - \gamma^q \delta T_0 = f_1^q + Fq_{1/2}^P \tag{656}$$

$$(U_{00}^g + \alpha^g)\delta T_0 - \gamma^{g1}\delta T_1 - \gamma^{g2}\delta q_1 = f_0^g - F\theta_{1/2} - LFq_{1/2} \tag{657}$$

Therefore, if we concatenate the three equations above, we get We can solve for the unknown variables $\delta T_1, \delta q_1, \delta T_0$. If we can solve these problems, we can then ([[solve-x]](#solve-x)) can be solved sequentially as x_2, x_3. Afterwards, the consistuous flux is applied to the obtained temperature

$$F\theta_c = F\theta_{1/2} + \gamma^T \delta T_0 - \alpha^T \delta T_1 \tag{658}$$
$$Fq_c^P = Fq_{1/2}^P + \gamma^q \delta T_0 - \alpha^q \delta q_1 \tag{659}$$

Calculate as. Here, we show the case where M is a general matrix, It is even simpler since it is actually a triple diagonal matrix.

During the program, For atmospheric parts in MODULE:[VFTND1(pimtx.F)], MODULE:[GNDHT1(pggnd.F)]for the underground part, the first half of the LU decomposition method. (where f_k is obtained), InMODULE:[SLVSFC(pgslv.F)], solve the equation of 3×3, Seeking $\delta q_1, \delta G_1, \delta T_0$. Then, inMODULE:[GNDHT2(pggnd.F) second half of the LU decomposition method is performed and the rate of change of temperature in the ground is solved, Correct the fluxes so that the balance is matched. Also, inMODULE:[VFTND2(pimtx.F)], for the atmosphere Solving the rate of temperature change, Fluxes are corrected withMODULE:[FLXCOR(pimtx.F)]‘.

3.8.6 Combined expression for time difference

The coupling formula for finding $\delta T_1, \delta q_1, \delta T_0$ is , Solve three times under different conditions as follows.

Solve for surface wetness β as 1. Surface temperature is a variable.

2. the surface wetness obtained by MODULE:[GNDBET]. Surface temperature is a variable .

3. the surface wetness obtained by MODULE:[GNDBET]. In case of snowmelt, the surface temperature is fixed at the freezing point.

The first calculation is performed to estimate the possible evaporation rate, Fq^P. (When the surface wetness is small, the energy balance of the model indicates that Using the obtained Fq, the possible evaporation rate can be calculated by $\widetilde{Fq^P} = Fq/\beta$ diagnosed as, would result in unrealistically large values). Possible evaporation rate is ,

$$Fq_c^P = Fq_{1/2}^P + \frac{\partial Fq_{1/2}^P}{\partial q_1}\delta q_1 2\Delta t + \frac{\partial Fq_{1/2}^P}{\partial T_0}\delta T_0 2\Delta t \tag{660}$$

That would be. The subscript c means after correction, This is a consistent flux to the obtained temperature etc.

In the second and subsequent calculations ,

1. to the amount of possible evaporation found in the first calculation Surface wetness (evaporation efficiency) β multiplied by the amount of evaporation Fq_1.

$$Fq = \beta Fq^P \tag{661}$$

2. evaporation quantity Fq_1 is

$$\beta\rho C_E|\mathbf{v}|(q_*(T_0) - q) \tag{662}$$

As required by , Once again, rebalancing the energy.

Two methods of calculating the amount of evaporation can be used (The standard uses the method in 1.). The third calculation is performed during snow and ice melt and sea ice formation in the mixed layer ocean. This is done in order to fix the surface temperature at the freezing point, for example, to unbalance the energy. In this case, the amount of energy used for the phase change of water, such as snowmelt, is diagnostically determined, It will be used later to calculate the amount of snow melt, etc.

The concrete form of the coupling formula is as follows.

$$\begin{pmatrix} U_{11}^T - \frac{\partial F\theta_{1/2}}{\partial T_1} 2\Delta t 0 - \left(\frac{\partial F\theta_{1/2}}{\partial T_0} + \frac{\partial FR_{1/2}}{\partial T_0} \right) 2\Delta t \\ 0 U_{11}^q - \beta \frac{\partial Fq_{1/2}^P}{\partial q_1} \Delta t - \beta \frac{\partial Fq_{1/2}^P}{\partial T_0} \Delta t \\ \frac{\partial F\theta_{1/2}}{\partial T_1} \Delta t L\beta \frac{\partial Fq_{1/2}}{\partial q_1} \Delta t U_{00}^g + \left(\frac{\partial F\theta_{1/2}}{\partial T_0} + L\beta \frac{\partial Fq_{1/2}}{\partial T_0} + \frac{\partial FR_{1/2}}{\partial T_0} \right) 2\Delta t \end{pmatrix} \begin{pmatrix} \delta T_1 \\ \delta q_1 \\ \delta T_0 \end{pmatrix} \tag{663}$$

$$= \begin{pmatrix} f_1^T + F\theta_{1/2} \\ f_1^q + \beta Fq_{1/2}^P \\ f_0^g - F\theta_{1/2} - L\beta Fq_{1/2}^P \end{pmatrix} \tag{664}$$

