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Due: 4/3/2020

Problem 35

a) $\text{rep}_D(-1 \quad 2) = \begin{bmatrix} -\frac{1}{3} \\ -2 \end{bmatrix}$ $\qquad$ $\text{rep}_D(1 \quad 1) = \begin{bmatrix} \frac{1}{3} \\ -1 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} y$ $\qquad$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} y$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ -2 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ -1 \end{bmatrix}$

$\text{rep}_{B,D}(\text{identity}) = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ -2 & -1 \end{bmatrix}$

b) Birthday vector $= \begin{bmatrix} 10 \\ 9 \end{bmatrix}$ $\qquad$ October 9^{th}

$B = \{(-1 \quad 2), (1 \quad 1)\}$

$\begin{bmatrix} 10 \\ 9 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} y$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ \frac{29}{3} \end{bmatrix}$

$\text{rep}_B \begin{bmatrix} 10 \\ 9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ \frac{29}{3} \end{bmatrix}$

$D = \{(3 \quad 0), (0 \quad -1)\}$

$\begin{bmatrix} 10 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} y$

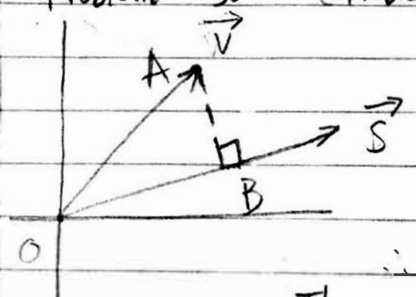
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{10}{3} \\ -9 \end{bmatrix}$

$\text{rep}_D \begin{bmatrix} 10 \\ 9 \end{bmatrix} = \begin{bmatrix} \frac{10}{3} \\ -9 \end{bmatrix}$

$\text{rep}_{B,D}(\text{identity}) \cdot \text{rep}_B \begin{bmatrix} 10 \\ 9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{3} \\ -2 & -1 \end{bmatrix} \begin{bmatrix} \frac{10}{3} \\ -9 \end{bmatrix}$

$= \begin{bmatrix} -\frac{10}{9} - \frac{9}{3} \\ -\frac{20}{3} + 9 \end{bmatrix} = \begin{bmatrix} -\frac{37}{3} \\ -\frac{7}{3} \end{bmatrix}$

Problem 36 (Problem 1.18, P.71)



Prove $|\vec{v} \cdot \vec{s}| \le \|\vec{v}\| \cdot \|\vec{s}\|$

The hypotenuse of a right triangle is the longest side.

$\therefore$ $BO < AO$

$\therefore$ The orthogonal projection of a vector into a line is shorter than the vector.

Problem 37

$C_1 = 3 - 5i \qquad C_2 = -1 + i$

a) $C_1 + C_2 = 3 - 5i - 1 + i$
$= 2 - 4i$

b) $C_1 C_2 = (3 - 5i)(-1 + i)$
$= -3 + 3i + 5i - 5i^2$
$= -3 + 8i - 5 \cdot (-1)$
$= -3 + 8i + 5$
$= 2 + 8i$

c) C_1 conjugate $3 + 5i$
C_2 conjugate $-1 - i$
$(3+5i)(3-5i) = 9 + 25 \cancel{- 3 \times 5i} \cancel{+ 3 \times 5i} = 34$
$(-1-i)(-1+i) = 1 + 1 \cancel{- i} \cancel{+ i} = 2$

When multiply the complex number by its conjugate, the complex number part will be cancelled out. It will only left with the square of two real numbers.
The general case: $(a + bi)(a - bi) = a^2 + b^2$

Problem 38

a) 3.26 P405

(a) $A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & -2 & 0 \\ -2 & 3-\lambda & 0 \\ 0 & 0 & 5-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (3-\lambda)\begin{bmatrix} 3-\lambda & 0 \\ 0 & 5-\lambda \end{bmatrix} - (-2)\begin{bmatrix} -2 & 0 \\ 0 & 5-\lambda \end{bmatrix} + 0$$

$$= (3-\lambda)^2(5-\lambda) - (-2)(-2)(5-\lambda)$$

$$= (3-\lambda+2)(3-\lambda-2)(5-\lambda)$$

$$0 = (5-\lambda)^2(1-\lambda)$$

$$\therefore \lambda_1 = 1 \quad \lambda_2 = 5$$

when $\lambda_1 = 1$,

$$\begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$3V_1 - 2V_2 = V_1$ $\qquad V_1 = V_2$

