

Resolution Exercise Solutions

2. Consider the following axioms:

1. Every child loves Santa.
 $\forall x (CHILD(x) \rightarrow LOVES(x, Santa))$
2. Everyone who loves Santa loves any reindeer.
 $\forall x (LOVES(x, Santa) \rightarrow \forall y (REINDEER(y) \rightarrow LOVES(x, y)))$
3. Rudolph is a reindeer, and Rudolph has a red nose.
 $REINDEER(Rudolph) \wedge REDNOSE(Rudolph)$
4. Anything which has a red nose is weird or is a clown.
 $\forall x (REDNOSE(x) \rightarrow WEIRD(x) \vee CLOWN(x))$
5. No reindeer is a clown.
 $\neg \exists x (REINDEER(x) \wedge CLOWN(x))$
6. Scrooge does not love anything which is weird.
 $\forall x (WEIRD(x) \rightarrow \neg LOVES(Scrooge, x))$
7. (Conclusion) Scrooge is not a child.
 $\neg CHILD(Scrooge)$

3. Consider the following axioms:

1. Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
 $\forall x (BUY(x) \rightarrow \exists y (OWNS(x, y) \wedge (RABBIT(y) \vee GROCERY(y))))$
2. Every dog chases some rabbit.
 $\forall x (DOG(x) \rightarrow \exists y (RABBIT(y) \wedge CHASE(x, y)))$
3. Mary buys carrots by the bushel.
 $BUY(Mary)$
4. Anyone who owns a rabbit hates anything that chases any rabbit.
 $\forall x \forall y (OWNS(x, y) \wedge RABBIT(y) \rightarrow \forall z \forall w (RABBIT(w) \wedge CHASE(z, w) \rightarrow HATES(x, z)))$

5. John owns a dog.

$$\exists x (DOG(x) \wedge OWNS(John,x))$$

6. Someone who hates something owned by another person will not date that person.

$$\forall x \forall y \forall z (OWNS(y,z) \wedge HATES(x,z) \rightarrow \neg DATE(x,y))$$

7. (Conclusion) If Mary does not own a grocery store, she will not date John.

$$((\neg \exists x (GROCERY(x) \wedge OWN(Mary,x))) \rightarrow \neg DATE(Mary,John))$$

4. Consider the following axioms:

1. Every Austinite who is not conservative loves some armadillo.

$$\forall x (AUSTINITE(x) \wedge \neg CONSERVATIVE(x) \rightarrow \exists y (ARMADILLO(y) \wedge LOVES(x,y)))$$

2. Anyone who wears maroon-and-white shirts is an Aggie.

$$\forall x (WEARS(x) \rightarrow AGGIE(x))$$

3. Every Aggie loves every dog.

$$\forall x (AGGIE(x) \rightarrow \forall y (DOG(y) \rightarrow LOVES(x,y)))$$

4. Nobody who loves every dog loves any armadillo.

$$\neg \exists x ((\forall y (DOG(y) \rightarrow LOVES(x,y))) \wedge \exists z (ARMADILLO(z) \wedge LOVES(x,z)))$$

5. Clem is an Austinite, and Clem wears maroon-and-white shirts.

$$AUSTINITE(Clem) \wedge WEARS(Clem)$$

6. (Conclusion) Is there a conservative Austinite?

$$\exists x (AUSTINITE(x) \wedge CONSERVATIVE(x))$$

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( ( (not (Austinite x)) (Conservative x) (Armadillo (f x)) )
  ( (not (Austinite x)) (Conservative x) (Loves x (f x)) ) )
( (not (Wears x)) (Aggie x) )
( (not (Aggie x)) (not (Dog y)) (Loves x y) )
( (Dog (g x)) (not (Armadillo z)) (not (Loves x z)) )
( (not (Loves x (g x))) (not (Armadillo z)) (not (Loves x z)) )
( (Austinite (Clem)) )
( (Wears (Clem)) )
( (not (Conservative x)) (not (Austinite x)) ) )
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5. Consider the following axioms:

1. Anyone whom Mary loves is a football star.

$$\forall x (LOVES(Mary, x) \rightarrow STAR(x))$$

2. Any student who does not pass does not play.

$$\forall x (STUDENT(x) \wedge \neg PASS(x) \rightarrow \neg PLAY(x))$$

3. John is a student.

$$STUDENT(John)$$

4. Any student who does not study does not pass.

$$\forall x (STUDENT(x) \wedge \neg STUDY(x) \rightarrow \neg PASS(x))$$

5. Anyone who does not play is not a football star.

$$\forall x (\neg PLAY(x) \rightarrow \neg STAR(x))$$

6. (Conclusion) If John does not study, then Mary does not love John.

$$\neg STUDY(John) \rightarrow \neg LOVES(Mary, John)$$

6. Consider the following axioms:

1. Every coyote chases some roadrunner.

$$\forall x (COYOTE(x) \rightarrow \exists y (RR(y) \wedge CHASE(x, y)))$$

2. Every roadrunner who says ``beep-beep" is smart.

$$\forall x (RR(x) \wedge BEEP(x) \rightarrow SMART(x))$$

3. No coyote catches any smart roadrunner.

$$\neg \exists x \exists y (COYOTE(x) \wedge RR(y) \wedge SMART(y) \wedge CATCH(x, y))$$

4. Any coyote who chases some roadrunner but does not catch it is frustrated.

$$\forall x (COYOTE(x) \wedge \exists y (RR(y) \wedge CHASE(x, y) \wedge \neg CATCH(x, y)) \rightarrow FRUSTRATED(x))$$

5. (Conclusion) If all roadrunners say ``beep-beep", then all coyotes are frustrated.