\brain[combin-eq]

Here, $U_{11}^T, U_{11}^q, U_{00}^g, U_{11}^T, U_{11}^q, U_{00}^g$ and $f_1^T, f_1^q, f_0^g, f_1^T, f_1^q, f_0^g$ are, The components of the matrices and vectors obtained by doing the first half of the LU decomposition method. When the ground is covered with snow or ice, instead of the latent heat L Using the Latent Heat of Sublimation $Ls = L + L_M$ L_M is the latent heat of melting of water. However, in the second calculation, If the first method is used as an estimate of evaporation, we get the following.

$$\begin{pmatrix} U_{11}^T - \frac{\partial F\theta_{1/2}}{\partial T_1} 2\Delta t 0 - \left(\frac{\partial F\theta_{1/2}}{\partial T_0} + \frac{\partial FR_{1/2}}{\partial T_0} \right) 2\Delta t \\ 0 U_{11}^q - \beta \frac{\partial F q_{1/2}^P}{\partial q_1} \Delta t \\ \frac{\partial F\theta_{1/2}}{\partial T_1} \Delta t U_{00}^g + \left(\frac{\partial F\theta_{1/2}}{\partial T_0} + \frac{\partial FR_{1/2}}{\partial T_0} \right) 2\Delta t \end{pmatrix} \begin{pmatrix} \delta T_1 \\ \delta q_1 \\ \delta T_0 \end{pmatrix} \tag{665}$$

$$= \begin{pmatrix} f_1^T + F\theta_{1/2} \\ f_1^q + \beta F q_c^P \\ f_0^g - F\theta_{1/2} - L\beta F q^P 0_c \end{pmatrix} . \tag{666}$$

In the third calculation, the concatenation equation for a fixed surface temperature is

$$\begin{pmatrix} U_{11}^T - \frac{\partial F\theta_{1/2}}{\partial T_1} 2\Delta t 0 \\ 0 U_{11}^q - \frac{\partial F q_{1/2}}{\partial q_1} 2\Delta t \end{pmatrix} \begin{pmatrix} \delta T_1 \\ \delta q_1 \end{pmatrix} \tag{667}$$

$$= \begin{pmatrix} f_1^T + +F\theta_{1/2} + \left(\frac{\partial F\theta_{1/2}}{\partial T_0} + \frac{\partial FR_{1/2}}{\partial T_0} \right) \delta_0 T_0 2\Delta t \\ f_1^q + \beta F q_{1/2}^P + \frac{\partial F q_{1/2}}{\partial T_0} \delta_0 T_0 2\Delta t \end{pmatrix} . \tag{668}$$

[combin-eq3]

Here, $\delta_0 T_0$ is the rate of change to the temperature to be fixed,

$$\delta_0 T_0 = (T_0^{fix} - T_0)/\Delta t \; . \tag{669}$$

T_0^{fix} is 273.15K for snow and ice melting, In the case of sea ice production it is 271.15K. If the second method of evaporation calculation is used, then Similarly, use Fq_c^P instead of $Fq_{1/2}^P$, Calculate the differential term of F_q as 0. In this case,

$$\Delta s = f_0^g - F\theta_{1/2} - L\beta F q_{1/2}^P - U_{00}^g - \left(\frac{\partial F\theta_{1/2}}{\partial T_0} + L\beta \frac{\partial F q_{1/2}}{\partial T_0} + \frac{\partial F R_{1/2}}{\partial T_0} \right) \delta_0 T_0 \Delta t \tag{670}$$

$$- \frac{\partial F\theta_{1/2}}{\partial T_1} \delta T_1 \Delta t - L\beta \frac{\partial F q_{1/2}^P}{\partial q_1} \delta q_1 \Delta t \tag{671}$$

Δs, calculated by the Surface Energy Balance, It is the amount of energy used for the phase change of water.

3.8.7 implicit Treatment of Time Steps in Time Differences

Although the implicit method is used for the time difference of the vertical diffusion term, In general, the diffusion coefficients are nonlinear, and we explicitly evaluate these coefficients This can cause problems of numerical instability. To improve stability, Kalnay and Kanamitsu (19?) following the I'm working on how to handle the time steps.