$-2V_1 + 3V_2 = V_2$ $\qquad \therefore V_2 = V_1$

$5V_3 = V_3$ $\qquad V_3 = 0$

Eigen space $\left\{ \begin{pmatrix} V_1 \\ V_2 \\ 0 \end{pmatrix} \middle| V_1, V_2 \in C \right\}$

Eigenvector $= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

when $\lambda_2 = 5$

$$\begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 5 \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$3V_1 - 2V_2 = 5V_1$ $\quad V_1 = -V_2$

$-2V_1 + 3V_2 = 5V_2$ $\quad V_2 = -V_1$

$5V_3 = 5V_3$ $\quad V_3 = V_3$

Eigenspace $\left\{ \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} V_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V_3 \;\middle|\; V_2, V_3 \in \mathbb{C} \right\}$

Eigenvector : $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$

$$B - \lambda I = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{bmatrix}$$

$$\det(B - \lambda I) = -\lambda \begin{bmatrix} -\lambda & 1 \\ -17 & 8-\lambda \end{bmatrix} - 1 \begin{bmatrix} 0 & 1 \\ 4 & 8-\lambda \end{bmatrix}$$

$$= \lambda^2(8-\lambda) + 4 - 17\lambda$$

$$0 = -1(\lambda - 4)(\lambda^2 - 4\lambda + 1)$$

$\therefore \lambda_1 = 4$

$\lambda_2 = \dfrac{4 + \sqrt{16-4}}{2} = 2 + \sqrt{3}$

$\lambda_3 = 2 - \sqrt{3}$

when $\lambda_1 = 4$.

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 4 \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$V_2 = 4V_1$

$V_3 = 4V_2$

$4V_1 - 17V_2 + 8V_3 = 4V_3$

$V_1 = \frac{1}{16} V_3$

$V_2 = \frac{1}{4} V_3$

Eigenspace $\left\{ \begin{bmatrix} \frac{1}{16} \\ \frac{1}{4} \\ 1 \end{bmatrix} V_3 \;\middle|\; V_3 \in C \right\}$

Eigenvector $\begin{bmatrix} 1 \\ 4 \\ 16 \end{bmatrix}$

when $\lambda_2 = 2 + \sqrt{3}$

$V_2 = (2+\sqrt{3})V_1$

$V_3 = (2+\sqrt{3})V_2$

$4V_1 - 17V_2 + 8V_3 = (2+\sqrt{3})V_3$

$V_1 = \frac{1}{(2+\sqrt{3})^2} V_3$

$V_2 = \frac{1}{(2+\sqrt{3})} V_3$

Eigenspace $\left\{ \begin{bmatrix} \frac{1}{(2+\sqrt{3})^2} \\ \frac{1}{(2+\sqrt{3})} \\ 1 \end{bmatrix} V_3 \;\middle|\; V_3 \in C \right\}$

Eigenvector $\begin{bmatrix} \frac{1}{(2+\sqrt{3})^2} \\ \frac{1}{2+\sqrt{3}} \\ 1 \end{bmatrix}$

when $\lambda_3 = 2 - \sqrt{3}$

Eigenspace $\left\{ \begin{bmatrix} \frac{1}{(2-\sqrt{3})^2} \\ \frac{1}{2-\sqrt{3}} \\ 1 \end{bmatrix} V_3 \;\middle|\; V_3 \in C \right\}$

Eigenvector $\begin{bmatrix} \frac{1}{(2-\sqrt{3})^2} \\ \frac{1}{2-\sqrt{3}} \\ 1 \end{bmatrix}$

b) 3.29, P405

$M_2 \rightarrow M_3$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \begin{bmatrix} 2c & a+c \\ b-2c & d \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2c \\ a+c \\ b-2c \\ d \end{bmatrix}$$

By using Wolframalpha. com I find

Eigenvalues: $\lambda_1 = -2$
$\lambda_2 = -1$
$\lambda_3 = 1$

when $\lambda_1 = -2$. Eigenvectors: $\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$

when $\lambda_2 = -1$ Eigenvectors: $\begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$

when $\lambda_3 = 1$, Eigenvectors. $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$