$$(\forall x (RR(x) \rightarrow BEEP(x)) \rightarrow (\forall y (COYOTE(y) \rightarrow FRUSTRATED(y))))$$

$$\begin{aligned} & ((\neg (Coyote\ x)) \quad (RR\ (f\ x))) \\ & (\neg (Coyote\ x)) \quad (Chase\ x\ (f\ x))) \\ & (\neg (RR\ x)) \quad (\neg (Beep\ x)) \quad (Smart\ x)) \\ & (\neg (Coyote\ x)) \quad (\neg (RR\ y)) \quad (\neg (Smart\ y)) \quad (\neg (Catch\ x\ y))) \\ & (\neg (Coyote\ x)) \quad (\neg (RR\ y)) \quad (\neg (Chase\ x\ y)) \quad (Catch\ x\ y) \\ & (Frustrated\ x)) \end{aligned}$$

((not (RR x)) (Beep x))
 ((Coyote (a)))
 ((not (Frustrated (a)))))

7. Consider the following axioms:

1. Anyone who rides any Harley is a rough character.
 $\forall x ((\exists y (HARLEY(y) \wedge RIDES(x,y))) \rightarrow ROUGH(x))$
2. Every biker rides [something that is] either a Harley or a BMW.
 $\forall x (BIKER(x) \rightarrow \exists y ((HARLEY(y) \vee BMW(y)) \wedge RIDES(x,y)))$
3. Anyone who rides any BMW is a yuppie.
 $\forall x \forall y (RIDES(x,y) \wedge BMW(y) \rightarrow YUPPIE(x))$
4. Every yuppie is a lawyer.
 $\forall x (YUPPIE(x) \rightarrow LAWYER(x))$
5. Any nice girl does not date anyone who is a rough character.
 $\forall x \forall y (NICE(x) \wedge ROUGH(y) \rightarrow \neg DATE(x,y))$
6. Mary is a nice girl, and John is a biker.
 $NICE(Mary) \wedge BIKER(John)$
7. (Conclusion) If John is not a lawyer, then Mary does not date John.
 $\neg LAWYER(John) \rightarrow \neg DATE(Mary,John)$

8. Consider the following axioms:

1. Every child loves anyone who gives the child any present.
 $\forall x \forall y \forall z (CHILD(x) \wedge PRESENT(y) \wedge GIVE(z,y,x) \rightarrow LOVES(x,z))$
2. Every child will be given some present by Santa if Santa can travel on Christmas eve.
 $TRAVEL(Santa,Christmas) \rightarrow \forall x (CHILD(x) \rightarrow \exists y (PRESENT(y) \wedge GIVE(Santa,y,x)))$
3. It is foggy on Christmas eve.
 $FOGGY(Christmas)$
4. Anytime it is foggy, anyone can travel if he has some source of light.
 $\forall x \forall t (FOGGY(t) \rightarrow (\exists y (LIGHT(y) \wedge HAS(x,y)) \rightarrow TRAVEL(x,t)))$

5. Any reindeer with a red nose is a source of light.

$$\forall x (RNR(x) \rightarrow LIGHT(x))$$

6. (Conclusion) If Santa has some reindeer with a red nose, then every child loves Santa.

$$(\exists x (RNR(x) \wedge HAS(Santa, x))) \rightarrow \forall y (CHILD(y) \rightarrow LOVES(y, Santa))$$

9. Consider the following axioms:

1. Every investor bought [something that is] stocks or bonds.

$$\forall x (INVESTOR(x) \rightarrow \exists y ((STOCK(y) \vee BOND(y)) \wedge BUY(x, y)))$$

2. If the Dow-Jones Average crashes, then all stocks that are not gold stocks fall.

$$DJCRASH \rightarrow \forall x ((STOCK(x) \wedge \neg GOLD(x)) \rightarrow FALL(x))$$

3. If the T-Bill interest rate rises, then all bonds fall.

$$TBRISE \rightarrow \forall x (BOND(x) \rightarrow FALL(x))$$

4. Every investor who bought something that falls is not happy.

$$\forall x \forall y (INVESTOR(x) \wedge BUY(x, y) \wedge FALL(y) \rightarrow \neg HAPPY(x))$$

5. (Conclusion) If the Dow-Jones Average crashes and the T-Bill interest rate rises, then any investor who is happy bought some gold stock.

$$(DJCRASH \wedge TBRISE) \rightarrow \forall x (INVESTOR(x) \wedge HAPPY(x) \rightarrow \exists y (GOLD(y) \wedge BUY(x, y)))$$

10. Consider the following axioms:

1. Every child loves every candy.

$$\forall x \forall y (CHILD(x) \wedge CANDY(y) \rightarrow LOVES(x, y))$$

2. Anyone who loves some candy is not a nutrition fanatic.

$$\forall x ((\exists y (CANDY(y) \wedge LOVES(x, y))) \rightarrow \neg FANATIC(x))$$

3. Anyone who eats any pumpkin is a nutrition fanatic.

$$\forall x ((\exists y (PUMPKIN(y) \wedge EAT(x, y))) \rightarrow FANATIC(x))$$

4. Anyone who buys any pumpkin either carves it or eats it.

$$\forall x \forall y (PUMPKIN(y) \wedge BUY(x, y) \rightarrow CARVE(x, y) \vee EAT(x, y))$$

5. John buys a pumpkin.

$$\exists x (PUMPKIN(x) \wedge BUY(John, x))$$

6. Lifesavers is a candy.

CANDY(Lifesavers)

7. (Conclusion) If John is a child, then John carves some pumpkin.

CHILD(John) $\rightarrow \exists x (PUMPKIN(x) \wedge CARVE(John, x))$

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