For simplicity, we will take the following ordinary differential equations as an example.

$$\frac{\partial X}{\partial t} = -K(X)X \tag{672}$$

The coefficient $K(X)$ represents the nonlinearity. If we evaluate only the coefficients explicitly and make them implicitly different, we get the following equation.

$$\frac{X^{m+1} - X^m}{\Delta t} = -K(X^m)X^{m+1} \tag{673}$$

[normal-fd]

However, consider the value of X two steps ahead, X^*,

$$\frac{X^* - X^m}{2\Delta t} = -K(X^m)X^* \tag{674}$$

[modify-fd1\blade]

$$X^{m+1} = \frac{X^* + X^m}{2} \tag{675}$$

[modify-fd2]

. Generally, (modify-fd1]), (modify-fd2](#modify-fd2)) is better ([normal-fd]) is known to be more stable than [normal-fd]).

([modifie-fd1](#modifie-fd1)), (modifie-fd2) to find the rate of change in time Rewriting it into a form yields the following.

$$\left(\frac{\Delta X}{\Delta t}\right)^* = -K(X^m)\left(X^m + \left(\frac{\Delta X}{\Delta t}\right)^* 2\Delta t\right) \tag{676}$$

$$X^{m+1} = X^m + \left(\frac{\Delta X}{\Delta t}\right)^* \Delta t \tag{677}$$

That is, the time step in determining the rate of change of the rate of change of time includes the following, Using twice the time integration step.

3.9 Gravitational wave resistance

3.9.1 Gravitational Wave Resistance Scheme Overview

The gravitational wave resistance scheme is, Excited by sub-grid scale terrain The upward momentum flux of the gravitational wave is represented, The horizontal wind deceleration associated with its convergence is calculated. The main input data are: east-west wind u, north-south wind v, and temperature T, The output data is the time rate of change for the east-west and north-south winds, $\partial u/\partial t, \partial v/\partial t$, is.

The outline of the calculation procedure is as follows.

1. the momentum flux at the ground surface. Dispersion of surface altitude, The horizontal wind speed at the lowest level, stratification stability, etc.

2. consider the upward propagation of gravitational waves with momentum fluxes. Momentum fluxes are determined from the critical fluid number If the critical flux is exceeded, Suppose a breaking wave occurs and the flux is at its critical value.

3. according to the convergence of the momentum flux at each layer. Calculate the time variation of the horizontal wind.

3.9.2 Relationship between local fluid number and momentum flux

The gravitational waves of surface origin Given the vertical flux of horizontal momentum, Flux at a certain altitude τ and Local Fluid Number $F_L = NH/U$ and ,

$$F_L = \left(\frac{\tau N}{E_f \rho U^3} \right)^{1/2} , \tag{678}$$

.

The relationship between the two is valid. Here, $N = g/\theta \partial\theta/\partial z$ is Brandt Vaisala Frequency, ρ is the density of the atmosphere, U corresponds to the wind speed and E_f corresponds to the horizontal scale of the ripples on the ground surface It is a proportional constant . Now..,

$$\tau = \frac{E_f F_L^2 \rho U^3}{N} \tag{679}$$

[p-grav:fl-tau]

Local Fluid Number F_L is , Assume that a certain value, the critical fluid number F_c, cannot be exceeded. Calculated from (£ p-grav:tau-fl] If the local fluid number exceeds the critical fluid number F_c The gravitational waves are supersaturated, Up to the momentum flux corresponding to the critical fluid number Flux decreases.

3.9.3 Momentum fluxes at the surface.

due to gravitational waves excited at the earth's surface. The magnitude of the vertical flux of horizontal momentum $\tau_{1/2}$ is , except for the local fluid number at the surface $(F_L)_{1/2} = N_1 h/U_1$. (p-grav:fl-tau]) by substituting into ,

$$\tau_{1/2} = E_f h^2 \rho_1 N_1 U_1 \ , \tag{680}$$

It is estimated that . Here, The $U_1 = |\mathbf{v}_1| = (u_1^2 + v_1^2)^{1/2}$ has a surface wind speed of , N_1, ρ_1 are the two most common types of data in the atmosphere near the earth's surface. It is stability and density. h is an indicator of the sub-grid surface elevation change, Assume that the standard deviation of the surface elevation is equal to Z_{SD}.

