目 次

1 Dynamics **2**
1.1 Basic Equations 2
1.1.1 Basic Equations 2
1.1.2 Boundary Conditions 3

1 Dynamics

1.1 Basic Equations

1.1.1 Basic Equations

The basic equations are a system of primitive equations at the spherical (λ, φ) and η coordinates, given as follows (Arakawa and Konor 1996).

1. Continuity equation

$$\frac{\partial m}{\partial t} + \nabla_\eta \cdot (m\mathbf{v}_H) + \frac{\partial (m\dot{\eta})}{\partial \eta} = 0 \tag{1}$$

2. Hydrostatic equation

$$\frac{\partial \Phi}{\partial \eta} = -\frac{RT_v}{p} m \tag{2}$$

3. Equation of motion

$$\frac{\partial \zeta}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial A_v}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(A_u \cos\varphi) - \mathcal{D}(\zeta) \tag{3}$$

$$\frac{\partial D}{\partial t} = \frac{1}{a\cos\varphi}\frac{\partial A_u}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(A_v \cos\varphi) - \nabla_\eta^2(\Phi + R\bar{T}\pi + E) - \mathcal{D}(D) \tag{4}$$

4. Thermodynamic equation

$$\frac{\partial T}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial uT'}{\partial \lambda} - \frac{1}{a}\frac{\partial}{\partial \varphi}(vT'\cos\varphi) + T'D \tag{5}$$

$$- \dot{\eta}\frac{\partial T}{\partial \eta} + \frac{\kappa T}{\sigma}\left[B\left(\frac{\partial \pi}{\partial t} + \mathbf{v}_H \cdot \nabla_\eta \pi\right) + \frac{m\dot{\eta}}{p_s}\right] + \frac{Q}{C_p} + \frac{Q_{diff}}{C_p} - \mathcal{D}(T) \tag{6}$$

5. Tracers

For any tracer whose mixing ratio is denoted as q,

$$\frac{\partial q}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial uq}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(vq\cos\varphi) + qD \tag{7}$$

$$- \dot{\eta}\frac{\partial q}{\partial \eta} + S_q - \mathcal{D}(q) \tag{8}$$

Here,

$$
\begin{aligned}
m &\equiv \left(\frac{\partial p}{\partial \eta}\right)_{p_s}, && (9)\\
\theta &\equiv T\,(p/p_0)^{-\kappa}, && (10)\\
\kappa &\equiv R/C_p, && (11)\\
\Phi &\equiv gz, && (12)\\
\pi &\equiv \ln p_S, && (13)\\
\dot{\eta} &\equiv \frac{\mathrm{d}\eta}{\mathrm{d}t}, && (14)\\
T_v &\equiv T(1+\epsilon_v q), && (15)\\
T &\equiv \bar{T}+T', && (16)\\
\bar{T} &\equiv 300 \text{ K}, && (17)\\
\zeta &\equiv \frac{1}{a\cos\varphi}\frac{\partial v}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(u\cos\varphi), && (18)\\
D &\equiv \frac{1}{a\cos\varphi}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(v\cos\varphi), && (19)\\
A_u &\equiv (\zeta+f)v - \dot{\eta}\frac{\partial u}{\partial \eta} - \frac{RT'}{a\cos\varphi}\frac{\partial \pi}{\partial \lambda} + \mathcal{F}_x, && (20)\\
A_v &\equiv -(\zeta+f)u - \dot{\eta}\frac{\partial v}{\partial \eta} - \frac{RT'}{a}\frac{\partial \pi}{\partial \varphi} + \mathcal{F}_y, && (21)\\
E &\equiv \frac{u^2+v^2}{2}, && (22)\\
\mathbf{v}_H\cdot\nabla &\equiv \frac{u}{a\cos\varphi}\left(\frac{\partial}{\partial \lambda}\right)_\sigma + \frac{v}{a}\left(\frac{\partial}{\partial \varphi}\right)_\sigma, && (23)\\
\nabla^2_\eta &\equiv \frac{1}{a^2\cos^2\varphi}\frac{\partial^2}{\partial \lambda^2} + \frac{1}{a^2\cos\varphi}\frac{\partial}{\partial \varphi}\left[\cos\varphi\frac{\partial}{\partial \varphi}\right]. && (24)
\end{aligned}
$$

$\mathcal{D}(\zeta), \mathcal{D}(D), \mathcal{D}(T), \mathcal{D}(q)$ are horizontal diffusion terms, $\mathcal{F}_\lambda, \mathcal{F}_\varphi$ are forces due to small-scale kinetic processes (treated as 'physical processes'), Q are forces due to radiation, condensation, small-scale kinetic processes, etc. Heating and temperature change due to 'physical processes', and S_q is a water vapor source term due to 'physical processes' such as condensation and small-scale motion. Q_{diff} is the heat of friction and

$$
Q_{diff} = -\mathbf{v}\cdot(\frac{\partial \mathbf{v}}{\partial t})_{diff}. \quad (25)
$$

$(\frac{\partial \mathbf{v}}{\partial t})_{diff}$ is a time-varying term of u, v due to horizontal and vertical diffusion.

1.1.2 Boundary Conditions

Upper and lower boundary conditions for the vertical velocity is:

$$
\dot{\eta} = 0 \quad at \quad \eta = 0,\ 1. \quad (26)
$$

The prognostic equation for p_s and the diagnostic equation for the vertical velocity can be derived by integrating the continuity equation and applying these boundary conditions.