Here, the local fluid number at the surface If $(F_L)_{1/2} = N_1 Z_{SD}/U_1$ is the critical fluid number When you exceed the F_c, The momentum flux is substituted for F_c into (p-grav:fl-tau]) Let's say it can be contained. Namely,

$$\tau_{1/2} = \min\left(E_f Z_{SD}^2 \rho_1 N_1 U_1, \ \frac{E_f F_c^2 \rho_1 U_1^3}{N_1} \right) \tag{681}$$

3.9.4 Momentum fluxes in the upper levels.

The momentum flux at level $k - 1/2$ is Suppose we are required to. $\tau_{k+1/2}$, when no saturation occurs Equal to $\tau_{k-1/2}$. This momentum flux, $\tau_{k-1/2}$, is , Momentum fluxes calculated from the critical fluid number at the $k + 1/2$ level In the case of a wave breaking event that exceeds The momentum flux decreases to the flux corresponding to the criticality.

$$\tau_{k+1/2} = \min\left(\tau_{k-1/2},\ \frac{E_f F_c^2 \rho_{k+1/2} U_{k+1/2}^3}{N_{k+1/2}}\right), \tag{682}$$

However, the $U_{k+1/2}$, . of the wind velocity vector at each layer, It is the magnitude of the projective component of the lowest level with respect to the direction of the horizontal wind,

$$U_{k+1/2} = \frac{\mathbf{v}_{k+1/2} \cdot \mathbf{v}_1}{|\mathbf{v}_1|} \tag{683}$$

3.9.5 The magnitude of the time variation of horizontal wind due to momentum convergence.

The temporal rate of change of the projective component of the horizontal wind, U_k, is

$$\frac{\partial U}{\partial t} = -\frac{1}{\rho}\frac{\partial \tau}{\partial z} = g\frac{\partial \tau}{\partial p} \tag{684}$$

as determined by i.e. ,

$$\frac{\partial U_k}{\partial t} = g\frac{\tau_{k+1/2} - \tau k - 1/2}{\Delta p}. \tag{685}$$

With this , The rate of change of the east-west and north-south winds over time is calculated as follows.

$$\frac{\partial u_k}{\partial t} = \frac{\partial U_k}{\partial t}\frac{u_1}{U_1} \tag{686}$$

$$\frac{\partial v_k}{\partial t} = \frac{\partial U_k}{\partial t}\frac{v_1}{U_1} \tag{687}$$

3.9.6 Other Notes.

1. when the wind speed at the lowest level is small ($U_1 \leq v_{min}$) and In the case of small undulations in the earth's surface ($Z_{SD} \leq (Z_{SD})_{min}$), Assuming no gravitational waves are excited at the earth's surface.

3.10 Drying convection regulation

3.10.1 Overview of Drying Convective Regulation

Drying convection control , Convective instability in the stratum between two successive levels, In other words, if the temperature decay rate is greater than the dry adiabatic decay rate The temperature reduction rate is adjusted to the dry adiabatic reduction rate. Water vapor and other substances are mixed in at this time. The main input data are temperature T and specific humidity q, The output data is the adjusted air temperature T and specific humidity q.

Essentially, if vertical diffusion is efficient, then The vertical convective instability should be basically removed. However, it may be in short supply in the stratosphere, A convection adjustment has been added to stabilize the calculation.

3.10.2 Drying convection regulation procedures.

The conditions for convective instability in the layers $(k-1, k)$ are

$$\frac{T_{k-1} - T_k}{p_{k-1} - p_k} > \frac{R}{C_p} \bar{T}_{k-1/2} = \frac{R}{C_p} \frac{\Delta p_{k-1} T_{k-1} + \Delta p_k T_k}{\Delta p_{k-1} + \Delta p_k} \tag{688}$$

Namely,

$$S = T_{k-1} - T_k - \frac{R}{C_p} \frac{\Delta p_{k-1} T_{k-1} + \Delta p_k T_k}{\Delta p_{k-1} + \Delta p_k} (p_{k-1} - p_k) > 0 \tag{689}$$

.

is a condition.

When this is satisfied ,

$$T_{k-1} \leftarrow \frac{\Delta p_k}{\Delta p_{k-1} + \Delta p_k} S \tag{690}$$

$$T_k \leftarrow \frac{\Delta p_{k-1}}{\Delta p_{k-1} + \Delta p_k} S \tag{691}$$

to compensate for the temperature. Furthermore,

$$q_{k-1}, q_k \leftarrow \frac{\Delta p_{k-1} q_{k-1} + \Delta p_k q_k}{\Delta p_{k-1} + \Delta p_k} \tag{692}$$

to average the values of specific humidity etc. in the two layers.

When you do this, The layers above and below it may become unstable. That's why, Repeating this operation from the lower level to the upper level. Repeat until there is no more layer of convective instability. However, considering the calculation error and so on, (as a condition of \p[p-adj:condo]), It is considered to have converged if S is less than or equal to some small finite value that is not zero.

Currently, the standard adjustment is between the second and third layer from the bottom and above.

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